Analytic approximations of $2 \to 2$ processes with massive internal particles

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National Laboratory of the Helmholtz Association

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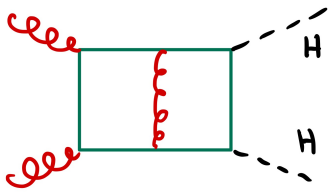
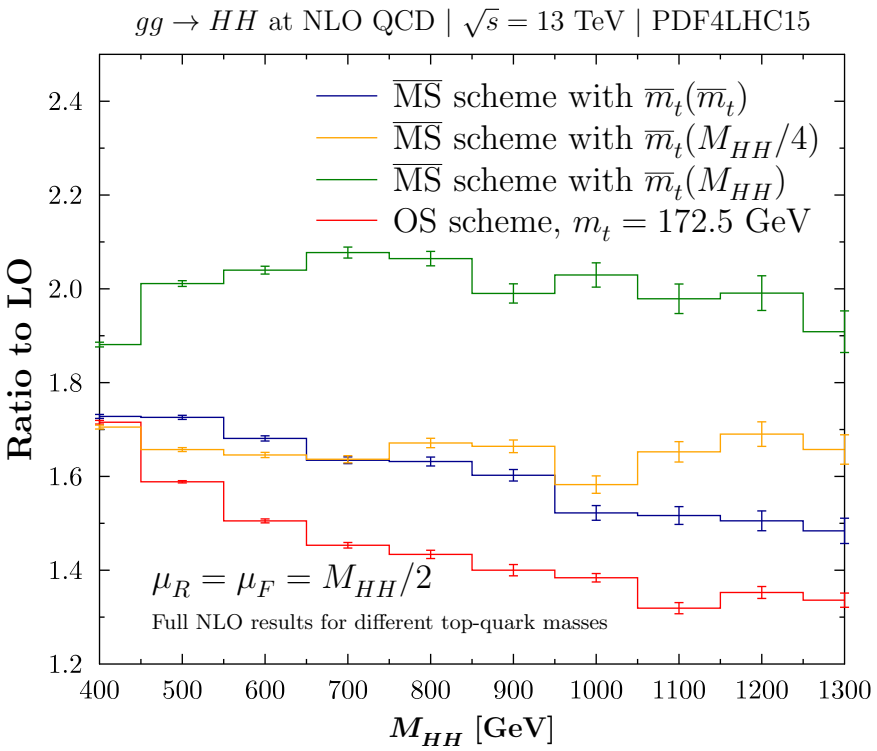
$gg \to HH$ and $gg \to ZH$ at NLO



[Baglio,Campanario,Glaus,Mühlleitner,Ronca,Spira'20]
[Borowka,Greiner,Heinrich,Jones,Kerner,Schlenk,Schubert,Zirke'16]



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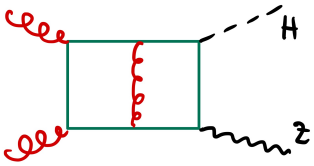

[Chen,Davies,Heinrich,Jones,Kerner,Mishima,Schlenk,Steinhauser'22]
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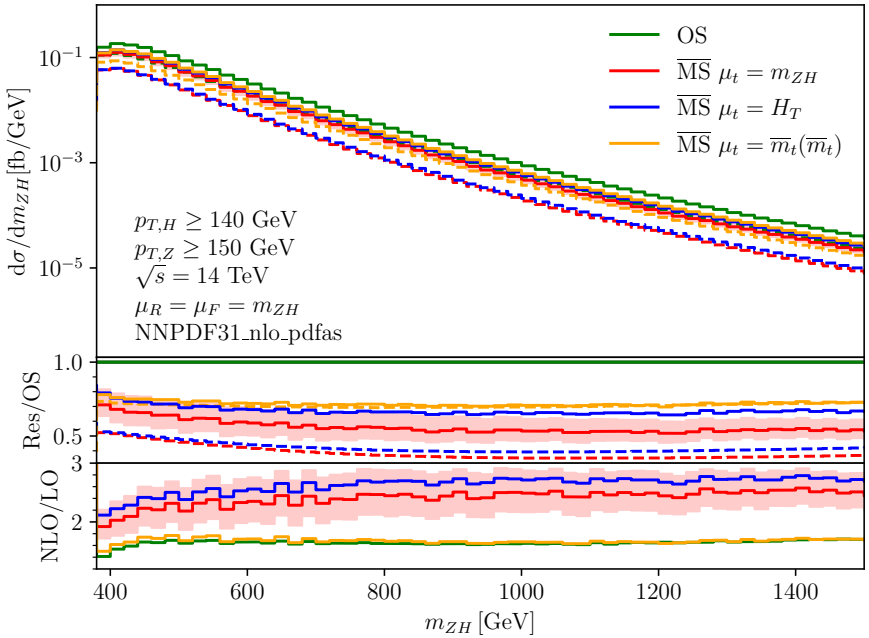

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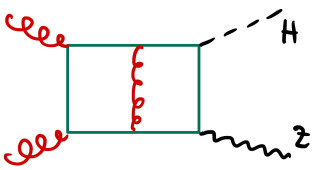

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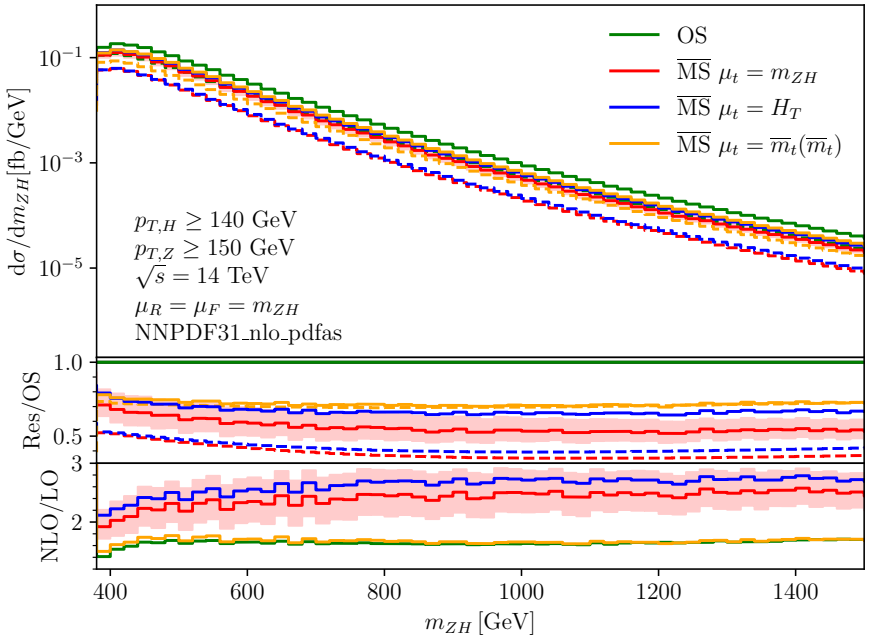
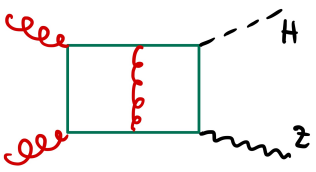


$\longrightarrow$ not satisfactory

$\longrightarrow$ NNLO (?)

