What we can learn from $B \to VV$ and Three Body Decays.

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Islay

Outline

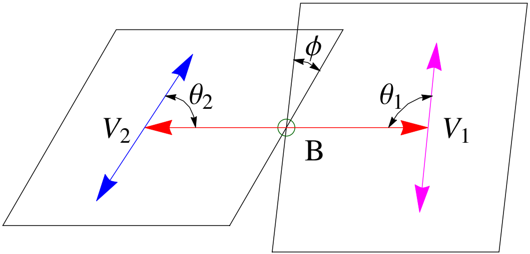
- B decays can be used to test the standard model (SM) for look for new physics(NP).
- $B \to V_1 V_2$ decays offer many probes of CP violation and in general NP.
- Experimentally not as easy as other B decays but these decays are being explored.
- Major part of the talk will be in $B \to V_1 V_2$ Decays.
- Three body decays contain a lot of information on CP violation and resonant structures.
- Briefly review three body decays.

CPV with $B \to V_1 V_2$ Decays

- $B \to V_1 V_2$ decays can be of several types:
- Both vector mesons are on-shell and observed through their decays to other final state particles.
- One or both the V can be off-shell. Example Semileptonic Decays: $B \to D^{(*)}(\rho) W^*$ with $W^* \to l\bar{\nu}_l$.
- The final state particles can be reached by both B_d^0 and $\bar{B}_d^0$ mesons (B_s^0 or $\bar{B}_s^0$).E.g. $B_d^0 \to K^* \bar{K}^*$ and $B_s^0 \to J/\psi\phi, \phi\phi$. Mixing effects have to be included and this becomes a time dependent problem.
- The final particles can be reached through a scalar background(resonant or non-resonant). Example: $B \to V_1 V_2 \to f$ and $B \to V_1 S \to f$. One has to include the interference effects.

Background

- Spin 0 meson (B) $\rightarrow$ 2 Spin 1 mesons (Vectors)
- Relative angular momentum : $L_{VV} = 0, 1, 2.$
- Vectors identified through their decay modes : Eg. $\phi \rightarrow K\bar{K}$.
- Angular analysis to separate out helicity amplitudes :
  1.) Functions of helicity angles θ_1, θ_2, and ϕ.
  2.) Observables can be dependent on time.



BVV Amplitudes

For the process:

- $B(p) \to V_1(k_1, \varepsilon) V_2(k_2, \eta)$

$$M_{\lambda_1,\lambda_2} = a\,\varepsilon^*_{\lambda_1} \cdot \eta^*_{\lambda_2} + \frac{b}{m_B^2}(p \cdot \varepsilon^*_{\lambda_1})(p \cdot \eta^*_{\lambda_2}) + i\,\frac{c}{m_B^2}\epsilon_{\mu\nu\rho\sigma} p^\mu q^\nu \varepsilon^{*\rho}_{\lambda_1} \eta^{*\sigma}_{\lambda_2},$$

  where $q \equiv k_1 - k_2$.

- The amplitude c is $L = 1$ and is parity-odd. The amplitudes a and b are combinations of $L = 0$ and $L = 2$ partial waves.
- Helicity conservations allows $M_{+,+}, M_{-,-}, M_{0,0}$. Use A_+, A_-, A_0.

BVV Transversity Amplitudes

- Another useful parametrization is the transversity basis:

$$M = A_0 \varepsilon_1^{*L} \cdot \varepsilon_2^{*L} - \frac{1}{\sqrt{2}} A_\| \vec{\varepsilon}_1^{*T} \cdot \vec{\varepsilon}_2^{*T} - \frac{i}{\sqrt{2}} A_\perp \vec{\varepsilon}_1^{*T} \times \vec{\varepsilon}_2^{*T} \cdot \hat{p} \, ,$$

where $\hat{p}$ is the unit vector along the direction of motion of V_2 in the rest frame of V_1, $\varepsilon_i^{*L} = \vec{\varepsilon}_i^{*} \cdot \hat{p}$, and $\vec{\varepsilon}_i^{*T} = \vec{\varepsilon}_i^{*} - \varepsilon_i^{*L} \hat{p}$.

- A_0, $A_\|$, $A_\perp$ are related to a, b and c of via

$$\begin{aligned} A_\| &= \sqrt{2} a \, , \quad A_0 = -ax - \frac{m_1 m_2}{m_B^2} b(x^2 - 1), \\ A_\perp &= 2\sqrt{2} \frac{m_1 m_2}{m_B^2} c \sqrt{x^2 - 1} \, , \end{aligned}$$

where $x = k_1 \cdot k_2 / (m_1 m_2)$.

- $A_+ = (A_\| + A_\perp)/\sqrt{2}$, $A_- = (A_\| - A_\perp)/\sqrt{2}$ and $M_{0,0} = A_0$.

$B \to V_1 V_2$: CP phases from $B \to V_1 V_2$ Decays

- In $B \to V_1 V_2$ decays an angular analysis is required to extract the different helicity amplitudes.
- Many correlations among the amplitudes appear in the angular distribution from which CPV phases can be extracted.
- These CPV phases can be in mixing or decay amplitude.
- Because there are many observables the CP structure of the SM or NP can be explored.

CPV in $B \to V_1 V_2$ Decays - Time Independent case

- In the angular distribution, besides the direct CP violation(DCPV) one can have another measurement of CP violation which is called the triple product asymmetry (TPA).
- DCPV $\sim \sin\Delta\phi \sin\Delta\delta$ while TPA $\sim \sin\Delta\phi \cos\Delta\delta$. Hence DCPV and TPA complement each other. If the strong phases are small then TPA are maximized.
- There is another measurement which is not CPV. Fake TP which go as $\sim \cos\Delta\phi \sin\Delta\delta$. This observable can constrain NP if the NP has the same weak phase as the SM. In this case DCPV and TPA vanish.

Triple Product Correlations

- In the B rest frame we can construct T.P

$$T.P = \vec{p}.(\vec{\epsilon}_1 \times \vec{\epsilon}_2).$$

- We can define a T-odd asymmetry

$$A_T = \frac{\Gamma[T.P > 0] - \Gamma[T.P < 0]}{\Gamma[T.P > 0] + \Gamma[T.P < 0]}.$$

- For true CP violation, we need to compare A_T and $\bar{A}_T$

$$A_{T.P}^{true} = A_T + \bar{A}_T \propto \sin\Delta\phi \cos\Delta\delta,$$

$$A_{T.P}^{fake} = A_T - \bar{A}_T \propto \cos\Delta\phi \sin\Delta\delta.$$

Measuring T.P.A.

