# Hunting for new physics with precision Lattice Calculations

Vera Gülpers

School of Physics and Astronomy The University of Edinburgh

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#### The Standard Model of Particle Physics



Higgs Boson discovered at LHC in 2012

 $\rightarrow$  Noble Prize 2013 for P. Higgs and F. Englert for prediction

#### The need for new physics

#### Many unsolved questions, for example:

► What is dark matter? Or dark energy?



- Why is there more matter than antimatter in the Universe?
- Why are there three generations of fermions?

#### New physics is out there!

#### Hunting for new physics

#### **High-Energy Frontier:**

## Searches at particle colliders such as the LHC at CERN



[https://cds.cern.ch/record/1295244]

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#### **High-Precision Frontier:**

New physics contributes via quantum

effects



"The closer you look the more there is to see" [F. Jegerlehner, The Anomalous Magnetic Moment of the Muon]

- precision measurements of properties of known particles
- precise calculation within the Standard Model
- ightarrow find (potential) discrepancies

- Quantum Chromo Dynamics (QCD)

   → theory of the strong interaction
- strong coupling  $\alpha_s \sim \mathcal{O}(1)$  at small energies
- quarks and gluons confined to hadrons

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#### Lattice QCD in a nutshell

- Discretize (Euclidean) space-time by a 4d lattice
- Quantize QCD using Euclidean path integrals

$$\langle A \rangle = \frac{1}{Z} \int \mathcal{D}[\Psi, \overline{\Psi}] \mathcal{D}[U] e^{-S_{E}[\Psi, \overline{\Psi}, U]} A(U, \Psi, \overline{\Psi})$$

gluonic expectation value: Monto Carlo techniques

• extrapolate to  $a \rightarrow 0$  and  $L \rightarrow \infty$ Vera Gülpers (Edinburgh) New Direction





#### QCD on the lattice

successfully used for hadronic observables, e.g. hadron spectrum



[S. Dürr et al, Science 322 (2008) 1224-1227]

 $\begin{array}{c} 10 \\ 8 \\ \hline \\ 8 \\ \hline \\ 8 \\ \hline \\ 8 \\ \hline \\ 9 \\ \hline \\ 8 \\ \hline \\ 0 \\ \hline 0 \\$ 

[Sz. Borsanyi et al, Science 347 (2015) 1452-1455]

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Quantities studied in Lattice calculations include

- Hadron Spectroscopy & interactions
- weak decays of Hadrons & quark-mixing CKM matrix
- Hadron Structure
- QCD phase diagram
- Beyond the Standard Model Physics
- ▶ ...

This talk: Lattice Calculations for Muon g-2

• magnetic moment  $\vec{\mu}$  of the muon due to its spin  $\vec{s}$  and electric charge e

$$ec{\mu} = g \, rac{e}{2m} \, ec{s}$$
 torque  $ec{ au} = ec{\mu} imes ec{B}$ 



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gyromagnetic-factor (g-factor) of the muon

without quantum effects:

$$g = 2$$



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**B** 

▶ gyromagnetic-factor (*g*-factor) of the muon

with quantum effects:

$$g=2.00233\ldots$$

anomalous magnetic moment of the muon

$$a_{\mu}=rac{\mathrm{g}-2}{2}$$



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$$que r = \mu \times D$$

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anomalous magnetic moment of the muon "Muon g-2"  $a_{\mu}=rac{g-2}{2}$ 



[https://upload.wikimedia.org /wikipedia/commons/a/aa/ Julian\_Schwinger\_headstone.JPG]

#### Muon g-2: Experimental measurement

Previous: Muon g-2 @ BNL (2006) [Phys.Rev. D73, 072003 (2006)]

New: Muon g-2 @ FNAL (2021) [PhysRevLett.126.141801 (2021)]

measure precession frequency of muons in magnetic field:





[https://commons.wikimedia.org/wiki/File: Fermilab\_g-2\_(E989)\_ring.jpg]

#### Muon g-2: Standard Model Prediction

White Paper (2020) of the Muon g-2 Theory initiative

[Phys.Rept. 887 (2020) 1-166] [https://muon-gm2-theory.illinois.edu/]



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electro-magnetism

## $11658471.8931(104) imes 10^{-10}$







 $O(10^4)$  diagrams at  $O(lpha^5)$ 

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 $\begin{array}{c} 11658471.8931(104) \times 10^{-10} \\ 15.36(10) \times 10^{-10} \end{array}$ 



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 $693.1(4.0) imes 10^{-10}$ 

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Hadronic Vacuum Polarisation (HVP)



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Hadronic Vacuum Polarisation (HVP) HVP $(\alpha^3, \alpha^4)$ 





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Hadronic Vacuum Polarisation (HVP)  $\mathrm{HVP}(lpha^3, lpha^4)$ 

Hadronic light-by-light scattering



 $\begin{array}{c} 11658471.8931(104)\times 10^{-10}\\ 15.36(10)\times 10^{-10}\\ 693.1(4.0)\times 10^{-10}\end{array}$ 

$$-8.59(7) imes 10^{-10}$$

 $9.2(1.8) imes10^{-10}$ 

Experiment vs Standard Model prediction



- SM:  $a_{\mu} = 0.00116591810(43)$ 
  - ► This could be new physics!



