Heavy-quark form factors

Peter Marquard

in collaboration with

J. Blümlein, A. de Freitas, N. Rana, C. Schneider

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Outline

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Introduction

Considering form factors at three-loop order for the process

$$X \to Q + \overline{Q}$$

coupling through one of the vertices

$$\{1, \gamma_5, \gamma^\mu, \gamma^\mu \gamma_5\}$$

here:

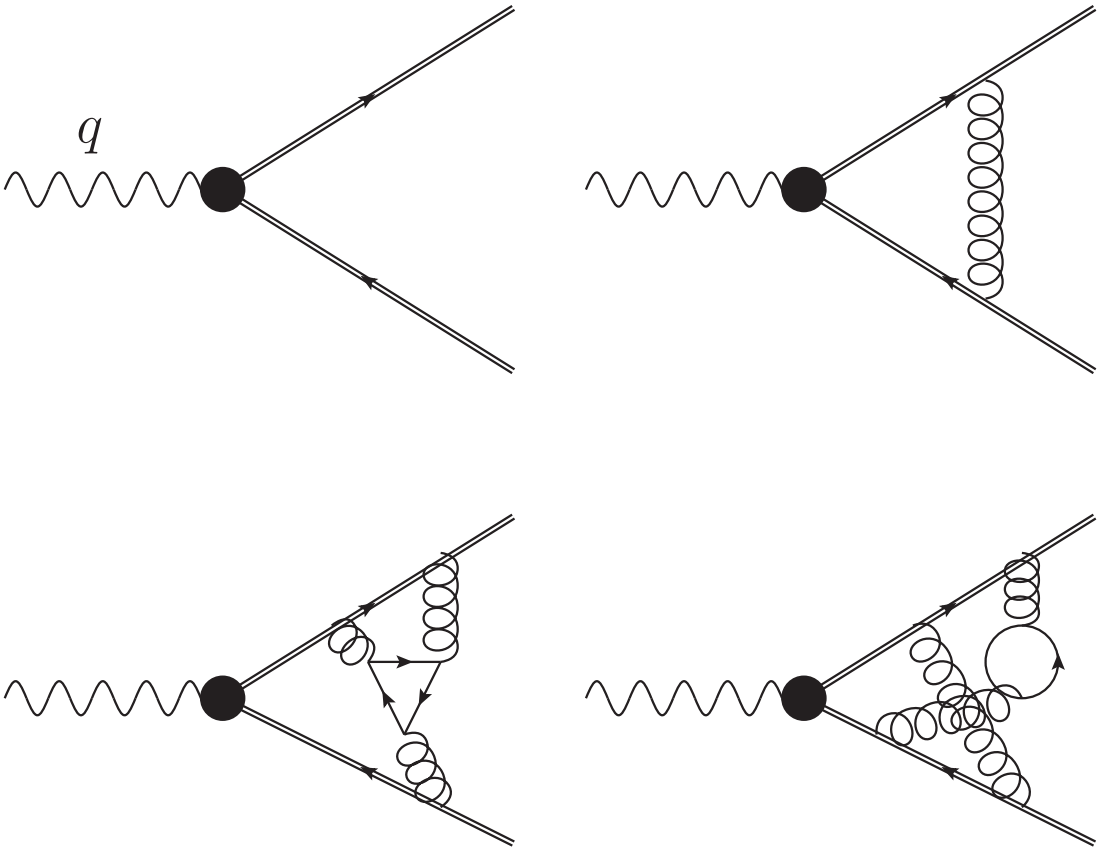
- only non-singlet contributions, i.e. the heavy-quark pair couples directly to the external current.
- at least one heavy-quark loop

Motivation

- heavy quark production
  - continuum production $e^+e^- \to t\bar{t}$
- particle decays
  - $H \to b\bar{b}$
  - $Z \to b\bar{b}$
  - $A \to t\bar{t}$
- technology development

Example: Vector case

$$\bar{\Psi}\Gamma_V^\mu\Psi = -i\bar{\Psi}\left(\gamma^\mu F_{V,1} + \frac{i}{2m}\sigma^{\mu\nu}q_\nu F_{V,2}\right)\Psi$$



History / Previous works

- two loop

  [Bernreuther,Bonciani,Gehrmann,Heinesch,Leineweber,Mastrolia,Remiddi '05]

  [Gluza,Mitov,Moch,Riemann '09]

  [Ablinger,Behring,Blümlein,Falcioni,De Freitas,PM,Rana,Schneider '18]

- three loop
  - light-fermionic contributions (HPLs)

    [Lee,Smirnov,Smirnov,Steinhauser'18]

    [Ablinger,Blümlein,PM,Rana,Schneider'18]

  - color-planar contributions (HPLs + cyclotomic HPLs)

    [Henn,Smirnov,Smirnov,Steinhauser '17]

    [Ablinger,Blümlein,PM,Rana,Schneider'18]

  - heavy-fermionic contributions

    [Blümlein,PM,Rana,Schneider'19]

  - full result including singlet/anomaly contributions [talk by K. Schönwald]

    [Fael,Lange,Schönwald,Steinhauser '22/'23]

- general infrared and high-energy structure [Ahmed,Henn,Steinhauser '17]

  [Blümlein, PM, Rana '18]

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Calculation

For the calculation of the form factors use the well-established multi-loop toolbox

- ✓ `QGRAF` for the generation of the diagrams
- ✓ use projectors to obtain scalar integrals
- ✓ `FORM` for the algebra
- ✓ use integration-by-parts identities [Chetyrkin,Tkachov]
  to reduce to an integral basis using `Crusher` [Seidel,PM]
  14 families, 104 master integral

??? calculate the required master integrals

- ✓ put everything together and renormalize
- ✓ final result still IR divergent – can be absorbed in cusp anom. dim.

