Two-loop QCD & QED corrections for light-by-light scattering

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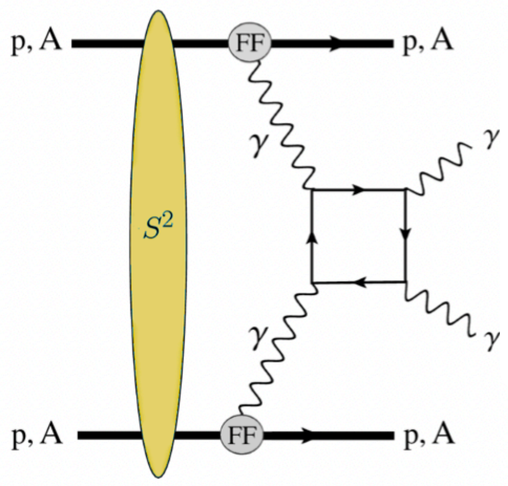
Based on work with: Ajjath AH, Hua-Sheng Shao

Physics motivation

- The LHC can accelerate not just protons but heavy ions with charges up to Z=82 for lead (Pb) ions. This enables many $\gamma\gamma$ collision measurements in ultra-peripheral pp, pn and nn collisions (UPCs).
- Light-by-light scattering one of the few photon-fusion processes observed for the first time at the LHC.

