Edinburgh workshop
2016

Near-conformal composite Higgs

or PNGB with partial compositeness?

with the Lattice Higgs Collaboration (L_{at}HC)

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Holography, conformal field theories, and lattice
Edinburgh workshop

June 30, 2016   Higgs Center, University of Edinburgh

What is our composite Higgs paradigm?

the Higgs doublet field

elementary scalar?

$$H = \frac{1}{\sqrt{2}} \begin{pmatrix} \pi_2 + i\,\pi_1 \\ \sigma - i\,\pi_3 \end{pmatrix} \qquad \frac{1}{\sqrt{2}}(\sigma + i\,\vec{\tau}\cdot\vec{\pi}) \equiv M$$

$$D_\mu M = \partial_\mu M - i\,g\,W_\mu M + i\,g' M\,B_\mu\,, \qquad \text{with} \qquad W_\mu = W_\mu^a \frac{\tau^a}{2}\,, \quad B_\mu = B_\mu \frac{\tau^3}{2}$$

The Higgs Lagrangian is

spontaneous symmetry breaking

Higgs mechanism

$$\mathcal{L} \;=\; \frac{1}{2}\mathrm{Tr}\left[D_\mu M^\dagger D^\mu M\right] - \frac{m_M^2}{2}\mathrm{Tr}\left[M^\dagger M\right] - \frac{\lambda}{4}\mathrm{Tr}\left[M^\dagger M\right]^2$$

strongly coupled gauge theory

fermions (Q) in gauge group reps in flavor/color space:

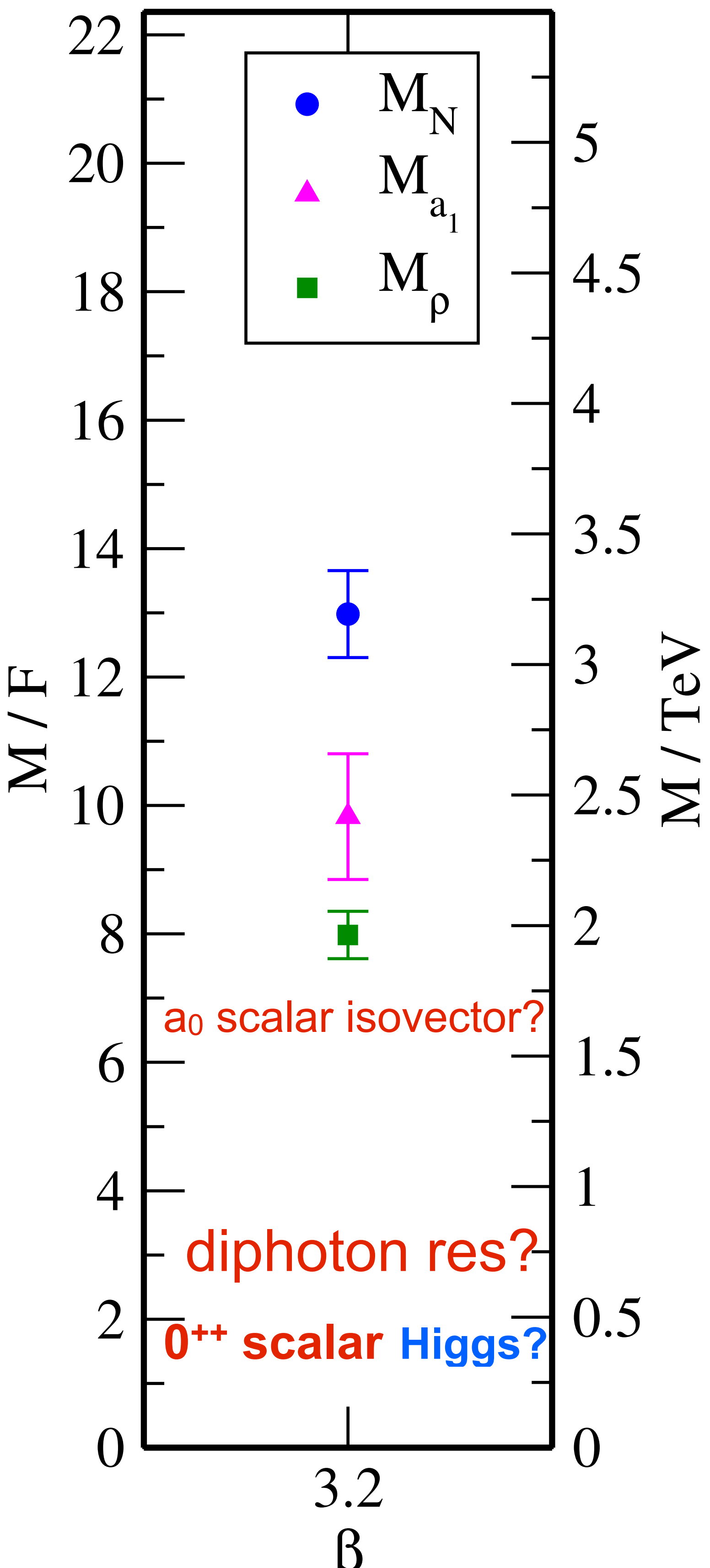
$$\mathcal{L}_{Higgs} \rightarrow -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + i\bar{Q}\gamma_\mu D^\mu Q + \ldots$$

light scalar separated from
2-3 TeV resonance spectrum

has to be unlike QCD
we learn field theory tools for LHC apps
in semi-realistic setting

The light 0++ scalar

BSM lattice challenges



We want to understand:

light scalar separated from 2-3 TeV resonance spectrum

multiple scalars in models close to CW?

Resonance spectrum?

what is the eta'?

entangled scalar-goldstone dynamics sigma model or dilaton?
how to decouple and isolate the light scalar?

bridge between UV and IR scale?
scale-dependent gauge coupling - high precision

what list of predictions independent of mass generation?
related phenomenology

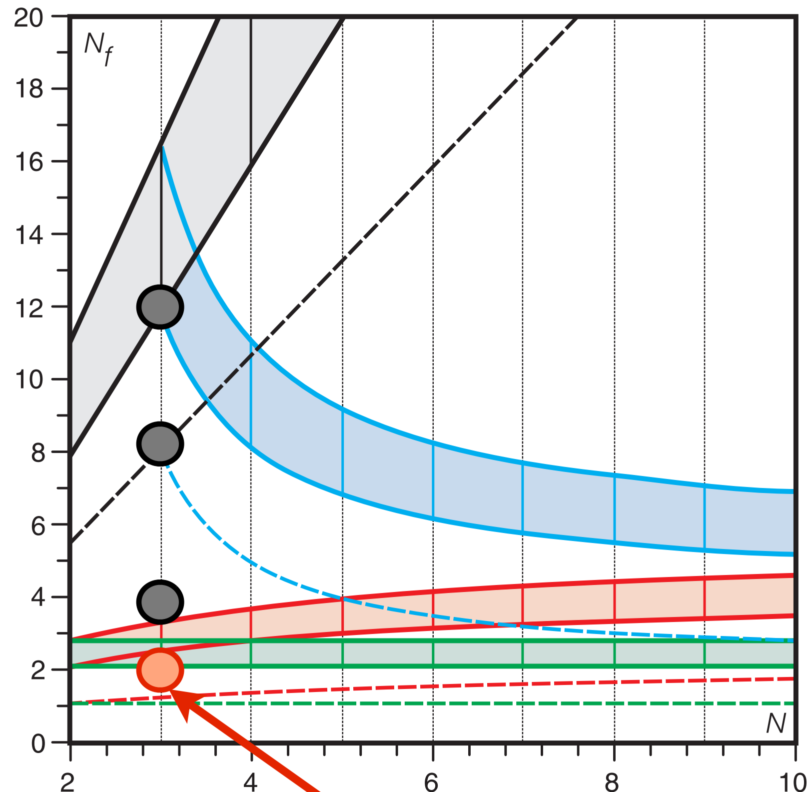
consistent EW embedding → dark matter

BSM needs new lattice tools → RMT and delta-regime

scaled up QCD cannot do the job

4 lattice BSM theories with light scalars and SU(3) color:

SCGT Theory Space



$\begin{bmatrix} u(+e/2 \\ d(-e/2) \end{bmatrix}$ minimal EW embedding

Nf=2 sextet rep
massless fermions
SU(2) doublet
3 Goldstones > weak bosons
minimal realization of Higgs mechanism
adding lepton doublets is a choice
adding EW singlet massive flavor
is also a choice

$\begin{bmatrix} u(+2/3) \\ d(-1/3) \end{bmatrix}$

QCD intuition for near-conformal
compositeness is wrong

Technicolor thought to be scaled up QCD
motivation of the project:
composite Higgs-like scalar close to the
conformal window with 2-3 TeV new physics

sextet from haystack:
Marciano in QCD
Sannino and Tuominen BSM

early lattice work:
Boulder/Tel Aviv
LatHC (also Kogut-Sinclair)
some recent CP^3 work

Nf = 4+4 and Nf=4+8 are popular in fundamental rep: LatKMI and LSD
talks at this workshop

Near the conformal window?

- β-function
- mass of the light composite scalar

Tag *minimal* for the sextet model?
though its gauge group is SU(3) and may need
fermion doublets in UV completion

our homework assignments

Conformal:

Exhibit zero in beta function

measure the scaling violation exponent ω

show that mass deformed spectroscopy works
including conformal scaling violation

Chiral Symmetry Breaking

Show that $F \cdot L \sim \sqrt{Nf}$

drive the running coupling g(L) into this volume
this excludes then any zeros in the beta function

decouple light scalar in p-regime PT and
drive to epsilon regime and RMT

the running coupling and the β function   finite volume

LatHC group introduced the running coupling and its β function from the gauge field gradient flow with the scale set by the finite volume
variations of it are becoming the standard approach

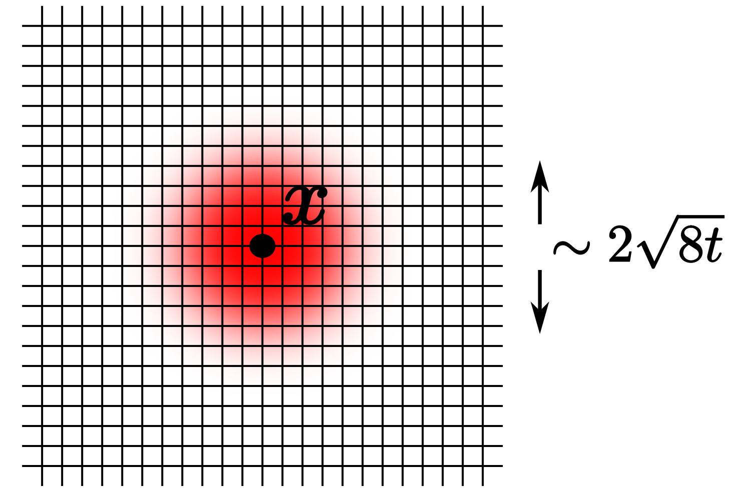
$$\dot{B}_\mu = D_\nu G_{\nu\mu} + \lambda D_\mu \partial_\nu B_\nu$$

$$B_{\mu,1}(t,x) = \int \mathrm{d}^D y \, K_t(x-y) A_\mu(y),$$

$$K_t(z) = \int \frac{\mathrm{d}^D p}{(2\pi)^D} \mathrm{e}^{ipz} \mathrm{e}^{-tp^2} = \frac{\mathrm{e}^{-z^2/4t}}{(4\pi t)^{D/2}}$$

Lüscher

earlier work by Neuberger



$$\langle E(t) \rangle = \frac{3}{4\pi t^2} \alpha(q) \{1 + k_1 \alpha(q) + O(\alpha^2)\}, \quad q = \frac{1}{\sqrt{8t}}, \quad k_1 = 1.0978 + 0.0075 \times N_f$$

t is the gradient flow time
Running coupling definition (range is $(8t)^{1/2}$) :

while holding $c = (8t)^{1/2}/L$ fixed: $\alpha_c(L) = \frac{4\pi}{3} \frac{\langle t^2 E(t) \rangle}{1 + \delta(c)}$

$$\delta(c) = \vartheta_3^4(e^{-1/c^2}) - 1 - \frac{c^4 \pi^2}{3}$$

3rd Jacobi function

three different boundary conditions are used in practice:
anti-periodic fermion fields
Schrödinger functional
twisted gauge fields and fermion fields

fundamental rep:

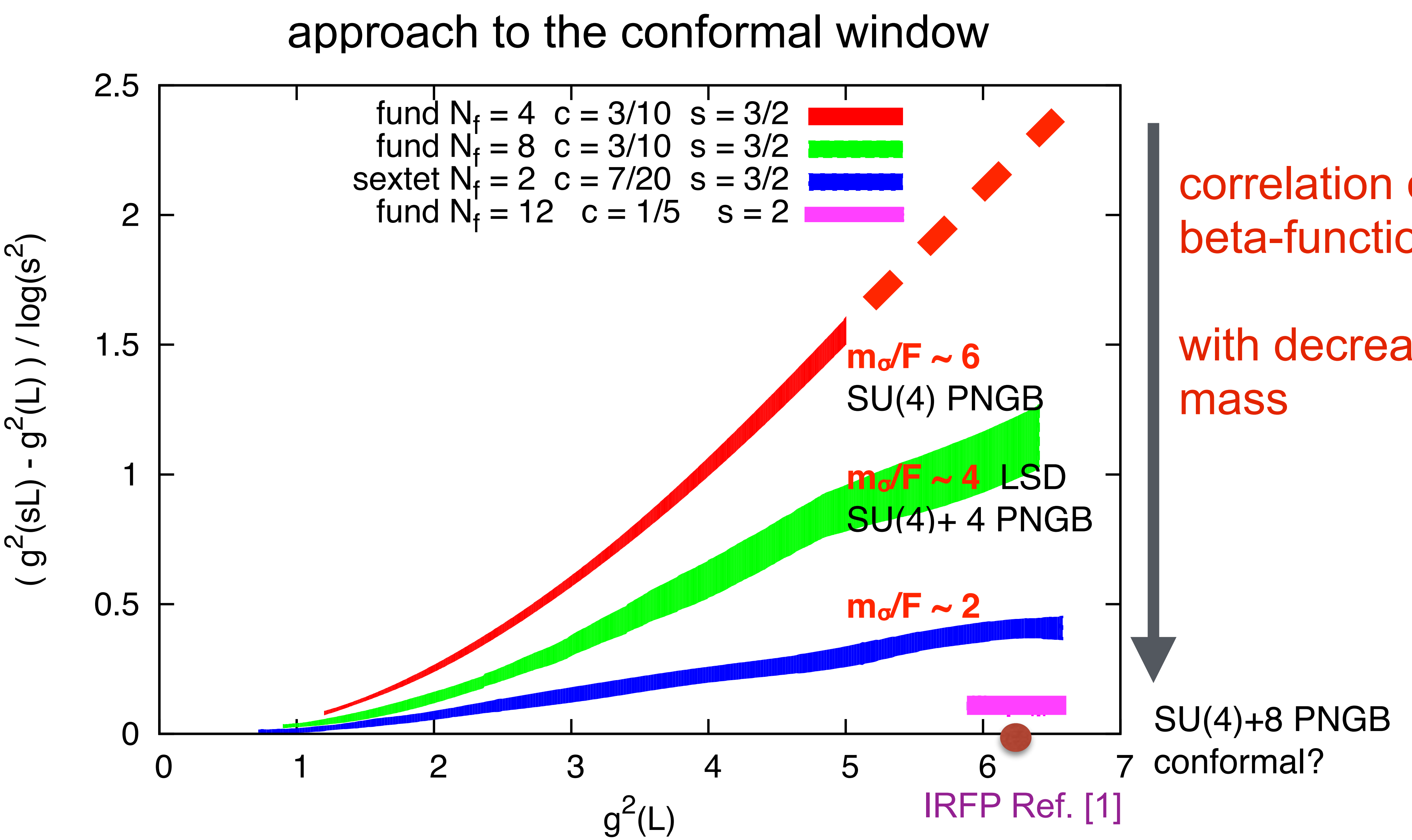
Nf=4/8 Boulder group and LatHC
Nf=12 Boulder group and Lin-Ramos