$gg \to HH$ and $gg \to ZH$ at NLO



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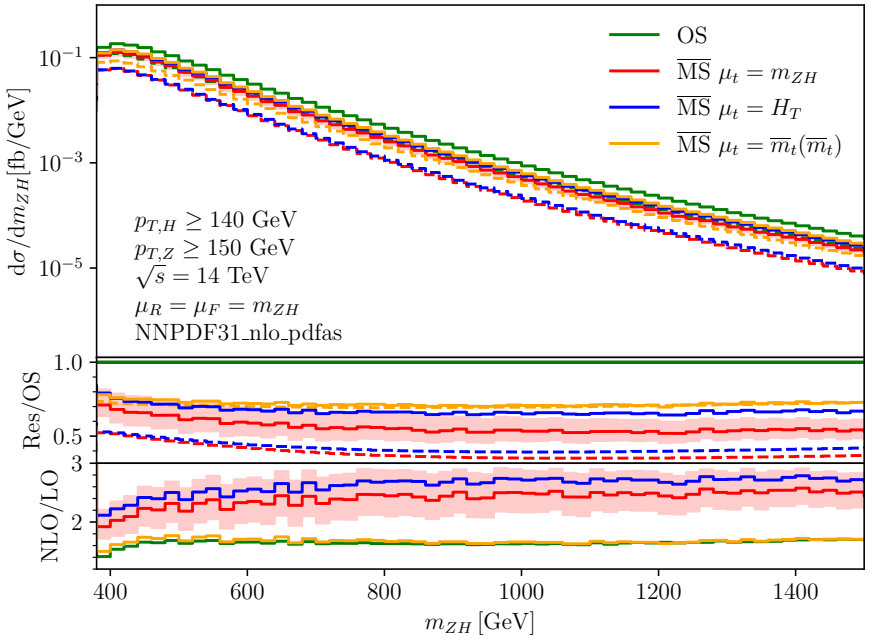


Numerical integration
– disadvantages

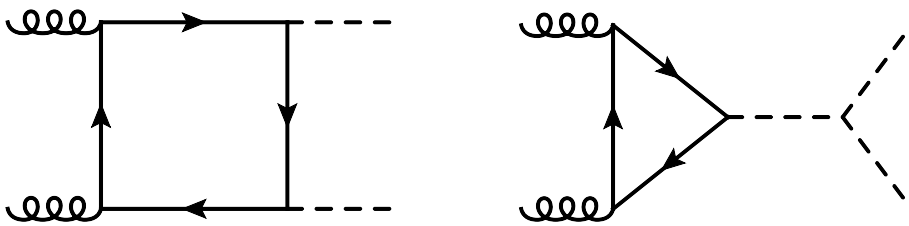
- runtime
- flexibility
- scheme change

$\longrightarrow$ not satisfactory
$\longrightarrow$ NNLO (?)

This talk

- Analytic approach to NLO $gg \to XY$ (2 loops)
- First steps towards NNLO (3 loops)

- $gg \to HH$
- 2 tensor structures
- form factors



$$\mathcal{M}_1 \sim \frac{3m_H^2}{s - m_H^2} F_{\text{tri}} + F_{\text{box1}}$$

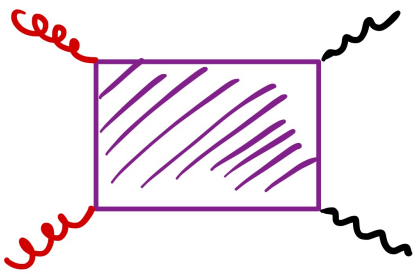
$$\mathcal{M}_2 \sim F_{\text{box2}}$$

- virtual corrections

$$\mathcal{V}_{\text{fin}} = \frac{\alpha_s^2(\mu)}{16\pi^2} \frac{G_F^2 s^2}{64} \left[C + 2\left(\tilde{F}_1^{(0)*} \tilde{F}_1^{(1)} + \tilde{F}_2^{(0)*} \tilde{F}_2^{(1)} + \tilde{F}_1^{(0)} \tilde{F}_1^{(1)*} + \tilde{F}_2^{(0)} \tilde{F}_2^{(1)*} \right) \right]$$

$$C = \left[\left| \tilde{F}_1^{(0)} \right|^2 + \left| \tilde{F}_2^{(0)} \right|^2 \right] \left(C_A \pi^2 - C_A \log^2 \frac{\mu^2}{s} \right)$$

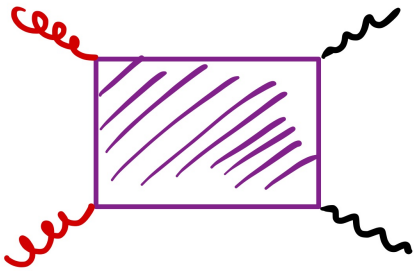
Analytic expansions



(s, t, m_t, m_X, m_Y)

- large-mass expansion: $m_t^2 \gg s, t, \ldots$
- high energy: $m_t^2 \ll s, t, \ldots$
- $t \to 0$

Analytic expansions



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  `exp` [Harlander,Seidensticker,Steinhauser'98]

  "simple": vacuum integrals and massless integrals
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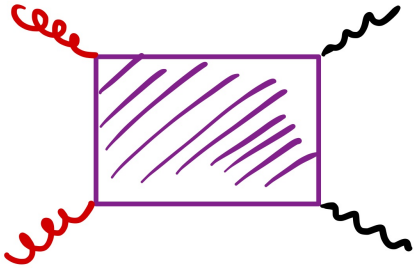

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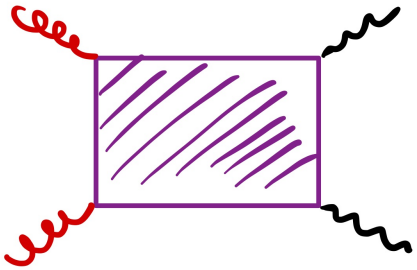

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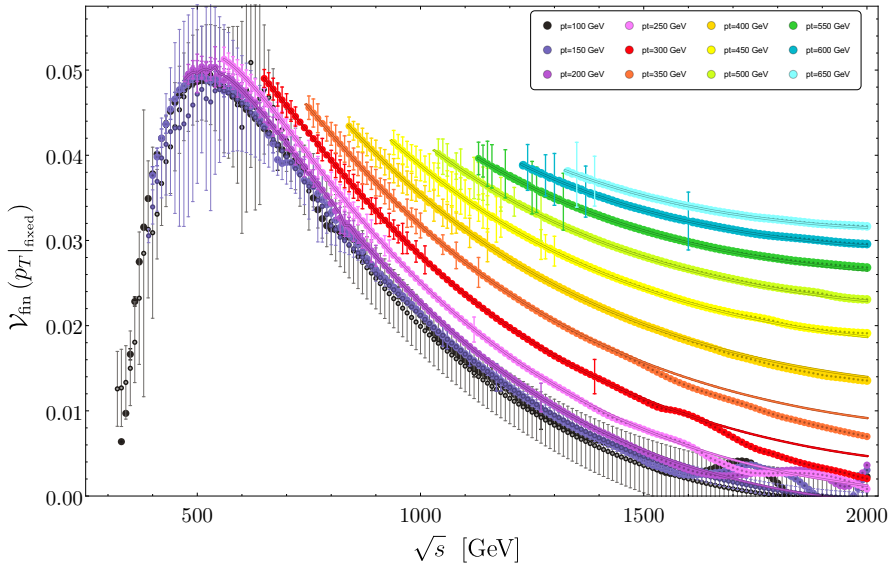
  Taylor expansion

