Context

2208.05029

Gravitational Tensor-Monopole Moment of Hydrogen Atom To Order $\mathcal{O}(\alpha)$

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We calculate the gravitational tensor-monopole moment of the momentum-current density T^{ij} in the ground state of the hydrogen atom to order $\mathcal{O}(\alpha)$ in quantum electrodynamics (QED). The result is

$$\tau_H/\tau_0 - 1 = \frac{4\alpha}{3\pi}(\ln\alpha - 0.028)$$

where $\tau_0 = \hbar^2/4m_e$ is the leading-order moment. The physics of the next-to-leading-order correction is similar to that of the famous Lamb shift for energy levels.

- Intriguing: a new atomic observable
- Is the physics really similar to the Lamb shift?
- Practically important: related to hadronic structure
- Experiments ongoing at JLab and planned in Electron-Ion Collider

Matrix elements of the energy-momentum tensor in the hydrogen atom

based on work with Yizhuang Liu, Jagiellonian University, Poland

Outline:

- Experimental relevance: hadronic physics (a new observable)
- Gravitational form-factors: the D-term
- D-term's sign vs stability of the system
- Logarithmic corrections: D-term vs Lamb shift

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June 1, 2023

Experimental consequences

Determining the gluonic gravitational form factors of the proton

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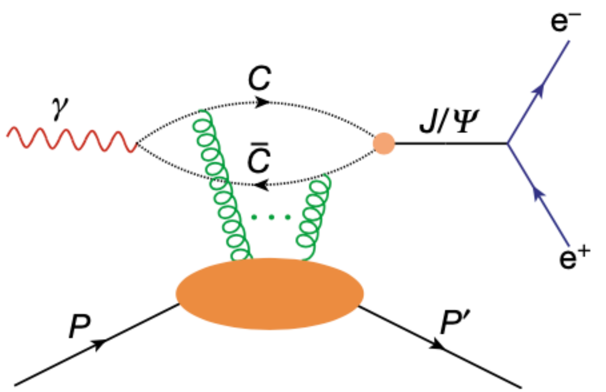
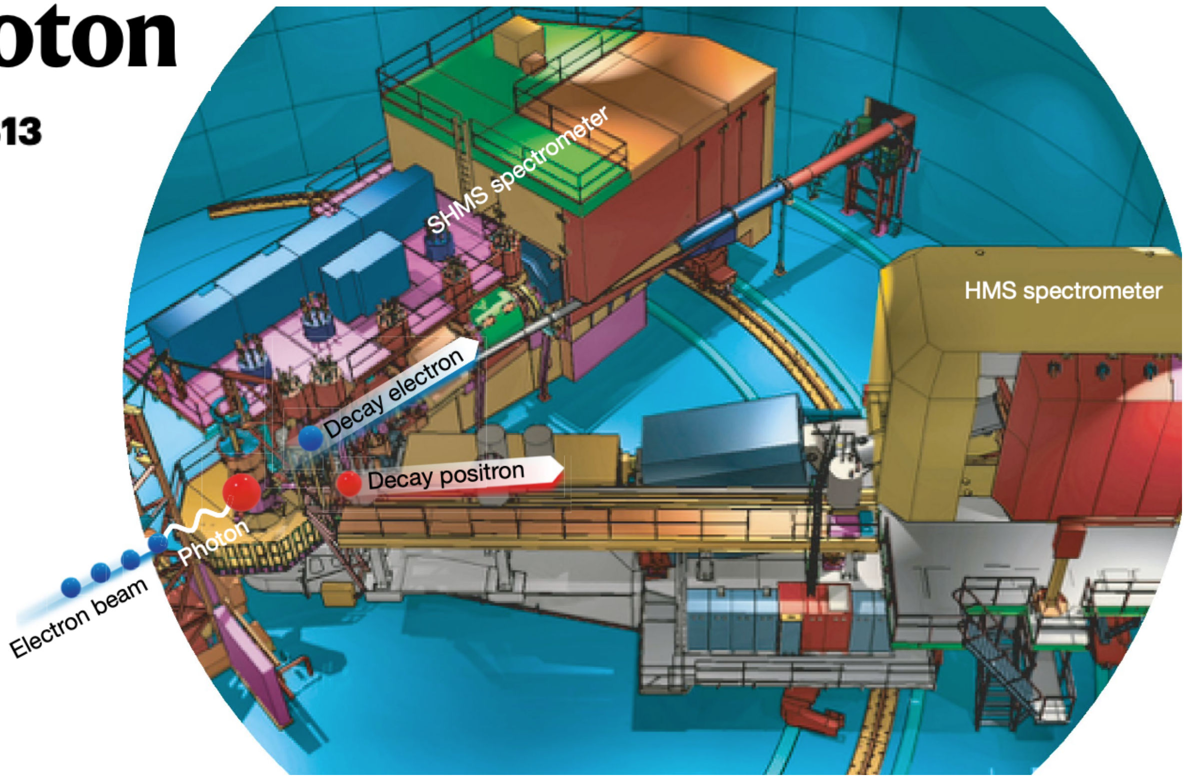
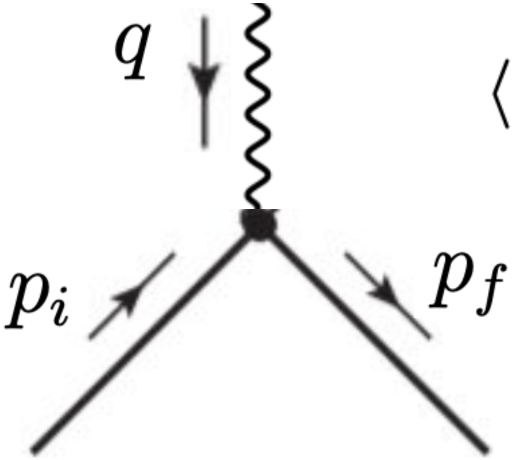