- The T.P appear in the angular distribution of $B \to V_1 V_2 \to (V_1 \to P_1 P_1')((V_2 \to P_2 P_2')$.
- We can define two T.P's

$$A_T^{(1)} \equiv \frac{\mathrm{Im}(A_\perp A_0^*)}{A_0^2 + A_\parallel^2 + A_\perp^2} \quad , \qquad A_T^{(2)} \equiv \frac{\mathrm{Im}(A_\perp A_\parallel^*)}{A_0^2 + A_\parallel^2 + A_\perp^2} \ .$$

- For the CP conjugate decay one defines two T.P's

$$\bar{A}_T^{(1)} \equiv -\frac{\mathrm{Im}(\bar{A}_\perp \bar{A}_0^*)}{\bar{A}_0^2 + \bar{A}_\parallel^2 + \bar{A}_\perp^2} \quad , \qquad \bar{A}_T^{(2)} \equiv -\frac{\mathrm{Im}(\bar{A}_\perp \bar{A}_\parallel^*)}{\bar{A}_0^2 + \bar{A}_\parallel^2 + \bar{A}_\perp^2} \ .$$

- For true CP violation, we need to compare A_T and $\bar{A}_T$

$$A_{T.P}^{true,1,2} = \frac{1}{2}\left(A_T^{(1,2)} + \bar{A}_T^{(1,2)}\right) \propto \sin\Delta\phi \cos\Delta\delta,$$

$$A_{T.P}^{fake,1,2} = \frac{1}{2}\left(A_T^{(1,2)} - \bar{A}_T^{(1,2)}\right) \propto \cos\Delta\phi \sin\Delta\delta.$$

T.P. General

- If the decay is dominated by a single amplitude (single weak phase):

$$A_h \approx a_h \, e^{i\phi} e^{i\delta_h}.$$

  Then $A_{T.P}^{true} \approx 0$ but $A_{T.P}^{fake}$ may be non-zero.
- If the transverse amplitudes $A_T << A_0$ then both the true and fake T.P are suppressed by $\frac{|A_\perp|}{|A_0|}$ and $\frac{|A_\perp A_\parallel|}{|A_0|^2}$ even in the presence of new CP violating sources.
- Since T.P. require large transverse amplitudes the interesting decays are penguin decays/penguin dominated decays which have large A_T.

Charmless $\bar{B} \to V_1 V_2$: Naive Amplitude Estimate in the SM

- $\bar{B} \to V_1 V_2$ when the vectors are light (charmless decays) there are naive estimates for: $A_L(A_0), A_-, A_+(A_\perp, A_\parallel.)$
- Consider $b \to f \bar{q} q$ where $f = s, d$ and $q = u, d, s$. Weak interactions are $(V - A)$ and so the weak transition is

$$b_L \quad \to \quad f_L \bar{q}_R q_L.$$

- Helicity A_0 no helicity flip $\sim O(1)$.
  A_- one helicity flip $\sim O(\Lambda_{QCD}/m_B)$.
  A_+ two helicity flips $\sim O(\Lambda^2_{QCD}/m^2_B)$.
- For $\bar{B} \to V_1 V_2$ where $V_{1,2}$ are light:

$$f_L >> f_- >> f_+.$$

$$f_i = \frac{\Gamma_i}{\Gamma_{total}}$$

Data violates Naive Polarization Pattern

- Large theoretical uncertainties in penguin amplitudes.

Decay	Final State	f_L
$B \to \phi K^*$	ϕK^{*0}	0.480 ± 0.030
	ϕK^{*+}	0.50 ± 0.05
$B_s \to \phi\phi$	$\phi\phi$	$0.348 \pm 0.18(stat) \pm 0.82$
$B \to \rho K^*$	$\rho^0 K^{*0}$	0.57 ± 0.12
	$\rho^+ K^{*0}$	0.48 ± 0.08
$B_d \to K^* \bar{K}^*$	$K^{*0} \bar{K}^{*0}$	$0.80^{+0.12}_{-0.13}$
	$K^{*+} \bar{K}^{*0}$	$0.75^{+0.16}_{-0.26}$
$B_s \to K^* \bar{K}^*$	$K^{*0} \bar{K}^{*0}$	$0.31 \pm 0.12 \pm 0.04$
$B \to \rho\rho$	$\rho^+ \rho^-$	$0.978^{+0.025}_{-0.022}$
	$\rho^0 \rho^0$	$0.75^{+0.12}_{-0.15}$
	$\rho^+ \rho^0$	0.950 ± 0.016

Table: Longitudinal polarization fraction f_L for various $B \to V_1 V_2$ decays

T.P. Estimates

- The transverse amplitudes are written in terms of helicity amplitudes

$$A_{\|} = \frac{1}{\sqrt{2}}(A_+ + A_-) ,$$
$$A_{\perp} = \frac{1}{\sqrt{2}}(A_+ - A_-) .$$

- Due to the fact that the weak interactions are left-handed, the helicity amplitudes obey the hierarchy

$$\left|\frac{A_+}{A_-}\right| = r_T = \frac{\Lambda_{QCD}}{m_b} .$$

- Thus, in the heavy-quark limit, $A_{\|} = -A_{\perp}$ which means $A_T^{(2)}$, which is proportional to $\mathrm{Im}(A_{\perp} A_{\|}^*)$, vanishes. Hence in the heavy quark limit both $A_{T.P}^{true,2}$ and $A_{T.P}^{fake,2}$ vanish. (Datta, Durisamy, London e-Print: arXiv:1103.2442).

$\bar{b} \to \bar{s}$ transitions within the SM -pure penguin

- Amplitude within the SM (pure penguin modes) : (Loosely : γ comes from phase of V^*_{ub})

  $A_h \; = \; e^{-i\phi_M/2} \left[P'_{tc,h} \; e^{i\delta_{tc,h}} \; + \; e^{i(\gamma+\phi_M/2)} \; P'_{uc,h} \; e^{i\delta_{uc,h}} \right]$. Example Decay $B \to \phi K^*$.

- Leading order in Wolfenstein Parameter λ : $P'_{tc,h} \; \propto \; |V^*_{tb} V_{ts}| \; \sim \; \mathcal{O}(\lambda^2)$.

- Next-to-leading order in λ : $P'_{uc,h} \; \propto \; |V^*_{ub} V_{us}| \; \sim \; \mathcal{O}(\lambda^4)$.

- If we neglect $P'_{uc,h}$ there there is only decay amplitude and so all CPV measurements- direct CP and Triple product asymmetries vanish.

- $A^{true,2}_{T.P} \sim \lambda^2 \frac{\Lambda_{QCD}}{m_b}$ and $A^{fake,2}_{T.P} \sim \frac{\Lambda_{QCD}}{m_b}$.

- $A^{true,1}_{T.P} \sim \lambda^2$ and $A^{fake,1}_{T.P} \sim 0(1)$.

NP in $\bar{b} \to \bar{s}$ decay

- Assume NP larger than sub-dominant SM term.
- Amplitude with (large) NP in the decay :

$$A_h = P_{tc,h} e^{i\delta_{tc,h}} (1 + R_h^{NP} e^{i\phi^{NP}} e^{i\Delta_h^{NP}}).$$

- Δ_h^{NP} is the difference between NP strong phase and $\delta_{tc,h}$.
- NP strong phases may themselves be helicity dependent.
- $R_h^{NP} = P^{NP,h}/P_{tc,h}$: $R_h^{NP} \gg R_h^{SM} \sim \mathcal{O}(\lambda^2) \Rightarrow$ New Physics.
- CP violation appears due to the interference of two terms.