High energy expansion

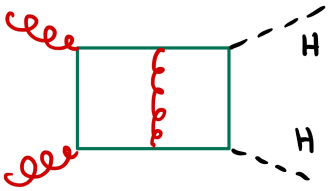
[Melnikov,Tancredi,Wever'16,..., Davies,Mishima,Steinhauser,Wellmann'18]

...applied to $gg \to HH$

- Taylor expansion in m_H
- differential equation in $\{s, t, m_t^2\}$
- BCs for $m_t \to 0$
- expansion up to $(m_t^2)^{16} \ldots (m_t^2)^{50} \ldots$



[Davies,Heinrich,Jones,Kerner,Mishima,Steinhauser,Wellmann'19]



Padé improvement

$$\sum_{k=0}^{N} c_k (m_t^2)^k \quad = \quad \frac{a_0 + \cdots + a_r (m_t^2)^r}{1 + b_1 + \cdots + b_s (m_t^2)^s} \quad = \quad [r/s](m_t^2) \qquad r + s = N$$

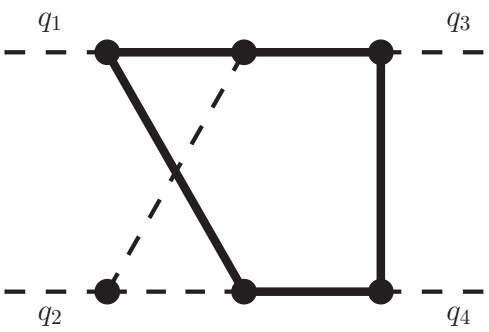
Uncertainty estimate:

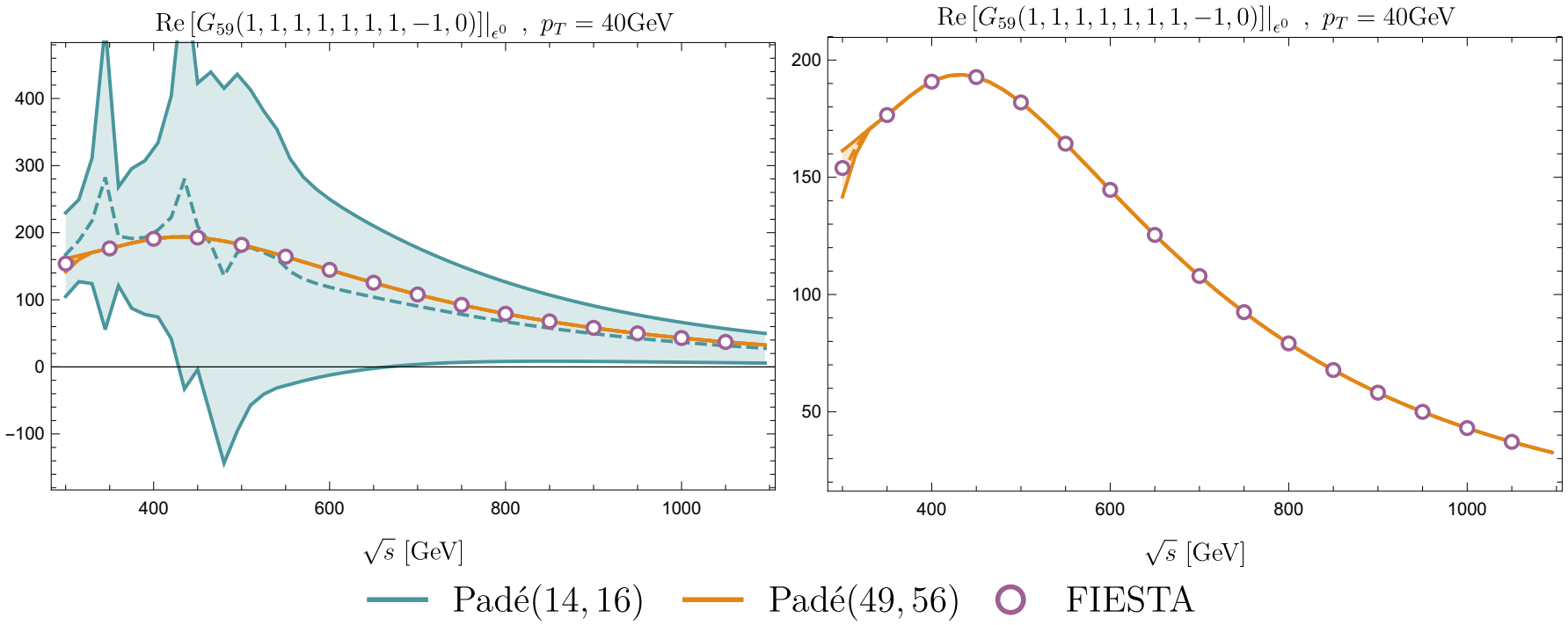
- more weight to PAs which have poles further away from the evaluation point
- more weight to PAs which are derived from a larger number of expansion terms
- more weight to near-diagonal PAs

➪ central value and corresponding uncertainty
for each phase-space point (s, p_T)

High energy expansion $\oplus$ PA

- expansion up to $N_{\max} = (m_t^2)^{56}$
- construct PAs with input for $(N_{\min}, N_{\max})$





