Prospects for heavy hadron lifetimes on the lattice

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Heavy Flavours 2016: Quo Vadis?
Ardbeg, Islay

B lifetimes

- Motivation
- Lifetime differences
- Lifetime ratios

[apologies for shortage of references]

Motivation

- Improve Standard Model predictions
- Ultimately search for new physics
- Test theoretical methods
- Categorize discrepancies

B mixing

$$i\frac{d}{dt}\begin{pmatrix}|B^0(t)\rangle\\|\bar{B}^0(t)\rangle\end{pmatrix} = \left(M - \frac{i}{2}\Gamma\right)\begin{pmatrix}|B^0(0)\rangle\\|\bar{B}^0(0)\rangle\end{pmatrix}$$

Flavour basis ≠ mass basis ⇒ M & Γ non-diagonal

Mixing governed by

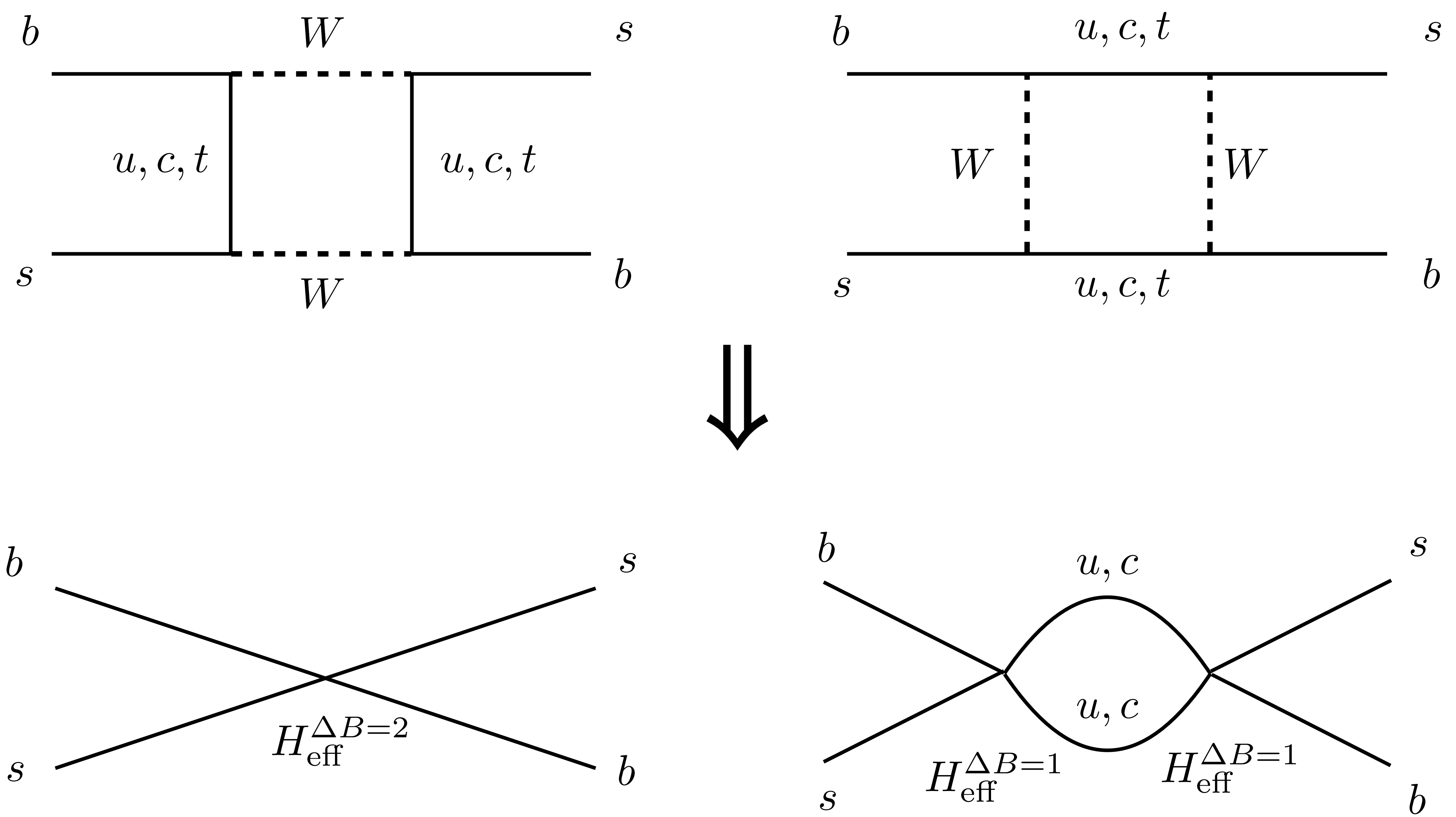
$$|M_{12}| \qquad |\Gamma_{12}| \qquad \phi = \arg\left(-\frac{M_{12}}{\Gamma_{12}}\right)$$

Observables

$$\Delta M = 2|M_{12}| \qquad \Delta\Gamma = 2|\Gamma_{12}|\cos\phi \qquad a_{\mathrm{fs}} = \frac{\Delta\Gamma}{\Delta M}\tan\phi$$

[up to corrections O(m_b^2/m_W^2)]

