Light-Cone Sum Rules: status, challenges and perspectives

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Questions to be addressed:

- current status and precision of LCSRs
- LCSR results vs lattice QCD results
- the challenge of unstable mesons

LCSRs for heavy flavours

- the method of QCD Light-Cone Sum Rules

  [I.I.Balitsky, V.M.Braun, A.V. Kolesnichenko (1986); V.L.Chernyak, I.Zhitnisky (1990)]

- important applications to heavy flavours:

  - transition form factors

$$F_a(q^2) = \langle h(p)|\bar{q}\,\Gamma_a Q|H(p+q)\rangle$$

$q = u, d, s, \quad Q = b, c,$

$H = B_{(s)}, \Lambda_b, D, ..$

h - a light (stable) hadron or photon

  - nonlocal contributions to weak semileptonic FCNC transitions

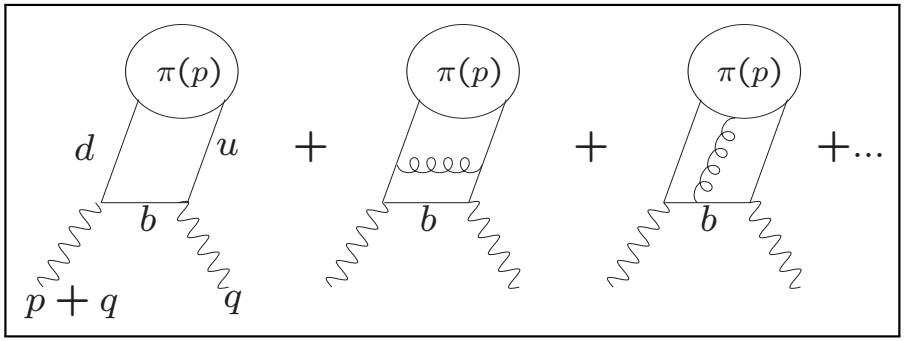
$$\mathcal{H}(q^2) = i\int d^4x e^{iqx}\langle h(p)|T\{\bar{q}\gamma_\mu q(x), O_i(0)\}|H(p+q)\rangle$$

$q = u, d, s, c,$

O_i effective heavy-light operator

- LCSRs valid at $q^2 \ll (m_H - m_h)^2$ (large recoil of h), finite m_Q

LCSR for $B \to \pi$ form factors



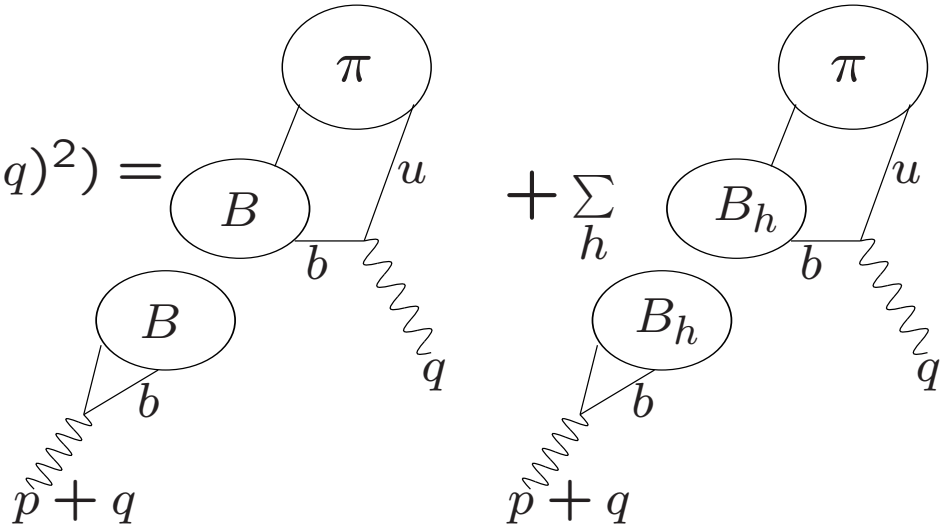
$\Leftarrow$ the correlation function

calculated in terms of
Operator Product Expansion
at $(p+q)^2, q^2 \ll m_b^2$

$\Downarrow$

hadronic dispersion relation $\Big\}$

$$F(q^2,(p+q)^2) =$$

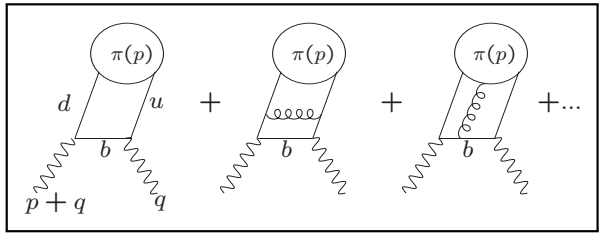


$f_B f^+_{B\pi}(q^2)$

$\sum_{B_h} \to duality\ (s_0^B)$

OPE calculation

- the correlation function $q^2 \ll m_b^2$



$$[F(q^2,(p+q)^2)]_{OPE} =$$

$$= \sum_{t=2,3,4,..} \int_0^1 \mathcal{D}u \; T^{(t)}(\alpha_s, m_b, m_q; q^2, (p+q)^2, u, \mu) \, \varphi_\pi^{(t)}(u,\mu)$$

$\Uparrow$ $\Uparrow$

{diagrams with b-propagator} $\otimes$ {pion Distribution Amplitudes}

- pion DA's, polynomial expansion:

$$\varphi_\pi^{(t)}(u,\mu) = f_\pi^{(t)}(\mu)\{C_0(u) + \sum_{n=1} a_n^{(t)}(\mu) C_n(u)\}$$

- accuracy of OPE
  - precision of the input: m_b, m_q, α_s, $f_\pi^{(t)}(\mu_0)$, $a_n^{(t)}(\mu_0)$
  - truncation level: $O(\alpha_s)$, $t \leq 4$, $n \leq 4$
  - variable scales: μ, $(p+q)^2 \to M^2 \sim m_b\chi$, $m_b \gg \chi \gg \Lambda_{QCD}$

Hadronic dispersion relation

- based on analyticity $\oplus$ unitarity in QFT

$$[F(q^2,(p+q)^2)]_{OPE} = \frac{m_B^2 f_B f_{B\pi}^{+}(q^2)}{m_B^2-(p+q)^2} + \int\limits_{(m_{B^*}+m_\pi)^2}^{\infty} ds\, \frac{\rho_h(s)}{s-(p+q)^2}$$

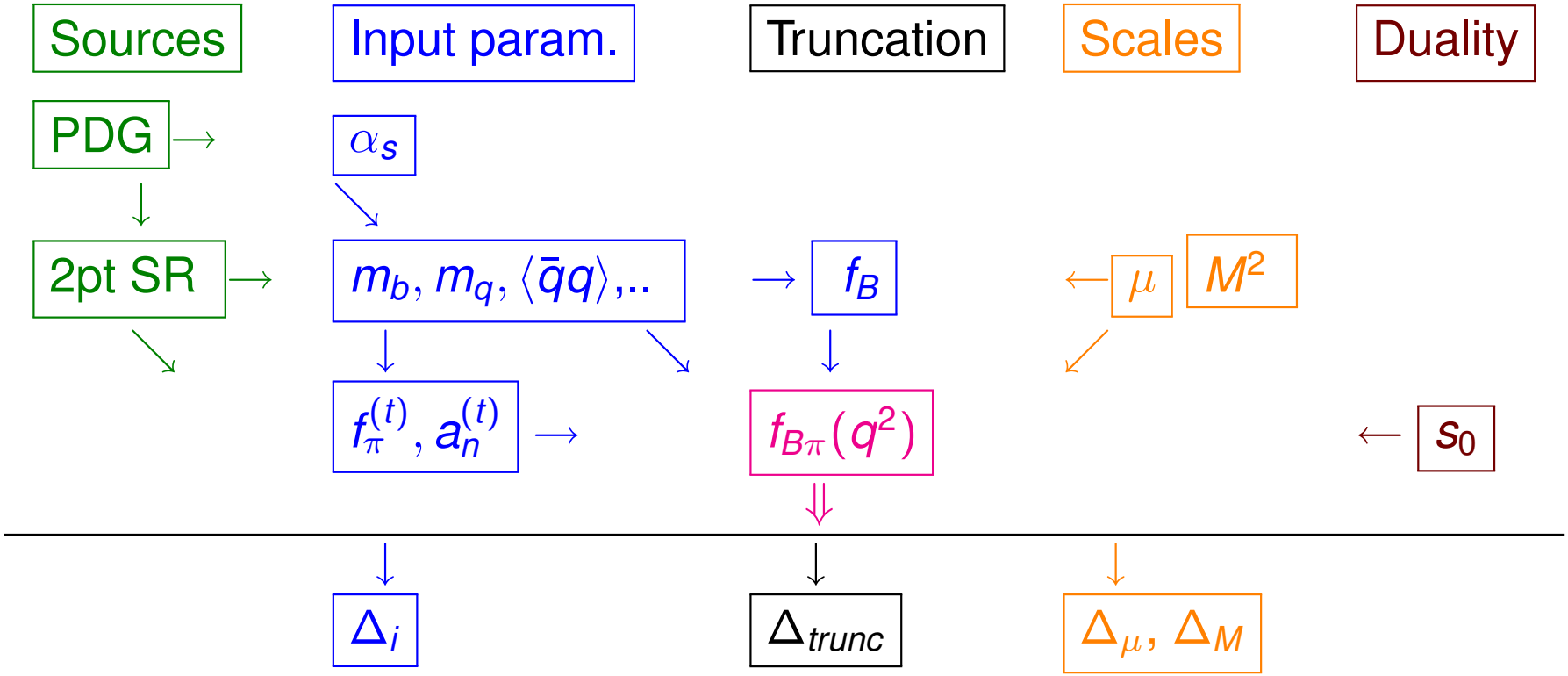
$$\Uparrow$$

- quark-hadron "semilocal" duality

$$\boxed{\int\limits_{(m_{B^*}+m_\pi)^2}^{\infty} ds\, \frac{\rho_h(s)}{s-(p+q)^2} = \int\limits_{s_0}^{\infty} ds\, \frac{[\mathrm{Im}F(q^2,s)]_{OPE}]}{s-(p+q)^2}}$$