sextet rep:
Nf=2 LatHC

4 lattice BSM theories with light scalars and SU(3) color:

scale-dependent coupling of the 4 lattice BSM models

gradient flow method with high accuracy



correlation of decreasing beta-function

with decreasing light scalar mass

Fate of the conformal fixed point with twelve massless fermions and SU(3) gauge group

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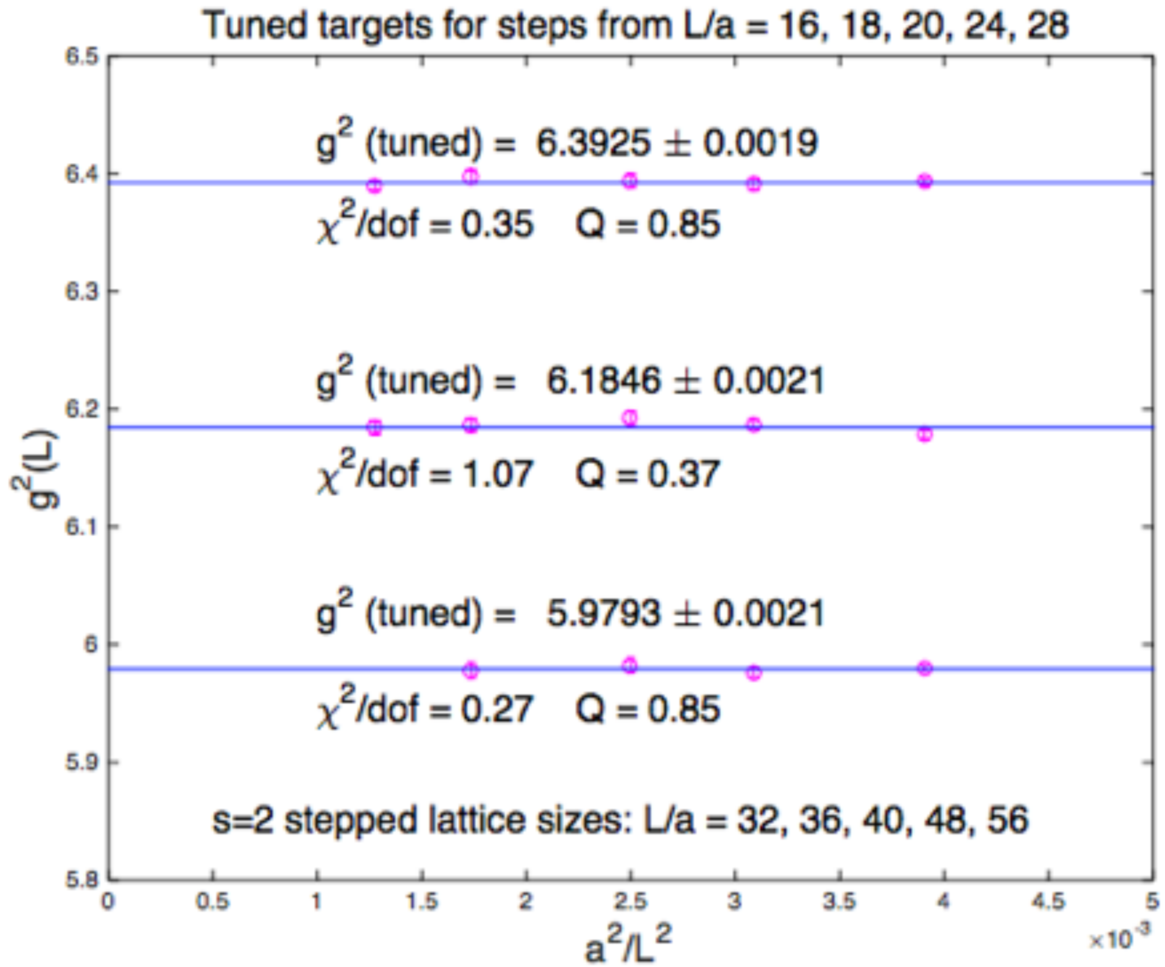
MTA-ELTE Lendulet Lattice Gauge Theory Research Group, Budapest 1117, Hungary
[5]*University of Wuppertal, Department of Physics, Wuppertal D-42097, Germany*

We report new results on the conformal properties of an important strongly coupled gauge theory, a building block of composite Higgs models beyond the Standard Model. With twelve massless fermions in the fundamental representation of the SU(3) color gauge group, an infrared fixed point of the β-function was recently reported in the theory [1] with uncertainty in the location of the critical gauge coupling inside the narrow $[6.0 < g_*^2 < 6.4]$ interval and widely accepted since as the strongest evidence for a conformal fixed point and scale invariance in the theory with model-building implications. Using the exact same renormalization scheme as the previous study, we show that no fixed point of the β-function exists in the reported interval. Our findings, under full control of the continuum limit, eliminate the only seemingly credible evidence for a conformal fixed point in the β-function of the $N_f = 12$ model whose infrared properties remain unresolved.