SM mixing



b
W
s
u, c, t
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$\Downarrow$
b
s
$H_{\mathrm{eff}}^{\Delta B=2}$
s
b
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u, c
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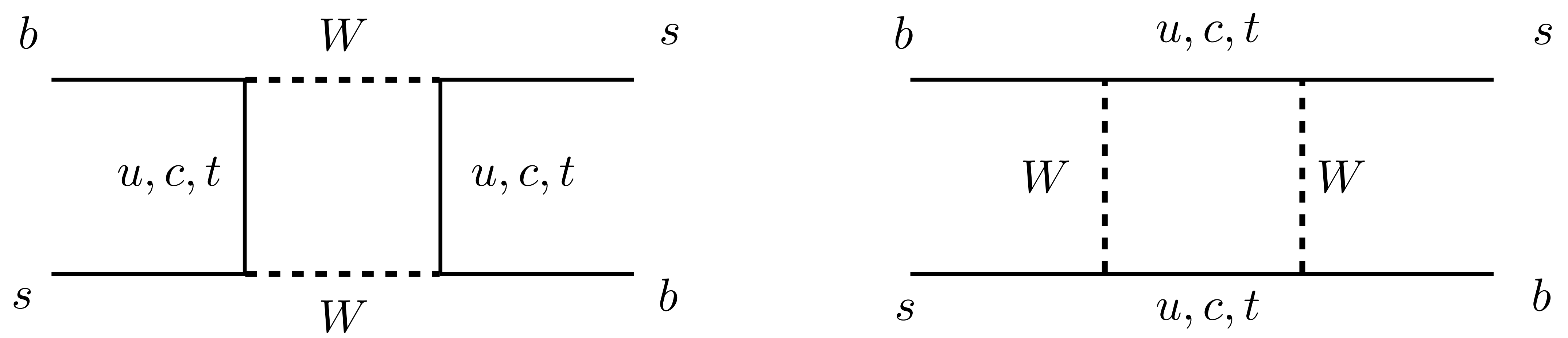
u, c
$H_{\mathrm{eff}}^{\Delta B=1}$
$H_{\mathrm{eff}}^{\Delta B=1}$
s
b

SM mixing



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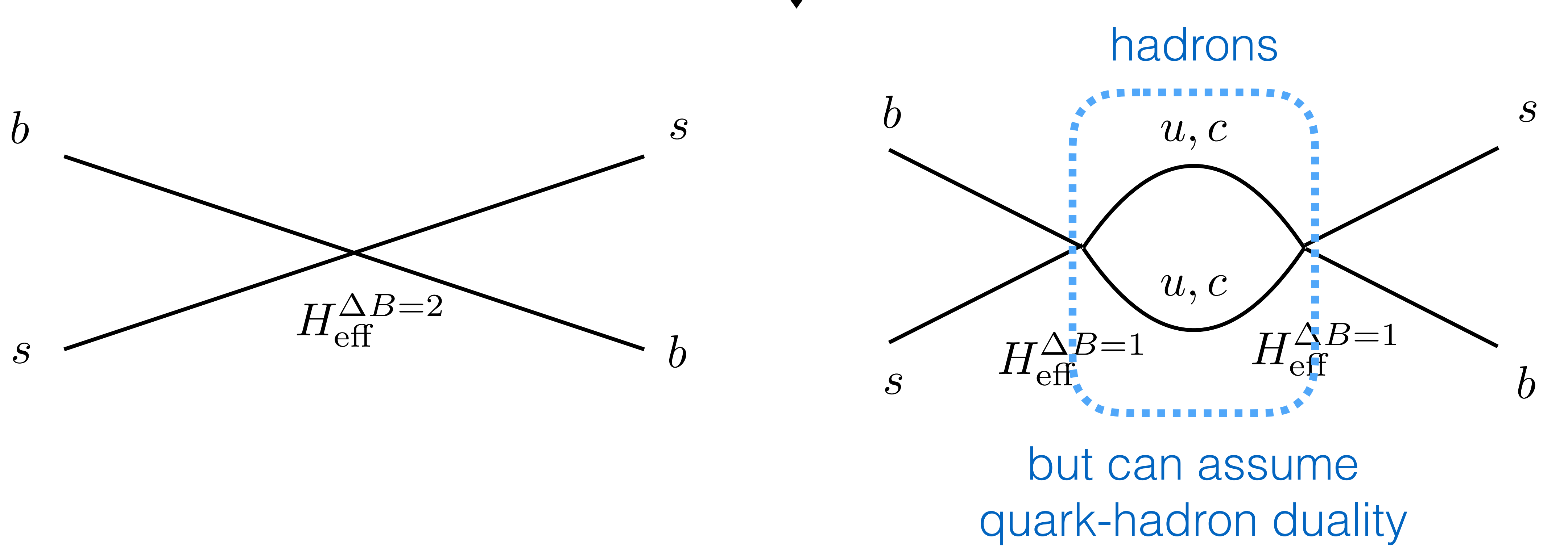


Mass difference

See talk by A. El-Khadra

$$\Delta M_s = \frac{1}{2m_{B_s}} \langle \bar{B}_s | H_{\text{eff}}^{\Delta B=2} | B_s \rangle$$