- accuracy:
  - f_B calculated from 2-point QCD SR)
  - variable scale: $(p+q)^2 \to M^2 \sim m_b\chi \to$ optimal interval of M^2
  - duality approximation, s_0 (determined by calculating m_B^2)

Summary on assumptions, input and error budget



- total uncertainty estimate:

$$\Delta f_{B\pi}(q^2) = \sqrt{\sum_i \Delta_i^2 + \Delta_{trunc}^2 + \Delta_\mu^2 + \Delta_M^2}$$

conservative, correlations neglected

Improving the error estimate for $f^+_{B\pi}(q^2)$

(within adopted theoretical approximation !)

I. S.Imsong, A.K., T. Mannel and D. van Dyk, JHEP **1502** (2015) 126 [arXiv:1409.7816 [hep-ph]]

- calculate the form factor $f^+_{B\pi}(q^2)$ from LCSR;
  use 2-point QCD SR for f_B
  LCSR analytic extressions from : G. Duplancic, A.K., T. Mannel, B. Melic and N. Offen, (2008)
- statistical (Bayesian) analysis:
  inputs (assumed uncorrelated) taken as priors,
  constructing theoretical likelihood by imposing $[m_B]_{SR}$ within 1% of m_B
- purely LCSR prediction
  (no parametrization/extrapolation involved)

$$\Delta\zeta\,(0, 12\mathrm{GeV}^2) = \frac{1}{|V_{ub}|^2} \int\limits_0^{12\,\mathrm{GeV}^2} dq^2 \frac{d\Gamma}{dq^2}(B \to \pi\ell\nu_\ell)$$

$$\equiv \frac{G_F^2}{24\pi^3} \int\limits_0^{12\,\mathrm{GeV}^2} dq^2 p_\pi^3 |f^+_{B\pi}(q^2)|^2 = (5.25^{+0.68}_{-0.54})\,\mathrm{ps}^{-1}\,,$$

LCSR results fitted to BCL parameterization

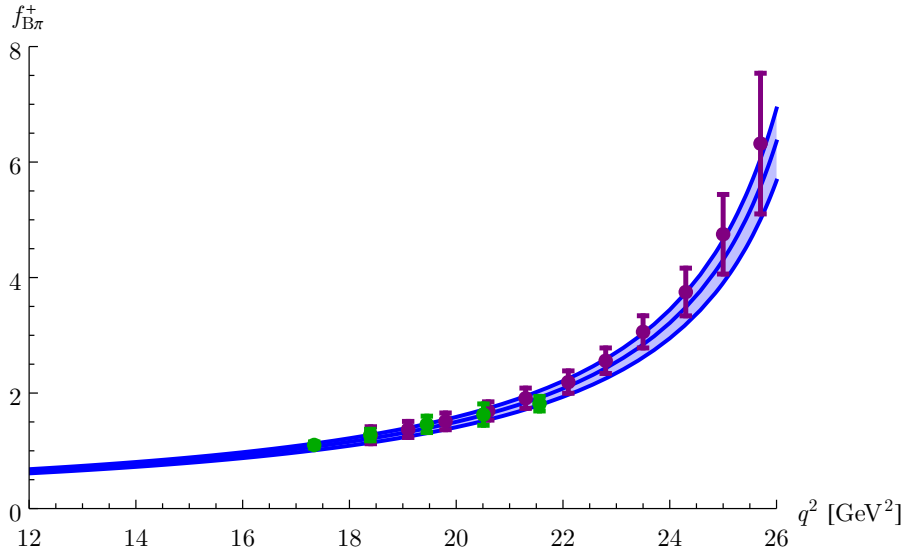
- z-series parameterization, including $f^+_{B\pi}(0)$ $q^2 \to z(q^2, t_0)$,
  mapping SL region to small z, *the BCL-version [Bourrely, Caprini, Lellouch, (2008)]*

$$f^+_{B\pi}(q^2) = \frac{f^+_{B\pi}(0)}{1 - q^2/m^2_{B^*}} \Big\{ 1 + b^+_1 \big[z(q^2,t_0) - z(0,t_0) - \frac{1}{3}\big(z(q^2,t_0)^3 - z(0,t_0)^3\big)\big]$$
$$+ b^+_2 \big[z(q^2,t_0)^2 - z(0,t_0)^2 + \frac{2}{3}\big(z(q^2,t_0)^3 - z(0,t_0)^3\big)\big] \Big\},$$

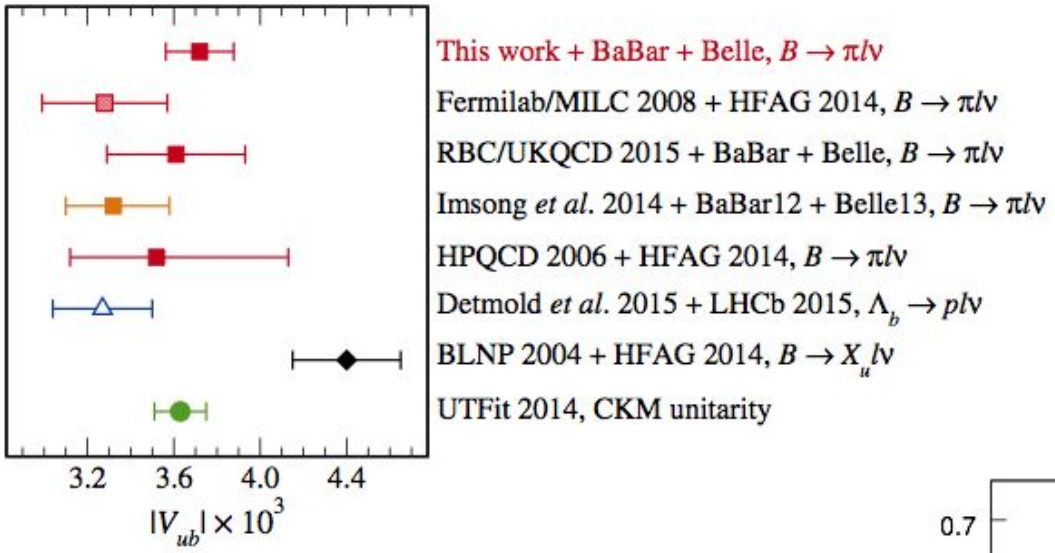
$f_{B\pi}(0) = 0.307 \pm 0.02$
$b^+_1 = -1.31 \pm 0.42$
$b^+_2 = -0.904 \pm 0.444$

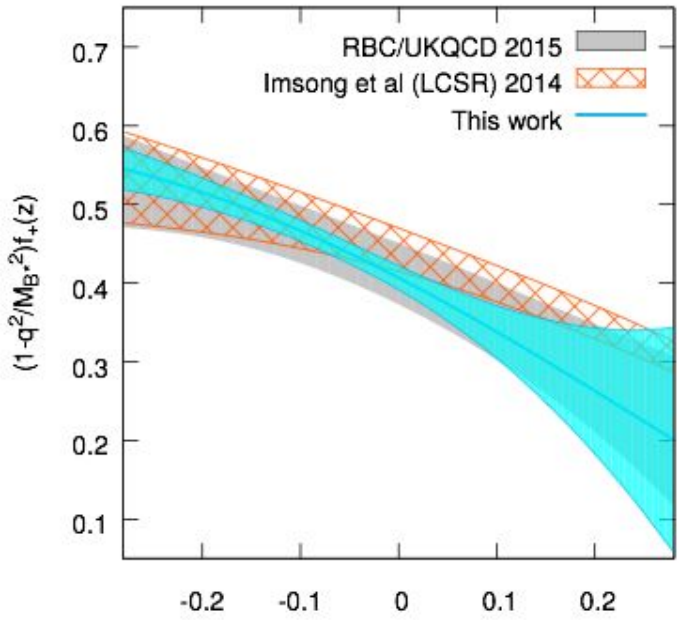
$$\rho^{BCL} = \begin{pmatrix} 1.000 & 0.503 & -0.391 \\ 0.503 & 1.000 & -0.824 \\ -0.391 & -0.824 & 1.000 \end{pmatrix}$$