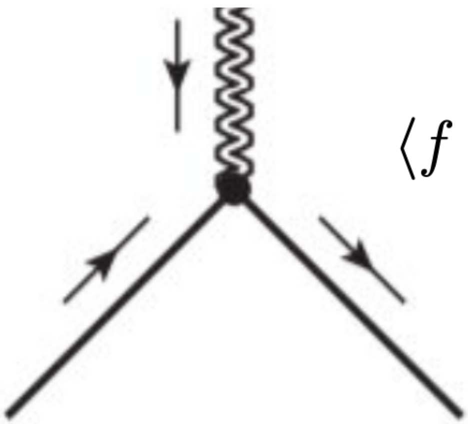
- Beyond earlier studies of the charge and spin distributions in the proton;
- New parameter: proton mass radius 0.52(3) fm.

Photon vs. graviton couplings



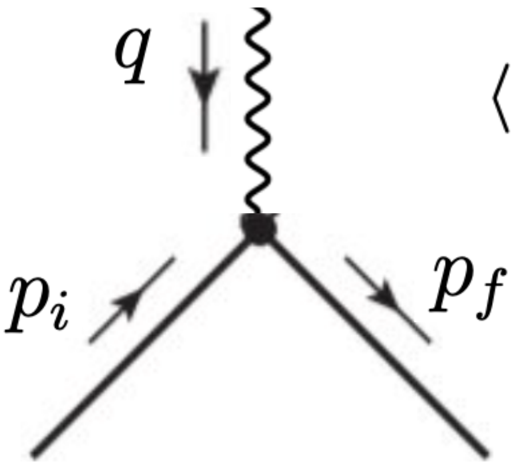
$$\langle f \left| j^{\mu}(0) \right| i \rangle \sim F\left(q^2\right) P^{\mu}$$