$$H_{\text{eff}}^{\Delta B=2} = (V_{ts}^* V_{tb})^2 \sum_{i=1}^{5} C_i Q_i$$

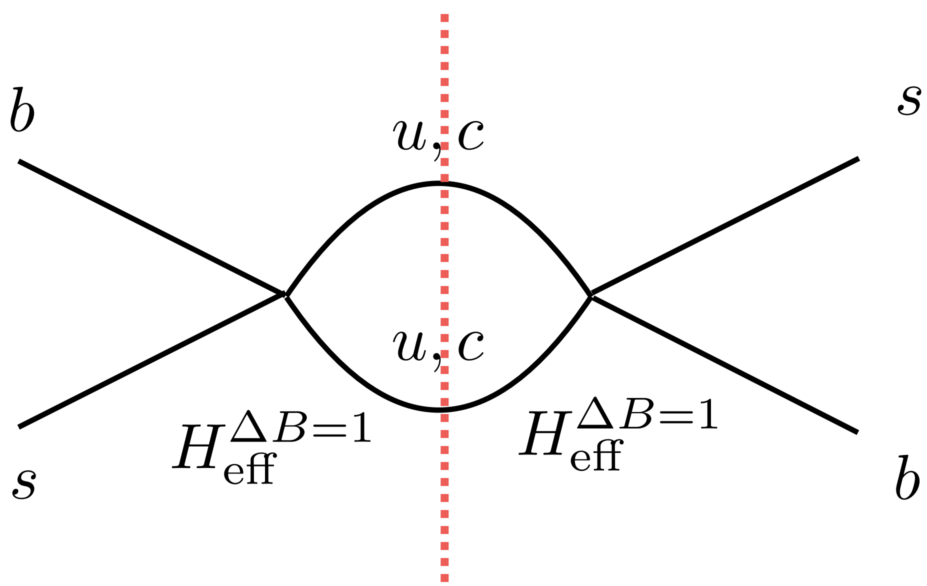
$$Q_1 = (\bar{b}^\alpha \gamma^\mu (1-\gamma^5) s^\alpha)(\bar{b}^\beta \gamma_\mu (1-\gamma^5) s^\beta)$$

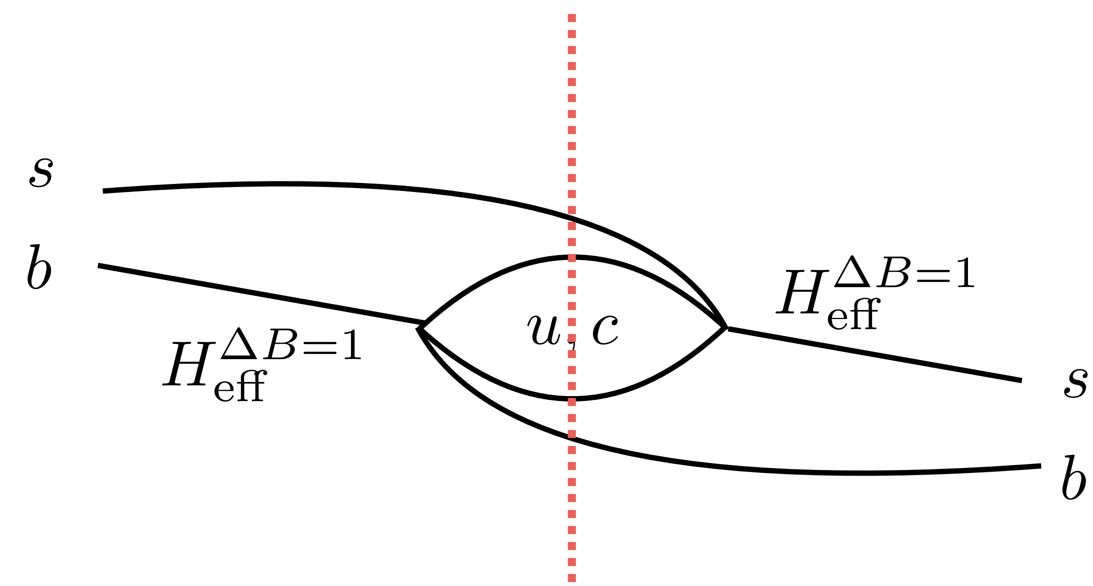
$$Q_2 = (\bar{b}^\alpha (1-\gamma^5) s^\alpha)(\bar{b}^\beta (1-\gamma^5) s^\beta) \qquad Q_4 = (\bar{b}^\alpha (1-\gamma^5) s^\alpha)(\bar{b}^\beta (1+\gamma^5) s^\beta)$$

$$Q_3 = (\bar{b}^\alpha (1-\gamma^5) s^\beta)(\bar{b}^\beta (1-\gamma^5) s^\alpha) \qquad Q_5 = (\bar{b}^\alpha (1-\gamma^5) s^\beta)(\bar{b}^\beta (1+\gamma^5) s^\alpha)$$

In the Standard Model: $C_i = 0$ for $i \geq 2$

Lifetime difference & HQE





- Γ_{12} from imaginary part (optical theorem)
- Assumes quark-hadron duality
- Large momentum through loop
- Operator product expansion: Heavy Quark Expansion (HQE)

HQE expressions

$$\Gamma_{12}^{s} = -\left[\,\lambda_c^2\,\Gamma_{12}^{cc} \;+\; 2\,\lambda_c\,\lambda_u\,\Gamma_{12}^{uc} \;+\; \lambda_u^2\,\Gamma_{12}^{uu}\,\right]$$

Leading order:

$$\Gamma_{12}^{ab} = \frac{G_F^2 m_b^2}{24\pi}\,M_{B_s}\,f_{B_s}^2\,\left[\left(G^{ab} + \frac{\alpha_2}{2}G_S^{ab}\right)\,\frac{8}{3}B \;+\; G_S^{ab}\alpha_1\,\frac{1}{3}\widetilde{B}_S'\right] \;+\; \widetilde{\Gamma}_{12,1/m_b}^{ab}$$

where lattice QCD can give matrix elements of Q_1 (B) and Q_3 $(\tilde{B}_S')$

Expressions from Lenz & Nierste, JHEP 06 (2007) 072

NLO

$$\widetilde{\Gamma}^{ab}_{12,1/m_b} = \frac{G_F^2 m_b^2}{24\pi M_{B_s}} \left[g_0^{ab} \langle B_s | R_0 | \overline{B}_s \rangle + \sum_{j=1}^{3} \left[g_j^{ab} \langle B_s | R_j | \overline{B}_s \rangle + \widetilde{g}_j^{ab} \langle B_s | \widetilde{R}_j | \overline{B}_s \rangle \right] \right]$$