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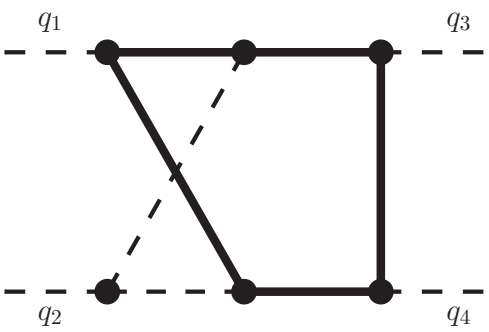
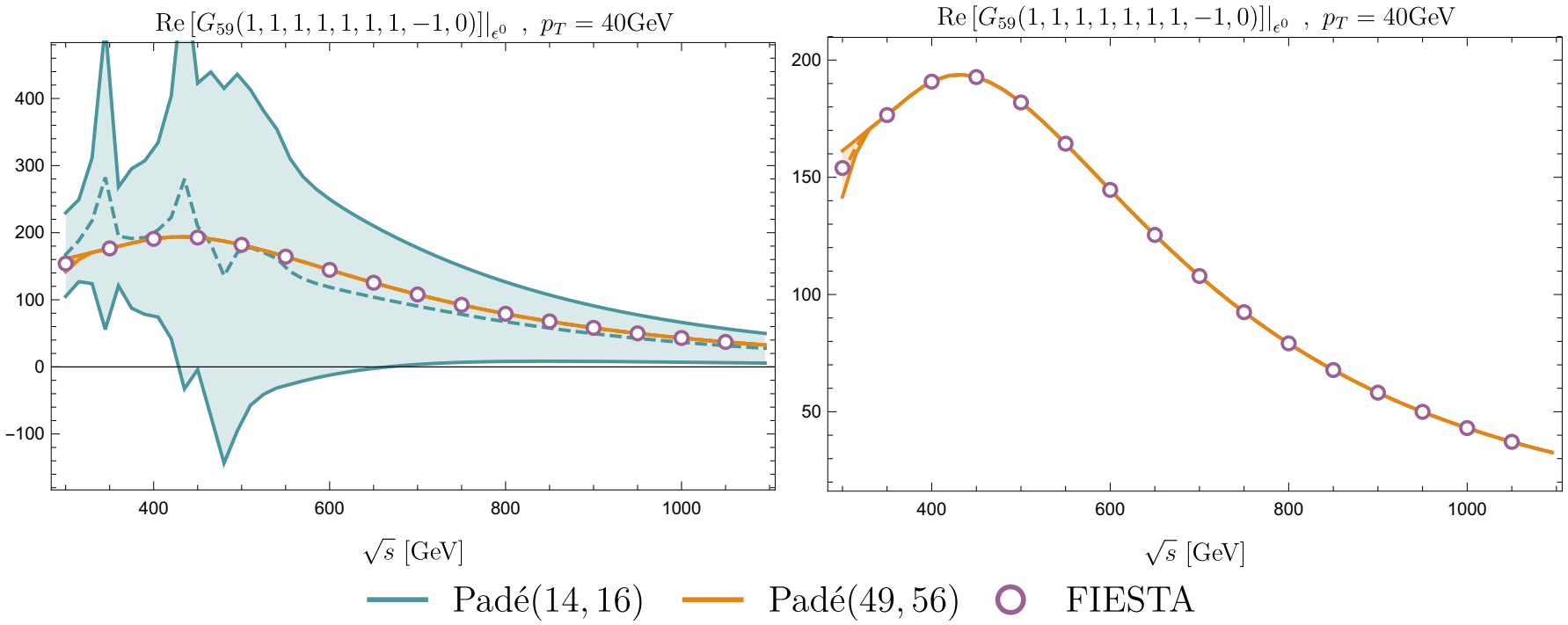
PA is a precision tool



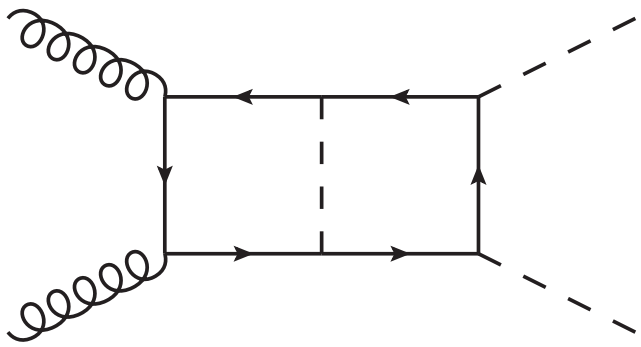


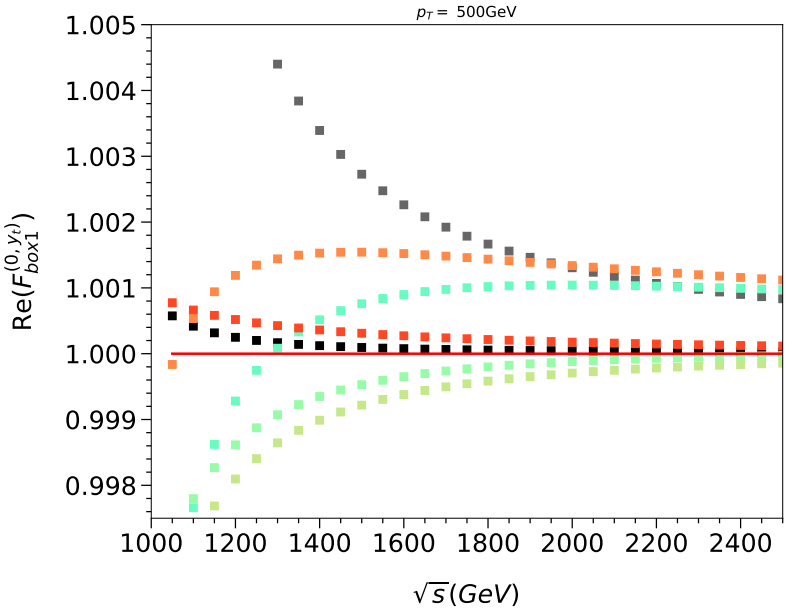
High energy expansion for $gg \to HH$

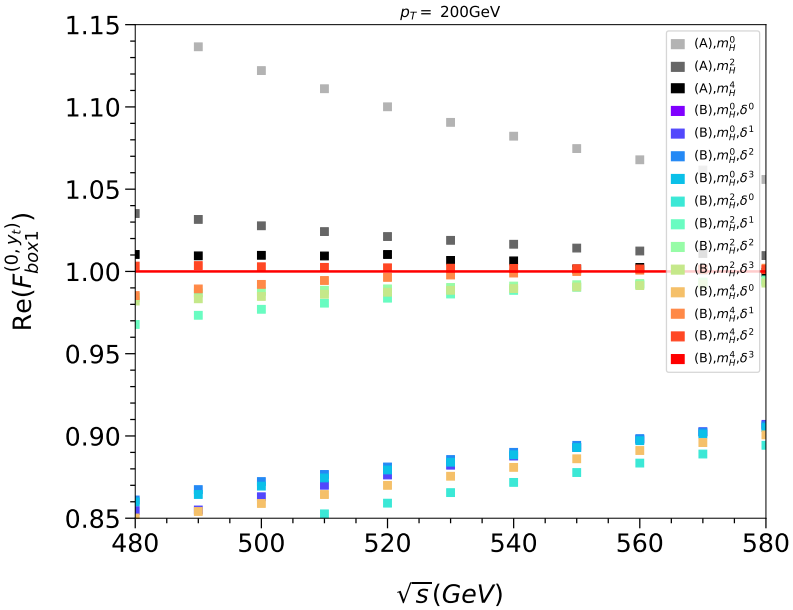
. . . electroweak correction: H exchange

- A: $s, t \gg m_t^2 \gg (m_H^{\rm int})^2, (m_H^{\rm ext})^2$
- B: $s, t \gg m_t^2 \approx (m_H^{\rm int})^2 \gg (m_H^{\rm ext})^2$
- expand in

  $\{\delta = 1 - \frac{m_H^{\rm int}}{m_t}, \frac{m_H^{\rm ext}}{m_t}\}$ or $\{\frac{m_H^{\rm int}}{m_t}, \frac{m_H^{\rm ext}}{m_t}\}$







[Davies,Mishima,Schönwald,Steinhauser,Zhang'22]

$t \to 0$ expansion

[Bonciani,Degrassi,Giardino,Gröber'18]
[Bellafronte,Degrassi,Giardino,Gröber,Vitti'22; . . .]
[Davies,Mishima,Schönwald,Steinhauser'23]

