Neutral B-meson mixing with lattice QCD

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Collaborators

Fermilab Lattice Collaboration:

Freeland, Gámiz, Gottlieb, Kronfeld, Laiho, Mackenzie, Neil,
Simone, Van de Water, AXK
Bouchard, Chang, Du, Zhou,…

MILC:

Bazavov, Bernard, DeTar, Gottlieb, Heller, Hetrick, Sugar, Toussaint,
…

~ dozen physics projects

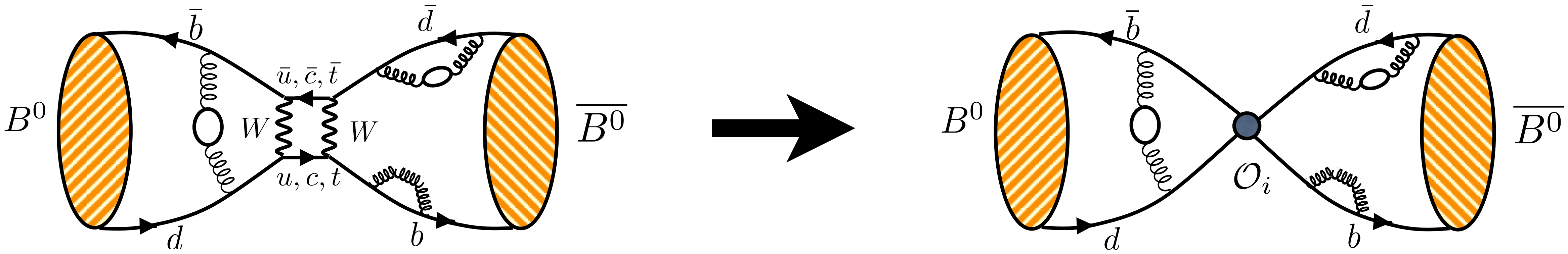
Computations done on:
NCSA (Blue Waters), Argonne, FNAL Lattice QCD clusters, …

Outline

- Introduction
- Lattice QCD introduction
  - heavy quark treatment
  - chiral-continuum extrapolation
  - systematic errors
- Results
- Phenomenological Implications
- Summary
- Quo Vadis?

Introduction

Standard Model



SM: $\Delta M_q = (\text{known}) \times |V_{tq}^* V_{tb}|^2 \times \langle \bar{B}_q^0 | \mathcal{O}_1 | B_q^0 \rangle$

also:

$$\frac{\Delta M_s}{\Delta M_d} = \frac{m_{B_s}}{m_{B\,d}} \times \left| \frac{V_{ts}}{V_{td}} \right|^2 \times \xi^2 \quad \text{with} \quad \xi \equiv \frac{f_{B_s}\sqrt{B_{B_s}}}{f_{B_d}\sqrt{B_{B_d}}}$$

$$\Delta\Gamma_q = \left[G_1 \langle \bar{B}_q^0 | \mathcal{O}_1 | B_q^0 \rangle + G_3 \langle \bar{B}_q^0 | \mathcal{O}_3 | B_q^0 \rangle \right] \cos\phi_q + O(1/m_b)$$

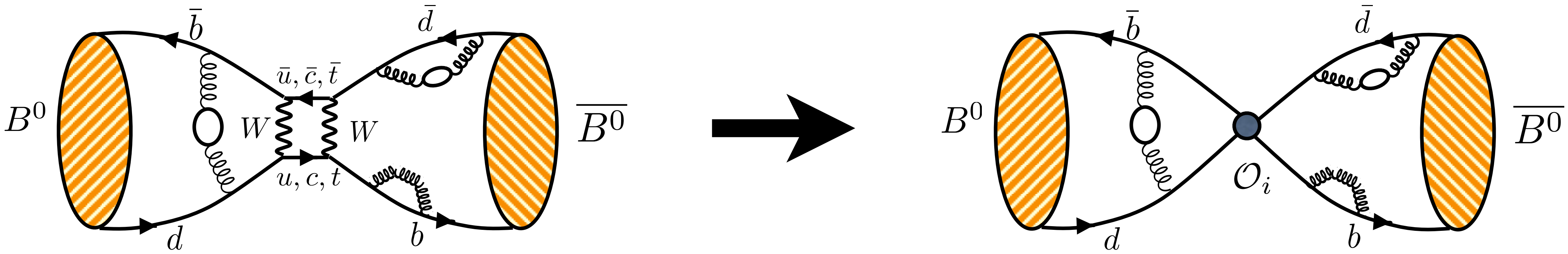
HFAG, PDG 2016 averages:

$$\Delta M_d = (0.5055 \pm 0.0020)\ \text{ps}^{-1}\ (0.4\%)$$
$$\Delta M_s = (17.575 \pm 0.021)\ \text{ps}^{-1}\ (0.1\%)$$

$$\Delta\Gamma_d/\Gamma_d = 0.001 \pm 0.010$$
$$\Delta\Gamma_s/\Gamma_s = 0.124 \pm 0.009\ \ (7.3\%)$$

Introduction

Standard Model



In general :

$$\mathcal{H}_{\rm eff} = \sum_{i=1}^{5} c_i(\mu)\mathcal{O}_i(\mu)$$

SM:

$$\mathcal{O}_1 = (\bar{b}^\alpha \gamma_\mu L q^\alpha)\ (\bar{b}^\beta \gamma_\mu L q^\beta)$$

$$\mathcal{O}_2 = (\bar{b}^\alpha L q^\alpha)\ (\bar{b}^\beta L q^\beta)$$

$$\mathcal{O}_3 = (\bar{b}^\alpha L q^\beta)\ (\bar{b}^\beta L q^\alpha)$$

BSM:

$$\mathcal{O}_4 = (\bar{b}^\alpha L q^\alpha)\ (\bar{b}^\beta R q^\beta)$$

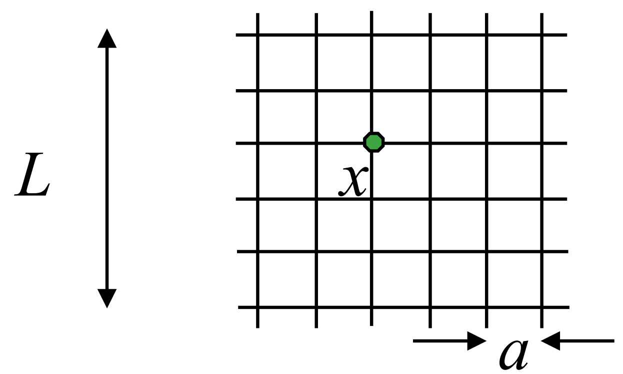
$$\mathcal{O}_5 = (\bar{b}^\alpha L q^\beta)\ (\bar{b}^\beta R q^\alpha)$$

$$\langle \mathcal{O}_i \rangle \equiv \langle \bar{B}_q^0 | \mathcal{O}_i | B_q^0 \rangle(\mu) = e_i\ m_{B_q}^2\ f_{B_q}^2\ B_{B_q}^{(i)}(\mu)$$

We calculate all five matrix elements.

Lattice QCD Introduction

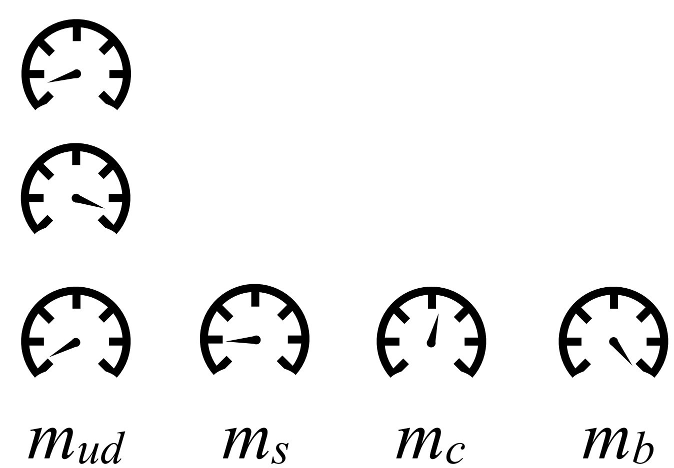
$$\mathcal{L}_{\rm QCD} = \sum_f \bar\psi_f (\not{D} + m_f)\psi_f + \frac{1}{4}{\rm tr} F_{\mu\nu}F^{\mu\nu}$$



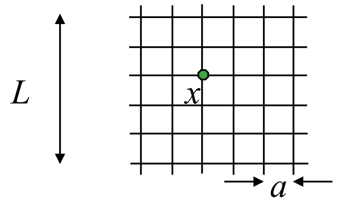
- ✦ discrete Euclidean space-time (spacing a)
  derivatives ➙ difference operators, etc…
- ✦ finite spatial volume (L)
- ✦ finite time extent (T)

adjustable parameters

- ❖ lattice spacing: $a \to 0$
- ❖ finite volume, time: $L \to \infty,\ T > L$
- ❖ quark masses (m_f): $M_{H,\rm lat} = M_{H,\rm exp}$
  tune using hadron masses $m_f \to m_{f,\rm phys}$
  extrapolations/interpolations



- ❖ also: n_f = number of sea quarks: 3 (2+1), 4 (2+1+1)



Lattice QCD Introduction

systematic error analysis

...of lattice spacing, chiral, heavy quark, and finite volume effects is based on EFT (Effective Field Theory) descriptions of QCD

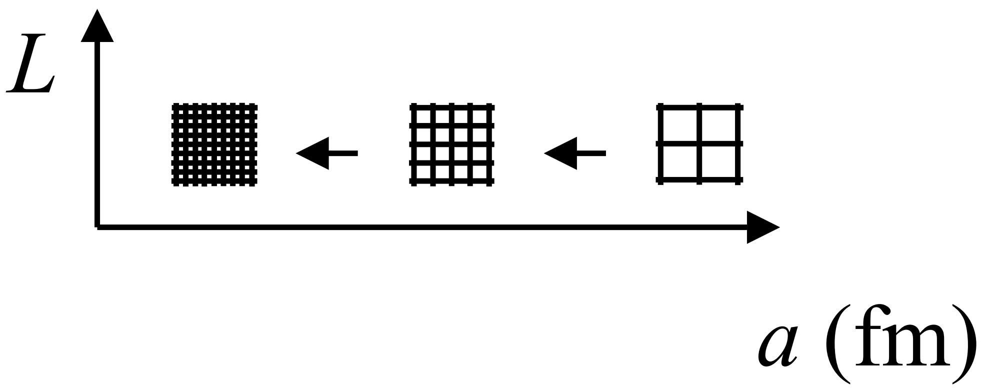
➙ ab initio

The EFT description:

- provides functional form for extrapolation (or interpolation)
- can be used to build improved lattice actions/methods
- can be used to anticipate the size of systematic effects

To control and reliably estimate the systematic errors

- repeat the calculation on several lattice spacings, light quark masses, spatial volumes, ...



Heavy Quark Treatment

- For light quarks ($m_\ell < \Lambda_{\rm QCD}$), discretization errors ~ $\alpha_s^k (a\Lambda_{\rm QCD})^n$
- For heavy quarks, discretization errors ~ $\alpha_s^k (am_h)^n$ with currently available lattice spacings

  for b quarks $am_b > 1$

  for charm $am_c \sim 0.15\text{-}0.6$

⇨ need effective field theory methods for *b* quarks
for charm can use light quark methods, if action is sufficiently improved

- avoid errors of $(am_b)^n$ in the action by using EFT:
  - ✦ relativistic HQ actions (Fermilab, Columbia, Tsukuba)
  - ✦ HQET
  - ✦ NRQCD

or

- use improved light quark actions for charm (HISQ, tmWilson, NP imp. Wilson,...) and for *b*:
  - ✦ use same LQ action as for charm but keep $am_h < 1$,
  - ✦ use HQET and/or static limit to extrapolate/interpolate to *b* quark mass

Heavy Quark Treatment

Relativistic Heavy Quarks - Fermilab formulation

- start with the relativistic Wilson action + $O(a)$ improvement
- with mass-dependent matching conditions, cut-off effects are

$$\alpha_s^k f(m_h a)(a\Lambda)^n \text{ with}$$

$$am_h \sim 1 : f(m_h a) \sim O(1)$$

FNAL/MILC implementation for action and currents:

tree-level tadpole $O(a)$ improved
mostly nonperturbative renormalization (mNPR)

Heavy-quark discretization errors

Fermilab formulation

- analyze cut-off effects with (continuum) HQET
- discretization errors arise due to mismatch of coefficients of the EFT descriptions of lattice and continuum matrix elements
- discretization errors take the form $\sim a^{d-4} f_k(am_0)\langle\mathcal{O}_k\rangle \sim f_k(am_0)(a\Lambda)^{d-4}$
- with tree-level tadpole $O(a)$ improvement we have errors $O(\alpha_s a\Lambda)$ and $O(a\Lambda)^2$

Renormalization and matching

Renormalization at one-loop in perturbation theory

$$\langle \mathcal{O}_i \rangle^{\mathrm{cont}}(\mu) = (\delta_{ij} + \alpha_s \zeta_{ij}) \langle \mathcal{O}_j \rangle^{\mathrm{lat}}(\mu) + O(\alpha_s^2)$$