Calculation / Strategy

Calculation of master integrals problematic since the heavy-fermionic and non-planar contributions contain structures beyond harmonic polylogarithms

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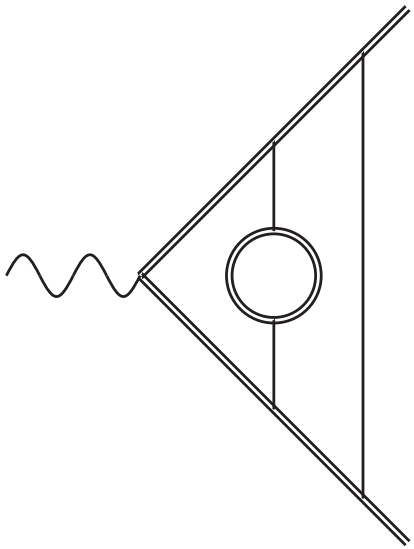
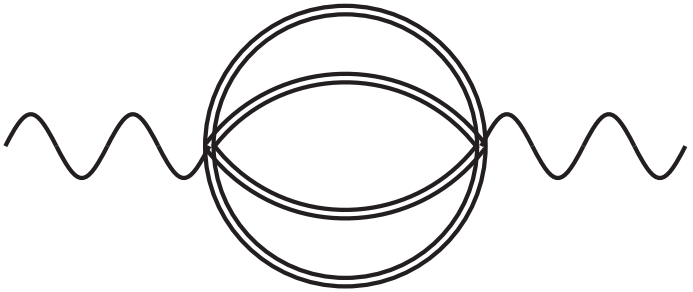


Reduction $\longrightarrow$



[see talk by S.Weinzierl]

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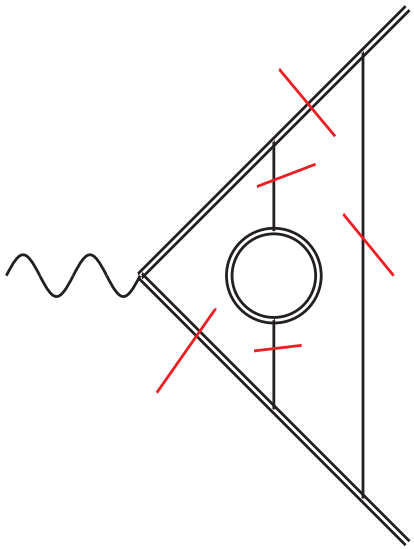

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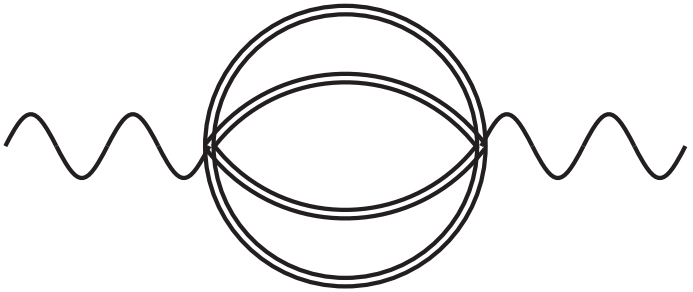

[see talk by S.Weinzierl]

Calculation / Strategy

- Strategy: Sum simpler then the individual parts!
- turn everything into recurrences by considering the expansion around $q^2 = 0$
- try to derive a recurrence for the whole form factor and find a analytic solution for that [Blümlein,Schneider '17]

Method

- choose a more appropriate variable

$$\frac{q^2}{m^2} = -\frac{(1-x)^2}{x}$$

$q^2 \to \pm\infty \equiv x \to 0_{\mp}$
$q^2 \to 0 \equiv x \to 1$

- around $q^2 = 0$, i.e. $x = 1$ the non-singlet form factors can be expanded in a simple power series

$$\mathcal{F} = \sum_{n=0} C_n \left(\frac{q^2}{m^2}\right)^n \quad \Leftrightarrow \quad \mathcal{F} = \sum_{n=0} D_n (1-x)^n = \sum_{n=0} D_n y^n$$

Method

- start from the coupled system of diff. eqn. for the master integrals
- insert the power series ansatz

$$\mathcal{M}_i = \sum_{j=0} M_j^{(i)} y^j$$

  and obtain recurrences for the coefficients $M_j^{(i)}$
- calculate 2,000 - 8,000 terms in the expansion for the master integrals
- use these to obtain 2,000 - 8,000 terms in the expansions of the full form factors
- as initial condition we need the values at $x = 1$, i.e. on-shell propagators

[Melnikov,v.Ritbergen]

Method

- the final expansion for the form factors has the form

$$\mathcal{F} = 1(\ldots) + \zeta_2(\ldots) + \zeta_3(\ldots) + \ln(2)(\ldots) + \mathrm{Li}_4(\tfrac{1}{2})(\ldots) + \cdots$$

  where $(\ldots)$ denote power series in y with rational coefficients
- this representation is unique
- can we do better?
  - Guess a recurrence [Kauers,Jaroschek,Johansson '15]
  - and try to solve it using `Sigma` [Schneider '07]
- if recurrence can be solved, i.e. first-order factorizing, one obtains (generalized) harmonic sums, which can be resummed using `HarmonicSums` [Ablinger '13]

Example

- start with the sequence for C_i in $\sum C_i y^i$

$$-2, 0, -\frac{1}{6}, -\frac{1}{6}, -\frac{3}{20}, -\frac{2}{15}, -\frac{5}{42}, -\frac{3}{28}, -\frac{7}{72}, -\frac{4}{45}, -\frac{9}{110}, -\frac{5}{66}, -\frac{11}{156},$$
$$-\frac{6}{91}, -\frac{13}{210}, -\frac{7}{120}, -\frac{15}{272}, -\frac{8}{153}, -\frac{17}{342}, -\frac{9}{190}, -\frac{19}{420}, \ldots$$