- extrapolation
  beyond the LCSR region
  lattice results (< 2015):
  • -HPQCD , • -Fermilab-MILC



Comparison with 2015 lattice results

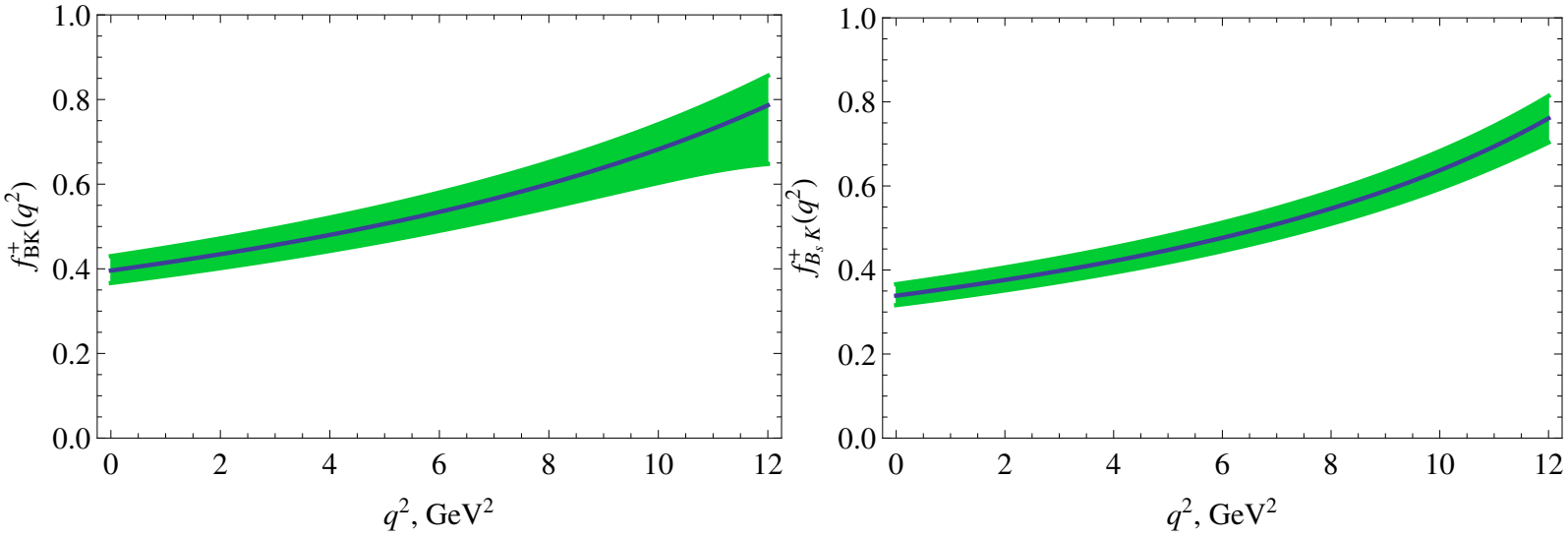




figs. from [J. A. Bailey *et al.* [Fermilab Lattice and MILC Collaborations], arXiv:1503.07839 [hep-lat].]

Update of LCSRs for $B \to K$, $B_s \to K$ form factors

PRELIMINARY! [AK, A.Rusov, work in preparation]



$$f^+_{B_sK}(0) = 0.34^{+0.033}_{-0.024}$$

- $|V_{ub}|$ from $B_s \to K\ell\nu_\ell$ decay

$$\Delta\zeta\,(0, 12\,\mathrm{GeV}^2) = \frac{1}{|V_{ub}|^2} \int\limits_0^{12\,GeV^2} dq^2 \frac{d\Gamma}{dq^2}(B_s \to K\ell\nu_\ell) = 6.92^{+1.09}_{-0.90}\ \mathrm{ps}^{-1}$$

Current status of LCSRs for $B \to h$ form factors

- at $0 < q^2 \leq 12 - 14$ GeV^2 the (parametrical) accuracy for $B \to \pi, K$ form factors at the level of $\pm 10\%$
- LCSRs for $B \to \rho, K^*$ form factors (zero-width ρ, K^*) currently at the same level of accuracy

  [P.Ball, R. Zwicky (2004), A.Bharucha, D.Straub, R.Zwicky (2015)]
- $\Lambda_b \to p$, less accurate, limited by proton DAs

  [AK, Th.Mannel, Ch. Klein, Y.-M. Wang (2011)]
- further improvement of $B \to \pi, K$ LCSRs ?
  - NLO, $O(\alpha_s)$ to the nonasymptotic twist-3 part challenging !
  - confirmed to be very small:

    NNLO $O(\alpha_s^2 \beta_0)$ correction to the twist-2 part [A. Bharucha (2012)]

    twist 5,6 term in factorizable approximation [A. Rusov, paper in preparation]
  - pion, kaon DAs: more accurate e.m. form factors

    BESS and Belle-2 data on $\gamma^* \gamma \to \pi^0$; JLab data on the pion, kaon e.m. FFs
  - to quantify the "systematic" uncertainty of semi-local duality ?

    (suppressed with Borel transformation, controlled by the m_B calculation)

Nonlocal effects in FCNC decays: example of $B \to \pi\ell\ell$

[C. Hambrock, A. K. , A. Rusov, PRD **92** (2015) [arXiv:1506.07760 [hep-ph]]],

- compact form:

$$A(B \to \pi\ell^+\ell^-) = \frac{G_F}{\sqrt{2}}\lambda_t \frac{\alpha_{\rm em}}{\pi} f^+_{B\pi}(q^2)\Bigg[\left(\bar{\ell}\gamma^\mu\ell\right) p_\mu \Bigg(C_9 + \Delta C_9^{(B\pi)}(q^2) + \frac{2m_b}{m_B+m_\pi} C_7^{\rm eff} \frac{f^T_{B\pi}(q^2)}{f^+_{B\pi}(q^2)}\Bigg) + \left(\bar{\ell}\gamma^\mu\gamma_5\ell\right) p_\mu C_{10}\Bigg],$$

- all "background" effects: the operators $O_{i=1,2,3,\ldots 6,8g}$ combined with e.m. lepton pair, collected in (process- and q^2-dependent)

$$\Delta C_9^{(B\pi)}(q^2) = -\frac{16\pi^2}{f^+_{B\pi}(q^2)}\left(\frac{\lambda_u}{\lambda_t}\mathcal{H}^{(u)}(q^2) + \frac{\lambda_c}{\lambda_t}\mathcal{H}^{(c)}(q^2)\right),$$

- defined via two nonlocal hadronic matrix elements

$$\mathcal{H}^{(p)}(q^2)\left[(p\cdot q)q_\mu - q^2 p_\mu\right] = i\int d^4x e^{iqx}\langle\pi(p)|{\rm T}\Bigg\{ j^{\rm em}_\mu(x), \Bigg[C_1\mathcal{O}^p_1(0) + C_2\mathcal{O}^p_2(0) + \sum_{k=3-6,8g} C_k\mathcal{O}_k(0)\Bigg]\Bigg\}|B(p+q)\rangle, \quad (p = u, c),$$

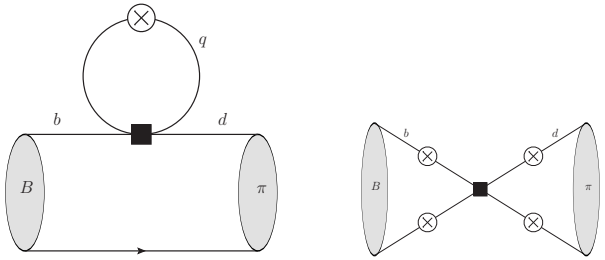
How do we obtain $\Delta C_9^{(B\pi)}(q^2)$?

the method used earlier for $B \to K\ell\ell$

[A. K., T. Mannel and Y. M. Wang, JHEP **1302** (2013) 010 [arXiv:1211.0234 [hep-ph]]]

- calculate $\mathcal{H}^{(u,c)}(q^2 < 0)$ at $|q^2| \gg \Lambda^2_{QCD}$

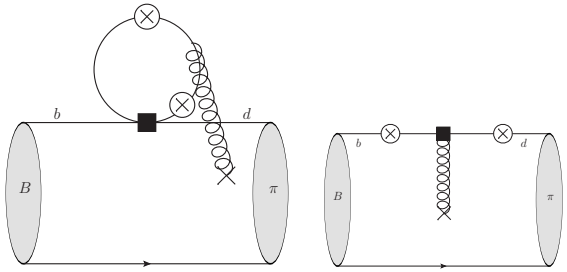
  - LO diagrams: factorizable loop, weak annihilation



  - soft-gluon nonfactorizable contributions

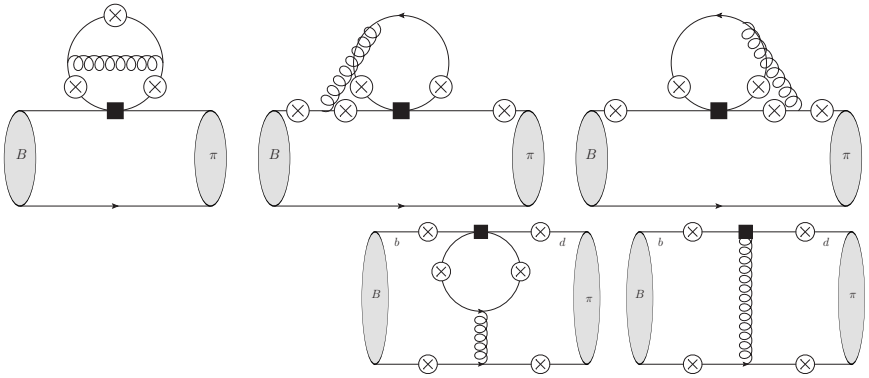
(LCSR with B-meson DA)

[A.K., T. Mannel, A. A. Pivovarov and Y.-M. Wang, JHEP **1009** (2010) 089 [arXiv:1006.4945 [hep-ph]]].