  $\Rightarrow$ CP-violating observables are proportional to R_h!

  Look for large CPV (direct, indirect, TP) for signals of NP in $\bar{b} \to \bar{s}$ decay.

Corrections to the heavy quark limit

- There are corrections to the prediction that $A_T^{(2)} = 0$, in $B \to \phi K^*$. The estimate for $A_T^{(2)}$ is, based on QCD factorization.

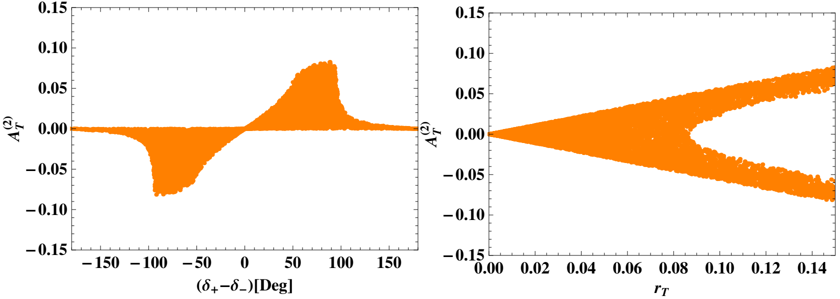


Figure: The left (right) panel of the figure shows $A_T^{(2)}$ for the decay $B_d \to \phi K^{*0}$ as a function of $(\delta_+ - \delta_-)$ and r_T.

- There we see that $|A_T^{fake,2}| \leq 9\%$ is predicted.

Corrections to the heavy quark limit

- The estimate for $A_T^{(1)}$ is :

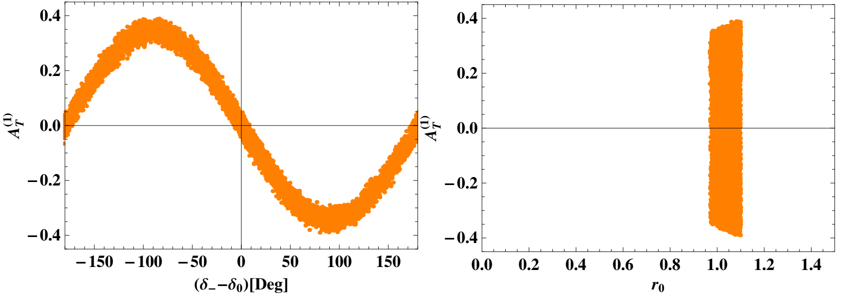


Figure: The left (right) panel of the figure shows $A_T^{(1)}$ for the decay $B_d \to \phi K^{*0}$ as a function of $(\delta_+ - \delta_-)$ (r_T).

- There we see that $|A_T^{fake,1}| \leq 40\%$ is predicted.
This prediction is not unexpected given the large size of the transverse amplitudes.

Experiments

- The relevant $B_d \to \phi K^{*0}$ polarization observables are shown in Table below.

Polarization fractions	
$f_L = 0.480 \pm 0.030$	$f_\perp = 0.241 \pm 0.029$
Phases	
$\phi_\parallel(rad) = 2.40^{+0.14}_{-0.13}$	$\phi_\perp(rad) = 2.39 \pm 0.13$
$\Delta\phi_\parallel(rad) = 0.11 \pm 0.13$	$\Delta\phi_\perp(rad) = 0.08 \pm 0.13$
CP asymmetries	
$A^0_{CP} = 0.04 \pm 0.06$	$A^\perp_{CP} = -0.11 \pm 0.12$

Table: $B_d \to \phi K^{*0}$ polarization observables .

- Note the T.P. are directly measurable from the angular distribution.
- $A^{fake,2}_{T.P} \sim \sin(\phi_\perp - \phi_\parallel)$ $\quad A^{true,2}_{T.P} \sim \sin(\Delta\phi_\perp - \Delta\phi_\parallel)$.
- $A^{fake,1}_{T.P} \sim \sin(\phi_\perp)$ $\quad A^{true,2}_{T.P} \sim \sin(\Delta\phi_\perp)$.

Experimental T.P's

- Using the numbers above we can calculate:

$$A_{T.P}^{fake,2} = \frac{1}{2}(A_{T,B}^{(2)} - \bar{A}_{T,\bar{B}}^{(2)}) = 0.002 \pm 0.049 \ ,$$
$$A_{T.P}^{fake,1} = \frac{1}{2}(A_{T,B}^{(1)} - \bar{A}_{T,\bar{B}}^{(1)}) = -0.23 \pm 0.03 \ .$$

- The measured value of $A_{T.P}^{fake,2}$ is therefore in agreement with the SM prediction in the heavy quark limit.
- The actual T.P are

$$A_{T.P}^{true,2} = \frac{1}{2}(A_{T,B}^{(2)} + \bar{A}_{T,\bar{B}}^{(2)}) = -0.004 \pm 0.025,$$
$$A_{T.P}^{true,1} = \frac{1}{2}(A_{T,B}^{(1)} + \bar{A}_{T,\bar{B}}^{(1)}) = 0.013 \pm 0.053.$$

Hence consistent with SM or with NP with same weak phase as the SM. No evidence for large NP contribution to the amplitude.

NP in $\bar{b} \to \bar{s}$ decay- Tree and Penguins

- The decays $B \to \rho K^*$ are interesting. There are Tree and Penguin contributions.

- They probe $b \to s u \bar{u}$ and $b \to s d \bar{d}$ transitions. They are the vector counterpart of the $B_d \to K\pi$ modes.

- $$\begin{aligned} A(B^+ \to \rho^+ K^{*0}) &= P'_{ct} \, , \\ \sqrt{2} A(B^+ \to \rho^0 K^{*+}) &= -P'_{ct} - T' \, e^{i\gamma} - P'_{EW} , \\ A(B^0 \to \rho^- K^+) &= -T' e^{i\gamma} - P'_{ct} \, , \\ \sqrt{2} A(B^0 \to \rho^0 K^0) &= P'_{ct} - P'_{EW} \, . \end{aligned}$$

- Large CPV in $B^0 \to K^+\pi^-$. What are the T.P in $B \to \rho K^*$ modes. Note these modes also have large transverse polarization. Can these modes shed light on the $K - \pi$ "puzzle".