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  $n^2 C_n - (n-1)(n+2) C_{n+1} = 0$

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  $-\frac{6}{91}, -\frac{13}{210}, -\frac{7}{120}, -\frac{15}{272}, -\frac{8}{153}, -\frac{17}{342}, -\frac{9}{190}, -\frac{19}{420}, \ldots$
- guess recurrence
  $n^2 C_n - (n-1)(n+2) C_{n+1} = 0$
- solve the recurrence
  $C_n = -\frac{n-1}{n(n+1)}$
- sum it
  $-2 - \sum_{n=1}^{\infty} \frac{n-1}{n(n+1)} y^n = -\frac{(y-2)\log(1-y)}{y} \overset{y \to 1-x}{=} \frac{(1+x)\log(x)}{1-x}$

Outline

Results

We could find analytic results for all terms but for $n_h \quad n_h\zeta_2 \quad n_h\zeta_3$

		degree	order	remaining order
F_V	$g_1 n_h$	1288	54	15
	$g_1 n_h \zeta_3$	409	29	10
	$g_1 n_h \zeta_2$	295	24	6
	$g_2 n_h$	1324	55	15
	$g_2 n_h \zeta_3$	430	30	10
	$g_2 n_h \zeta_2$	273	23	6
F_S	n_h	1114	50	15
	$n_h \zeta_3$	350	27	10
	$n_h \zeta_2$	230	22	6

For leading color we could also solve the term $\propto N_c^2 n_h \zeta_2$

Results – Scalar form factor

$$
\begin{aligned}
F_S \;=\; & -\frac{1}{\varepsilon^3}\frac{1}{2(1+x)^2}\Bigg\{ n_h^2\left[-\frac{64}{27}(1+x)^2+\frac{64(1+x)\left(1+x^2\right)}{27(1-x)}H_0\right] \\
& +n_h\left[\frac{4}{27}\left(997+1418x+997x^2\right)-\frac{32H_0P_8^{(5)}}{27(1-x^2)}\right. \\
& \left.-n_l\left[\frac{32}{9}(1+x)^2-\frac{64(1+x)\left(1+x^2\right)}{27(1-x)}H_0\right]+\frac{256\left(1+x^2\right)^2}{27(1-x)^2}H_0^2\right]\Bigg\}
\end{aligned}
$$

Results – Scalar form factor cont'd

$$
\begin{aligned}
&-\frac{1}{\varepsilon^2}\frac{1}{2(1+x)^2}\Bigg\{n_h^2\Bigg[-\frac{832}{81}(1+x)^2-\frac{256x(1+x)H_0}{27(1-x)}-\frac{128(1+x)(1+x^2)}{27(1-x)}H_{-1}H_0\\
&+\frac{32(1+x)(1+x^2)}{27(1-x)}H_0^2+\frac{128(1+x)(1+x^2)}{27(1-x)}H_{0,-1}-\frac{64(1+x)(1+x^2)}{27(1-x)}\zeta_2\Bigg]\\
&+n_h\Bigg[\frac{16}{27}(897+1786x+897x^2)+n_l\Bigg[-\frac{64}{3}(1+x)^2+\frac{64(1+x)(5-24x+5x^2)}{81(1-x)}H_0\\
&-\frac{256(1+x)(1+x^2)}{27(1-x)}H_{-1}H_0+\frac{64(1+x)(1+x^2)}{27(1-x)}H_0^2+\frac{256(1+x)(1+x^2)}{27(1-x)}H_{0,-1}\\
&-\frac{128(1+x)(1+x^2)}{27(1-x)}\zeta_2\Bigg]+\left(\frac{128H_{-1}P_7^{(5)}}{27(1-x^2)}-\frac{16P_{13}^{(5)}}{27(1-x^2)}\right)H_0+\Bigg(\frac{64P_{26}^{(5)}}{27(1-x)^2(1+x)}\\
&-\frac{1024(1+x^2)^2}{27(1-x)^2}H_{-1}\Bigg)H_0^2-\frac{128(-2+x^2)(1+x^2)}{27(1-x)^2}H_0^3-\frac{128(1+x)(1+x^2)}{3(1-x)}H_0H_1\\
&+\left(\frac{128(1+x)(1+x^2)}{3(1-x)}-\frac{128(1+x^2)^2}{3(1-x)^2}H_0\right)H_{0,1}-\left(\frac{128P_7^{(5)}}{27(1-x^2)}-\frac{2176(1+x^2)^2}{27(1-x)^2}H_0\right)\\
&\times H_{0,-1}+\frac{256(1+x^2)^2}{3(1-x)^2}H_{0,0,1}-\frac{256(1+x^2)^2}{3(1-x)^2}H_{0,0,-1}+\Bigg(\frac{64P_5^{(5)}}{27(1-x^2)}\\
&-\frac{64(1+x^2)(-1+35x^2)}{27(1-x)^2}H_0\Bigg)\zeta_2-\frac{64(1+x^2)^2}{3(1-x)^2}\zeta_3\Bigg]\Bigg\}
\end{aligned}
$$