$$R_0 = Q_2 + \alpha_1 Q_3 + \frac{1}{2} \alpha_2 Q_1$$

$$\begin{aligned}
R_1 &= \frac{m_s}{m_b} \, \overline{s}_\alpha (1+\gamma_5) b_\alpha \, \overline{s}_\beta (1-\gamma_5) b_\beta \;\; = \frac{m_s}{m_b} Q_4 \\
R_2 &= \frac{1}{m_b^2} \, \overline{s}_\alpha \overleftarrow{D}_\rho \gamma^\mu (1-\gamma_5) D^\rho b_\alpha \, \overline{s}_\beta \gamma_\mu (1-\gamma_5) b_\beta \\
R_3 &= \frac{1}{m_b^2} \, \overline{s}_\alpha \overleftarrow{D}_\rho (1+\gamma_5) D^\rho b_\alpha \, \overline{s}_\beta (1+\gamma_5) b_\beta
\end{aligned}$$

Status

Artuso, Borissov, Lenz, arXiv:1511.09466v1

$$\Delta\Gamma_s^{\mathrm{SM},2015} = (0.088 \pm 0.020)\ \mathrm{ps}^{-1}$$

Dominent uncertainties:

- 15% due to matrix element of R_2 (bag factor = 1 ± 0.5)
- 14% due to matrix element of Q_1 (FLAG, but see new FNAL/MILC)
- 8% due to renormalization scale

HPQCD calculation

C Davies, GP Lepage, C Monahan, J Shigemitsu, MW

- Extends ongoing calculation of matrix elements of dimension-6 $\Delta B=2$ operators ($Q_1 \ldots Q_5$)
- MILC highly improved staggered quark (HISQ) gauge field configurations (2+1+1 sea quarks)
- Nonrelativistic bottom quark, HISQ strange quark

Matching schemes

Continuum QCD

Lattice NRQCD

$$\langle Q_i \rangle_{\overline{\mathrm{MS}}} = \langle \hat{Q}i \rangle_L + \langle \hat{Q}i1 \rangle_L + \ldots$$

where lattice NRQCD is a 1/M expansion.

$$\hat{Q}i = (\bar{\Psi}_Q \Gamma_1 \Psi_q)(\bar{\Psi}_{\bar{Q}} \Gamma_2 \Psi_q) + (\bar{\Psi}_{\bar{Q}} \Gamma_1 \Psi_q)(\bar{\Psi}_Q \Gamma_2 \Psi_q).$$

$$\begin{aligned}\hat{Q}i1 = \frac{1}{2M} [&(\vec{\nabla}\bar{\Psi}_Q \cdot \boldsymbol{\gamma}\Gamma_1 \Psi_q)(\bar{\Psi}_{\bar{Q}} \Gamma_2 \Psi_q) \\ &+ (\bar{\Psi}_Q \Gamma_1 \Psi_q)(\vec{\nabla}\bar{\Psi}_{\bar{Q}} \cdot \boldsymbol{\gamma}\Gamma_2 \Psi_q) \\ &+ (\vec{\nabla}\bar{\Psi}_{\bar{Q}} \cdot \boldsymbol{\gamma}\Gamma_1 \Psi_q)(\bar{\Psi}_Q \Gamma_2 \Psi_q) \\ &+ (\bar{\Psi}_{\bar{Q}} \Gamma_1 \Psi_q)(\vec{\nabla}\bar{\Psi}_Q \cdot \boldsymbol{\gamma}\Gamma_2 \Psi_q)].\end{aligned}$$

Perturbative matching

Match continuum and lattice at $O(\alpha_s)$

$$\langle Q_i \rangle_{\overline{\mathrm{MS}}} = \langle \hat{Q}i \rangle + \alpha_s \rho_{ij} \langle \hat{Q}j \rangle + \langle \hat{Q}i1 \rangle^{\mathrm{sub}}$$

taking into account power-law "mixing down" at $O\left(\frac{\alpha_s}{aM}\right)$