Time dependent Angular Distribution: $B \to V_1 V_2$

- Decays like $B_d \to \rho^0 K^0$, $B_d \to K^* \bar{K}^*$, $B_s \to J/\psi\phi, \phi\phi(\bar{b} \to \bar{s}s\bar{s}), K^*\bar{K}^*$, the final state can be reached by both B_q and $\overline{B}_q$ decays so mixing effects have to be included.
- Assuming that $V_{1,2}$ both decay into pseudoscalars (i.e. $V_1 \to P_1 P_1'$, $V_2 \to P_2 P_2'$), the angular distribution of the decay is then given in terms of the vector $\vec{\omega} = (\cos\theta_1, \cos\theta_2, \Phi)$:

$$\frac{d^3\Gamma(t)}{d\vec{\omega}} = \frac{9}{32\pi}\sum_{i=1}^{6} K_i(t) f_i(\vec{\omega}) \; .$$

- Functions $K_i(t)$ are expressed in terms of ϕ_q , Γ_q , $\Delta\Gamma_q$, the B_q^0 oscillation frequency Δm_q and transversity amplitudes $A_{i(=0,\|,\perp)}$.

Time-integrated untagged distribution

The time-integrated untagged angular distribution can be obtained by integrating the $K_i(t) + \bar{K}_i(t)$ observables over time:

$$\frac{d^3\langle\Gamma(B_s^0 \to f)\rangle}{d\vec{\omega}} = \frac{9}{32\pi}\sum_{i=1}^{6}\langle K_i\rangle f_i(\vec{\omega}) ,$$

$$\langle\Gamma(B_s^0 \to f)\rangle = \frac{1}{2}\int_0^\infty dt(\Gamma^{B_s} + \Gamma^{\bar{B}_s}) , \quad \langle K_i\rangle = \frac{1}{2}\int_0^\infty dt(K_i(t) + \bar{K}_i(t)) .$$

The general structure is

$$\langle K_i\rangle \propto \mathcal{A}_i^{ch} + \mathcal{A}_i^{sh} y_q,$$

where $y_q = \frac{\Delta\Gamma_q}{2\Gamma_q}$. The $\mathcal{A}_i^{ch}$ are used to extract the polarization fractions and triple products.

Time-integrated untagged distribution

- If y_q is small (e.g. y_d) then

$$\langle K_i \rangle \propto \mathcal{A}_i^{ch}$$

  The polarization fractions and triple products can be extracted from $\langle K_i \rangle$ which appear in the angular distribution.
- If y_q cannot be neglected (e.g. y_s) then we need the input $\mathcal{A}^i_{\Delta\Gamma} \equiv \mathcal{A}_i^{sh}/\mathcal{A}_i^{ch}$ (known in SM, Fleischer et.al.).
- Use:

$$\tau_{B_s}^{eff,i} = \frac{\int_0^\infty t(K_i(t) + \bar{K}_i(t))dt}{\int_0^\infty (K_i(t) + \bar{K}_i(t))dt} = \frac{\tau_{B_s}}{(1-y_s^2)} \frac{(1 + 2\mathcal{A}^i_{\Delta\Gamma} y_s + y_s^2)}{(1 + \mathcal{A}^i_{\Delta\Gamma} y_s)}.$$

Polarization Fractions

In the SM (one amp): We have $\mathcal{A}_i^{sh} = \mp\mathcal{A}_i^{ch}$, where the minus sign is for $i = 1, 2, 5$, the plus sign for $i = 3$, and both quantities vanish when $i = 4, 6$. With NP these relations are no longer true.

The polarization fractions can be extracted from $\langle K_i \rangle$, $i = 1, 2, 3$

$$\begin{aligned}\langle K_h \rangle &= \frac{\tau_{B_s}}{2(1-y_s^2)} \Big[\Big(|A_h|^2 + |\bar{A}_h|^2 \Big) \\ &\quad -\eta_h 2 \Big(\mathrm{Re}(A_h^* \bar{A}_h) \cos\phi_s + \mathrm{Im}(A_h^* \bar{A}_h) \sin\phi_s \Big) y_s \Big].\end{aligned}$$

and $\eta_h = \eta_{0,\parallel,\perp} = (1, 1, -1)$.

$$f_h = \frac{|A_h|^2 + |\bar{A}_h|^2}{|A_0|^2 + |\bar{A}_0|^2 + |A_\parallel|^2 + |\bar{A}_\parallel|^2 + |A_\perp|^2 + |\bar{A}_\perp|^2} = \frac{\mathcal{A}_h^{ch}}{\sum_{i=1,2,3} \mathcal{A}_i^{ch}} .$$

Triple Products

We now turn to the measurement of TP's

$$\begin{aligned}
\langle K_4 \rangle &= \frac{\tau_{B_s}}{2(1-y_s^2)} \Big[\Big(\mathrm{Im}(A_\perp A_\parallel^*) - \mathrm{Im}(\bar{A}_\perp \bar{A}_\parallel^*) \Big) \\
&\quad - \Big((\mathrm{Im}(A_\perp \bar{A}_\parallel^*) - \mathrm{Im}(\bar{A}_\perp A_\parallel^*)) \cos\phi_s \\
&\qquad + (\mathrm{Re}(A_\perp \bar{A}_\parallel^*) + \mathrm{Re}(\bar{A}_\perp A_\parallel^*)) \sin\phi_s \Big) y_s \Big] , \\
\langle K_6 \rangle &= \frac{\tau_{B_s}}{2(1-y_s^2)} \Big[\Big(\mathrm{Im}(A_\perp A_0^*) - \mathrm{Im}(\bar{A}_\perp \bar{A}_0^*) \Big) \\
&\quad - \Big((\mathrm{Im}(A_\perp \bar{A}_0^*) - \mathrm{Im}(\bar{A}_\perp A_0^*)) \cos\phi_s \\
&\qquad + (\mathrm{Re}(A_\perp \bar{A}_0^*) + \mathrm{Re}(\bar{A}_\perp A_0^*)) \sin\phi_s \Big) y_s \Big] .
\end{aligned}$$

The TP's in the untagged distribution can be from the decay as well as mixing and can be measured by constructing asymmetries involving the angular variables (Rosner, Gronau).

Triple Products- Untagged Decays

We begin with $i = 4$, for which $f_4(\vec{\omega}) = -2\sin^2\theta_1 \sin^2\theta_2 \sin 2\Phi$. We define $u \equiv \sin 2\Phi$. Construct the T.P as an asymmetry in u.

$$\begin{aligned}
\mathcal{A}_u &= \frac{1}{2}\Big[\frac{\langle\Gamma(B_s^0 \to \phi\phi), u > 0\rangle - \langle\Gamma(B_s^0 \to \phi\phi), u < 0\rangle}{\langle\Gamma(B_s^0 \to \phi\phi), u > 0\rangle + \langle\Gamma(B_s^0 \to \phi\phi), u < 0\rangle}\Big] \\
&= -\frac{2}{\pi}[\mathcal{A}_T^{(2,true)}]_{exp} \quad , \quad [\mathcal{A}_T^{(2,true)}]_{exp} = \frac{\langle K_4\rangle}{\langle\Gamma(B_s^0 \to \phi\phi)\rangle} \ .
\end{aligned}$$