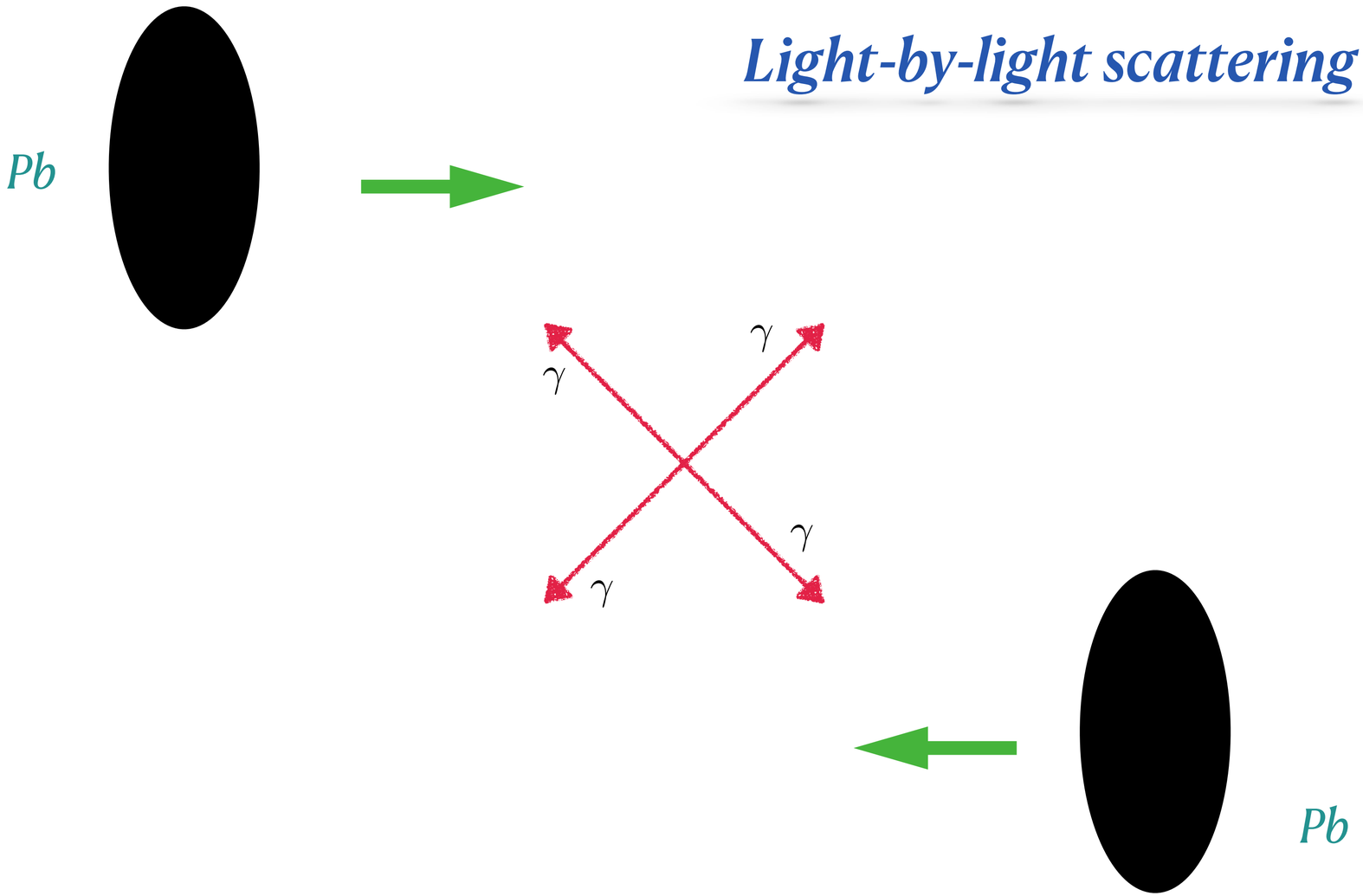


Light-by-light scattering at UPCs
[Shao, d'Enterria,`22]

[ATLAS collaboration `17,`19, `20]
[Klusek-Gawenda, Schaefer, Szczurek,`16]
[Beloborodov, Kharlamova, Telnov,`23]

Important for studies of anomalous quartic gauge couplings, axion-like particles, Born-Infeld extensions of QED or anomalous tau electromagnetic moments as well as for important SM and BSM studies.

Light-by-light scattering



Gold-plated SM and BSM processes accessible via photon-photon collisions in UPCs at hadron colliders. [Shao, d'Enterria,`22]

Process	Physics motivation
$\gamma\gamma \to e^+e^-, \mu^+\mu^-$	"Standard candles" for proton/nucleus γ fluxes, EPA calculations, and higher-order QED corrections
$\gamma\gamma \to \tau^+\tau^-$	Anomalous τ lepton e.m. moments [29–32]
$\gamma\gamma \to \gamma\gamma$	aQGC [25], ALPs [27], BI QED [28], noncommut. interactions [36], extra dims. [37],...
$\gamma\gamma \to \mathcal{T}_0$	Ditauonium properties (heaviest QED bound state) [38, 39]
$\gamma\gamma \to (c\bar{c})_{0,2}, (b\bar{b})_{0,2}$	Properties of scalar and tensor charmonia and bottomonia [40, 41]
$\gamma\gamma \to$ XYZ	Properties of spin-even XYZ heavy-quark exotic states [42]
$\gamma\gamma \to$ VM VM	(with VM = ρ, ω, ϕ, J/ψ, Υ): BFKL-Pomeron dynamics [43–46]
$\gamma\gamma \to$ W$^+$W$^-$, ZZ, Z$\gamma, \cdots$	anomalous quartic gauge couplings [11, 26, 47, 48]
$\gamma\gamma \to$ H	Higgs-γ coupling, total H width [49, 50]
$\gamma\gamma \to$ HH	Higgs potential [51], quartic $\gamma\gamma$HH coupling
$\gamma\gamma \to t\bar{t}$	anomalous top-quark e.m. couplings [11, 49]
$\gamma\gamma \to \tilde{\ell}\tilde{\ell}, \tilde{\chi}^+\tilde{\chi}^-$, H^{++}H^{--}	SUSY pairs: slepton [11, 52, 53], chargino [11, 54], doubly-charged Higgs bosons [11, 55].
$\gamma\gamma \to a, \phi, \mathcal{MM}, G$	ALPs [27, 56], radions [57], monopoles [58–61], gravitons [62–64],...

Gamma UPC

- Scale we are probing is 5 GeV (tau, lepton, bottom should be massive)
- Observed recently [ATLAS collaboration `17,`19, `20]
- Gamma_UPC has been integrated into automated event generators Madgraph5_aMC@NLO for NLO

C. Light-by-light scattering in Pb-Pb UPCs at $\sqrt{s_{NN}}$ = 5.02 TeV

The loop-induced LbL signal is generated with gamma-UPC plus MadGraph5_aMC@NLO v2.6.6 [75, 138] with the virtual box contributions computed at leading order. Table XIV compares the integrated fiducial cross sections measured by ATLAS [15] with the gamma-UPC using EDFF and ChFF γ fluxes and the Superchic predictions. The measured cross section is about 2 standard deviations above the gamma-UPC and Superchic predictions.

TABLE XIV: Fiducial light-by-light cross sections measured in Pb-Pb UPCs at $\sqrt{s_{NN}}$ = 5.02 TeV (with $E_T^\gamma > 2.5$ GeV , $|\eta^\gamma| < 2.4$, $m_{\gamma\gamma} > 5$ GeV, $p_{T,\gamma\gamma} < 1$ GeV), compared to the theoretical gamma-UPC results obtained with EDFF and ChFF γ fluxes (and their average), as well as with the Superchic MC prediction.