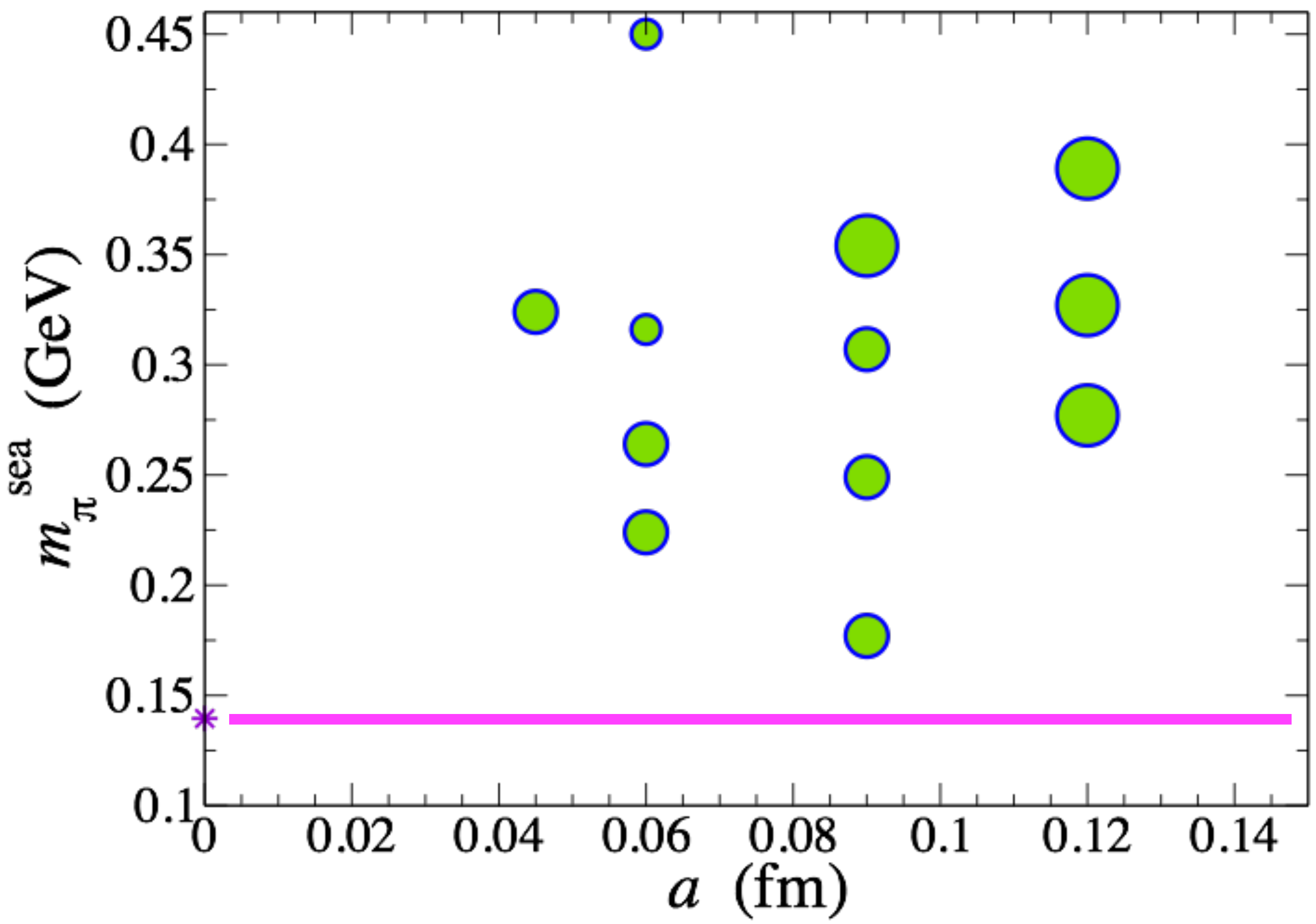
- mixing between operators due HQET matching
- $\zeta_{ij} = \zeta_{ij}(\mu, m_b, am_b) = Z_{ij}^{\mathrm{cont}} - Z_{ij}^{\mathrm{lat}}$
- calculated in lattice perturbation theory
- $\overline{\mathrm{MS}}$ -NDR scheme
- $\alpha_s = \alpha_V(2/a)$
- $\mu = m_b$

- mostly nonperturbative method (mNPR):

$$\langle \mathcal{O}_i \rangle^{\mathrm{cont}}(\mu) = Z_{V_{bb}^4} Z_{V_{\ell\ell}^4} (\delta_{ij} + \alpha_s \rho_{ij}) \langle \mathcal{O}_j \rangle^{\mathrm{lat}}(\mu) + O(\alpha_s^2)$$

chiral-continuum extrapolation

A. Bazavov et al(FNAL/MILC, arXiv:1602.03560, PRD 2016)



- 14 MILC asqtad ensembles
  4 lattice spacings
  ~ 4 sea quark masses per lattice spacing
  ~ 600 - 2000 configurations
  $\times\, 4$ time-sources per configuration

- asqtad light valence quarks
  ~ 7 light valence masses per ensemble

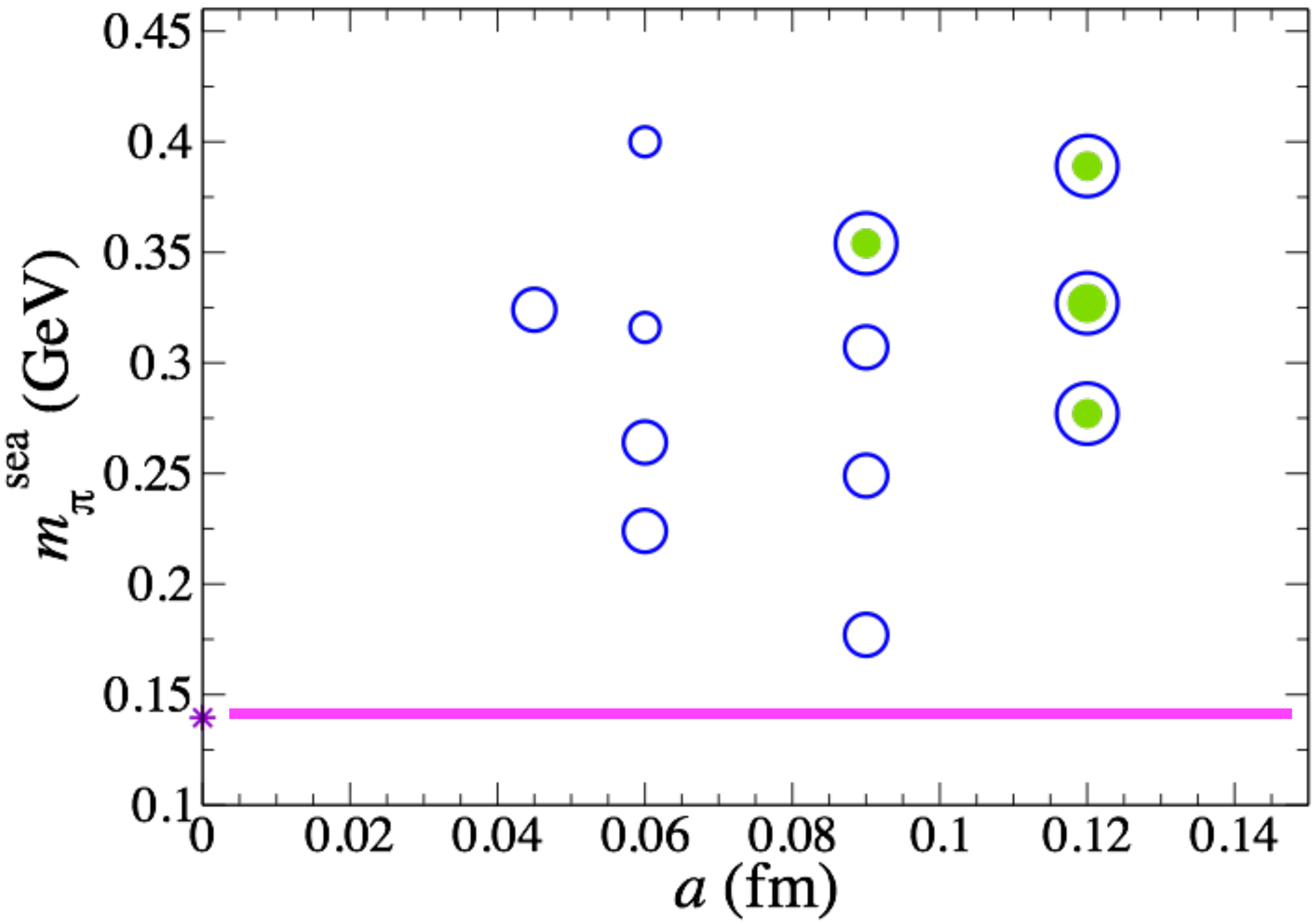
- Fermilab b quarks

- $O(a)$ improved four-quark operators

- mNPR renormalization

chiral-continuum extrapolation

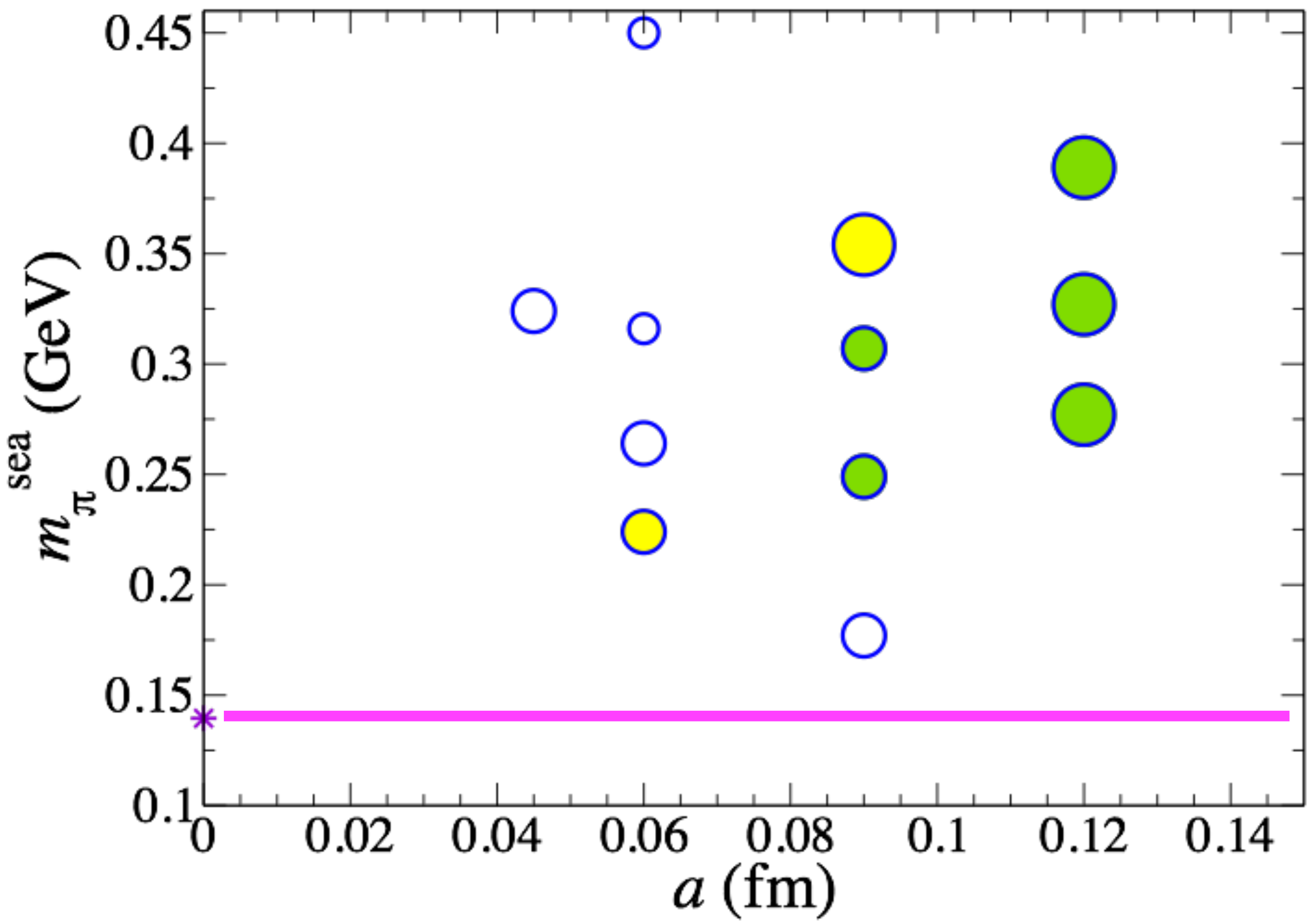
A. Bazavov et al (FNAL/MILC, arXiv:1205.7013, PRD 2012) - “old data”



- 6 MILC asqtad ensembles
  2 lattice spacings
  $4(2)$ sea quark masses per lattice spacing
  ~ 600 configurations
  $\times 4$ time-sources per ensemble
- asqtad light valence quarks
  ~ 7 light valence masses per ensemble
- Fermilab b quarks
- $O(a)$ improved four-quark operators

chiral-continuum extrapolation

C. Bouchard et al. (arXiv:1112.5642, Lattice 2011 proceedings)



- 6+3 (partial) MILC asqtad ensembles
  3 lattice spacings
  ~4 sea quark masses per lattice spacing
  ~ 600 - 2000 configurations
  $\times$ 4 time-sources per ensemble
- asqtad light valence quarks
  ~ 7 light valence masses per ensemble
- Fermilab b quarks
- $O(a)$ improved 4-quark operators

chiral-continuum extrapolation

Ensembles used here still have

$$m_{\text{light}} > 1/2\,(m_u + m_d)_{\text{phys}}$$

χPT guides the extrapolation to the physical point.

