What is the ultimate precision of mixing variables?

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IP3

[1] based on arXiv:1603.07770 – Jubb, MK, Lenz,Tetlalmatzi-Xolocotzi

Outline

1 Flavour Physics – motivations

2 Background
   Common assumptions in theory
   What is duality?

3 Duality violation with B mesons
   Violation in decays
   How accurate can theory get?

4 Duality violation in charm sector
   Charm vs. HQE
   Duality violation to the rescue?

5 Summary and Outlook

Why do flavour physics?

- To test our understanding of QCD
- To develop theoretical tools (e.g. SMEFT, SCET)
- Determining parameters of SM (around half are relevant for flavour)

On a more practical level:

- There is plenty of data to go around
- Our theories work well (but not too well!)

Underlying assumptions

What assumptions should we revisit?

- Size of penguin contributions
- How large can NP at tree-level be?
- How well does QCD factorisation work?
- To what extent does quark-hadron duality work?

⋮

What is quark-hadron duality?

What does quark-hadron duality mean?
Idea dates from over 40 years ago

- 1970: e-p scattering – Blom, Gilman
- 1979: $e^- e^+ \rightarrow$ hadrons – Poggio, Quinn, Weinberg

What do *we* mean by duality?

Quark-hadron duality corresponds to Heavy Quark Expansion (HQE), and duality violation to deviations from it.

HQE and duality violation

HQE is a Taylor expansion in $\frac{\Lambda}{m_b}$.

E.g. decay rate

$$\Gamma = \Gamma_0 + \frac{\Lambda^2}{m_b^2}\Gamma_2 + \frac{\Lambda^3}{m_b^3}\Gamma_3 + \dots$$

Imagine a term like $\exp(-m_b/\Lambda)$ – Taylor expansion is exactly 0.

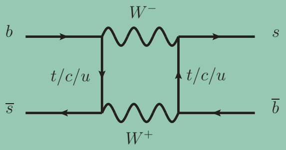
HQE and duality violation

Expansion parameter is really $\Lambda/\sqrt{m_i^2 - m_f^2}$ – channel dependent

Channel	Expansion parameter x	$\exp[-1/x]$
$\mathrm{b} \to \mathrm{c\bar{c}s}$	$\Lambda/\sqrt{m_\mathrm{b}^2 - 4m_\mathrm{c}^2} \approx 0.05 - 0.6$	$10^{-8} - 0.18$
$\mathrm{b} \to \mathrm{c\bar{u}s}$	$\Lambda/\sqrt{m_\mathrm{b}^2 - m_\mathrm{c}^2} \approx 0.045 - 0.5$	$10^{-10} - 0.13$
$\mathrm{b} \to \mathrm{u\bar{u}s}$	$\Lambda/\sqrt{m_\mathrm{b}^2} \approx 0.04 - 0.5$	$10^{-11} - 0.12$

We see that a “non-perturbative” term can easily give 20–30% corrections

Meson mixing

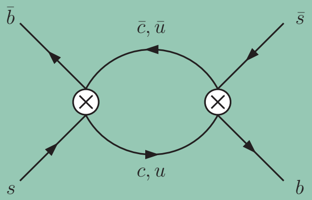


- Mass difference $\Delta M \approx 2|M_{12}|$ – due to off-shell particles, so can get contributions from heavy NP.
- Decay rate difference $\Delta\Gamma \approx 2|\Gamma_{12}|\cos\phi$ – due to on-shell particles, so free from NP (at least at first sight).

Large hadronic uncertainties in M_{12} and Γ_{12} – take ratios to improve theory predictions

- $\Delta\Gamma/\Delta M = -\,\mathrm{Re}(\Gamma_{12}/M_{12})$
- $a_{sl} = \mathrm{Im}(\Gamma_{12}/M_{12})$

Decay difference calculation



The decay rate difference gets three contributions from internal (cc, uc, uu) quarks, with CKM factors $\lambda_q = V_{qb} V_{qs}^*$

$$\Gamma_{12} = -\lambda_c^2 \Gamma_{12}^{cc} - 2\lambda_c \lambda_u \Gamma_{12}^{uc} - \lambda_u^2 \Gamma_{12}^{uu}$$

Use CKM unitarity to show GIM and CKM suppression

$$\frac{\Gamma_{12}}{M_{12}} = -\frac{\Gamma_{12}^{cc}}{\widetilde{M}_{12}} - 2\frac{\lambda_u}{\lambda_t} \frac{(\Gamma_{12}^{cc} - \Gamma_{12}^{uc})}{\widetilde{M}_{12}} - \frac{\lambda_u^2}{\lambda_t^2} \frac{(\Gamma_{12}^{cc} - 2\Gamma_{12}^{uc} + \Gamma_{12}^{uu})}{\widetilde{M}_{12}}$$