Process, system	ATLAS data [15]	gamma-UPC σ			Superchic σ
		EDFF	ChFF	average	
$\gamma\gamma \to \gamma\gamma$, Pb-Pb at 5.02 TeV	120 ± 22 nb	63 nb	76 nb	70 ± 7 nb	78 ± 8 nb

Cross-sections @NLO

Effective two-photon luminosity

$$\sigma(A\ B \xrightarrow{\gamma\gamma} A\ X\ B) = \int \frac{dE_{\gamma_1}}{E_{\gamma_1}} \frac{dE_{\gamma_2}}{E_{\gamma_2}} \frac{d^2 N^{(AB)}_{\gamma_1/Z_1,\gamma_2/Z_2}}{dE_{\gamma_1} dE_{\gamma_2}} \sigma_{\gamma\gamma\to X}(W_{\gamma\gamma})$$

$|\mathcal{M}|^2$

- Loop-induced in SM $\longrightarrow$ pure quantum effect!
- 2-loop QCD & QED corrections in the ultra-relativistic limit (s,t,u>> m^2, massless internal lines) [Bern, De Freitas, Dixon, Ghinculov & Wong, 2001]
- Two-loop corrections to light-by-light scattering in supersymmetric QED [Binoth, Glover, Marquard, & van Der Bij, 2002]
- Two-loop integrals for light-by-light scattering including massive loop corrections studied in [Caron-huot & Henn, 2014] (presence of square-roots)
- Monte carlo framework ready, needs the cross sections [Shao, d'Enterria,`22]

Amplitude computation

$$\gamma(p_1,\lambda_1)+\gamma(p_2,\lambda_2)+\gamma(p_3,\lambda_3)+\gamma(p_4,\lambda_4)\to 0$$

$$\mathcal{M}=\varepsilon_{1,\mu_1}\varepsilon_{2,\mu_2}\varepsilon_{3,\mu_3}\varepsilon_{4,\mu_4}\mathcal{M}^{\mu_1\mu_2\mu_3\mu_4}(p_1,p_2,p_3,p_4)$$

$$\begin{aligned}\mathcal{M}^{\mu_1\mu_2\mu_3\mu_4} =& A_1 g^{\mu_1\mu_2}g^{\mu_3\mu_4}+A_2 g^{\mu_1\mu_3}g^{\mu_2\mu_4}+A_3 g^{\mu_1\mu_4}g^{\mu_2\mu_3}+\sum_{j_1,j_2=1}^{3}\big(B^1_{j_1j_2}g^{\mu_1\mu_2}p^{\mu_3}_{j_1}p^{\mu_4}_{j_2}\\ &+B^2 j_1j_2 g^{\mu_1\mu_3}p^{\mu_2}_{j_1}p^{\mu_4}_{j_2}+B^3 j_1j_2 g^{\mu_1\mu_4}p^{\mu_2}_{j_1}p^{\mu_3}_{j_2}+B^4 j_1j_2 g^{\mu_2\mu_3}p^{\mu_2}_{j_1}p^{\mu_4}_{j_2}\\ &+B^5 j_1j_2 g^{\mu_2\mu_4}p^{\mu_1}_{j_1}p^{\mu_3}_{j_2}+B^6 j_1j_2 g^{\mu_3\mu_4}p^{\mu_1}_{j_1}p^{\mu_2}_{j_2}\big)\\ &+\sum_{j_1,j_2,j_3,j_4=1}^{3}C_{j_1j_2j_3j_4}p^{\mu_1}_{j_1}p^{\mu_2}_{j_2}p^{\mu_3}_{j_3}p^{\mu_4}_{j_4}.\end{aligned}$$

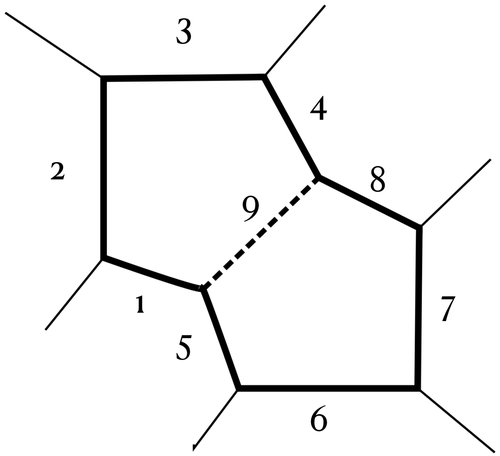
$\varepsilon_j\cdot p_j=0$
Bose symmetry
Gauge symmetry

Number of independent functions

$A_1(s,t,u)$ $\quad B^1_{11}(s,t,u)$ $\quad C_{2111}(s,t,u)$

[Binoth, Glover, Marquard, & van Der Bij, 2002]