Results – Scalar form factor – unsolved recurrences

$$F_S = \ldots + n_h F^{(0)}_{S,1}(x) + n_h \zeta_2 F^{(0)}_{S,2}(x) + n_h \zeta_3 F^{(0)}_{S,3}(x)$$

$$F^{(0)}_{S,1}(x) = -\frac{874750}{243} - \frac{731018}{729} y^2 - \frac{731018}{729} y^3 - \frac{316061833}{437400} y^4 - \frac{96756433}{218700} y^5 + \cdots + O(y^{2001})$$

$$F^{(0)}_{S,2}(x) = \frac{343864}{81} + \frac{2421832}{3645} y^2 + \frac{2421832}{3645} y^3 + \frac{16041283}{36450} y^4 + \frac{3932123}{18225} y^5 + \cdots + O(y^{2001})$$

$$F^{(0)}_{S,3}(x) = \frac{62968}{27} - \frac{22516}{81} y^2 - \frac{22516}{81} y^3 - \frac{21262303}{97200} y^4 - \frac{7752703}{48600} y^5 + \cdots + O(y^{2001}).$$

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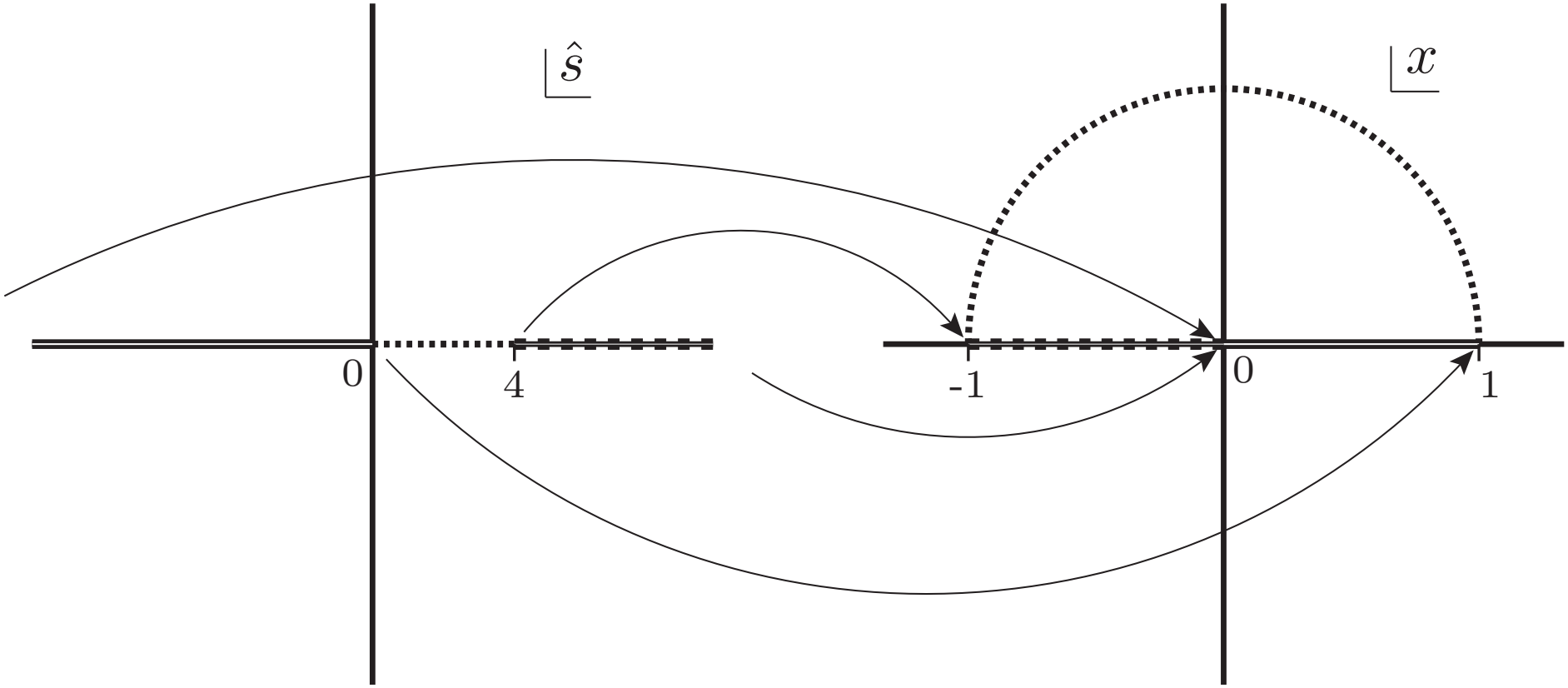
What to do with them?

- Expansions about $x = 0$
- Expansions about $x = -1 \leftrightarrow q^2 = 4m^2$