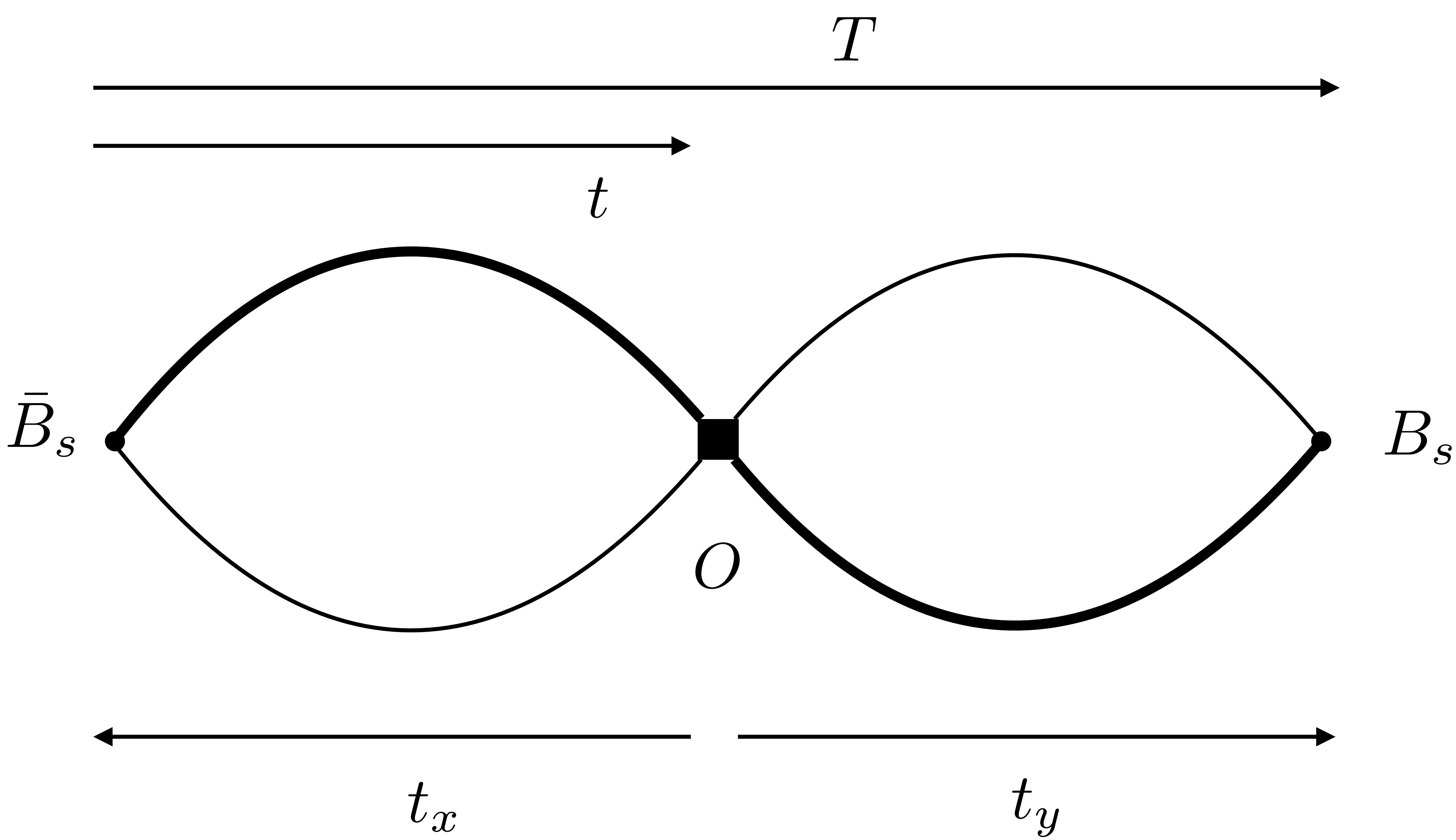
$$\langle \hat{Q}i1 \rangle^{\mathrm{sub}} = \langle \hat{Q}i1 \rangle - \alpha_s \zeta_{ij} \langle \hat{Q}j \rangle$$

Monahan, Gámiz, Horgan, Shigemitsu, PRD90 (2014)

Similarly we have now computed coefficients in

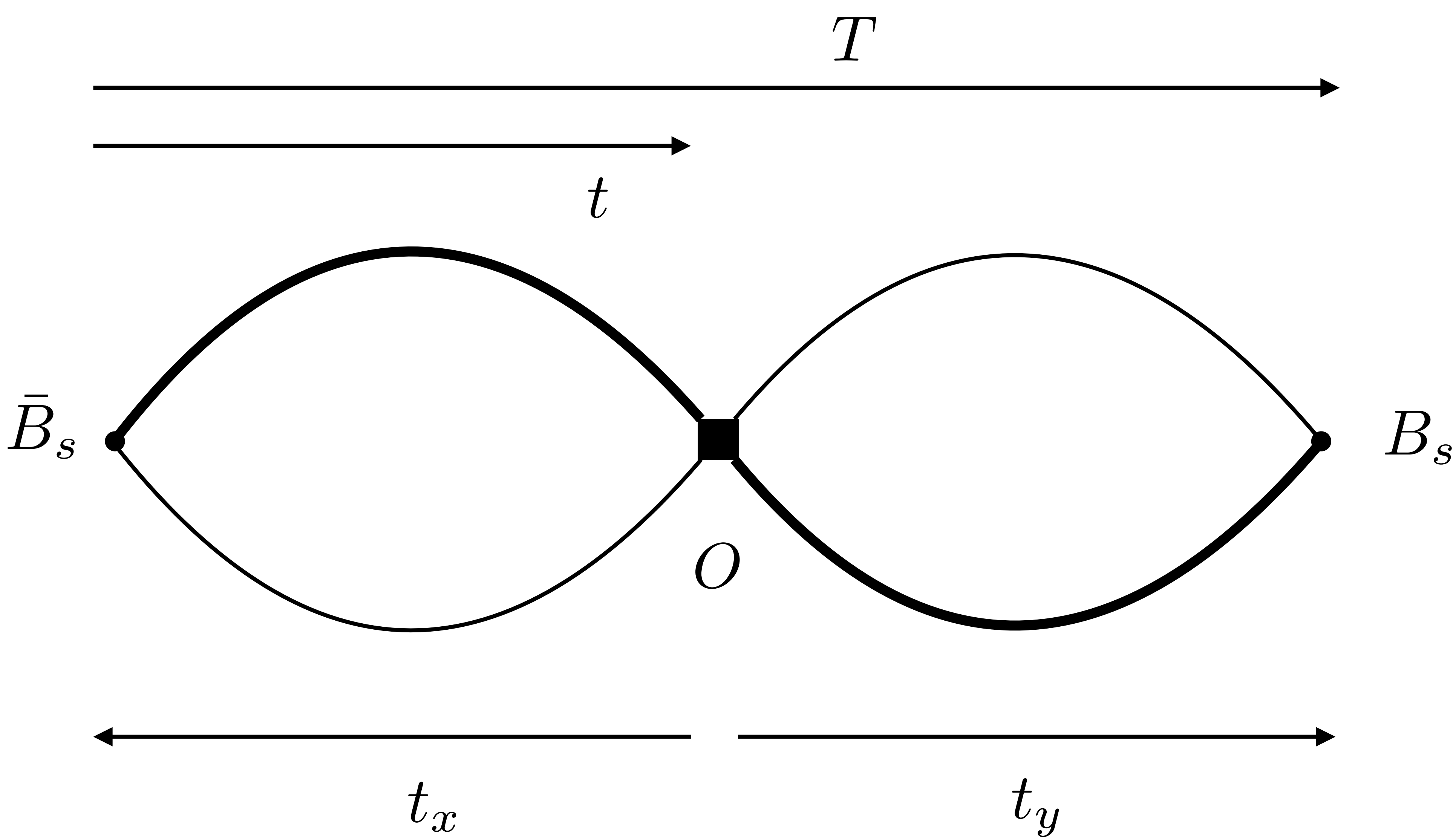
$$\langle \hat{R}_i \rangle^{\mathrm{sub}} = \langle \hat{R}_i \rangle - \alpha_s \xi_{ij} \langle \hat{Q}j \rangle$$