$$P = \frac{p_i + p_f}{2}, \quad q = p_f - p_i$$



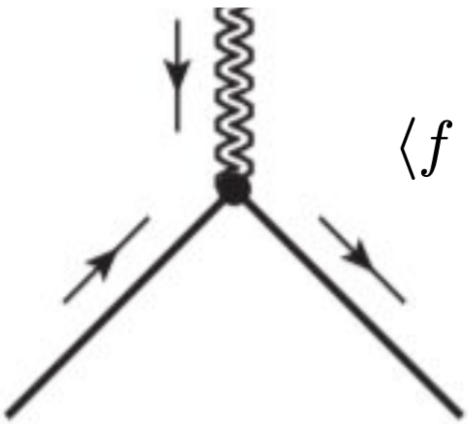
$$\langle f \left| T^{\mu\nu}(0) \right| i \rangle \sim A\left(q^2\right) P^{\mu} P^{\nu} + D\left(q^2\right)\left(q^{\mu} q^{\nu} - q^2 g^{\mu\nu}\right)$$

Photon vs. graviton couplings



$$\langle f \left| j^{\mu}(0) \right| i \rangle \sim F\left(q^2\right) P^{\mu}$$

$$j = \begin{pmatrix} \rho \\ j^x \\ j^y \\ j^z \end{pmatrix}$$



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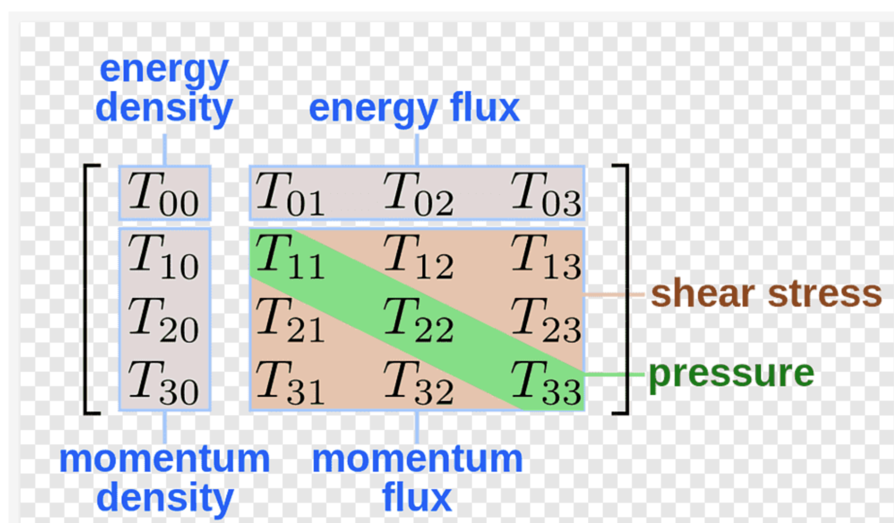
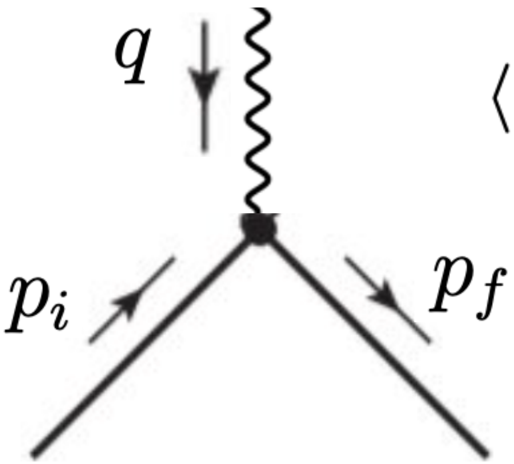
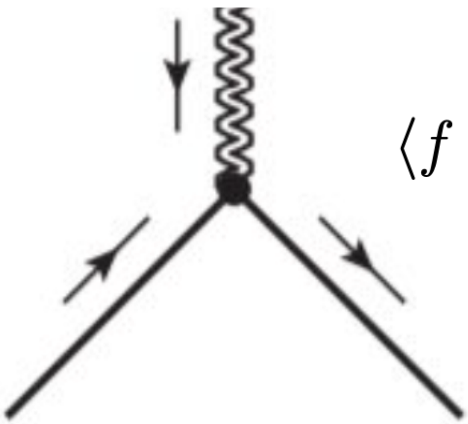