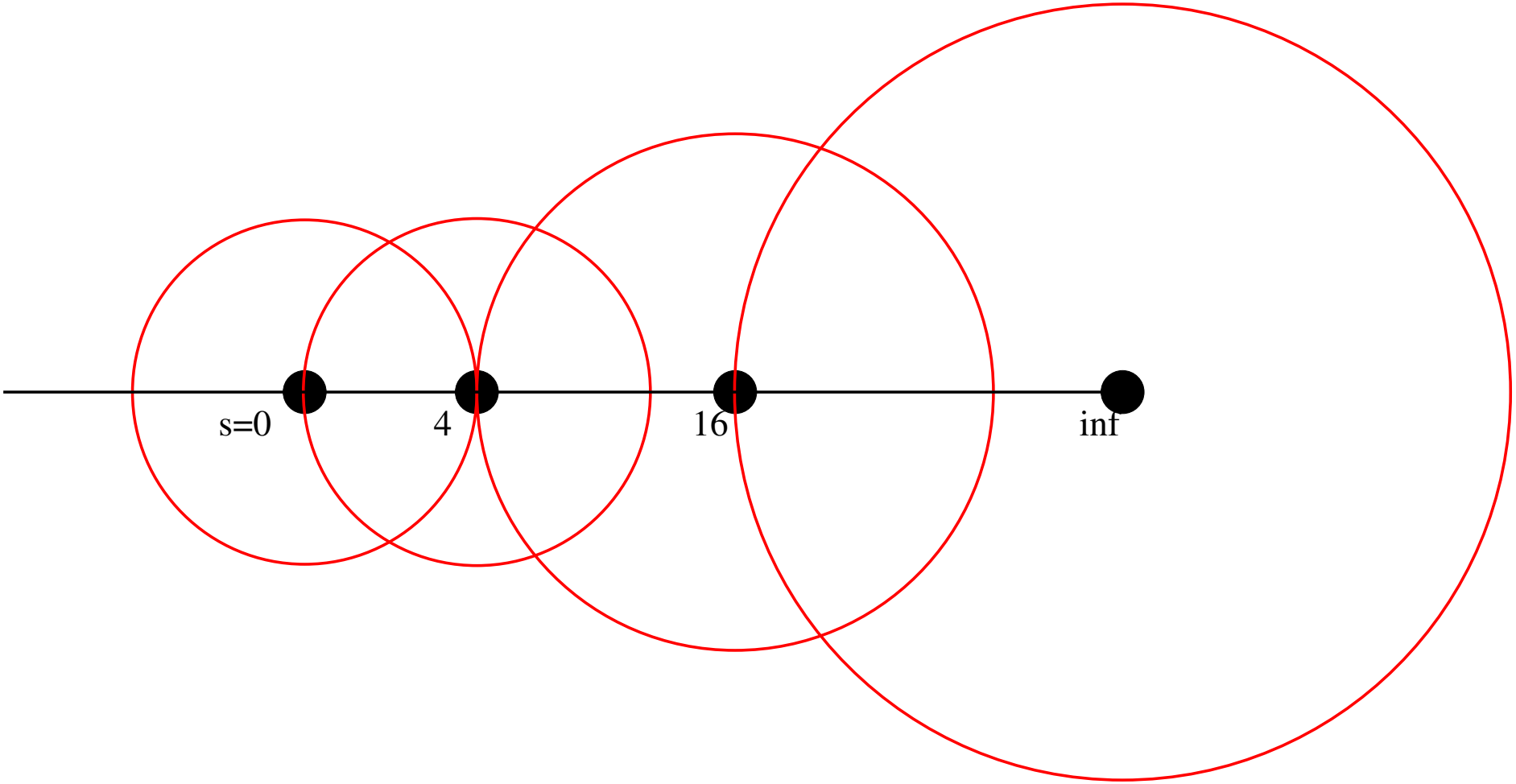
Outline

Change of variable

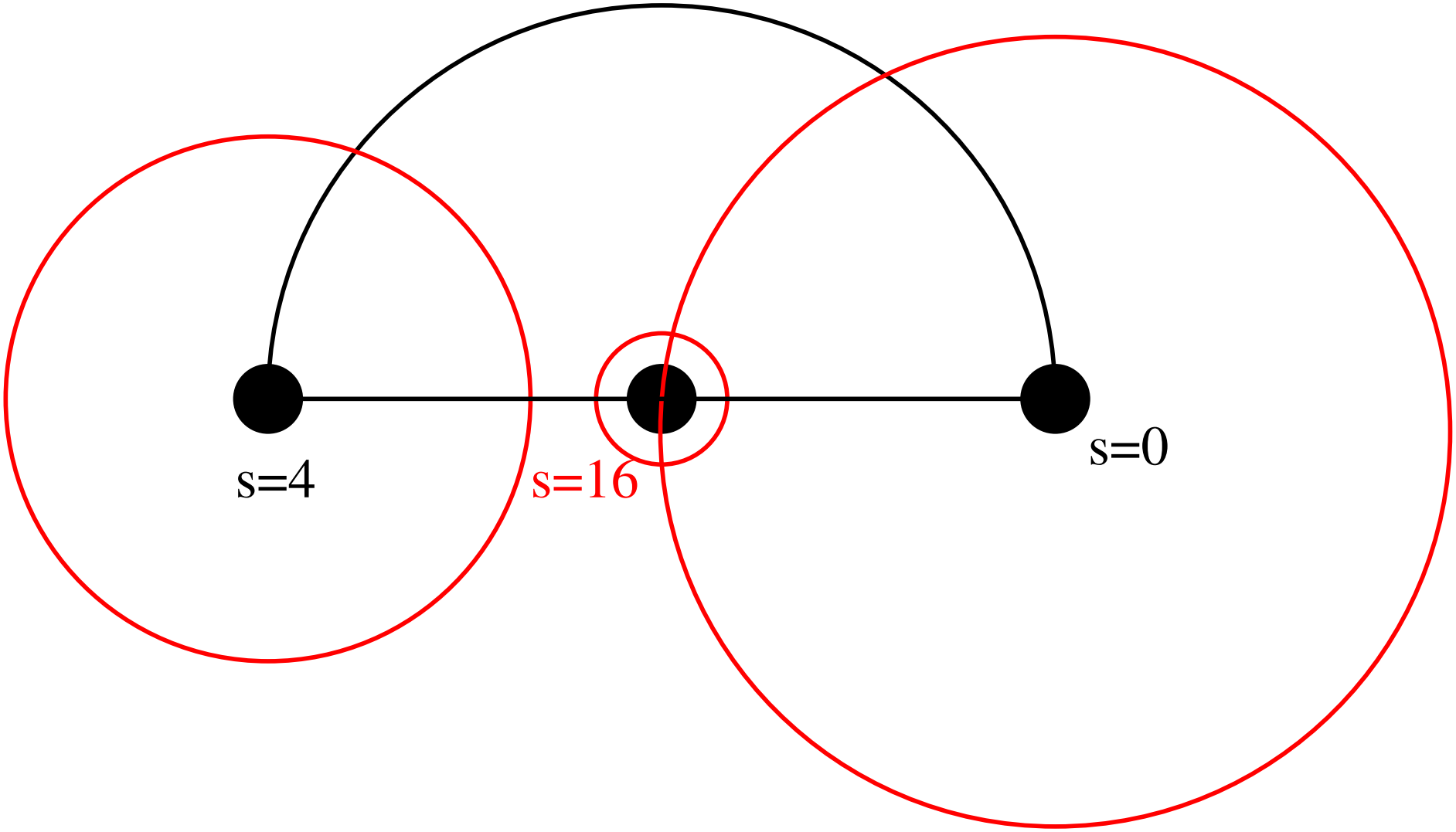


$$\hat{s} = \frac{q^2}{m^2} = -\frac{(1-x)^2}{x}$$

Radii of convergence: $\hat{s}$ space



Radii of convergence: x space



Expansions about other points

Take two points x_1 and x_2
Expansions about x_1 known

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need to determine expansion coefficients at other point x_2

$$f(x) = \sum_{n=0}^{N} D_n (x - x_2)^n$$

$$D_n = ?$$

Expansions about other points

- both series should have some overlap in their range of convergence
- equate the two series at number of points y_i

$$f(y_i) = \sum_{n=0}^{N} C_n (y_i - x_1)^n = \sum_{n=0}^{M} D_n (x - x_2)^n$$

- fit D_n

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- fit D_n
  $\mathcal{O}(1000) C_n \to \mathcal{O}(1000) D_n$?

Example cont'd

Recurrence fixed to the expansion point
$\Rightarrow$ but can also guess a differential equation

$$f''(x) = \left(-\frac{1}{x-1} - \frac{2}{x} + \frac{2}{x-2}\right) f'(x) + \left(-\frac{2}{x-1} + \frac{2}{x-2} - \frac{2}{(x-2)^2}\right) f(x)$$

Solving this diff. eqn. gives the same result as before.

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Here, we can perform variable transformation to any kinematic point and insert the appropriate expansions and obtain the corresponding recurrences.

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Here, we can perform variable transformation to any kinematic point and insert the appropriate expansions and obtain the corresponding recurrences.

Need only (degree of diff eqn) initial conditions for the recurrences

Diff. eqs. for the non-solvable rec

Orders of the diff eqns obtained

i	$F^{(0)}_{v,1,i}$	$F^{(0)}_{v,2,i}$	$F^{(0)}_{a,1,i}$	$F^{(0)}_{a,2,i}$	$F^{(0)}_{s,i}$	$F^{(0)}_{p,i}$