Breaking GIM suppression with duality violation

- Non-leading terms in Γ_{12} are GIM suppressed
- We expect duality violation to be stronger in certain decay channels
- This breaks the GIM suppression – duality violation could give potentially large change in observables

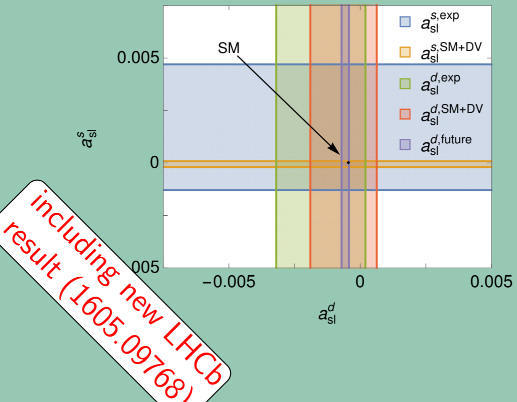
We take

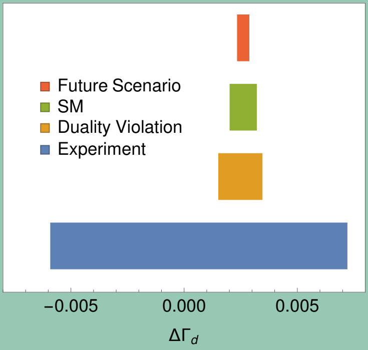
$$\begin{aligned}\Gamma_{12}^{cc} &\to \Gamma_{12}^{cc}(1+\delta^{cc})\\ \Gamma_{12}^{uc} &\to \Gamma_{12}^{uc}(1+\delta^{uc})\\ \Gamma_{12}^{uu} &\to \Gamma_{12}^{uu}(1+\delta^{uu})\end{aligned}$$

with $\delta^{cc} \geq \delta^{uc} \geq \delta^{uu}$.

Limits on duality violation from $\Delta\Gamma_s$ – future possibilities

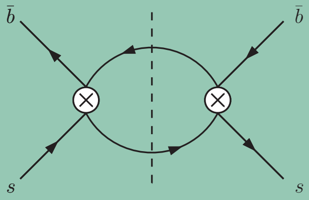
Currently, our duality violating parameters can go up to 30% – this bound is dominated by theory error. Duality violation then can lead to factor ~ 3 increase in a_{sl}^s.



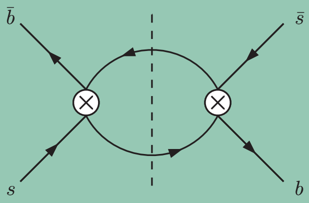


Limits on duality violation from B lifetimes

Very similar diagrams contribute to B lifetimes as to Γ_{12}.



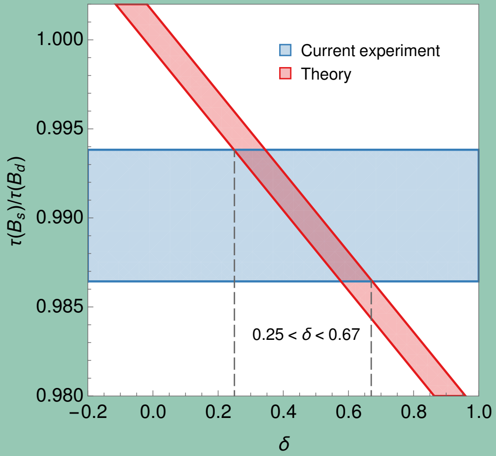
(a) $\tau(B_s)$



(b) Γ_{12}

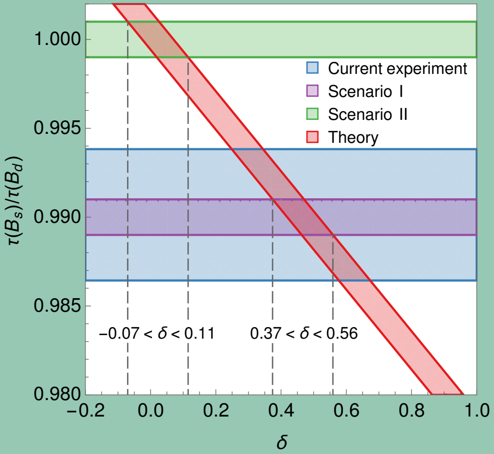
BUT: in (a) all decay modes of B_s contribute, while in (b) only modes shared by B_s and $\overline{B}_s$ are involved.

Limits on duality violation from B lifetimes



Take simplified model for duality violation
$(\delta^{cc} = 4\delta^{uu}, \delta^{uc} = 2\delta^{uu})$

Future limits



Reduction in error from experiment would allow much better constraints on duality violation.