Interpretation of form factors at q^2=0



$$\langle f \left| j^{\mu}(0) \right| i \rangle \sim F\left(q^2\right) P^{\mu}$$

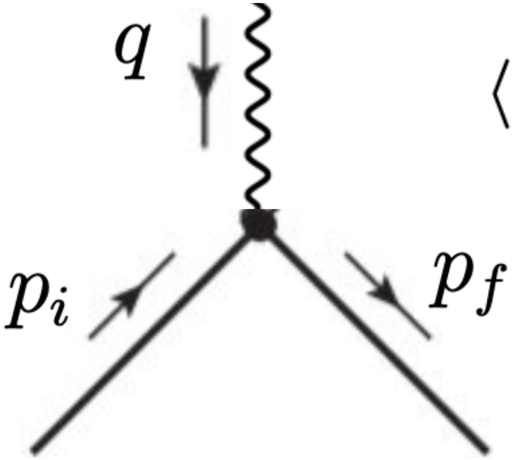
$$F(0) \sim \text{charge}$$



$$\langle f \left| T^{\mu\nu}(0) \right| i \rangle \sim A\left(q^2\right) P^{\mu} P^{\nu} + D\left(q^2\right)\left(q^{\mu} q^{\nu} - q^2 g^{\mu\nu}\right)$$

$$A(0) \sim \text{mass} \qquad D(0) \sim ?$$

Interpretation of form factors at $q^2=0$



$$\langle f \left| j^\mu (0) \right| i \rangle \sim F\left(q^2\right) P^\mu$$

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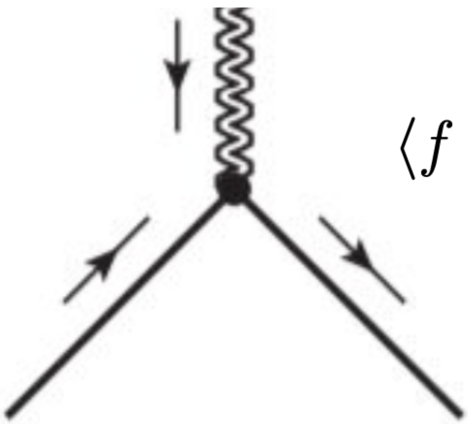


$$\langle f \left| T^{\mu\nu} (0) \right| i \rangle \sim A\left(q^2\right) P^\mu P^\nu + D\left(q^2\right)\left(q^\mu q^\nu - q^2 g^{\mu\nu}\right)$$

$$A(0) \sim \text{mass} \qquad D(0) \sim \int \mathrm{d}^3 r \; r^2 p(r)$$

Two integrals: moments of pressure distribution

$$0 = \int p(r)\mathrm{d}^3r$$

Max von Laue's stability condition

$$D \sim \int p(r)r^2\mathrm{d}^3r$$

assuming spherical symmetry

Max von Laue's stability condition

Energy-momentum conservation → in terms of the EMT $\partial^\mu T_{\mu\nu} = 0$

In a stationary state: no time dependence, $\nabla^i T_{i\nu} = 0$

Integral form (n: normal to an enclosing surface): $\int_\sigma T^{ij} n_j \mathrm{d}\sigma = 0$

Choose the surface as a cross-section of the system in the x-plane, closed at infinity:

$$\int T^{xi} \mathrm{d}y\mathrm{d}z = 0$$

Finally, integrate over x:

$$\int T^{xi} \mathrm{d}^3 r = 0$$

Diagonal element like i=x: pressure $0 = \int p(r) \mathrm{d}^3 r$

Zur Dynamik der Relativitätstheorie
von M. Laue. 1911

The sign of the D-term and system's stability

On LHCb pentaquarks as a baryon-$\psi(2S)$ bound state – prediction of isospin $\frac{3}{2}$ pentaquarks with hidden charm *