Correlation functions



Strange quark “source” at operator *O.*
Derivative source (finite difference) for *R* operators

Correlation functions



$$C_{ab}^{\rm 3pt}(t,T) = \sum_{i,j} X_{a,i} V_{nn,ij} X_{b,j} \exp(-E_i t) \exp(-E_j (T-t))$$

Correlation functions

$$C_{ab}^{\mathrm{3pt}}(t,T) = \sum_{i,j} X_{a,i} V_{nn,ij} X_{b,j} \exp(-E_i t) \exp(-E_j (T-t))$$

$$X_{a,0} V_{nn,00} X_{b,0} = \frac{\langle 0|\Phi_a|B_s\rangle\langle \bar{B}_s|a^6 O_{comp}|B_s\rangle\langle B_s|\Phi_b|0\rangle}{(2m_{B_s}a^3)^2}$$

Remove unwanted factors using 2-point functions

$$C_{ab}^{\mathrm{2pt}}(t) = \sum_{i} X_{a,i} X_{b,i} \exp(-E_i t)$$

$$X_{a,0} X_{b,0} = \frac{\langle 0|\Phi_a|B_s\rangle\langle B_s|\Phi_b|0\rangle}{2m_{B_s}a^3}$$

Status

- Matrix elements computed on 2 ensembles
  - will do 5 ensembles: 3 lattice spacings, including some with physically light quark masses
- Statistical errors 5-10% for $\langle R_i \rangle$ but will be larger for $\langle R_i \rangle^{\mathrm{sub}}$
- Systematic uncertainty dominated by tree-level matching between lattice and continuum: 20-30%
- Hope to show preliminary results in 2 weeks @ Lattice 2016

Lifetime ratios

For general lifetime calculations, need matrix elements of $\Delta B = 0$ operators

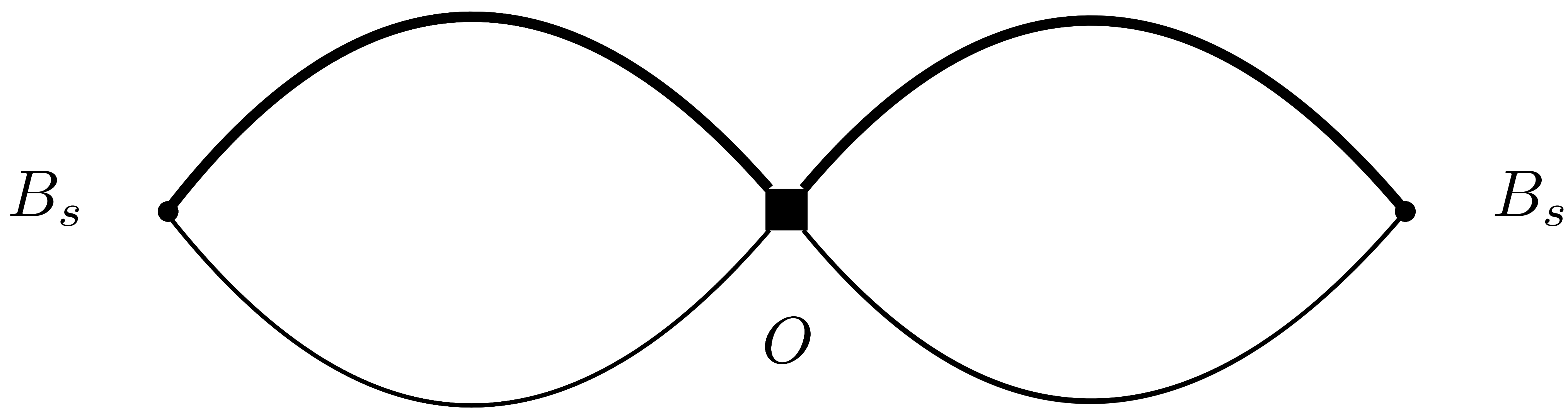
$$Q_1 = (\bar{b}^\alpha \gamma^\mu (1-\gamma^5) s^\alpha)(\bar{s}^\beta \gamma_\mu (1-\gamma^5) b^\beta)$$