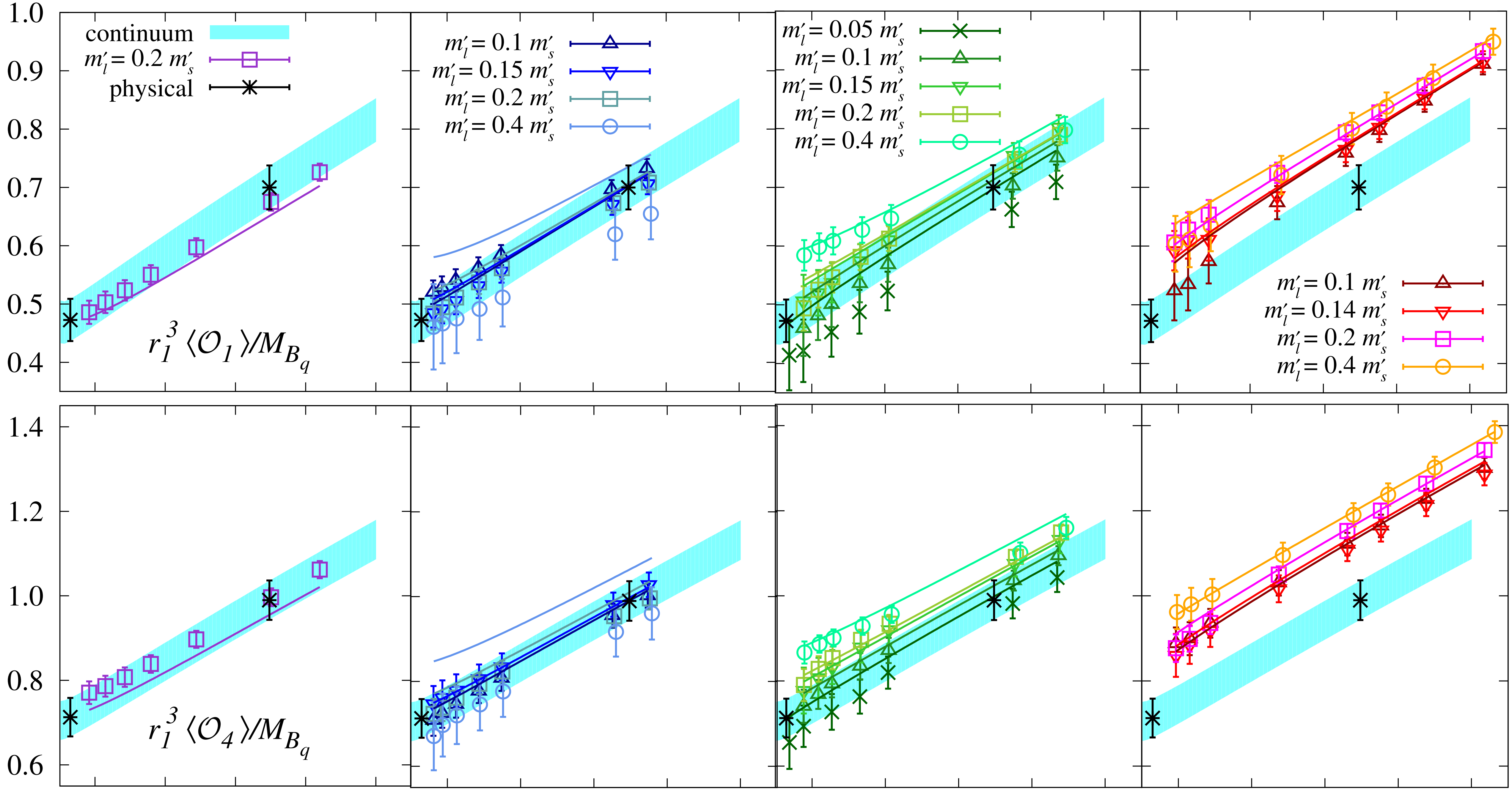
- include (light quark) discretization effects (for example, staggered χPT)
- combined continuum-chiral extrapolation
- Heavy meson χPT: $\chi\text{PT} + 1/M$ expansion
- also add HQ discretization terms to chiral-continuum fits

chiral-continuum extrapolation

SU(3) heavy-meson partially-quenched rooted staggered χPT

- NLO chiral logs + staggered discretization corrections
- + analytic terms (up to $\mathrm{N^3LO}$)
- + leading $1/M$ terms in HM expansion
- + HQ discretization terms
- + higher order PT terms (up to $O(\alpha_s)^3$)

chiral-continuum extrapolation

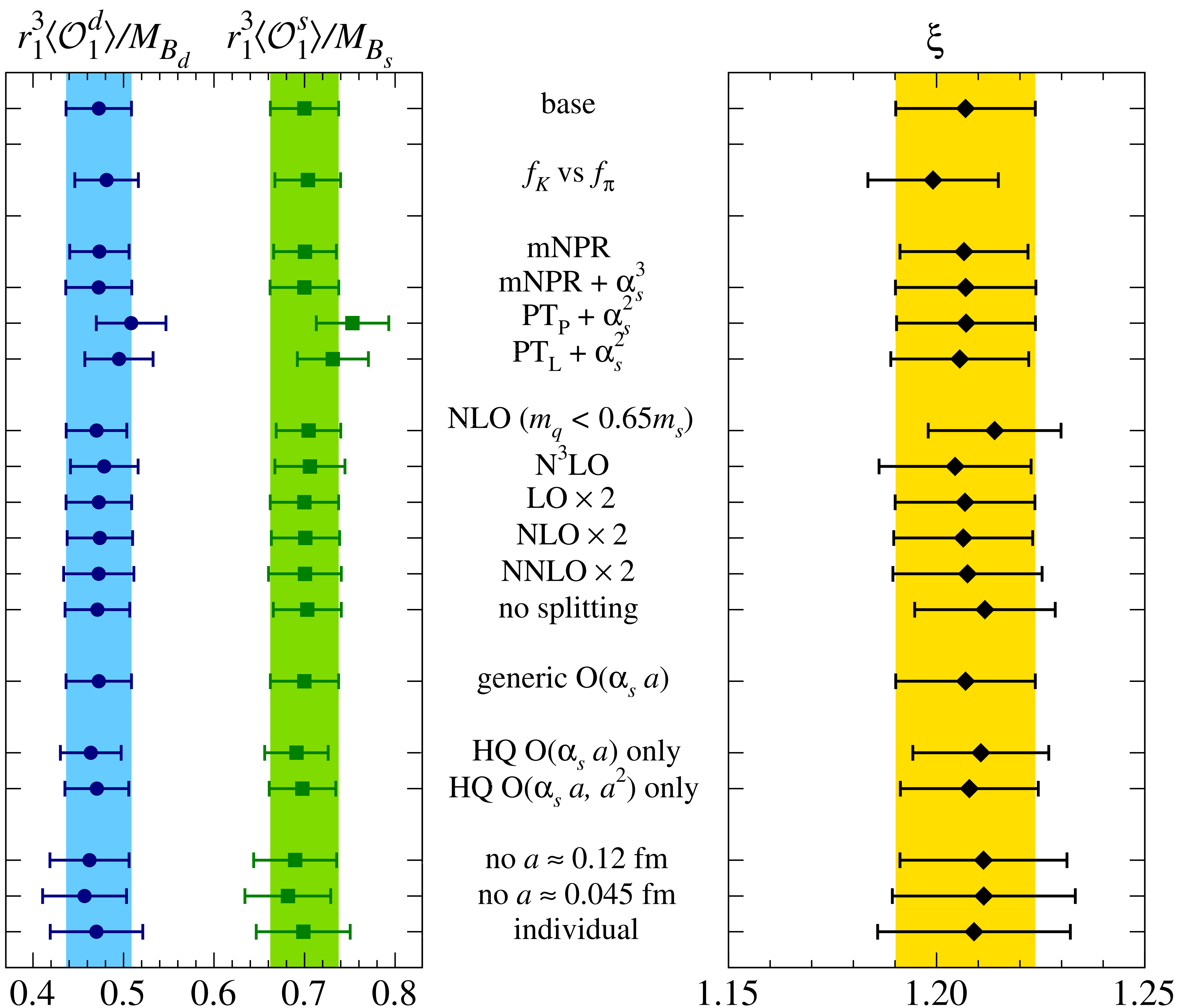


systematic error study

test stability of chiral-continuum extrapolation under changes of

- fit function: removing or adding higher order terms for
  - chiral expansion
  - heavy meson expansion
  - light quark discretization effects
  - HQ discretization effects
  - renormalization (perturbative expansion)
- data included
- inputs

systematic error study

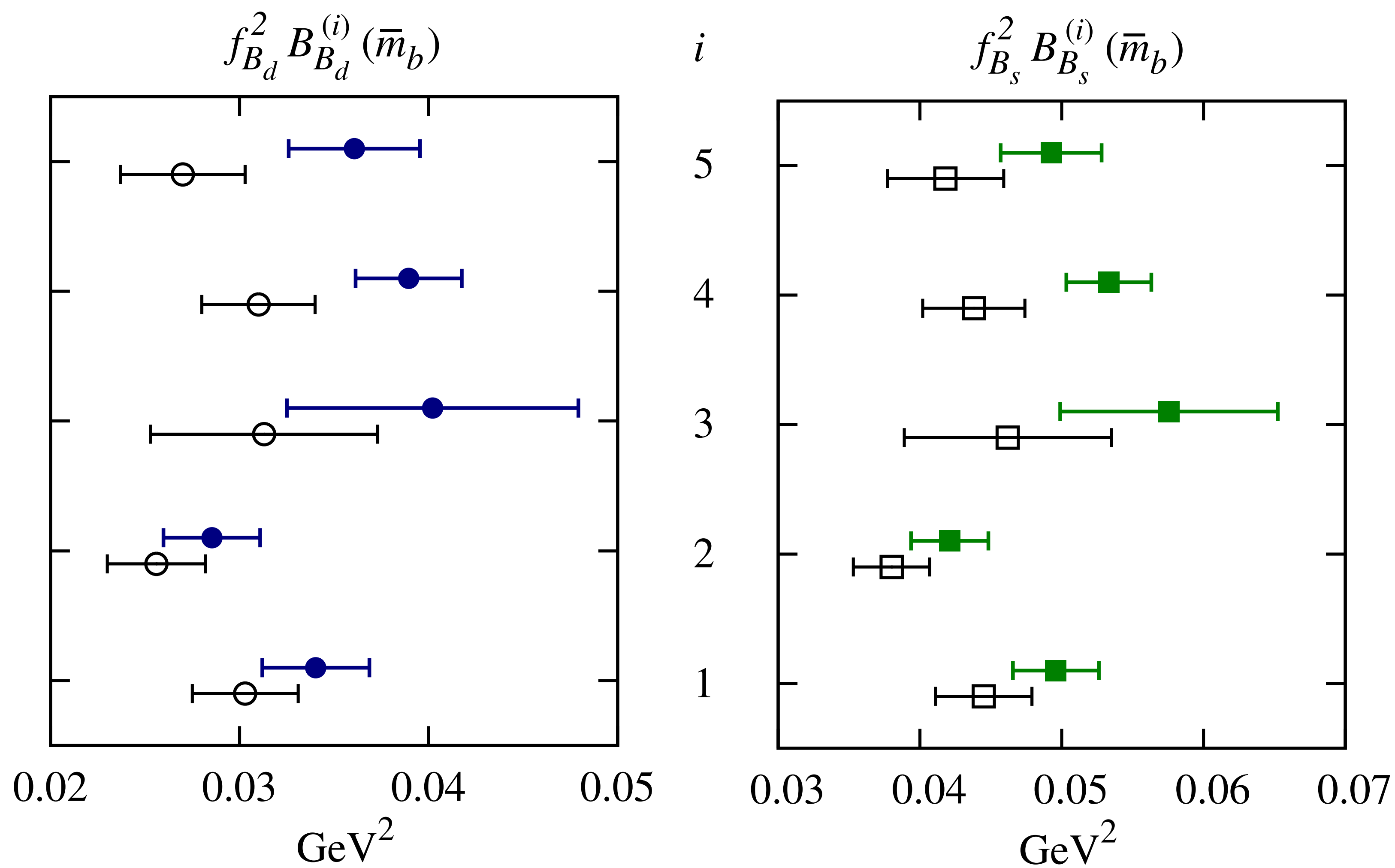


systematic error budget

	ξ		$f^2_{B_q} B^{(1)}_{B_q}$		$f^2_{B_q} B^{(i)}_{B_q}$		
source	2012	2016	2011	2016	2011	2016	
comb stat. χPT- cont.	3.7 +3.2	1.4	7 15	5.4 7.7	3-11 4.3-16	5-13 6-19	B_s B_d
HQ disc.	0.3	included	4	included (3-5)	4	included (3-10)	
inputs	0.7	included	5.1	included	5.1	included	
scale	in inputs	0.6	in inputs	3	in inputs	3	
matching/ renorm	0.5	included (0.5)	8	included (2-3)	8	included (2-12)	
FV	0.5	< 0.1	1	1	1	< 0.3	
EM	-	0.04	-	0.2	-	0.2	
total	**5**	**1.5**	**12** **18**	**6.1** **8.3**	**10-15** **11-19**	**6-13** **8-19**	B_s B_d
charm sea	-	0.5	-	2	-	2	

