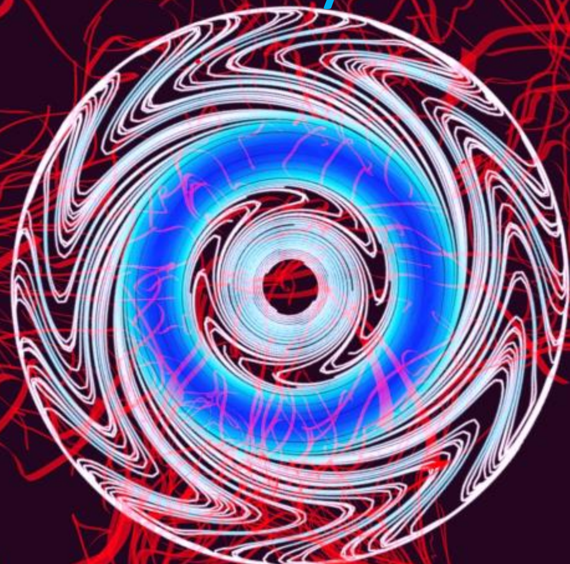
Beyond chaos: new directions in the statistical mechanics of turbulence

Nigel Goldenfeld, University of California San Diego



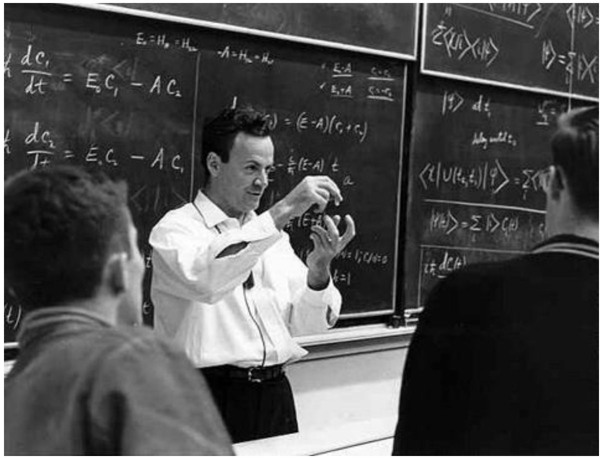
Transition to turbulence: Hong-Yan Shih, Tsung-Lin Hsieh, Xueying Wang, Bjorn Hof, Gregoire Lemoult, Mukund Vasudevan, Jose M. Lopez, Gaute Linga, Joachim Mathiesen

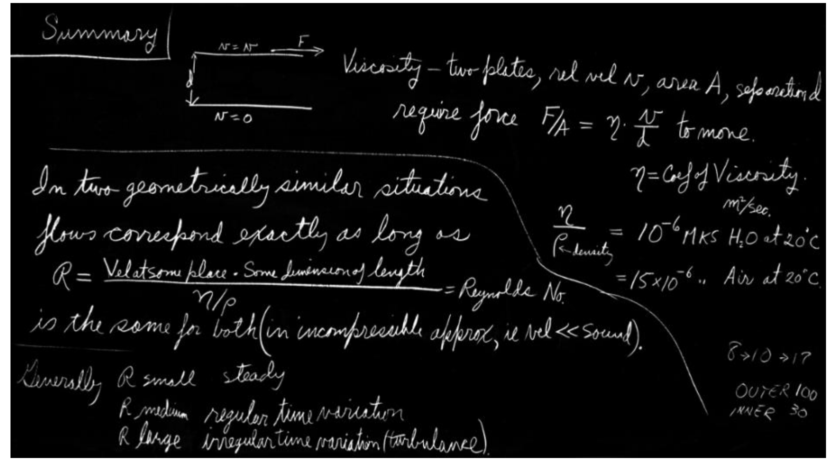
Spontaneous stochasticity: Greg Eyink, Dmytro Bandak, Alexei Mailybaev

Supported by Simons Foundation Grants: 663054 (G.E.), 62960 (BH.) & 662985(N.G.)

Propaganda

Feynman's vision: RG & Turbulence

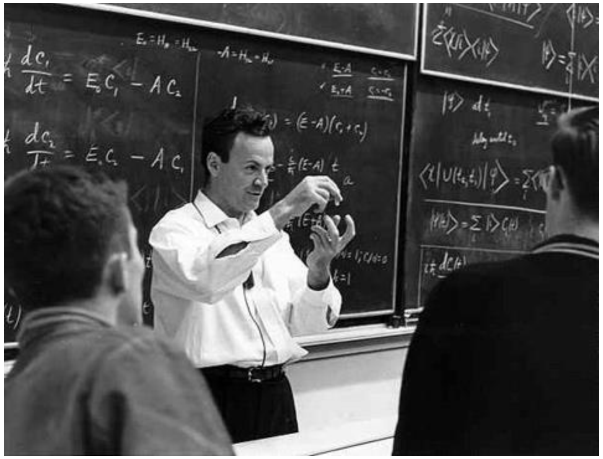


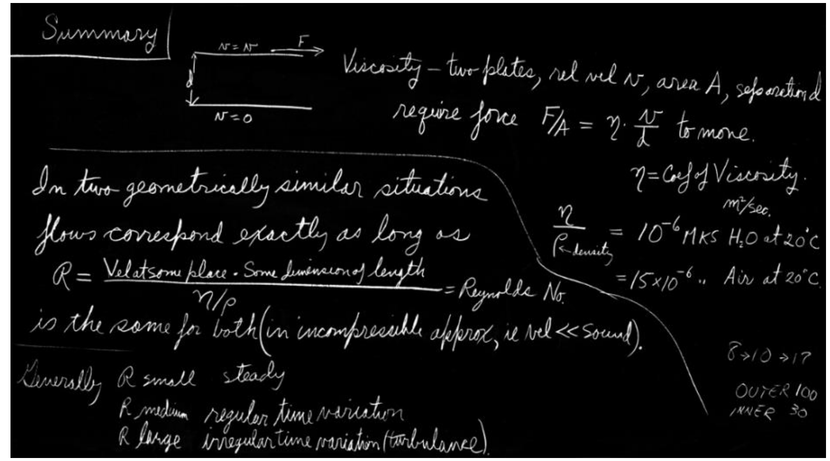


We have written the equations of water flow. From experiment, we find a set of concepts and approximations to use to discuss the solution—vortex streets, turbulent wakes, boundary layers. When we have similar equations in a less familiar situation, and one for which we cannot yet experiment, we try to solve the equations in a primitive, halting, and confused way to try to determine what new ***qualitative*** features may come out, or what new ***qualitative*** forms are a consequence of the equations.

http://www.feynmanlectures.caltech.edu/II_41.html Scientific American

Feynman’s vision: RG & Turbulence





The next great era of awakening of human intellect may well produce a method of understanding the ***qualitative*** content of equations. Today we cannot. Today we cannot see that the water flow equations contain such things as the barber pole structure of turbulence that one sees between rotating cylinders. Today we cannot see whether Schrödinger’s equation contains frogs, musical composers, or morality—or whether it does not.

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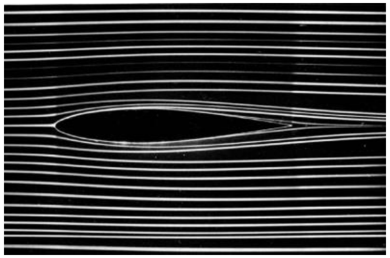
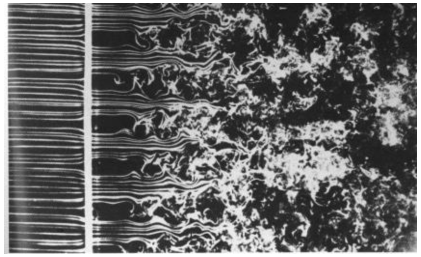
Goal

- The qualitative behavior of matter is the domain of statistical mechanics
- **Our goal: expose the qualitative content of the equations of fluid mechanics, from a statistical mechanical perspective.**
- A start: novel predictions and perspectives based on statistical mechanics.
  - Transitional flows
  - Fluctuation-dissipation relations
  - Thermal fluctuations and spontaneous stochasticity

Phase diagram of turbulence





laminar flow

steady

predictable

laminar-turbulent transition

critical behavior

fully-developed turbulence

fluctuating

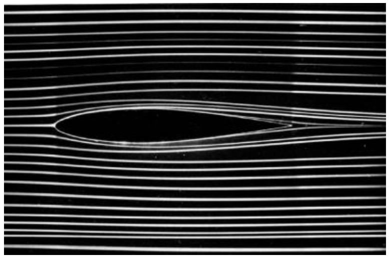
unpredictable

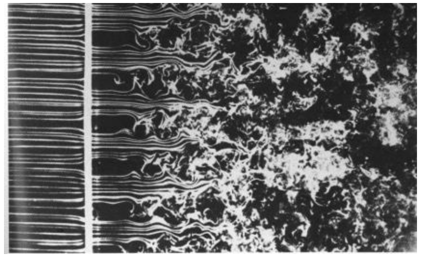
Re < 1600

Re ~ 2000

Re > 10^5

What is turbulence?





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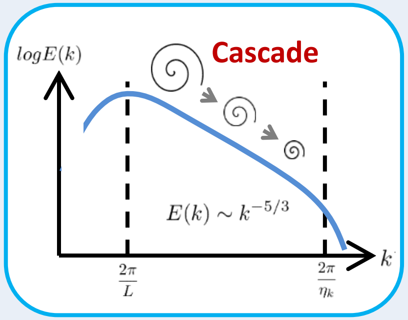
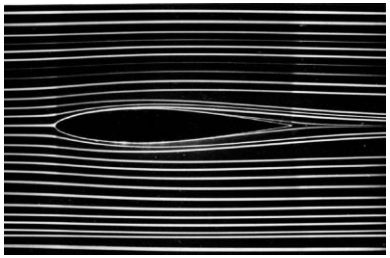
Fully-developed turbulence

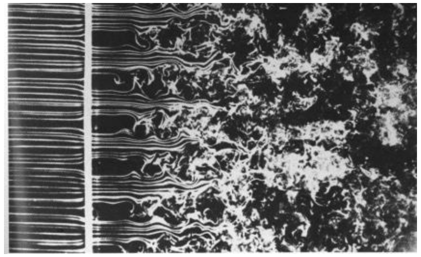
- **Energy cascade: Non-equilibrium steady state**
  Energy transfers step by step → power law
- **Dissipative anomaly:**
  energy dissipation rate = 0 when viscosity = 0
  energy dissipation rate ≠ 0 when viscosity → 0

$$\epsilon = \nu |\nabla u|^2 \neq 0 \text{ as } \nu \to 0$$



Phase diagram of turbulence





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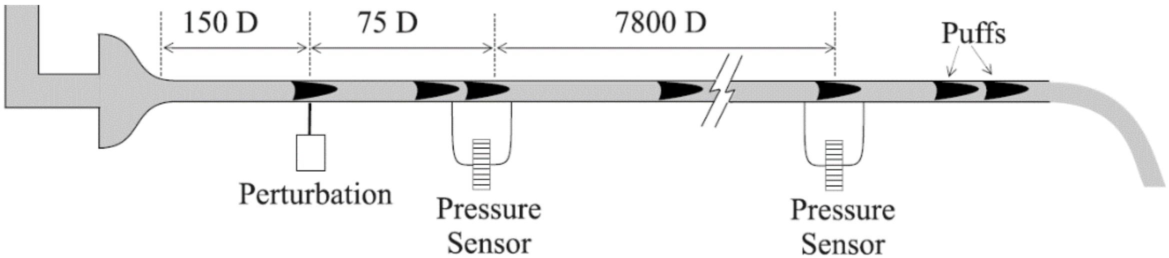
- Boundary conditions <u>not</u> periodic
- Walls need to be included
- Turbulence generated by instabilities
  - Linear and long-wavelength
  - Nonlinear and spatially-localized
  - Not artificial noise
- Turbulence interacts with mean flow
  - Turbulence can generate emergent mean flows

Phase diagram of pipe turbulence



Masudevan & Hof (2018)

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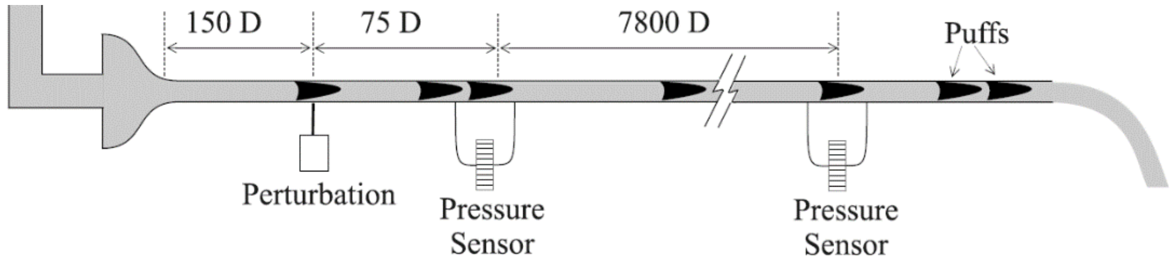

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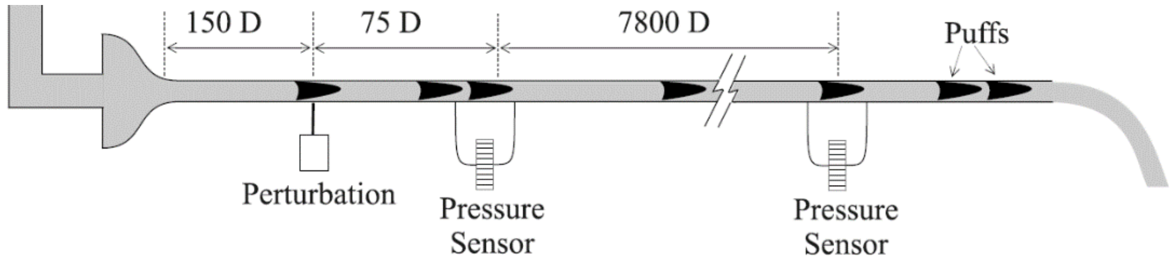

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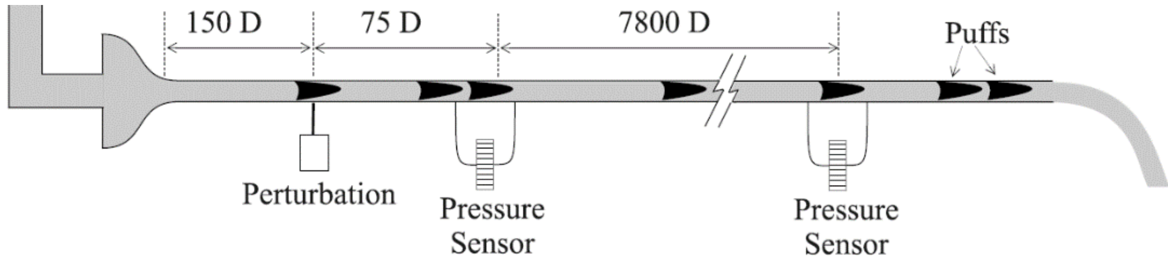
Directed percolation

Phase diagram of pipe turbulence



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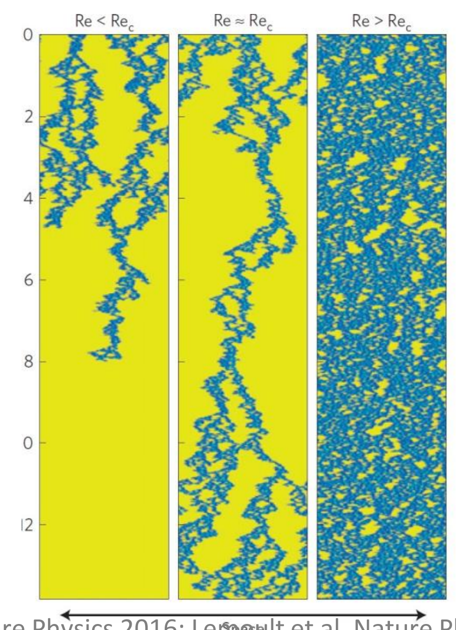
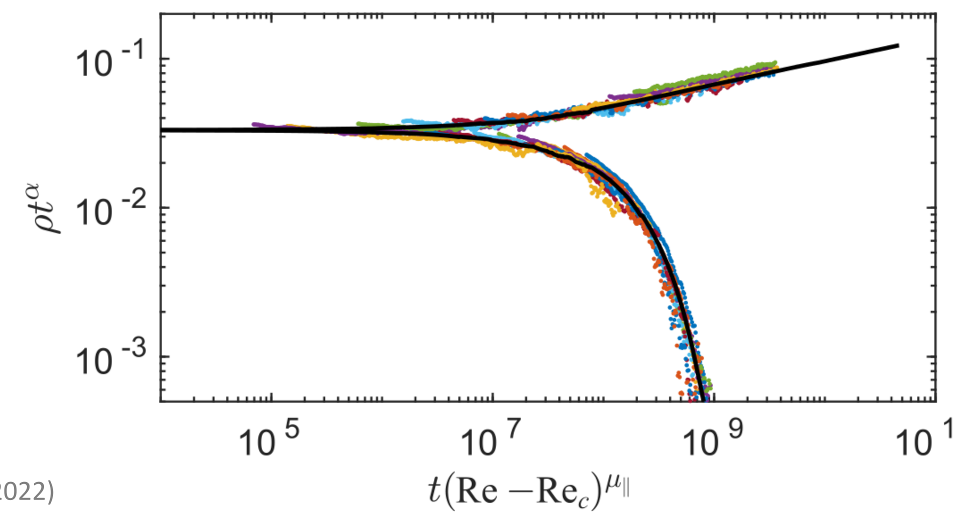
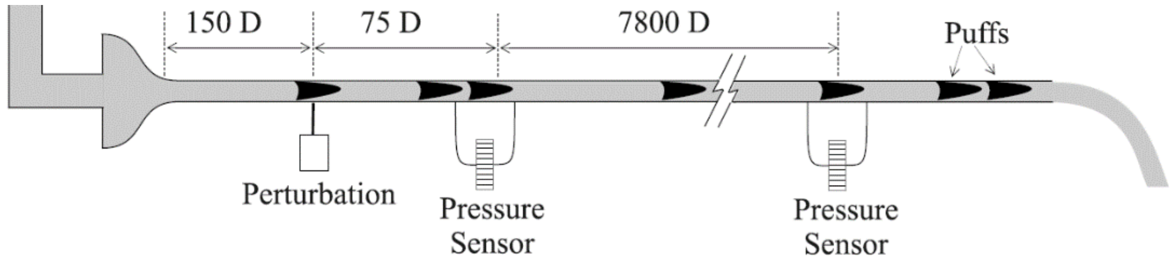


Directed percolation



Lemoult et al, Nature Physics 2016; Lemoult et al. Nature Physics (in review, 2022)

Phase diagram of pipe turbulence



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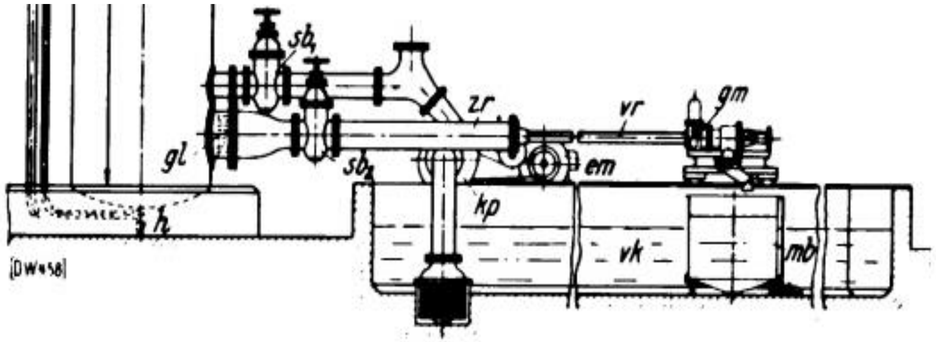
Directed percolation

Critical fixed point controls drag and relates it to spectral properties of flow

Phase diagram of pipe turbulence



Nikuradse 1933

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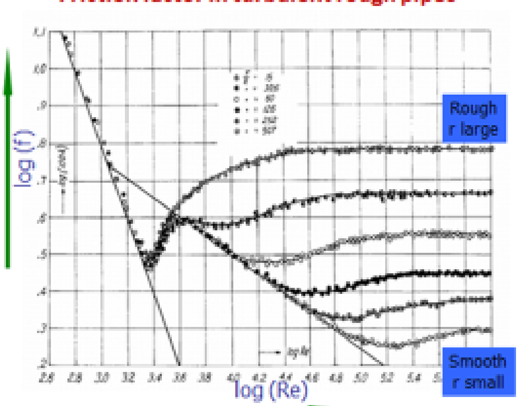
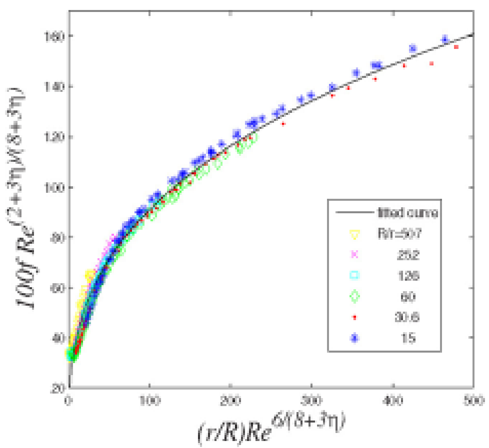
Friction factor in turbulent rough pipes





Critical fixed point controls drag and relates it to spectral properties of flow

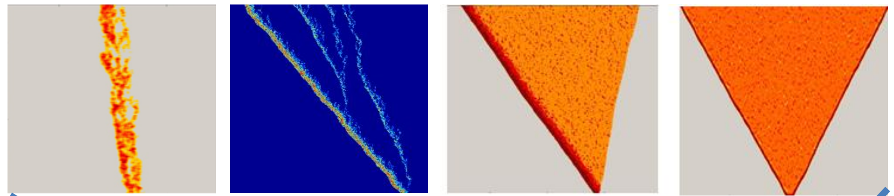
$$f = \mathrm{Re}^{-(2+3\eta)/(8+3\eta)}\, g\left(\frac{r}{R}\,\mathrm{Re}^{6/(8+3\eta)}\right)$$

Intermittency corrections included
Value of η ~ 0.02 consistent with spectral estimates

Goldenfeld, Phys. Rev. Lett. (2006); Mehrafarin & Pourtolami, Phys. Rev. E. (2008)

Phase diagram of pipe turbulence

Wang, Shih, NG PRL 2022



laminar flow

laminar-turbulent transition

fully-developed turbulence

Puffs and Slugs

Re < 1600

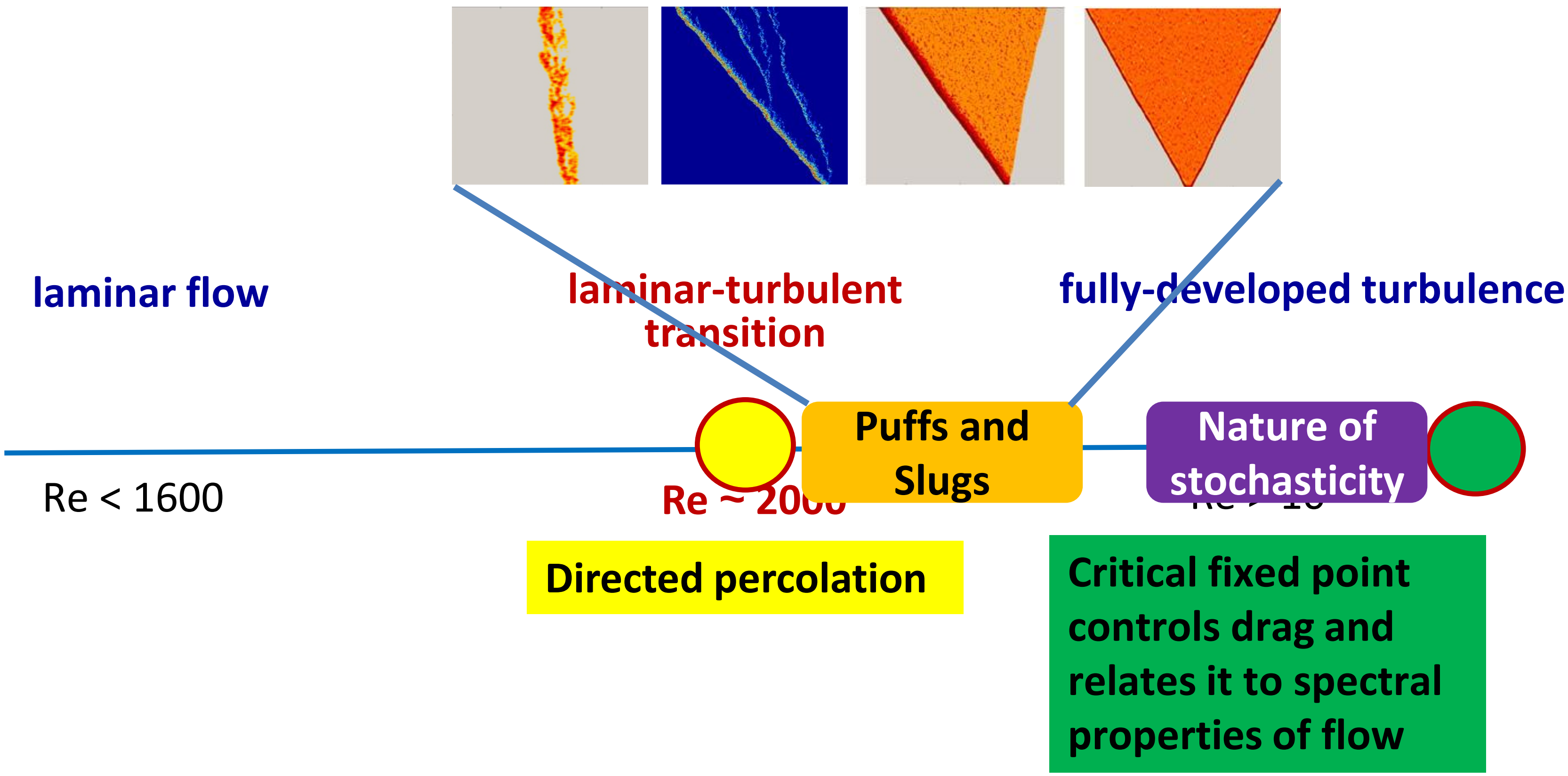
Re ~ 2000

Re > 10^5

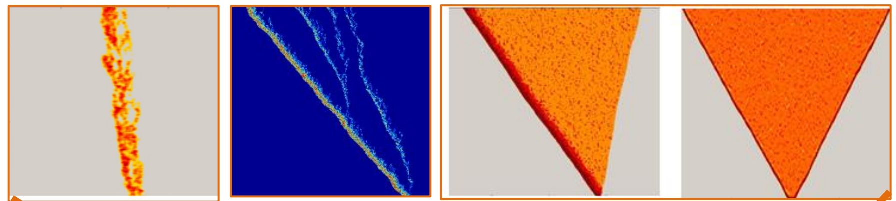
Directed percolation

Critical fixed point controls drag and relates it to spectral properties of flow

Phase diagram of pipe turbulence



Phase diagram of turbulence



laminar flow | **laminar-turbulent transition** | **fully-developed turbulence**

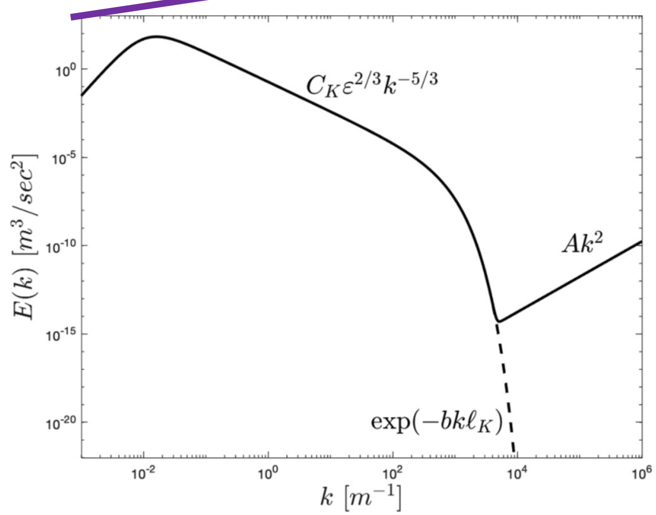
Re < 1600

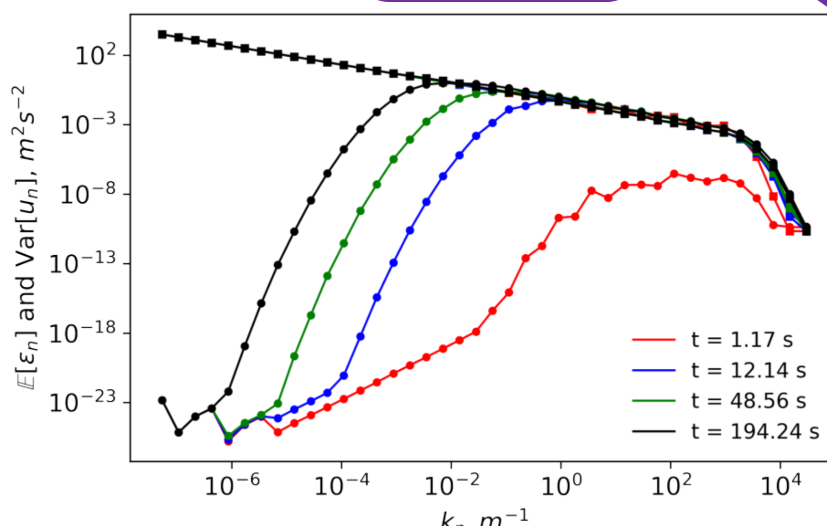
Re ~ 2000

Puffs and Slugs

Thermally-triggered stochastic wave

Directed percolation





Bandak et al., PRE (2022); Bandak et al. (in review, PNAS, 2022)

Outline

1. Laminar-turbulence transition
   – Puff decay and splitting lifetimes
   – Minimal stochastic model
   – Directed percolation predictions
   – Directed percolation experimental confirmation
2. Beyond the critical point
   – Growth of turbulence slugs
   – Bistable fronts, structural stability and Kolmogorov-Petrovsky-Piscunov-Fisher fronts
3. Thermal fluctuations and spontaneous stochasticity
   – Length scale where thermal fluctuations are relevant
   – Thermal fluctuations trigger spontaneous stochasticity

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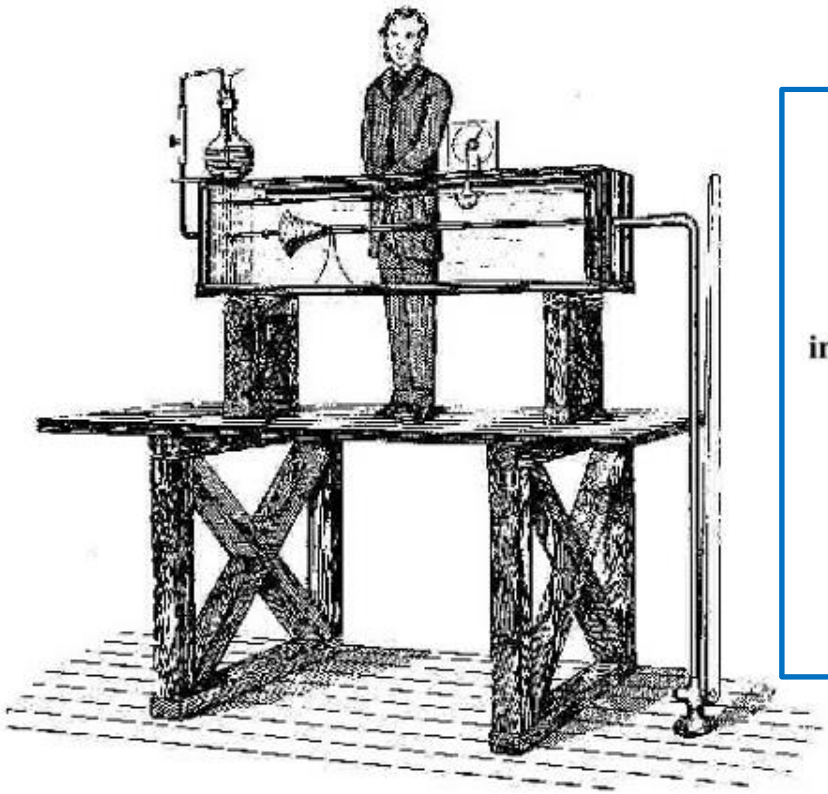
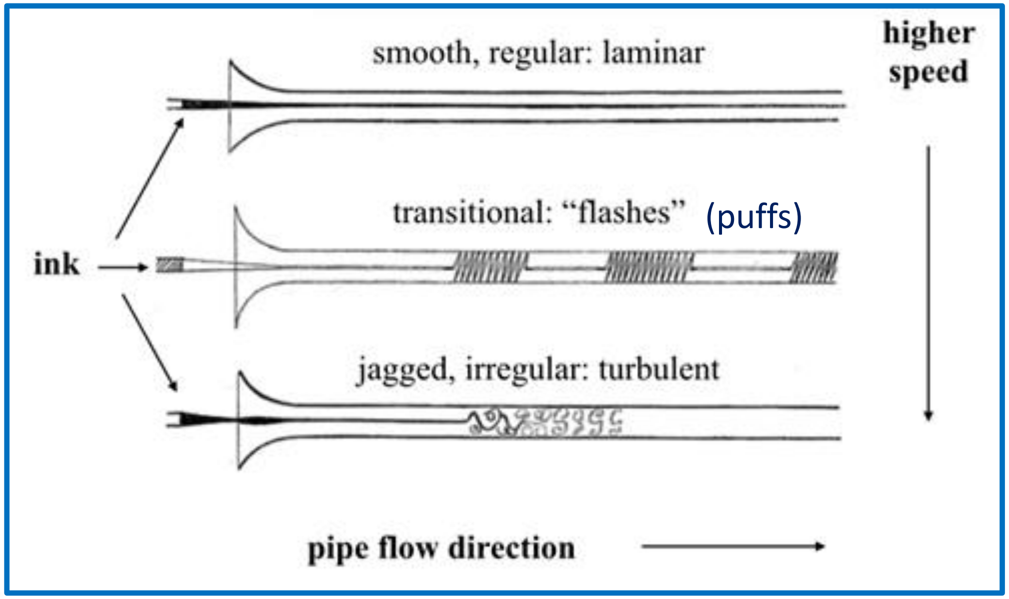
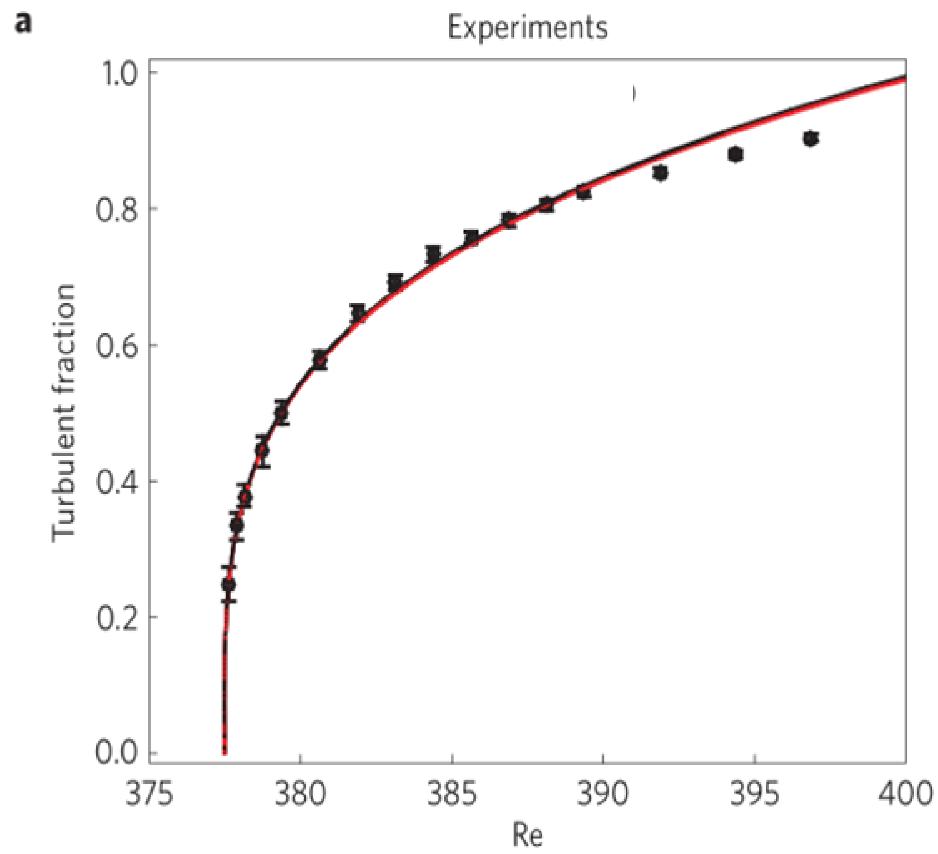
Transitional turbulence in pipe flow: puffs

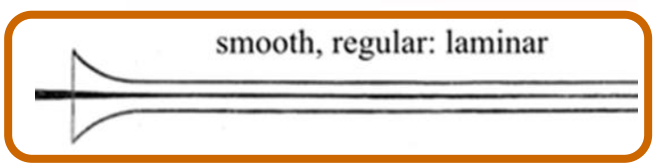
- Reynolds' original pipe turbulence (1883) reports on the transition
- Defined Reynolds' number $\mathrm{Re} = \dfrac{UL}{\nu}$

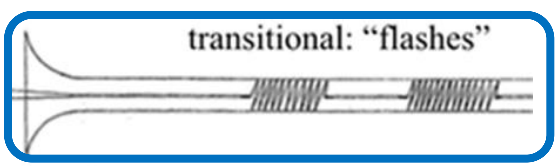


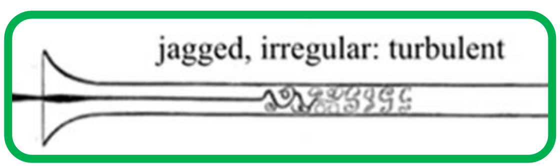


How much turbulence is in the pipe?