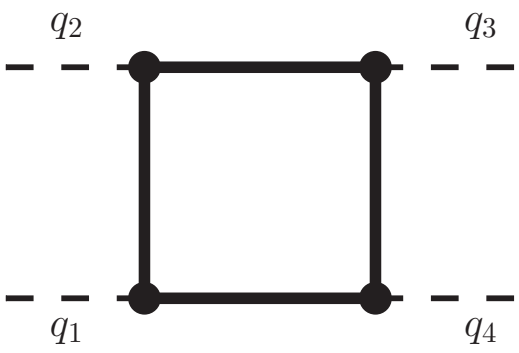
- forward scattering kinematics
- Taylor expansion

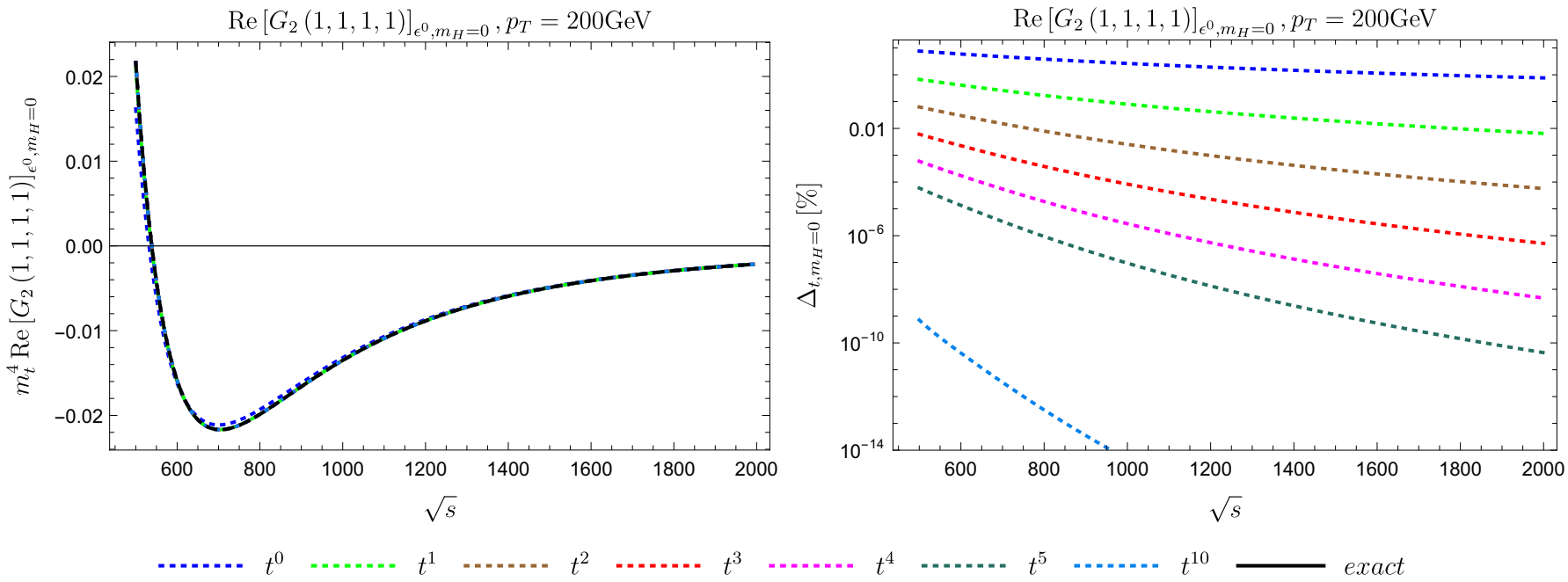
- same differential equations as for high-energy expansion $(\{s, t, m_t^2\})$
- construct for each MI expansion in t
- BC at $t = 0$: $f(s/m_t^2)$
- compute $f(s/m_t^2)$ with "expand and match" [Fael,Lange,Schönwald,Steinhauser'21'22]

  [see talks by K. Schönwald and M. Egner]

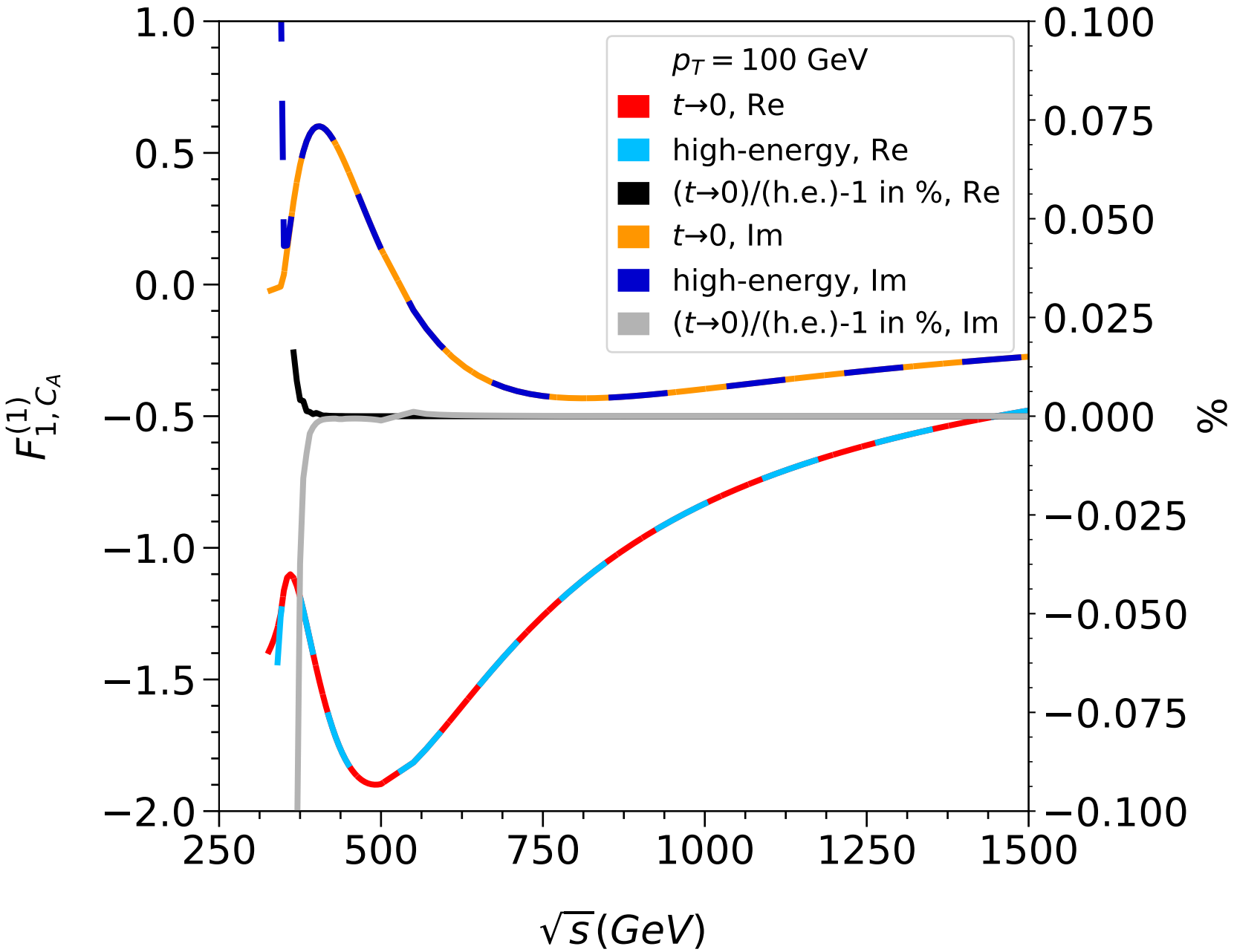
  with starting point: $m_t \to \infty$ ("simple")