Irina A. Perevalova,[1] Maxim V. Polyakov,[2,3] and Peter Schweitzer[4,5]

arXiv:1607.07008

A less trivial local criterion can be obtained by considering that at any chosen distance r the force exhibited by the system on an infinitesimal piece of area $\mathrm{d}A\, e_r^i$ must be directed outwards. If this was not the case, the system would collapse. Since this force is $F^i(\boldsymbol{r}) = T^{ij}(\boldsymbol{r})\mathrm{d}A\, e_r^j = [\frac{2}{3}\, s(r) + p(r)]\mathrm{d}A\, e_r^i$ we obtain the criterion

$$\frac{2}{3}\, s(r) + p(r) > 0\,. \tag{18}$$

We checked that the condition (18) is satisfied in all systems we are aware of where EMT densities were studied [9, 10, 25–31]. As this includes unstable systems, apparently also (18) is a necessary but not sufficient condition for stability. Due to its local character, it provides a stronger criterion than the von Laue condition (14) and will play an important role below. Interestingly, the criterion (18) allows one to draw a conclusion on the sign of the D-term. We see that

$$0 < 4\pi \int_0^\infty \mathrm{d}r\; r^4 \left(\frac{2}{3}\, s(r) + p(r)\right) = -\frac{2d_1}{M_N} + \frac{4d_1}{5M_N} = -\,\frac{6d_1}{5M_N}\;. \tag{19}$$

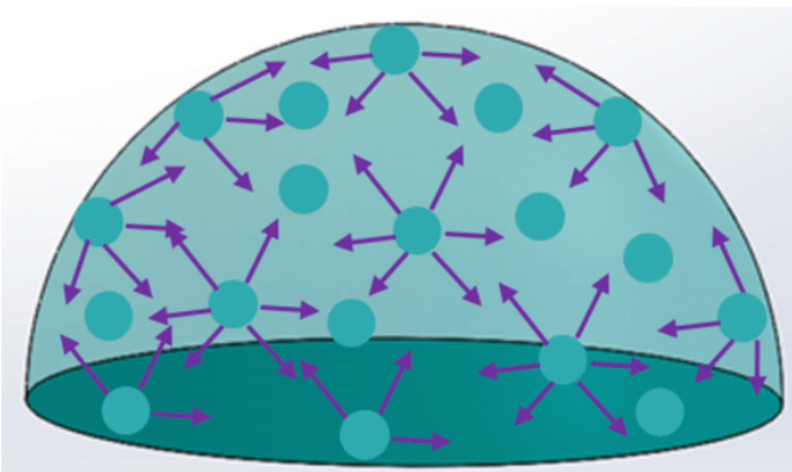
Thus, if a system satisfies the local stability criterion (18), then it must necessarily have a negative D-term (but a negative D-term does not imply that $s(r)$ and $p(r)$ satisfy (18), so the opposite is in general not true). Indeed, in all systems studied so far the D-terms were found to be negative [9, 10, 25–31].

Two types of stable systems:

$$0 = \int p(r) \mathrm{d}^3 r$$

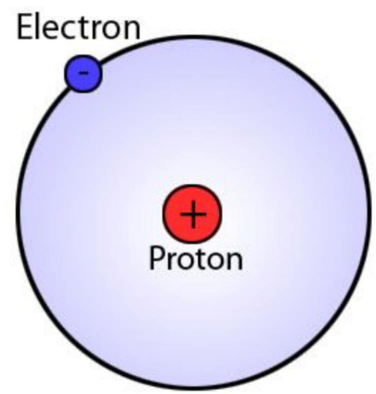
Max von Laue's stability condition

”Liquid droplet”