results in comparison

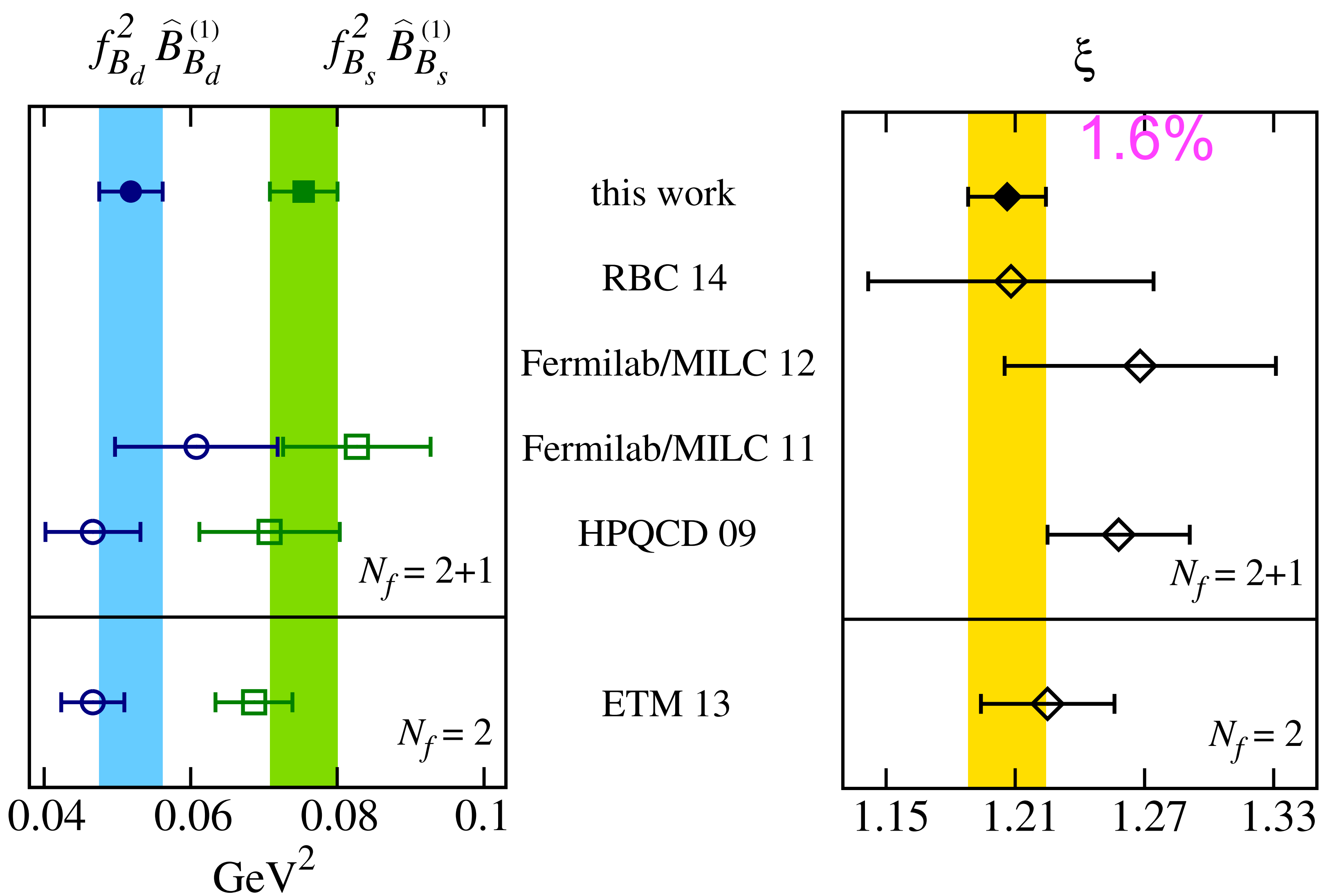
ETM (n_f=2, arXiv:1308.1851, JHEP 2014) vs. FNAL/MILC (n_f=3, arXiv:1602.03560, PRD 2016)



⭐ **First** three flavor LQCD results for all five matrix elements including the correlations between all 10 MEs.

results in comparison

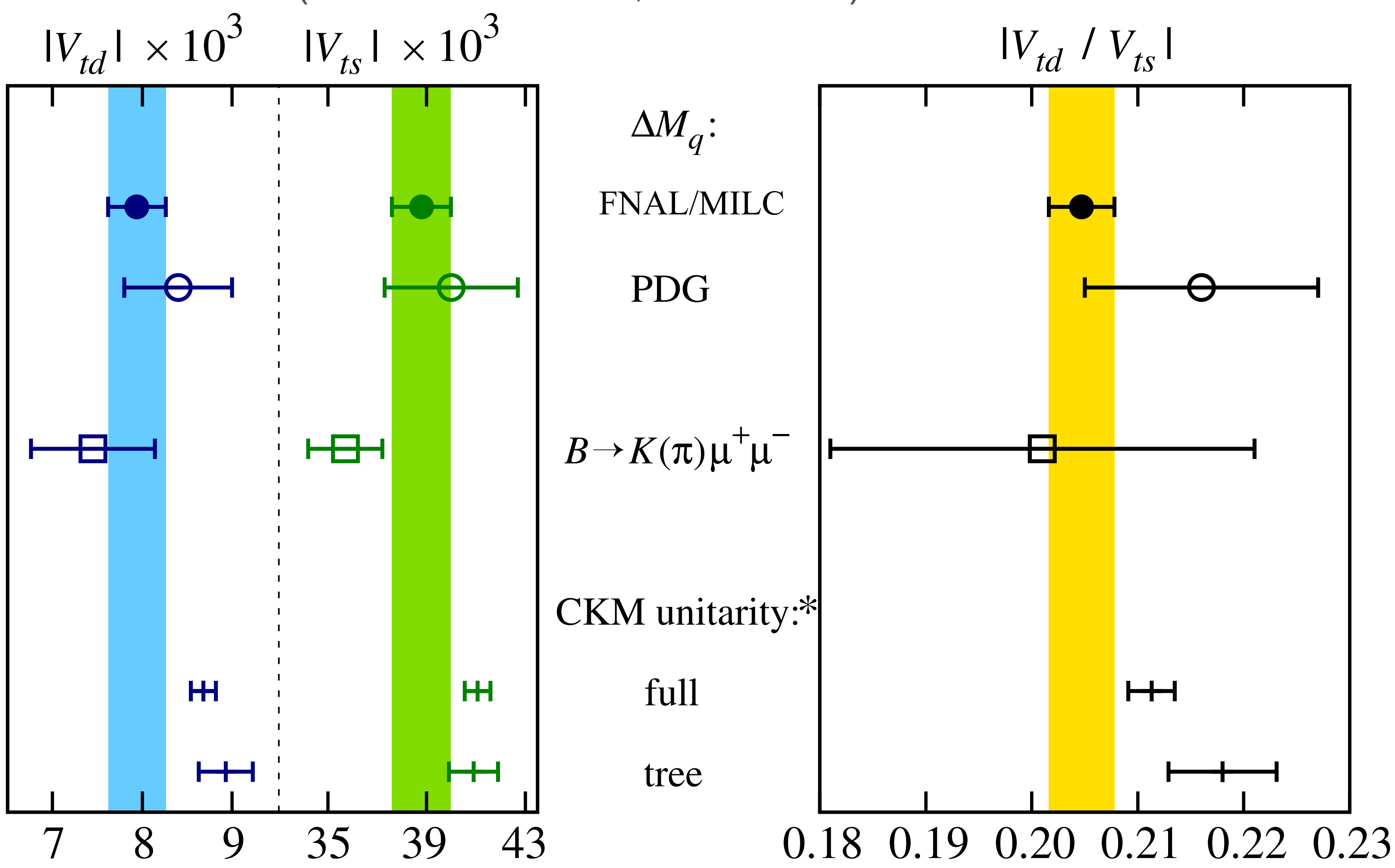
FNAL/MILC (arXiv:1602.03560, PRD 2016)



Significant reduction of errors compared to previous three flavor results, especially for ξ

Implications for $|V_{ts}|$, $|V_{td}|$, $|V_{td}/V_{ts}|$

D. Du et al (arXiv:1510.02349, PRD 2016)



Blanke & Buras:
(arXiv:1602.04020, EPJC 2016)

tension between $\Delta M_{s,d}$ & ϵ_K inconsistent with CMFV (Constrained Minimal Flavor Violation)

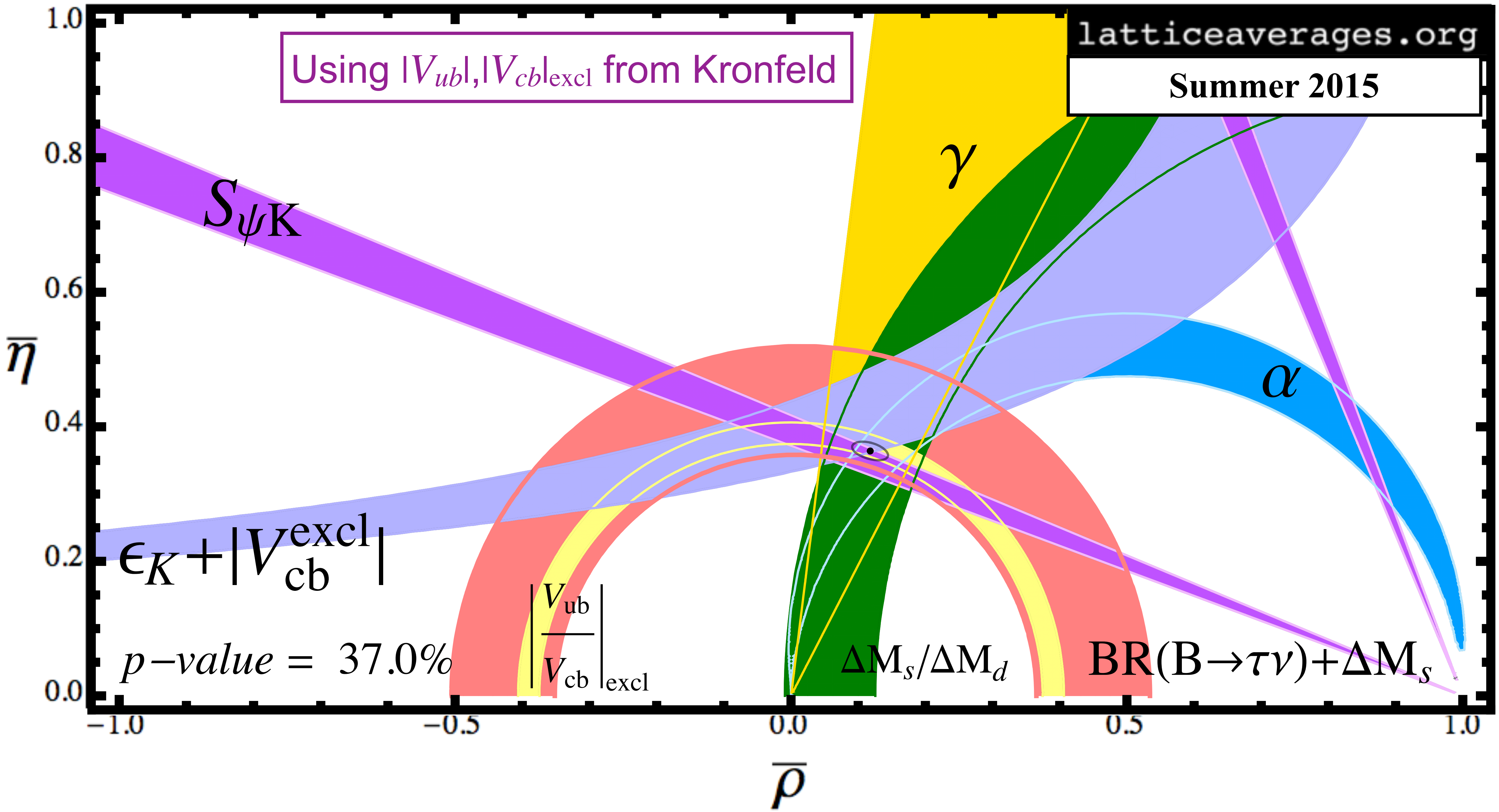
Buras & De Fazio:
(arXiv:1604.02334)
implications for "331" models

$\sim 2\sigma$ tensions between loop processes and CKM unitarity.

*from CKMfitter 2015 (hep-ph/0406186, http://ckmfitter.in2p3.fr)

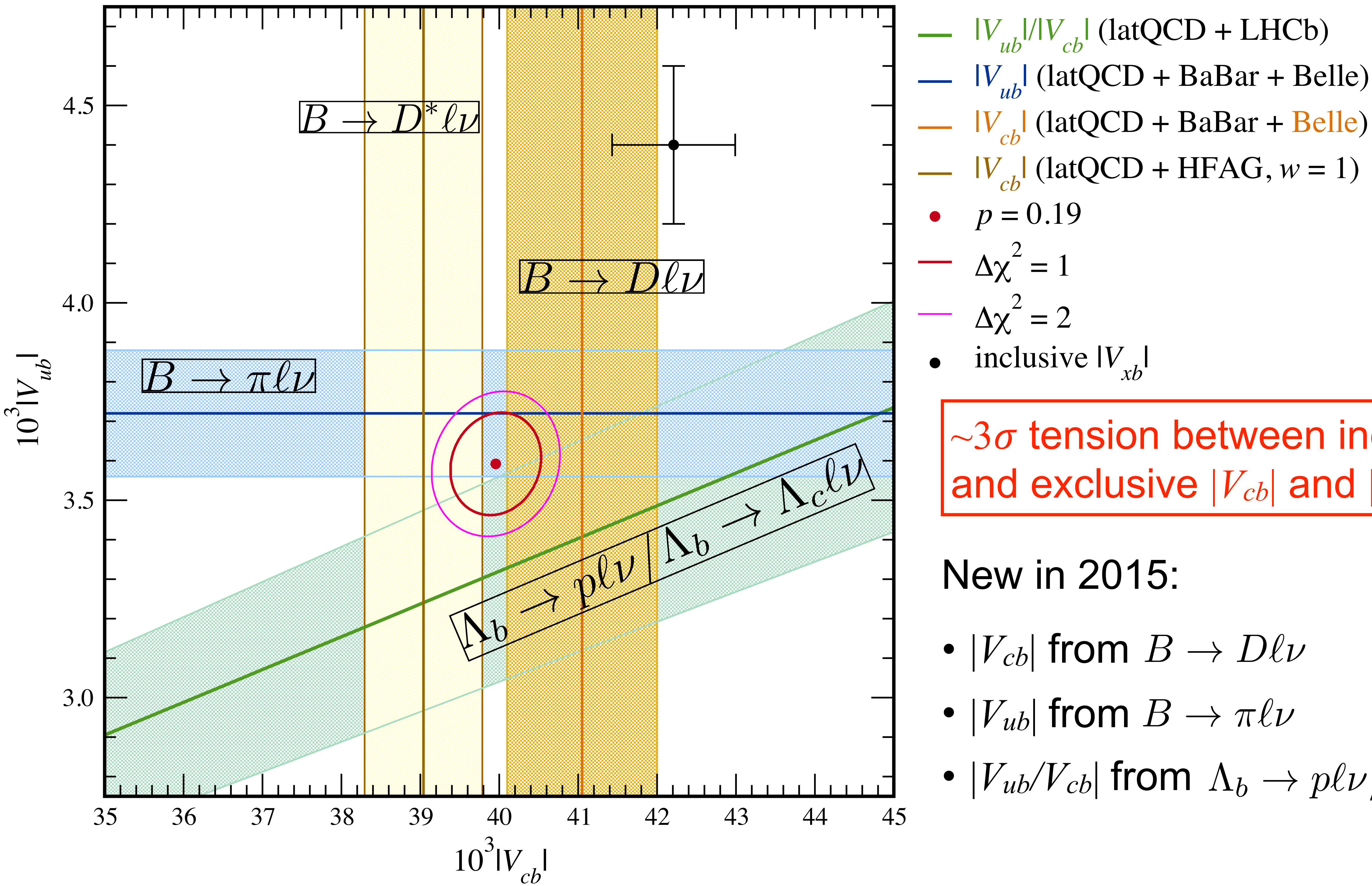
UT analysis

Laiho, Lunghi & Van de Water (Phys.Rev.D81:034503,2010), E. Lunghi, private comm.