Aggressive theory predictions

Observable	SM – conservative	SM - aggressive	Experiment
ΔM_s	$(18.3 \pm 2.7)\,\mathrm{ps}^{-1}$	$(20.11 \pm 1.37)\,\mathrm{ps}^{-1}$	$(17.757 \pm 0.021)\,\mathrm{ps}^{-1}$
$\Delta\Gamma_s$	$(0.088 \pm 0.020)\,\mathrm{ps}^{-1}$	$(0.098 \pm 0.014)\,\mathrm{ps}^{-1}$	$(0.082 \pm 0.006)\,\mathrm{ps}^{-1}$
a_{sl}^s	$(2.22 \pm 0.27) \cdot 10^{-5}$	$(2.27 \pm 0.25) \cdot 10^{-5}$	$(-7.5 \pm 4.1) \cdot 10^{-3}$
$\Delta\Gamma_s/\Delta M_s$	$48.1(1 \pm 0.173) \cdot 10^{-4}$	$48.8(1 \pm 0.125)$	$46.2(1 \pm 0.073) \cdot 10^{-4}$
ΔM_d	$(0.528 \pm 0.078)\,\mathrm{ps}^{-1}$	$(0.606 \pm 0.056)\,\mathrm{ps}^{-1}$	$(0.5055 \pm 0.0020)\,\mathrm{ps}^{-1}$
$\Delta\Gamma_d$	$(2.61 \pm 0.59) \cdot 10^{-3}\,\mathrm{ps}^{-1}$	$(2.99 \pm 0.52) \cdot 10^{-3}\,\mathrm{ps}^{-1}$	$(0.658 \pm 6.579) \cdot 10^{-3}\,\mathrm{ps}^{-1}$
a_{sl}^d	$(-4.7 \pm 0.6) \cdot 10^{-4}$	$(-4.90 \pm 0.54) \cdot 10^{-4}$	$(-1.5 \pm 1.7) \cdot 10^{-3}$
$\Delta\Gamma_d/\Delta M_d$	$49.4(1 \pm 0.172) \cdot 10^{-4}$	$49.3(1 \pm 0.49)$	$13.0147(1 \pm 10) \cdot 10^{-3}$

Our aggressive estimates use the recent lattice results from Fermilab-MILC[1] for dimension-6 operators, which also inspire our estimates for dimension-7 bag parameters.

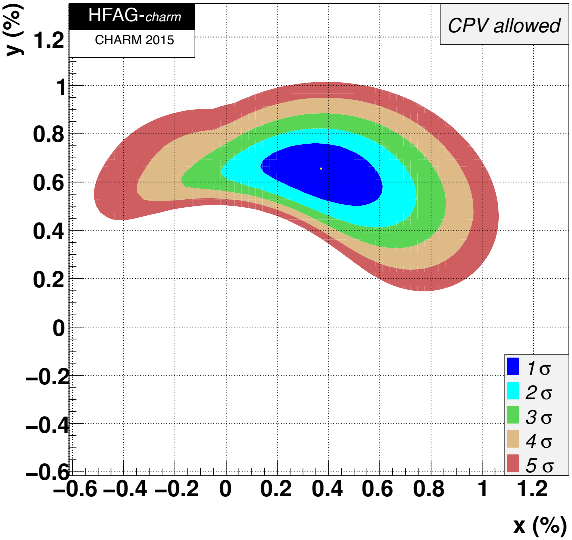
[1] 1602.03560

Status of charm mixing

- In 2012 (courtesy of LHCb), charm mixing established at 9σ
- HFAG 2015 result:
  $x = (3.7 \pm 1.6) \cdot 10^{-3}, y = 6.6^{+0.7}_{-1.0} \cdot 10^{-3}$

Status of charm mixing

- In 2012 … ed at 9σ
- HFAG 2… $x = (3.7$…



Charm vs. the HQE

- HQE calculation of charm mixing gives a result around 3 order of magnitude too small
- In contrast, exclusive approach gives correct ballpark figure, but not a first principles approach (e.g. Falk, Grossman, Ligeti, (Nir,) Petrov[1])

[1] hep-ph/0110317, (hep-ph/0402204)

Why doesn't HQE work?

- Are hadronic effects to blame? Can be tested with HQE prediction of D lifetimes – Lenz, Rauh[1]
- Do we need to calculate higher dimensional terms with less GIM suppression? Bigi, Uraltsev[2]; Bobrowski, Lenz, Riedl, Rohrwild[3]
- Or is new physics to blame?