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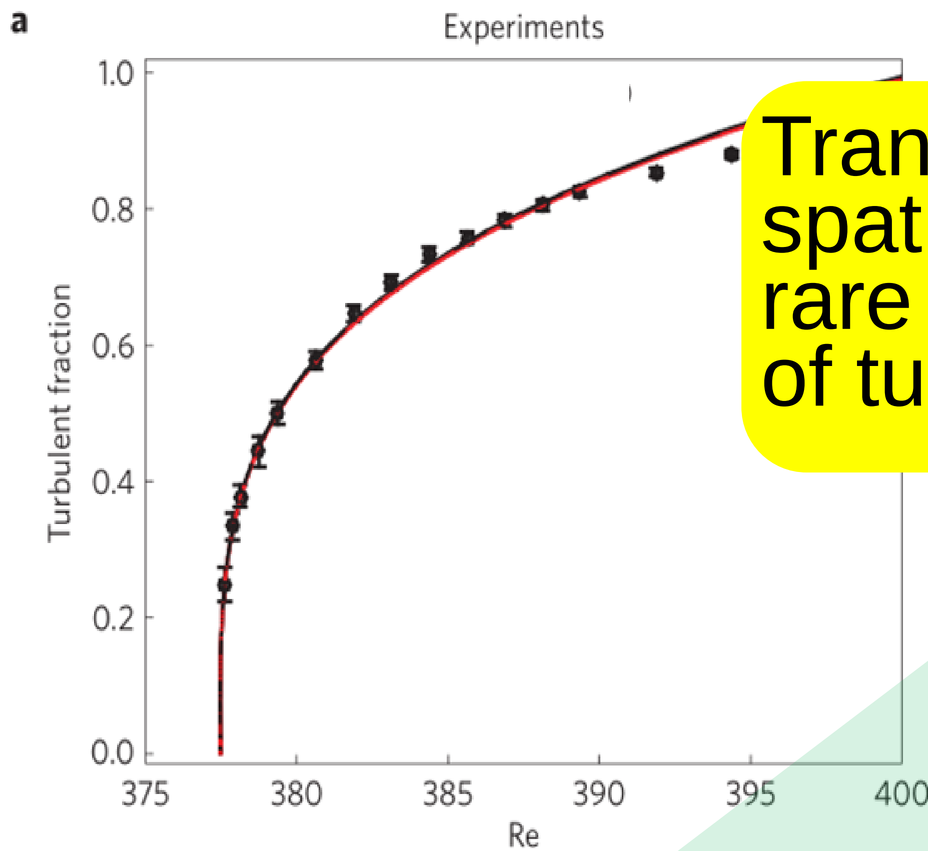
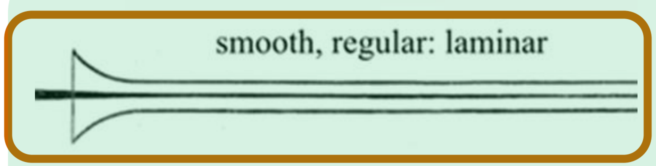
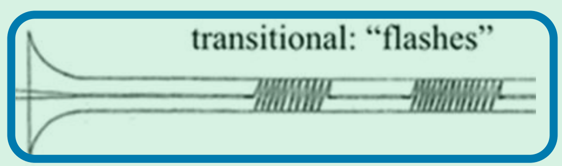
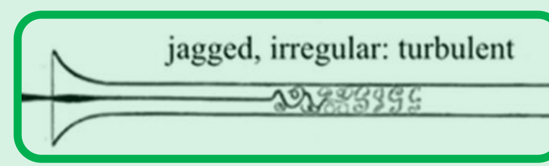


Transition not spatially uniform: rare sharp bursts of turbulence







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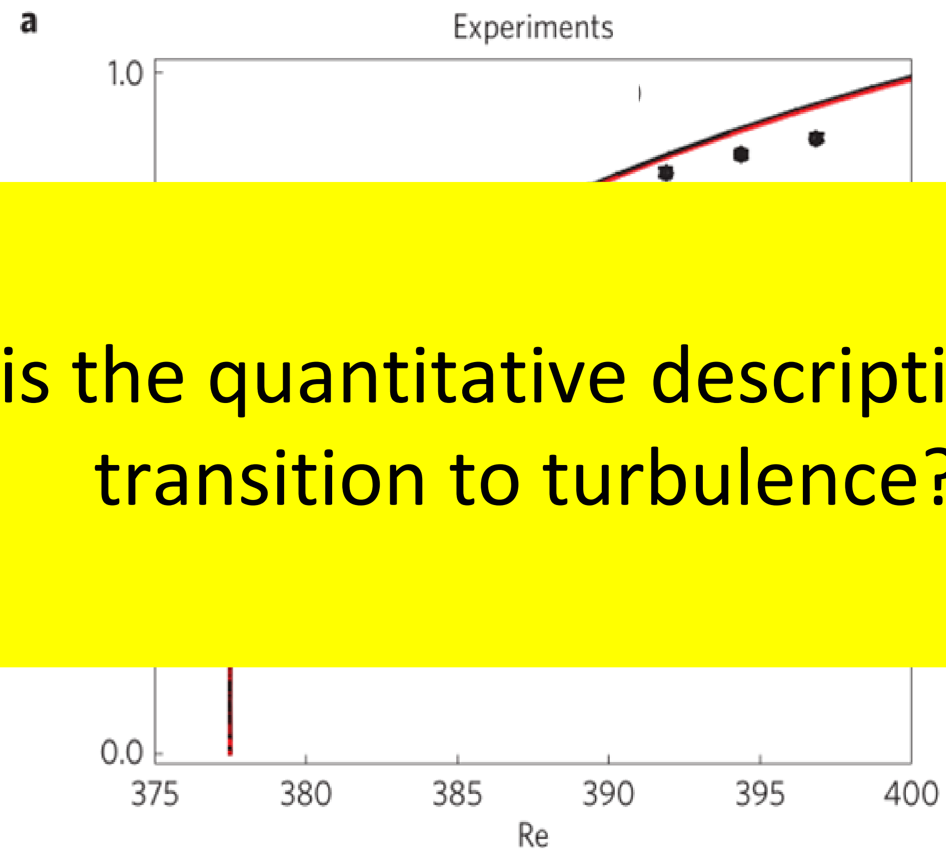
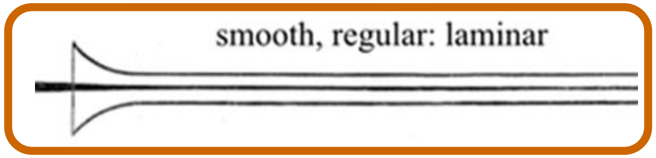
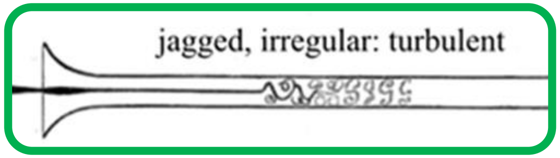
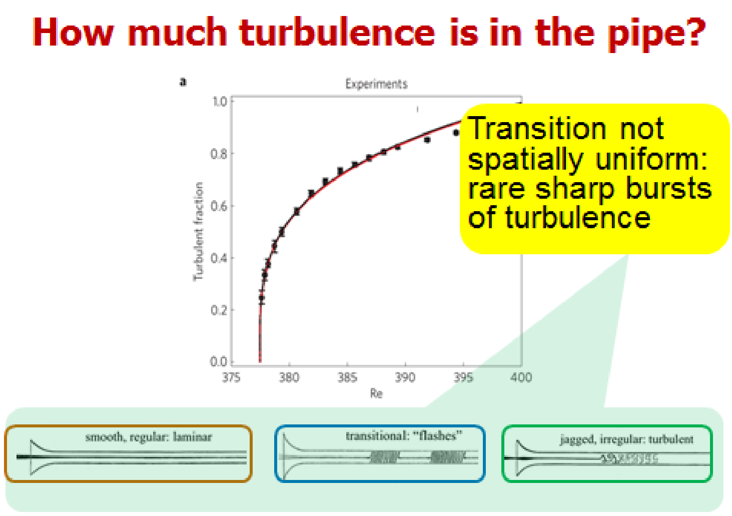


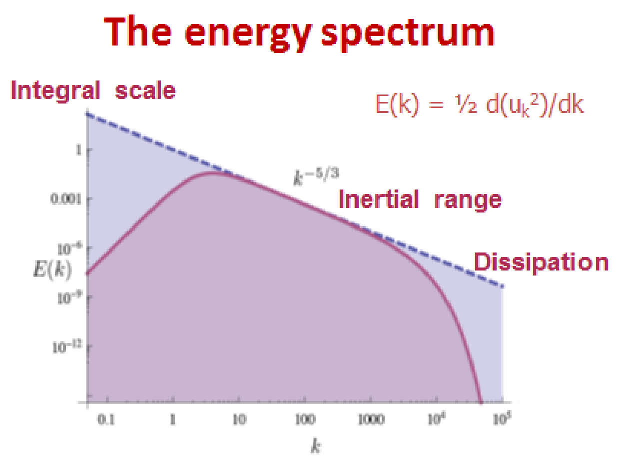
What is the quantitative description of the transition to turbulence?





Turbulence & Phase Transitions





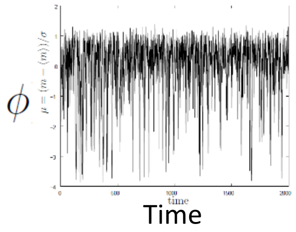
- Turbulent fraction as a function of Re is reminiscent of a continuous phase transition
- Strong power-law correlated fluctuations is reminiscent of a continuous phase transition

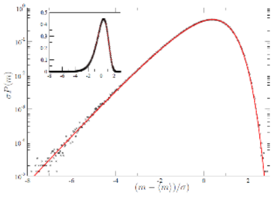
Why is fully-developed turbulence hard?

Why is turbulence unsolved?

Why are phase transitions hard?

- **Strong interactions + fluctuations** in the order parameter ϕ :
  - Very non-Gaussian
  - Intermittency





- **No usable small parameter!**

Landau free energy

$$\mathcal{H} = \int d^d\mathbf{r} \left[\frac{1}{2}\gamma \left(\nabla\phi\right)^2 + \frac{1}{2} r_0 \phi^2 + \frac{1}{4} u_0 \phi^4 \right]$$

$$t = (T - T_c)/T_c \qquad r_0 \sim t$$

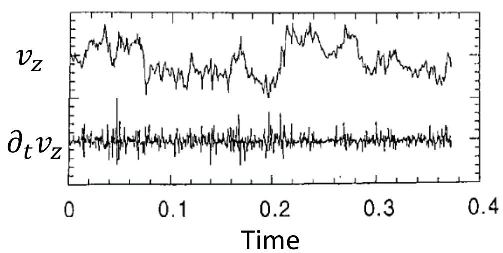
rescaling $\overline{u}_0 \sim u_0 t^{(d-4)/2} \begin{cases} d < 4 : & \overline{u}_0 \to \infty \\ d > 4 : & \overline{u}_0 \to 0 \end{cases}$

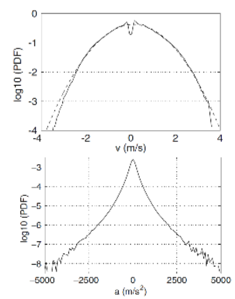
The coefficient of the **interaction** becomes relatively large at **phase transition** $t \to 0$

Why is turbulence unsolved?

Why is turbulence hard?

- **Strong interactions + fluctuations** in the velocity derivatives $\partial_i v_j$:
  - Very non-Gaussian
  - Intermittency: intervals of weak fluctuations interspersed with bursts of strong fluctuations





- **No usable small parameter!**

The Navier-Stokes equation

$$\partial_t \mathbf{v} + (\mathbf{v}\cdot\nabla)\,\mathbf{v} = -\frac{1}{\rho}\nabla p + \nu\nabla^2\mathbf{v} + \mathbf{f}$$

Turbulence: $\mathrm{Re}\to\infty,\ \nu\to 0$

The coefficient of the **nonlinearity** becomes relatively large as the **viscosity** $\nu\to 0$

Why are phase transitions hard?

- **Strong interactions + fluctuations** in the order parameter ϕ :
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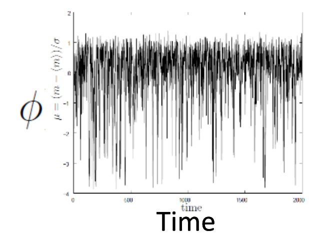
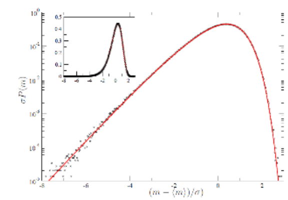
  - Intermittency





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The coefficient of the **interaction** becomes relatively large at **phase transition** $t\to 0$

Transition to turbulence

Stability of laminar flow

1924. № 15.

ANNALEN DER PHYSIK.
VIERTE FOLGE. BAND 74.

1. *Über Stabilität und Turbulenz von Flüssigkeitsströmen;*
von Werner Heisenberg.

Einleitung.

Das Turbulenzproblem, das ganz allgemein den Gegenstand der folgenden Untersuchungen bilden soll, ist im Laufe der Zeit in so vielen Arbeiten von so vielen verschiedenen Gesichtspunkten aus behandelt worden, daß es nicht unsere Absicht sein kann, einleitend über die bisherigen Resultate eine Übersicht zu geben. Wir verweisen zu diesem Zweck auf eine Arbeit von Noether[1]) über den heutigen Stand des Turbulenzproblems, in welcher auch die meisten Literaturangaben zu finden sind.

Für unseren Zweck genügt es, den gegenwärtigen Stand des Turbulenzproblems in ganz groben Umrissen zu skizzieren: die bisherigen Untersuchungen zerfallen in zwei Teile: die einen von ihnen befassen sich mit der Stabilitätsuntersuchung irgendwelcher laminaren Bewegung, die anderen mit der turbulenten Bewegung selbst.

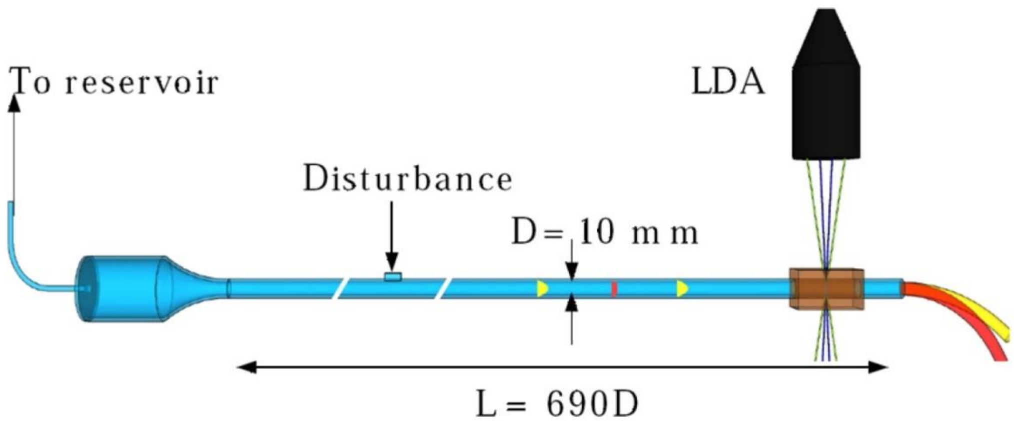
Die ersteren führten anfangs zu dem negativen Resultat, daß alle untersuchten Laminarbewegungen stabil seien. v. Mises[2]) und Hopf[3]) bewiesen auf Grund eines Ansatzes von Sommerfeld[4]) die Stabilität des der Couetteschen Anordnung entsprechenden linearen Geschwindigkeitsprofils, Blumenthal[5]) gelangte bei einem von Noether zur Diskussion gestellten Profile 3. Grades zu demselben Ergebnis. Dagegen gelang es später Noether[6]), ein labiles Profil anzugeben — allerdings

1) F. Noether, Zeitschr. f. angew. Math. u. Mech. 1. S. 125, 1921.
2) R. v. Mises, Beitrag z. Oszillationsprobl.: Heinr. Weber-Festschrift. 1912. S. 252.
3) L. Hopf, Ann. d. Phys. 44. S. 1. 1914.
4) A. Sommerfeld, Atti d. IV. congr. int. dei Mathem. Rom 1909.
5) O. Blumenthal, Sitzungsber. d. bayr. Akad. d. Wiss. S. 563. 1913.
6) F. Noether, Nachr. d. Ges. d. Wiss. Göttingen 1917.

Werner Heisenberg

Precision measurement of turbulent transition

Q: will a turbulent puff survive to the end of the pipe?



Many repetitions ➔ **Survival probability = P(Re, t)**

Hof et al., *PRL* **101**, 214501 (2008)

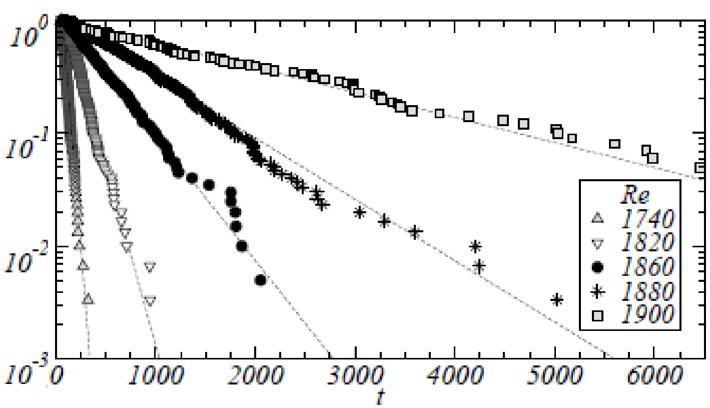
Pipe flow turbulence

Decaying single puff

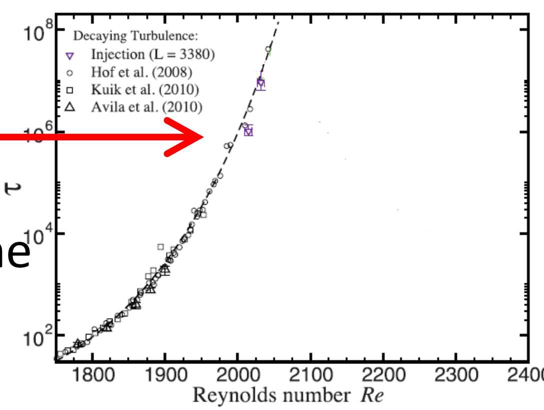
laminar	metastable puffs	spatiotemporal intermittency	expanding slugs

Re: 1775, 2050, 2500

Survival probability $P(\mathrm{Re},t) = e^{-\frac{t-t_0}{\tau(\mathrm{Re})}}$

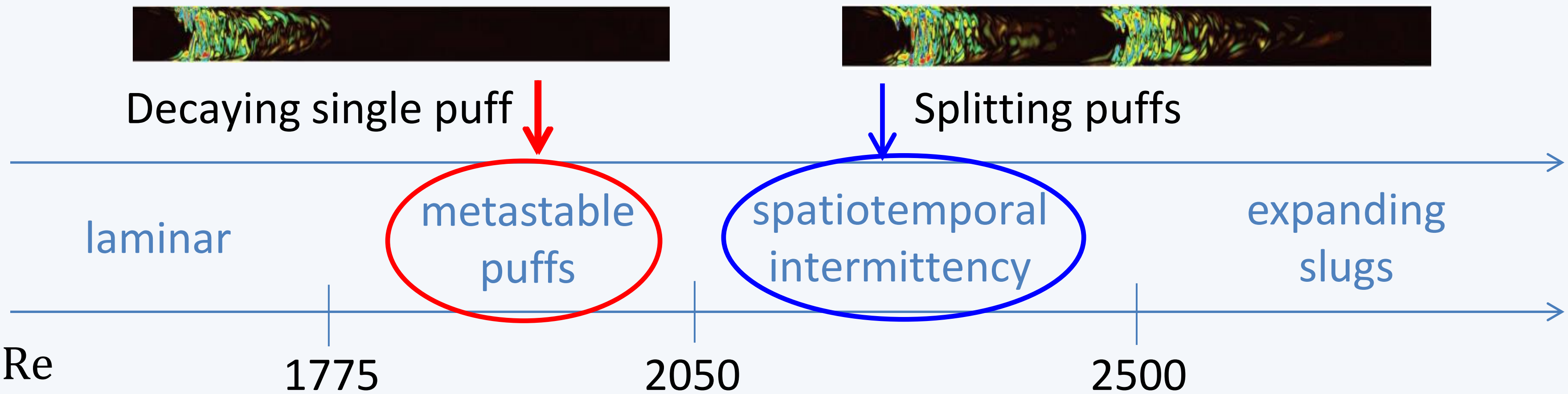


Puff lifetime

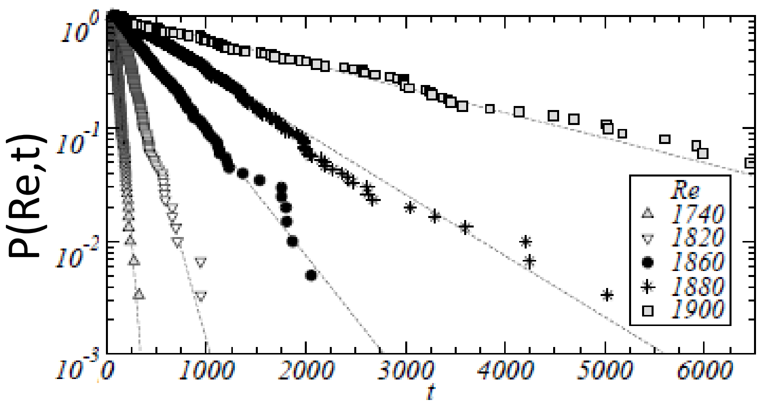


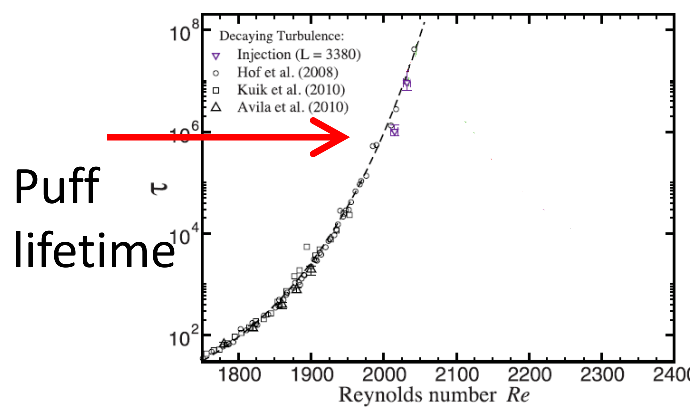
Avila *et al.*, (2009) Avila *et al.*, *Science* **333**, 192 (2011) Hof *et al.*, *PRL* **101**, 214501 (2008)

Pipe flow turbulence



Survival probability $P(\mathrm{Re},t) = e^{-\frac{t-t_0}{\tau(\mathrm{Re})}}$

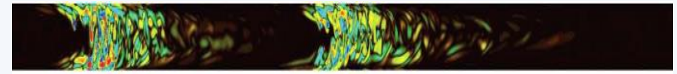




Avila *et al.*, (2009) Avila *et al.*, *Science* **333**, 192 (2011) Hof *et al.*, *PRL* **101**, 214501 (2008)

Pipe flow turbulence

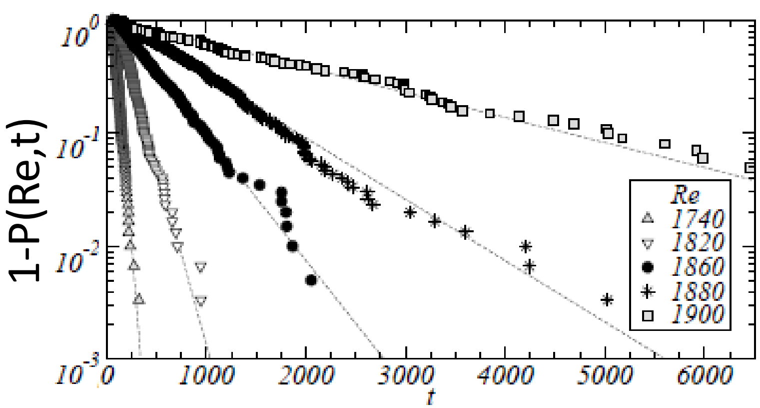
Decaying single puff



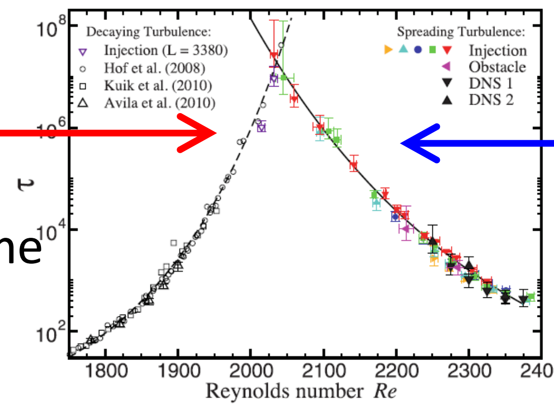
Splitting puffs

Re	laminar	metastable puffs	spatiotemporal intermittency	expanding slugs
		1775	2050	2500

Splitting probability $1 - P(\mathrm{Re}, t) = e^{-\frac{t - t_0}{\tau(\mathrm{Re})}}$



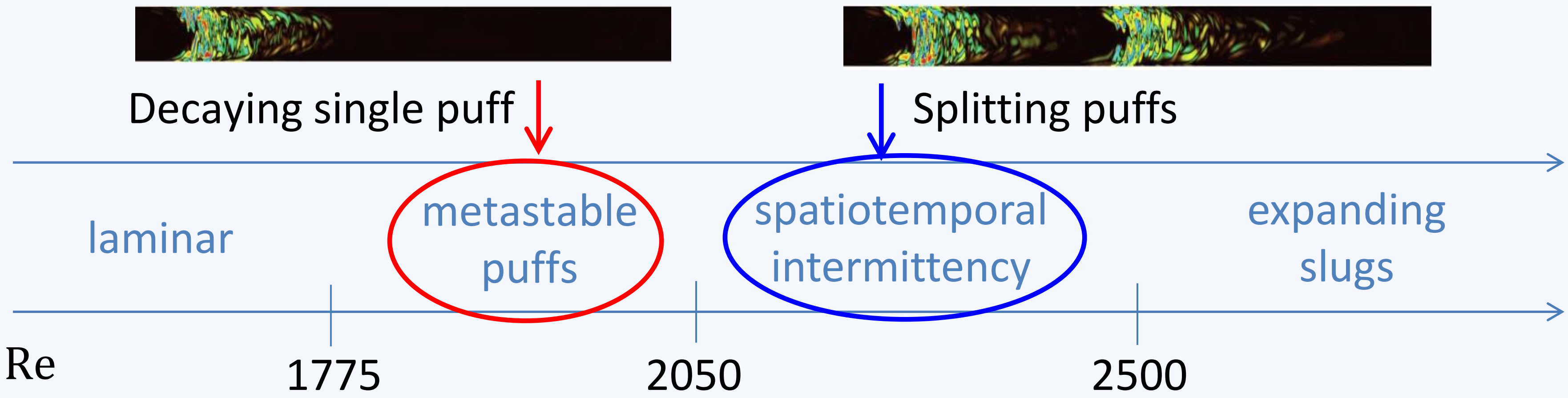
Puff lifetime



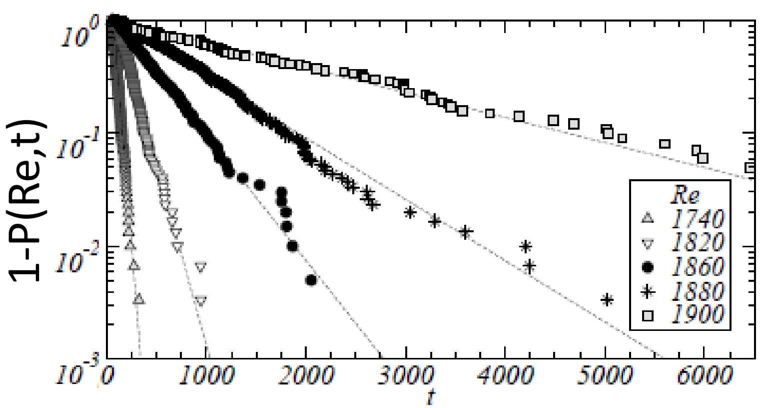
Mean time between split events

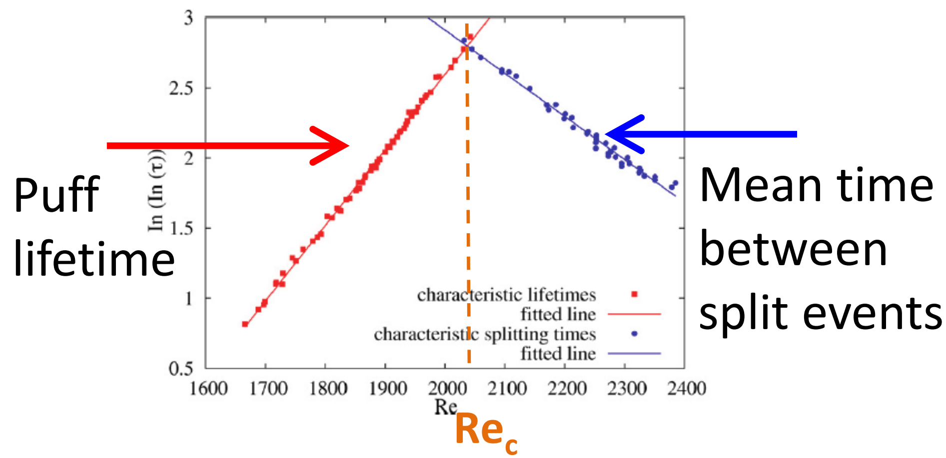
Avila *et al.*, (2009) Avila *et al.*, *Science* **333**, 192 (2011) Hof *et al.*, *PRL* **101**, 214501 (2008)

Pipe flow turbulence



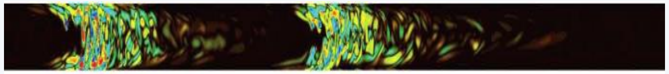
Splitting probability $1 - P(\mathrm{Re}, t) = e^{-\frac{t - t_0}{\tau(\mathrm{Re})}}$





Avila *et al.*, (2009) Avila *et al.*, *Science* **333**, 192 (2011) Hof *et al.*, *PRL* **101**, 214501 (2008)

Pipe flow turbulence



Decaying single puff

Splitting puffs

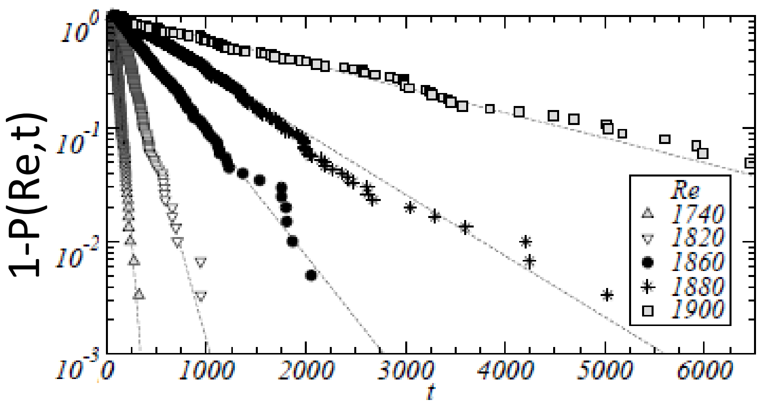
laminar | metastable puffs | spatiotemporal intermittency | expanding slugs

Re

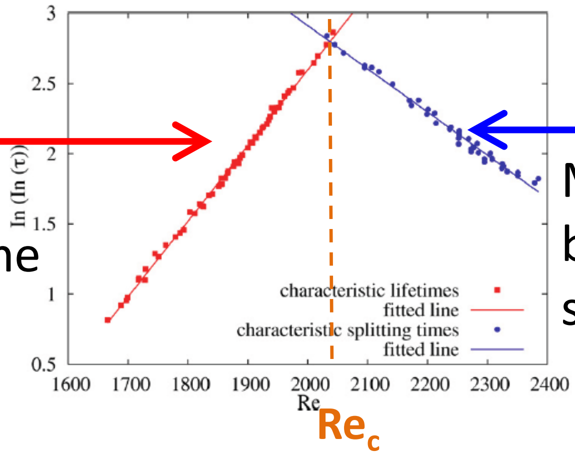
Super-exponential scaling:

$$\frac{\tau}{\tau_0} \sim \exp(\exp \mathrm{Re})$$

Splitting p



Puff lifetime



Mean time between split events

Avila *et al.*, (2009) Avila *et al.*, *Science* **333**, 192 (2011) Hof *et al.*, *PRL* **101**, 214501 (2008)

Theory for the laminar-turbulent transition in pipe flow

Logic of modeling phase transitions

Magnets

Electronic structure

↓

Ising model

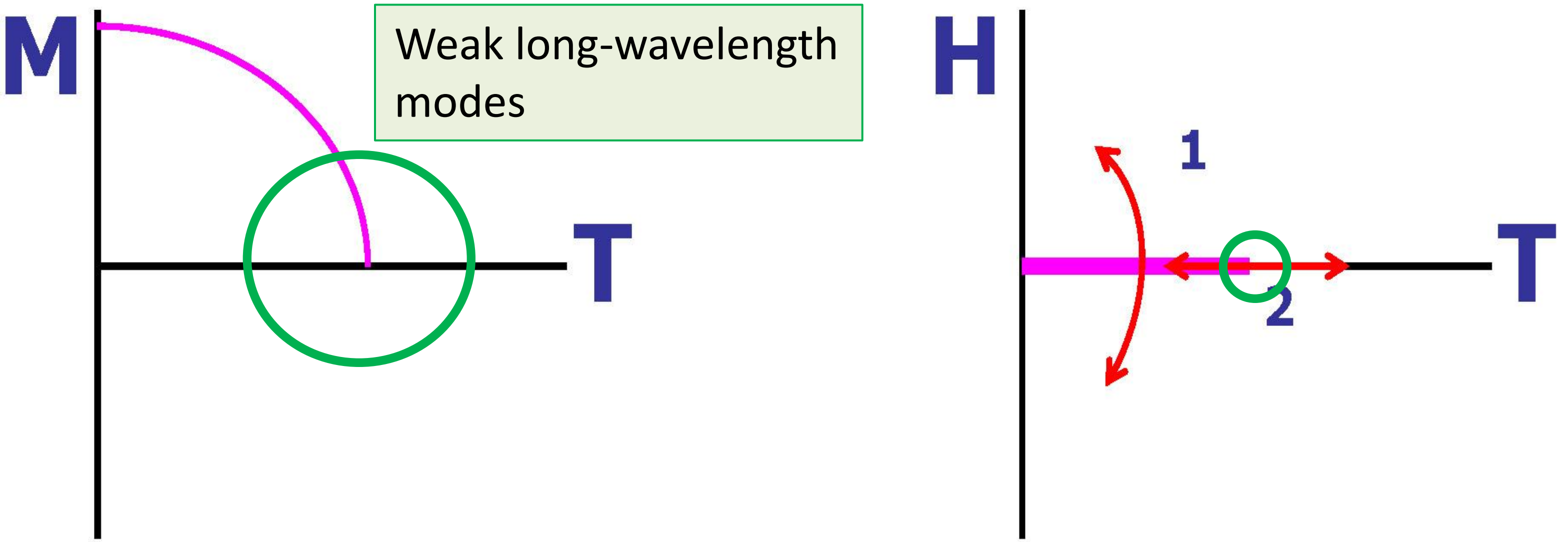
↓

Landau theory

↓

Renormalization group
universality class
(Ising universality class)

Critical phenomena in magnets



$M \sim M_0[|T - T_c|/T_c]^\beta$ for $H = 0$ as $T \to T_c$

Critical isotherm: $M \sim H^{1/\delta}$ for $T = T_c$

- Widom (1963) pointed out that both these results followed from a *similarity formula*:

$$M(t,h) = |t|^\beta f_M(h/t^\Delta)$$

where $t \equiv (T - T_c)/T_c$ for some choice of exponent Δ and scaling function $f_M(x)$

Universality at a critical point

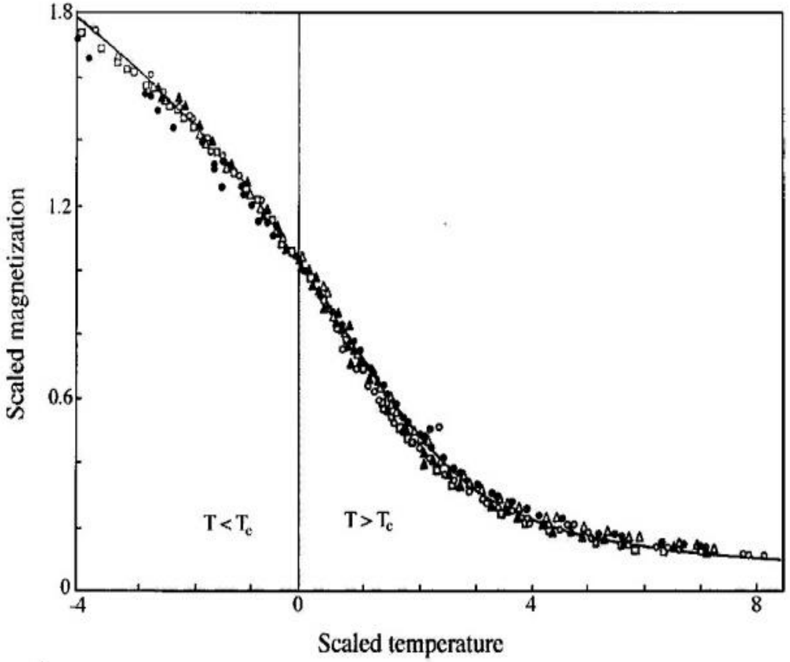


FIG. 1. Experimental *MHT* data on five different magnetic materials plotted in scaled form. The five materials are $CrBr_3$, EuO, Ni, YIG, and Pd_3Fe. None of these materials is an idealized ferromagnet: $CrBr_3$ has considerable lattice anisotropy, EuO has significant second-neighbor interactions. Ni is an itinerant-electron ferromagnet, YIG is a ferrimagnet, and Pd_3Fe is a ferromagnetic alloy. Nonetheless, the data for all materials collapse onto a single scaling function, which is that calculated for the $d=3$ Heisenberg model [after Milošević and Stanley (1976)].

Stanley (1999)

- Magnetization M of a material depends on temperature T and applied field H
  - M(H,T) ostensibly a function of two variables
- Plotted in appropriate scaling variables get ONE universal curve
- Scaling variables involve critical exponents

A theoretical physics success

A model …

Gives a precise prediction in agreement with experiment!

materials collapse onto a single scaling function, which is that calculated for the $d = 3$ Heisenberg model [after Milošević and Stanley (1976)].

Stanley (1999)

A theoretical physics success

A model of a model of a model of a model of a model

Landau theory

Classical Heisenberg

Quantum Heisenberg

Electronic structure

Quantum chemistry

Gives a precise prediction in agreement with experiment!

Non-systematic approximations

materials collapse onto a single scaling function, which is that calculated for the $d=3$ Heisenberg model [after Milošević and Stanley (1976)].

Stanley (1999)

Logic of modeling phase transitions

Magnets

Electronic structure

↓

Ising model

↓

Landau theory

↓

Renormalization group universality class (Ising universality class)

Turbulence

Kinetic theory

↓

Navier-Stokes eqn

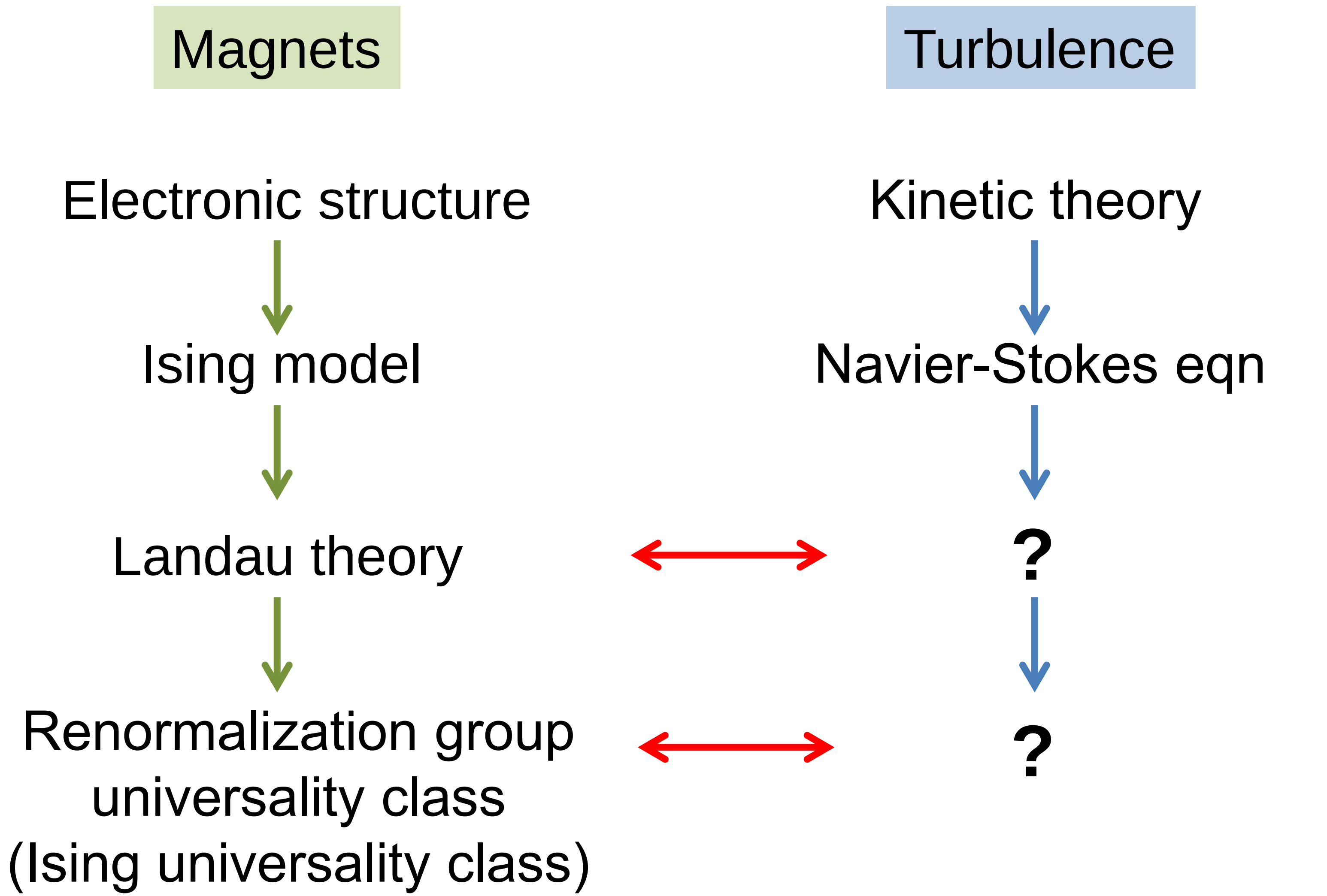
↓

?