1	46	48	46	43	43	43
2	20	22	20	18	18	18
3	25	26	25	23	23	23

$\Rightarrow$ Need to fit at most 48 coefficients.

High-energy region: $x = 0$

Input:

- 500 000 terms in the expansion about $x = 1$
- ansatz with 3 000 terms in the expansion about $x = 0$
  $\hookrightarrow$ converges only up to $x = 7 - 4\sqrt{3} \approx 0.072$

Fit to

$$\sum_k \ln^k x \sum_i C_{k,i} x^i$$

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Output:

- $\mathcal{O}(1\,000)$ digits for the coefficients fitted
- use PSLQ to fit to set of constants

$$\zeta_2,\ \zeta_3,\ \zeta_4,\ \zeta_5,\ \zeta_6,\ \zeta_2\zeta_3,\ \zeta_3^2,\ \ln^4 2,\ \zeta_2 \ln^4 2,\ \zeta_2 \ln^2 2,\ \zeta_4 \ln^2 2,$$
$$\zeta_3 \ln 2,\ \zeta \mathrm{Li}_4(1/2) \ln 2,\ \zeta_2\zeta_3 \ln 2,\ \mathrm{Li}_4(1/2),\ \zeta_2 \mathrm{Li}_4(1/2),\ \ldots$$

Result: high-energy limit

$$
\begin{aligned}
F_S^{\text{non-sol}} &= +\ln(x)\left(\frac{512\text{Li}_4\left(\frac{1}{2}\right)}{9} - \frac{51260\,\zeta(3)}{81} + \frac{22184}{27} - \frac{2938\pi^2}{729}\right.\\
&\left. + \frac{5432\pi^4}{1215} - \frac{256}{27}\pi^2\ln^2(2) + \frac{64\ln^4(2)}{27} + \frac{6704}{81}\pi^2\ln(2)\right)\\
&+ \ln^2(x)\left(-\frac{512\zeta\,(3)}{9} + \frac{77548}{243} + \frac{760\pi^2}{243} + \frac{64}{27}\pi^2\ln\,(2)\right)\\
&- \frac{176}{81}\ln^5(x) + \frac{2888\ln^4(x)}{243} + \left(\frac{5452}{81} - \frac{16\pi^2}{9}\right)\ln^3(x)\\
&+ \frac{3964\zeta(5)}{9} - \frac{820\pi^2\zeta(3)}{27} + \frac{107668\,\zeta(3)}{81} + \frac{2581\pi^4}{405} + \frac{914054\pi^2}{729}\\
&- \frac{1676170}{243} - \frac{304\ln^4(2)}{243} - \frac{128}{243}\pi^2\ln^2(2) - \frac{256}{9}\pi^4\ln(2)\\
&+ \frac{104800}{243}\pi^2\ln\,(2) - \frac{2432\,\text{Li}_4\left(\frac{1}{2}\right)}{81} + \mathcal{O}(x)
\end{aligned}
$$

Agrees numerically with [Fael,Lange,Schönwald,Steinhauser '22]