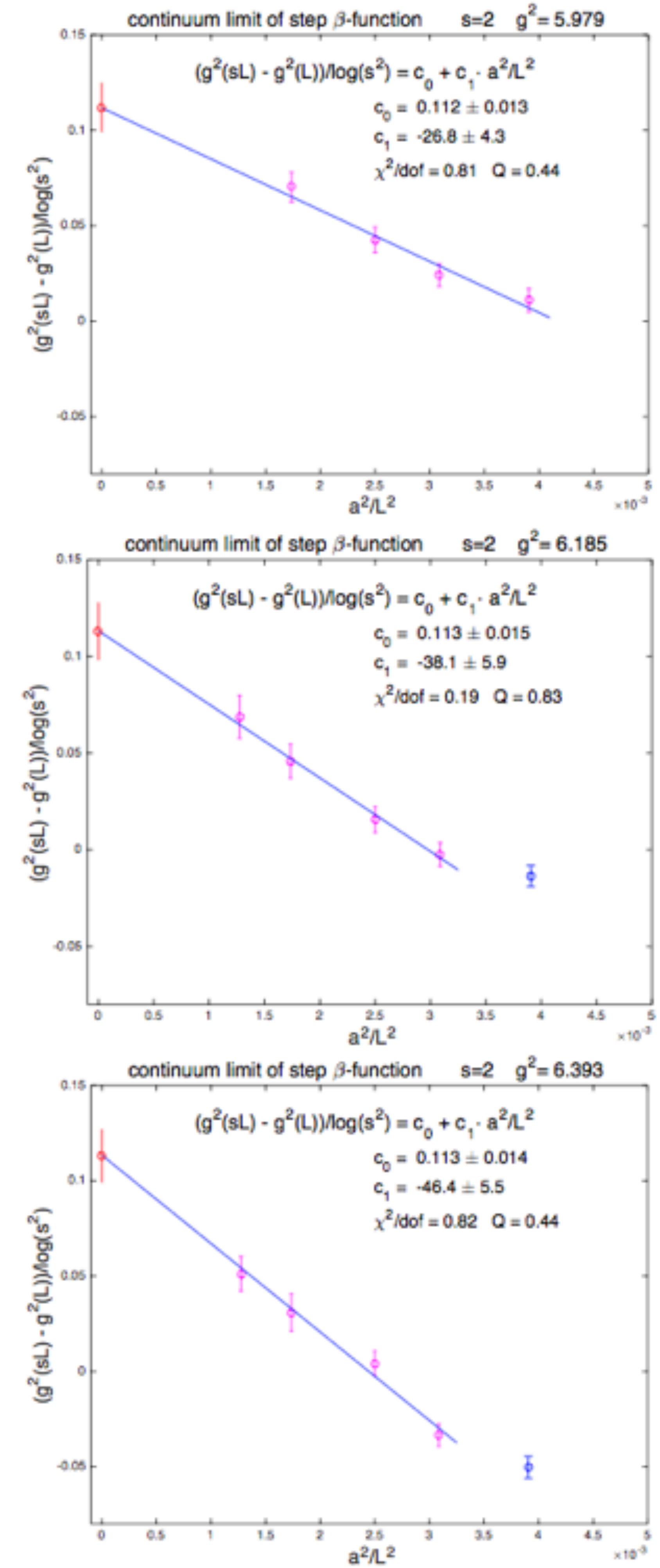
in our new Nf=12 work:

- interpolations is eliminated by tuned targeting in the previously published range in [1]
- statistics with ‰ accuracy in the renormalized coupling
- large volumes are used for credible continuum extrapolation



	Target A		Target B		Target C	
L/a	$6/g_0^2$	g^2	$6/g_0^2$	g^2	$6/g_0^2$	g^2
16	3.1519	5.9801(29)	3.0830	6.1786(39)	3.0110	6.3930(30)
32	3.1519	5.9952(79)	3.0830	6.1597(64)	3.0110	6.3233(74)
18	3.1510	5.9767(40)	3.0785	6.1871(37)	3.0055	6.3909(51)
36	3.1510	6.0101(71)	3.0785	6.1840(81)	3.0055	6.3446(64)
20	3.1499	5.9828(64)	3.0704	6.1922(64)	2.9896	6.3942(59)
40	3.1499	6.0419(73)	3.0704	6.2137(67)	2.9896	6.4000(67)
24	3.1480	5.9784(68)	3.0680	6.1861(55)	2.9800	6.3976(60)
48	3.1480	6.0758(84)	3.0680	6.2497(109)	2.9800	6.4404(122)
28			3.0698	6.1839(58)	2.9819	6.3900(37)
56			3.0698	6.2792(142)	2.9819	6.4610(124)

TABLE I. The final 28 runs are tabulated with 14 tuned runs and 14 paired steps.