  - NLO (hard-gluon) contributions from QCD factorization (at $q^2 < 0$)

M. Beneke, T. Feldmann,D. Seidel, (2001)

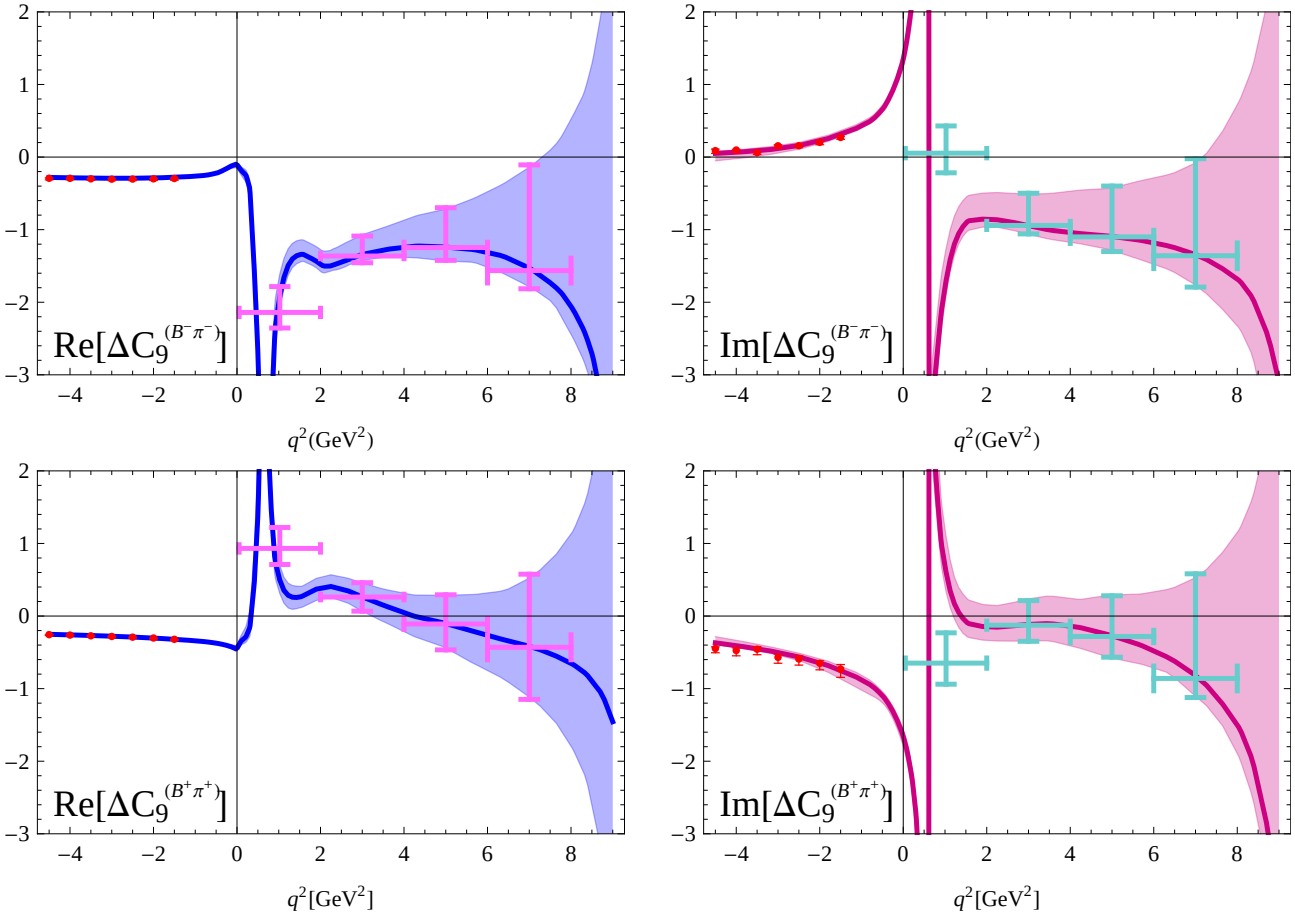


two-loop diagrams taken from

H. H. Asatryan, C. Greub and M. Walker,(2002)

$\Delta C_9(q^2)$ for $B^\pm \to \pi^\pm \ell^+ \ell^-$

- $\mathcal{H}^{(u,c)}(q^2 > 0)$ obtained matching the OPE result at $q^2 < 0$ to the hadronic dispersion relation, continuing to $q^2 > 0$
- including $V = \rho, \omega, \phi, J/\psi, \psi(2S)$ resonances;
- inputs: decay constants of V's, $B \to V\pi$ nonleptonic amplitudes



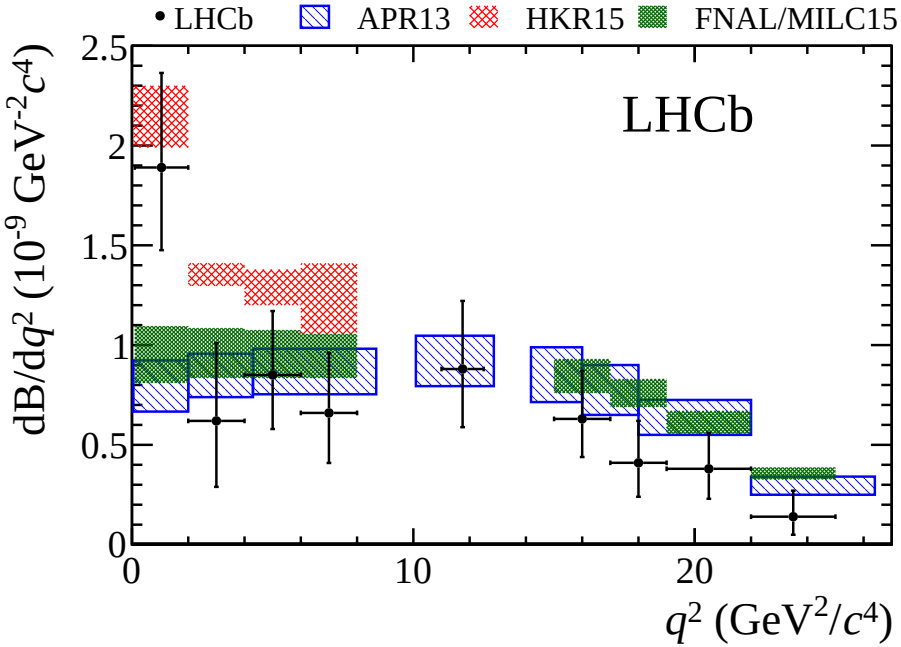
points: calculated $\Delta C_9^{(B\pi)}(q^2 < 0)$, crosses: binned $\Delta C_9^{(B\pi)}(q^2 > 0)$, solid line/shaded area - fit/errors

Observables in $B \to \pi \ell^+ \ell^-$

- the dilepton invariant mass spectrum:

$$\frac{d\mathrm{Br}(B \to \pi \ell^+ \ell^-)}{dq^2} = \frac{G_F^2 \alpha_{\mathrm{em}}^2 |\lambda_t|^2}{1536 \pi^5 m_B^3} |f_{B\pi}^+(q^2)|^2 \lambda^{3/2}(m_B^2, q^2, m_\pi^2)$$
$$\times \left\{ \left| C_9 + \Delta C_9^{B\pi}(q^2) + \frac{2m_b}{m_B + m_\pi} C_7 \frac{f_{B\pi}^T(q^2)}{f_{B\pi}^+(q^2)} \right|^2 + |C_{10}|^2 \right\} \tau_B \,.$$

- recent measurement of $B^\pm \to \pi^\pm \mu^+ \mu^-$ at LHCb



Extracting $|V_{td}/V_{ts}|$ from the ratio of FCNC decays

- the ratio of the differential widths:

$$R(q^2) = \frac{d\Gamma(B \to \pi \ell^+ \ell^-)/dq^2}{d\Gamma(B \to K \ell^+ \ell^-)/dq^2} = \left|\frac{V_{td}}{V_{ts}}\right|^2 \frac{|f^+_{B\pi}(q^2)|^2}{|f^+_{BK}(q^2)|^2} \frac{\lambda^{3/2}(m_B^2, q^2, m_\pi^2)}{\lambda^{3/2}(m_B^2, q^2, m_K^2)}$$
$$\times \frac{\left|C_9 + \Delta C_9^{B\pi}(q^2) + \frac{2m_b}{m_B+m_\pi} C_7 \frac{f^T_{B\pi}(q^2)}{f^+_{B\pi}(q^2)}\right|^2 + |C_{10}|^2}{\left|C_9 + \Delta C_9^{BK}(q^2) + \frac{2m_b}{m_B+m_\pi} C_7 \frac{f^T_{BK}(q^2)}{f^+_{BK}(q^2)}\right|^2 + |C_{10}|^2}$$

- the ratio in the second line contains different nonlocal contributions, the numerator also depends on V_{ub} !