Threshold region: $\hat{s} = 4 \leftrightarrow x = -1$

- use same procedure
- but: expansion about $x = 1$ does not converge on the arc
  $\hookrightarrow$ go back to expansion about $\hat{s} = q^2/m^2 \approx 0$
  $\hookrightarrow$ converges up to threshold
- fit to
$$\sum_k \ln^k(z) \sum_i C_{ki} z^i, \qquad z = \sqrt{4 - \hat{s}}$$
- 100 000 / 3 000 terms $\rightarrow \mathcal{O}(1800)$ digits
- fit works, but PSLQ reconstruction fails due to unknown basis of constants $\rightarrow \tilde{\kappa}_i$

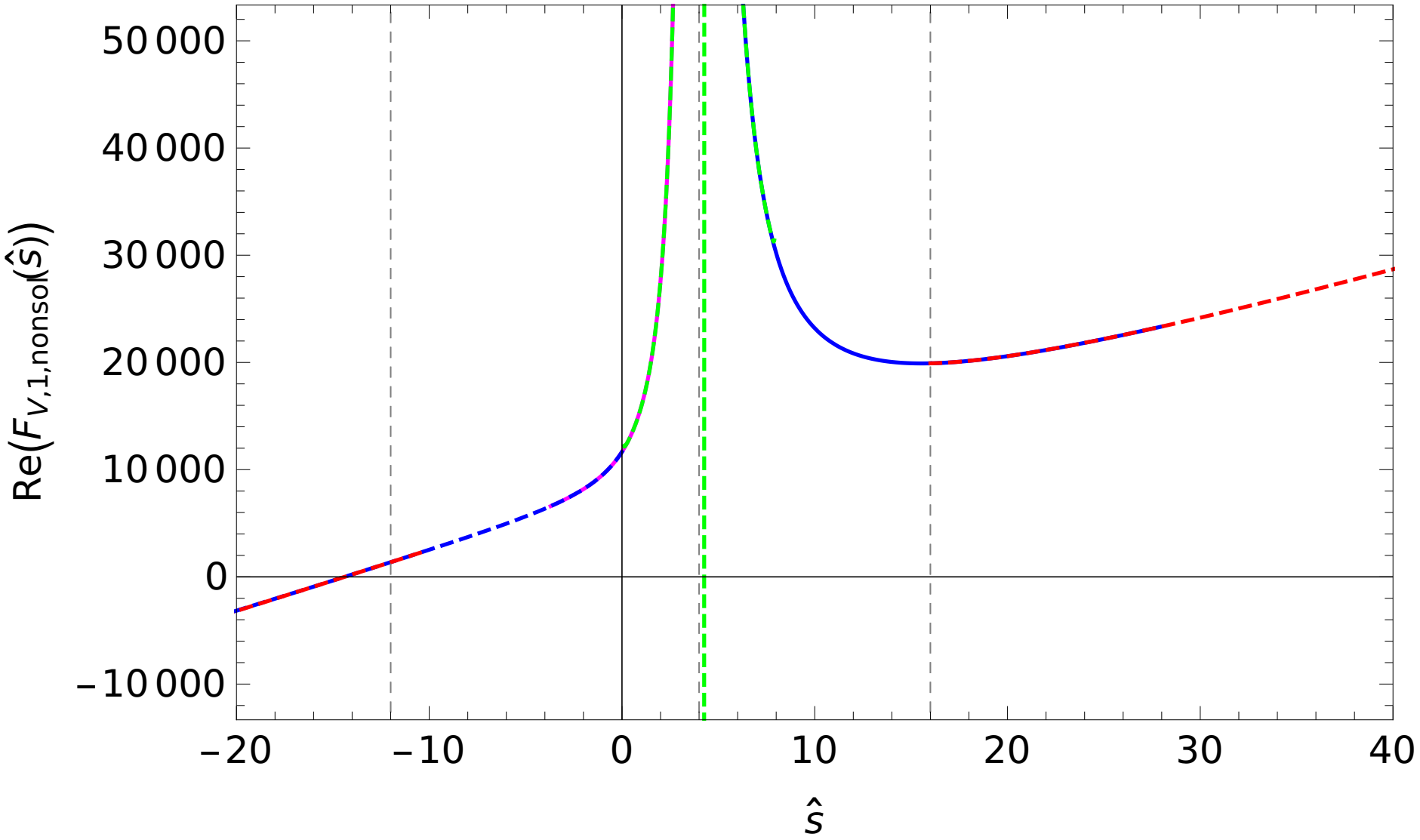
$s = 4$: new constants

$$
\begin{aligned}
\tilde{\kappa}_1 &= -1264.9432224278092329957750523362172006762421108620998611129 6, \\
\tilde{\kappa}_2 &= -26176.4667608724683949216820111127329755051498931864672207674, \\
\tilde{\kappa}_3 &= -2729.2992177505811200034225906925106691569743552187882961646 1, \\
\tilde{\kappa}_4 &= 55185.6670430603029362317458218280389428429637759659305766923, \\
\tilde{\kappa}_5 &= -231417.5433206241973350291333548322775137629566421686709169 34, \\
\tilde{\kappa}_6 &= 27058.0674155939392402733850737674942036176399269710732266681, \\
\tilde{\kappa}_7 &= 37228.1393096283192321319569484136035748028723926780936227023, \\
\tilde{\kappa}_8 &= -13339.4468993806410955294285663003095854470302119871183700172, \\
\tilde{\kappa}_9 &= 36376.0677825693690120778060832493123585788086425881920483389, \\
\tilde{\kappa}_{10} &= 44168.3670154020748917804528969924640054915510728808520969412, \\
\tilde{\kappa}_{11} &= 216837.1191056016042984235154720743505272680683085353849252 74, \\
\tilde{\kappa}_{12} &= -5730.8715584389448171926403934422566463660438060546199600 9706, \\
\tilde{\kappa}_{13} &= -135665.0668062562684803898005593667922857694812717318247235 68, \\
\tilde{\kappa}_{14} &= 25026.2194317039528218591802514512389969169209802143192666245.
\end{aligned}
$$