Exclusive vs. inclusive $|V_{cb}|$ and $|V_{ub}|$

A. Kronfeld (priv. communication)



$\sim 3\sigma$ tension between inclusive and exclusive $|V_{cb}|$ and $|V_{ub}|$

New in 2015:

- $|V_{cb}|$ from $B \to D\ell\nu$
- $|V_{ub}|$ from $B \to \pi\ell\nu$
- $|V_{ub}/V_{cb}|$ from $\Lambda_b \to p\ell\nu/\Lambda_b \to \Lambda_c\ell\nu$

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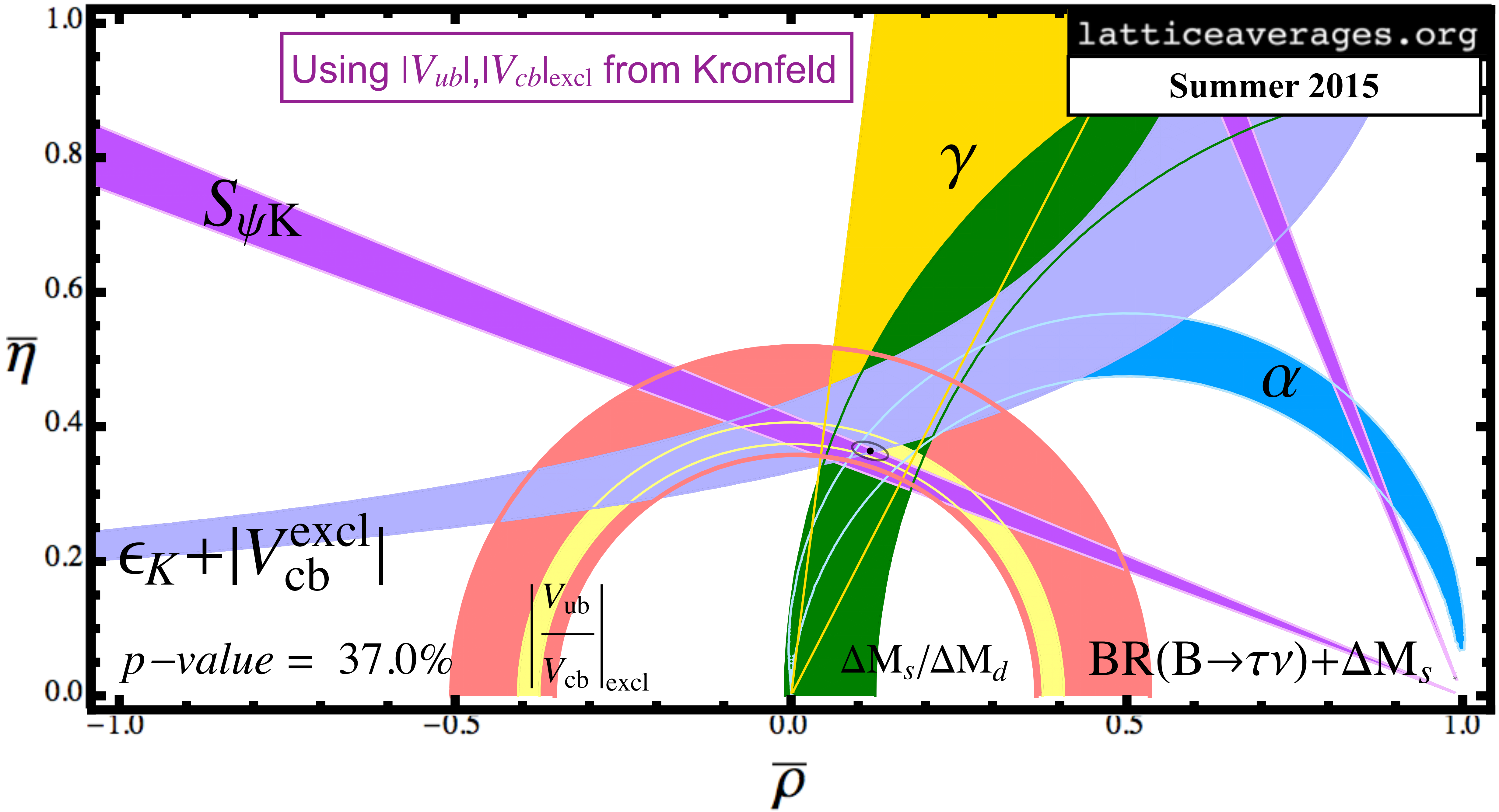

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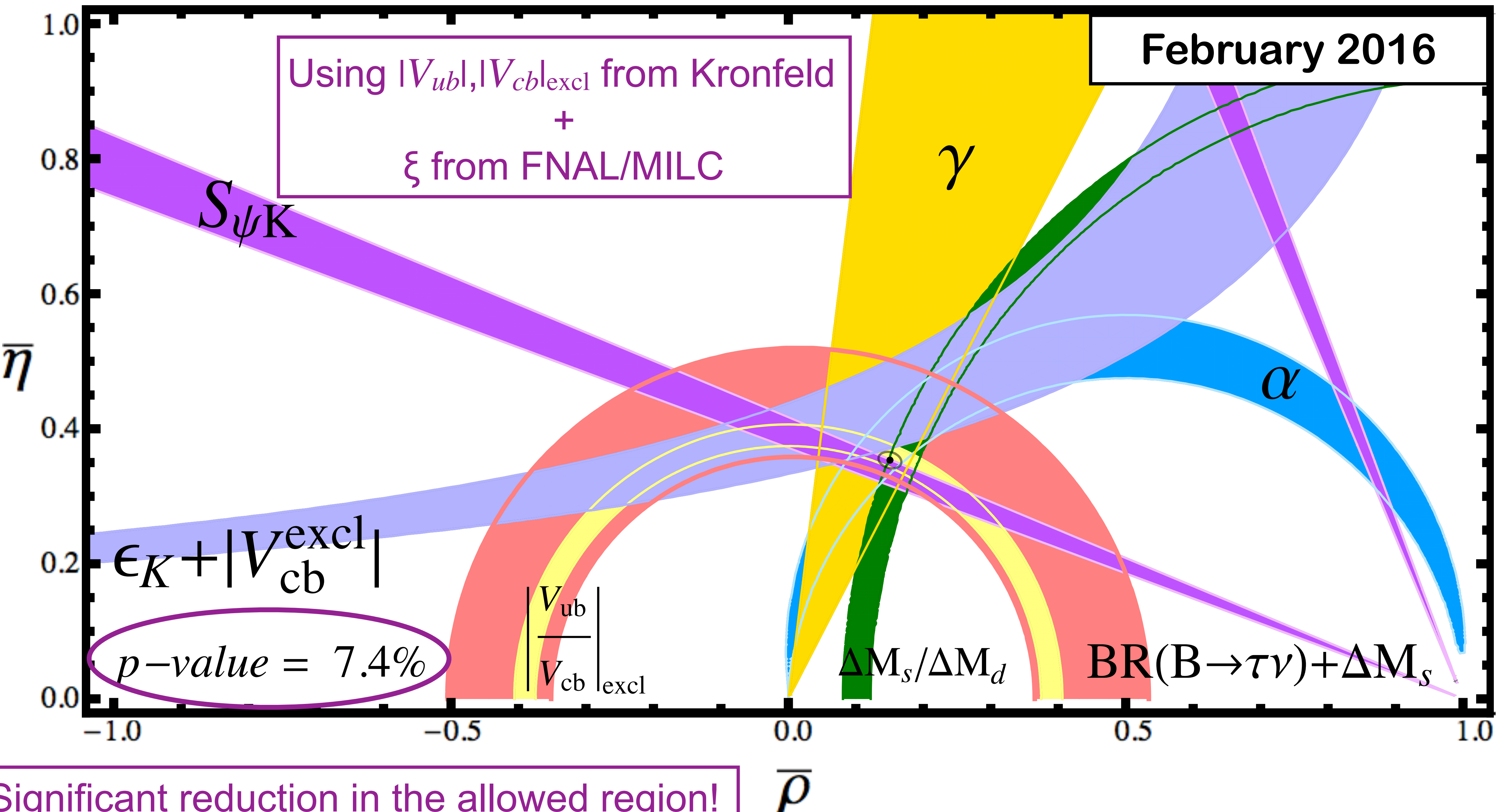
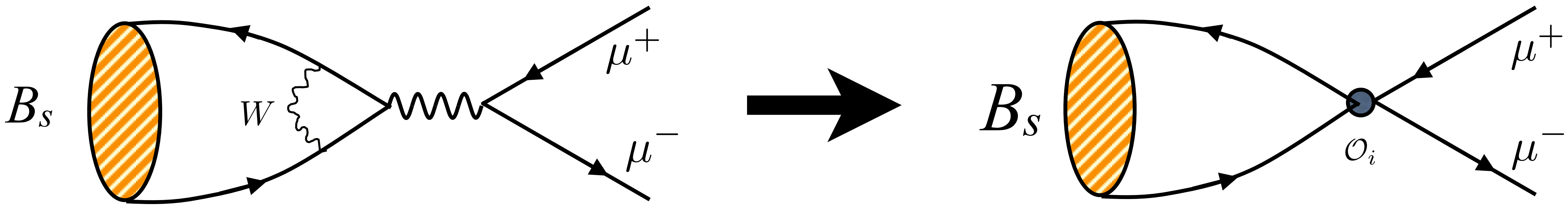


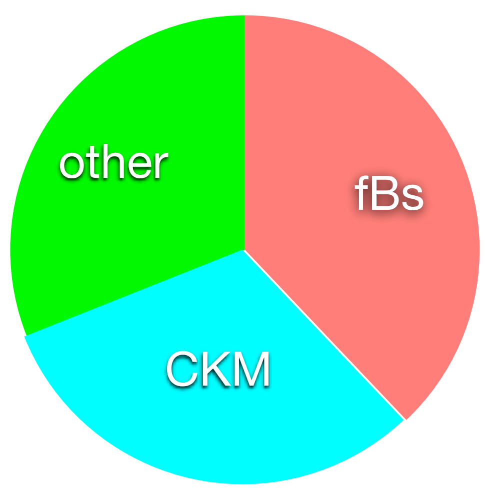
Significant reduction in the allowed region!