For $i = 6$ with $f_6(\vec{\omega}) = -\sqrt{2}\sin 2\theta_1 \sin 2\theta_2 \sin\Phi$. We define $v \equiv \mathrm{sign}(\cos\theta_1 \cos\theta_2)\sin\Phi$, which has the following associated TP asymmetry :

$$\begin{aligned}
\mathcal{A}_v &= \frac{1}{2}\Big[\frac{\langle\Gamma(B_s^0 \to \phi\phi), v > 0\rangle - \langle\Gamma(B_s^0 \to \phi\phi), v < 0\rangle}{\langle\Gamma(B_s^0 \to \phi\phi), v > 0\rangle + \langle\Gamma(B_s^0 \to \phi\phi), v < 0\rangle}\Big] \\
&= -\frac{\sqrt{2}}{\pi}[\mathcal{A}_T^{(1,true)}]_{exp} \quad , \quad [\mathcal{A}_T^{(1,true)}]_{exp} = \frac{\langle K_6\rangle}{\langle\Gamma(B_s^0 \to \phi\phi)\rangle} \ .
\end{aligned}$$

Triple Products- Untagged Decays

The relation between $[\mathcal{A}_T^{(2,1)}]_{exp}$ and the theoretical expression for the TP in the decay are.

$$[\mathcal{A}_T^{(2,true)}]_{exp} = \frac{1}{2}\left(\mathrm{Im}(A_\perp A_\parallel^*) - \mathrm{Im}(\bar{A}_\perp \bar{A}_\parallel^*)\right) \left[\frac{(1+A_{\Delta\Gamma}^{(4)} y_s)}{(1-y_s^2)} \frac{\tau_{B_s}}{\langle \Gamma(B_s^0 \to \phi\phi)\rangle}\right].$$

$$[\mathcal{A}_T^{(1,true)}]_{exp} = \frac{1}{2}\left(\mathrm{Im}(A_\perp A_0^*) - \mathrm{Im}(\bar{A}_\perp \bar{A}_0^*)\right) \left[\frac{(1+A_{\Delta\Gamma}^{(6)} y_s)}{(1-y_s^2)} \frac{\tau_{B_s}}{\langle \Gamma(B_s^0 \to \phi\phi)\rangle}\right].$$

$A_{\Delta\Gamma}^{(4,6)} = \mathcal{A}_{4,6}^{sh}/\mathcal{A}_{4,6}^{ch}.$

Measuring β_s, Penguin Pollution

- B_s^0 and $\bar{B}_s^0$ can decay to $f \equiv J/\psi\phi$.
- The indirect CPA measures

$$\mathrm{Im}\left(\frac{q}{p}\frac{\bar{\mathcal{A}}_s^f}{\mathcal{A}_s^f}\right) ,$$

  where $\mathcal{A}_s^f$ and $\bar{\mathcal{A}}_s^f$ are the amplitudes for $B_s^0 \to f$ and $\bar{B}_s^0 \to f$, respectively.
- $q/p = (V_{tb}^* V_{ts}/V_{tb}V_{ts}^*) = \exp(2i\,\arg(V_{tb}^* V_{ts}))$. This is phase-convention dependent.
- Assuming $\mathcal{A}_s^f$ is dominated by a single decay amplitude. $\bar{\mathcal{A}}_s^f/\mathcal{A}_s^f = (V_{cb}V_{cs}^*/V_{cb}^*V_{cs}) = \exp(2i\,\arg(V_{cb}V_{cs}^*))$, which is also phase-convention dependent.

- However, the product of these two quantities is

$$\frac{q}{p}\frac{\bar{\mathcal{A}}_s^f}{\mathcal{A}_s^f} = \frac{V_{tb}^* V_{ts}}{V_{cb}^* V_{cs}}\frac{V_{cb} V_{cs}^*}{V_{tb} V_{ts}^*} = e^{2i\beta_s} ~,$$

where

$$\beta_s \equiv \arg\left[-\frac{V_{tb}^* V_{ts}}{V_{cb}^* V_{cs}}\right] ~.$$

This is phase-convention independent, and hence physical. The indirect CPA measures $\sin 2\beta_s$.

- Including "penguin pollution" or new physics or both(Bhattacharya, Datta, London 1209.1413).

$$
\begin{aligned}
A^h &= \lambda_c^{(s)}(C' + P'_{ct} - \frac{2}{3}P'_{EW}) + \lambda_u^{(s)}(P'_{ut} - \frac{2}{3}P'_{EW}) + A_{NP} \\
&\equiv e^{i\arg(V_{cb}^* V_{cs})}\left[A_1^h + e^{i\gamma}A_2^h\right] ,
\end{aligned}
$$

  where $\lambda_q^{(q')} \equiv V_{qb}^* V_{qq'}$. This holds for the four helicities $h = 0, \perp, \|, S$.
- Experimental analysis: Assumes A_2^h=0 for all helicities. There are 8 assumptions. Using a convention when the overall phase vanishes then we have $A_h = \bar{A}_h$.
- The tagged angular distribution has enough observables to fit for β_s.
- The point is the 8 assumptions are not needed.

- $$\frac{d^4\Gamma(B_s^0 \to J/\psi\phi)}{dtd\vec{\Omega}} \propto \sum_{k=1}^{10} h_k(t) f_k(\vec{\Omega}) \ .$$

- $$\begin{aligned} h_k(t) &= \frac{1}{2} e^{-\Gamma_s t} \left[c_k \cos\Delta m_s t + d_k \sin\Delta m_s t \right. \\ &\qquad \left. + \, a_k \cosh\left(\Delta\Gamma_s/2\right)t + b_k \sinh\left(\Delta\Gamma_s/2\right)t \right] \ . \end{aligned}$$

- By measuring the time-dependent angular distribution and fitting to the four time-dependent functions, Γ_s and $\Delta\Gamma_s$ can be determined, as well as the coefficients a_k-d_k.

- By applying the angular analysis to the full amplitudes A_h and $\bar{A}_h$, one can still extract $\phi_s^{c\bar{c}s}$, even if there is PP or NP.
- In the general case there are two complex set A_h and $\bar{A}_h$. The a_k-d_k are expressed in terms of 16 unknown parameters: the magnitudes of the A_h and $\bar{A}_h$ (8), their relative phases (7), and $\phi_s^{c\bar{c}s}$.
- The angular observables can be used to get 15 of these parameters.
- For the phase differences we define

$$
\begin{aligned}
\delta_{ij} &\equiv \arg(A_i) - \arg(A_j) \ , \\
\bar{\delta}_{ij} &\equiv \arg(\bar{A}_i) - \arg(\bar{A}_j) \ , \\
D_{ij} &\equiv \arg(\bar{A}_i) - \arg(A_j) \ ,
\end{aligned}
$$

where i, j are any of the 4 helicities $0, \|, \perp, S$.

- In the one amplitude method $A_h = \bar{A}_h \Rightarrow D_{ii} = 0$.
- The point is only theory input is necessary: e.g. an estimate of $D_{0,0}$.
- If there is evidence of deviation from the SM we would like to know if the NP is in the decay or mixing.
- NP in the decay can be explored by the CPV quantities in the angular distribution, $a_k - d_k$. If the second amplitude (PP or NP) is tiny all these quantities are also very small.