↓

?

Logic of modeling phase transitions

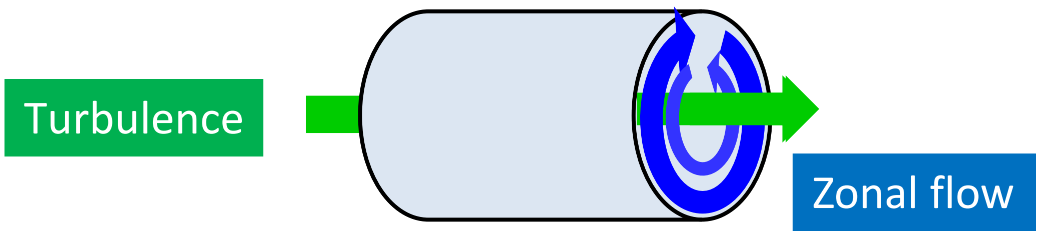


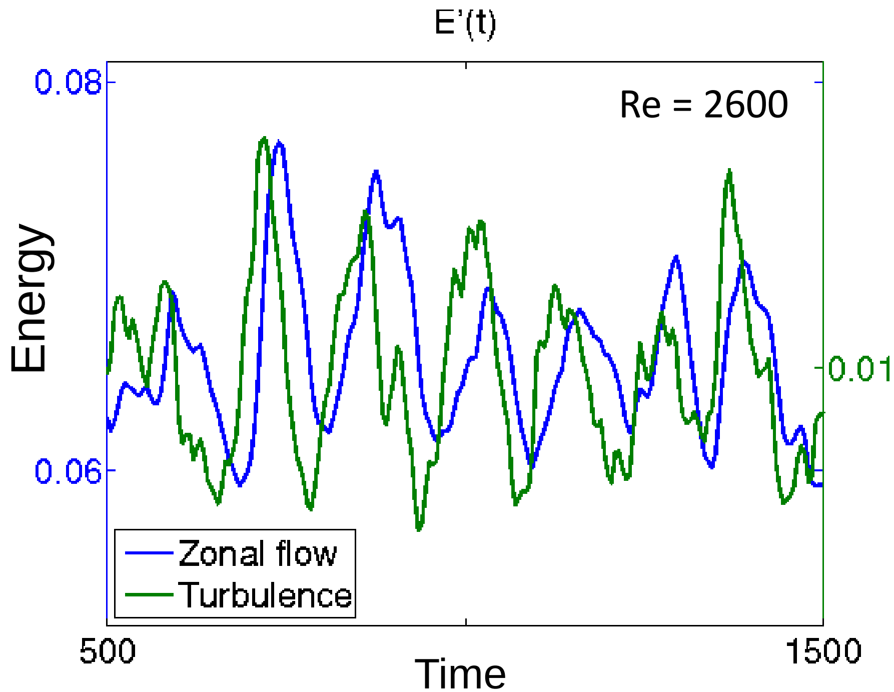
Identification of long-wavelength collective modes at the laminar-turbulent transition

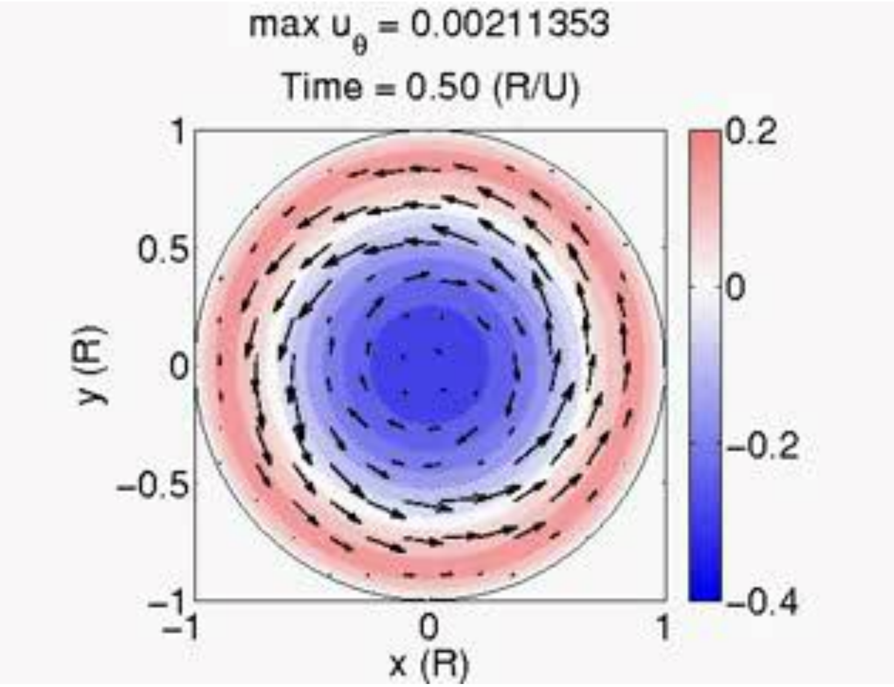
To avoid uncontrolled approximations, we use **direct numerical simulation** of the **Navier-Stokes equations**

H.-Y Shih, T.L. Hsieh, NG, Nature Physics 2016

Predator-prey oscillations in pipe flow

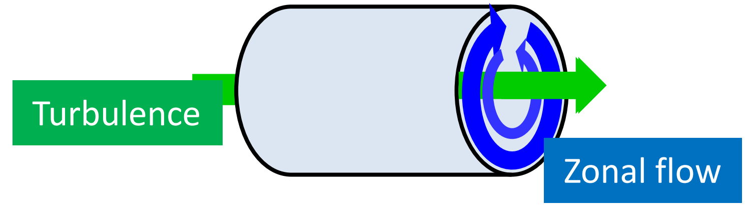






Simulation based on the open source
code by Ashley Willis: openpipeflow.org

What drives the zonal flow?

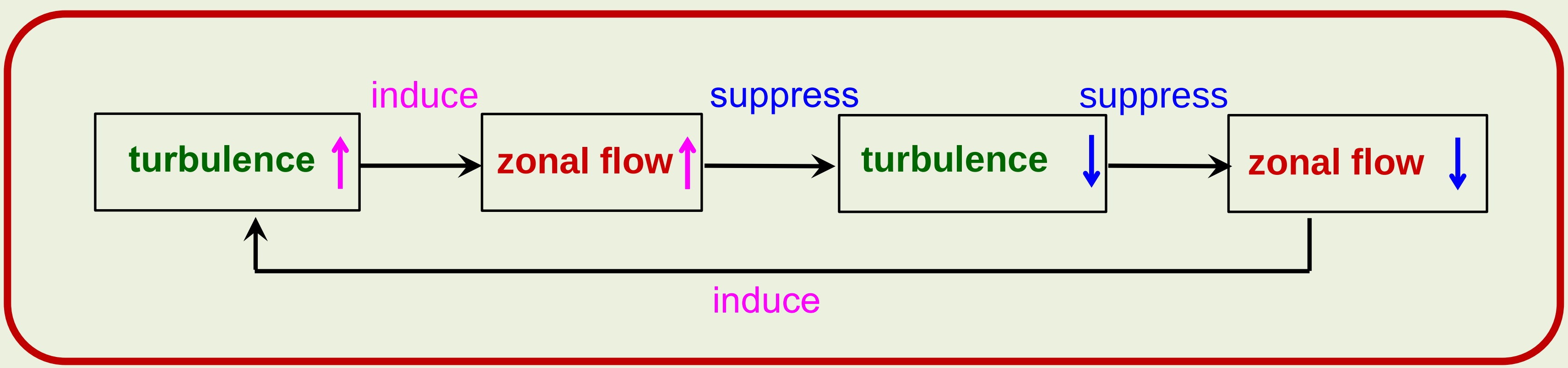


- **Interaction in two fluid model**
  - Turbulence, small-scale (k>0)
  - Zonal flow, large-scale (k=0,m=0): induced by turbulence and creates shear to suppress turbulence

1) Anisotropy of turbulence creates Reynolds stress which generates the mean velocity in azimuthal direction

$$\partial_t \langle v_\theta \rangle = -\partial_r \langle (\widetilde{v}_\theta \cdot \widetilde{v}_r) \rangle - \mu \langle v_\theta \rangle$$

2) Mean azimuthal velocity decreases the anisotropy of turbulence and thus suppress turbulence



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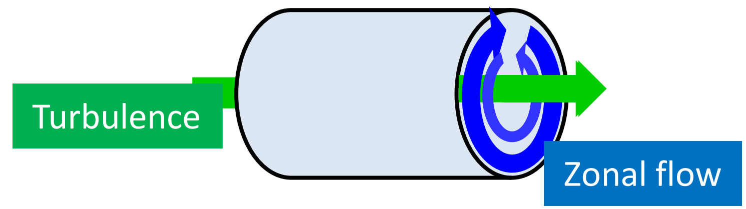

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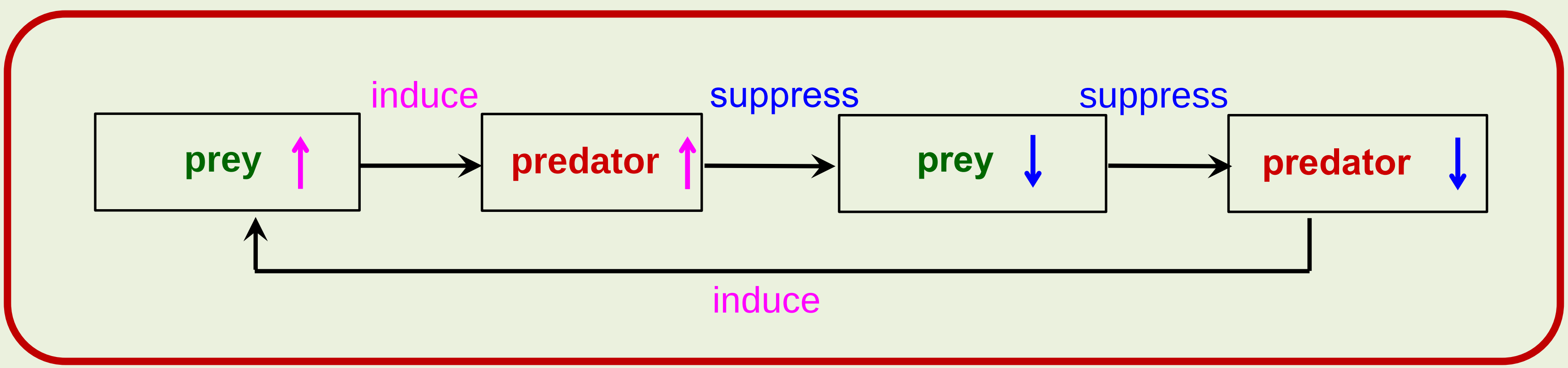
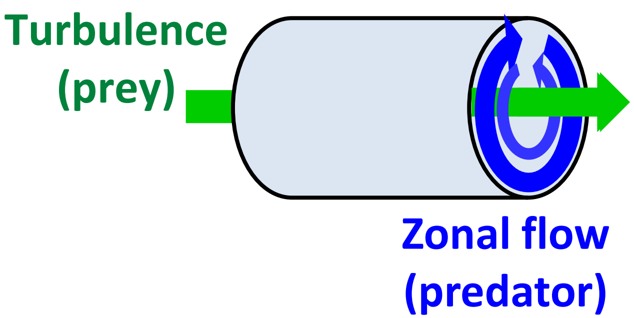
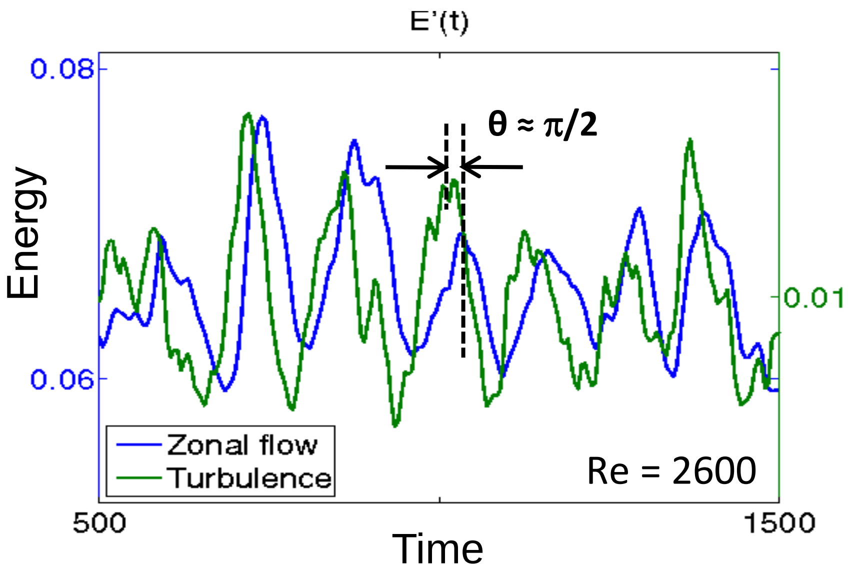


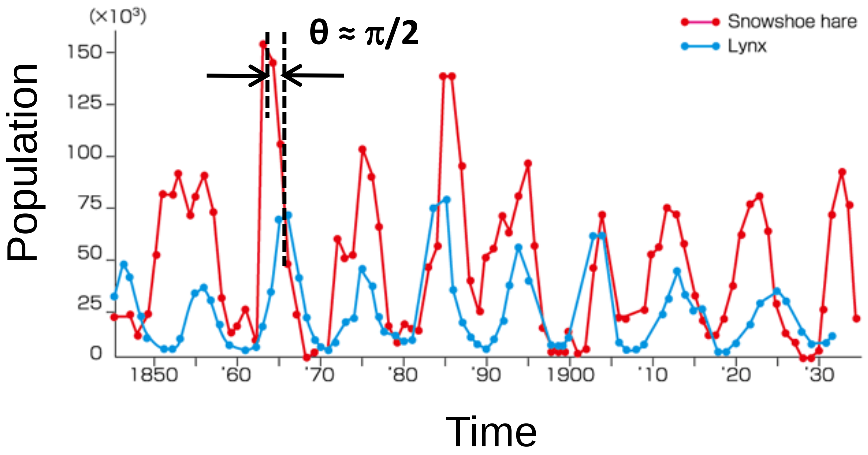
Pipe flow near transition to turbulence

?
=

Predator-prey ecosystem



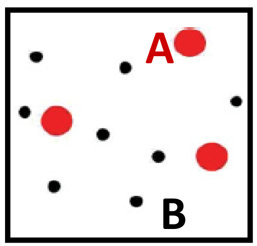




Stochastic model of predator-prey dynamics

- **Stochastic individual-level model**
  fluctuations in number → demographic stochasticity that induces **quasi-cycles**

A = predator
B = prey
E = food or
available space

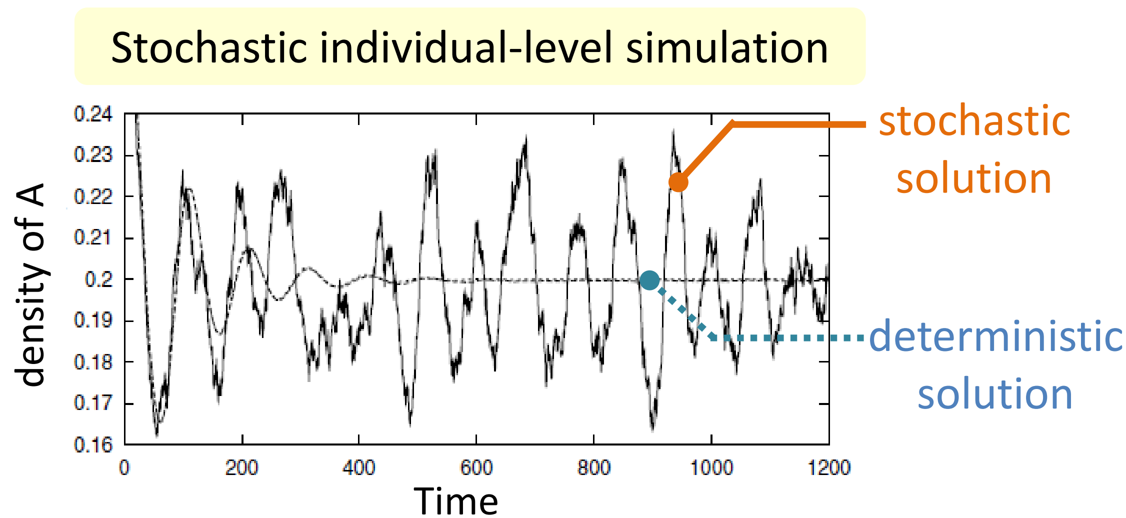


$$B + E \xrightarrow{b} B + B$$

$$B + B \xrightarrow{c} B + E$$

$$A + B \xrightarrow{p} A + A$$

$$A \xrightarrow{d_A} E \qquad B \xrightarrow{d_B} E$$



McKane & Newman (2005) Butler & Goldenfeld (2009)

Landau theory for transitional pipe turbulence is stochastic predator-prey

Bill Wyld (1928-2013)

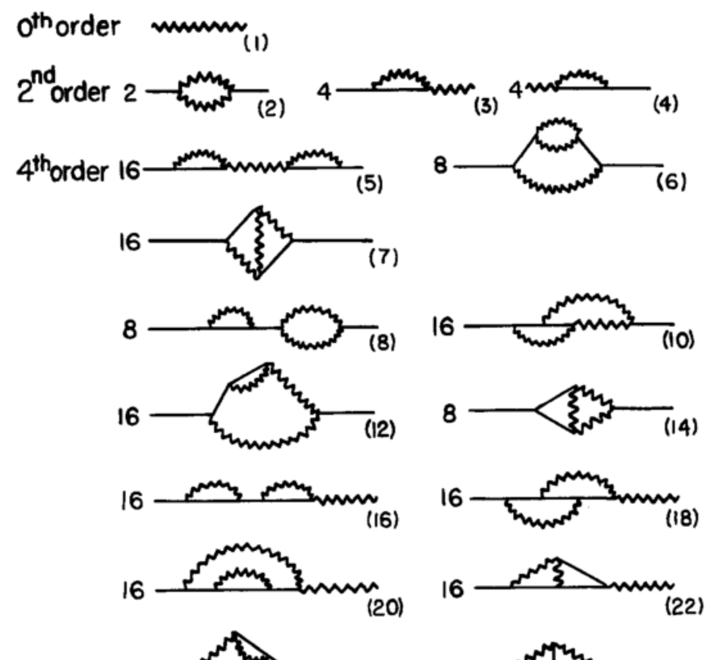
ANNALS OF PHYSICS: **14:** 143–165 (1961)

Formulation of the Theory of Turbulence in an Incompressible Fluid

H. W. WYLD, JR.

Physics Department, University of Illinois, Urbana, Illinois and Space Technology Laboratories, Los Angeles, California

The theory of turbulence in an incompressible fluid is formulated using methods similar to those of quantum field theory. A systematic perturbation theory is set up, and the terms in the perturbation series are shown to be in one to one correspondence with certain diagrams analogous to Feynman diagrams. From a study of the diagrams it is shown that the perturbation series can be rearranged and partially summed in such a way as to reduce the problem to the solution of three simultaneous integral equations for three functions, one of which is the second order velocity correlation function. The equations have the form of infinite power series integral equations, and the first few terms in the power series are derived from an analysis of the diagrams to sixth order. Truncation of the integral equations at the lowest nontrivial order yields Chandrasekhar's equation, and truncation at a higher order yields the equations discussed by Kraichnan.

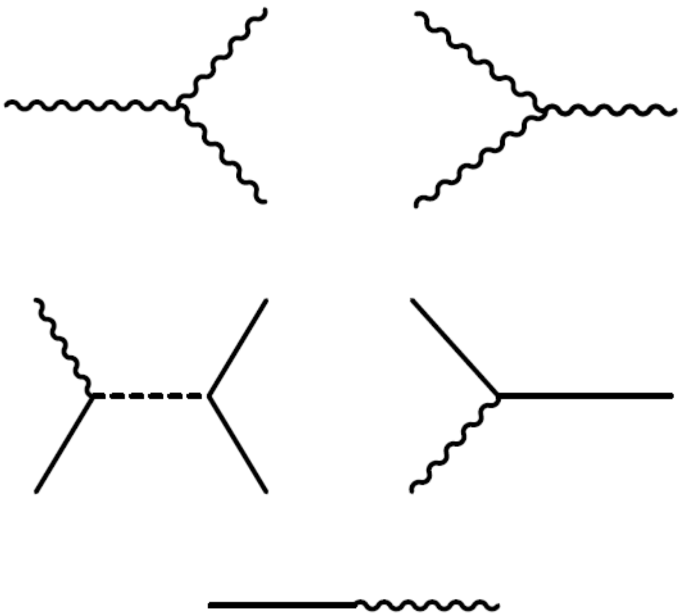


Derivation of predator-prey equations

Zonal flow   Turbulence
Vacuum = Laminar flow

A = predator B =prey
E = food/empty state

Zonal flow-turbulence



Predator-prey

$$B + E \xrightarrow{b} B + B$$

$$B + B \xrightarrow{c} B + E$$

$$A + B \xrightarrow{p} A + A$$

$$A + B \xrightarrow{p'} A + E$$

$$B \xrightarrow{m} A$$

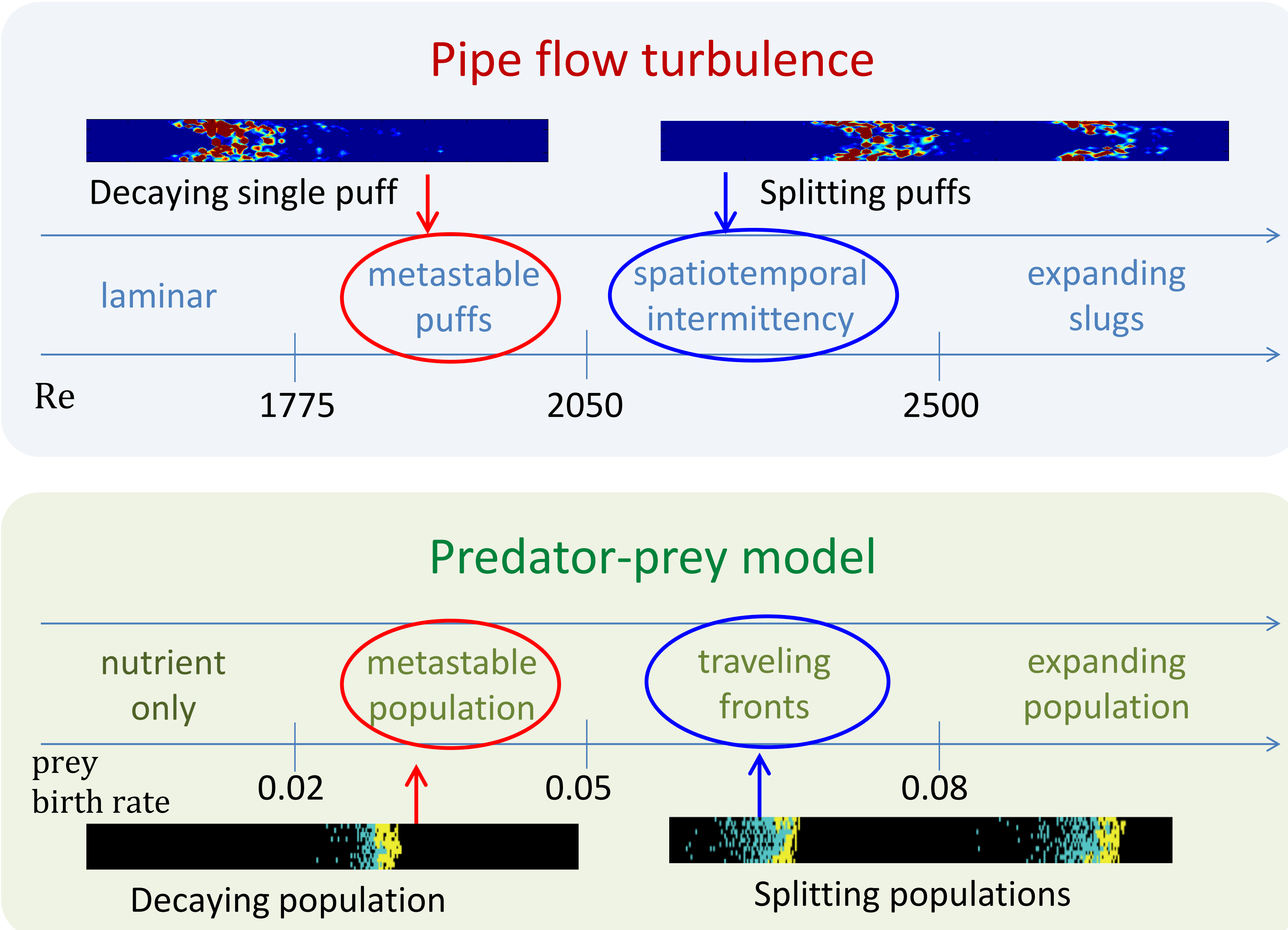
$$A \xrightarrow{d_A} E \quad B \xrightarrow{d_B} E$$

Stochastic predator-prey recapitulates turbulence data

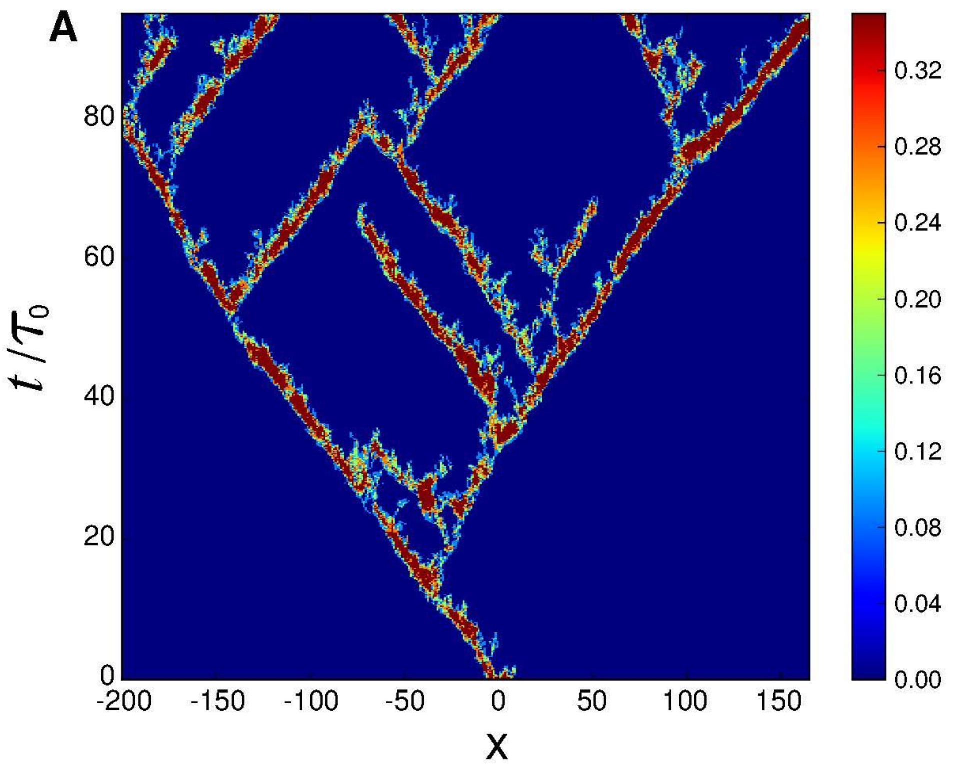
Phase diagram

Lifetime statistics

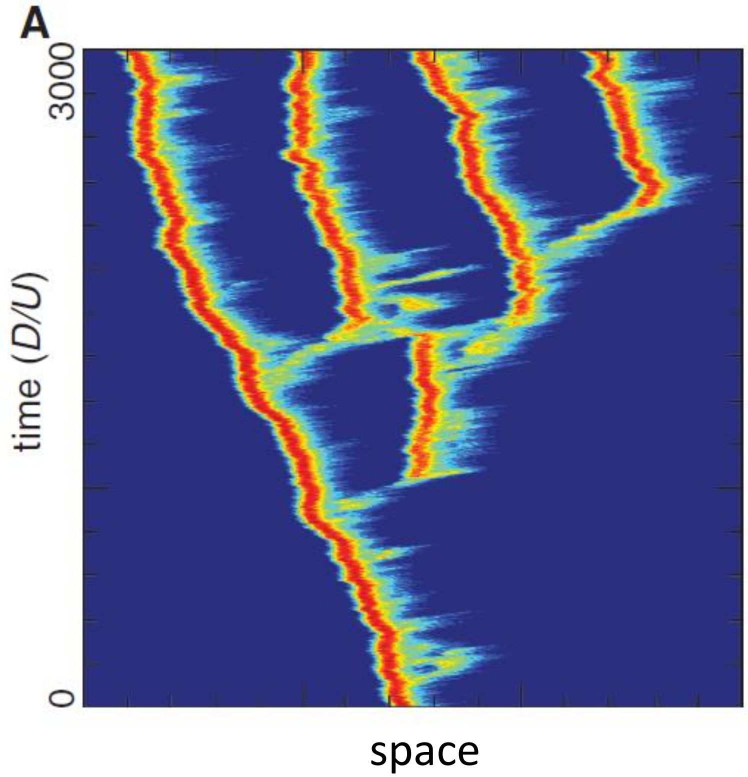
Universality class prediction


Pipe flow turbulence
Decaying single puff
Splitting puffs
laminar
metastable puffs
spatiotemporal intermittency
expanding slugs
Re
1775
2050
2500
Predator-prey model
nutrient only
metastable population
traveling fronts
expanding population
prey birth rate
0.02
0.05
0.08
Decaying population
Splitting populations

“Puff splitting” in predator-prey systems

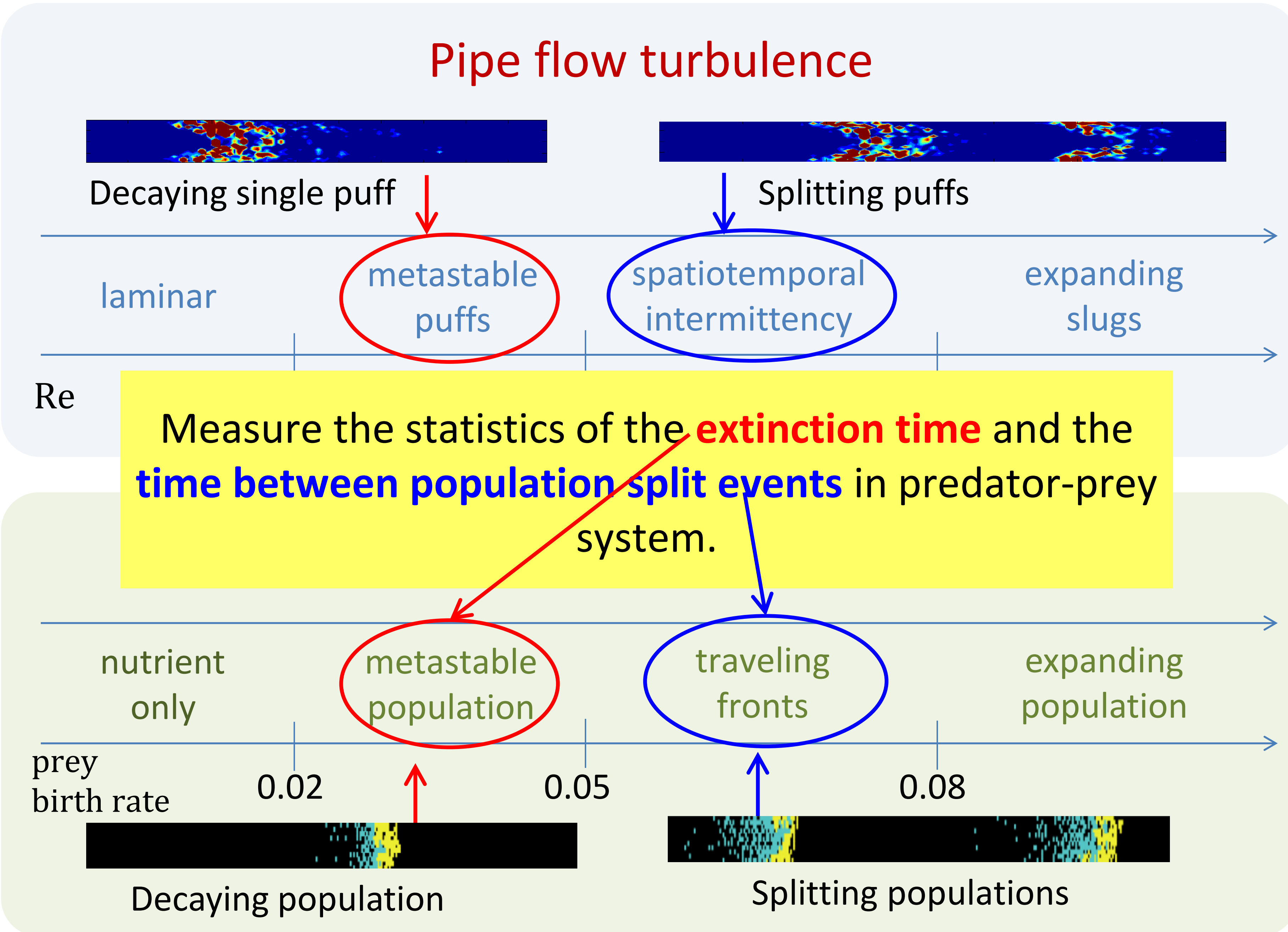


Population-splitting in predator-prey ecosystem



Puff-splitting in pipe turbulence

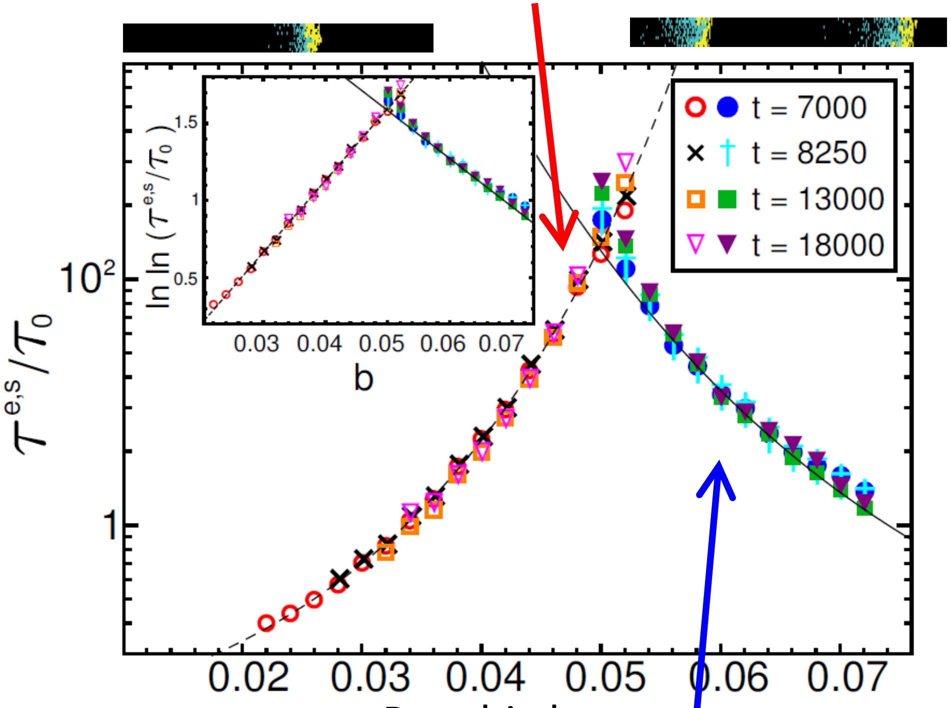
Avila et al., Science (2011)

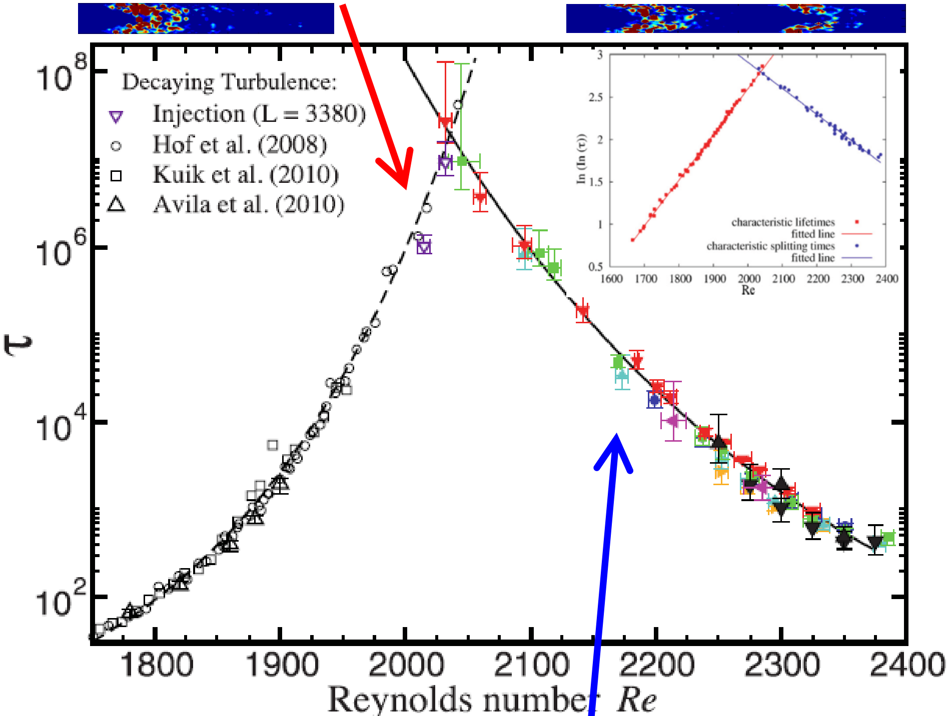

Pipe flow turbulence
Decaying single puff
Splitting puffs
laminar
metastable puffs
spatiotemporal intermittency
expanding slugs
Re
Measure the statistics of the extinction time and the time between population split events in predator-prey system.
nutrient only
metastable population
traveling fronts
expanding population
prey birth rate
0.02
0.05
0.08
Decaying population
Splitting populations

Predator-prey vs. transitional turbulence

Prey lifetime

Turbulent puff lifetime





Mean time between population split events

Mean time between puff split events

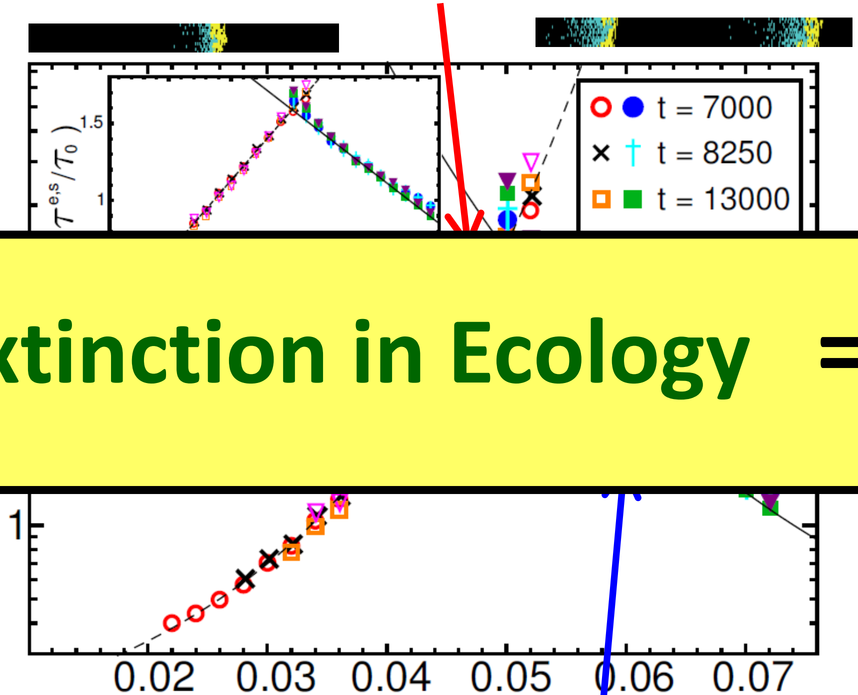
Shih, Hsieh and Goldenfeld
Nature Physics **12**, 245 (2016)

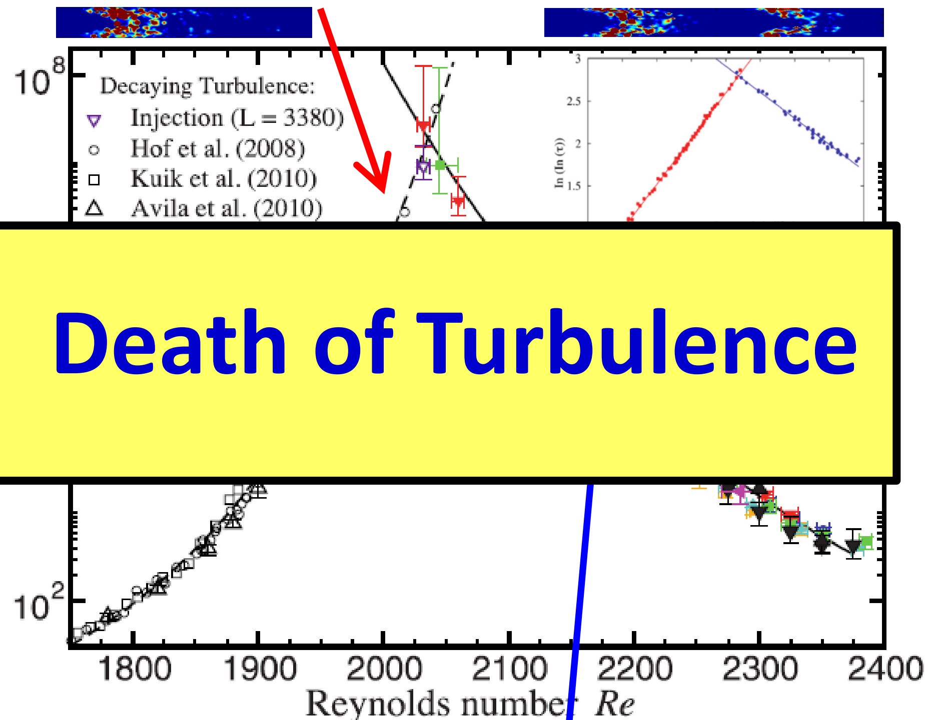
Avila *et al.*, *Science* **333**, 192 (2011)
Song *et al.*, *J. Stat. Mech.* 2014(2**)**, P020010

Predator-prey vs. transitional turbulence

Prey lifetime

Turbulent puff lifetime





Extinction in Ecology = Death of Turbulence

Mean time between population split events

Mean time between puff split events

Shih, Hsieh and Goldenfeld
Nature Physics **12**, 245 (2016)

Avila *et al.*, *Science* **333**, 192 (2011)
Song *et al.*, *J. Stat. Mech.* 2014(2), P020010

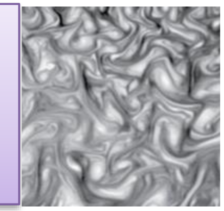
Roadmap: Universality class of laminar-turbulent transition

Universality class

?