Rare leptonic decay $B_s \to \mu^+\mu^-$

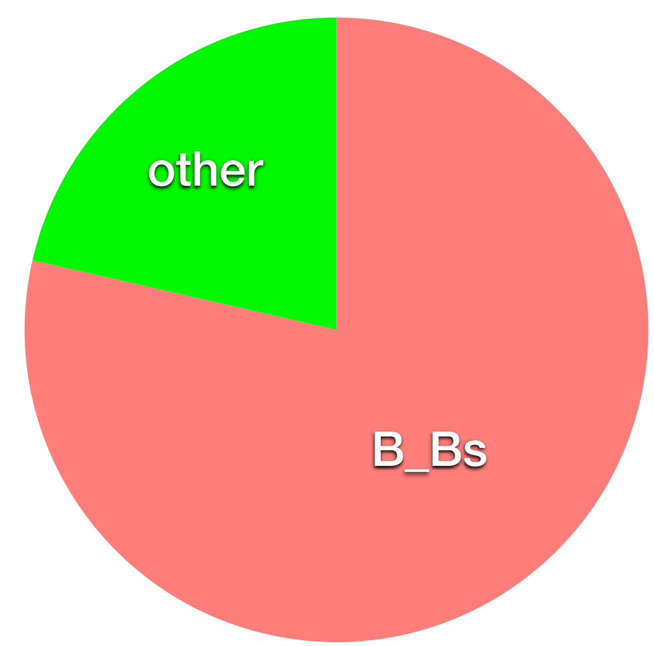


Standard Model prediction: Buras, et al (arXiv:1303.3820, JHEP 2013), Bobeth, et al (arXiv:1311.0903, PRL 2014)

$$\bar{\mathcal{B}}(B_s \to \mu^+\mu^-) = 3.53(11)(9)(9) \times 10^{-9}$$



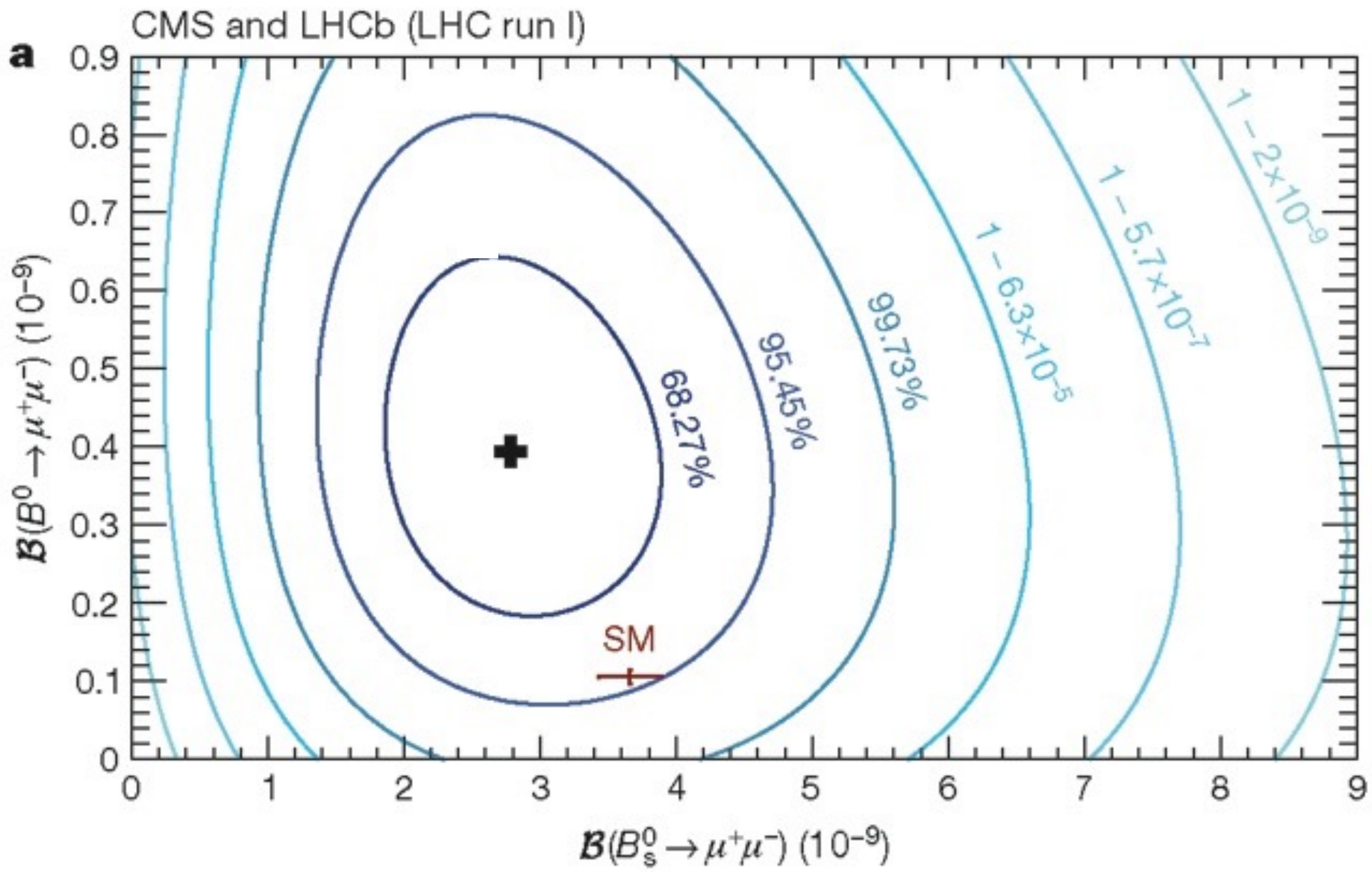
$$\bar{\mathcal{B}}(B_s \to \mu^+\mu^-) = 3.22(22)(6) \times 10^{-9}$$



FNAL/MILC (arXiv:1602.03560)

BSM phenomenology $B_{s(d)} \to \mu^+\mu^-$

CMS+LHCb combined (arXiv:1411.4413, Nature 2015)

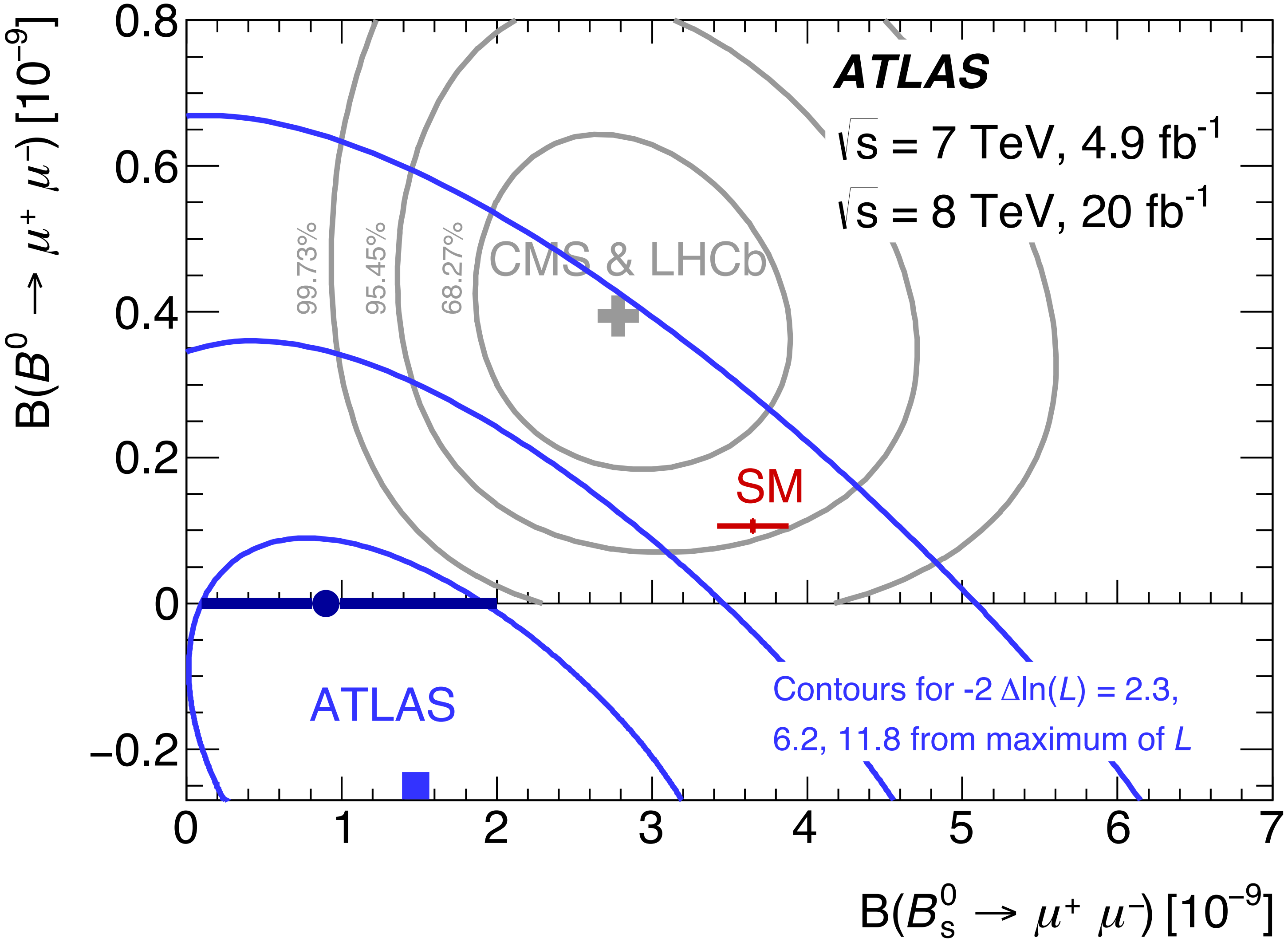


exp. measurements consistent with SM expectations, but with ample room for NP.

SM predictions depend on $f_{B(s)}$ or $\hat{B}_{B_s}$

BSM phenomenology $B_{s(d)} \to \mu^+\mu^-$

CMS+LHCb combined (arXiv:1411.4413, Nature 2015) and ATLAS (arXiv:1604.04263)



exp. measurements consistent with SM expectations, but with ample room for NP.

Summary

- ☆ new FNAL/MILC results for neutral B meson mixing matrix elements with significantly smaller theory uncertainties than before…
  … but still larger than experimental errors …
  Note: Errors on bag parameters will improve when companion f_B analysis is final.

- ☆ Precise LQCD results for semileptonic form factors for $B \to \pi, K, D$ transitions
  ➢ SM pre/postdictions with theory errors that are commensurate with experimental uncertainties

  ➠ emerging $\sim 2\sigma$ tensions between loop processes and CKM unitarity

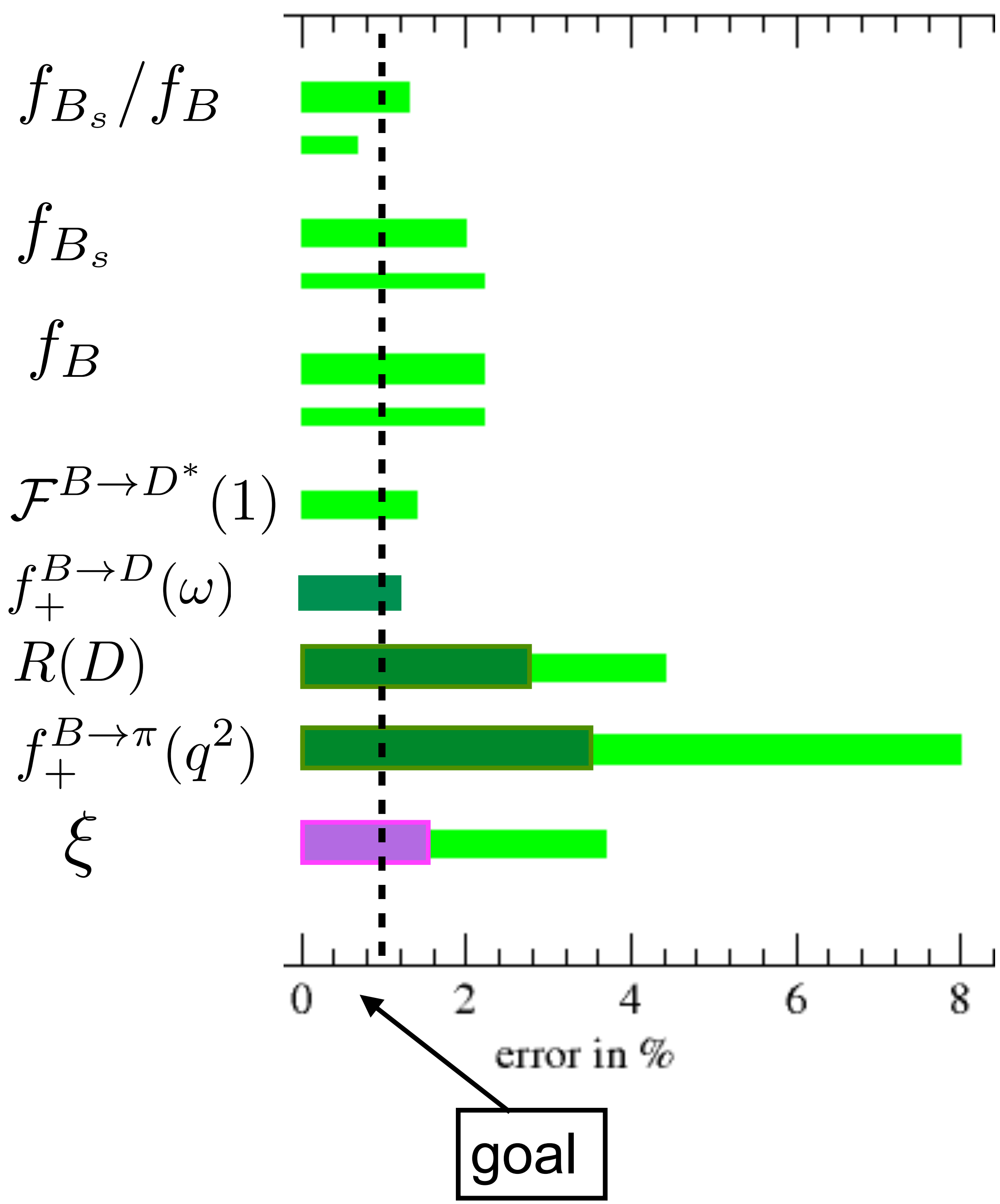
- ☆ tension for $|V_{cb}|$ and $|V_{ub}|$ between exclusive and inclusive determinations remains, but new $B \to D$ analysis with LQCD form factors at nonzero recoil brings $|V_{cb}|$ exclusive closer to inclusive result.