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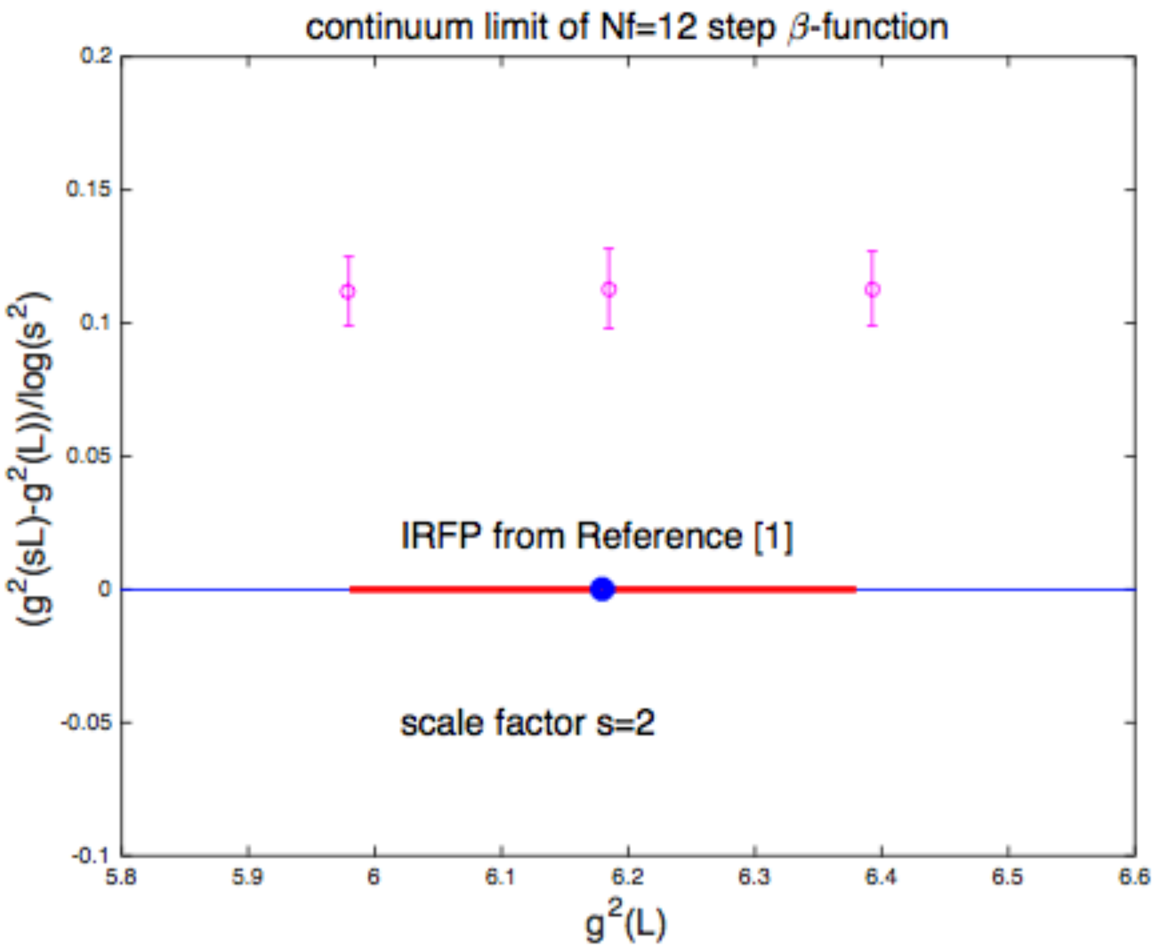
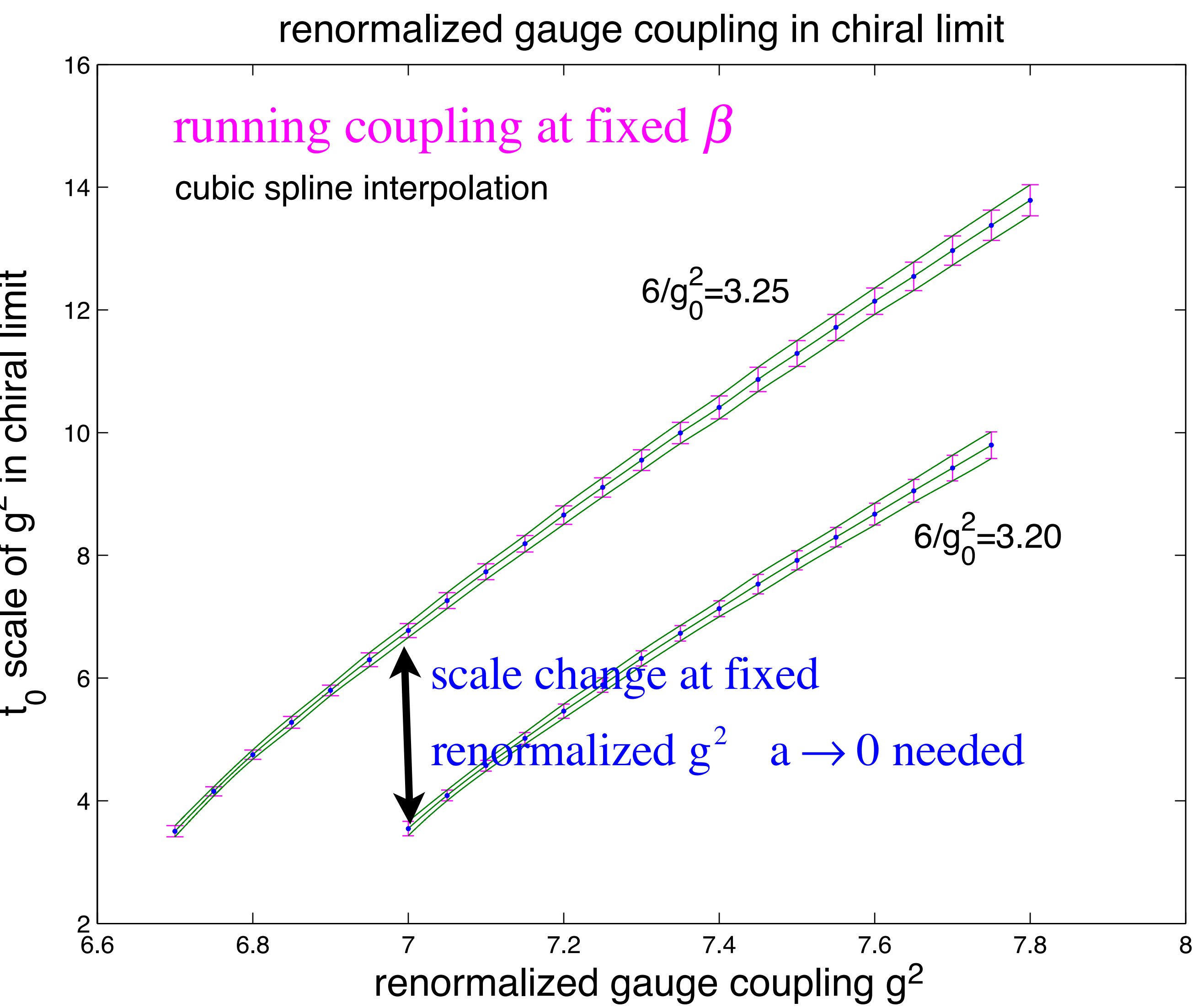
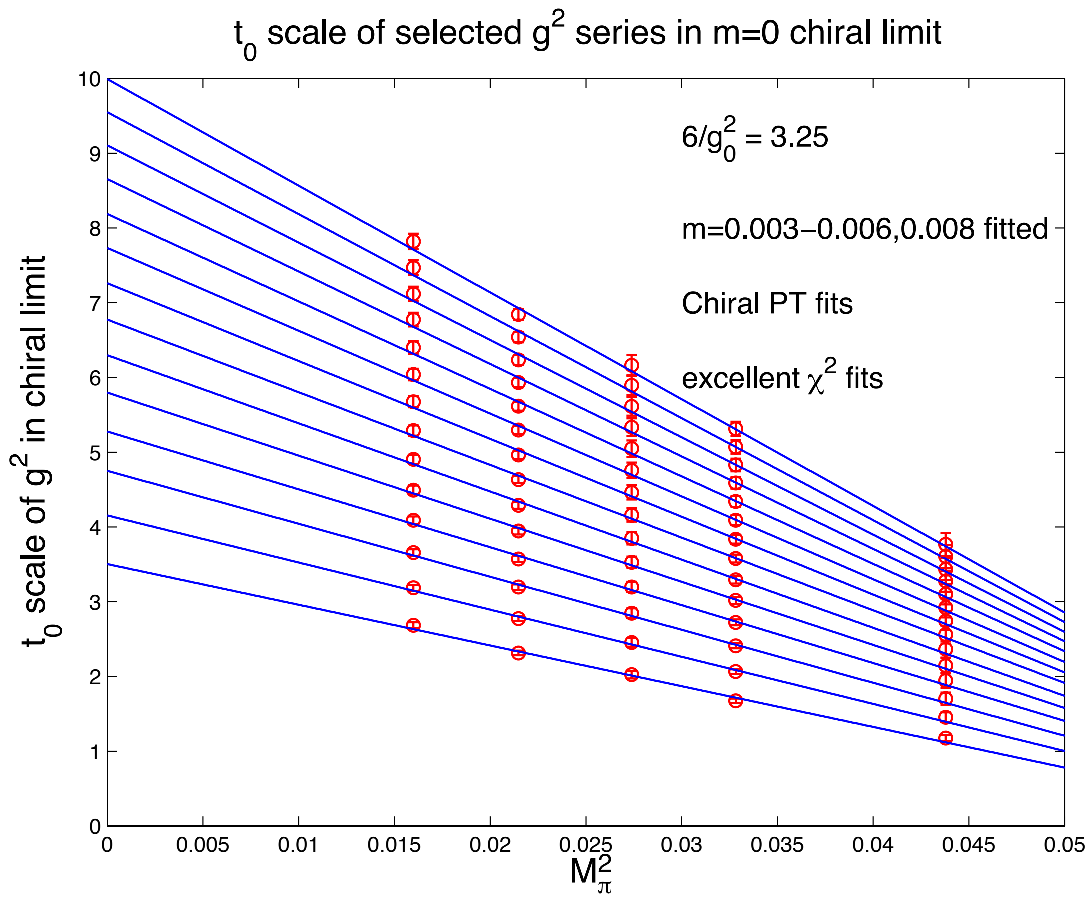