(Classical) Turbulence

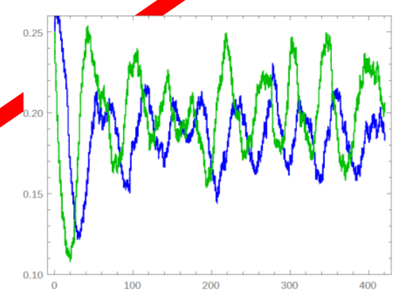
(Boffetta and Ecke, 2012)



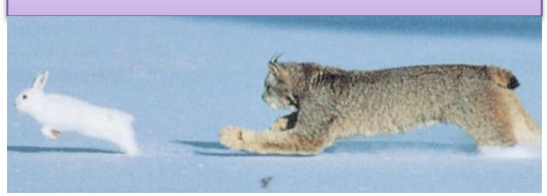
Direct Numerical Simulations of Navier-Stokes

Two-fluid model

Turbulence + Zonal flow

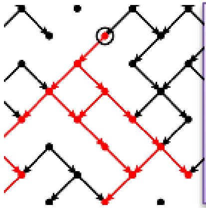


Predator-Prey



(Pearson Education, Inc., 2009)

Roadmap: Universality class of laminar-turbulent transition



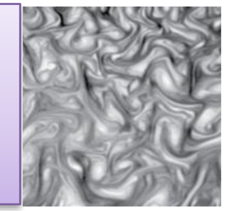
Directed Percolation

(Wikimedia Commons)

?

(Boffetta and Ecke, 2012)

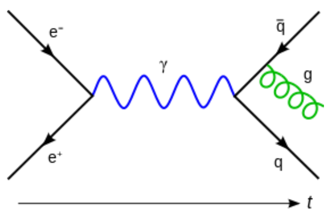
(Classical) Turbulence



Reggeon field theory
(Janssen, 1981)

Direct Numerical Simulations
of Navier-Stokes

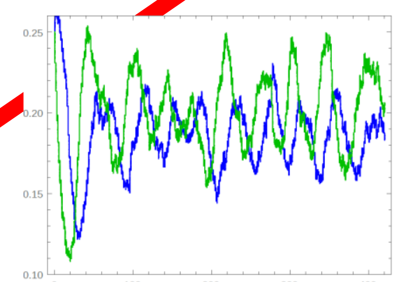
Field Theory



(Wikimedia Commons)

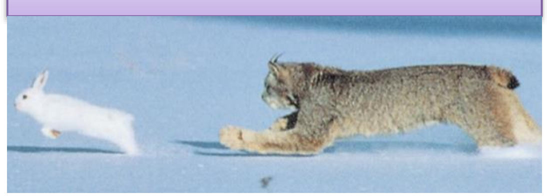
Two-fluid model

Turbulence + Zonal flow



Extinction transition
(Mobilia et al., 2007)

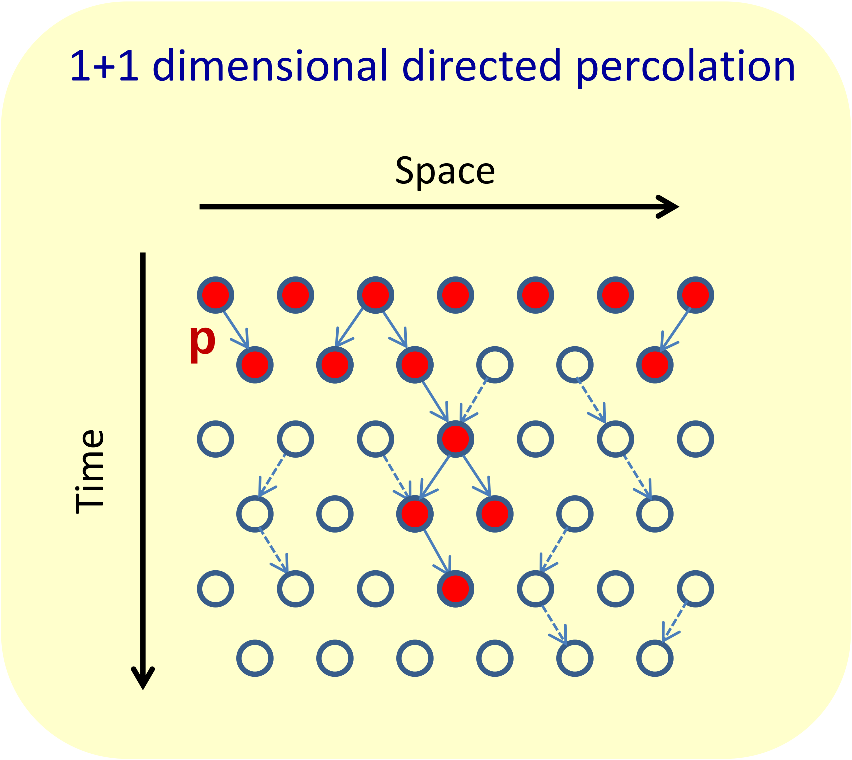
Predator-Prey

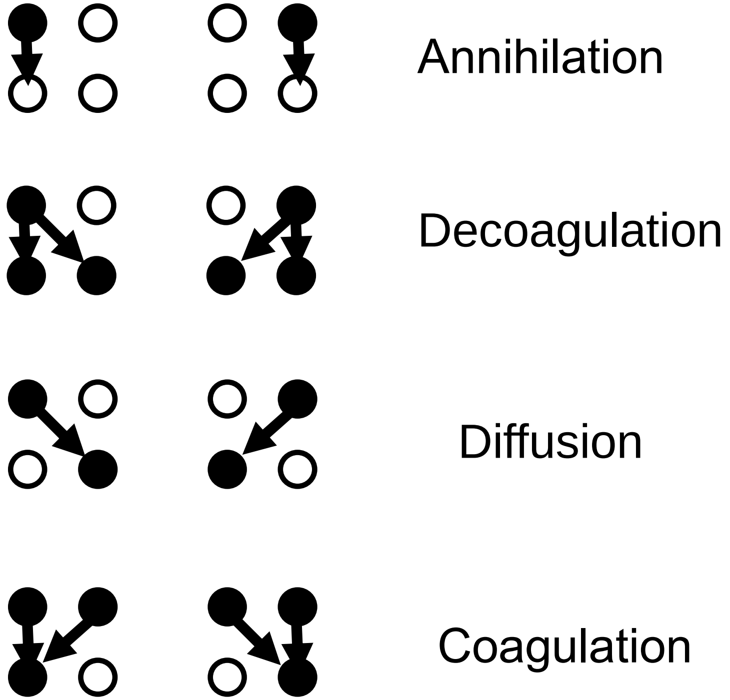


(Pearson Education, Inc., 2009)

Directed percolation & the laminar-turbulent transition

- Turbulent regions can spontaneously relaminarize (go into an absorbing state).
- They can also contaminate their neighbourhood with turbulence. (Pomeau 1986)

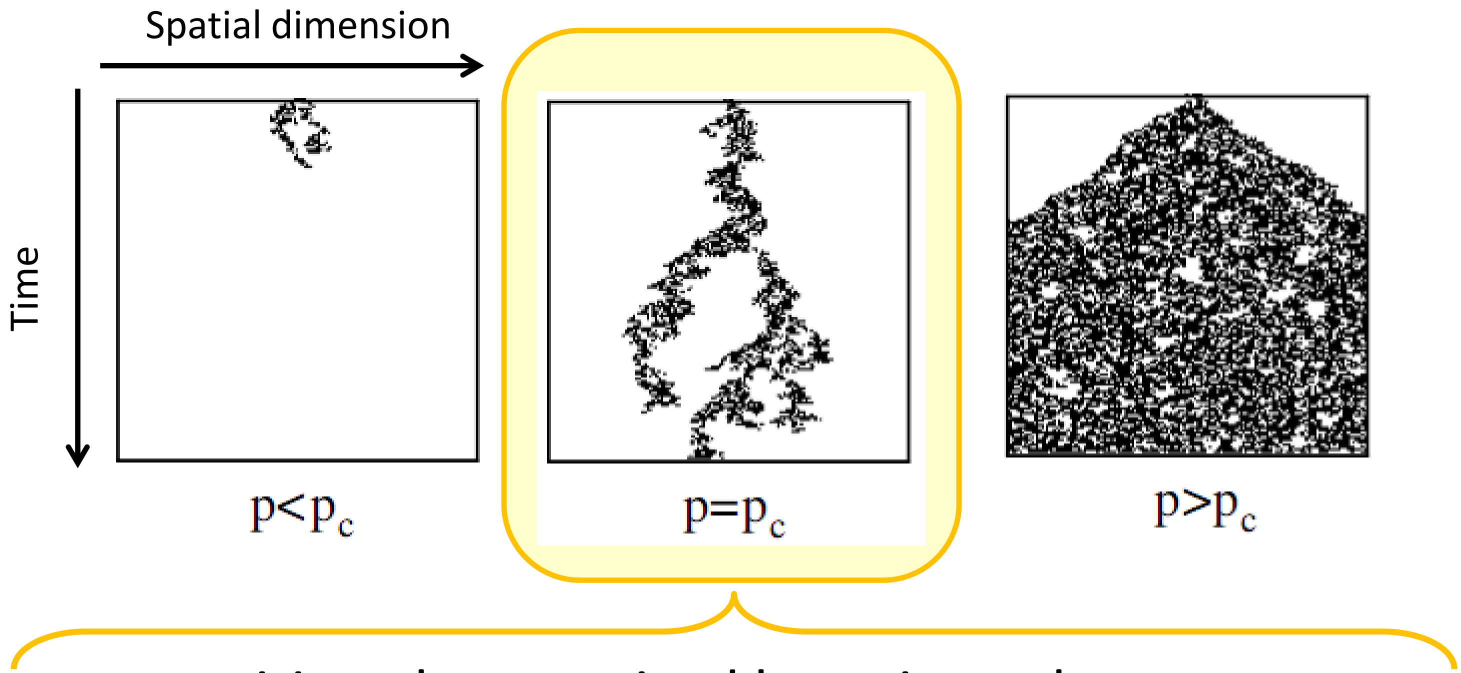




Hinrichsen, Adv. in Physics (2000)

Directed percolation transition

- A continuous phase transition occurs at p_c.

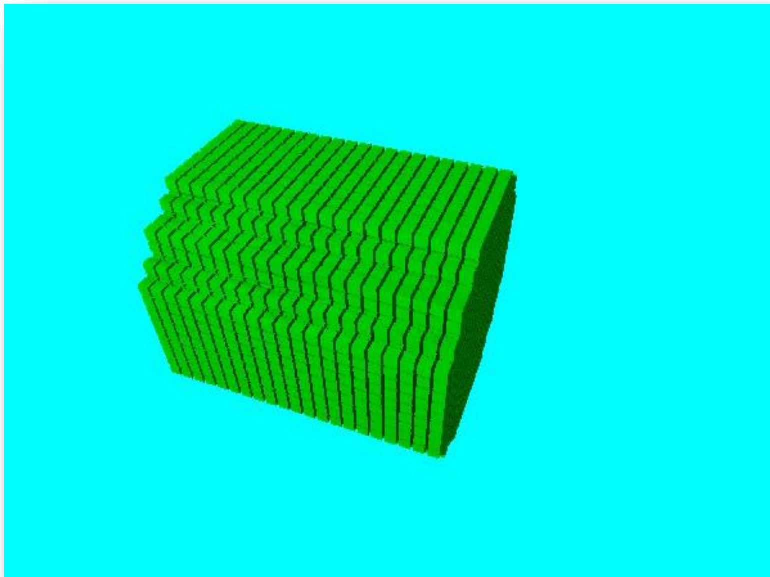


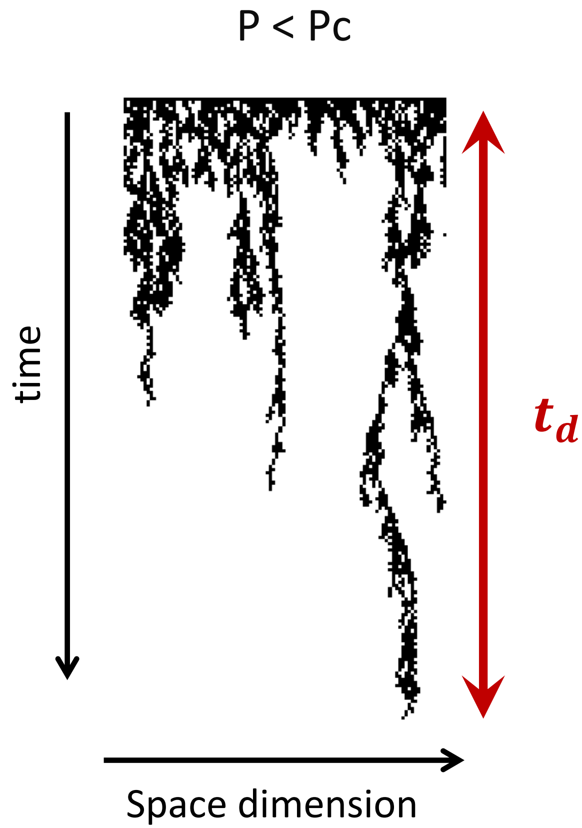
- Phase transition characterized by universal exponents:

$$\rho \sim (p - p_c)^{\beta} \qquad \xi_{\perp} \sim (p - p_c)^{-\nu_{\perp}} \qquad \xi_{\parallel} \sim (p - p_c)^{-\nu_{\parallel}}$$

Hinrichsen, Adv. in Physics (2000)

DP in 3 + 1 dimensions in pipe



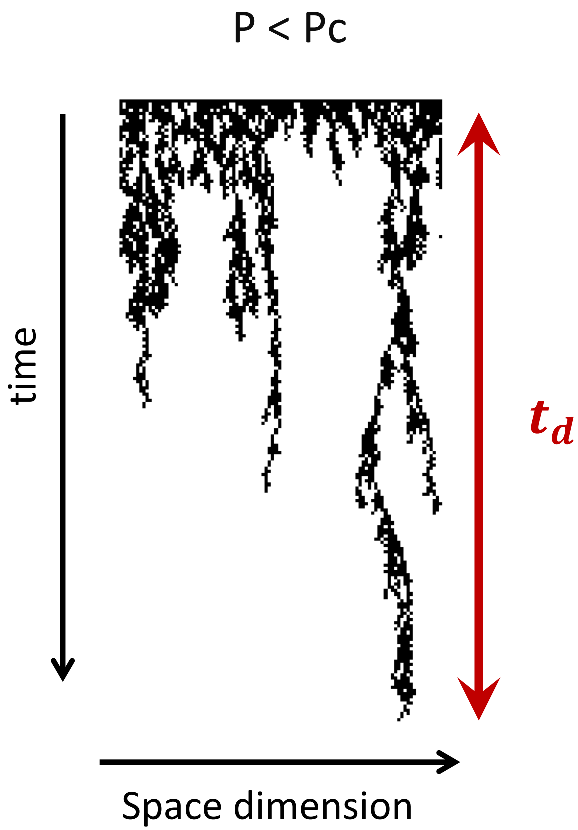


Puff decay 3 + 1

Puff decay 1 + 1

Sipos and Goldenfeld, *PRE* **84**, 035304(R) (2011)

Origin of superexponential scaling

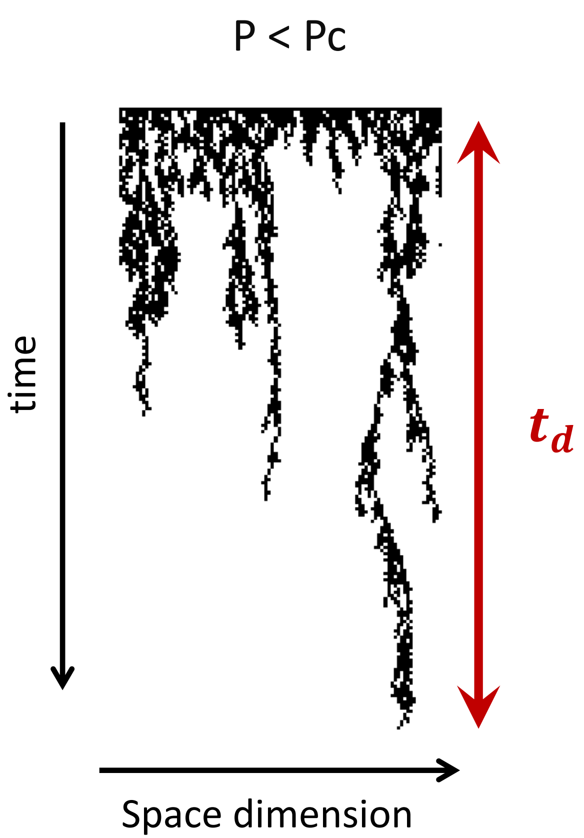


- Active state persists until **the most long-lived** percolating "strands" decay.
- In other words, **the longest path** = **the maximum** among **independent** random percolating paths

→ **Extreme value statistics**

Goldenfeld et al. (2010); Sipos & Goldenfeld (2011); Shih & Goldenfeld (in preparation)

Origin of superexponential scaling



- Active state persists until **the most long-lived** percolating "strands" decay.
- In other words, **the longest path** = **the maximum** among **independent** random percolating paths

→ **Extreme value statistics**

$$X_{\max} \propto \max(\{X_i\})$$

$$P(X_{\max} < y) = \exp\left(-\exp(-y)\right)$$

- Near phase transition: correlated fluctuations are NOT i.i.d.

Goldenfeld et al. (2010); Sipos & Goldenfeld (2011)

Universality class of predator-prey system near extinction

Basic individual processes in predator (A) and prey (B) system:

Death	$B_i \xrightarrow{d_B} E_i$	$A_i \xrightarrow{d_A} E_i$ $B_i \xrightarrow{m} A_i$
Birth	$B_i + E_j \xrightarrow[\langle ij \rangle]{b} B_i + B_j$	$A_i + B_j \xrightarrow[\langle ij \rangle]{p} A_i + A_j$
Diffusion	$B_i + E_j \xrightarrow[\langle ij \rangle]{D} E_i + B_j$	$A_i + E_j \xrightarrow[\langle ij \rangle]{D} E_i + A_j$
Carrying capacity	$B_i + B_j \xrightarrow[\langle ij \rangle]{c} B_i + E_j$	

Universality class of predator-prey system near extinction

Near the transition to prey extinction, the prey (B) population is very small and no predator (A) can survive; A ~ 0.

Death $B_i \xrightarrow{d_B} E_i$ ~~$A_i \xrightarrow{d_A} E_i$~~ ~~$B_i \xrightarrow{m} A_i$~~

Birth $B_i + E_j \xrightarrow[\langle ij \rangle]{b} B_i + B_j$ ~~$A_i + B_j \xrightarrow[\langle ij \rangle]{p} A_i + A_j$~~

Diffusion $B_i + E_j \xrightarrow[\langle ij \rangle]{D} E_i + B_j$ ~~$A_i + E_j \xrightarrow[\langle ij \rangle]{D} E_i + A_j$~~

Carrying capacity $B_i + B_j \xrightarrow[\langle ij \rangle]{c} B_i + E_j$

Universality class of predator-prey system near extinction

Near the transition to prey extinction, the prey (B) population is very small and no predator (A) can survive; A ~ 0.

Death $$B_i \xrightarrow{d_B} E_i$$

Birth $$B_i + E_j \xrightarrow[\langle ij \rangle]{b} B_i + B_j$$

Diffusion $$B_i + E_j \xrightarrow[\langle ij \rangle]{D} E_i + B_j$$

Carrying capacity $$B_i + B_j \xrightarrow[\langle ij \rangle]{c} B_i + E_j$$

Universality class of predator-prey system near extinction

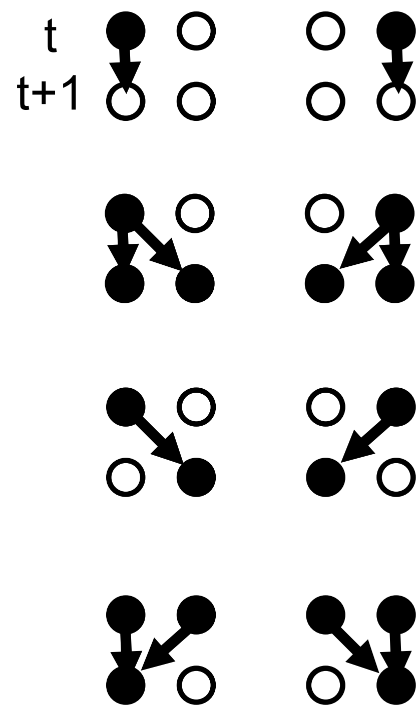
Near the transition to prey extinction, the prey (B) population is very small and no predator (A) can survive; A ~ 0.

Death $B_i \xrightarrow{d_B} E_i$ — Annihilation

Birth $B_i + E_j \xrightarrow[\langle ij \rangle]{b} B_i + B_j$ — Decoagulation

Diffusion $B_i + E_j \xrightarrow[\langle ij \rangle]{D} E_i + B_j$ — Diffusion

Carrying capacity $B_i + B_j \xrightarrow[\langle ij \rangle]{c} B_i + E_j$ — Coagulation



Universality class of predator-prey system near extinction

Near the transition to prey extinction, the prey (B) populatio… n
survive; …

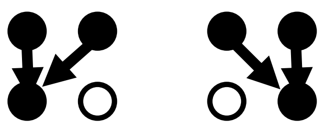
Near the extinction transition, stochastic predator-prey dynamics reduces to directed percolation

Death $B_i \xrightarrow{d_B}$ … …ation

Birth $B_i + E$ … …ulation

Diffusion $B_i + E_j$ … $\langle ij \rangle$ … …usion

Carrying capacity $B_i + B_j \xrightarrow[\langle ij \rangle]{c} B_i + E_j$

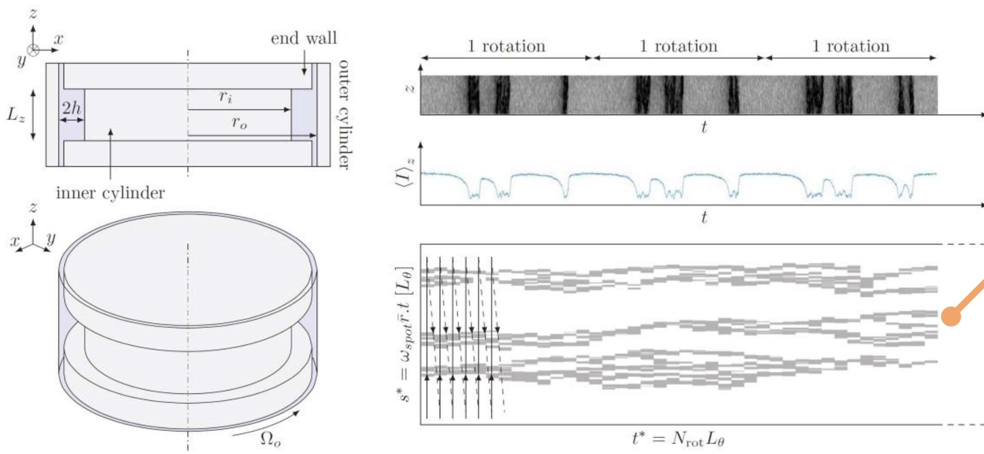


Coagulation

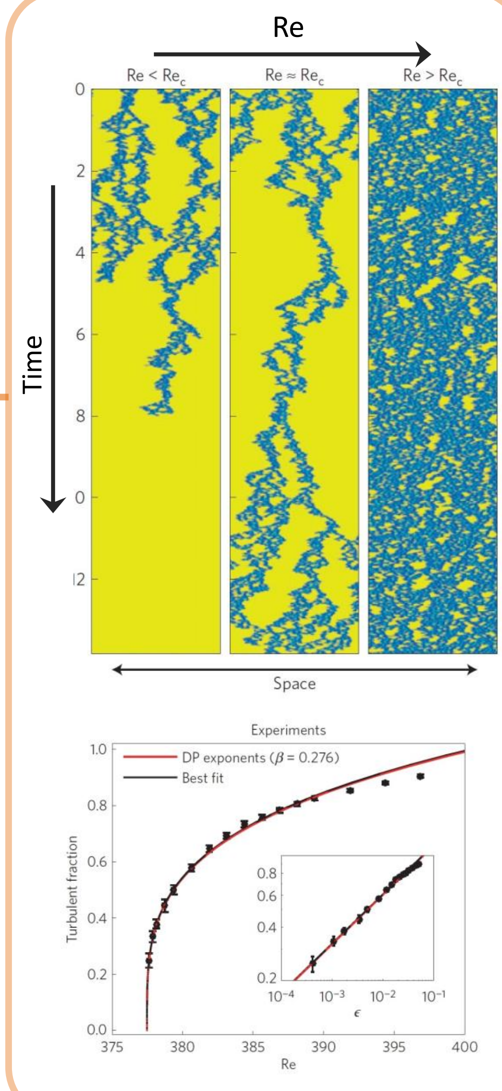
Experimental test of DP in transitional turbulence

Directed percolation in turbulence experiments

- Turbulent strands in **Couette flow** form long-time dynamics like (1+1) DP



Lemoult et al., *Nature Physics* (2016)



- Turbulent patches in **2D Taylor-Couette** form clusters like (2+1) DP

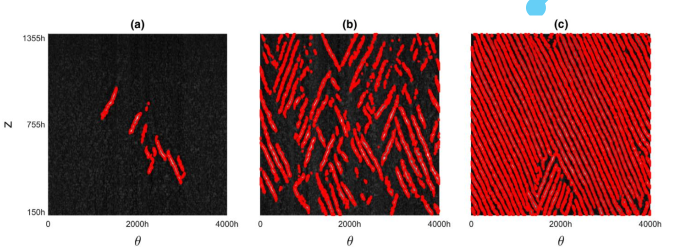
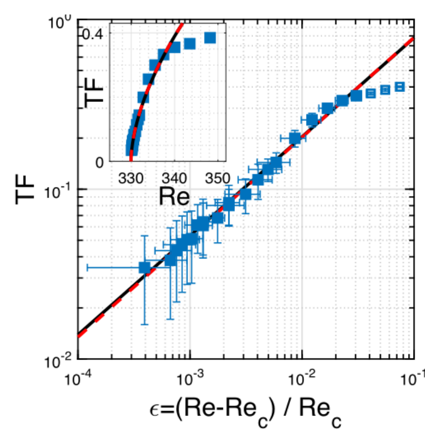


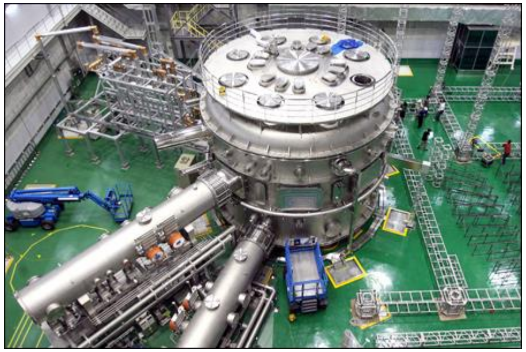
FIG. 2. Turbulent stripes in the axial-azimuthal cross section. Flow visualizations for (a) Re = 331, (b) Re = 333, and (c) Re = 349. Turbulent phase is marked by red contours. The laminar regions (black) expand as the Reynolds number is decreased and stripes become sparse.

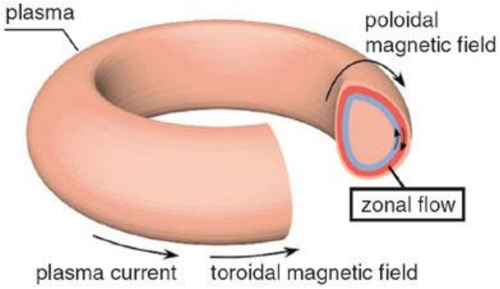


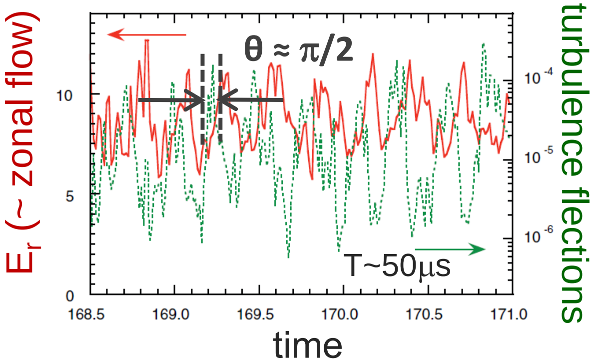
Klotz et al. PRL (2022)

Observation of predator-prey dynamics in magneto-hydrodynamics

- Low-High confinement transition in fusion plasmas in a tokamak
- Interplay between drift-wave turbulence along the axis of the tokomak and an azimuthal zonal flow that emerges from the turbulence and simultaneously suppresses it
- Observations follow theoretical predictions by P. Diamond and collaborators







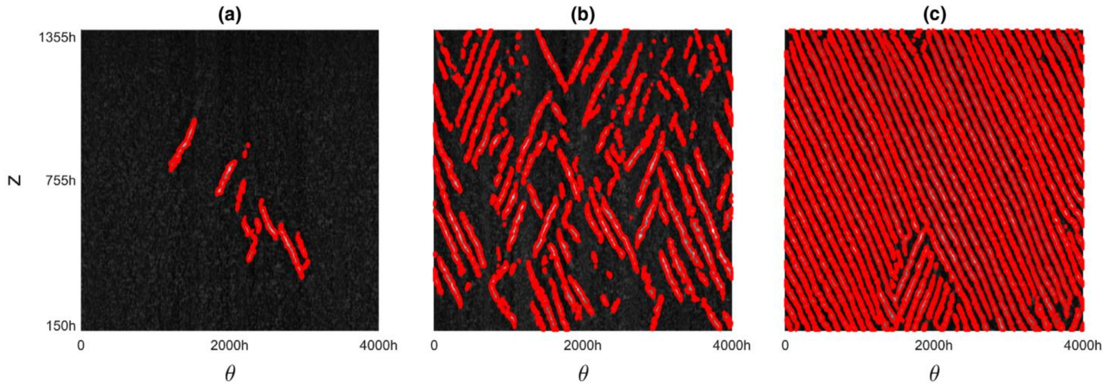
Diamond et al. *PRL (1994)*)   Estrada et al. *EPL* (2012)

http://alltheworldstokamaks.wordpress.com/gallery-of-external-views/kstar-completed/

Summary of transitional turbulence

Theoretical results predict that the laminar-turbulence transition is a non-equilibrium critical point in the universality class of directed percolation.

Confirmed in two Taylor-Couette flows D=1, D=2 experimentally.



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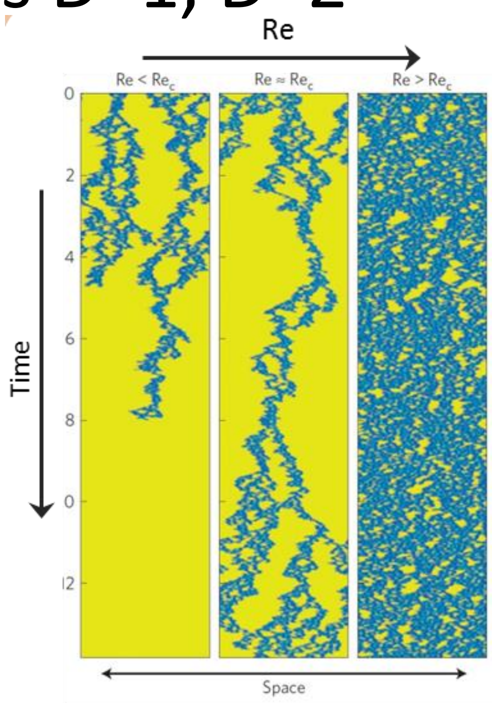


Phase transition with puff interactions

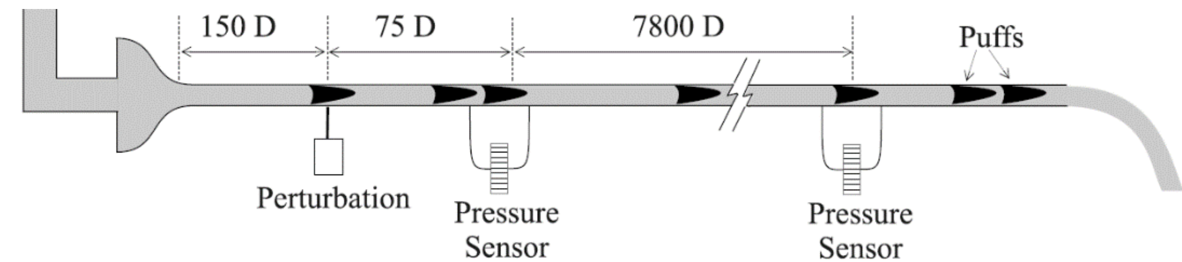
Grégoire Lemoult (Normandie U),

Mukund Vasudevan (ISTA), Jose M.Lopez (ISTA), Björn Hof (ISTA),

Gaute Linga (U of Oslo), Joachim Mathiesen (NBI)

Hong-Yan Shih (Academia Sinica), Nigel Goldenfeld (UCSD)

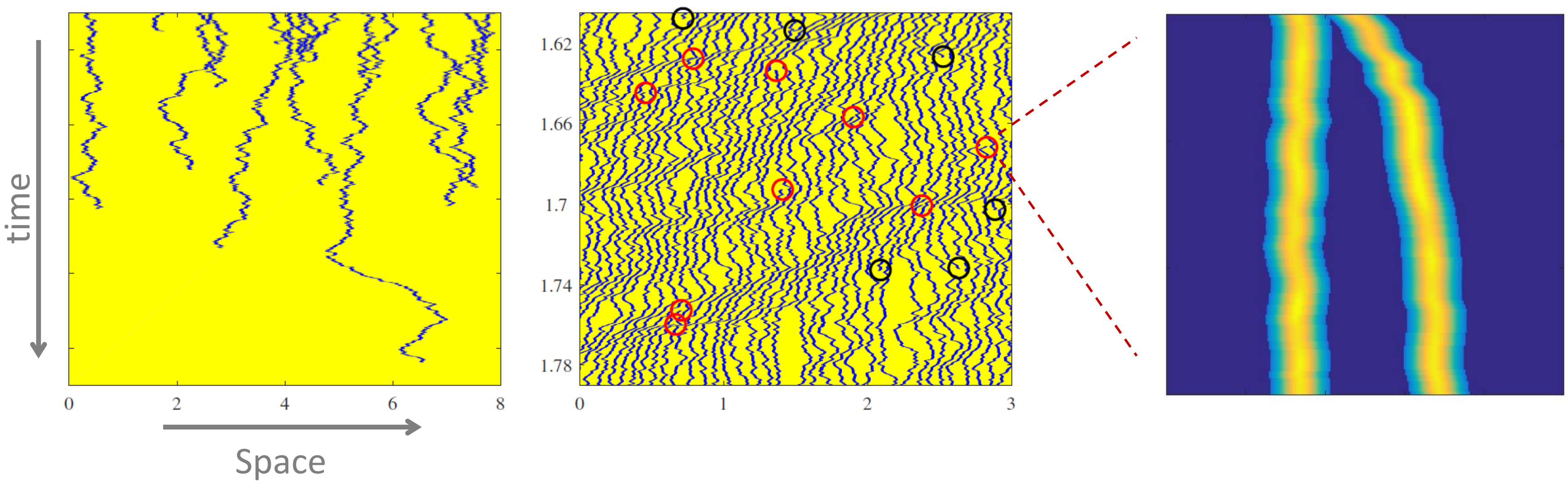
Pipe flow with interacting turbulent puffs



Masudevan & Hof (2018)

Re < Re_c

Re > Re_c

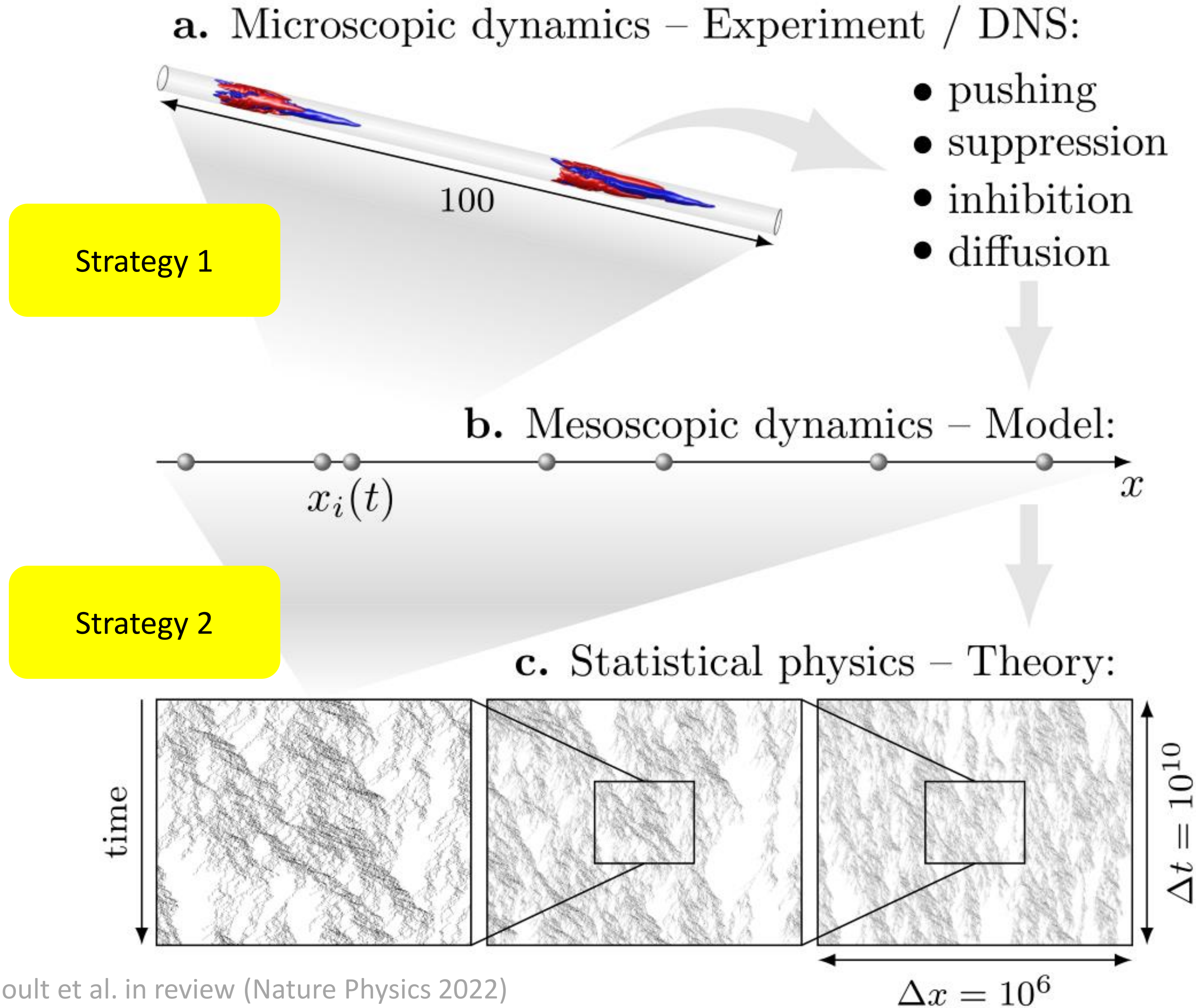


Q: Do puff interactions prevent the directed percolation transition in pipes?