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## Experiment vs Standard Model prediction

- Exp:  $a_{\mu} = 0.00116592061(41)$
- SM:  $a_{\mu} = 0.00116591810(43)$ 
  - ▶ This could be new physics!

## What's next?

▶ FNAL reduce error by factor ~ 4, new upcoming experiment @JPARC

175 180 185 190 195 200 205 210 215

Muon g-2 Coll., Phys. Rev. Lett. 126, 141801

**4.2**σ

a...×10<sup>9</sup>-1165900

BNL g-2

Breakdown of Standard Model Prediction



#### The HVP from R-ratio



#### Lattice calculation of HVP

Comparision of available lattice QCD calculations of HVP



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calculate hadronic part on the lattice



#### vector two-point function

$$C_{\mu
u}(t) = \sum_{ec{x}} \langle J_{\mu}(t,ec{x}) J_{
u}(0) 
angle$$

electromagnetic current

$$J_{\mu} = \frac{2}{3}\overline{u}\gamma_{\mu}u - \frac{1}{3}\overline{d}\gamma_{\mu}d - \frac{1}{3}\overline{s}\gamma_{\mu}s + \dots$$

 $\blacktriangleright$   $a_\mu$  from  $\mathcal{C}(t)$  [T. Blum, Phys.Rev.Lett.91, 052001 (2003); Bernecker and Meyer, Eur.Phys.J.A47, 148 (2011)]

$$a_{\mu}^{\text{HVP}} = \sum_{t} w_t C_{ii}(t)$$
 with kernel function  $w_t$ 

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▶ flavour decomposition (isospin symmetric QCD  $u = d = \ell$ )

$$C(t) = \frac{5}{9}C^{\ell}(t) + \frac{1}{9}C^{s}(t) + \frac{4}{9}C^{c}(t)$$

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#### Precision Challenges for the HVP









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**recent progress:** [VG *et al*, PRL 121, 022003 (2018); D. Giusti *et al*, Phys. Rev. D 99, 114502 (2019); S. Borsanyi *et al*, Nature 593, 51 (2021); M. Cè *et al*, Phys. Rev. D 106, 114502 (2022]

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Disconnected contribution needs stochastic evaluation  $\rightarrow$  noisy



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#### Light-quark contribution

- main challenges:
  - statistical noise at large t
  - finite volume effects
     (largest at large t)
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summary of available lattice results



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summary of available lattice results



Vera Gülpers (Edinburgh)

schematic lat

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w(t)C(t)

#### Lattice Cross Checks - Window method

•  $a_{\mu}^{\text{HVP}}$  from intermediate window  $a_{\mu} = a_{\mu}^{\text{SD}} + a_{\mu}^{\text{W}} + a_{\mu}^{\text{LD}}$ [T. Blum, P. Boyle, VG *et al* Phys.Rev.Lett. 121 (2018) 022003]

$$a_{\mu}^{\mathbb{W}} = \sum_{t} w_{t} C(t) [\theta(t, t_{0}, \Delta) - \theta(t, t_{1}, \Delta)]$$

e.g.  $t_0 = 0.4$  fm to  $t_1 = 1.0$  fm



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#### compare *R*-ratio with lattice using window quantity







- "R-ratio Scenario": lattice consistent with R-ratio (unlikely?)
- ► "BMW Scenario": Other (full) lattice calculations agree with BMW → tension between lattice and *R*-ratio
- update of the Theory whitepaper in progress



Sep 22 @Higgs Centre





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Will we discover new physics with Muon g-2?



Sep 22 @Higgs Centre





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#### Will we discover new physics with Muon g-2? Maybe. Maybe not.



Sep 22 @Higgs Centre

#### Conclusion

#### Summary

- Standard Model very successful, but leaves many open questions
- ► low-energy precision test of the Standard Model → QCD: first principles calculations using Monte Carlo (Lattice QCD)
- Muon g-2: promising quantity for finding new physics
  - $\rightarrow$  lattice calculation closer to experiment, in tension with  $\textit{\textbf{R}}\textsc{-}ratio$
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## Thank you!