scale-dependent coupling of lattice BSM model

bridge between UV scale and IR scale





the two scale dependent couplings to be matched to leave no room for further speculations on conformal fixed points

leading dependence of $g^2(t,m)$ on M_π^2 is linear

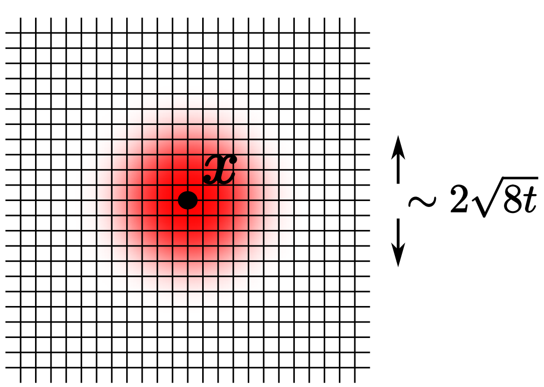
based on gradient flow chiPT Bär and Golterman

works better than expected

chiral logs are not detectable

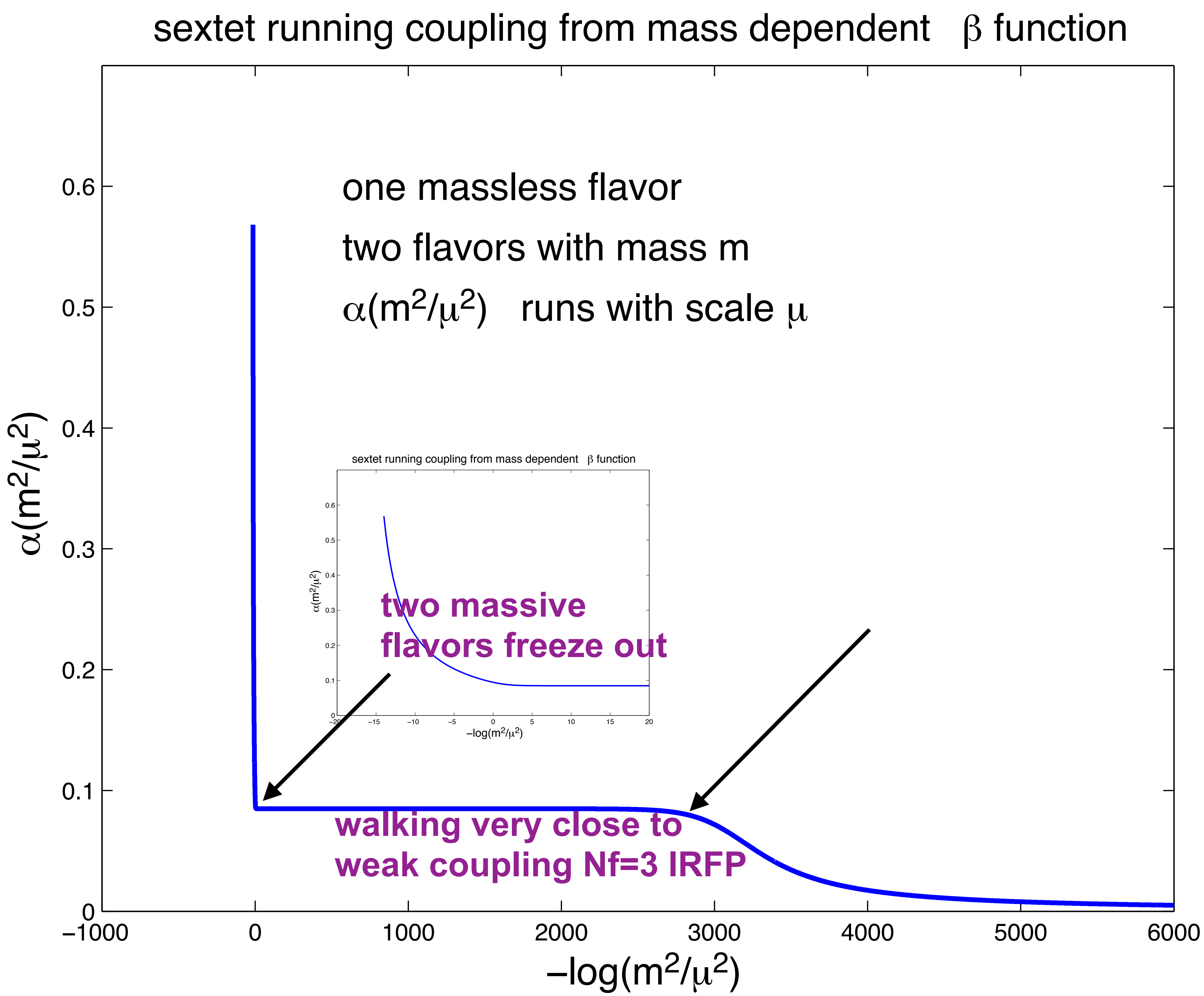
decoupling of the scalar has

to be better understood



unsolved: how to do this right in ChiPT with low lying scalar coupled to Goldstone dynamics?

scale-dependent coupling   mass dependent tuning?



in 1+2 freeze-out scenario anything to learn about strong coupling dynamics of single massless flavor?

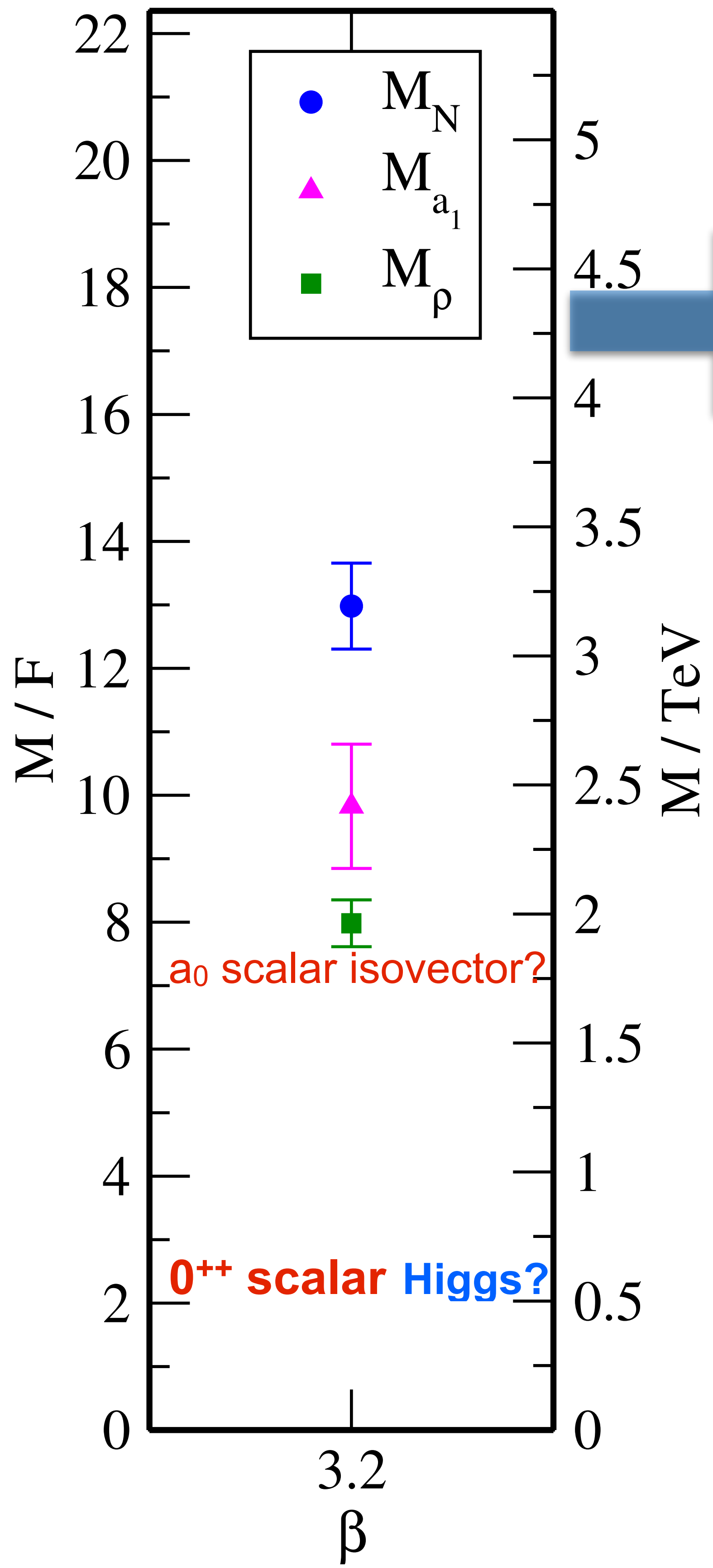
Similarly, in 2+1 freeze-out scenario anything to learn about strong coupling dynamics of doublet massless flavor?