Do puff interactions affect transition?

- Strategy 1:
  - Measure puff interactions accurately in experiment
  - Use the empirically-determined interactions as input to molecular dynamics calculations of puff dynamics
  - Measure turbulent fraction, correlations etc
  - Determine critical Re and scaling
- Strategy 2:
  - Abstractly represent puff interactions in a field theory model of stochastic puff dynamics
  - Are the puff interactions relevant at the fixed point controlling directed percolation scaling?

Lemoult et al. in review (Nature Physics 2022)



Lemoult et al. in review (Nature Physics 2022)

Why this is hard

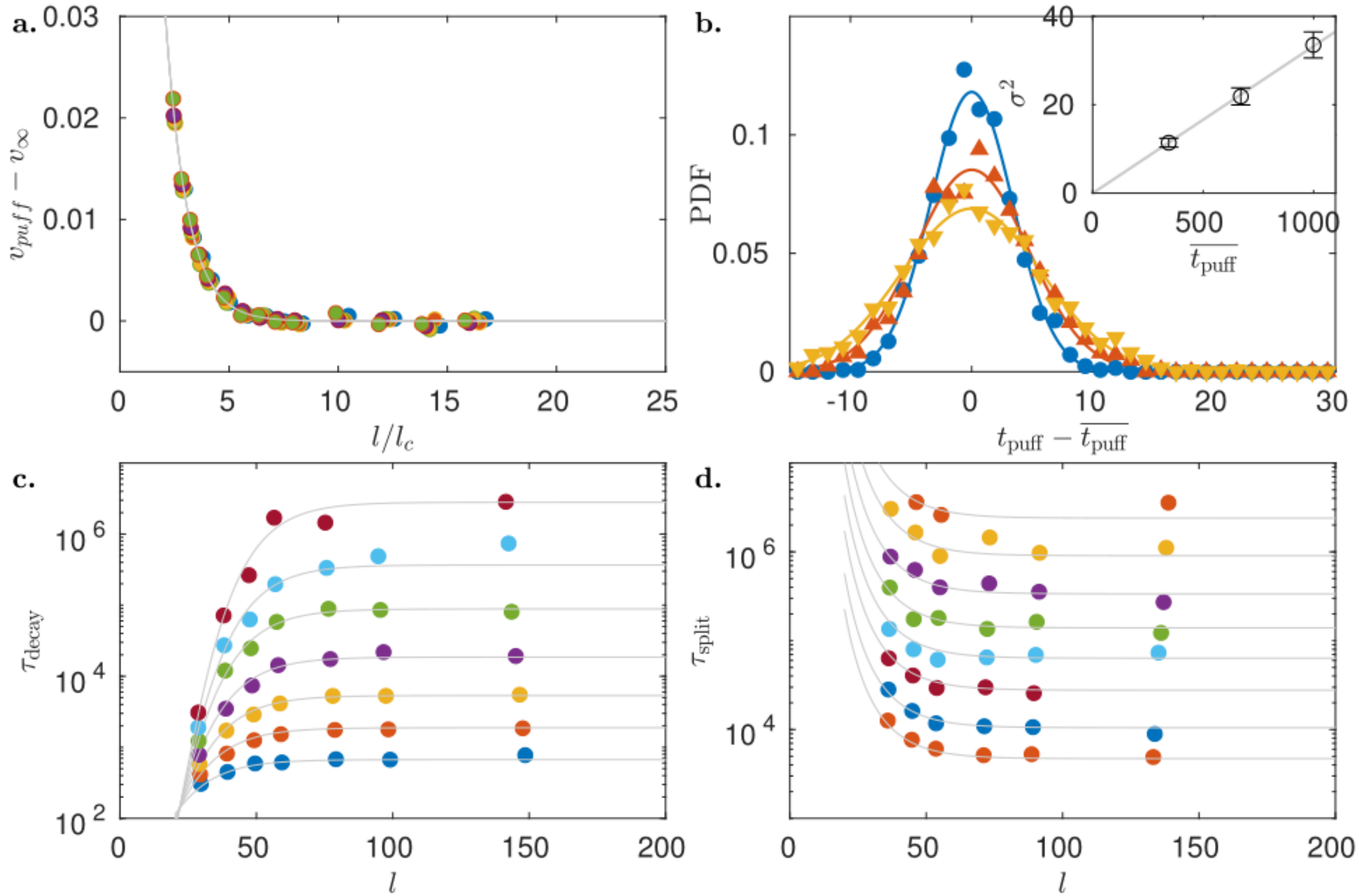
- Near critical points, need large system sizes and long times for equilibration

- We estimate that to get a single measurement of turbulent fraction near the critical point requires a pipe of length $L=10^6$ D and time $T=10^{11}$ advective time units
  - L = 4000 m
  - T = 100 years

- The critical regime is ~ 0.05% of Re_c. In jammed phase, relaxation is much faster, and we can estimate accurately turbulent fraction ~0.1% of Re_c.

Lemoult et al. in review (Nature Physics 2022)

Strategy 1: empirical puff interactions

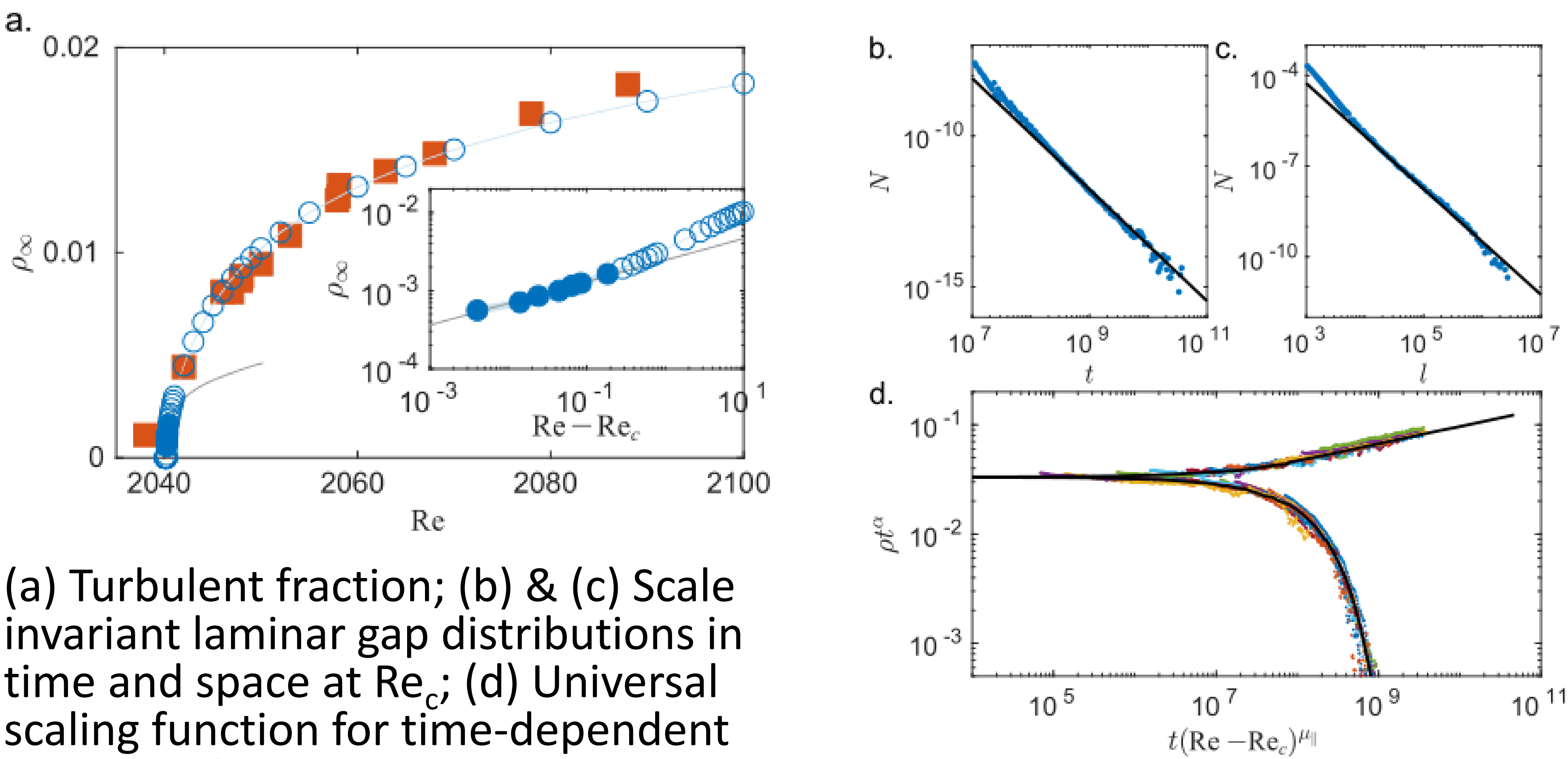


(a) Puff speed as function of distance to upstream puff; (b) Puff arrival times yield puff diffusion coefficient; (c) Decay time as function of distance to upstream puff; (d) Splitting time as function of distance to downstream puff.

Lemoult et al. in review (Nature Physics 2022)

Strategy 1: puff molecular dynamics

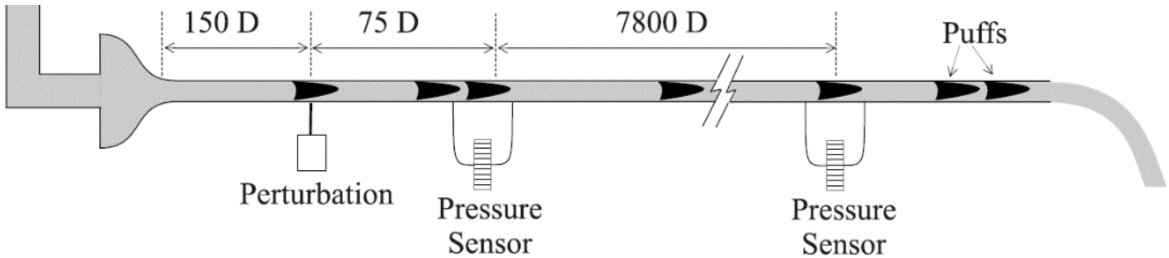
$$\frac{\mathrm{d}x_i}{\mathrm{d}t} = v_{\mathrm{puff}}(x_i - x_{i-1}) + \sqrt{D_{\mathrm{puff}}}\xi_i(t)$$ + splitting and decay



(a) Turbulent fraction; (b) & (c) Scale invariant laminar gap distributions in time and space at Re_c; (d) Universal scaling function for time-dependent turbulent fraction. Solid lines are DP exponents

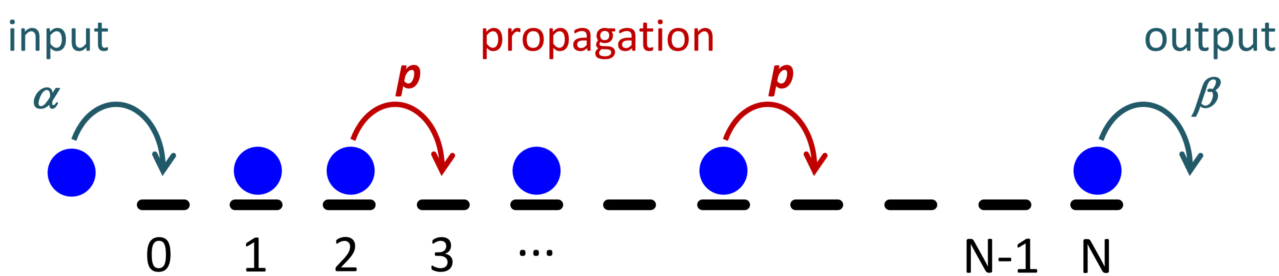
Lemoult et al. in review (Nature Physics 2022)

Strategy 2: Simple lattice model for interacting particles



Masudevan & Hof (2018)

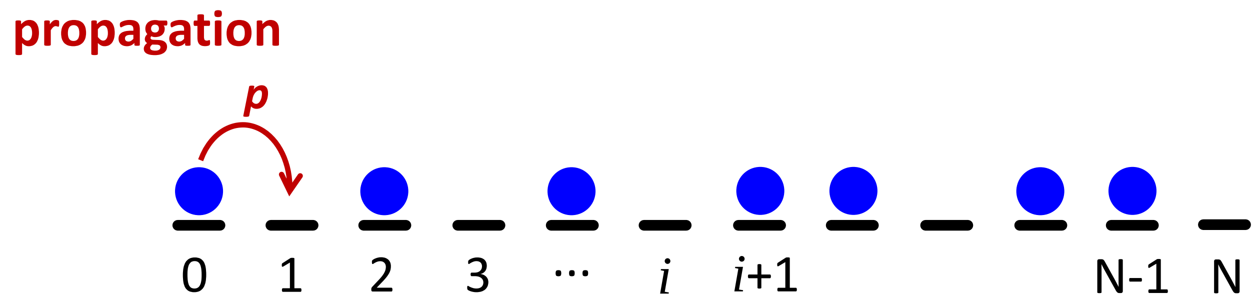
Lattice model: Totally Asymmetric Simple Exclusion Process (TASEP)



Idea: Turbulent puff in the pipe flow =

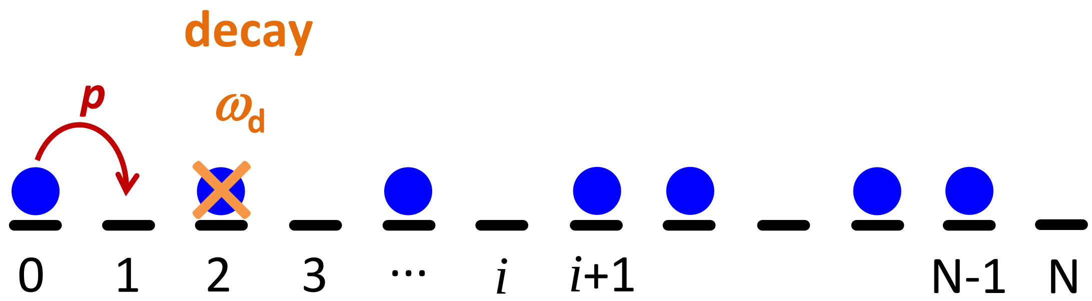
Lemoult et al. in review (Nature Physics, 2022)

Simple lattice model for interacting particles



Lemoult et al. in review (Nature Physics, 2022)

Simple lattice model for interacting particles



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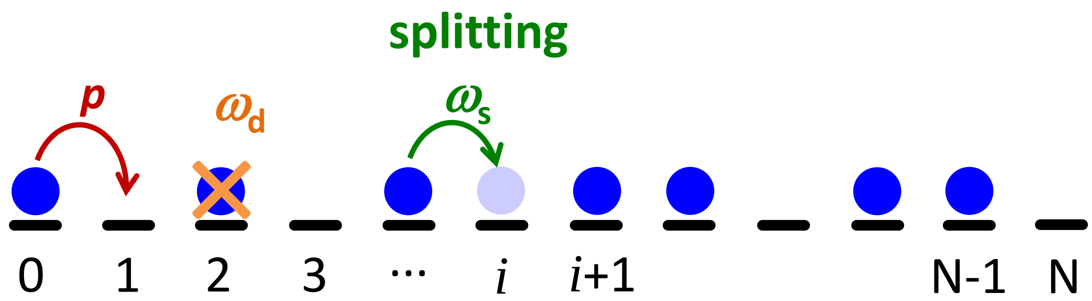

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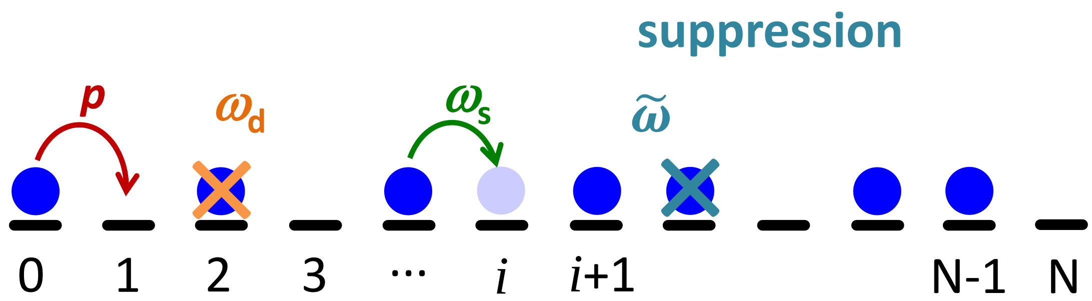

Lemoult et al. in review (Nature Physics, 2022)

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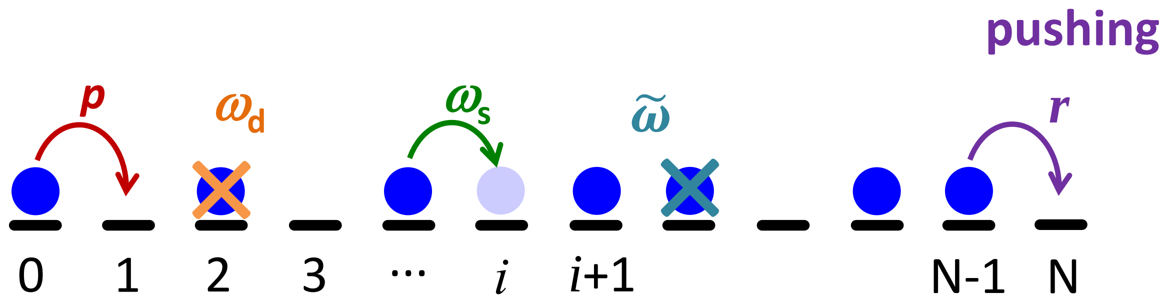

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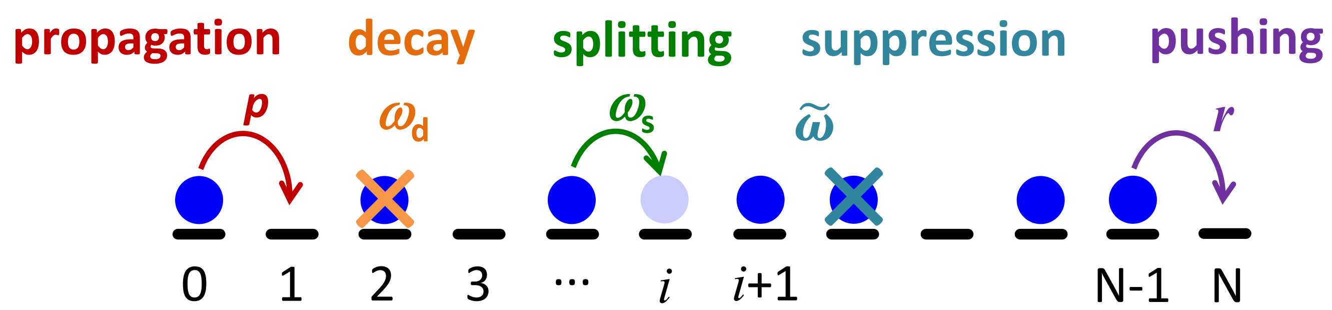
Simple lattice model for interacting particles



Mean-field equation

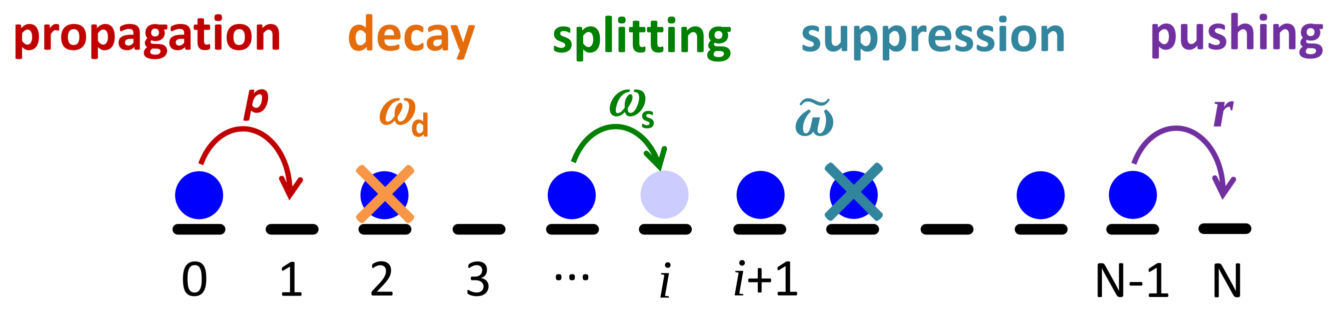
$$\partial_t \langle n_i \rangle = -p\langle n_i(1-n_{i+1})\rangle + p\langle n_{i-1}(1-n_i)\rangle - \omega_d \langle n_i\rangle + \omega_s \langle n_{i-1}(1-n_i)\rangle$$
$$-\tilde{\omega}\langle n_{i-1} n_i\rangle - r\langle n_i n_{i-1}(1-n_{i+1})\rangle + r\langle (1-n_i) n_{i-1} n_{i-2}\rangle$$

Stochastic hydrodynamics

$$\partial_t \rho = -\omega_d \rho + \frac{1}{2}(p+\omega_s)\partial_x^2 \rho - (p+\omega_s)\partial_x \rho + \omega_s \rho(1-\rho)$$
$$+(2p+\omega_s)\rho\partial_x\rho - \frac{\omega_s}{2}\rho\partial_x^2\rho + \sqrt{\rho(1-\rho)}\eta$$

Lemoult et al. in review (Nature Physics, 2022)

Simple lattice model for interacting particles



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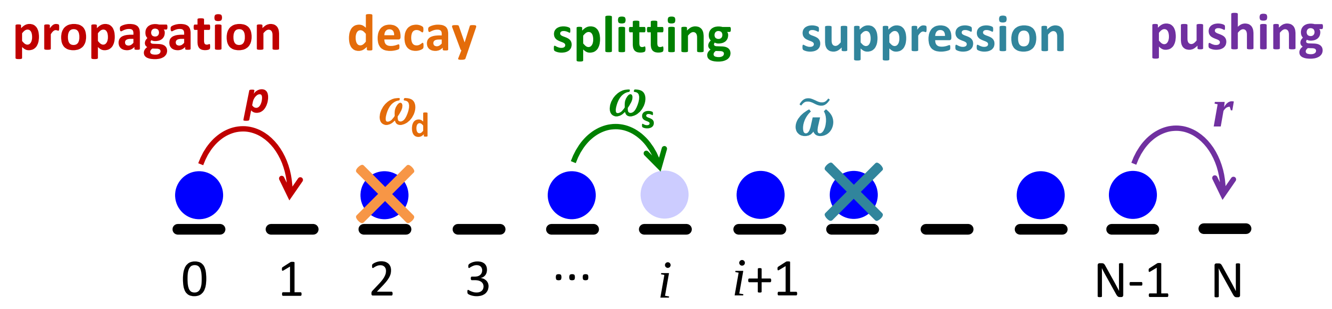
additional terms from interactions that are not in directed percolation

Lemoult et al. in review (Nature Physics, 2022)

Simple lattice model for interacting particles



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$$+(2p+\omega_s)\rho\partial_x\rho - \frac{\omega_s}{2}\rho\partial_x^2\rho + \sqrt{\rho(1-\rho)}\eta$$

Q: How do puff interaction terms affect **DP** universality class?

Lemoult et al. in review (Nature Physics, 2022)

Universality class for interacting puffs on lattice model

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Phase transition logic: **Renormalization group** analysis **around the DP fixed point**

Lemoult et al. in review (Nature Physics, 2022)

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Phase transition logic: **Renormalization group** analysis **around the DP fixed point**

(1) Start from DP path integral action:

$$A(\rho,\tilde{\rho}) = \int d^d x\, dt \left[\tilde{\rho}\left(\partial_t + D(r-\partial_x^2)\right)\rho - u_3 \tilde{\rho}(\rho-\tilde{\rho})\rho + u_4 \tilde{\rho}^2 \rho^2 + O(\rho^5)\right]$$

(2) Dimensional analysis:

(3) **Puff interacting model**: look at the general form of terms in the action

Lemoult et al. in review (Nature Physics, 2022)

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Q: How do puff interaction terms affect **DP** universality class?

$$\partial_t \rho = -\omega_d \rho + \frac{1}{2}(p+\omega_s)\partial_x^2 \rho \boxed{-(p+\omega_s)\partial_x \rho} + \omega_s \rho(1-\rho)$$
$$\boxed{+(2p+\omega_s)\rho\partial_x \rho - \frac{\omega_s}{2}\rho\partial_x^2 \rho} + \sqrt{\rho(1-\rho)}\eta$$

Phase transition logic: **Renormalization group** analysis **around the DP fixed point**

(1) Start from DP path integral action:

$$A(\rho,\tilde{\rho}) = \int d^d x\, dt\, \left[\tilde{\rho}\left(\partial_t + D(r-\partial_x^2)\right)\rho - u_3 \tilde{\rho}(\rho-\tilde{\rho})\rho + u_4 \tilde{\rho}^2 \rho^2 + O(\rho^5)\right]$$

(2) Dimensional analysis:

$$[x] = \mu^{-1},\ [t] = \mu^{-2},\ [\rho] = \mu^{d/2}, [D] = \mu^0,$$
$$[r] = \mu^2,\ [u_3] = \mu^{\boxed{2-d/2}},\ [u_4] = \mu^{2-d}$$

→ $d_c = 4$

(3) **Puff interacting model**: look at the general form of terms in the action

Lemoult et al. in review (Nature Physics, 2022)

Universality class for interacting puffs on lattice model

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(3) **Puff interacting model**: look at the general form of terms in the action

$$I_{\alpha\beta\gamma} \equiv \lambda_{\alpha\beta\gamma}\rho^\alpha \left(\partial_x^\beta \rho\right)^\gamma \ ,\ \ [\lambda_{\alpha\beta\gamma}] = \mu^{y_{\alpha\beta\gamma}}$$ with $y_{\alpha\beta\gamma} = 2(2-\alpha-\gamma) - \beta\gamma$

e.g. $[\lambda_1 \rho \partial_x \rho] = \mu^{2+d/2}$, $\alpha=\beta=\gamma=1$ $\Rightarrow y_{\alpha\beta\gamma} < 0$ **irrelevant at DP critical point**

Lemoult et al. in review (Nature Physics, 2022)

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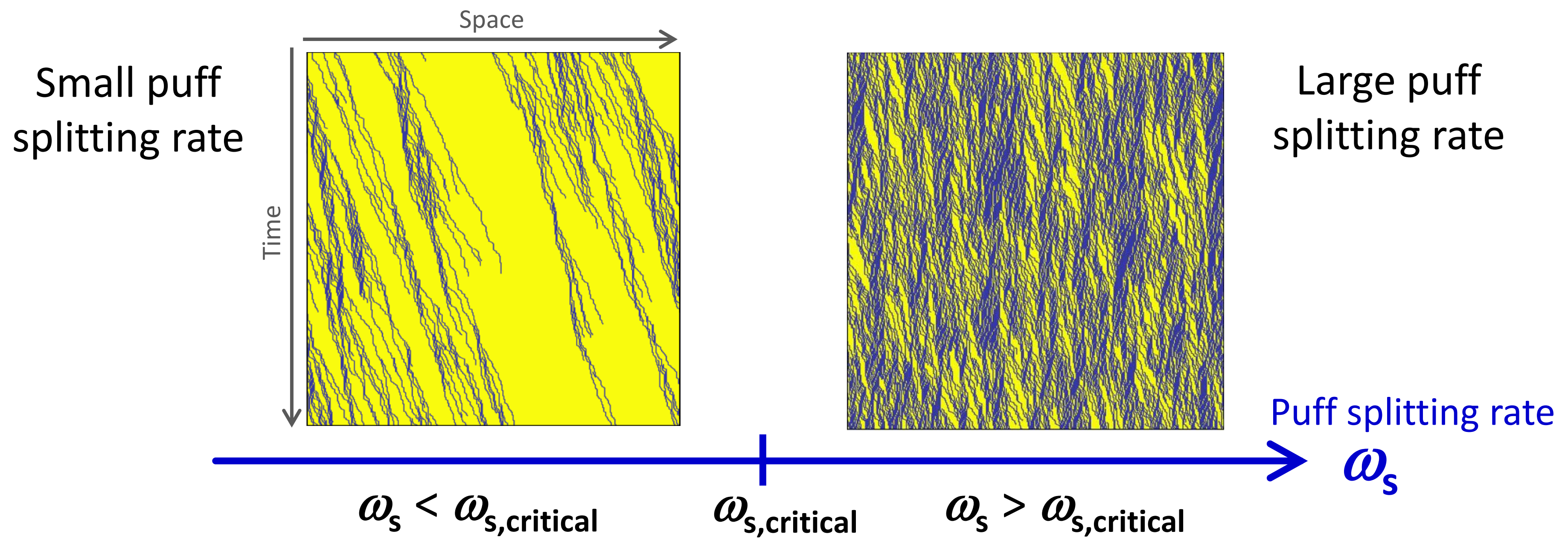
(3) **Puff interacting model**: look at the general form of terms in the action

A: All puff interaction terms are irrelevant at the **DP** fixed point.

Transitional pipe turbulence with interacting puffs = DP

Lemoult et al. in review (Nature Physics, 2022)

Numerical results of simple lattice model for interacting particles



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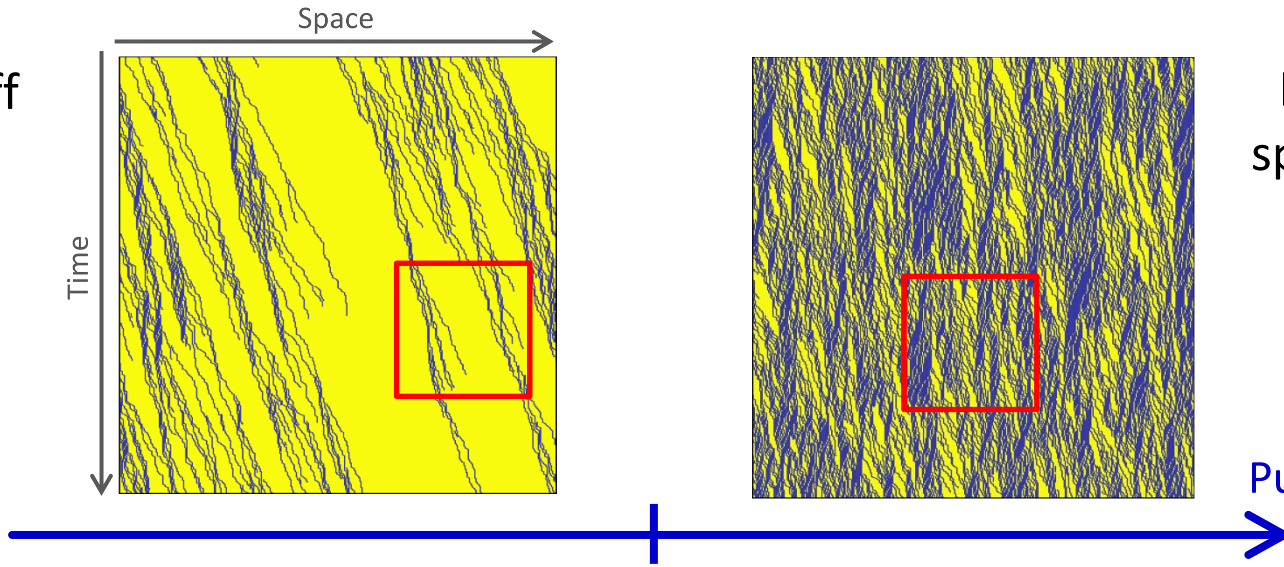

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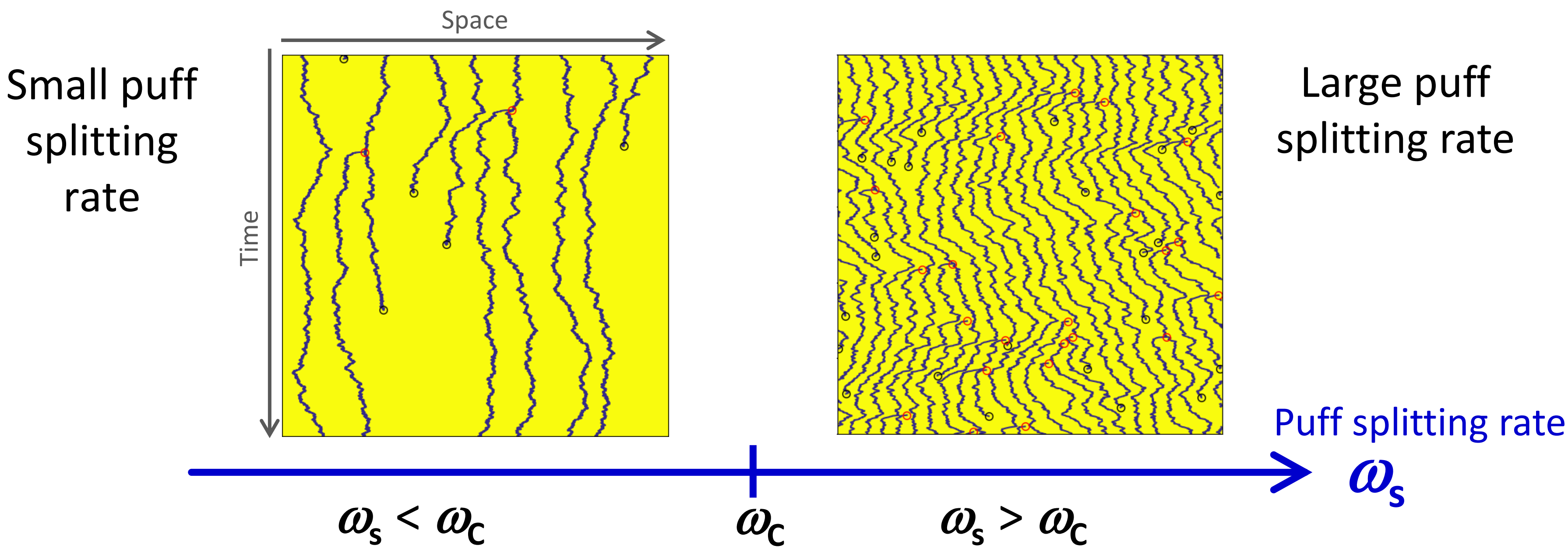

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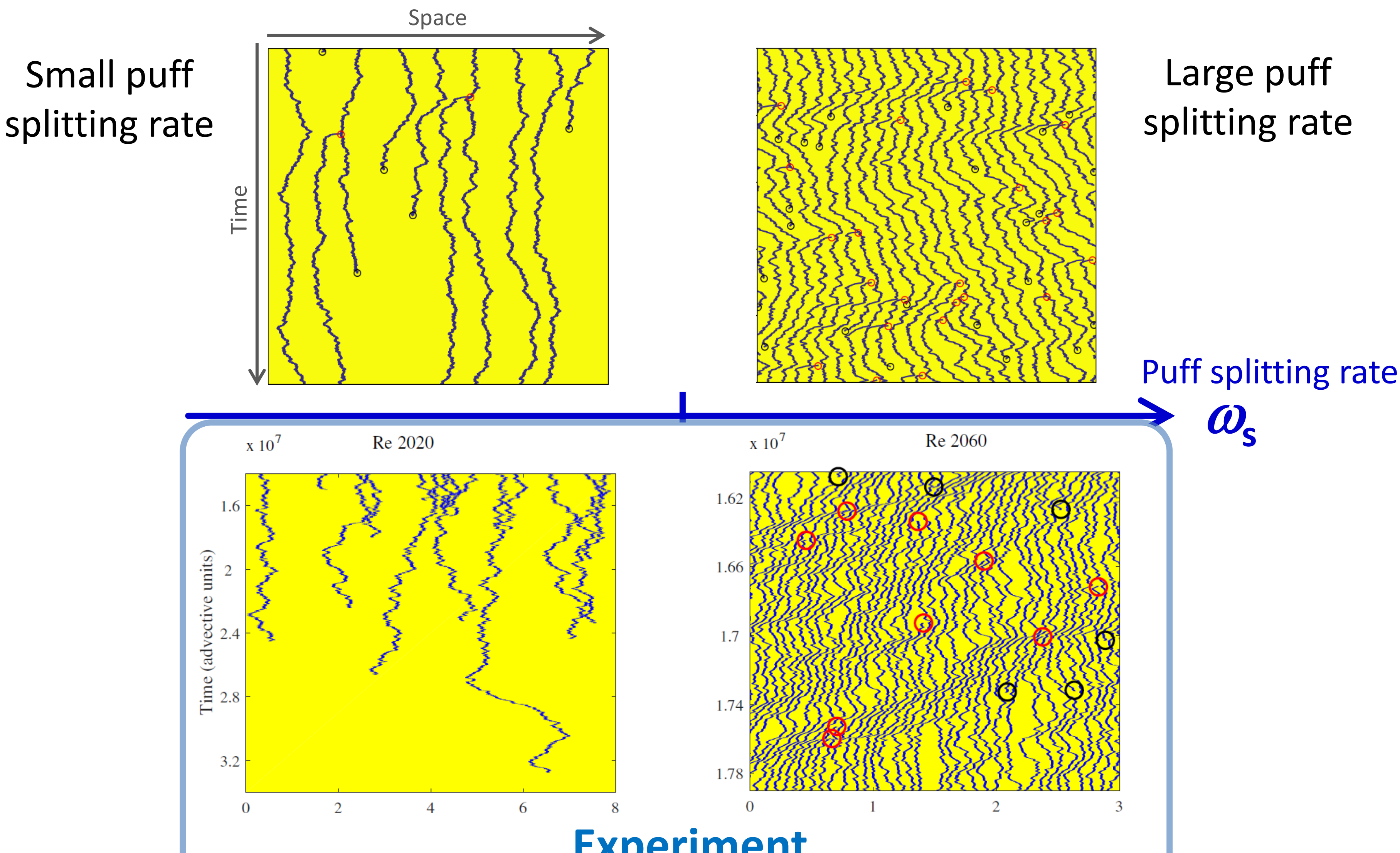
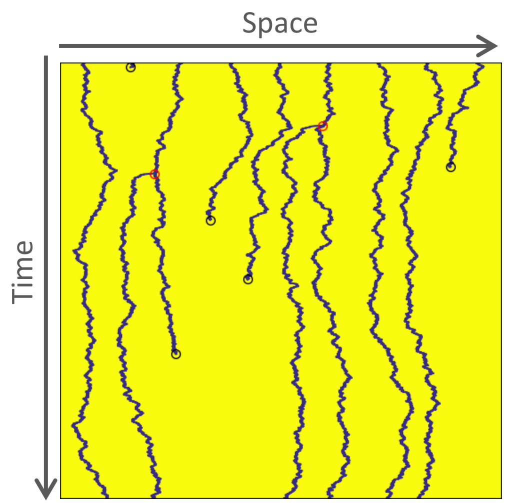
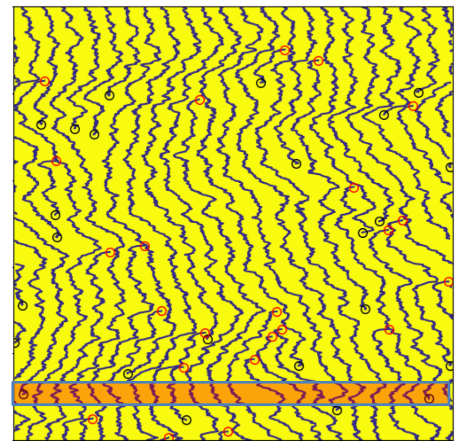


Lemoult et al. in review (Nature Physics, 2022)

Numerical results of simple lattice model for interacting particles

Small puff splitting rate

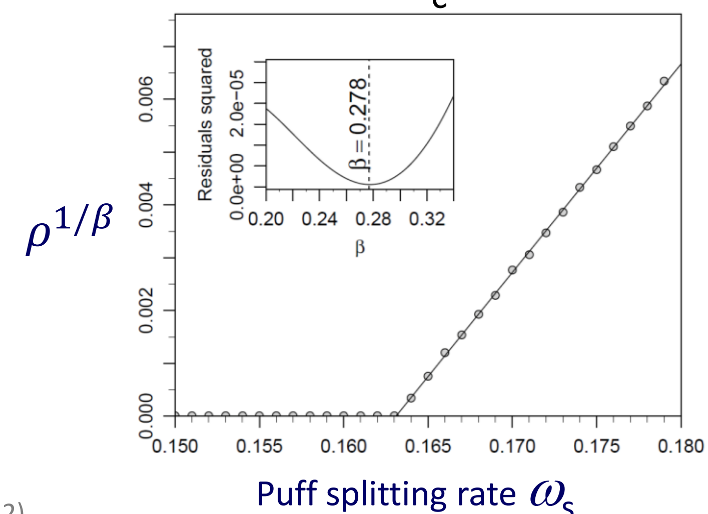




Large puff splitting rate

Puff splitting rate ω_s

ω_c



Steady state density at T ≫ 1

$$\langle \rho \rangle = \frac{1}{L} \left\langle \sum_i^L n_i \right\rangle$$

$$\langle \rho \rangle \sim (\omega_s - \omega_c)^{-\beta}$$

$\beta \sim 0.276$ (1+1 DP exponent)

Lemoult et al. in review (Nature Physics, 2022)

Summary

- Puff interactions lead to traffic jams and local regions of puff crystallization.
  - Close to Re_c the crystals melt due to fluctuations and the DP universality class is observable.
    - Turbulent fraction
    - Laminar gap exponents at Re_c
    - Universal scaling functions
  - Precise measurement of critical behavior emerging from a purely fluid dynamical system!