Reduction to MIs & Simplification



$$I_{a_1,\cdots,a_9} = \left(\frac{e^{\epsilon\gamma_E}}{i\pi^{\frac{d}{2}}}\right)^2 \int \prod_{i=1}^{2} d^d k_i \frac{D_4^{a_4} D_6^{a_6}}{D_1^{a_1} D_2^{a_2} D_3^{a_3} D_5^{a_5} D_7^{a_7} D_8^{a_8} D_9^{a_9}} ,$$

$$k_1^2 - m_t^2, (k_1+p_1)^2 - m_t^2, (k_1+p_1+p_2)^2 - m_t^2, (k_1+p_1+p_2+p_3)^2 - m_t^2,$$
$$(k_2)^2 - m_t^2, (k_2+p_1)^2 - m_t^2, (k_2+p_1+p_2)^2 - m_t^2, (k_2+p_1+p_2+p_3)^2 - m_t^2,$$
$$(k_2-k_1)^2 \quad .$$

60 diagrams in total

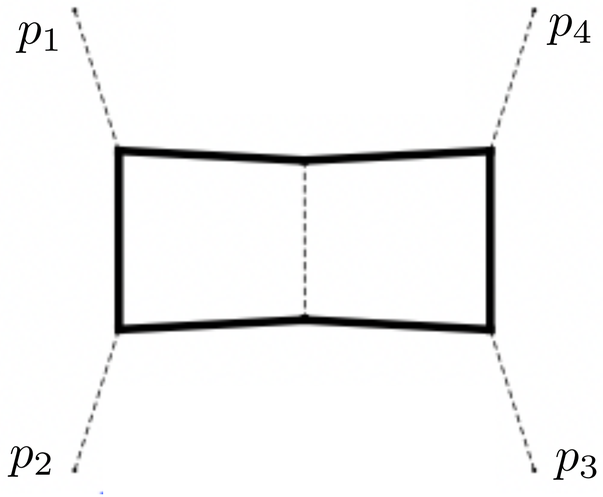
7798 integrals before IBP

18 top-level sectors

Can be mapped into the 2-loop diagram shown on the right

LiteRed (FiniteFlow), KIRA

[Lee, `13] [Peraro, `19] [Klappert, Lange, Maierhöfer, Usovitsch, `20]



Analytic computation of the MIs

- 29 MIs; use of differential equations;
- Choice of a canonical basis [Caron-huot, Henn, 14] J. Henn's talk

$$\partial_s \vec{f} = \epsilon A_s \vec{f}$$
$$\partial_t \vec{f} = \epsilon A_t \vec{f}$$

- Square roots:

$$\sqrt{s(s-4m^2)} \quad \sqrt{t(t-4m^2)} \quad \sqrt{st(st-4m^2(s+t))} \quad \sqrt{s(m^4 s - 2m^2 t(s+2t) + st^2)}$$

- Choice of variables

$$s = -\frac{4(w-z)^2}{(1-w^2)(1-z^2)} \qquad t = -\frac{(w-z)^2}{wz} \qquad m^2 = 1$$

$$\sqrt{-2wz + z^2 + w^4 z^2 - 2w^3 z^3 + w^2(1+z^2+z^4)}$$ *Non-rationalizable??..*

Analytic representation

Chen's iterated integrals

Let $\mathcal{M}$ be the manifold, and $\omega's$ be the one-forms.

The pull-back of the differential one-forms to the unit interval [0,1] $\gamma^*(\omega_i) = f_i(t)dt$

Iterated integrals of $\omega's$ along γ is defined by

$$\int_\gamma I(\omega_1 ... \omega_n; \lambda) = \int_0^\lambda d\lambda_1 f_1(\lambda_1) \int_0^{\lambda_1} d\lambda_2 f_2(\lambda_2) ... \int_0^{\lambda_{k-1}} d\lambda_k f_k(\lambda_k)$$

S. Weinzierl's, D. Chicherin,'s S. Zoia's talk

Goncharov's Polylogarithms

$$G(z_1, z_2, ..., z_k; y) = \int_0^y \frac{dy_1}{y_1 - z_1} G(z_2, ... z_k; y_1)$$

Boundary condition:

$f_i(s = 0, t = 0; m^2) = \delta_{i,1}$

- Solutions in terms of Chen's iterated integrals
- One-forms: rational, with square root
- Numerical evaluation using G and one-fold integrations

The alphabet

$$sq = \sqrt{-2wz + z^2 + w^4z^2 - 2w^3z^3 + w^2(1 + z^2 + z^4)}$$ [Caron-huot, Henn, 14]

$$1 - w, 1 + w, 1 - wz, w - z, w, w + z, 1 + w - z + wz, 1 - w + z + wz, 1 + wz, 1 - z, 1 + z,$$
$$z, \frac{-sq + w - z - wz + w^2z - wz^2}{sq + w - z - wz + w^2z - wz^2}, \frac{-sq + w^2 - 3wz + z^2}{sq + w^2 - 3wz + z^2}, \frac{-1 - sq + w^2z - wz^2}{-1 + sq + w^2z - wz^2},$$
$$\frac{1 - sq + w^2z - wz^2}{1 + sq + w^2z - wz^2}$$

6 master integrals containing all 4 square roots in the integrand at weight-4.