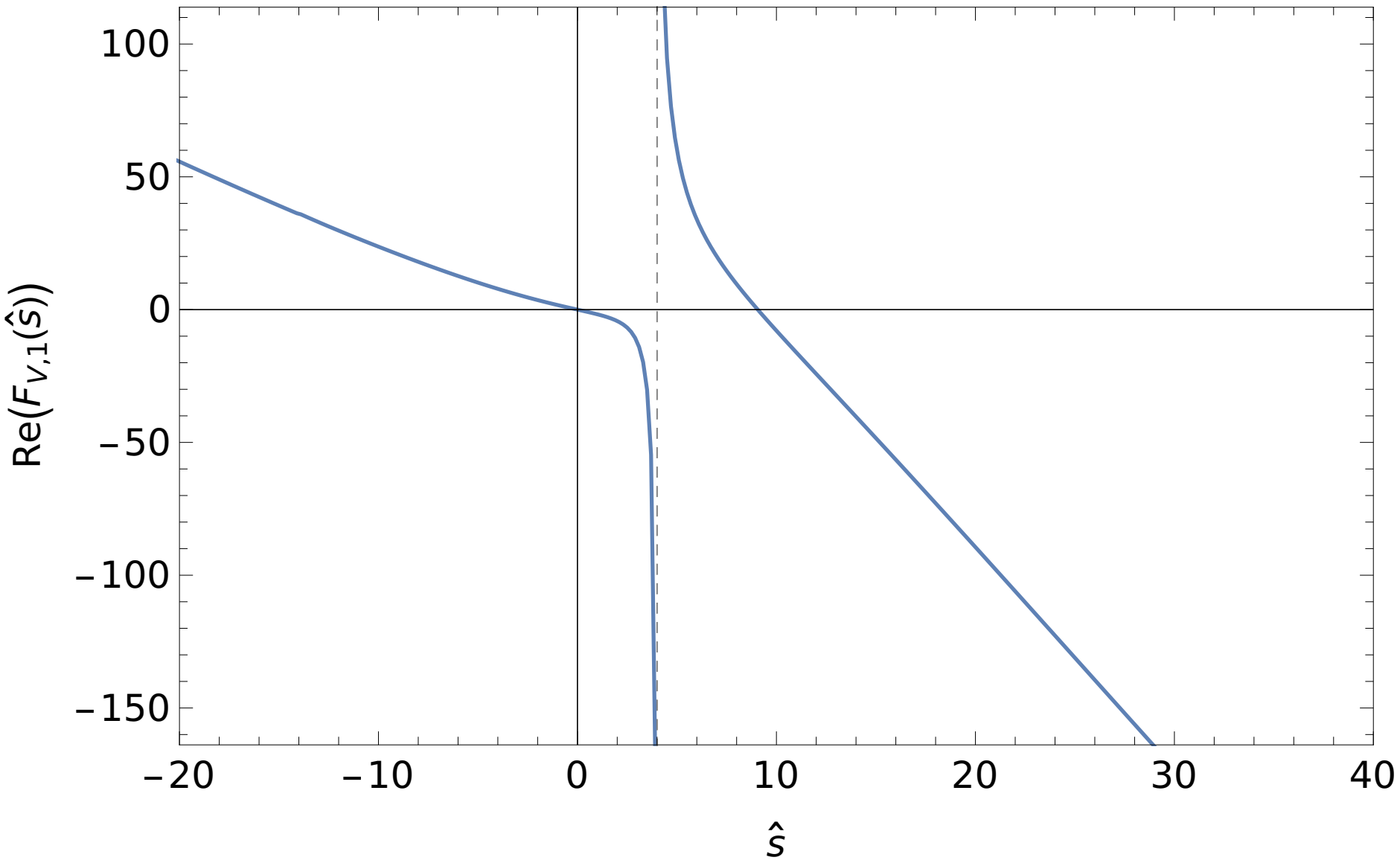
4-particle threshold: $\hat{s} = 16 \leftrightarrow x \approx -0.072$

- fit to
$$\sum_i c_i \bar{z}^i, \qquad \bar{z} = \sqrt{16 - \hat{s}}$$
- match with expansion at $x = 0$
- no intermediate $\bar{z}^{-n}, \ln \bar{z}$
- constants from the solvable part indicate constants $H(\{\ldots\}, 4\sqrt{3} - 7)$
- did not try PSLQ fit

Results


50 000
40 000
30 000
20 000
10 000
0
−10 000
−20
−10
0
10
20
30
40
$\hat{s}$
$\mathrm{Re}(F_{V,1,\mathrm{nonsol}}(\hat{s}))$

Results



Conclusions

- Extended the results for the heavy-fermionic contributions to the massive form factors to cover the whole kinematic range
- Expansions about $\hat{s} = 0, 4, 16, \infty$ are sufficient
- High-energy expansion with analytic coefficients
- Numerical agreement with [Fael,Lange,Schönwald,Steinhauser '22]
- ToDo: missing gluonic contributions
  $\hookrightarrow$ For $\hat{s} = 0$ agreement with [Fael,Lange,Schönwald,Steinhauser '22]

LOOPS AND LEGS IN QUANTUM FIELD THEORY

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Organizing committee:

P. Marquard
M. Steinhauser

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