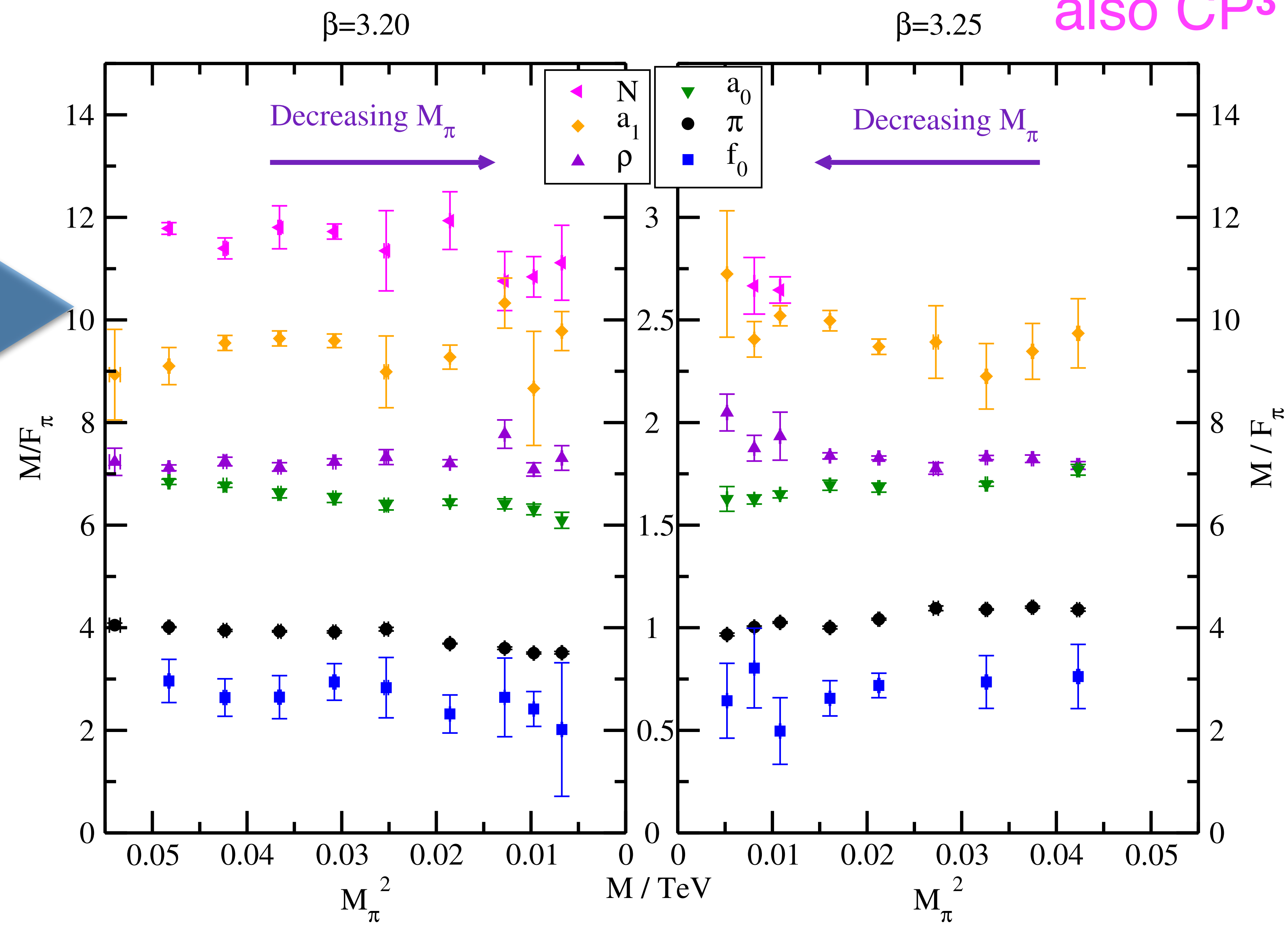
Not likely that light scalar mass can be tuned effectively helping non perturbative results