- preliminary estimate [AK, A.Rusov, work in preparation]

3 bins $[2-4], [4-6], [6-8]$	1 bin $[1-6]$
$\lvert V_{td}/V_{ts}\rvert = 0.22 \pm 0.03$	$\lvert V_{td}/V_{ts}\rvert = 0.24 \pm 0.04$

- Fermilab Lattice and MILC (using Lattice $B \to P$ form factors):

$$|V_{td}/V_{ts}| = 0.20 \pm 0.02$$

- PDG 2014 (from the mixing of $B_d^0 \leftrightarrow \bar{B}_d^0$ and $B_s^0 \leftrightarrow \bar{B}_s^0$):

$$|V_{td}/V_{ts}| = 0.216 \pm 0.011$$

Nonlocal effects in FCNC decays, current status

- for $B \to K\ell^+\ell^-$ and $B \to \pi\ell^+\ell^-$ at large recoil ($q^2 < m^2_{J/\psi}$)
  $\Delta C_9(q^2)$ has an accuracy $\sim \pm 30 - 40\%$
- OPE: future improvements possible:
  using LCSRs for all nonlocal effects at $q^2 < 0$
  (currently, perturbative NLO parts in QCDF approach)
- model-dependence of the dispersion relation ansatz unavoidable
- nonlocal effects for $B \to K^*\ell^+\ell^-$ or $B \to \rho\ell^+\ell^-$ accessible (K^* or ρ in zero-width approximation)

  soft-gluon charm-loop contribution, relatively large $\Delta C_9(q^2)$ at small q^2,
  an accurate analysis demands separation of flavours in NLO contributions to match dispersion relations properly
- no lattice QCD calculations of nonlocal effects, in future?

Transitions to unstable mesons

- $B \to R$ form factors with strongly decaying $R \to h_1 h_2$

  examples: $\rho \to \pi\pi$, $K^* \to K\pi$,...

- practical problem: separate ρ or K^* from "nonresonant" background in $B \to \pi\pi\ell\nu_\ell$ or $B \to K\pi\ell\ell$?
- in the theory language:
  - define $B \to \pi\pi$ form factors, e.g.,:

$$\langle \pi^+(k_1)\pi^0(k_2)|\bar{u}\gamma^\mu(1-\gamma_5)b|\bar{B}^0(p)\rangle = -F_\perp(q^2,k^2,\zeta)\,\frac{4}{\sqrt{k^2\lambda_B}}\,i\epsilon^{\mu\alpha\beta\gamma}\,q_\alpha\,k_{1\beta}\,k_{2\gamma} + ...$$

$$(2\zeta - 1) = (1 - 4m_\pi^2/k^2)^{1/2} cos\theta_\pi, \text{ in dipion c.m.}$$

  - expand in partial waves, isolate dipion P-wave

$$F_\perp(q^2,k^2,\zeta) \Rightarrow F_\perp^{(\ell=1)}(q^2,k^2)$$

  - hadronic dispersion relation in dipion invariant mass

- dispersion relation for the $B \to \pi\pi$ vector FF:

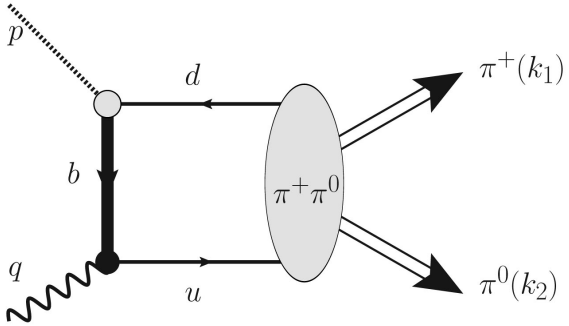
$$\frac{\sqrt{3}F_{\perp}^{(\ell=1)}(q^2,k^2)}{\sqrt{k^2}\sqrt{\lambda_B}} = \frac{g_{\rho\pi\pi}}{m_\rho^2 - k^2 - im_\rho\Gamma_\rho(k^2)}\frac{V^{B\to\rho}(q^2)}{m_B+m_\rho}$$
$$+\frac{g_{\rho'\pi\pi}}{m_{\rho'}^2 - k^2 - im_{\rho'}\Gamma_{\rho'}(k^2)}\frac{V^{B\to\rho'}(q^2)}{m_B+m_{\rho'}} +$$
$$+\frac{g_{\rho''\pi\pi}}{m_{\rho''}^2 - k^2 - im_{\rho''}\Gamma_{\rho''}(k^2)}\frac{V^{B\to\rho''}(q^2)}{m_B+m_{\rho''}} + \ldots$$

- inspired by the timelike pion e.m. form factor in $e^+e^- \to \pi^+\pi^-$ or in $\tau \to \pi^-\pi^0\nu_\tau$: modelled at $\sqrt{k^2} \lesssim 1.5$ GeV to a sum of ρ,$\rho'(1450)$,$\rho''(1750)$
- calculate $B \to \pi\pi$ or $B \to K\pi$ form factors with QCD methods $\rho, \rho', \ldots$ have to be "embedded" in this calculation
- model-dependence of the input is unavoidable

LCSRs for $B \to \pi\pi$ form factors

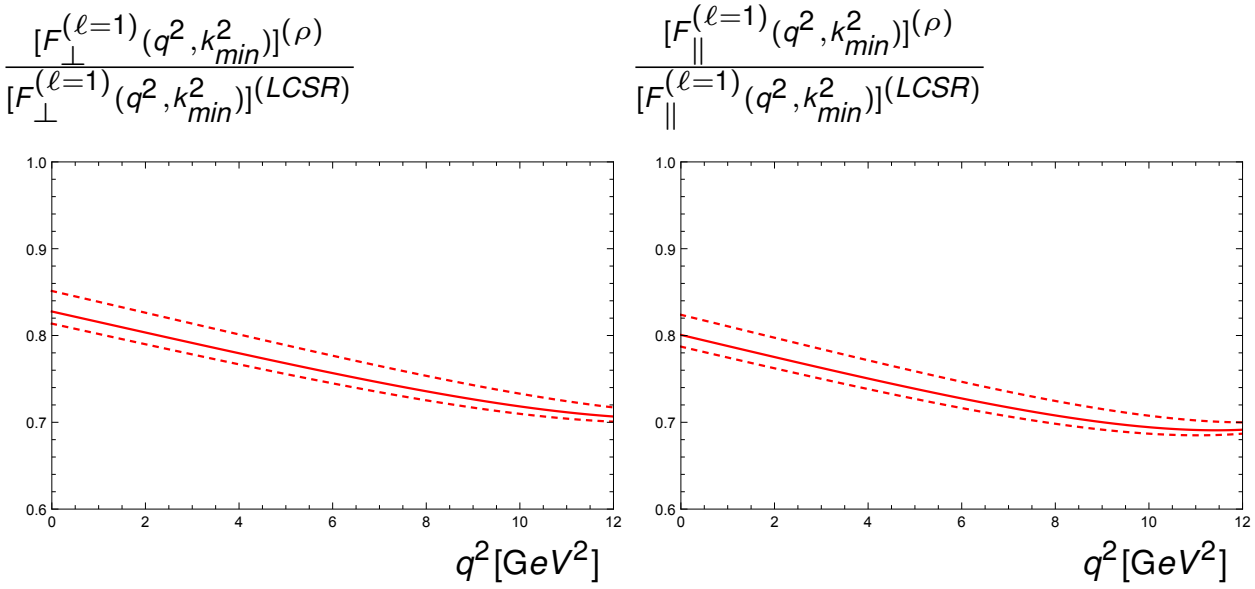
[Ch. Hambrock, AK, Nucl. Phys. B (2016); 1511.02509 [hep-ph]]

- the method: similar to the LCSR for $B \to \pi$ form factors,
- applicable for dipion with a small invariant mass and large recoil: $k^2 \lesssim 1$ GeV^2, $0 \leq q^2 \leq$ 12-14 GeV^2.
- nonperturbative input: dipion distribution amplitudes (DAs)
- vacuum $\to$ on-shell dipion hadronic matrix elements of nonlocal $\bar{u}(x)d(0)$ operators
  FSI including the ρ-meson "embedded" in DAs



- we consider only $\bar{B}^0 \to \pi^+\pi^0\ell^-\nu_\ell$,
  isospin 1, $L = 1, 3, , ..$
- only LO, twist-2 approximation for dipion DAs available
- quark-hadron duality in the B-channel, $\Rightarrow$ effective threshold s_0, Borel transformation , $p^2 \to M^2$

Numerical estimates



Relative contribution of ρ-meson to the $B \to \pi^+\pi^0$ P-wave form factors $F_{\perp}^{(\ell=1)}(q^2, k^2_{min})$ (left panel) and $F_{\parallel}^{(\ell=1)}(q^2, k^2_{min})$ (right panel) from LCSRs.