$B \to V_1 V_2$ with scalar background

- Penguin-dominated decays : Eg. $B_s \to \phi\phi, K^* \bar{K}^*$. Scalar background contribute to the final state.(arXiv:1306.1911, Bhattacharya, Datta, Duraisamy, London).
- Vectors detected via hadronic decay come with scalar backgrounds

  Eg. $\phi \to K^+ K^-$, Background Scalar : $K^+ K^-$ s-wave.
- Additional contributions to Amplitude :

  $A(B \to V_1 V_2) + A(B \to V_1 S_2) + A(B \to S_1 V_2) + A(B \to S_1 S_2)$.
- 3 helicity amplitudes in $B \to VV$: 1 Longitudinal and 2 transverse.
- Scalar background adds additional helicities : (SV, VS, SS).

  Identical final-state vector mesons : 2 additional helicities $(VS = -SV)$.

  Distinguishable final-state vector mesons : 3 additional helicities.

The differential decay rate

- Most general amplitude has the following terms :

$$A_{VV} : A_0 \cos\theta_1 \cos\theta_2 + \frac{A_{\|}}{\sqrt{2}} \sin\theta_1 \sin\theta_2 \cos\phi + i\frac{A_{\perp}}{\sqrt{2}} \sin\theta_1 \sin\theta_2 \sin\phi$$

$$A_{VS} : -\frac{A_+^{(VS)}}{\sqrt{6}}(\cos\theta_1 - \cos\theta_2) - \frac{A_-^{(VS)}}{\sqrt{6}}(\cos\theta_1 + \cos\theta_2)$$

$$A_{SS} : -\frac{A_s}{3};$$

- $A_{\pm}^{(VS)} = (A_{VS} \pm A_{SV})/\sqrt{2}$. $A_-^{(VS)}$ and $A_+^{(VS)}$ are CP even and CP odd.
- If $V_1 = V_2$ then $A_+^{(VS)} \equiv 0$.

The differential decay rate

- The differential decay rate is then :

$$\frac{d^4\Gamma}{dt\ d\vec{\Omega}} \propto |A_{VV} + A_{VS} + A_{SS}|^2$$

- Triple product and DCPV can be constructed from $(A_\perp)$ and $A_+^{(VS)}$ which are CP-odd amplitudes.
- CP-violating terms are the result of interference between CP-odd and CP-even amplitudes.

- Angular distribution with six helicities : $({}^6C_2 + 6 = 21)$

$$\frac{d^4\Gamma}{dt\ d\vec{\Omega}} = \frac{9}{8\pi} \sum_{i=1}^{21} K_i(t)\ X_i(\theta_1, \theta_2, \phi)$$

$$\text{where } K_i(t) = \tfrac{1}{2}\ e^{-\Gamma t} \left[a_i \cosh\left(\frac{\Delta\Gamma}{2} t\right) + c_i \cos(\Delta m t) \right.$$
$$\left. + b_i \sinh\left(\frac{\Delta\Gamma}{2} t\right) + d_i \sin(\Delta m t) \right]$$

- Appropriately integrate over phase space to extract K_i 's using :

$$\int X_i(\vec{\Omega})\ f_j(\vec{\Omega})\ d\vec{\omega} = \delta_{ij}$$

- Note: It is not possible to distinguish between $\mathrm{Re}[A_S A_0^*]$ and $|A_+^{(VS)}|^2$ - $|A_-^{(VS)}|^2$ since the angular function is the same : $X \propto \cos\theta_1 \cos\theta_2$.

- Time-dependent fit to K_i's give the observables : a_i, b_i, c_i, d_i - 84 of them!.

$B_s \to K^{*0} \bar{K}^{*0}$

- CP conjugate K_i's can be obtained from :

$$\overline{K}_i(t) = \frac{1}{2}\, e^{-\Gamma t} \left[\overline{a}_i \cosh\left(\frac{\Delta\Gamma}{2} t\right) + \overline{c}_i \cos(\Delta m t) + \overline{b}_i \sinh\left(\frac{\Delta\Gamma}{2} t\right) + \overline{d}_i \sin(\Delta m t) \right]$$

where $\overline{a}_i = a_i, \quad \overline{b}_i = b_i, \quad \overline{c}_i = -c_i, \quad \overline{d}_i = -d_i$

- Untagged analysis angular distribution
- Asymmetric integration over helicity angles obtain :

$$K_i^{\rm untagged} = K_i + \overline{K}_i = e^{-\Gamma t} \left[a_i \cosh\left(\frac{\Delta\Gamma}{2} t\right) + b_i \sinh\left(\frac{\Delta\Gamma}{2} t\right) \right].$$

- Observables a_i and b_i from time-dependent fit to $K_i^{\rm untagged}$

$\bar{b} \to \bar{s}$ transitions within the SM- $B_s \to \phi\phi$

- Amplitude within the SM (pure penguin modes) : (Loosely : γ comes from phase of V^*_{ub})

  $A_h = e^{-i\phi_M/2}\left[P'_{tc,h}\, e^{i\delta_{tc,h}} + e^{i(\gamma+\phi_M/2)}\, P'_{uc,h}\, e^{i\delta_{uc,h}}\right]$. Example Decay $B \to \phi K^*$.

- Leading order in Wolfenstein Parameter λ : $P'_{tc,h} \propto |V^*_{tb}V_{ts}| \sim \mathcal{O}(\lambda^2)$.
- Next-to-leading order in λ : $P'_{uc,h} \propto |V^*_{ub}V_{us}| \sim \mathcal{O}(\lambda^4)$.
- If we neglect $P'_{uc,h}$ there there is only decay amplitude and so all CPV measurements- direct CP and Triple product asymmetries vanish.
- $A^{true,2}_{T.P} \sim \mathcal{A}_u \sim \lambda^2 \frac{\Lambda_{QCD}}{m_b}$ and $A^{fake,2}_{T.P} \sim \frac{\Lambda_{QCD}}{m_b}$.
- $A^{true,1}_{T.P} \sim\sim \mathcal{A}_v \sim \lambda^2$ and $A^{fake,1}_{T.P} \sim 0(1)$.

$B_s \to \phi\phi$

- Two identical vectors in the final state :

  5 helicity amplitudes $(3VV, SS, VS_{-})$
- Studied by LHCb in detail : arXiv:1407.2222 (published in PRD)
- LHCb(new) results for $B_s^0 \to \phi\phi$.