Outline

1. Laminar-turbulence transition
   – Puff decay and splitting lifetimes
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   – Length scale where thermal fluctuations are relevant
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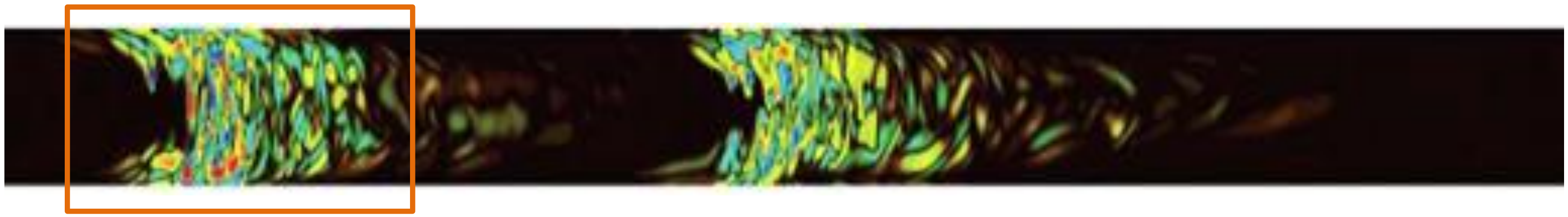
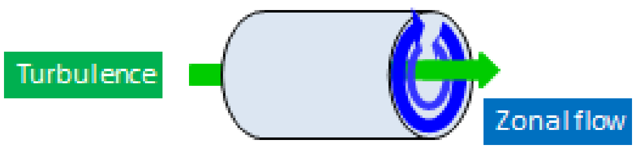
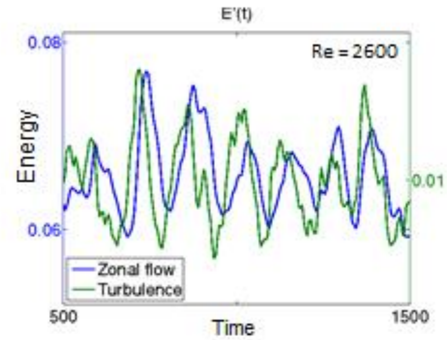
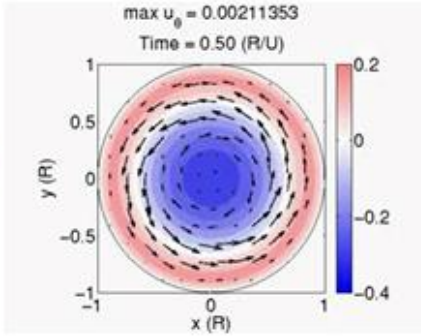
Beyond the critical point

Transitional turbulence





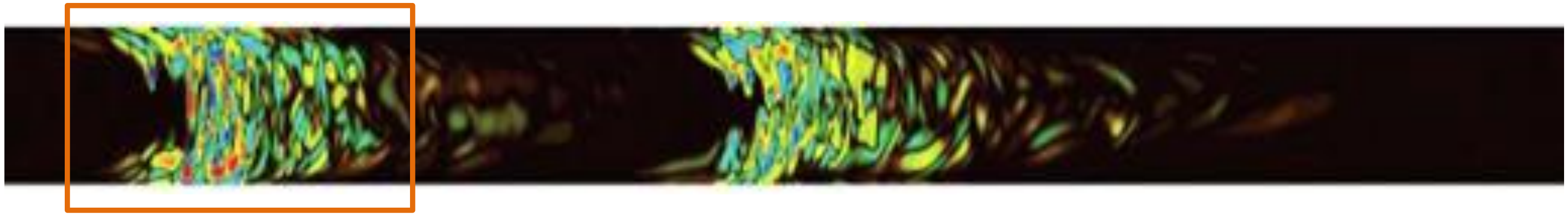


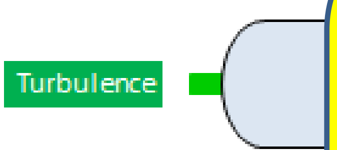


- Dynamics of single puff
  - Nascent turbulence generates an emergent mean flow (zonal flow)
  - Zonal flow inhibits the turbulence by shearing
  - "Predator-prey" oscillations
  - Long-wavelength fluctuations of soft mode -> second order phase transition
- Single puff spatial extent is actually determined by the transfer of laminar shear into turbulence

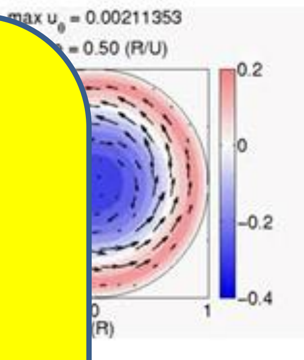
H.-Y. Shih et al. (2016)

Transitional turbulence





What happens if we include mean shear as well?



- Dynamics of
  - Nascent t … flow)
  - Zonal flow
  - “Predator
  - Long-wavele … phase transition
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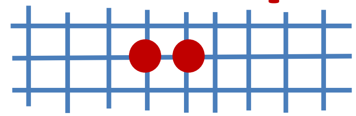
H.-Y. Shih et al. (2016)

The three trophic-level predator-prey model

A: predator(zonal flow) B: prey(turbulence)
N: nutrient(mean flow energy) Φ:null



Predation $A_i + B_j \xrightarrow[<ij>]{p} A_i + A_j$

Diffusion $A_i \xrightarrow[<ij>]{D_A} A_j$ $B_i \xrightarrow[<ij>]{D_B} B_j$

Death $A_i \xrightarrow{d_A} \phi_i$ $B_i \xrightarrow{d_B} \phi_i$

Competition $2A_i \xrightarrow{c_A} A_i$ $2B_i \xrightarrow{c_B} B_i$

Mutation $B_i \xrightarrow{m} A_i$

Two trophic-level reactions

New reactions

Birth $B_i + N_j \xrightarrow[<ij>]{b} B_i + B_j$

Recovery of nutrient $\phi_i \xrightarrow{g} N_i$

Advection of nutrient $N_i \xrightarrow[velocity=U]{prob.=1} N_{i+1}$

X. Wang, H.-Y. Shih, NG PRL 2022

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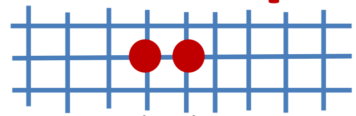

Nearest neighbor sites

Predation $A_i + B_j \xrightarrow[<ij>]{p} A_i + A_j$

Diffusion $A_i \xrightarrow[<ij>]{D_A} A_j$ $B_i \xrightarrow[<ij>]{D_B} B_j$

Death $A_i \xrightarrow{d_A} \phi_i$ $B_i \xrightarrow{d_B} \phi_i$

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Advection of nutrient $N_i \xrightarrow[velocity=U]{prob.=1} N_{i+1}$

$$Re = \frac{UD}{\upsilon}$$

Various *Re* number can be reached by changing the advection speed

X. Wang, H.-Y. Shih, NG PRL 2022

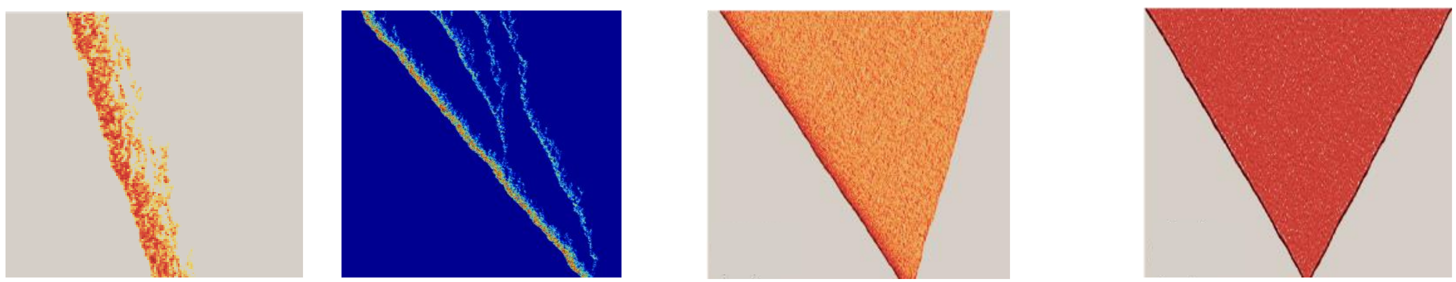
Summary of transitional turbulence

laminar	metastable puffs	spatiotemporal intermittency	weak slugs	strong slugs

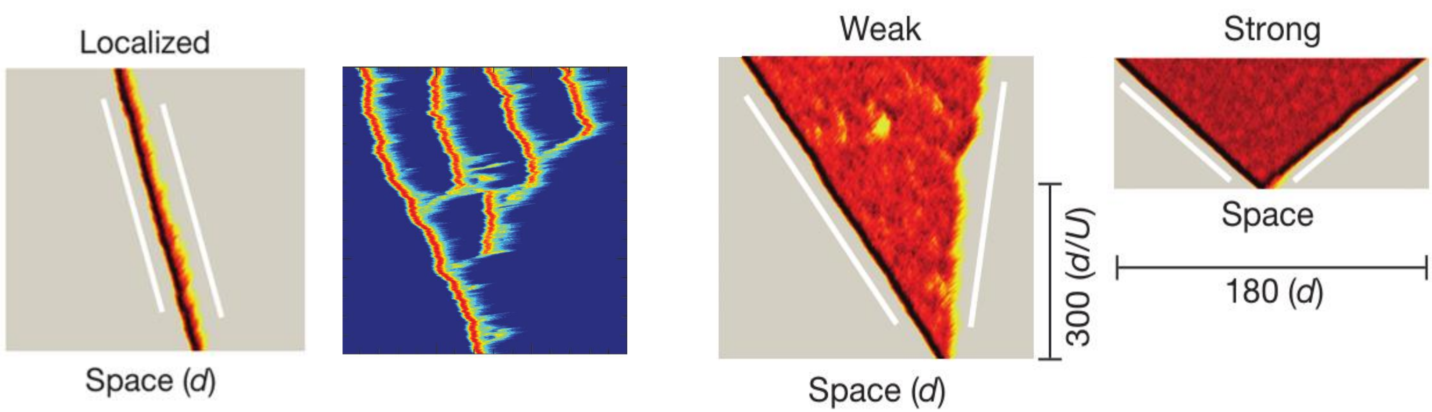
Re: 1775, 2050, 2250, 4500

Stochastic model

X. Wang, H.-Y. Shih, NG PRL 2022

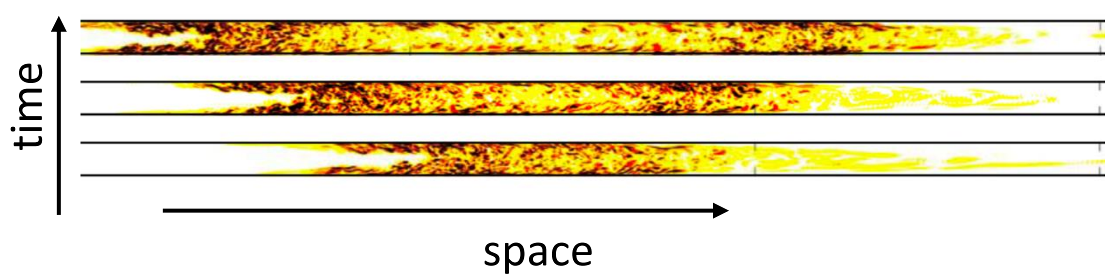


Navier-Stokes DNS



D. Barkley et al. Nature 2015

D. Barkley, JFM 2016



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Thermal fluctuations and spontaneous stochasticity

Bandak et al. submitted to PNAS; Bandak et al. PRE 2022

David Ruelle, “Microscopic fluctuations and turbulence”, Physics Letters A, 72 (2), 81-82 (1979)

$$t \sim \omega(l_0)^{-1} \log((\nu/c) N^{1/2} l_0^{1/2}) \quad (4)$$

The logarithm in (4) is not very sensitive to its argument. For air we have $\nu = 0.16\ \mathrm{cm}^2\ \mathrm{s}^{-1}$, $c = 34 \times 10^3\ \mathrm{cm\ s}^{-1}$, $N = 25 \times 10^{18}\ \mathrm{cm}^{-3}$. If we take $l_0 = 1$ cm, we get

$$\log((\nu/c) N^{1/2} l_0^{1/2}) = 10$$

(and also, using (1), $\omega(l_0)^{-1} = 6.25$ s). Therefore (4) implies that the time it takes for thermal fluctuations to affect the macroscopic motion is of the order of the characteristic time associated with the smallest eddies, multiplied by 10. For larger eddies the result would not be very different.

Edward Lorenz, “The predictability of a flow which possesses many scales of motion,” Tellus XXI (3), 289-307 (1969)

It is proposed that certain formally deterministic fluid systems which possess many scales of motion are observationally indistinguishable from indeterministic systems; specifically, that two states of the system differing initially by a small “observational error” will evolve into two states differing as greatly as randomly chosen states of the system within a finite time interval, which cannot be lengthened by reducing the amplitude of the initial error.

Thermal noise and Navier-Stokes

Thermal noise is unavoidable. Its effects on turbulent flows can be modeled by the Landau-Lifschitz fluctuating hydrodynamic equations:

$$\partial_t \mathbf{u} + P_\Lambda(\mathbf{u}\cdot\nabla)\mathbf{u} = -\nabla p + \nu\Delta\mathbf{u} + \sqrt{\frac{2\nu k_B T}{\rho}}\nabla\cdot\boldsymbol{\eta} + \mathbf{f}, \nabla\cdot\mathbf{u} = 0$$

$$\langle \eta_{ij}(\mathbf{x},t)\eta_{kl}(\mathbf{x}',t')\rangle = \left(\delta_{ik}\delta_{jl} + \delta_{il}\delta_{jk} - \frac{2}{3}\delta_{ij}\delta_{kl}\right)\delta^3_\Lambda(\mathbf{x}-\mathbf{x}')\delta(t-t')$$

An ultraviolet cutoff Λ is physically required in these equations, where cutoff length Λ^{-1} must be chosen between the Kolmogorov length $\eta=\nu^{3/4}/\varepsilon^{1/4}$ and mean-free-path λ_{mfp}.

Non-dimensionalizing with integral length L and r.m.s. velocity $U=(\varepsilon L)^{1/3}$

$$\partial_t \mathbf{u} + P_{\hat{\Lambda}}(\mathbf{u}\cdot\nabla)\mathbf{u} = -\nabla p + \frac{1}{Re}\Delta\mathbf{u} + \frac{1}{Re^{15/8}}\sqrt{2\theta_K}\nabla\cdot\boldsymbol{\eta} + \mathfrak{f}_L\mathbf{f}$$

with dimensionless number groups

$$Re = \frac{UL}{\nu}, \theta_K = \frac{k_B T}{\rho\nu^{11/4}\varepsilon^{-1/4}}, \mathfrak{f}_L = \frac{f_{rms}L}{U}, \hat{\Lambda} = \Lambda L \in [Re^{3/4}, Re/Ma]$$

Typically $\theta_K \sim 10^{-8}$ so thermal noise is very weak and not directly relevant in the inertial range.

Bandak et al. submitted to PNAS; Bandak et al. PRE 2022

Thermal noise in far dissipation range

θ_η is the ratio of the thermal energy to the energy of the Kolmogorov-scale velocity fluctuations u_η in a spatial region of diameter $\sim\eta$.

Atmospheric boundary layer

$$\varepsilon = 400 \text{ cm}^2/\text{s}^3, \quad \nu = 0.15 \text{ cm}^2/\text{s},$$

$$\rho = 1.2 \times 10^{-3} \text{ g/cm}^3, \quad T = 300^\circ\text{K}$$

$$\eta = 0.54 \text{ mm}, \quad u_\eta = 2.78 \text{ cm/s}, \quad \theta_\eta = 2.83 \times 10^{-8}$$

Below the Kolmogorov scale $u_\ell \sim u_\eta \exp(-\eta/\ell)$

$$\theta_\ell = \frac{k_B T}{\rho u_\ell^2 \ell^3} \sim \theta_\eta \left(\frac{\eta}{\ell}\right)^3 \exp(\eta/\ell)$$

$$\theta_\ell \sim 1 \qquad\qquad \boxed{\ell_{\text{eq}} \sim \eta/11}$$

- For atmospheric boundary this length is 49 microns >> the mean free path of 68 nm!
- This result implies that stochastic Landau-Lifshitz equations must be used in far dissipation range, not Navier-Stokes
- This conclusion was first made by R. Betchov (1957) in a long-forgotten paper

Bandak et al. submitted to PNAS; Bandak et al. PRE 2022

Thermal noise in far dissipation range

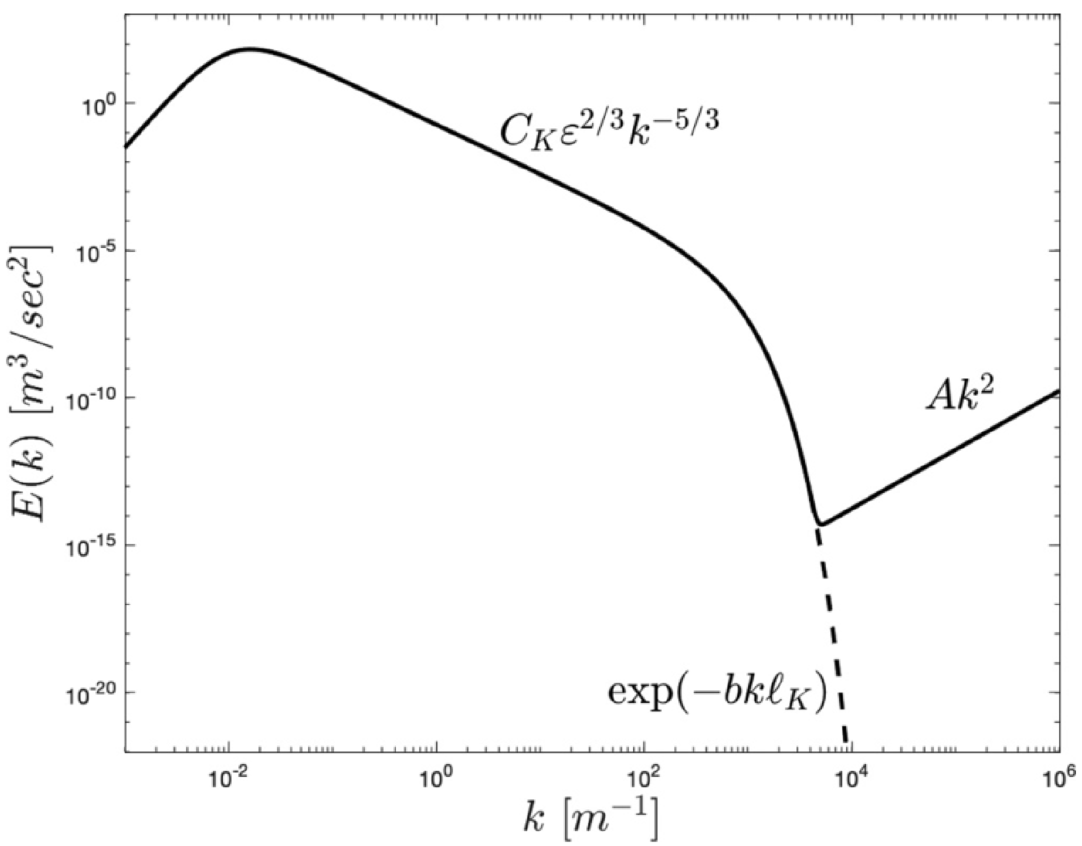


FIG. 1. Plot of the model turbulent energy spectrum Eq. (2.18) for the parameters Eq. (2.11) of the atmospheric boundary layer and for a typical Reynolds number Re $= 10^7$, as a solid line. The dashed line is the spectrum with no thermal noise.

Bandak et al. submitted to PNAS; Bandak et al. PRE 2022

Thermal noise washes out dissipation range intermittency

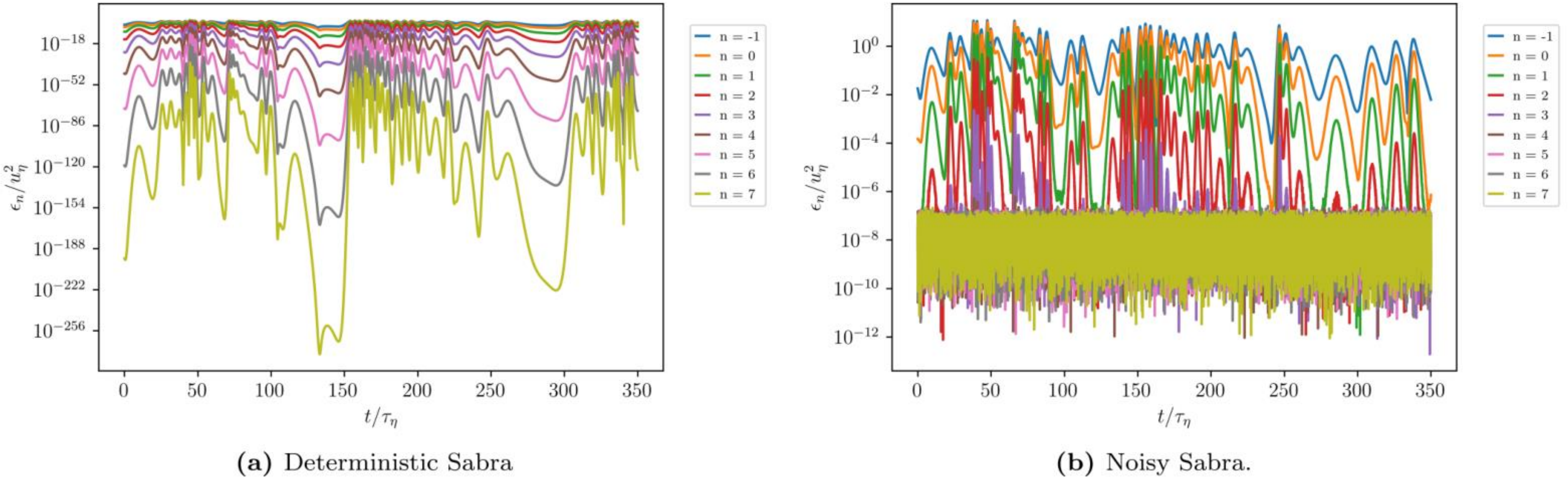


FIG. 7. Plots of modal energies $\epsilon_n(t)$ versus time t for shells in the dissipation range with $n = -1$ to $n = 7$. (a) The deterministic Sabra model results, exhibiting the extreme intermittency predicted by Kraichnan [39], and (b) the stochastic model results, with the large intermittent fluctuations completely washed out by Gaussian thermal fluctuations.

- Inertial range intermittency can cause the thermal equipartition scale to fluctuate!
- Even still, it remains 100 x mean free path of air

Bandak et al. submitted to PNAS; Bandak et al. PRE 2022

Spontaneous stochasticity: Historical origins

- Insight from unpredictability of weather due to Lorentz

- Sensitivity of solutions of DEs to change in initial conditions

- **Scenario 1**: The difference between two copies of the system can be made arbitrarily small by sufficiently reducing the initial separation

- **Scenario 2** The states diverge in finite time, no matter how close the initial conditions are taken

Chaos **Determinism**

Spontaneous stochasticity

- Exponential separation of trajectories
- ”Effective stochasticity”

- Intrinsic stochasticity

E. N. Lorenz, “The predictability of a flow which possesses many scales of motion,” Tellus, vol. 21, no. 3, pp. 289–307, 1969.

Lagrangian spontaneous stochasticity

Lagrangian particle in noisy medium

$$\frac{dR_i(t)}{dt} = v_i^{(\nu)}(\vec{R}, t) + \sqrt{2D}\eta(t)$$

$v_i^{(\nu)}(\vec{R}, t)$ is a solution of Navier-Stokes

1 $D \to 0$ **Deterministic limit**

$$P(\vec{R}_f, t|\vec{R}_i; 0) = \delta(\vec{R}_f - \vec{R}(t))$$

2 $\nu \to 0,\ D \to 0$ ⇒ **Trajectories remains stochastic!** ⇔ **Spontaneous stochasticity**

- **Order of taking limits is crucial**
- **Noise is akin to symmetry breaking field**
- **"Stochastic anomaly"**

- **Hölder continuous drift rather then differentiable (weaker condition)**

$$|\vec{v}(\vec{x} + \vec{r}) - \vec{v}(\vec{x})| < \text{const} \cdot r^h$$

$$h \in (0, 1)$$

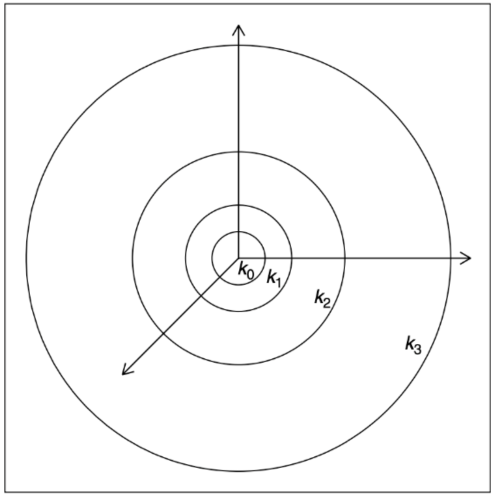
Shell models of turbulent cascade

Navier-Stokes

$$\partial_t u_\alpha(\mathbf{k},t) = -\frac{1}{\mathrm{Re}} k^2 u_\alpha(\mathbf{k},t) + f_\alpha(\mathbf{k},t) - \frac{i}{2}\left\{k_\beta P_{\alpha\gamma}(\mathbf{k}) + k_\gamma P_{\alpha\beta}(\mathbf{k})\right\} \sum_{\mathbf{p}} u_\beta(\mathbf{p}) u_\gamma(\mathbf{k}-\mathbf{p})$$

$$P_{\alpha\beta}(\mathbf{k}) = \delta_{\alpha\beta} - \frac{k_\alpha k_\beta}{k^2}$$

Reduced models of cascade (shell models)



- Drastic reduction in the number of degrees of freedom
- Exponential spacing in k-space
- Coupling local in k-space

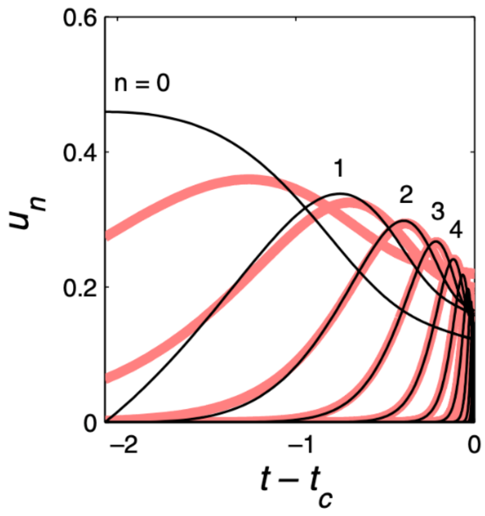
Sabra model

$$du_n/dt = B_n[u] - \nu k_n^2 u_n + f_n$$

$$B_n[u] = i[k_{n+1} u_{n+2} u_{n+1}^* - (1/2) k_n u_{n+1} u_{n-1}^* + (1/2) k_{n-1} u_{n-1} u_{n-2}]$$

P.D. Ditlevsen "Turbulence and shell models", 2001

Spontaneous stochasticity in Sabra model: Origin



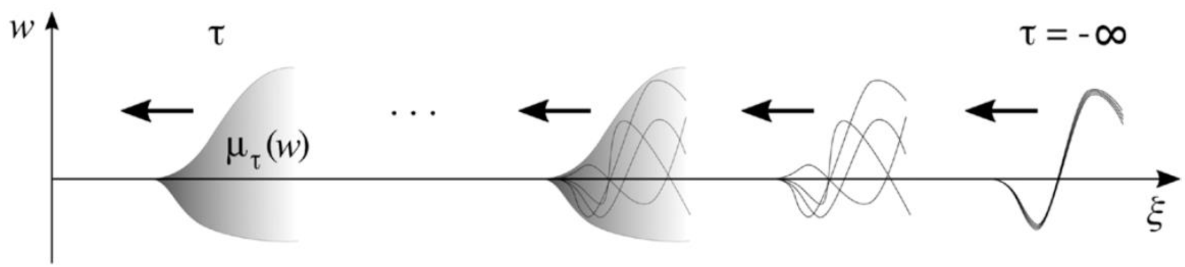
Singular states encountered in the dynamics

Self-similar blow-up in finite time

$$u_n(t) = -i e^{i\theta_n} k_n^{z-1} U(k_n^z (t - t_b))$$

Traveling wave in renormalized variables
(shell velocities and time)

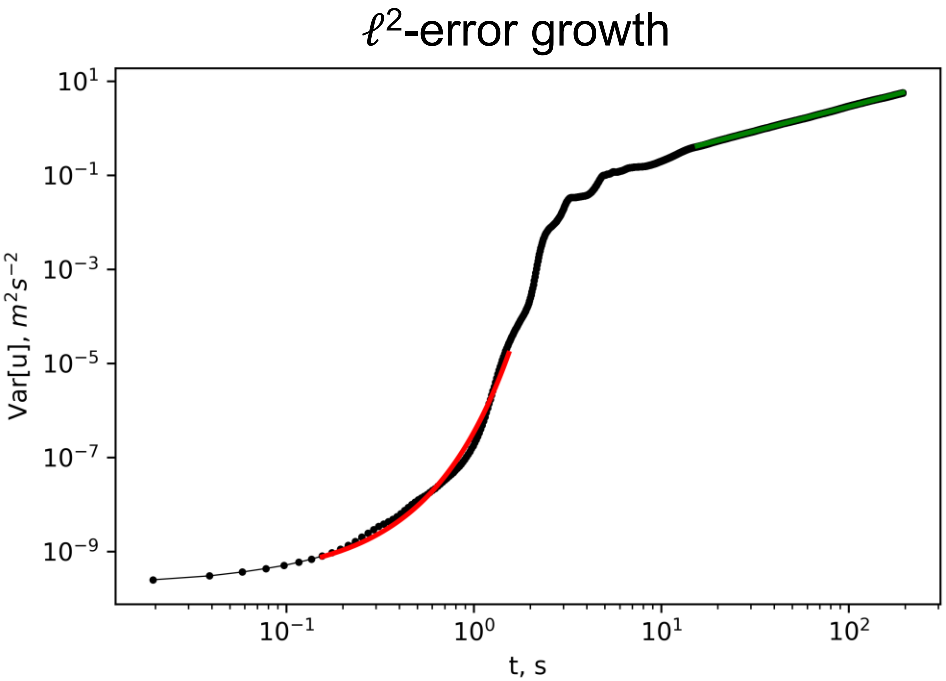
Stochastic front after blow-up

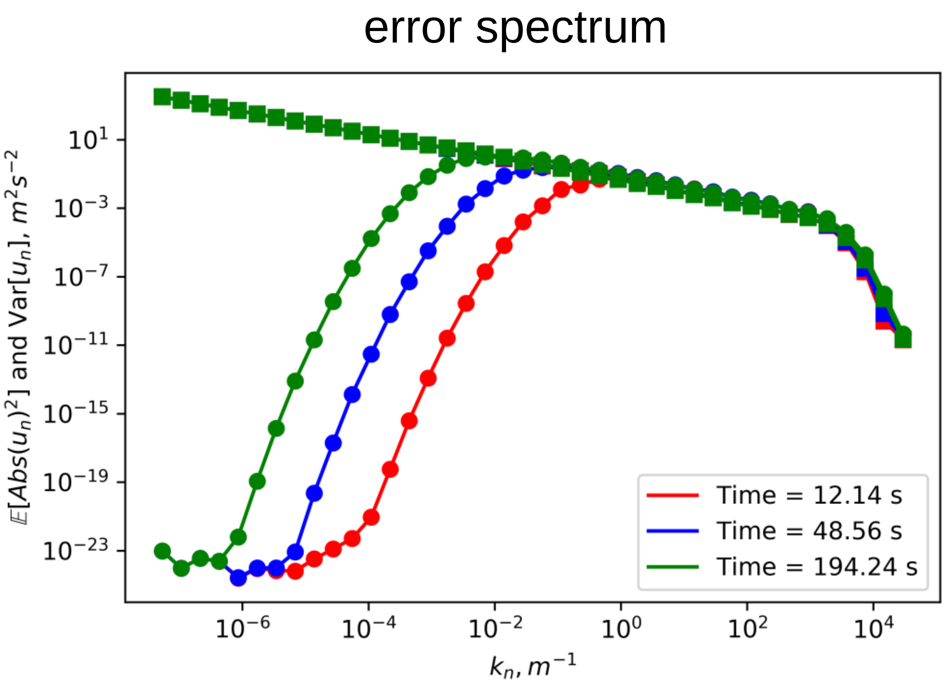


- **Stochastic propagating wave (error)** in renormalized variables
- Inviscid solutions should be understood in the **probabilistic sense**

Mailybaev, A. A. Spontaneously stochastic solutions in one-dimensional inviscid systems. Nonlinearity 29, 2238 (2016)

Variance growth in Sabra model





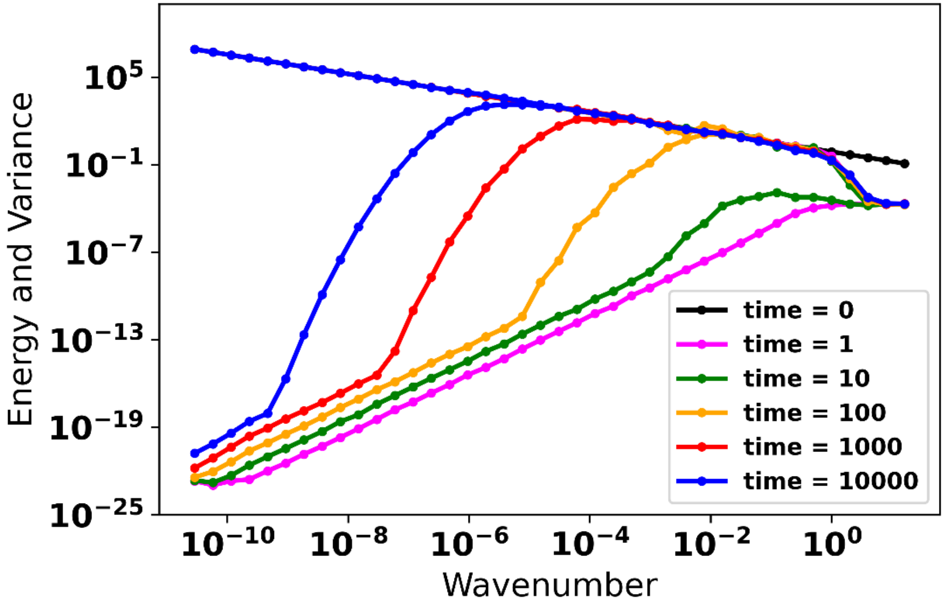
Simulations were performed with flow parameters of the atmospheric boundary layer
Here we plot the total ℓ^2-variance

$$\mathrm{Var}[u] = \sum_{n=1}^{N} \mathrm{Var}[u_n]$$
$$= \sum_{n=1}^{N} \mathbb{E}[|u_n - \mathbb{E}u_n|^2]$$

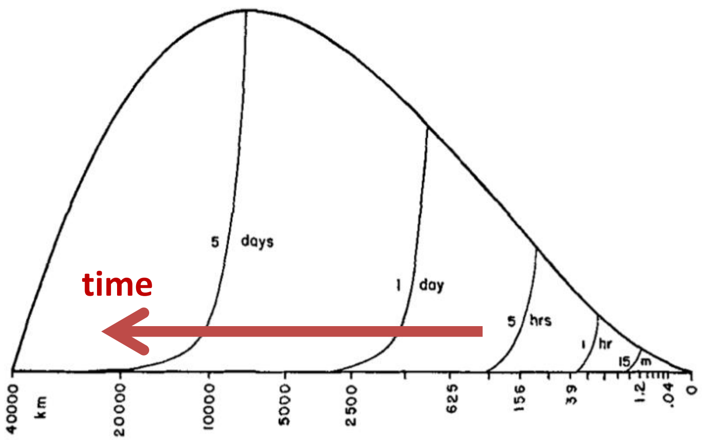
as a function of time and the error spectrum $\mathrm{Var}[u_n]$ versus n at several times.