Dashed lines - the uncertainty due to the variation of the Borel parameter.

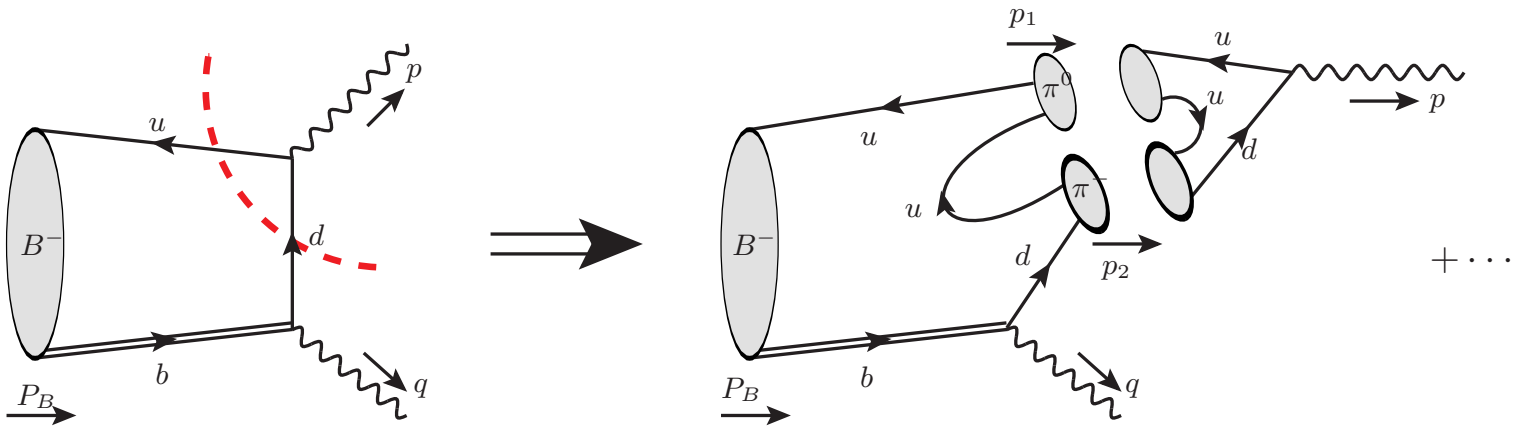
Alternative method to access $B \to \pi\pi$ FFs

[S.Cheng, AK, J.Virto, work in progress]

- LCSRs with B-meson DA and $\bar{u}\gamma_\mu d$ interpolating current
- the method introduced to calculate $B \to P, V$ form factors,

[A.K., N. Offen, Th. Mannel (2006)] , *"SCET sum rules", [F. De Fazio, Th. Feldmann, T.Hurth (2006)]*

NLO corrections to $B \to \pi$,[Y-M. Wang, Y-L.Shen (2015)



- insert a dispersion.relation for $B \to 2\pi$ form factors
  and a (dispersion rel. $\oplus$ experiment) parametrization for F_π
- not a direct calculation, given the shape of the $B \to 2\pi$ form factors, these sum rules can provide normalization

LCSRs: Brief Summary

- "where they are?":
  - $B \to \pi, K, \ldots$ form factors at large recoil with $O(10\%)$ accuracy
  - nonlocal hadronic effects in FCNC decays, but less accurately
  - $B \to \pi\pi$ form factors at small dipion mass and large recoil
    - LCSRs provide quantitative estimates for P-wave dominance, ρ-meson dominance in P-wave, etc.
    - "will they be competitive with the lattice QCD?"
  - they certainly are now,
  - the LCSRs will cease to compete if:
    - lattice calculations can be extended (and not extrapolated !) to large recoil region
    - nonlocal effects will be accessible on the lattice

Backup Slides

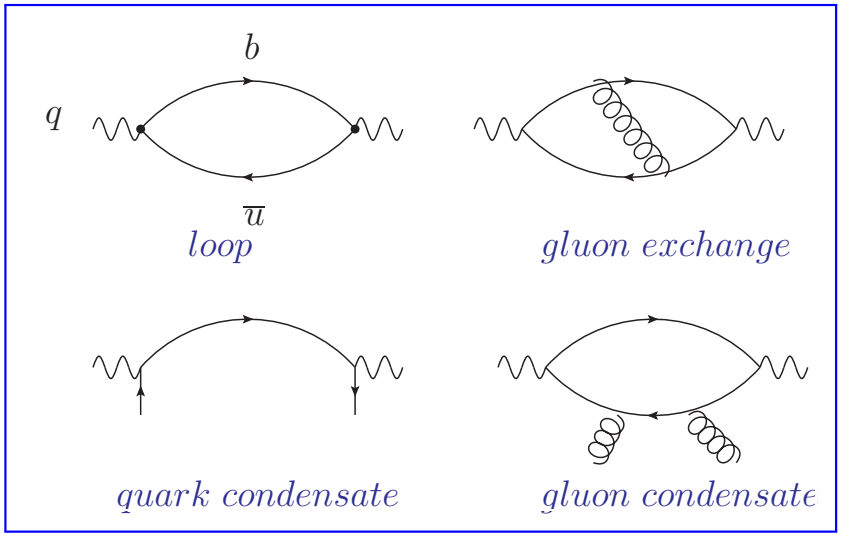
Sources of input parameters for LCSR

- QCD sum rule from $\langle 0|\bar{b}\gamma_\mu, b(x)\bar{b}\gamma_\nu b(0)|0\rangle$ saturated by Υ states

  $\Rightarrow$ non-lattice determination of m_b (in $\overline{MS}$ scheme)

- various 2pt SRs with kaon currents: $\Rightarrow m_s$, $f_K^{(t)}, a_n^{(t)}$

- 2pt SR from $\langle 0|\bar{b}\gamma_5 u(x), \bar{u}\gamma_5 b(0)|0\rangle$:



$$= \overbrace{\frac{\langle 0|\bar{b}\gamma_5 u|B\rangle\langle B|\bar{u}\gamma_5 b|0\rangle}{m_B^2 - q^2}}^{f_B^2}$$

$$\underbrace{+\sum_{B_h} \frac{\langle 0|\bar{b}\gamma_5 u|B_h\rangle\langle B_h|\bar{u}\gamma_5 b|0\rangle}{m_{B_h}^2 - q^2}}_{\text{quark-hadron duality}}$$

- LCSRs for pion and kaon e.m. form factors

- cross-check of the method: $D \to \pi$, $D \to K$ form factors

$B_{(s)}$ and $D_{(s)}$ decay constants

[P.Gelhausen, AK, A.A.Pivovarov, D.Rosenthal, 1305.5432 hep/ph]

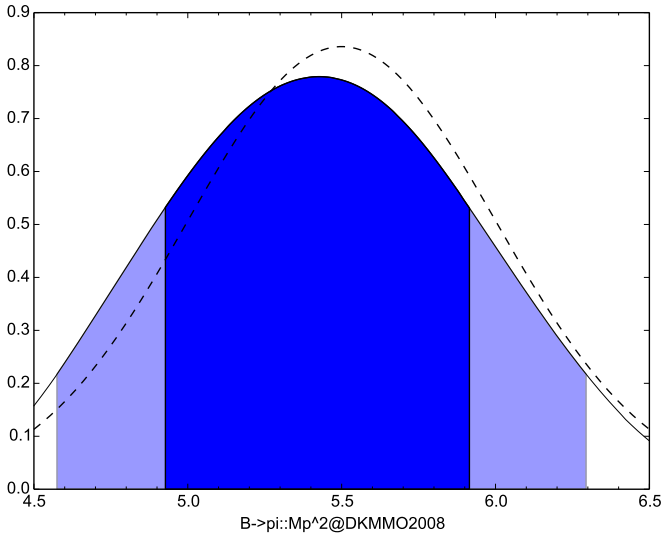
Decay constant	Lattice QCD [ref.]	this work
f_B[MeV]	196.9 ± 9.1 [1] 186 ± 4 [2]	207^{+17}_{-9}
f_{B_s}[MeV]	242.0 ± 10.0 [1] 224 ± 5 [2]	242^{+17}_{-12}
f_{B_s}/f_B	1.229 ± 0.026 [1] 1.205 ± 0.007 [2]	$1.17^{+0.04}_{-0.03}$
f_D[MeV]	218.9 ± 11.3 [1] 213 ± 4 [2]	201^{+12}_{-13}
f_{D_s}[MeV]	260.1 ± 10.8 [1] 248.0 ± 2.5 [2]	238^{+13}_{-23}
f_{D_s}/f_D	1.188 ± 0.025 [1] 1.164 ± 0.018 [2]	$1.15^{+0.04}_{-0.05}$