[1] 1305.3588
[2] hep-ph/0005089
[3] 1002.4794

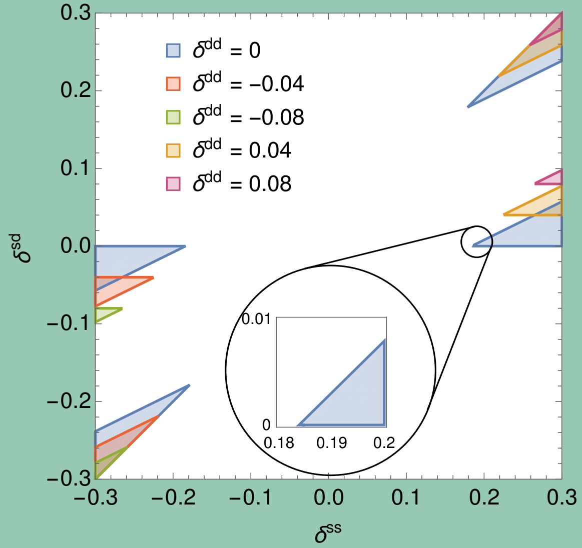
How does duality violation affect D mixing?

Similar to B system, take

$$\begin{aligned}\Gamma_{12}^{ss} &\to \Gamma_{12}^{ss}(1+\delta^{ss})\\ \Gamma_{12}^{sd} &\to \Gamma_{12}^{sd}(1+\delta^{sd})\\ \Gamma_{12}^{dd} &\to \Gamma_{12}^{dd}(1+\delta^{dd})\end{aligned}$$

with $\delta^{ss} \geq \delta^{sd} \geq \delta^{dd}$.

How does duality violation affect D mixing?



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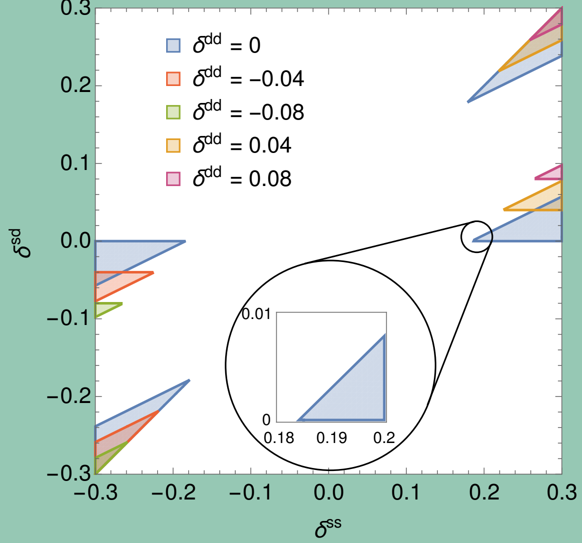

Duality violation of as little as 20% can match experimental result – factor 1000 increase!

Summary

- Best constraints on duality violation come from $\Delta\Gamma_s/\Delta M_s$
- From these limits, a_{sl}^s cannot be enhanced by more than factor of ~ 3
- Complementary bounds from studying $\tau(B_s)/\tau(B_d)$ – currently consistent
- New lattice results reduce errors, but shift slight away from experiment
- Charm mixing could be evidence of small duality violation

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- Test HQE in lifetimes, calculate higher dimensional contributions to mixing

Thanks!

Backup

Aggressive assumptions

- Most recent lattice results (Fermilab-MILC, arXiv:1602.03560)
- Shows VIA works very well for dim-6 operators ($B \in [0.8, 1.2]$)
  $\Rightarrow$ use smaller errors for dim-7 operators ($B = 1 \pm 0.2$)
- Most recent CKM inputs
- Use exact equations of motion for dim-7 operators

$\tau(\mathrm{B_s})/\tau(\mathrm{B_d})$ – colour suppressed operators

$$\tau(\mathrm{B_s})/\tau(\mathrm{B_d}) = 1.0005 \pm 0.0011$$

80% of error from colour suppressed operators, $\epsilon_{1,2}$

$$\left\langle B \,\middle|\, (\bar{\mathrm{b}}\gamma_\mu(1-\gamma^5)T^a q) \otimes (\bar{q}\gamma^\mu(1-\gamma^5)T^a \mathrm{b}) \,\middle|\, B \right\rangle = f_{\mathrm{B}}^2 M_{\mathrm{B}}^2 \epsilon_1$$

$$\left\langle B \,\middle|\, (\bar{\mathrm{b}}(1-\gamma^5)T^a q) \otimes (\bar{q}(1-\gamma^5)T^a \mathrm{b}) \,\middle|\, B \right\rangle = f_{\mathrm{B}}^2 M_{\mathrm{B}}^2 \epsilon_2$$

2001 determination (Becirevic, hep-ph/0110124):
$\epsilon_1 = -0.02 \pm 0.02$, $\epsilon_2 = 0.03 \pm 0.01$