$t \to 0$ at 1 loop



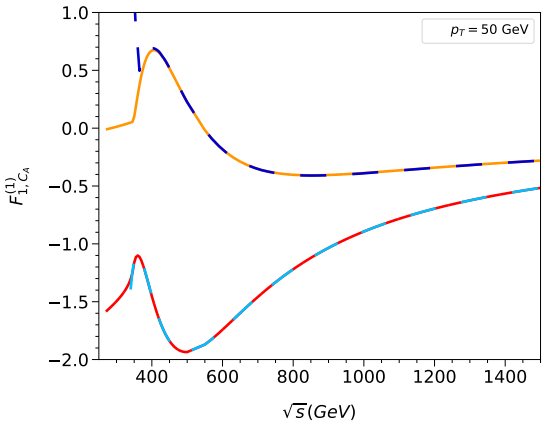


Combine: $t \to 0$ and h.e. at 2 loops

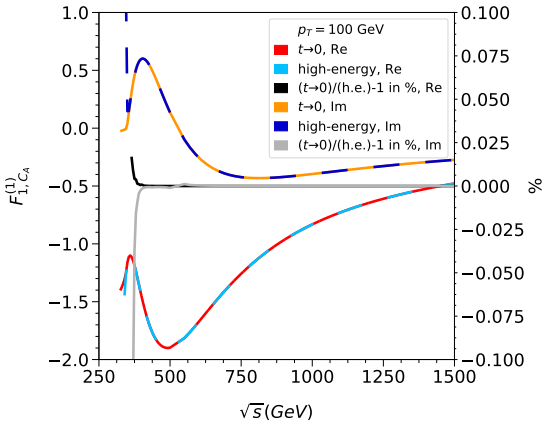


Combine: $t \to 0$ and h.e. at 2 loops

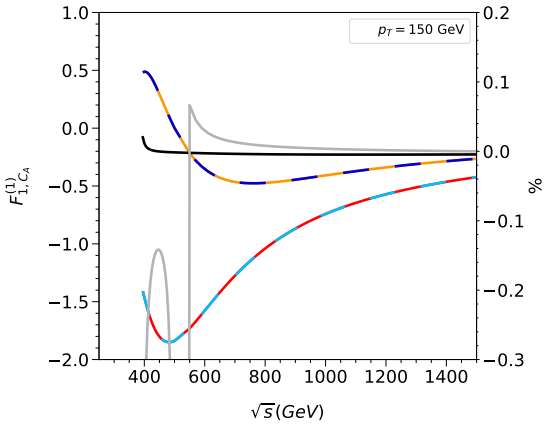
$p_T = 50$ GeV

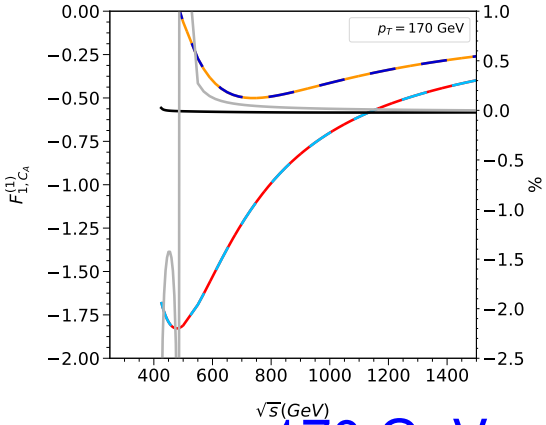


$p_T = 100$ GeV

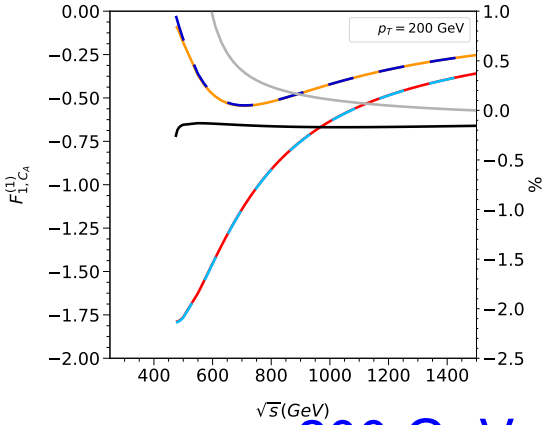


$p_T = 150$ GeV





$p_T = 170$ GeV



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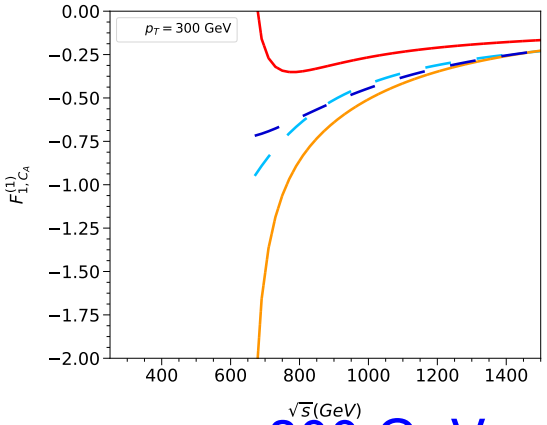
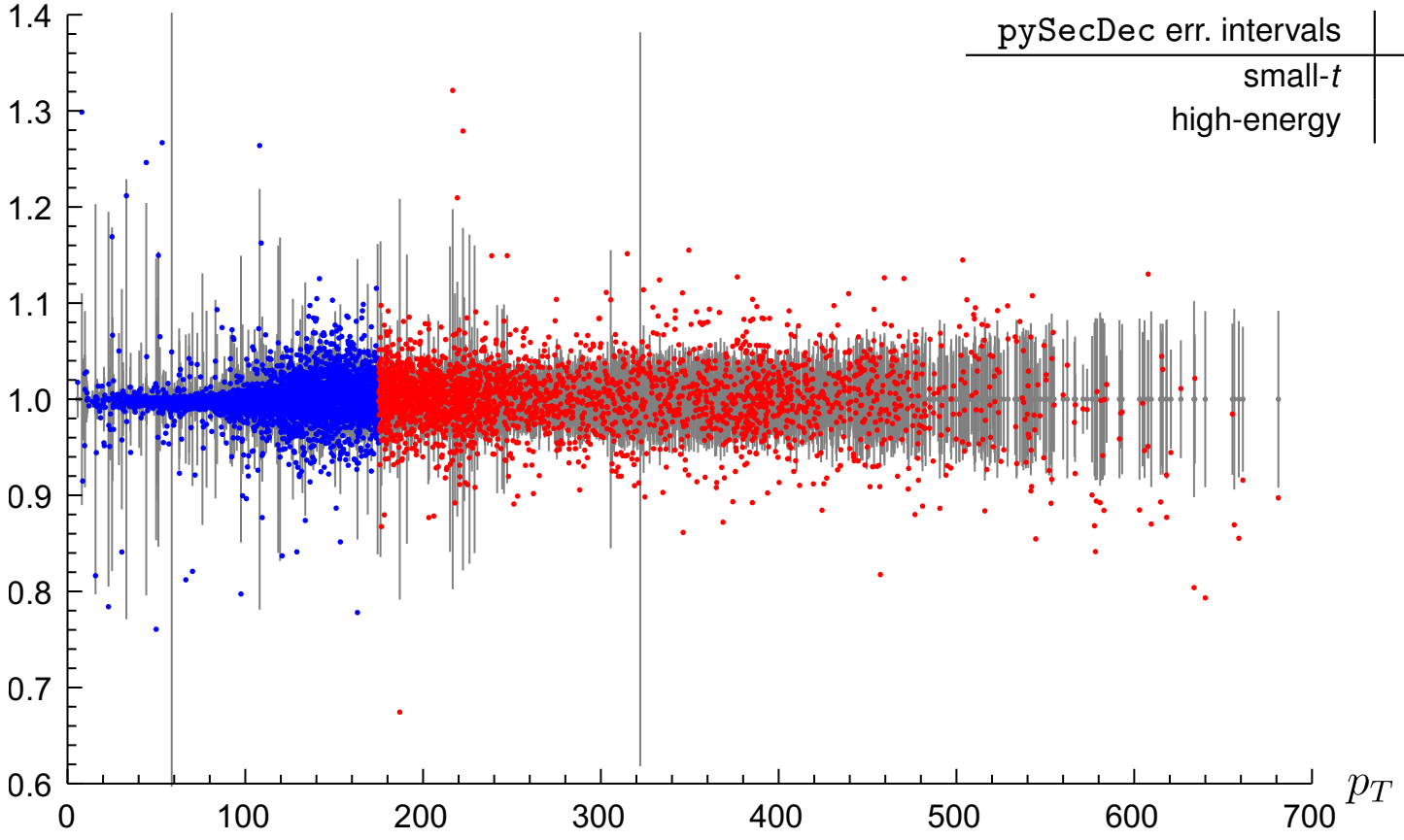