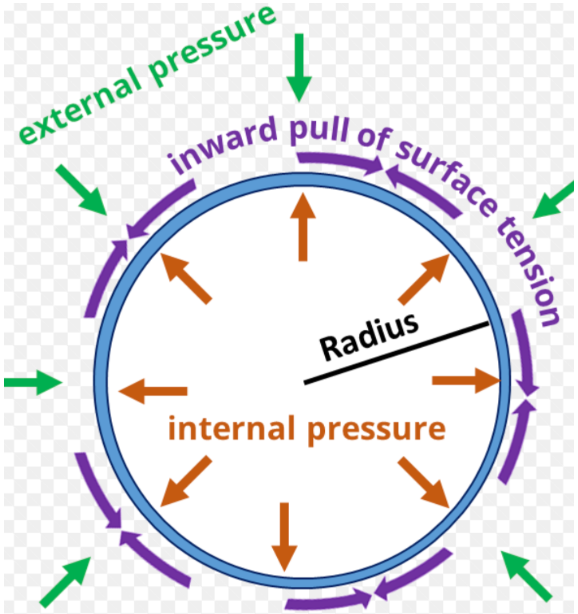
tension at large r: D < 0

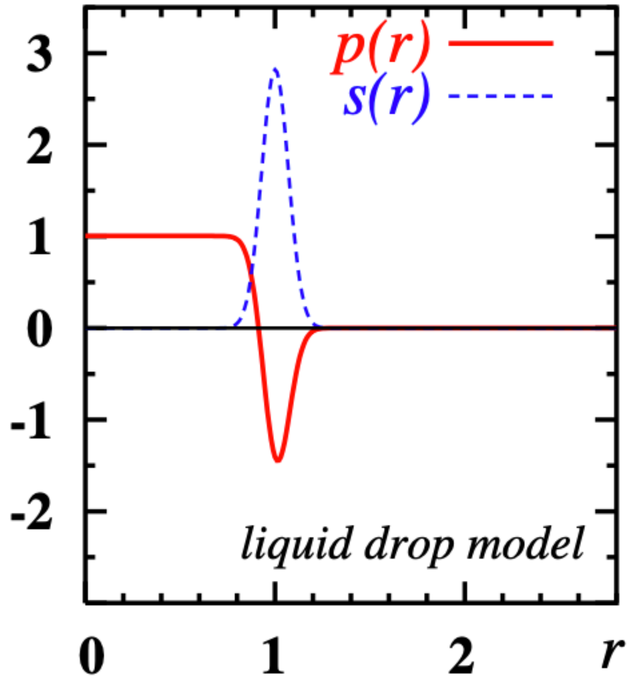
“Atom”



tension at small r: D > 0

von Laue's stability example 1: liquid droplet





$$p(r) = p_0\theta(r-R) - \frac{p_0 R}{3}\delta(r-R)$$

$$\int p(r)\mathrm{d}^3r = \frac{4\pi R^3}{3}p_0 - \frac{p_0 R}{3}4\pi R^2 = 0$$

Forces inside hadrons: pressure, surface tension, mechanical radius, and all that

Maxim V. Polyakov[1,2] and Peter Schweitzer[3] 1805.06596

von Laue's stability example 2: hydrogen atom

$$T^{ij}(\vec{r}) = mv^i v^j \delta^3(\vec{r} - \vec{x}(t)) - E^i E^j + \frac{\delta^{ij}}{2}\vec{E}^2$$

Electron's motion

Electron's and proton's electric field

We want to show $\int d^3\vec{r}\, T^{ii}(\vec{r}) = 2T + V = 0$ as in virial theorem

$$\vec{E} \approx \frac{e}{4\pi}\frac{\vec{r}}{r^3} - \frac{e}{4\pi}\frac{\vec{r} - \vec{x}(t)}{|\vec{r} - \vec{x}(t)|^3} \equiv \vec{E}_p + \vec{E}_e$$

$$\int d^3\vec{r}\, \vec{E}_e \cdot \vec{E}_p = -\frac{\alpha}{|\vec{x}(t)|} = -\frac{\alpha}{R} = V$$

Example: D-term of a liquid drop

$$p(r) = p_0\theta(r-R) - \frac{p_0 R}{3}\delta(r-R)$$

$$\begin{aligned} D &= m4\pi^2\left(\int_0^R \mathrm{d}r\ r^4 p_0 - \frac{p_0 R}{3}R^4\right) \\ &= -\frac{8\pi^2}{15}mp_0 \end{aligned}$$

It is negative because the negative pressure region is at the outer boundary.