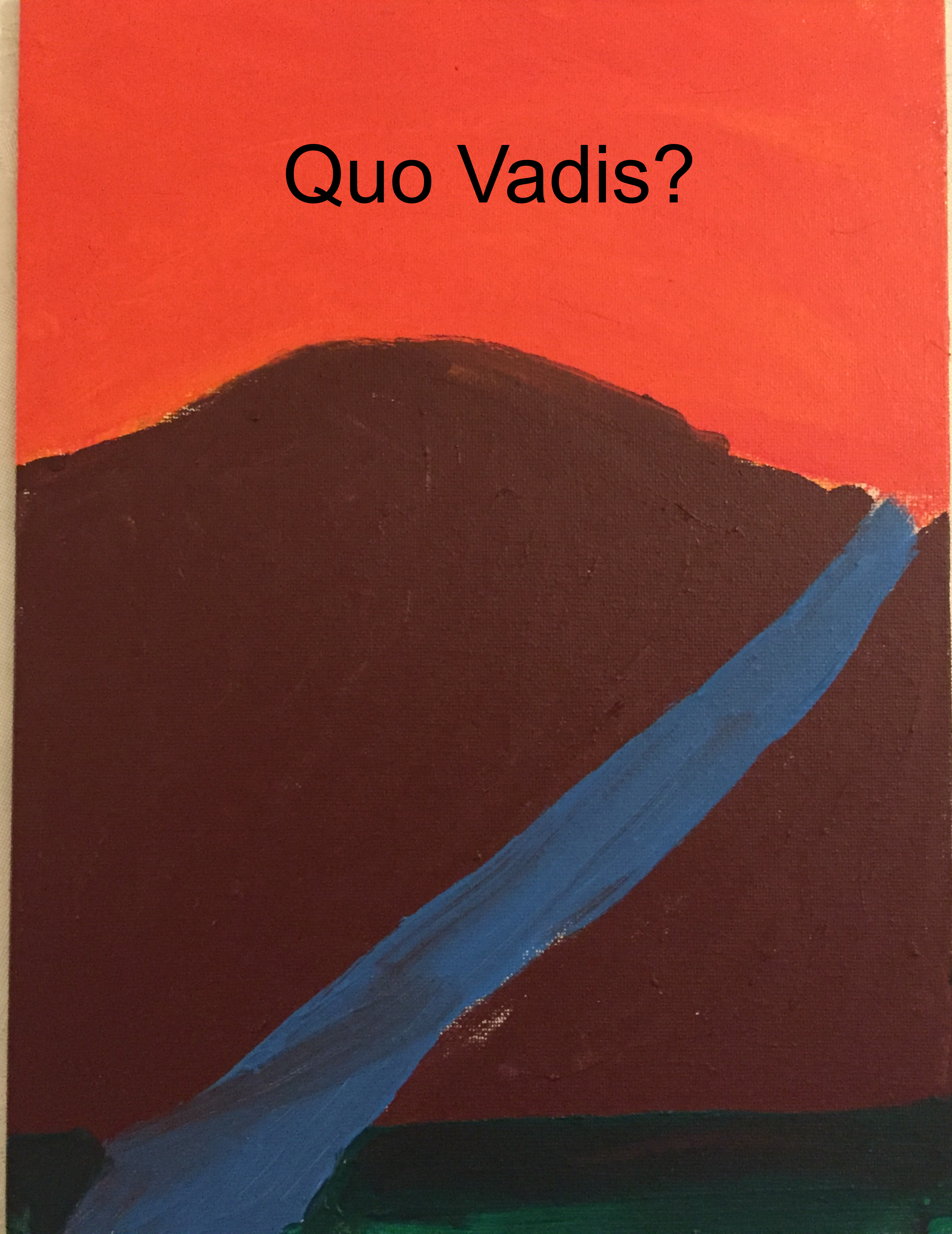
  ➠ need LQCD form factors for $B \to D^*$ at nonzero recoil

- ☆. Note: we still need to reduce theory errors and extend LQCD calculations to include more quantities….

Summary

errors (in %) **FLAG-2/3 averages** + **new results**





Amala Willenbrock

Quo Vadis?

Near term:

- ☆ FNAL/MILC: new bag parameters in upcoming f_B paper
  - ➠ cancellation/reduction of correlated errors in ratio
- ☆ gauge field ensembles with light sea quarks at their physical masses have already been used extensively for LQCD calculations of kaon and D meson quantities. First results also for f_B (HPQCD, FNAL/MILC) and $B \to \pi$ (HPQCD).
  - ➠ removes chiral extrapolation errors

  HPQCD: preliminary results on physical mass ensembles with NRQCD b quarks
  FNAL/MILC: plans to repeat B mixing calculation on new ensemble set
- ☆ Renormalization/matching errors are difficult to reduce to below ~few % with NRQCD or Fermilab b quarks.

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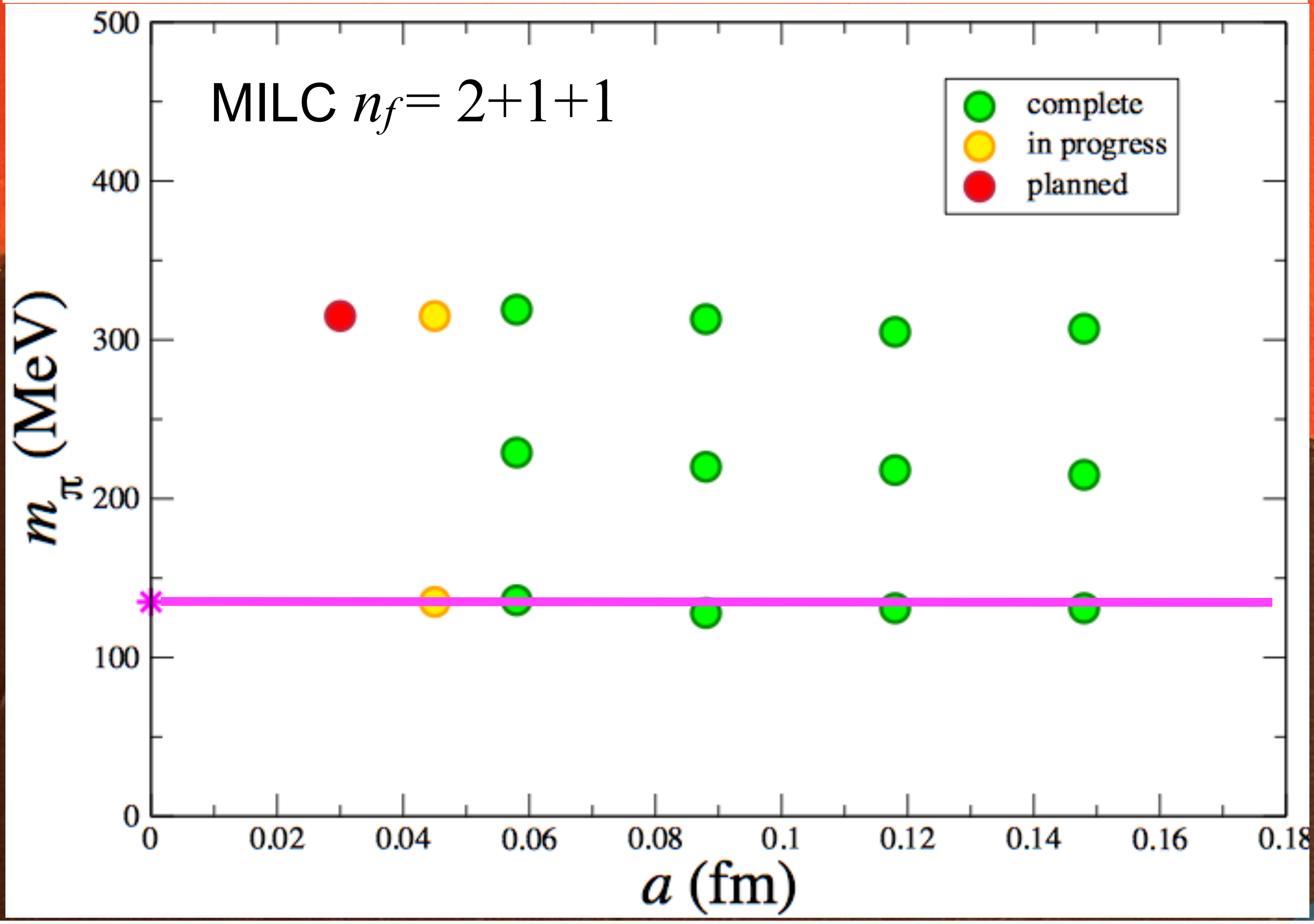
Quo Vadis?

New set of ensembles by MILC collaboration



Five collaborations have now generated sets of ensembles that include sea quarks with physical light-quark masses:

PACS-CS, BMW, MILC, RBC/UKQCD, ETM

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Quo Vadis?

Near term:

☆ FNAL/MILC: new bag parameters in upcoming f_B paper

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☆ gauge field ensembles with light sea quarks at their physical masses have already been used extensively for LQCD calculations of kaon and D meson quantities. First results also for f_B (HPQCD, FNAL/MILC) and $B \to \pi$ (HPQCD).

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HPQCD: preliminary results on physical mass ensembles with NRQCD b quarks
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☆ Renormalization/matching errors are difficult to reduce to below ~few % with NRQCD or Fermilab b quarks.

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Quo Vadis?

Long term: How do we get to 1% total errors (or below)?

☆ physical mass ensembles are essential

☆ need ensembles at very small lattice spacings where $am_b \lesssim 0.6$ — already in progress (FNAL/MILC)

➠ can use highly improved light quark actions with multiplicatively renormalized four-fermion operators

➠ calculate renormalizations nonperturbatively

☆ will also need small statistical errors (straightforward, but expensive)

☆ will eventually need to include

✦ strong isospin breaking ($m_u \neq m_d$) effects ✓

✦ QED effects

program being developed for kaon quantities, muon g-2

Amala Willenbrock

Thank you!

Farah Willenbrock

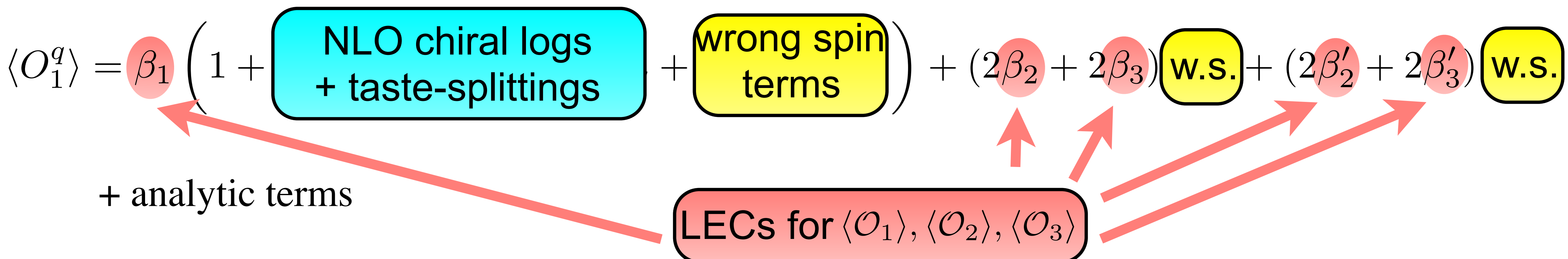
Backup slides

Chiral-continuum extrapolation

SU(3) heavy-meson partially-quenched rooted staggered χPT

- NLO chiral logs + taste-splittings + “wrong-spin” corrections
- + analytic terms (up to N^{3}LO)
- + B-meson hyperfine and flavor splittings
- + HQ discretization terms
- + higher order PT terms (up to $O(\alpha_s)^3$)