Spontaneous stochasticity in Sabra model: Numerical Results

Average energy and variance per shell



Original plot of Lorentz



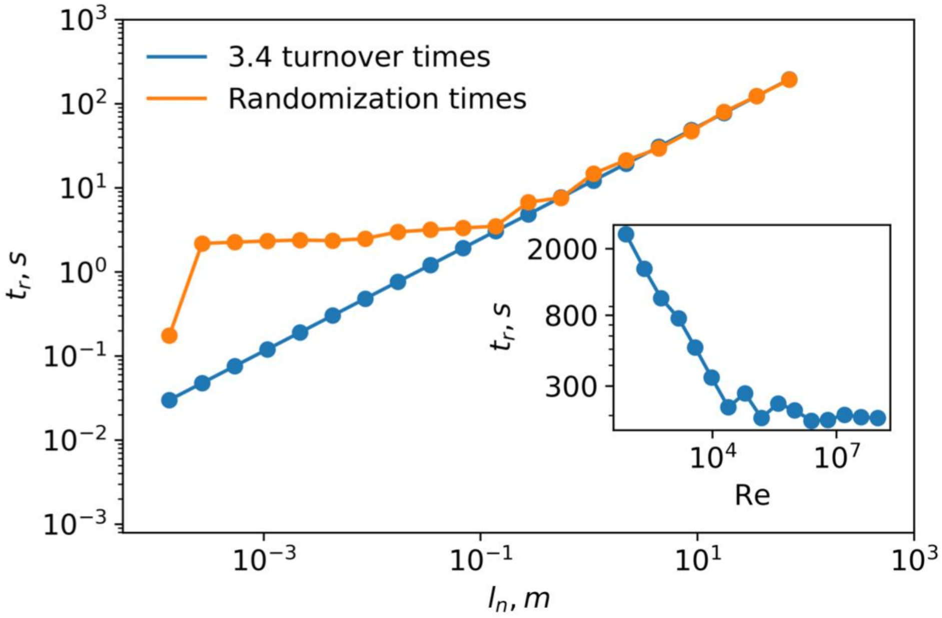
Lorenz, 1969

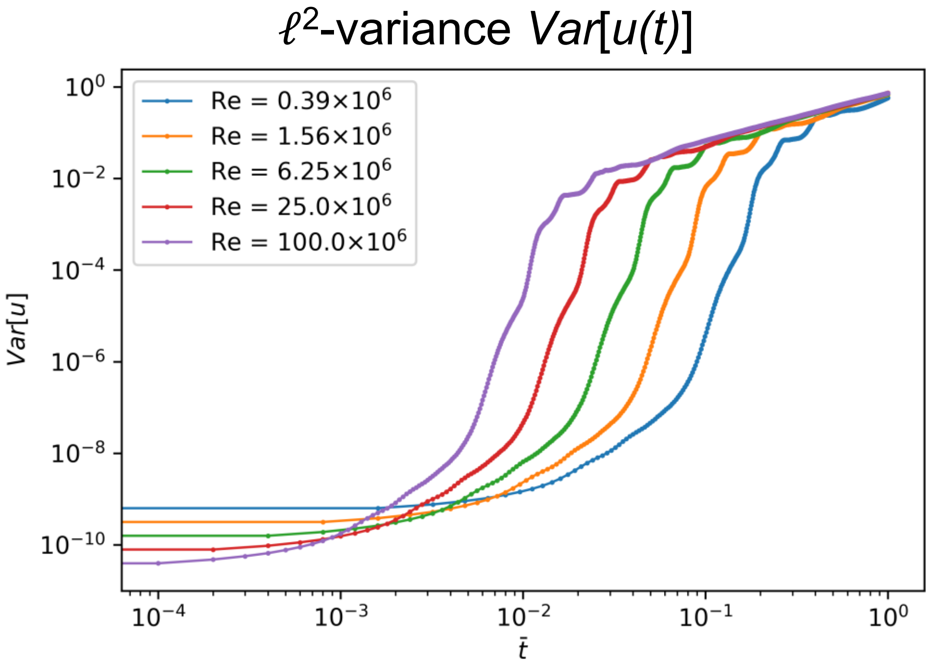
For each shell (scale):

- variance grows linearly at first
- then stochastic front reaches the scale
- statistics becomes fully turbulent

Randomization time

Crucial for spontaneous stochasticity is the Re-dependence. We define the <u>randomization time</u> $t_r(n)$ as the time when the shell's variance $\mathrm{Var}[u_n(t)]$ reaches the ensemble average energy $E[(1/2)|u_n(t)|^2]$. At the flow parameters of the ABL we see that $t_r(n) \sim 3.4\varepsilon^{-1/3}\,\ell_n^{2/3}$ for lengths above 10 cm and, furthermore, approaches that limiting time for <u>any</u> shellnumber n as $Re \to \infty$.



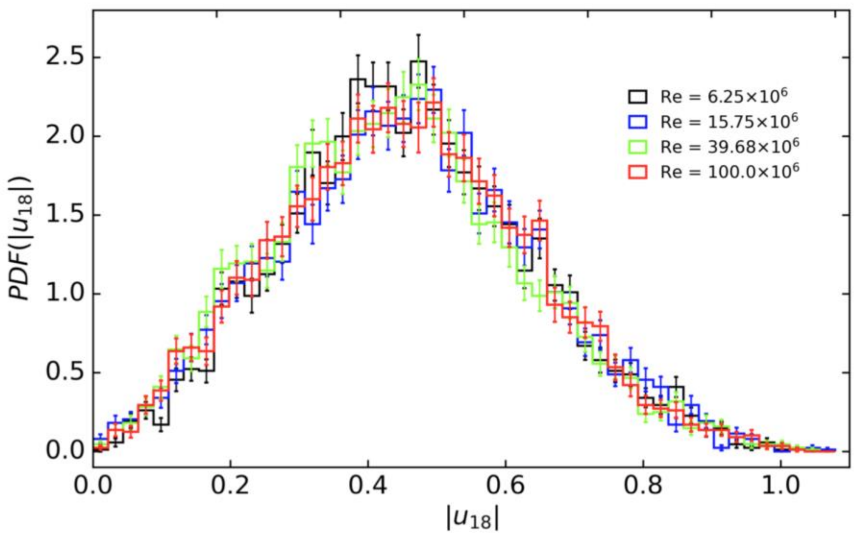


This holds because the variance $\mathrm{Var}[u_n(t)]$ at each shellnumber n approaches a positive limit as $Re \to \infty$. Note the similarity of the plot on the right to Richardson dispersion!

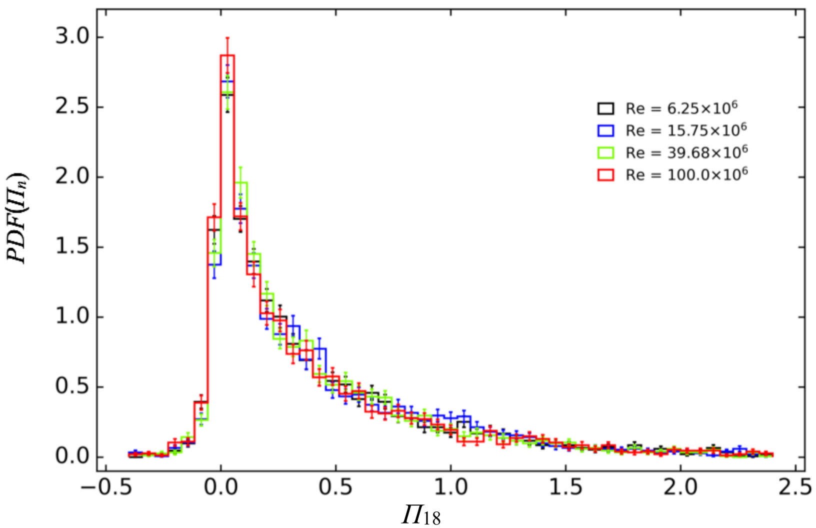
Bandak et al. submitted to PNAS; Bandak et al. PRE 2022

Universal statistics at large Re

PDF of $|u_n(t)|$, n=18, t=1



PDF of energy flux $\Pi_n(t)$, n=18, t=1



This is the clear signature of spontaneous stochasticity in the limit $Re \to \infty$.

Recall the statement of Lorenz (1969) that some "formally deterministic fluid systems… will be operationally indistinguishable from indeterministic systems" and their evolution will result in "states differing as greatly as randomly chosen states of the system within a finite time interval, which cannot be lengthened by reducing the amplitude of the initial error."

Bandak et al. submitted to PNAS; Bandak et al. PRE 2022

Summary

- Thermal noise modifies the energy spectrum at scales about a tenth of the Kolmogorov scale, well above the mean free path
  - Smooths out fluctuations in far dissipation range

- Thermal noise can trigger spontaneous stochasticity, leading to propagation of error to large scales in a few eddy turnover times
  - LES, weather and climate modeling should not attempt higher and higher resolution
  - Stochastic ensemble modeling is fundamentally appropriate

Bandak et al. submitted to PNAS; Bandak et al. PRE 2022

Conclusion

- Stochasticity controls universal asymptotic behavior in turbulence through two fixed points
  - Sub-critical transition to turbulence is a non-equilibrium phase transition
  - Near transitional behavior and the spread of turbulence is intrinsically stochastic

- At large Re, thermal fluctuations and spontaneous stochasticity randomize turbulence with infinite Lyapunov exponents, in a few eddy turnover times

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- Nigel Goldenfeld. Roughness-induced criticality in a turbulent flow. *Phys. Rev. Lett.* **96**, 044503:1-4 (2006)
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THERMAL NOISE AND SPONTANEOUS STOCHASTICITY

- Dmytro Bandak, Nigel Goldenfeld, Alexei A. Mailybaev, and Gregory Eyink. Dissipation-range fluid turbulence and thermal noise. Phys. Rev. E 105, 065113 (2022).
- Dmytro Bandak, Gregory Eyink, Alexei Mailybaev and Nigel Goldenfeld. Spontaneous stochasticity amplifies thermal noise to the largest scales of turbulence in a few eddy turnover times. Submitted to PNAS.

Fluctuations and dissipation

Nikuradse's pipe experiment (1933) to measure the friction factor *f*

$$f = \Delta P / l \rho U^2$$

Pipe diameter is 25-100 mm
Pipe length is ~ 70 diameters

Monodisperse sand grains 0.8mm glued to sides of pipe

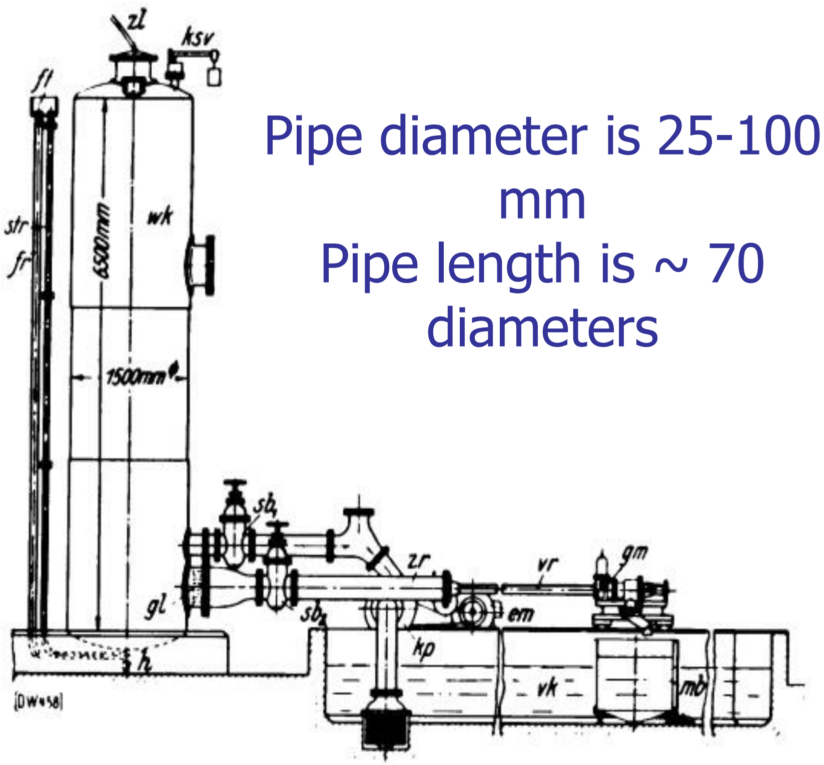


Figure 3.- Test apparatus.

em = electric motor
kp = centrifugal pump
vk = supply canal
wk = water tank
vr = test pipe
zl = supply line
str = vertical pipe
fr = overflow pipe
ft = trap
h = outlet valve
zr = feed line
mb = measuring tank
gm = velocity measuring device
ksv = safety valve on water tank
sb_1 = gate valve between wk and kp
sb_2 = gate valve between wk and zr
gl = baffles for equalizing flow

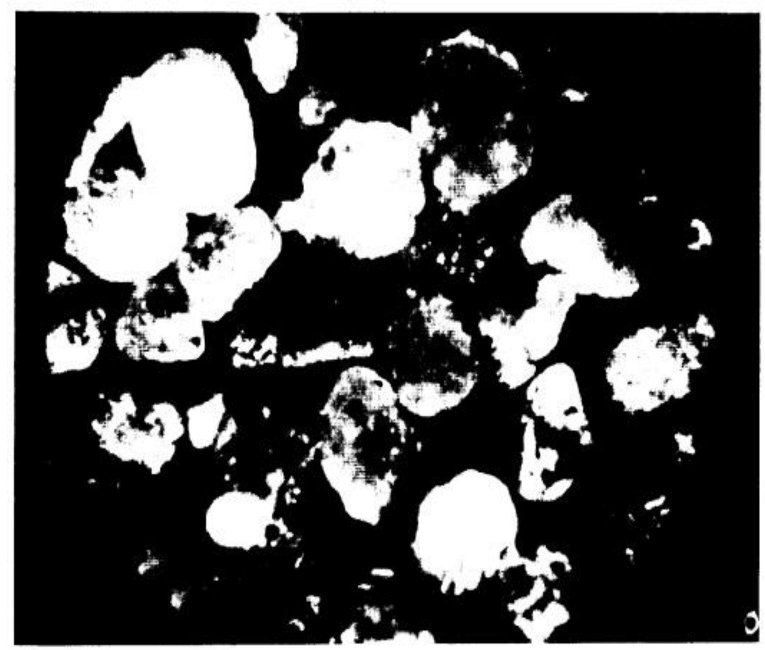
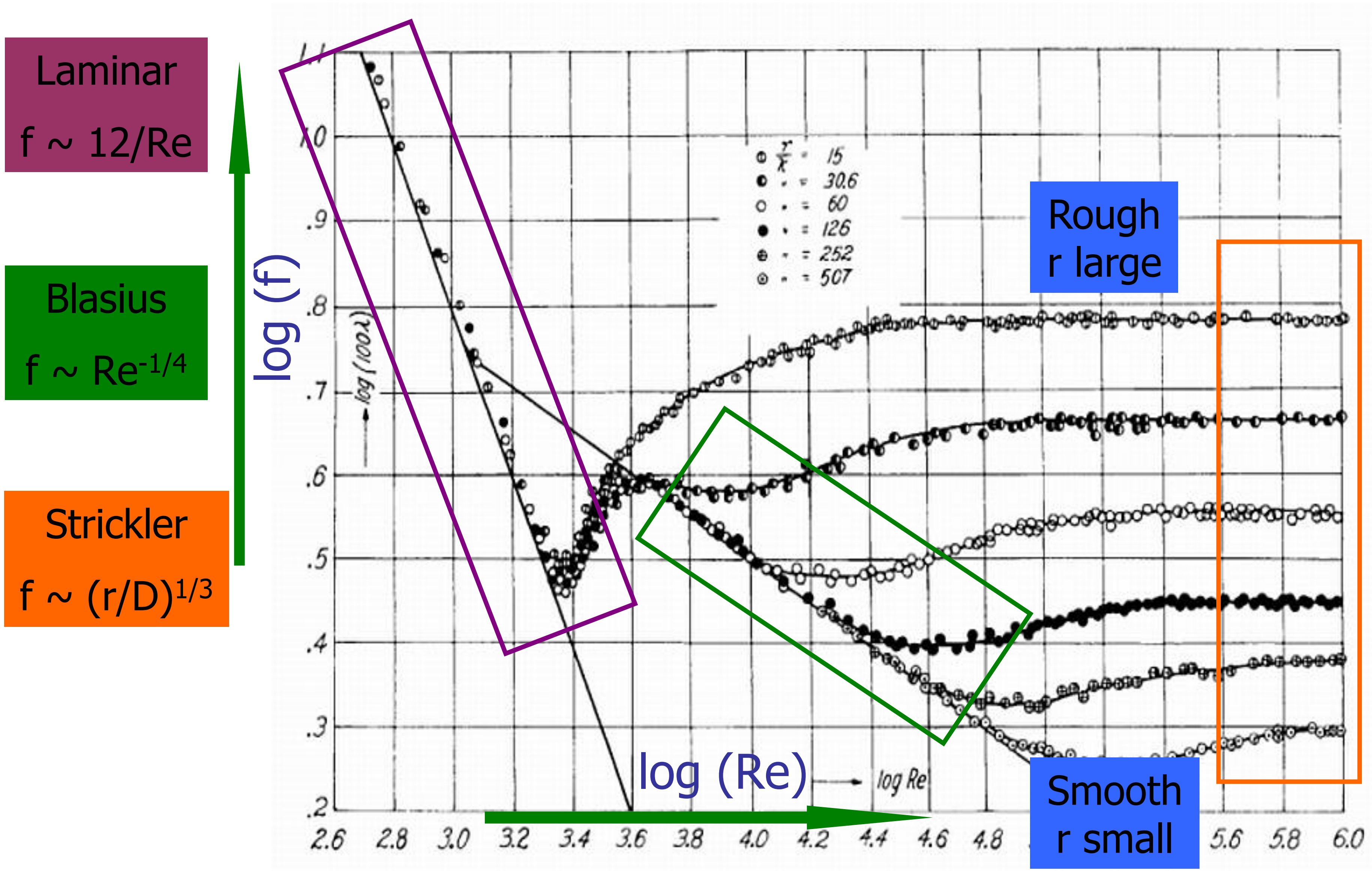


Figure 4.- Microphotograph of sand grains which produce uniform roughness. (Magnified about 20 times.)

Friction factor in turbulent rough pipes



Friction factor in turbulent rough pipes

Laminar

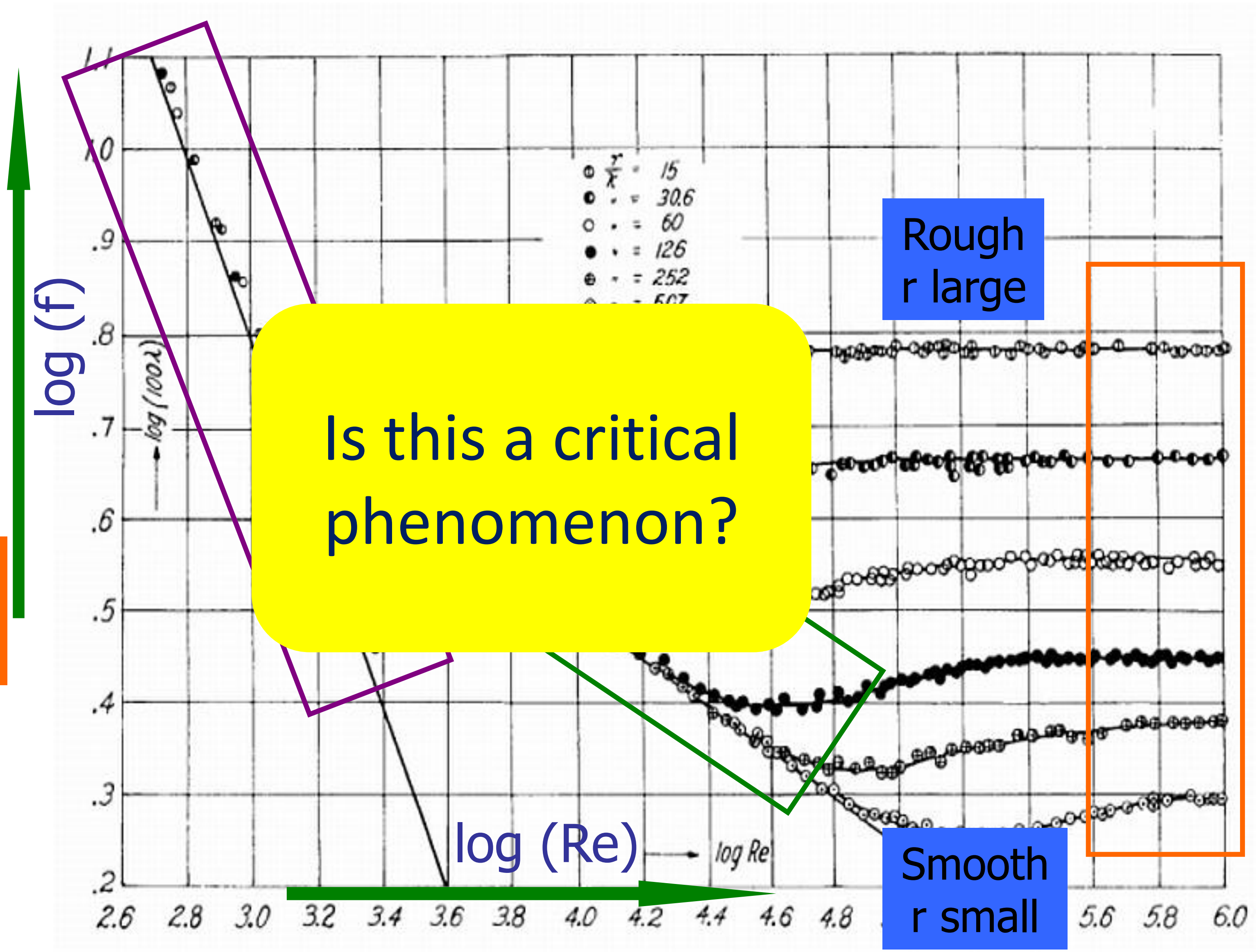
$f \sim 12/Re$

Blasius

$f \sim Re^{-1/4}$

Strickler

$f \sim (r/D)^{1/3}$



Critical phenomena and turbulence

	Critical phenomena	Turbulence
Correlations	$G(k) \sim k^{-2}$	$E(k) \sim k^{-5/3}$
Large scale thermodynamics	Data Collapse	?

Universality at a critical point

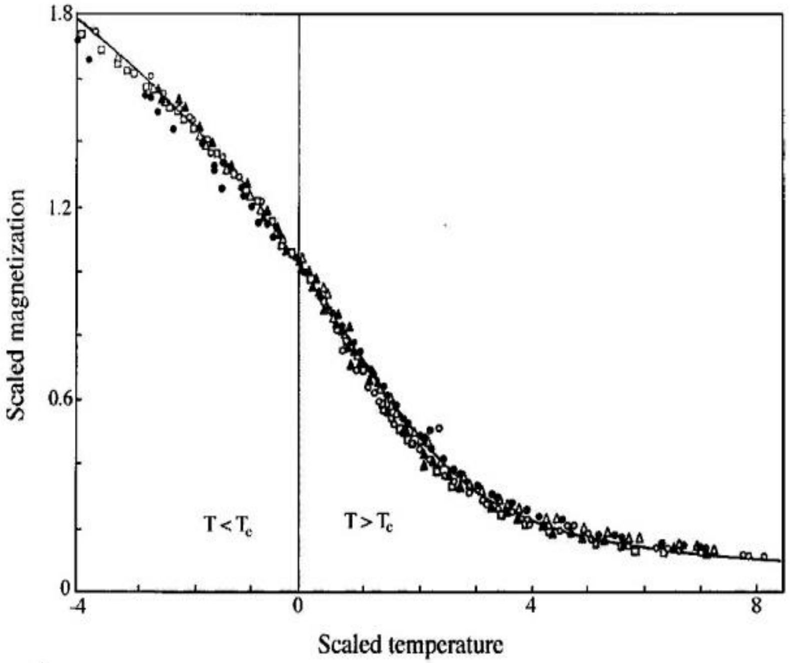


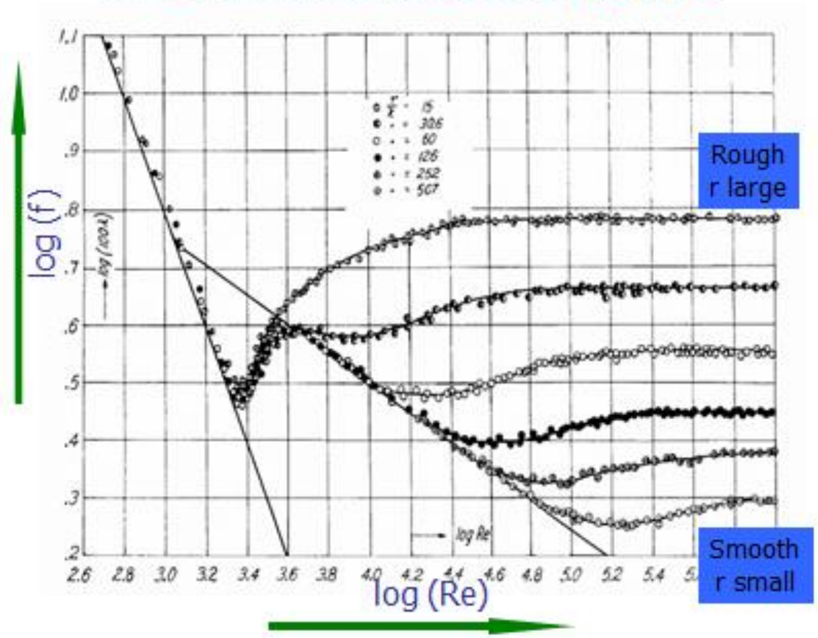
FIG. 1. Experimental *MHT* data on five different magnetic materials plotted in scaled form. The five materials are $CrBr_3$, EuO, Ni, YIG, and Pd_3Fe. None of these materials is an idealized ferromagnet: $CrBr_3$ has considerable lattice anisotropy, EuO has significant second-neighbor interactions. Ni is an itinerant-electron ferromagnet, YIG is a ferrimagnet, and Pd_3Fe is a ferromagnetic alloy. Nonetheless, the data for all materials collapse onto a single scaling function, which is that calculated for the $d=3$ Heisenberg model [after Milošević and Stanley (1976)].

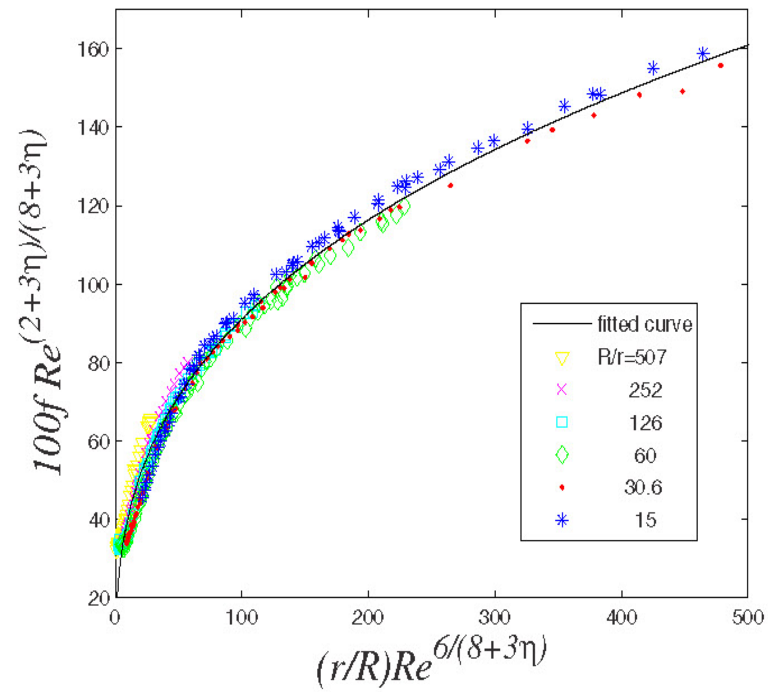
Stanley (1999)

- Magnetization M of a material depends on temperature T and applied field H
  - M(H,T) ostensibly a function of two variables
- Plotted in appropriate scaling variables get ONE universal curve
- Scaling variables involve critical exponents

Data collapse of friction factor

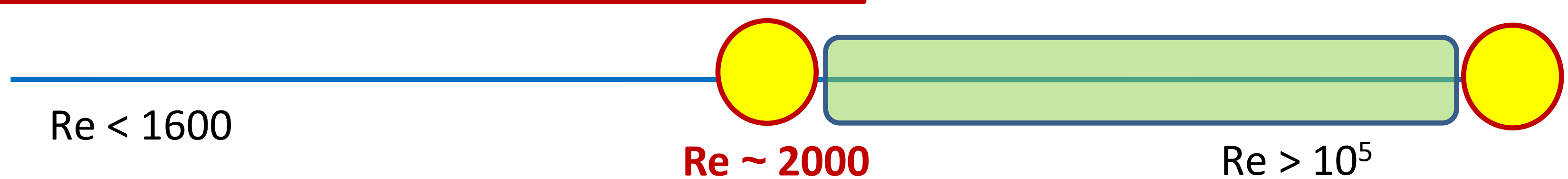
Friction factor in turbulent rough pipes





$$f = \mathrm{Re}^{-(2+3\eta)/(8+3\eta)}\, g\left(\frac{r}{R}\,\mathrm{Re}^{6/(8+3\eta)}\right)$$

Intermittency corrections included
Value of η ~ 0.02 consistent with spectral estimates



Goldenfeld, Phys. Rev. Lett. (2006); Mehrafarin & Pourtolami, Phys. Rev. E. (2008)

Critical phenomena and turbulence

	Critical phenomena	Turbulence
Correlations	$G(k) \sim k^{-2}$	$E(k) \sim k^{-5/3}$
Large scale thermodynamics	$M(h,t) = \|t\|^{\beta} f_M(ht^{-\beta\delta})$	$f = \mathrm{Re}^{-(2+3\eta)/(8+3\eta)} g\left(\frac{r}{R}\mathrm{Re}^{6/(8+3\eta)}\right)$

By simply measuring the pressure drop across a pipe, Nikuradse in 1933 measured the anomalous spectral exponents (intermittency corrections) 8 years before Kolmogorov's mean field theory!

Friction factor → intermittency corrections to velocity fluctuations

Connection between the velocity fluctuations at small scales and the friction factor or dissipation is called the "spectral link" and is an example of a fluctuation-dissipation relation

Friction factor in turbulent rough pipes

Laminar

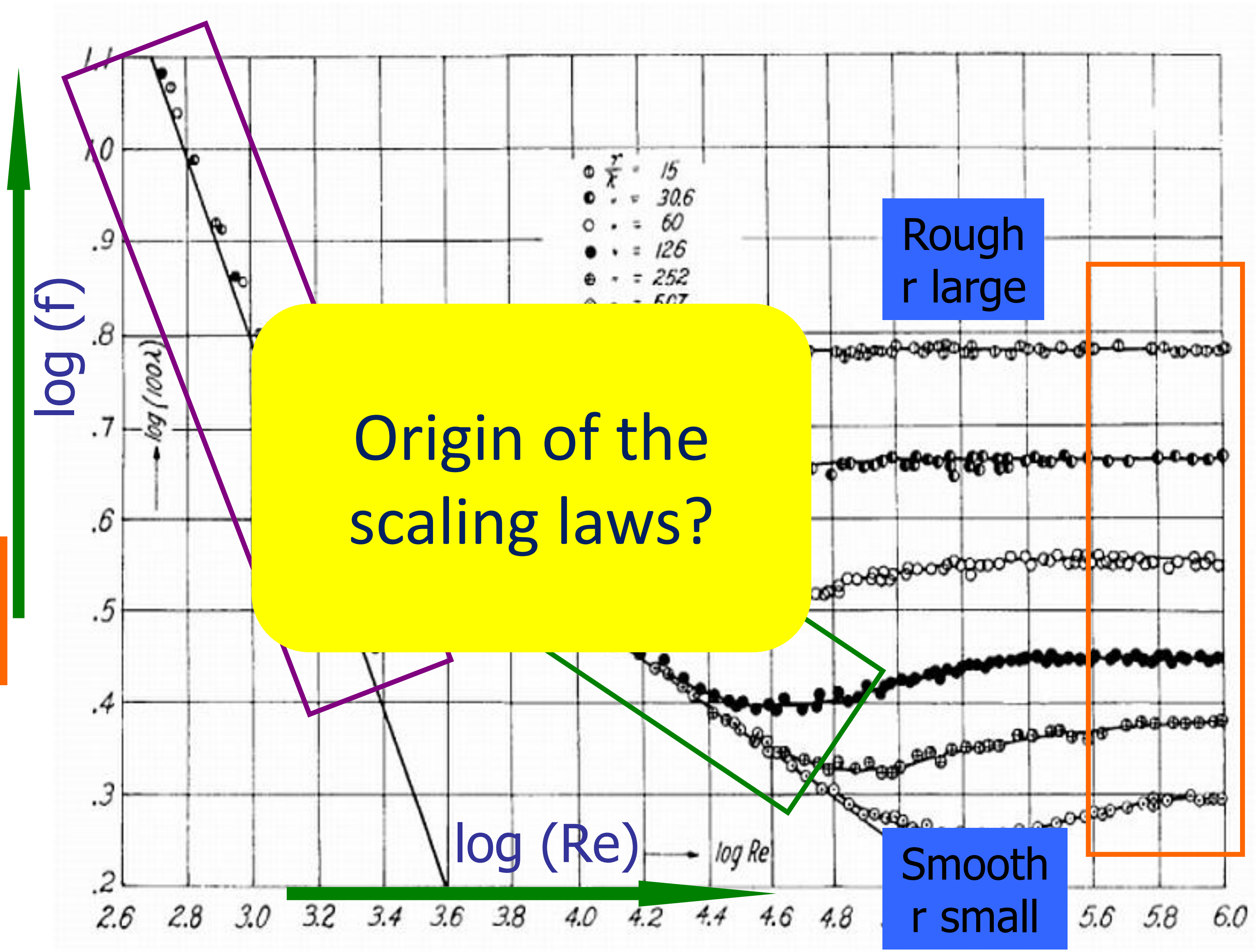
$f \sim 12/Re$

Blasius

$f \sim Re^{-1/4}$

Strickler

$f \sim (r/D)^{1/3}$



Calculating scaling exponents

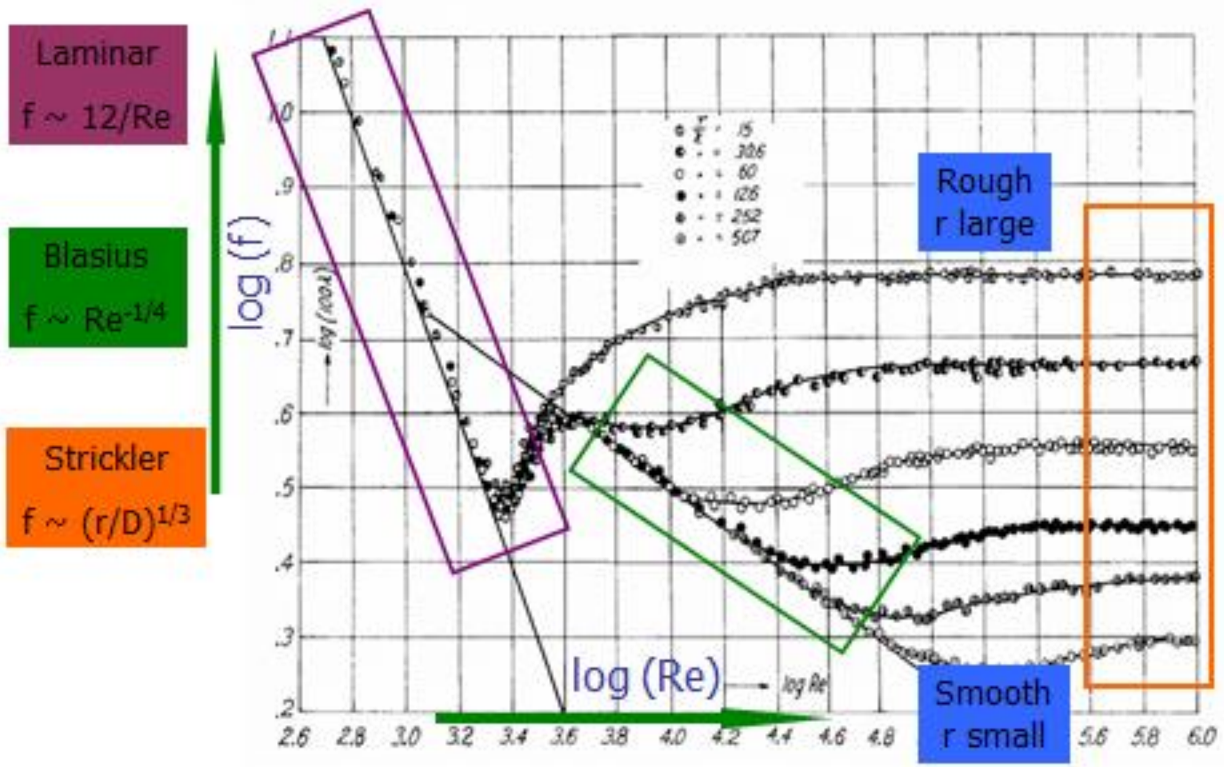
- Basic idea: $f \sim \rho V u_s / \rho V^2$
  - u_s is the RMS velocity at a characteristic scale s determined by main source of dissipation
  - For weak turbulence dissipation comes from the flow, i.e. the Kolmogorov scale $s \sim Re^{-3/4}$
  - For strong turbulence, dissipation because fluid strongly affected by wall roughness $s \sim r$
  - Model this by $s \sim r + Re^{-3/4}$

$$f \propto u_s \propto \left(\int_{1/s}^{\infty} E(k)dk\right)^{1/2}$$

Dissipation **Fluctuation**

Gioia & Chakraborty PRL (2006); Guttenberg & Goldenfeld (2009); Gioia, Guttenberg, Goldenfeld, Chakraborty PRL (2010)

Friction factor in turbulent rough pipes



- **K41: Use E(k) ~ $k^{-5/3}$**

$$f \sim \left(\frac{r}{R} + ab\,\mathrm{Re}^{-3/4}\right)^{1/3}$$

- **Large Re: s ~ r and f ~ $(r/D)^{1/3}$ Strickler law predicted!**
- **Small Re: s ~ η and f ~ $Re^{-1/4}$ Blasius law predicted!**