Observable	Measurement
$\|A_0\|^2_{exp}$	$0.365 \pm 0.022\ (\mathrm{stat}) \pm 0.012\ (\mathrm{syst})$
$\|A_\perp\|^2_{exp}$	$0.291 \pm 0.024\ (\mathrm{stat}) \pm 0.010\ (\mathrm{syst})$
$\|A_\parallel\|^2_{exp}$	$0.344 \pm 0.024\ (\mathrm{stat}) \pm 0.014\ (\mathrm{syst})$
$\cos(\delta_\parallel - \delta_0)$	$-0.844 \pm 0.068\ (\mathrm{stat}) \pm 0.029\ (\mathrm{syst})$
$\mathcal{A}_u$	$-0.003 \pm 0.017\ (\mathrm{stat}) \pm 0.006\ (\mathrm{syst})$
$\mathcal{A}_v$	$-0.017 \pm 0.017\ (\mathrm{stat}) \pm 0.006\ (\mathrm{syst})$

$B_s \to \phi\phi$

- $\phi_s = -0.17 \pm 0.15 \pm 0.03$.
- Scalar background consistent with zero.
- No evidence of new CP violation in $b \to s\bar{s}s$ transitions as in $B_d \to \phi K^*$.
- As more experimental precision is achieved the discarded subleading SM amplitude will have to be included.
- We can move on to $b \to s\overline{d}d$. Motivated by the $K-\pi$ puzzle. Examples are $B_d \to \rho K^*$, $B_s \to K^*\bar{K}^*$.

$B_s \to K^{*0}\bar{K}^{*0}$

- Final state has distinguishable vectors : 6 helicity amplitudes.
- The same final state is accessible to both B_s and $\bar{B}_s$.
- $K^{*0}(890)$ is identified through its decay to $K^+\pi^-$.
- Scalar background : $K^{*0}(1430)$ (Large width) is noticed.
- Time-dependent tagged analysis could be difficult.
- Interesting physics even in untagged time-dependent analysis.

$B_s \to K^{*0}\bar{K}^{*0}$ LHCb

- LHCb (1503.05362) measured 8 CP violating observables in the time integrated untagged decay.
- Triple product constructed with $(A_\perp)$ and DCPV with $A_+^{(VS)}$
- $A_T^i \sim \mathrm{Im}\left[A_\perp A_i^* - \bar{A}_\perp \bar{A}_i^*\right]$ where $A_i = A_0, A_\parallel, A_-^{(VS)}, A_S$.
- $A_D^i \sim \mathrm{Re}\left[A_+^{(VS)} A_i^* - \bar{A}_+^{(VS)} \bar{A}_i^*\right]$ where $A_i = A_0, A_\parallel, A_-^{(VS)}, A_S$.
- Found large scalar background from $K_0^*(1430)$ and $K_0^*(800)$.

New-Physics Scenarios

- Typical effective NP operator :

  $H_{AB}^{NP} \sim (\bar{b}\,\gamma_A\,s)(\bar{q}\,\gamma_B\,q)$ where A, B stands for left(L) or right(R)
- Expansion parameters : Λ_{QCD}/m_B and R_h^{NP}
- RR and LL operators only contribute to $A_{\|}, A_{\perp}$, and A_{SS}

  $\Rightarrow$ Direct CPV involving $A^{(VS)+}$ are suppressed

  Reasonable triple products
- RL and LR operators don't contribute to VS helicities

  $\Rightarrow$ Triple products involving $A_{\|}$ and $A_{\perp}$ are small

  Other CP violating observables are reasonable, including direct CPV
- a_i, b_i, c_i, d_i can help distinguish between different NP scenarios

Table: Triple product and direct CP asymmetries measured in this analysis. The first uncertainties are statistical and the second systematic.

Asymmetry	Value
A_T^1	$0.003 \pm 0.041 \pm 0.009$
A_T^2	$0.009 \pm 0.041 \pm 0.009$
A_T^3	$0.019 \pm 0.041 \pm 0.008$
A_T^4	$-0.040 \pm 0.041 \pm 0.008$
A_D^1	$-0.061 \pm 0.041 \pm 0.012$
A_D^2	$0.081 \pm 0.041 \pm 0.008$
A_D^3	$-0.079 \pm 0.041 \pm 0.023$
A_D^4	$-0.081 \pm 0.041 \pm 0.010$

Three Body Decays

- Multibody B and D decays are being explored experimentally.
- Lot on interesting physics in these decays- measure CP phases, study resonances e.t.c.
- Theoretically difficult to study. Model calculations exist. See for example archive:1308.5139, Cheng and Chua.
- In general difficult to apply QCD factorization results: see archive: 1505.04111, Krankl, Mannel, Virto.
- One can use flavor symmetry to extract CP phases and predict CP violation: See for example: 1303.0846 (Bhattacharya, Imbeault, London), 1306.2625(Bhattacharya, Gronau, Rosner).

An example: Extraction of γ: Battacharya, Imbeault, London

- γ is obtained by combining information from the Dalitz plots for $B_d^0 \to K^+\pi^0\pi^-$, $B_d^0 \to K^0\pi^+\pi^-$, $B^+ \to K^+\pi^+\pi^-$, $B_d^0 \to K^+K^0K^-$, and $B_d^0 \to K^0K^0\bar{K}^0$.

- The method applies to each point in the Dalitz plot. The value of γ is independent of momentum, so that the method really represents *many* independent measurements of γ.

- The isobar model is used to model the amplitude in the Dalitz plot.

- Flavor symmetry is used to relate amplitudes.

Diagrams: Rey-LeLorier, Imbeault, London, archive: 1011.49

- Express amplitudes in terms of diagrams as in two body decays. Diagrams are T (tree), C(color-suppressed tee), P (QCD penguin) and P_{EW} (Electoweak penguin).

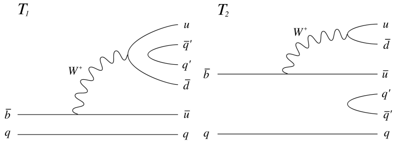
- 

Figure: Color allowed Tree diagrams contributing to $B \to \pi\pi\pi$.

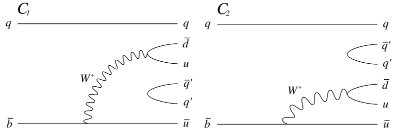


Figure: Color Suppressed tree diagrams contributing to $B \to \pi\pi\pi$.

Penguins

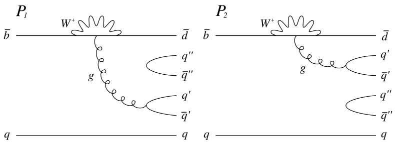


Figure: QCD penguin diagrams contributing to $B \to \pi\pi\pi$.

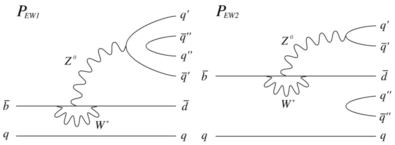


Figure: P_{EW} diagrams contributing to $B \to \pi\pi\pi$.

Penguins

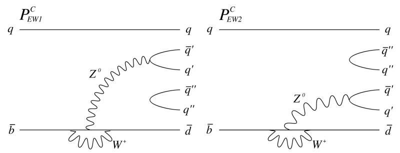


Figure: P_{EWC} diagrams contributing to $B \to \pi\pi\pi$.