Example: D-term of the hydrogen atom

Consider $\int d^3\vec{r}\; r^2 T^{ii}$

In dimensional regularization, terms homogeneous in r vanish.
Potential energy contributions give two integrals,

$$I = I_1 + I_2 \equiv \int d^3\vec{r}\left(-\frac{2}{|\vec{r}||\vec{r}-\vec{R}|} + \frac{2\vec{R}\cdot(\vec{r}-\vec{R})}{r|\vec{r}-\vec{R}|^3}\right)$$

$$\begin{aligned} I_1(D) &= -\frac{2}{\pi}\int \frac{d\alpha_1 d\alpha_2}{\sqrt{\alpha_1\alpha_2}}\int d^D\vec{r} e^{-(\alpha_1+\alpha_2)r^2 - \frac{\alpha_1\alpha_2}{\alpha_1+\alpha_2}R^2} \\ &\to 4\pi R|_{D=3}\ , \\ I_2(D) &= -2R^2\frac{2}{\sqrt{\pi}}\frac{1}{\sqrt{\pi}}\int \frac{d\alpha_1 d\alpha_2\sqrt{\alpha_1\alpha_2}}{\alpha_1+\alpha_2}\int d^D\vec{r} e^{-(\alpha_1+\alpha_2)r^2 - \frac{\alpha_1\alpha_2}{\alpha_1+\alpha_2}R^2} \\ &\to -4\pi R\ . \end{aligned}$$

Together with the kinetic energy contribution, we get

$$\int d^3\vec{r}\; r^2\, T^{ii}(\vec{r}) = mv^2R^2 + \frac{e^2}{32\pi^2}\Big(I_1 + I_2\Big) = \alpha R$$

This is positive, reflecting electron-proton attraction (rather than surface tension).

Logarithmic corrections: Lamb vs D-term

Vacuum fluctuations smear electron's position, $\overline{\delta^2} = \frac{4\alpha}{\pi m^2} \ln \frac{1}{\alpha}$

Lamb

$$E = E^{(0)} \left(1 - \frac{16\alpha^3}{3\pi} \ln \frac{1}{\alpha} \right)$$

$$\frac{\Delta E}{E^{(0)}} \sim \frac{\overline{\delta^2}}{a_B^2}$$

Only S-states are affected

D-term

$$D = D^{(0)} \left(1 - \frac{4\alpha}{3\pi} \ln \frac{1}{\alpha} \right)$$

$$\frac{\Delta D}{D^{(0)}} \sim \frac{\overline{\delta^2}}{\lambda_C^2}$$

Universal log-correction: all states

Welton's interpretation of Lamb shift

Electron's position in the H-atom modified by vacuum fluctuations.
This changes the potential experienced by the electron,

$$\langle U_c(\boldsymbol{r}+\boldsymbol{q})\rangle = U_c(r) + \underbrace{\langle \boldsymbol{q}\rangle}_{0} \cdot \boldsymbol{\nabla} U_c + \frac{1}{2}\underbrace{\left\langle q^i q^j \right\rangle}_{\frac{\delta_{ij}}{3}\langle q^2\rangle} \nabla^i \nabla^j U_c + \ldots,$$

$$\delta U = \langle U_c(\boldsymbol{r}+\boldsymbol{q})\rangle - U_c(r) \simeq \frac{1}{6}\left\langle q^2\right\rangle \nabla^2 U_c = \frac{1}{6}\left\langle q^2\right\rangle \alpha 4\pi \delta^3(\boldsymbol{r}).$$