Gioia & Chakraborty (2006)

Test in 2D

Experimental test in 2D

- Two cascades in 2D turbulence, an inverse one of energy, a forward one of enstrophy

$$E(k) \propto \lambda^{2/3} k^{-3}$$

2D forward cascade

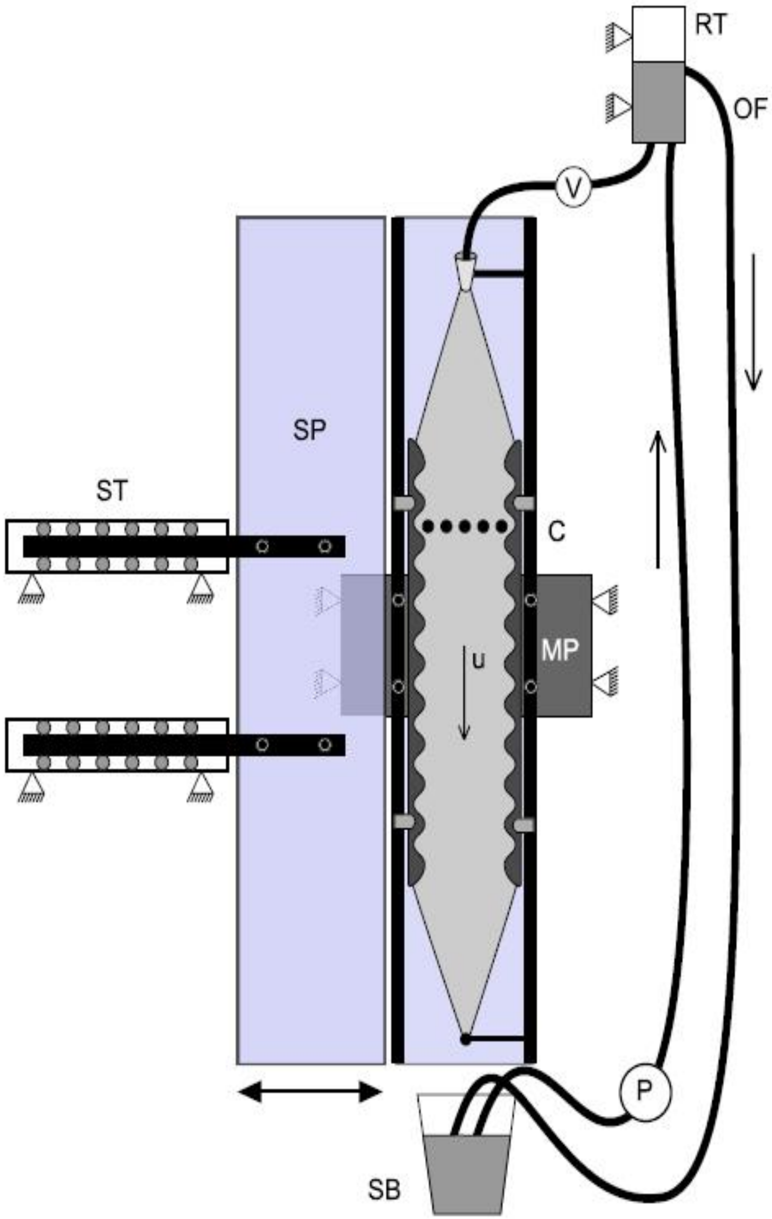
$$E(k) \propto \epsilon^{2/3} k^{-5/3}$$

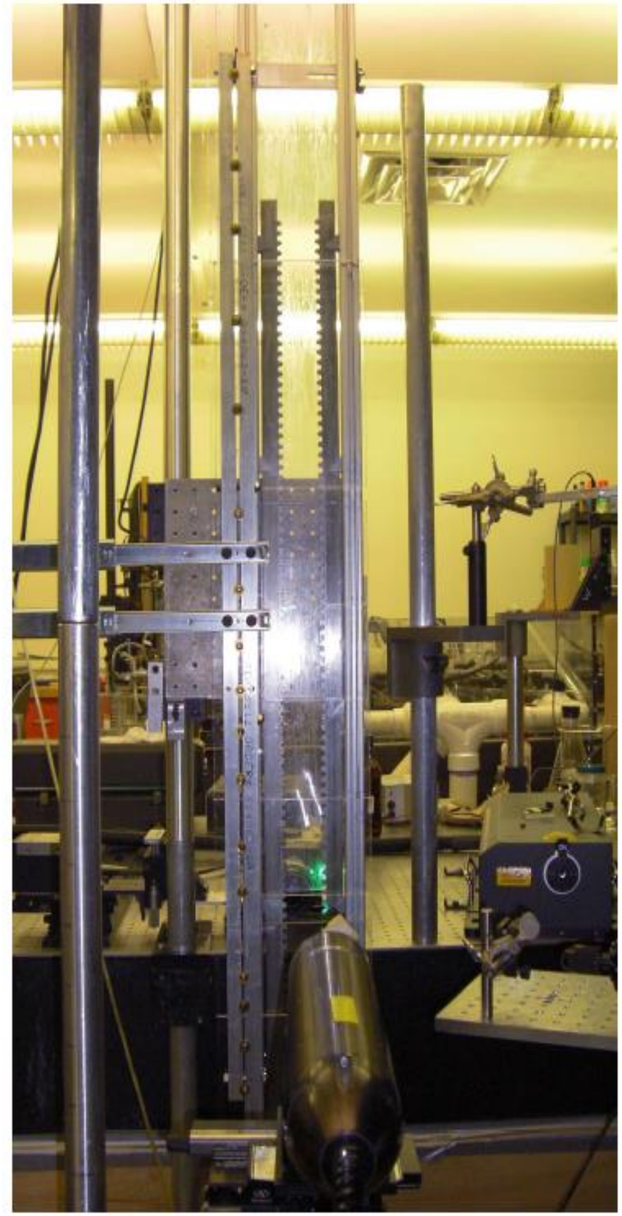
2D inverse cascade

Enstrophy cascade
f ~ $Re^{-1/2}$ (Blasius)
f ~ (r/D) (Strickler)

Inverse cascade
f ~ $Re^{-1/4}$ (Blasius)
f ~ $(r/D)^{1/3}$ (Strickler)

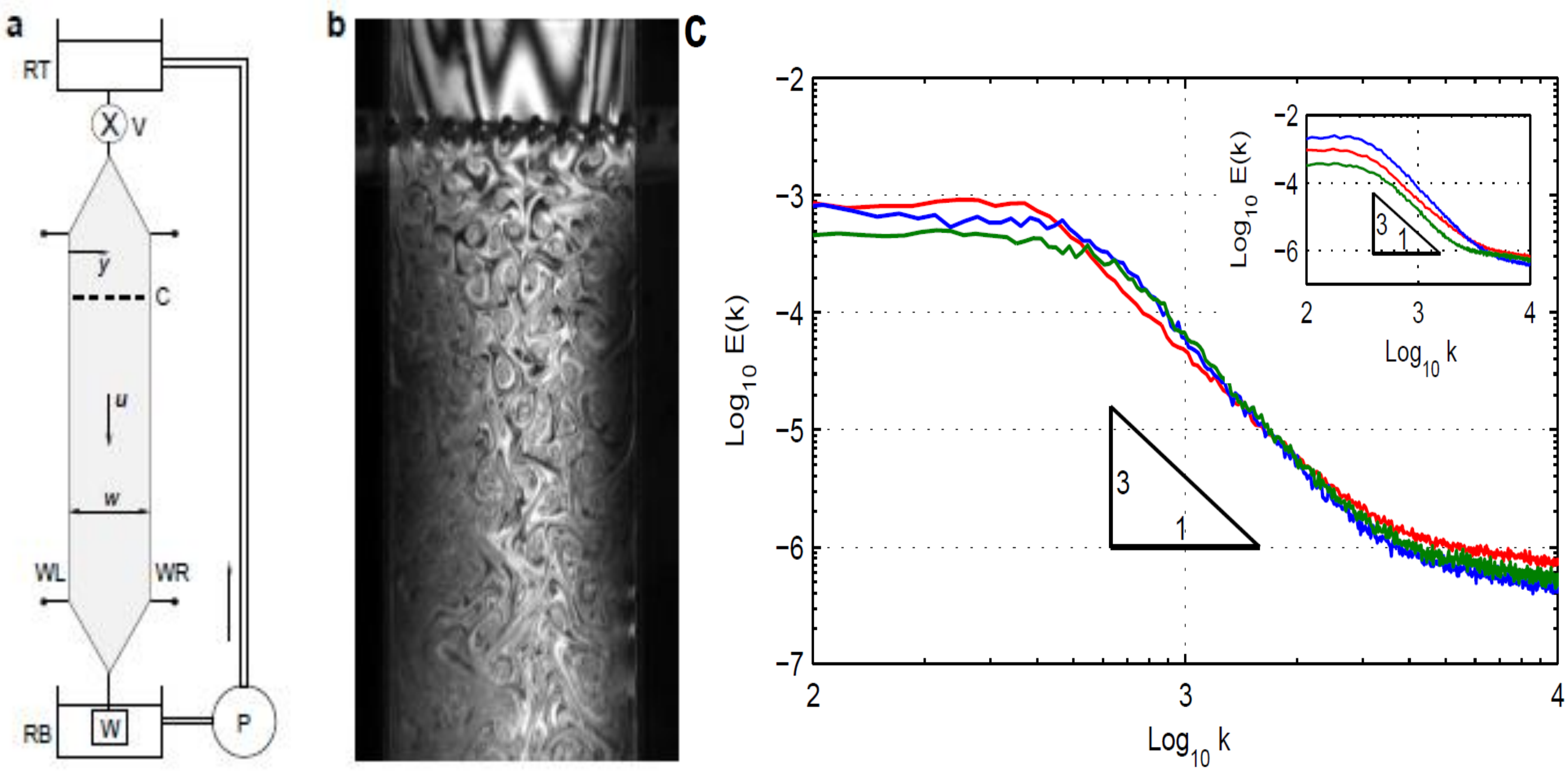
Guttenberg & NG (2009)





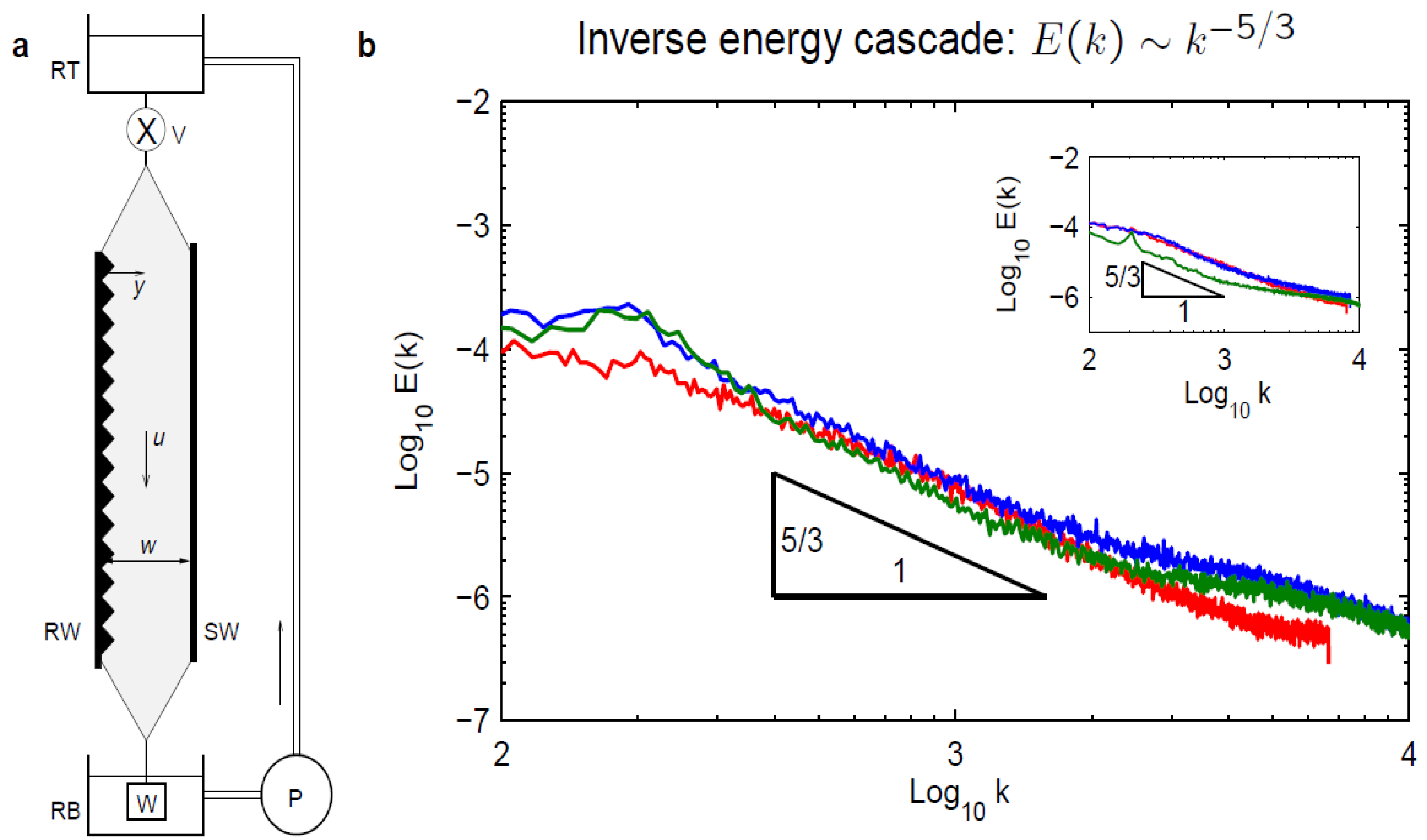
Tran et al., Nature Physics (2010); Kellay et al. Phys. Rev. Lett. (2012)

Forward cascade in 2D soap films



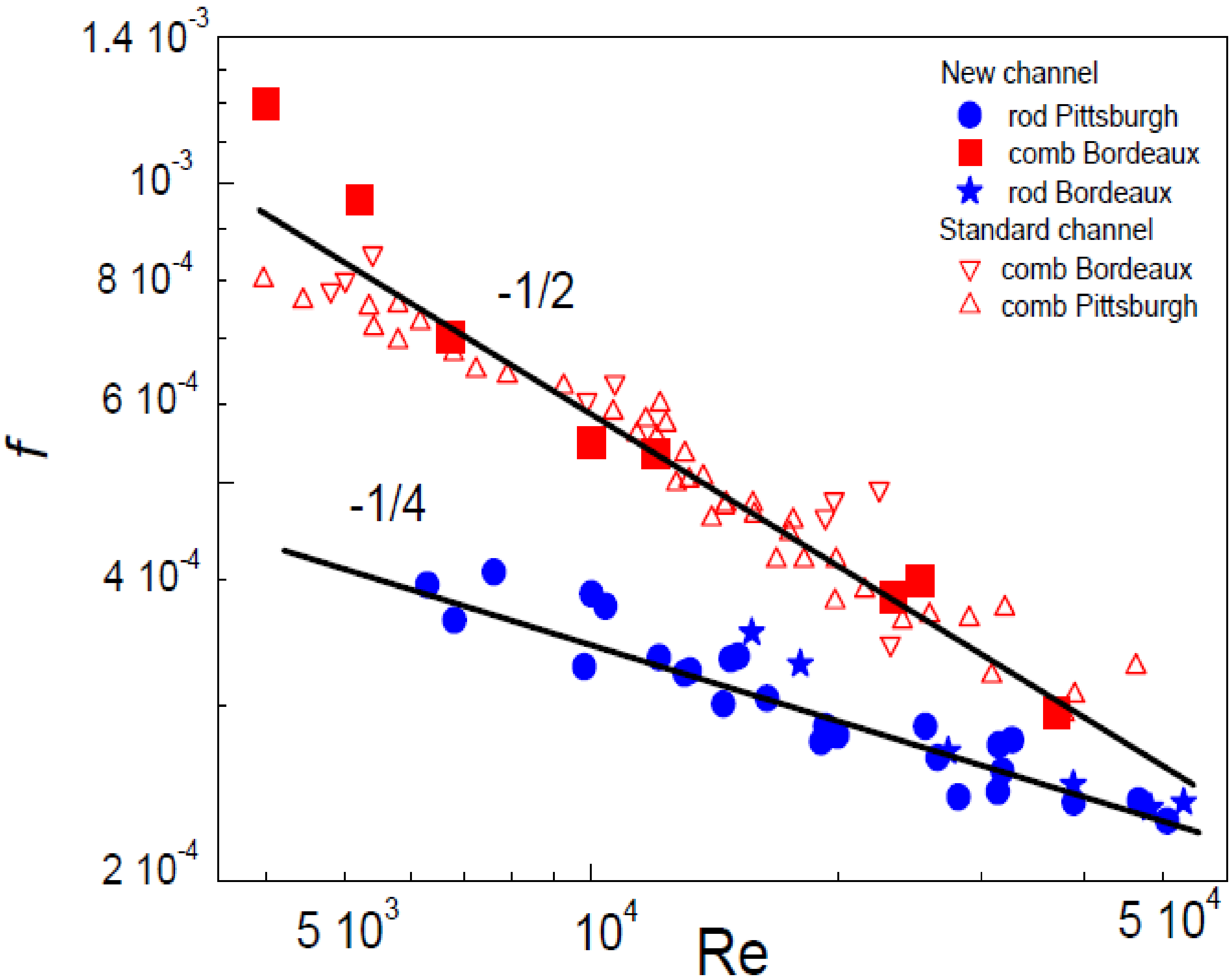
Tran et al., Nature Physics (2010); Kellay et al. Phys. Rev. Lett. (2012)

Inverse cascade in 2D soap films



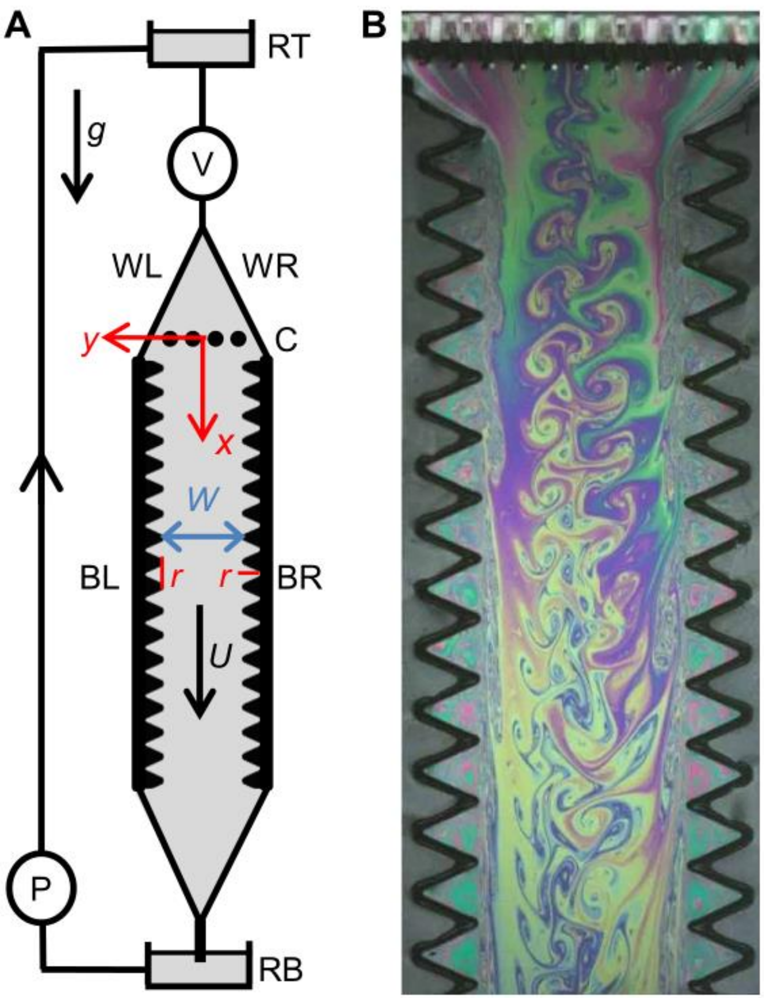
Tran et al., Nature Physics (2010); Kellay et al. Phys. Rev. Lett. (2012)

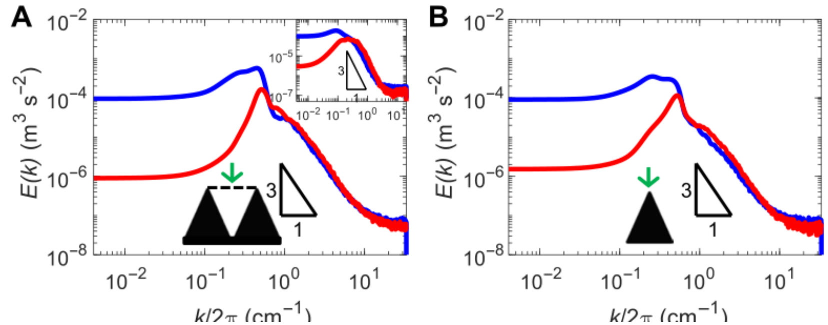
Friction factor depends on cascade



Tran et al., Nature Physics (2010); Kellay et al. Phys. Rev. Lett. (2012)

Beyond Blasius regime in 2D





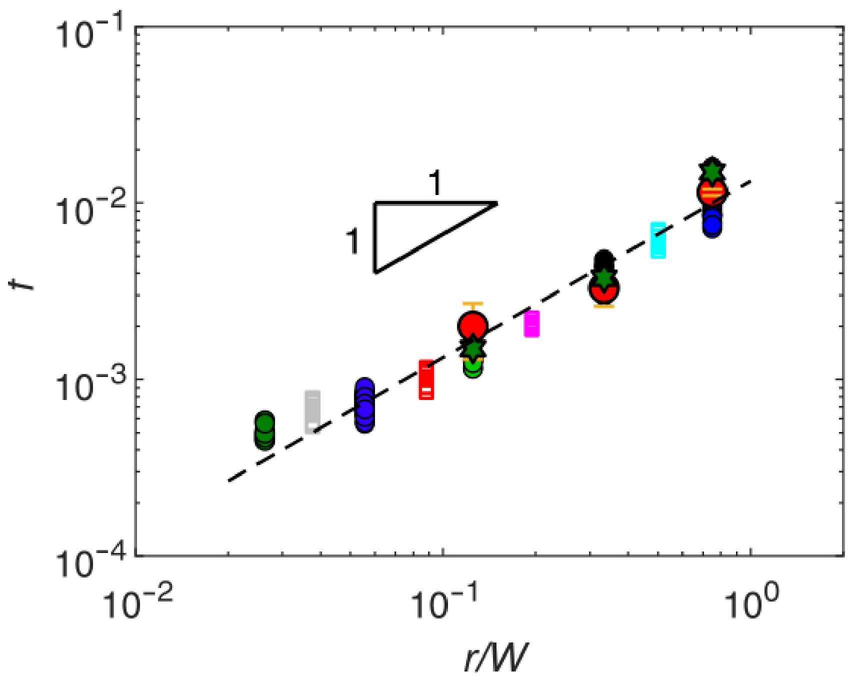


Fig. 5. Log-log plot of the data points (*r*/*W*, *f*). The data points are transferred from Fig. 4 (squares and small circles) or computed via the direct numerical simulations described in the main text (large red circles and green stars).

Vilquin, et al. …. Kellay (2021)

Roughness-induced criticality in 2D

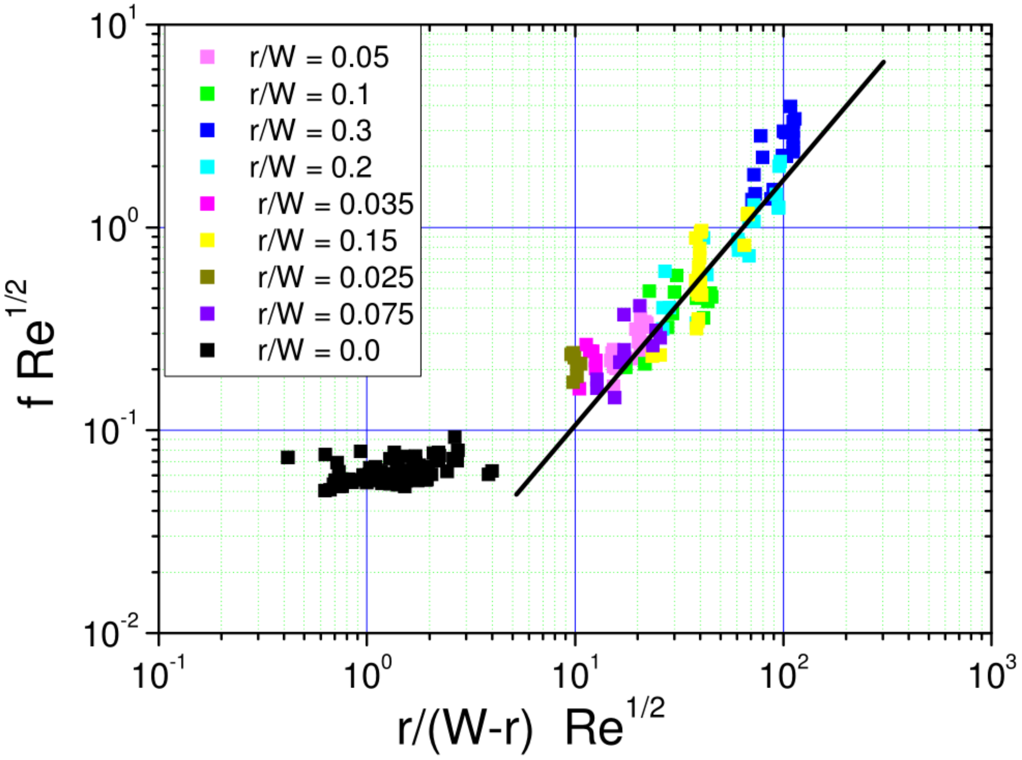
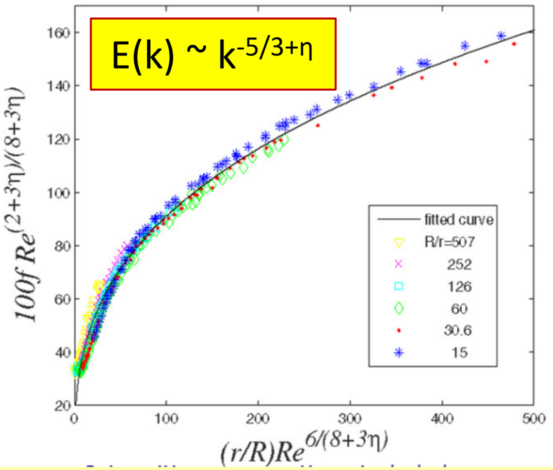


Figure 7: Goldenfeld's scaling for the enstrophy cascade using all the different roughness values. The black squares are for the smooth channel with a fictitious r/w (0.01) instead of r=0.

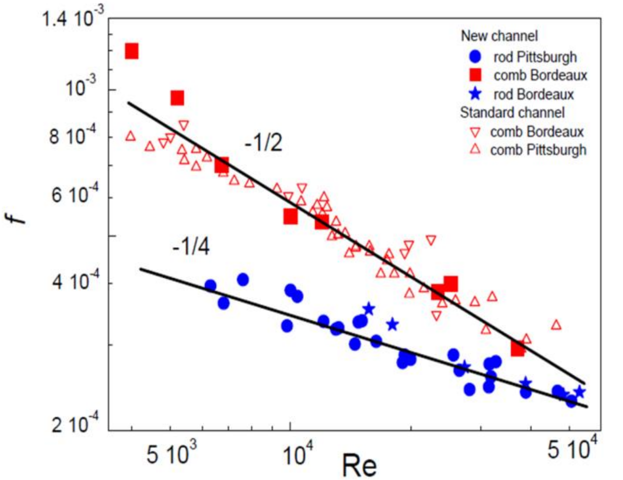
Vilquin, et al. …. Kellay (2021)

Fluctuations and Dissipation

The drag experienced at large scales reflects the very nature of the turbulent state at smaller scales. We make two predictions, both confirmed experimentally



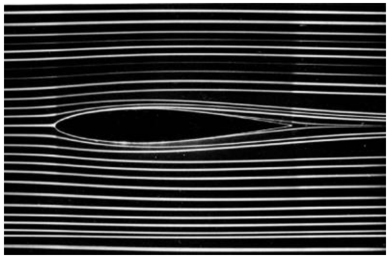
Anomalous scaling exponent from data collapse of friction factor

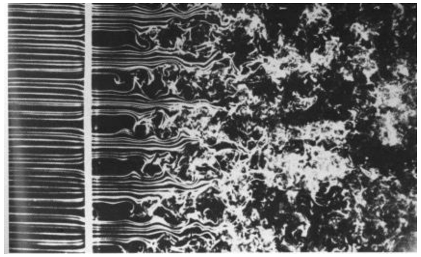


Friction factor depends on energy spectrum/cascade, measured in 2D

Goldenfeld, Phys. Rev. Lett. (2006); Mehrafarin & Pourtolami, Phys. Rev. E. (2008); Gioia & Chakraborty, Phys. Rev. Lett. (2006); Kellay et al. Phys. Rev. Lett. (2012)

Take-home: 2 types of universality in turbulence





laminar flow

steady
predictable

Re < 1600

laminar-turbulent transition

critical behavior

Re ~ 2000

fully-developed turbulence

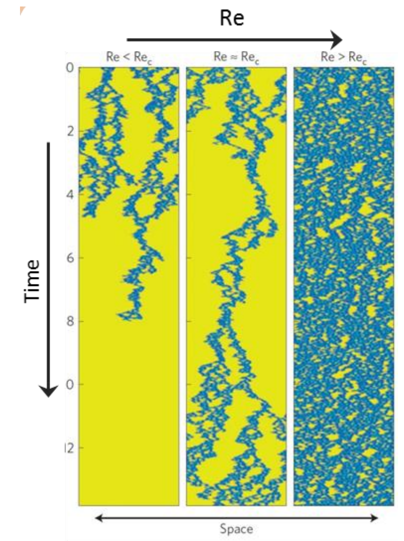
fluctuating
unpredictable

Re > 10^5

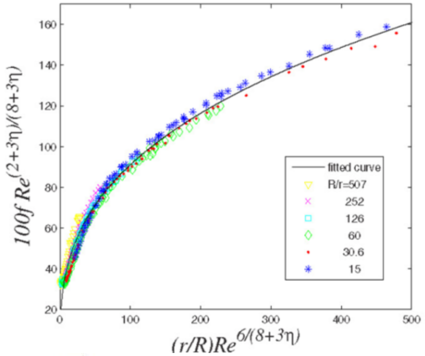
Critical points with their own scaling laws, crossovers and universality classes

Unifying concepts of fluctuation-dissipation, rare events, mean-flow interactions

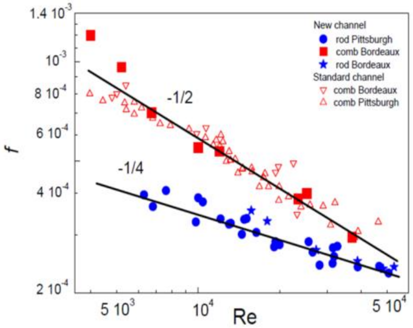
Summary



- The transition to turbulence seems to be a non-equilibrium phase transition in the universality class of directed percolation
  - Theory → phase diagram and universality class
  - Experiments in quasi-1D Taylor-Couette
  - DNS in 2D Waleffe flow



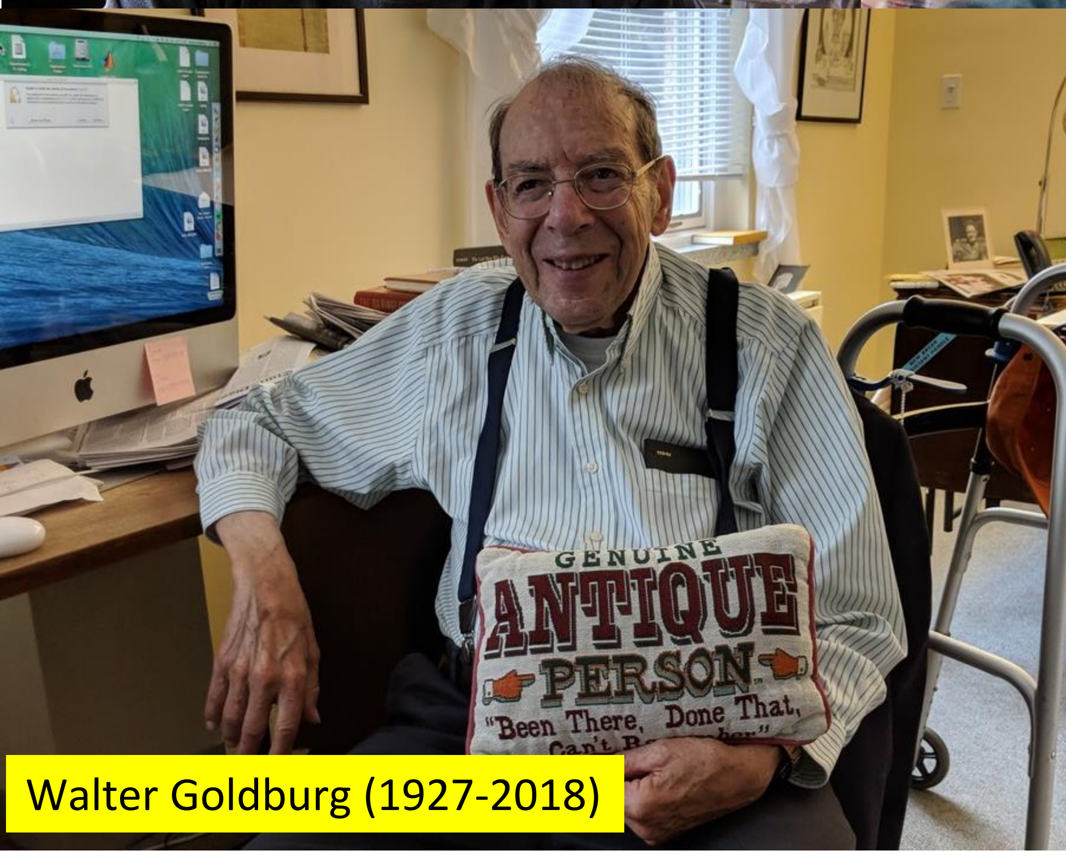
- Widom scaling of the friction factor suggests that fully-developed turbulence is controlled by a critical point and anomalous dimensions can be extracted from small Re data



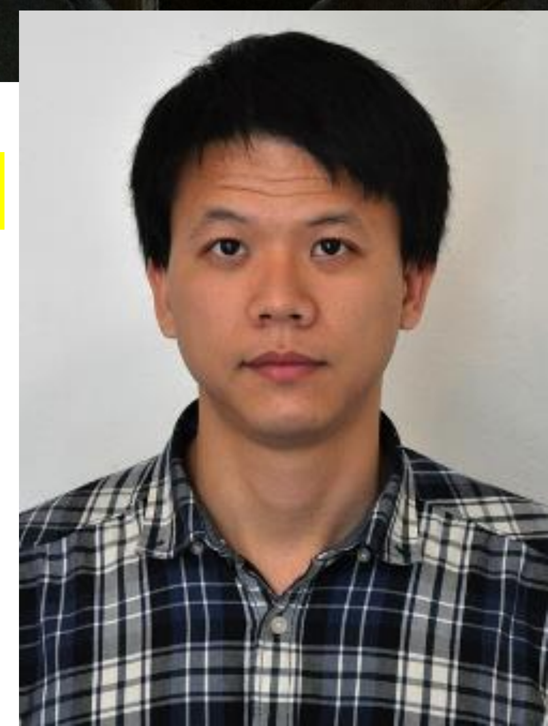
- Friction factor depends on the nature of the small scale velocity fluctuations
  - Predicted and observed dependence on spectrum in quasi-2D turbulence

Hamid Kellay

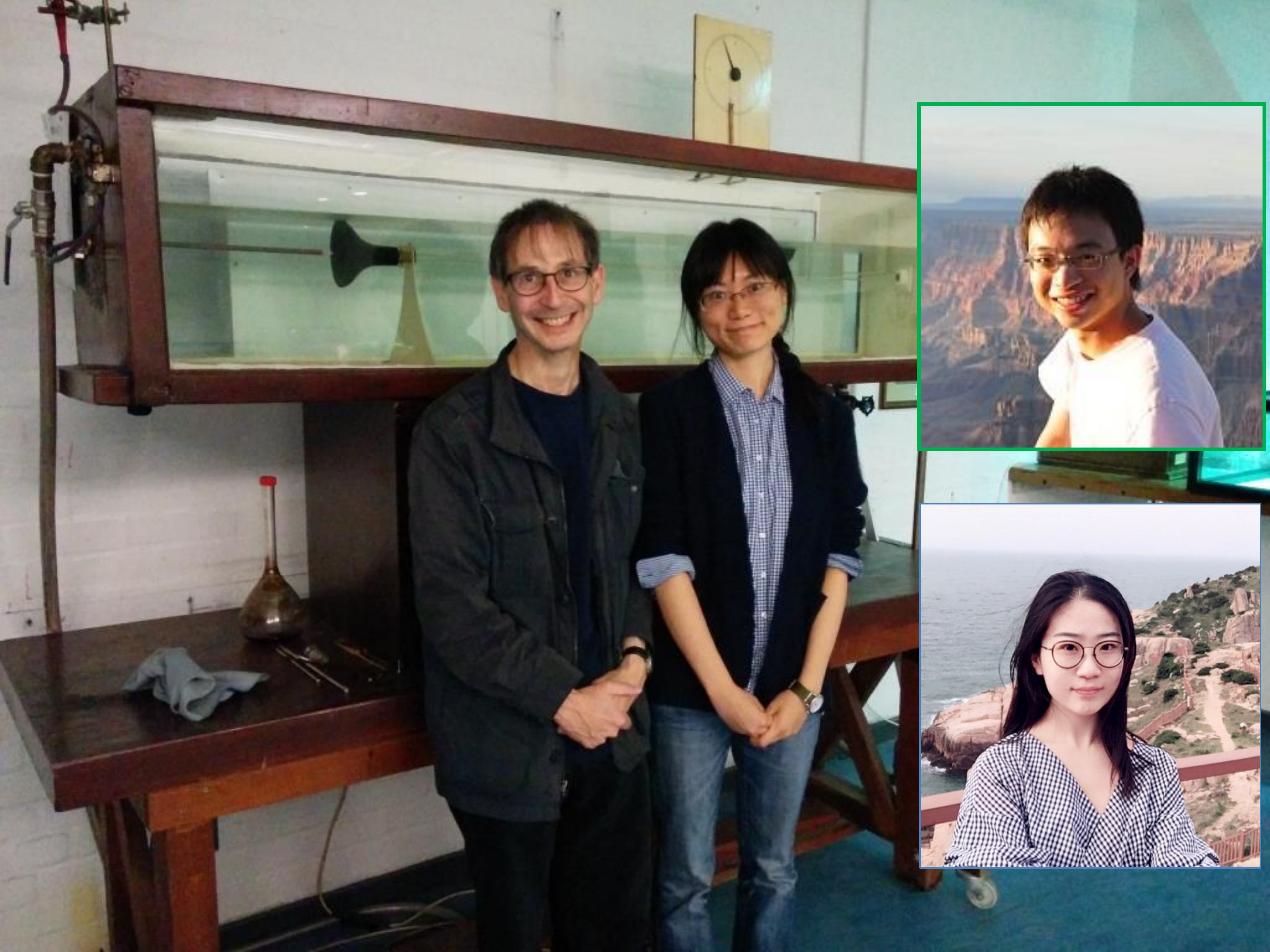
Gustavo Gioia, NG, Pinaki Chakraborty



Walter Goldburg (1927-2018)



Tuan Tran



Turbulence is a life force. It is opportunity. Let's love turbulence and use it for change.

Lucky Numbers 34, 15, 28, 4, 19, 20

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