Under flavor SU(3) there are relations between the electroweak penguin (EWP) and tree diagrams for $\bar{b} \to \bar{s}$ transitions. These take the simple form

$$P'_{EWi} = \kappa T'_i \ , \quad P'^C_{EWi} = \kappa C'_i \ \ (i = 1, 2) \ \ ; \qquad \kappa \equiv -\frac{3}{2} \frac{|\lambda_t^{(s)}|}{|\lambda_u^{(s)}|} \frac{c_9 + c_{10}}{c_1 + c_2} \ ,$$

where the c_i are Wilson coefficients and $\lambda_p^{(s)} = V^*_{pb} V_{ps}$. The EWP-tree relations hold only for the state that is fully symmetric under exchanges of the final-state particles.

The amplitudes for the five decays are written in terms of diagrams. Using the four effective diagrams:

$$a \equiv -\tilde{P}'_{tc} + \kappa\left(\frac{2}{3}T'_1 + \frac{1}{3}C'_1 + \frac{1}{3}C'_2\right) ,$$
$$b \equiv T'_1 + C'_2 , \quad c \equiv T'_2 + C'_1 , \quad d \equiv T'_1 + C'_1 .$$

$$\begin{aligned}
2A(B_d^0 \to K^+\pi^0\pi^-)_{\rm fs} &= be^{i\gamma} - \kappa c , \\
\sqrt{2}A(B_d^0 \to K^0\pi^+\pi^-)_{\rm fs} &= -de^{i\gamma} - \tilde{P}'_{uc}e^{i\gamma} - a + \kappa d , \\
\sqrt{2}A(B^+ \to K^+\pi^+\pi^-)_{\rm fs} &= -ce^{i\gamma} - \tilde{P}'_{uc}e^{i\gamma} - a + \kappa b , \\
\sqrt{2}A(B_d^0 \to K^+K^0K^-)_{\rm fs} &= \alpha_{SU(3)}(-ce^{i\gamma} - \tilde{P}'_{uc}e^{i\gamma} - a + \kappa b) , \\
A(B_d^0 \to K^0K^0\bar{K}^0)_{\rm fs} &= \alpha_{SU(3)}(\tilde{P}'_{uc}e^{i\gamma} + a) ,
\end{aligned}$$

where $\alpha_{SU(3)}$ measures the amount of flavor-SU(3) breaking. $\alpha_{SU(3)}$=1 in the symmetric limit.

Dalitz Plot

- $B \to P_1 P_2 P_3$ (the P_i are pseudoscalar mesons). Denoting by p_i the momentum of each P_i, one defines the three Mandelstam variables $s_{ij} \equiv (p_i + p_j)^2$. These are not independent, but obey $s_{12} + s_{13} + s_{23} = m_B^2 + m_1^2 + m_2^2 + m_3^2$.
- Use the isobar model to construct the amplitude from experiment

$$\mathcal{M}(s_{12}, s_{13}) = \mathcal{N}_{\rm DP} \sum_j c_j e^{i\theta_j} F_j(s_{12}, s_{13}) \ ,$$

- Finally construct the fully symmetric state and use the amp relations

$$\begin{aligned} \mathcal{M}_{\rm fs} \ &= \ \frac{1}{\sqrt{6}} \left[\mathcal{M}(s_{12}, s_{13}) + \mathcal{M}(s_{13}, s_{12}) + \mathcal{M}(s_{12}, s_{23}) \right. \\ & \qquad \left. + \, \mathcal{M}(s_{23}, s_{12}) + \mathcal{M}(s_{23}, s_{13}) + \mathcal{M}(s_{13}, s_{23}) \right] \ . \end{aligned}$$

Fit to γ

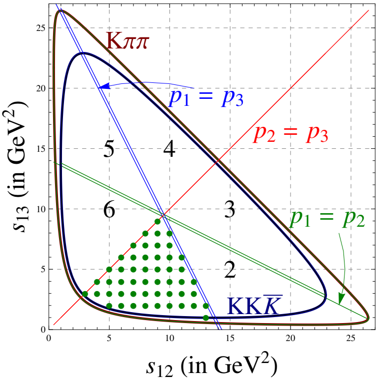


Figure: Kinematic boundaries and symmetry axes of $B \to K\pi\pi$ and $B \to KKK$ Dalitz plots. The symmetry axes divide each plot into six zones, five of which are marked 2-6. The fifty dots in the region of overlap of the first of six zones from all Dalitz plots are used for the γ measurement.

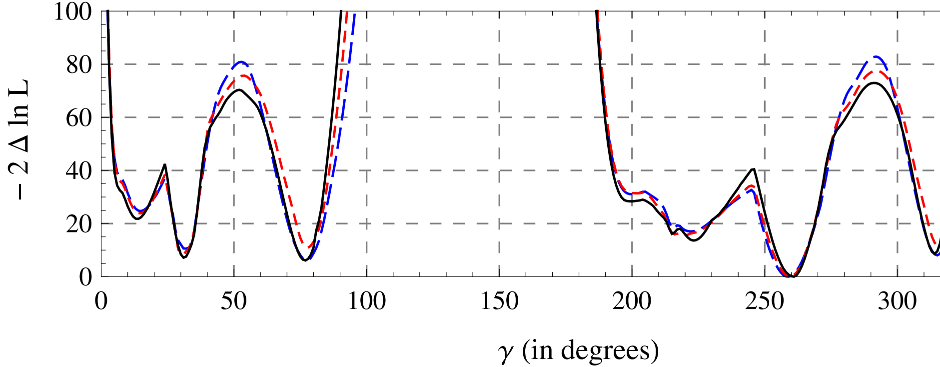


Figure: Results of maximum-likelihood fits. The solid (black) curve represents the fit assuming flavor-SU(3) symmetry. The short dashes (red) represent the fit where flavor-SU(3) breaking is fixed by a point-by-point comparison of Dalitz plots for $B^+ \to K^+\pi^+\pi^-$ and $B^0 \to K^+K^0K^-$. The long dashes (blue) represent the fit with inputs from five Dalitz plots and an extra hadronic fit parameter $|\alpha_{SU(3)}|$.

Solution	Fit 1	Fit 2	Fit 3
I	31^{+2}_{-1}	31^{+1}_{-2}	32 ± 2
II	77 ± 2	78 ± 2	77 ± 2
III	261^{+2}_{-3}	259^{+3}_{-2}	259^{+2}_{-3}
IV	314 ± 2	315 ± 2	315 ± 2

One value is close to the SM value. The other solutions may point to $B \to K\pi\pi$, $B \to KKK$ "puzzle".

Conclusions

- $B \to V_1 V_2$ offer many probes of CP violation.
- Many observables can be used to probe the SM or NP.
- Multibody decays can be used to study CP violation, resonance structures.
- Experiments have measured observables in $B \to V_1 V_2$ and three body decays and will continue to do so with more precision. Challenge is to study carefully the implications of these measurements.