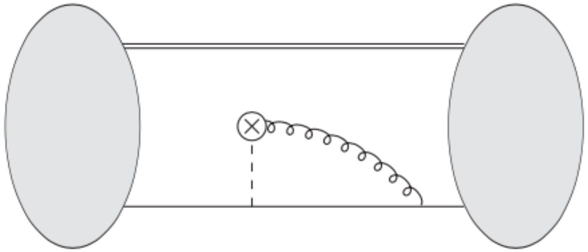
Mean-squared displacement <q^2>:

$$\begin{aligned}\left\langle q^2\right\rangle &= 2\int \left(\frac{e}{m\omega^2}\right)^2 \frac{V \mathrm{d}^3 k}{(2\pi)^3} E_{\boldsymbol{k}}^2 \\ &= \frac{2\alpha}{\pi m^2}\int \frac{\mathrm{d}k}{k}.\end{aligned}$$

$$\left\langle q^2\right\rangle = \frac{2\alpha}{\pi m^2}\ln\frac{1}{\alpha^2} + \text{non-logarithmic terms.}$$

$$\langle \delta U\rangle_{2S} = \frac{m}{3\pi}\alpha^5 \ln\frac{1}{\alpha}$$

Log correction to the D-term



Ji & Liu 2022

$$D_{\mathrm{NLO}} = \frac{\alpha}{6\pi} \sum_M \frac{2\vec{v}_{0M} \cdot \vec{v}_{M0}}{D(E_M - E_0)} \left(\ln \frac{4(E_M - E_0)^2}{m_e^2} - \frac{1}{4} \right)$$

dimension $D = 3$

Coefficient of the log: $$\sum_M \frac{2\vec{v}_{0M} \cdot \vec{v}_{M0}}{D(E_M - E_0)} \equiv \frac{1}{m_e}$$

Summary

- There are interesting observables in atoms, related to the energy-momentum tensor, in addition to the usually studied electromagnetic current.
- Atomic examples help understand properties of the EMT
- Sign of the D-term can be positive for a stable system
- Logarithmic corrections to the D-term are universal, affecting not only S-states.

Michael Eides' recent work on energy-momentum tensor

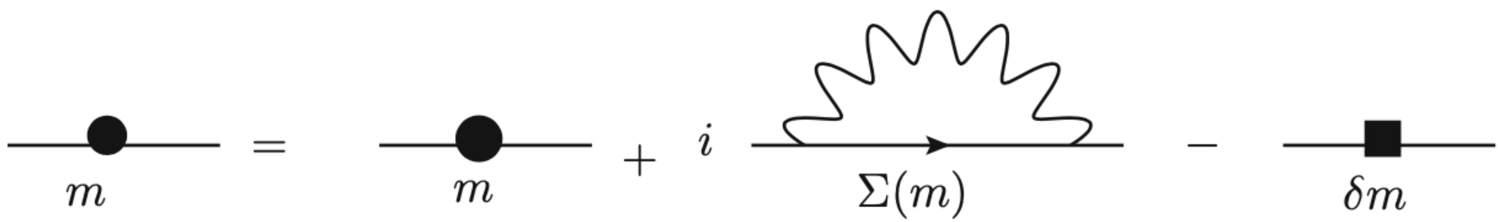
Motivation:

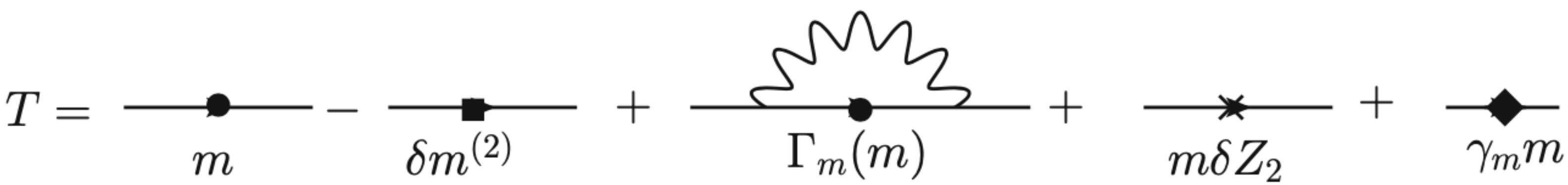
A new insight into the EMT properties could arise from consideration of EMT in theories which allow perturbative [treatment]

One-loop electron mass and QED trace anomaly

Michael I. Eides[a]

Eur. Phys. J. C (2023) 83:356





Graviton-atom interaction:
Of recent interest because can be probed in scattering experiments, via Generalized Parton Distributions [X. Ji; A. V. Radyushkin]