[1]-Fermilab/MILC, [2]-HPQCD

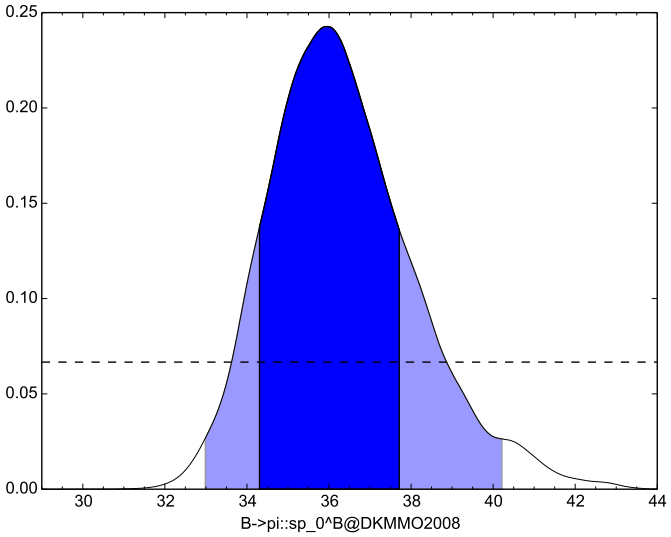
Some results

- Posterior of parameter space: one-dimension marginal PDF's

$M^2\,(\mathrm{GeV}^2)$



$s_0^B\,(\mathrm{GeV}^2)$



(prior: dashed lines, blue: 68%, light-blue: 95%)

- 6 quantities obtained from LCSR:
  $f_{B\pi}^{+}(q^2)$ + first + second derivative (value, slope, curvature) at $q^2 = 0, 10\ \mathrm{GeV}^2$,
  output approximately gaussian with large correlations

Determination of $|V_{ub}|$ from $B \to \pi \ell \nu_\ell$ data

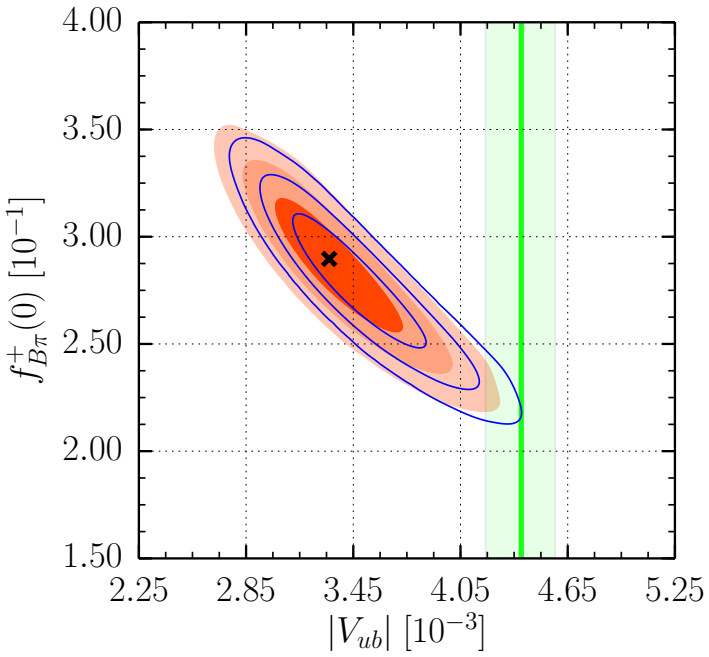
fit of LCSR with the combined BaBar/Belle data at $0 < q^2 < 12$ GeV^2

(2010): $|V_{ub}| = \left(3.43^{+0.27}_{-0.23}\right) \cdot 10^{-3}$

(2013): $|V_{ub}| = \left(3.32^{+0.26}_{-0.22}\right) \cdot 10^{-3}$

blue lines: 68%, 95%, 99% prob. contours for 2010 data
red area: 68%, 95%, 99% prob. contours for 2013 data

green line/area - inclusive determination:
central value / 68% CL interval for GGOU/HFAG



- the $b \to d\ell^+\ell^-$ effective Hamiltonian in SM

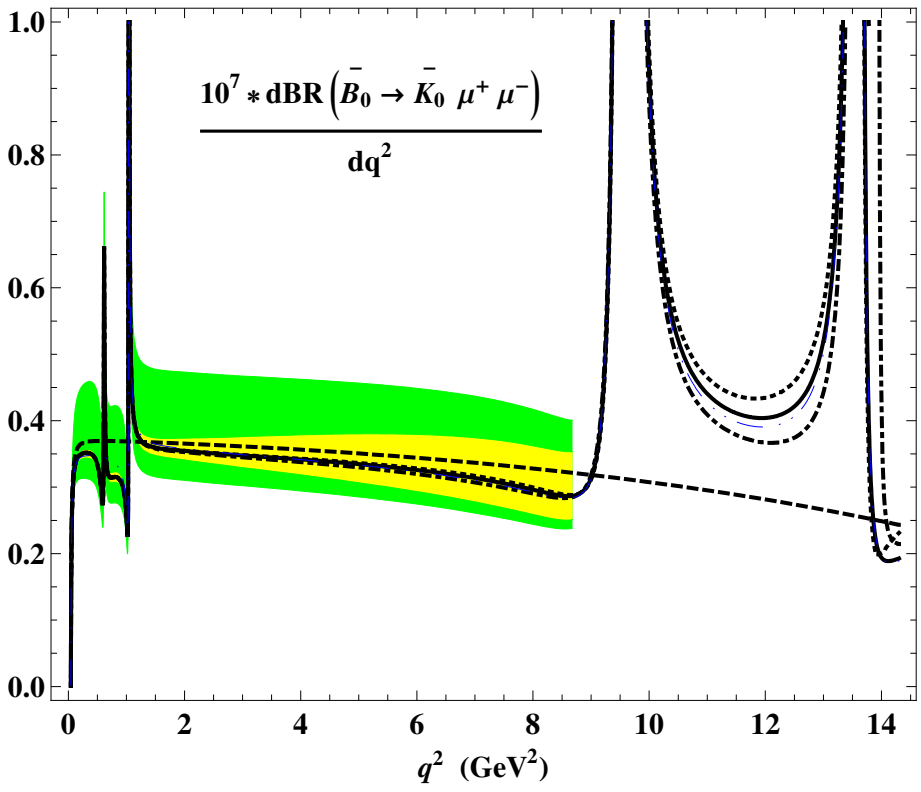
$$H_{\rm eff}^{b\to d} = \frac{4G_F}{\sqrt{2}}\Big[-\lambda_t(C_9\mathcal{O}_9 + C_{10}\mathcal{O}_{10} + C_7\mathcal{O}_{7\gamma})$$
$$-\lambda_t \sum_{i=3,4,5,6,8g} C_i\,\mathcal{O}_i + \lambda_c \sum_{i=1,2} C_i\,\mathcal{O}_i^c + \lambda_u \sum_{i=1,2} C_i\,\mathcal{O}_i^u\Big] + h.c.\,,$$

$\lambda_p = V_{pb}V_{pd}^*$, $(p = u, c, t)$, $\qquad |\lambda_u| \lesssim |\lambda_c| \sim |\lambda_t|$,

- hereafter $\lambda_t = -(\lambda_u + \lambda_c)$, with $\sim \lambda_u$ and $\sim \lambda_c$ parts separated
- direct CP violation (CKM phase in λ_u) a noticeable effect in SM
- $H_{\rm eff}^{b\to s}$ governing the $b \to s\ell^+\ell^-$ transitions: CKM enhanced,
  has a less "rich" structure, with srongly suppressed $\sim \lambda_u$ part and $\lambda_t \simeq -\lambda_c$

- the dilepton invariant mass spectrum:

from A. K., T. Mannel and Y. M. Wang, JHEP **1302** (2013) 010 [arXiv:1211.0234 [hep-ph]],

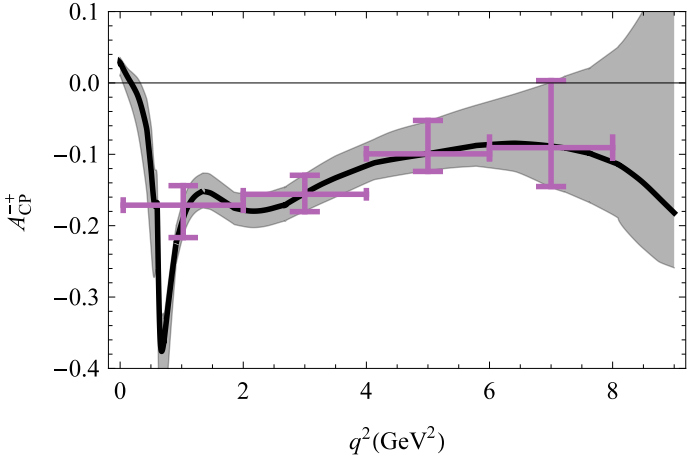


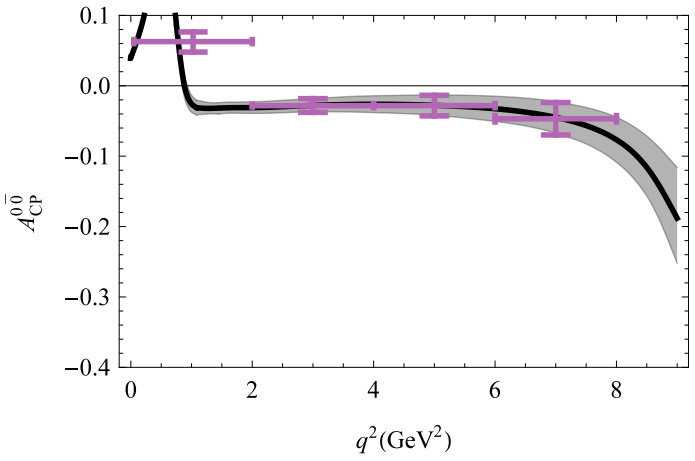
green (yellow) band -uncertainties of $B \to K$ form factors (of nonlocal effects)