Schematically

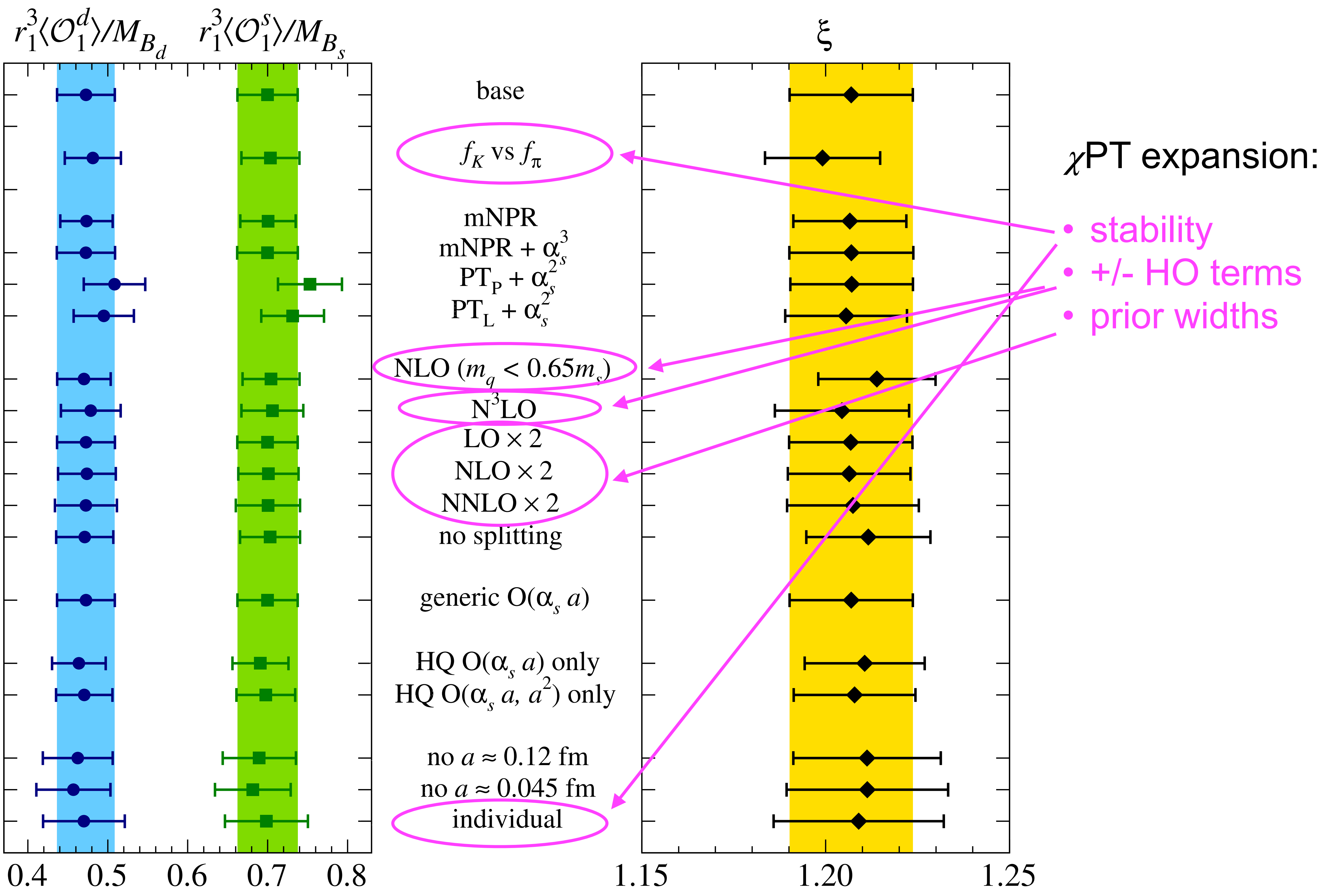


C. Bernard (Phys.Rev. D87 (2013) 114503, arXiv: 1303.0435)

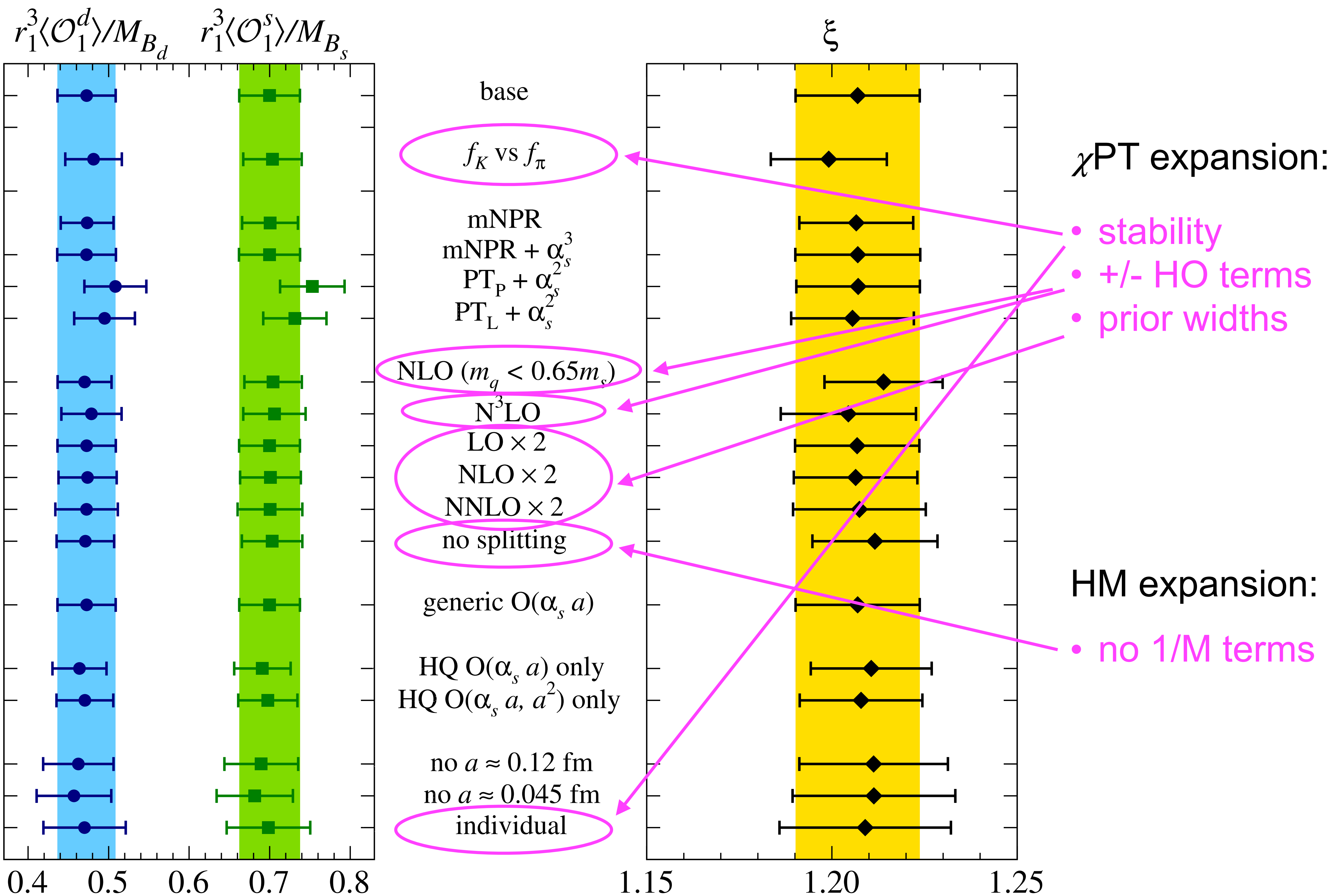
- no new LECs with simultaneous fits to the operators that mix at NLO

  $[\langle \mathcal{O}_1 \rangle, \langle \mathcal{O}_2 \rangle, \langle \mathcal{O}_3 \rangle]$ and $[\langle \mathcal{O}_4 \rangle, \langle \mathcal{O}_5 \rangle]$

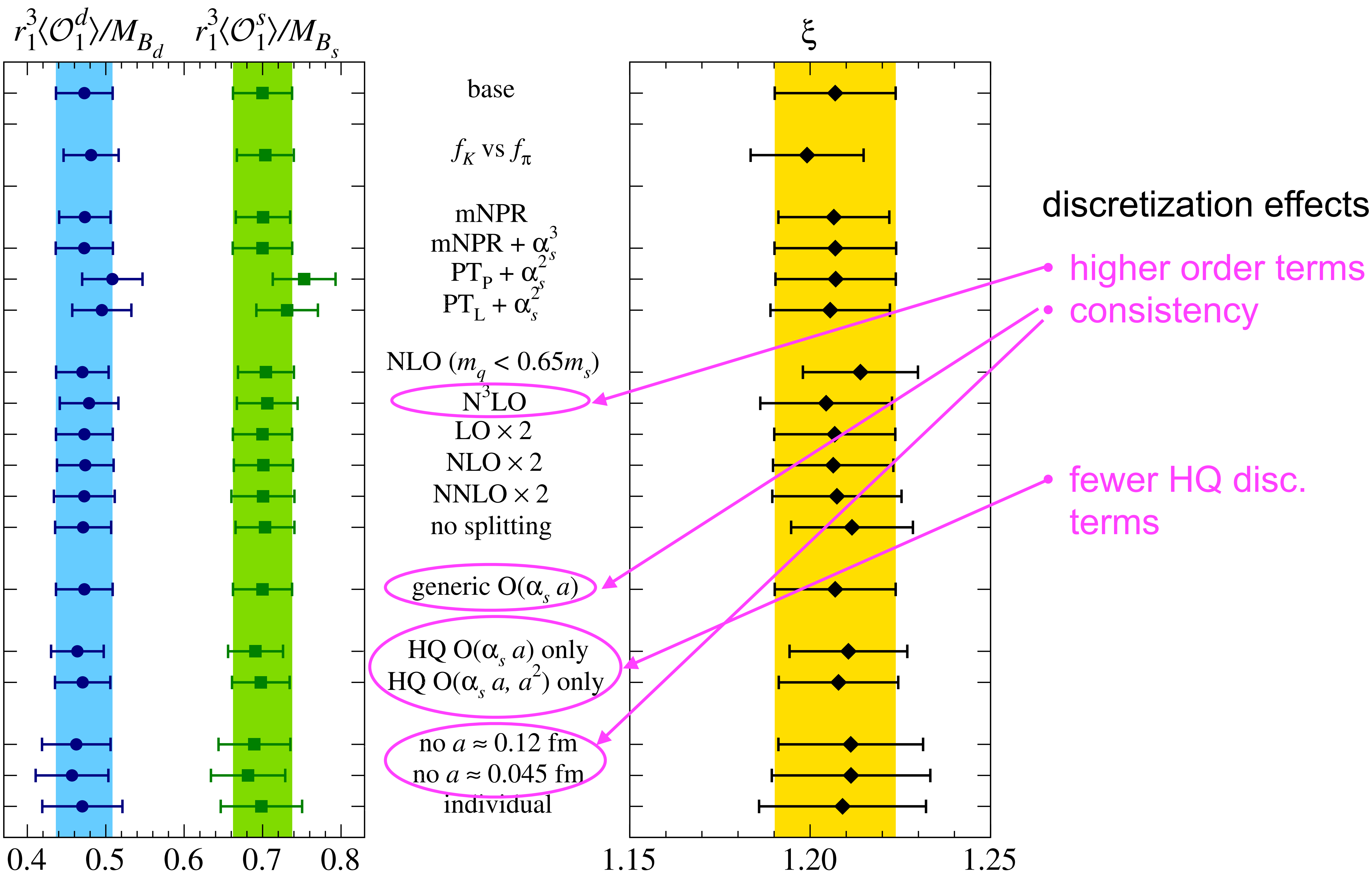
systematic error study



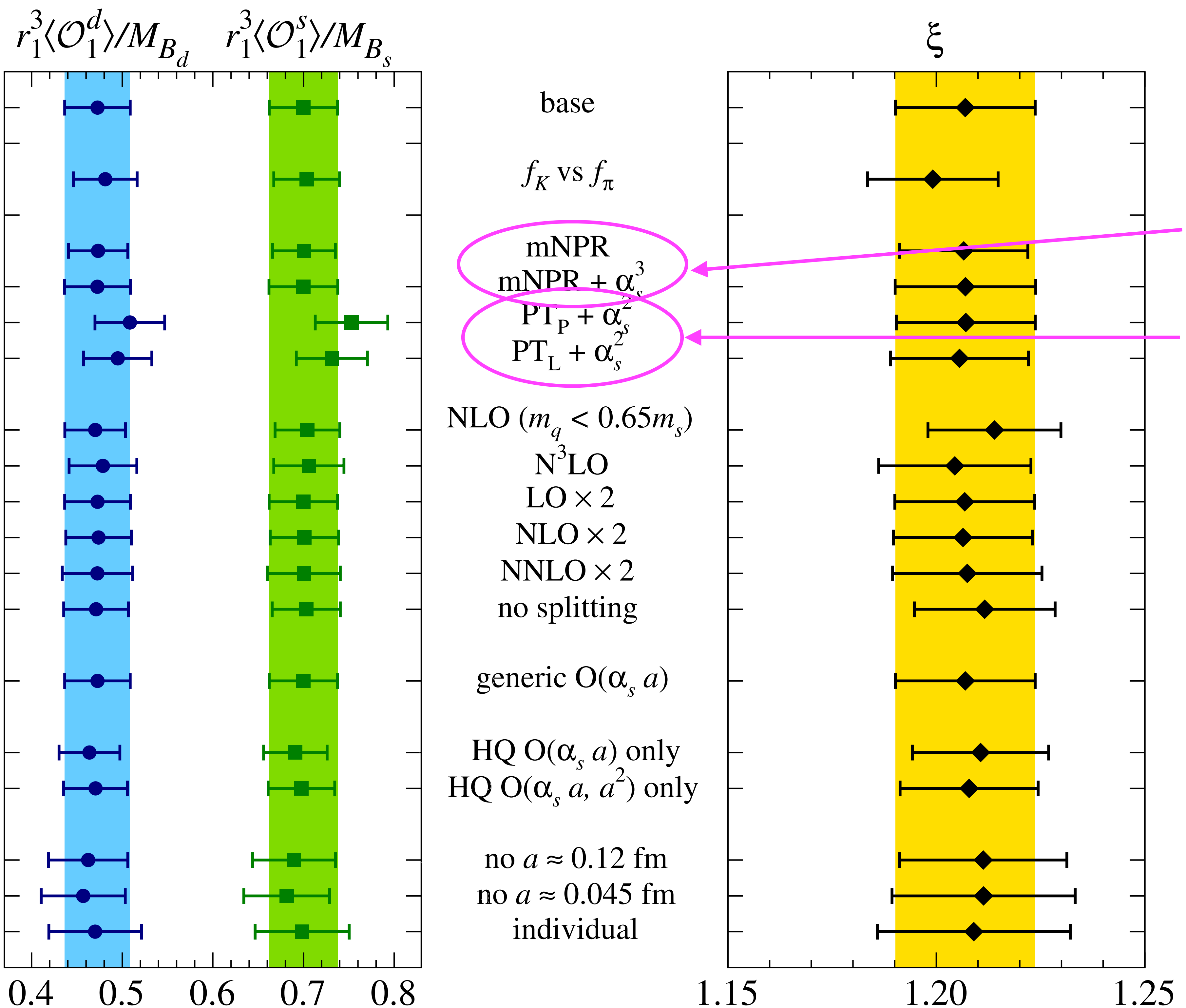
systematic error study



systematic error study



systematic error study



renormalization effects

- -/+ higher order terms
- without NPR

Heavy Quark Treatment

HISQ action for charm:

- like asqtad, the HISQ action is a tree-level tadpole improved staggered action, with discretization errors for light quarks:

$$\alpha_s (a\Lambda)^2, (a\Lambda)^4$$

- HISQ action is highly improved for charm quarks:

$$\sim \alpha_s \Lambda/m_h (am_h)^2, (\Lambda/m_h)^2 (am_h)^4$$

- can also be used for heavier than charm