$p_T = 300$ GeV

[Davies,Mishima,Schönwald,Steinhauser'23]

$\mathcal{V}_{\text{fin}}$



pySecDec err. intervals	1σ	2σ	3σ
small-t	0.57	0.85	0.92
high-energy	0.65	0.94	0.99

https://github.com/mppmu/hhgrid

[Borowka,Greiner,Heinrich,Jones,Kerner,Schlenk,Schubert,Zirke'16]

Can we go to 3 loops?

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large-mass expansion: DONE [Davies,Steinhauser'19]
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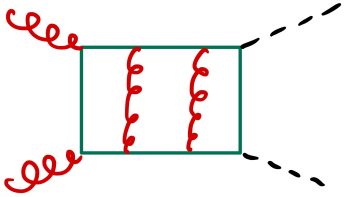
$t \to 0$: YES, if we can do the reduction for  $\{s, t, m_t^2\}$

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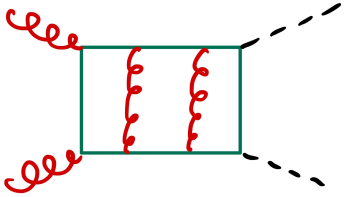
$t \to 0$: YES, if we can do the reduction for  $\{s, t, m_t^2\}$

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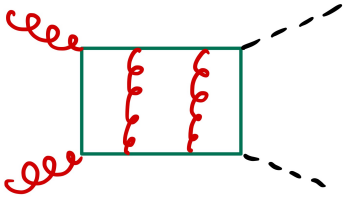
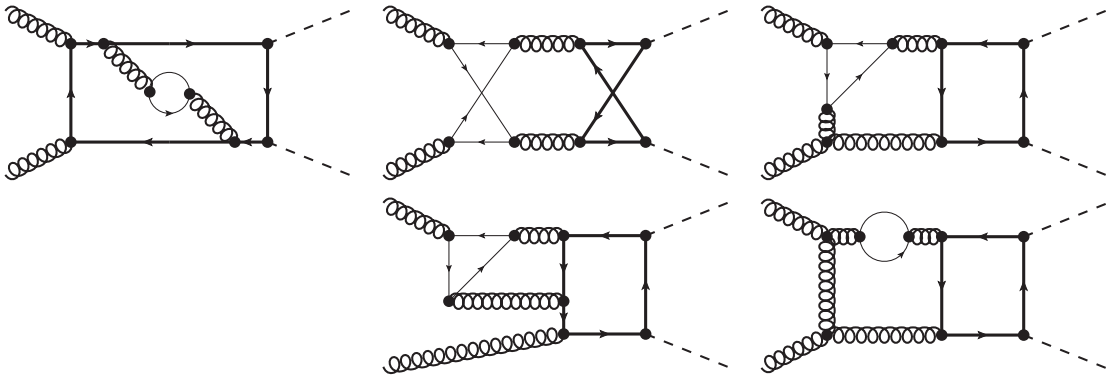
But: invert order:

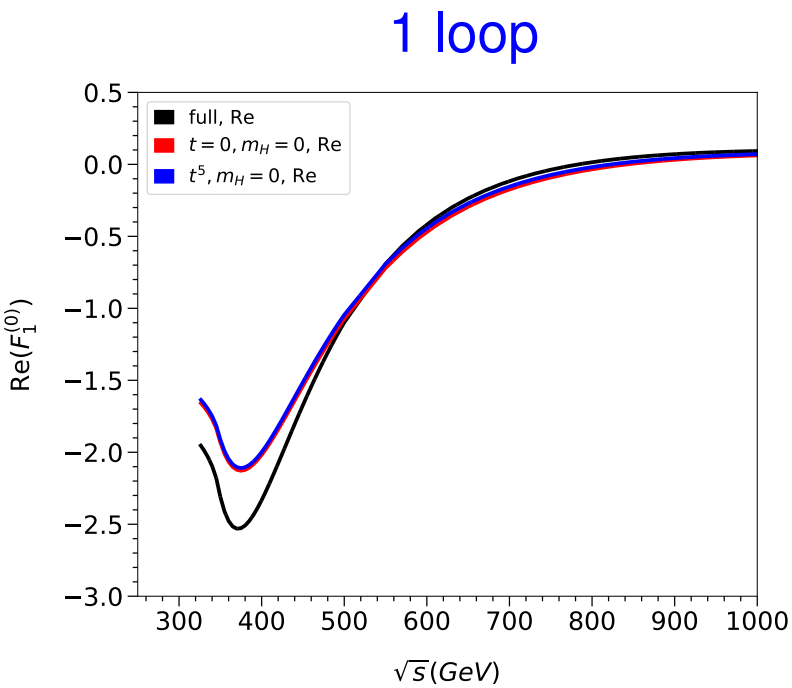
1. expand in t ➪ no t dependence
2. reduce:  $\{s, m_t^2\}$ (still challenging)