- a larger error of $B \to K$ form factor, less complicated nonlocal effects , binned BR somewhat larger than central exp. [LHCb'12] values

Direct CP-violation in $B \to \pi\ell^+\ell^-$ in $B \to \pi\ell^+\ell^-$

- direct CP-asymmetry





- Binned observables: *CP*-asymmetry and isospin asymmetry

Bin $[GeV^2]$	[0.05, 2.0]	[2.0, 4.0]	[4.0, 6.0]	[6.0, 8.0]	[1.0, 6.0]
$\mathcal{A}_{CP}^{(-+)}$	$-0.171^{+0.027}_{-0.045}$	$-0.156^{+0.027}_{-0.024}$	$-0.099^{+0.047}_{-0.025}$	$-0.091^{+0.093}_{-0.053}$	$-0.143^{+0.035}_{-0.029}$
$\mathcal{A}_{CP}^{(\bar{0}0)}$	$0.063^{+0.014}_{-0.015}$	$-0.028^{+0.010}_{-0.010}$	$-0.028^{+0.015}_{-0.015}$	$-0.047^{+0.023}_{-0.023}$	$-0.008^{+0.013}_{-0.013}$
$\mathcal{A}_I$	$-0.195^{+0.033}_{-0.035}$	$-0.020^{+0.031}_{-0.032}$	$-0.021^{+0.035}_{-0.053}$	$-0.021^{+0.060}_{-0.100}$	$-0.063^{+0.033}_{-0.040}$

- LHCb (1509.00414 [hep-ex]): measured total

  $\mathcal{A}(B^\pm \to \pi^\pm\mu^+\mu^-) = -0.11 \pm 0.12 \pm 0.01$

Dipion light-cone DAs

- introduced and developed for $\gamma^*\gamma \to 2\pi$ processes

  [M. Diehl, T. Gousset, B. Pire and O. Teryaev, (1998)
  D. Müller, D. Robaschik, B. Geyer, F.-M. Dittes and J. Horejsi, (1994)
  M. V. Polyakov, (1999)]

- twist-2 DAs:

$$\langle \pi^+(k_1)\pi^0(k_2)|\bar{u}(x)\gamma_\mu[x,0]d(0)|0\rangle = -\sqrt{2}k_\mu \int_0^1 du\, e^{iu(k\cdot x)}\Phi_{\parallel}^{I=1}(u,\zeta,k^2)\,,$$

$$\langle \pi^+(k_1)\pi^0(k_2)|\bar{u}(x)\sigma_{\mu\nu}[x,0]d(0)|0\rangle = 2\sqrt{2}i\frac{k_{1\mu}k_{2\nu}-k_{2\mu}k_{1\nu}}{2\zeta-1}\int_0^1 du\, e^{iu(k\cdot x)}\Phi_{\perp}^{I=1}(u,\zeta,k^2)$$

  - the "angular" variable: $\zeta = k_1^+/k^+,\ \ 1-\zeta = k_2^+/k^+\,,\ \ \zeta(1-\zeta) \geq \frac{m_\pi^2}{k^2}\,.$

$$q\cdot\bar{k} = \frac{1}{2}(2\zeta-1)\lambda^{1/2}(p^2,q^2,k^2)\,,\ \text{in dipion c.m.}\ \ (2\zeta-1) = (1-4m_\pi^2/k^2)^{1/2}cos\theta_\pi\,,$$

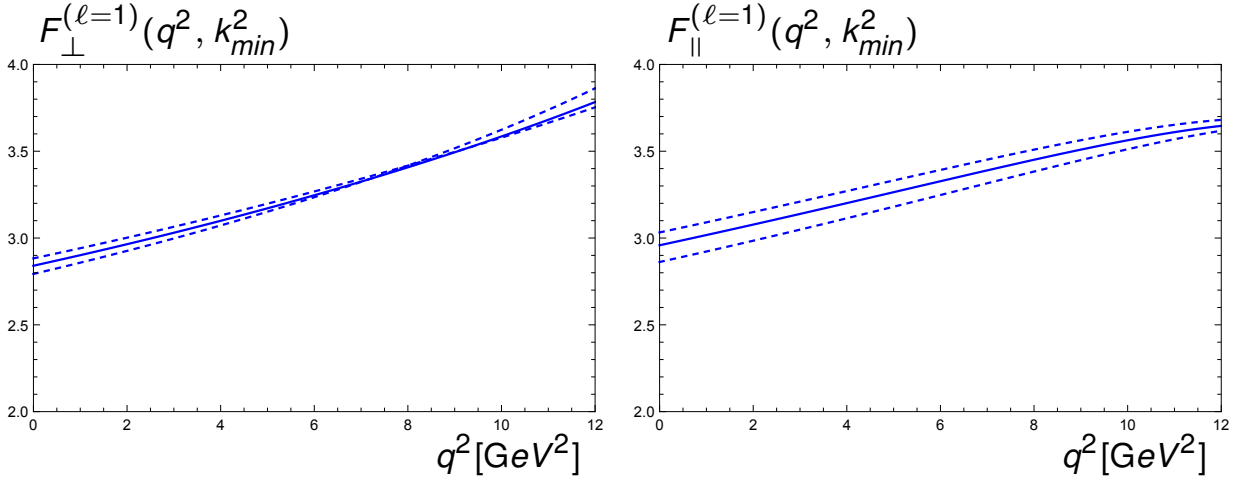
  - normalization conditions $\to$ pion timelike form factors ,

$$\int_0^1 du \left\{ \begin{array}{l} \Phi_{\parallel}^{I=1}(u,\zeta,k^2) \\ \Phi_{\perp}^{I=1}(u,\zeta,k^2) \end{array} \right. = (2\zeta-1)\left\{ \begin{array}{ll} F_\pi^{em}(k^2) & \text{pion e.m. form factor} \\ F_\pi^{t}(k^2) & \text{pion "tensor" form factor} \end{array} \right.$$

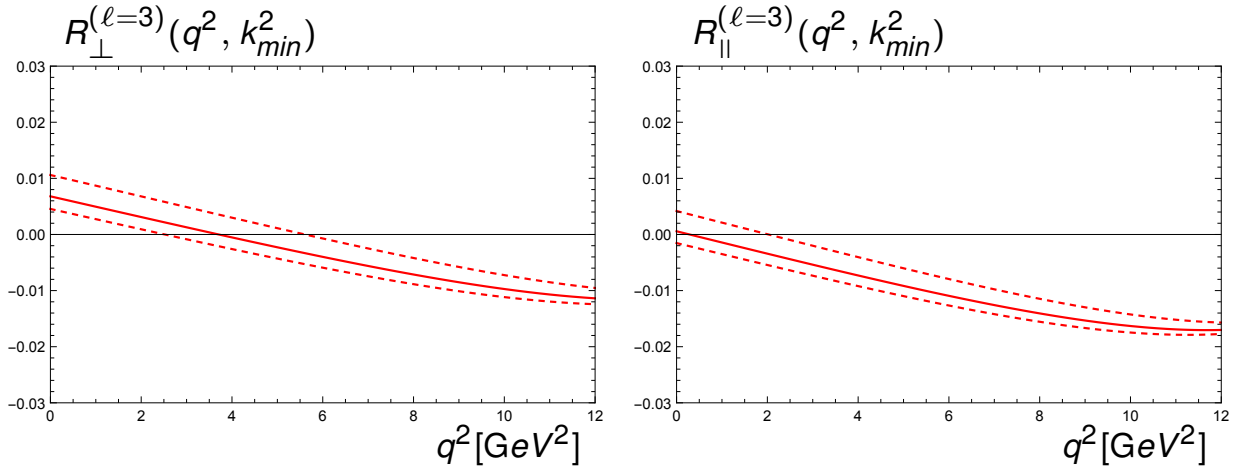
  - $\Phi_{\perp,\parallel}(u,\zeta,k^2)$ at $k^2 > 4m_\pi^2$ contain Im part
  - $F_\pi^{em}(0) = 1$, • "tensor" charge of the pion $F_\pi^t(0) = 1/f_{2\pi}^{\perp}$

Numerical results

- P-wave form factors: (only twist-2)



- P-wave dominance: ratios of F- and P-wave form factors



- - - - uncertainties from the variation of M^2.