We keep the analytic result in terms of iterated integrals with dog one-form derived using [Heller, von Manteuffel, Schabinger, `20]

We convert all the integral in terms of 1-dimensional integrations [Caron-huot, Henn, 14] [Chicherin, Sotnikov `21] [Chicherin, Sotnikov, Zoia `22]

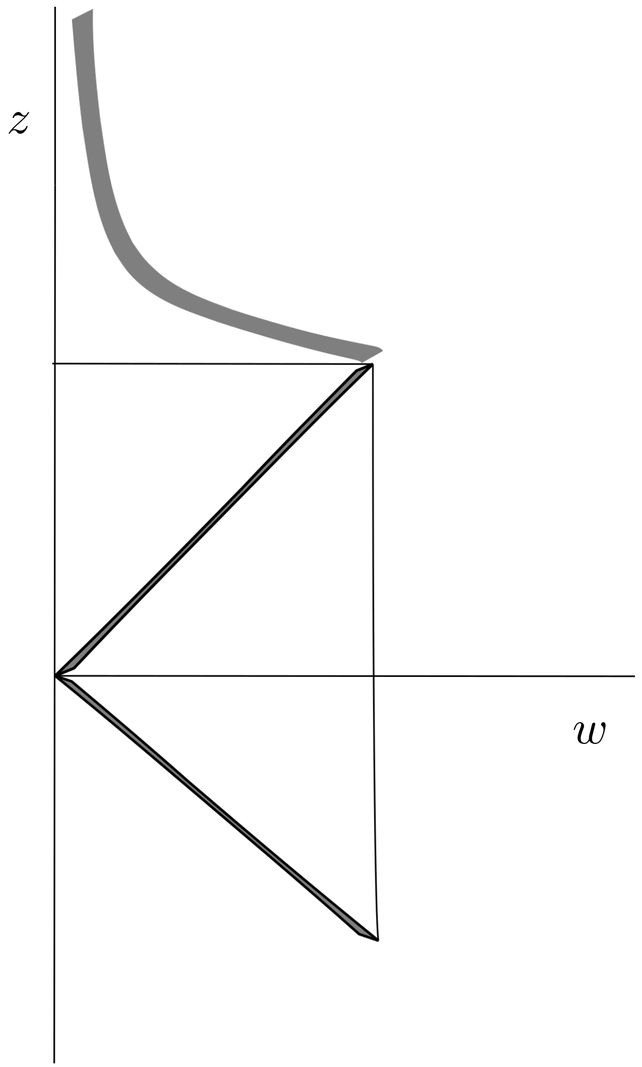
We also express the first two orders in terms of logs and classical polylogs by matching symbols [Duhr, Gangl, Rhodes, `11]

$$\int_\gamma I(\omega_1 ... \omega_4; \lambda) = \int_0^\lambda d\lambda_1 f_1(\lambda_1) \int_0^{\lambda_1} d\lambda_2 f_2(\lambda_2) \underbrace{\int_0^{\lambda_2} d\lambda_3 f_3(\lambda_3) \int_0^{\lambda_3} d\lambda_4 f_4(\lambda_4)}_{\{\mathrm{Log}^2(z), \mathrm{Li}_2(z)\}}$$

Numerical evaluation

Physical phase-space regions of interest

$$0 < w < 1 \;\; \& \;\; (0 < z < w \mid w < z < 1)$$
$$0 < w < 1 \;\; \& \;\; (1 < z < \frac{1}{w} \mid z > \frac{1}{w})$$
$$0 < w < 1 \;\; \& \;\; (-1 < z < -w \mid -w < z < 0)$$



We obtain different analytic representations of the results valid in different regions

Conclusions

- We are aiming for a completely analytic representation for the squared matrix element at 2-loop; checks and comparisons are in progress.
- NLO cross section with massive contributions for light-by-light scattering at UPC within reach!

- Joining forces to compute the NNLO corrections by computing the 3loop corrections using local unitarity methods (Z. Capatti's talk)