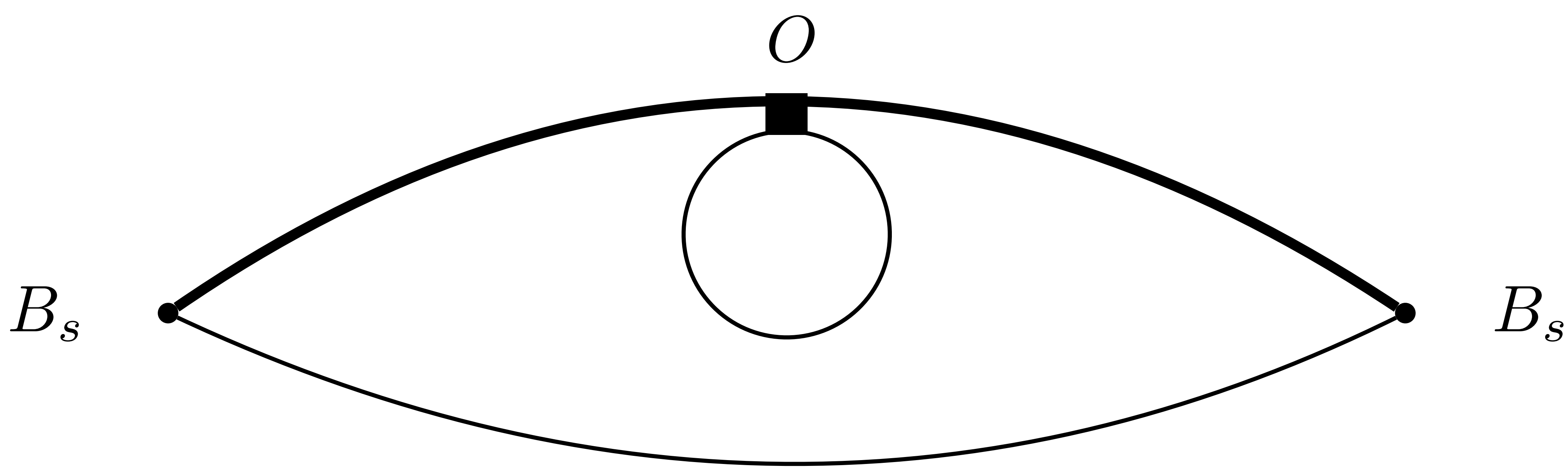
$$Q_2 = (\bar{b}^\alpha (1-\gamma^5) s^\alpha)(\bar{s}^\beta (1+\gamma^5) b^\beta)$$

$$Q_3 = (\bar{b}^\alpha \gamma^\mu (1-\gamma^5) t^a s^\alpha)(\bar{s}^\beta \gamma_\mu (1-\gamma^5) t^a b^\beta)$$

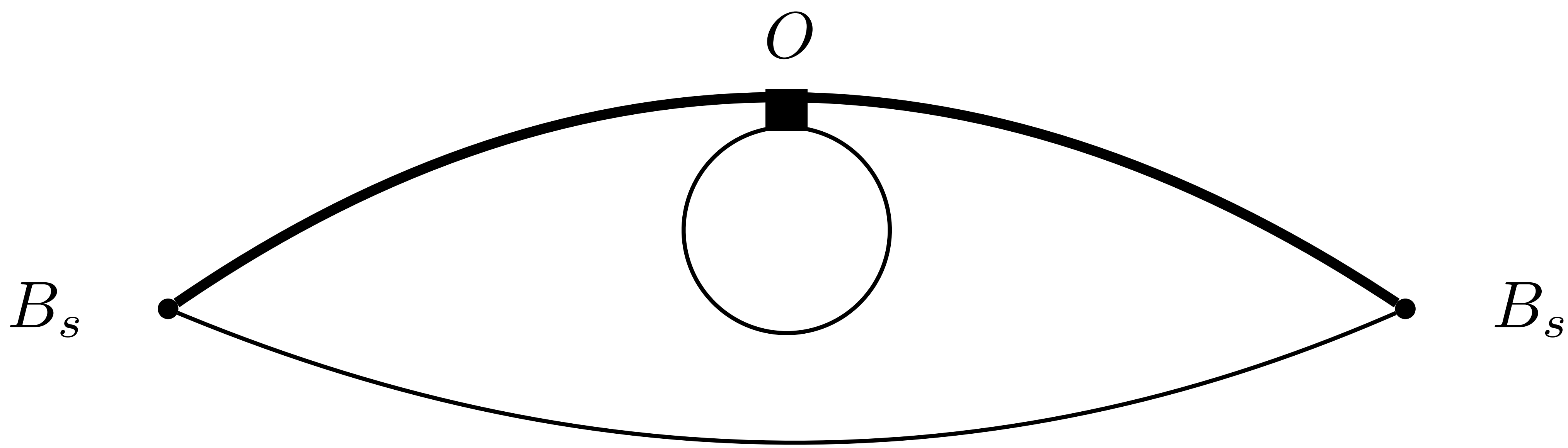
$$Q_4 = (\bar{b}^\alpha (1-\gamma^5) t^a s^\alpha)(\bar{s}^\beta (1+\gamma^5) t^a b^\beta)$$

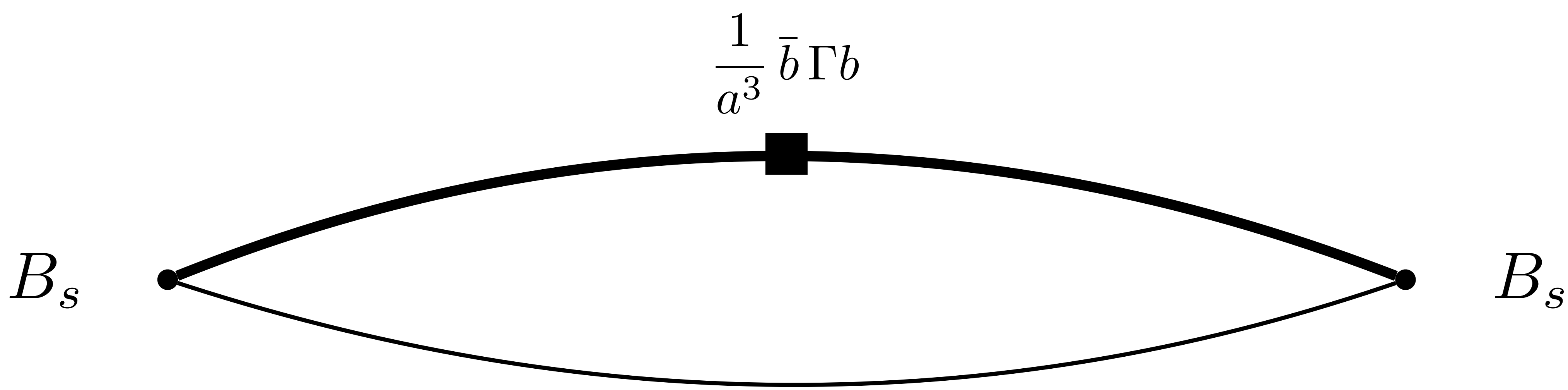
Contractions





Mixing





Severe fine tuning necessary to determine relevant contribution to lifetimes

Approximation

- In SU(3)$_F$ symmetric limit, spectator effects should cancel in ratios [Neubert & Sachrajda, NPB483 (1997)]