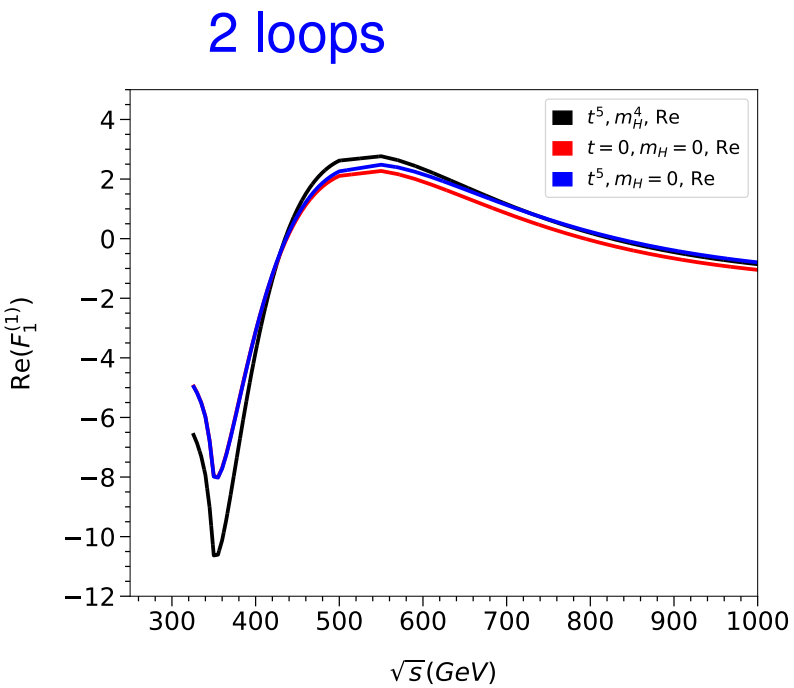
Fermionic corrections



- $t = 0$, $m_H = 0$
- 31 integral families — tapir: [Gerlach,Herren,Lang]
- 176 MIs — kira: [Klappert,Lange,Maierhöfer,Usovitsch'20]
- useful: `LiteRed` [Lee], `LIMIT` [Herren], `Feynson` [Magerya]
- reduction: about 1 week for most complicated family

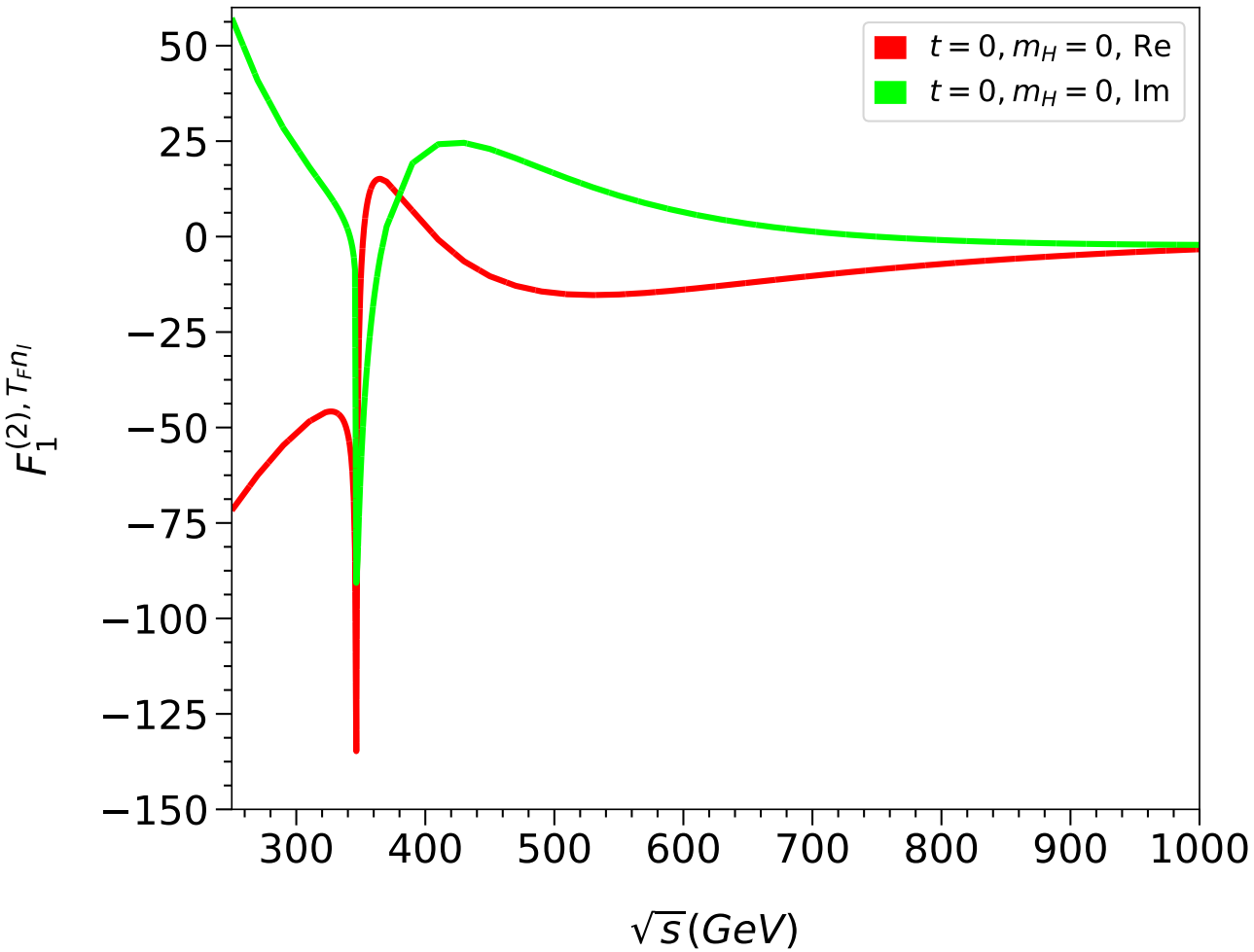
1- and 2-loop results for $t = 0$

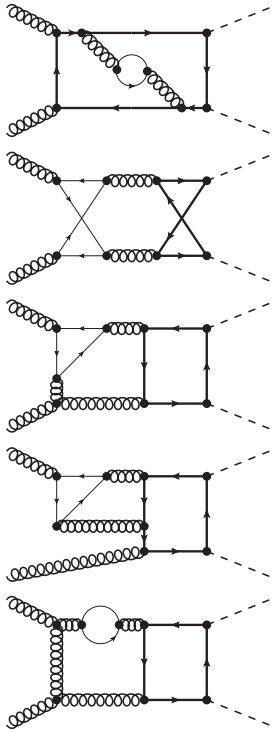




3-loop n_l for $t = 0, m_H = 0$

PRELIMINARY [Davies,Schönwald,Steinhauser]





At threshold:

$v = \sqrt{1 - 4m_t^2/s}$

NLO: $v \log v, v^2 \log v, v^3 \log^2 v, \ldots$

NNLO: $v \log^2 v, v^2 \log^2 v, v^3 \log^3 v, \ldots$

Conclusions

- Combination of expansions (large-m_t, high-energy, $t \to 0$) successful
- Do we still need numerical integration for NLO $2 \to 2$ processes?
  YES! . . . but less
- 3-loop $gg \to HH$, $gg \to ZH$, $gg \to ZZ$, . . .
  (with massive m_t, m_H, m_Z) in reach
  semi-analytic $t \to 0$ expansion