Big!

$$\frac{\tau(B^+)}{\tau(B^0)} = 1 \; + \; 16\pi^2 \frac{f_B^2 m_B}{m_b^3 c_3(m_b)} \left\{ G_1^{ss}(m_b)\; B_1(m_b) \; + \boxed{G_1^{oo}(m_b)}\, \varepsilon_1(m_b) \right.$$
$$\left. + \; G_2^{ss}(m_b)\; B_2(m_b) \; + \; G_2^{oo}(m_b)\; \varepsilon_2(m_b) + \bar{\delta}_{1/m_b} \right\}$$

- Lattice calculations exist [Becirevic, JHEP Proceedings 2001] but are dated (and only in proceedings)

- *B* factors =1.0 ± 20-30%, *ε* factors = few ×10^{-2} with 100% uncertainty

- A new calculation in this approximation is possible, but would take effort. Would we get a signal for the *ε* factors? Would it have an impact? …

Lifetime ratios

Artuso, Borissov, Lenz, arXiv:1511.09466v1

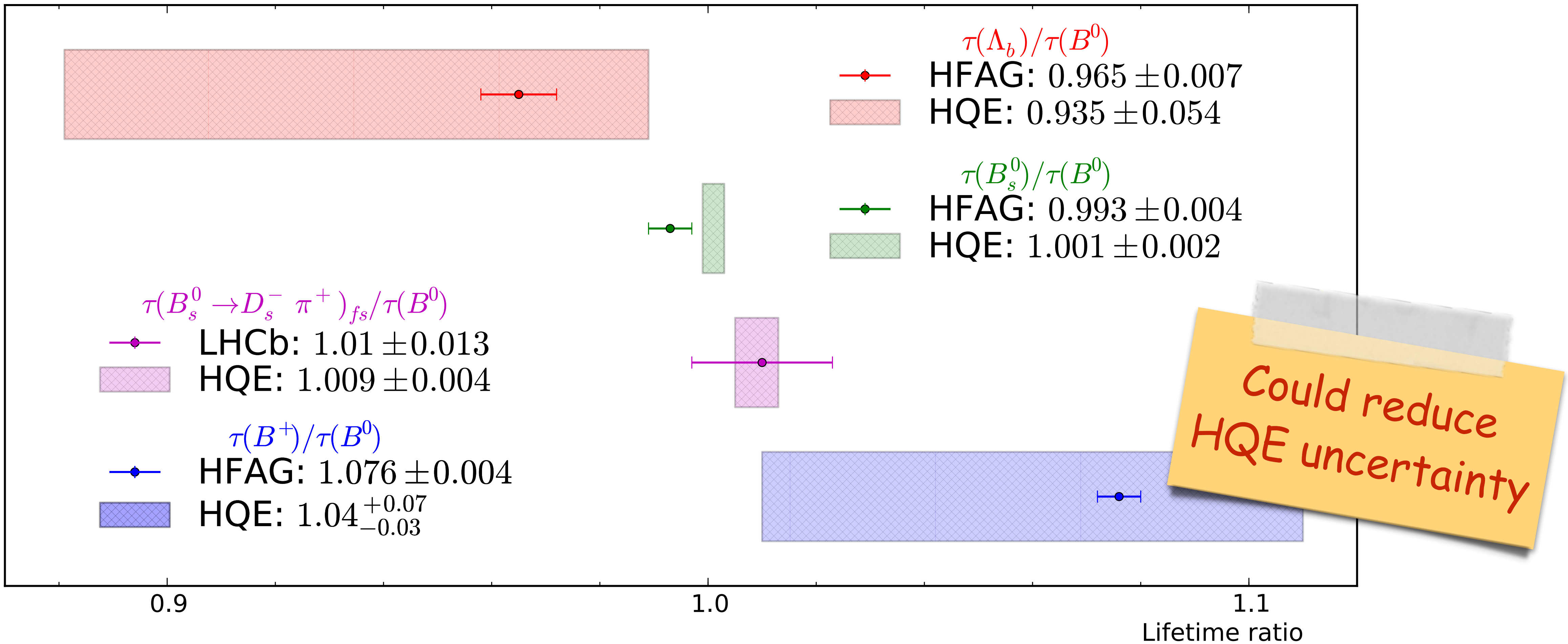


FIG. 3 Comparison of HQE predictions for lifetime ratios of heavy hadrons with experimental values. The theory values are taken from (Lenz, 2014). Experimental numbers are taken from HFAG ((Amhis *et al.*, 2014)).

Conclusions

- $\Delta\Gamma_s$: Calculation of R_2 & R_3 matrix elements
  - Preliminary results at Lattice 2016 (2 spacings)
  - Running s-quark done on 4 of 5 lattices
  - To do: full error estimates
- $\tau(B^+)/\tau(B_d)$: Can only do part of the calculation
- $\tau(D^+)/\tau(D^0)$: Test limits of HQE [Lenz, arXiv:1405.3601]