light 0++ scalar and spectrum

sextet model LatHC

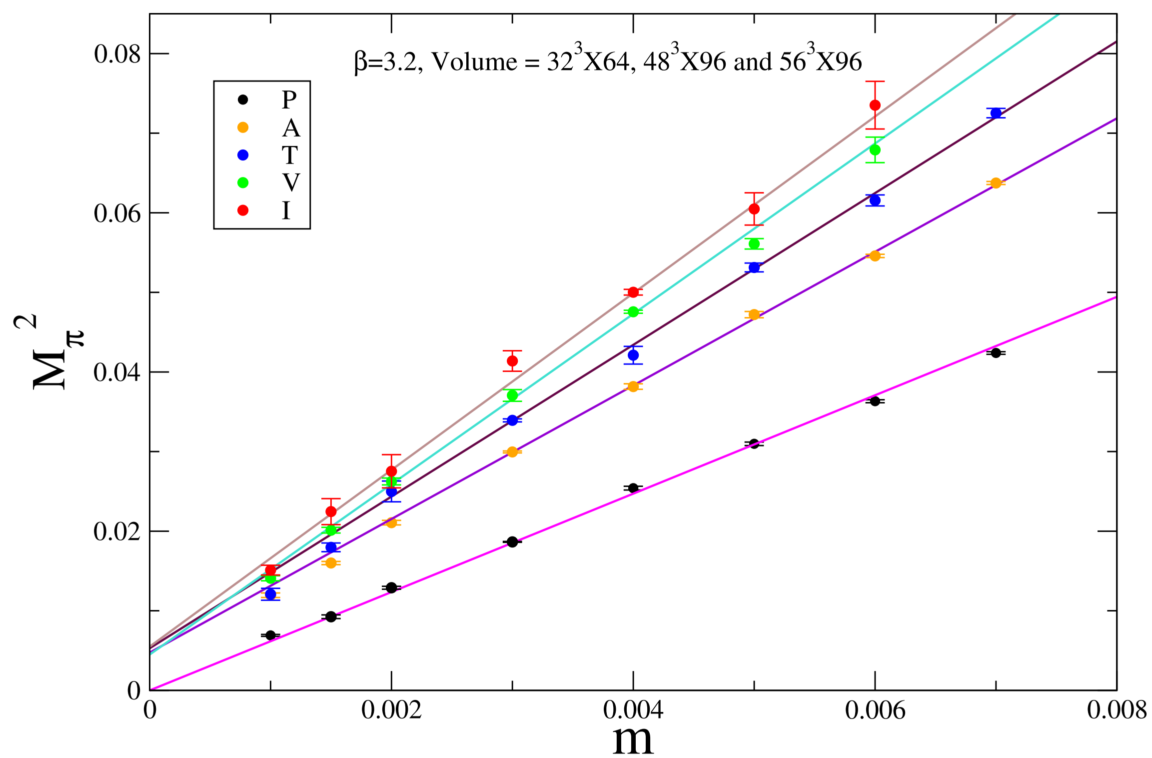
also CP3

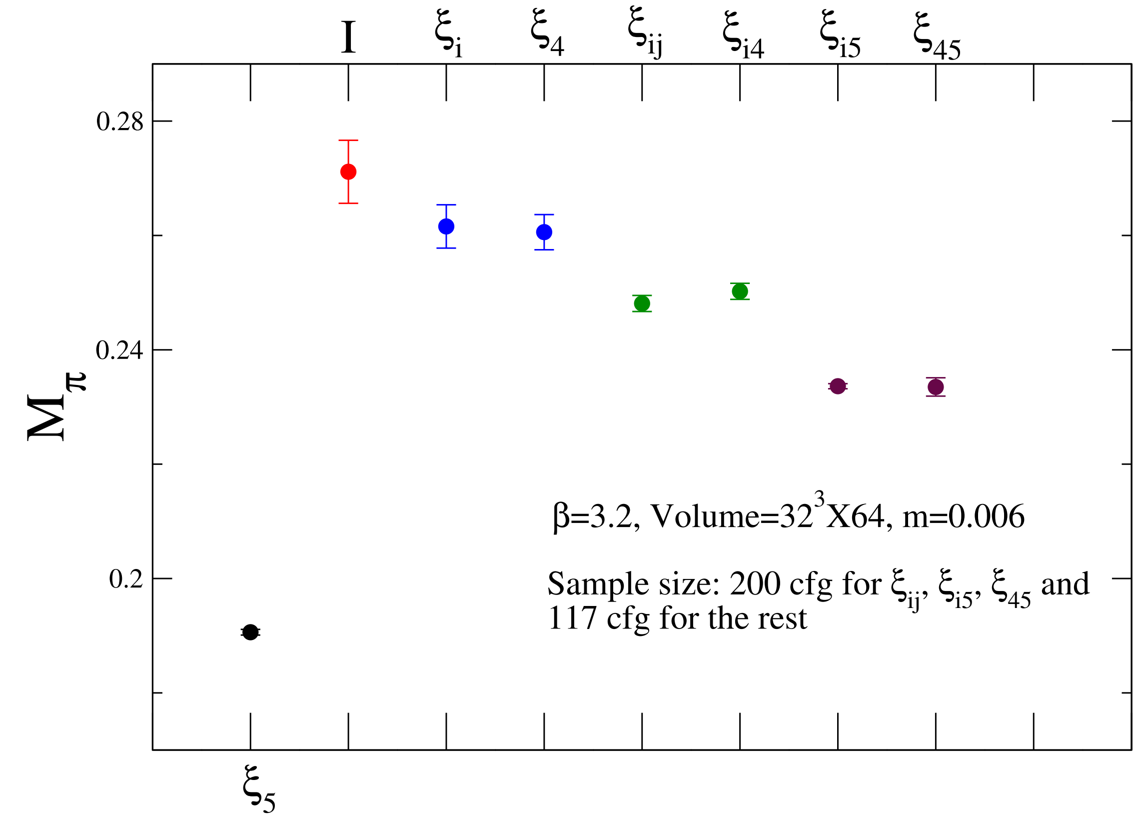


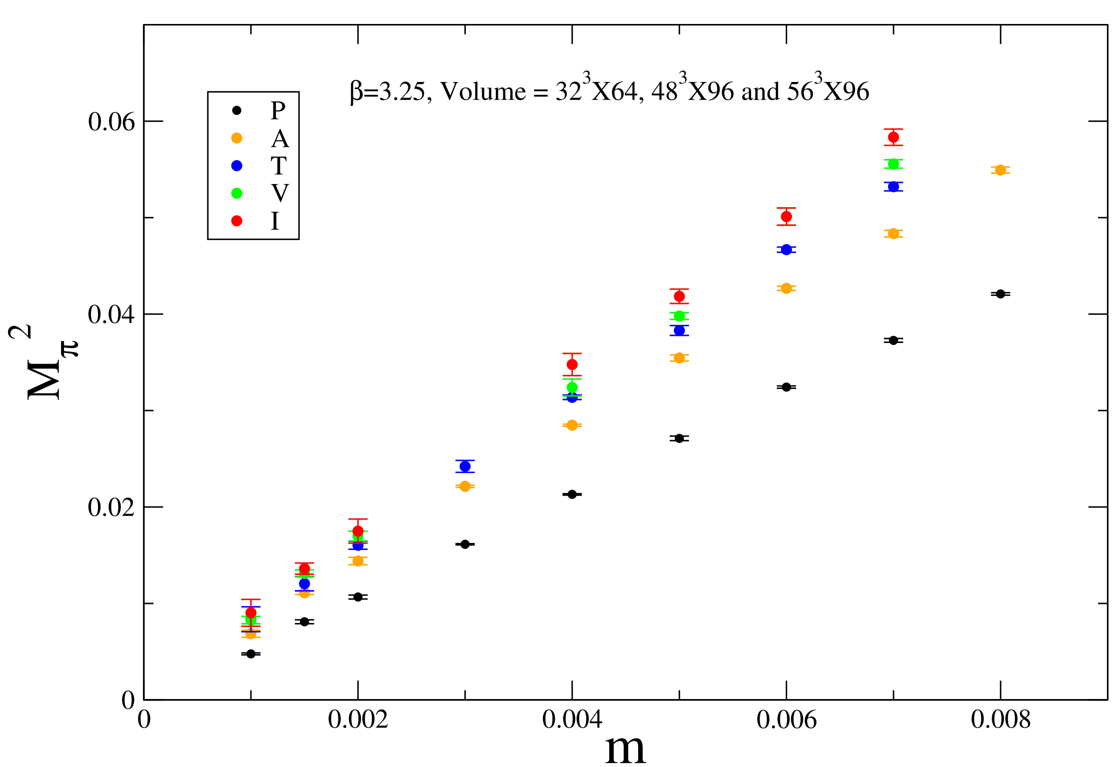


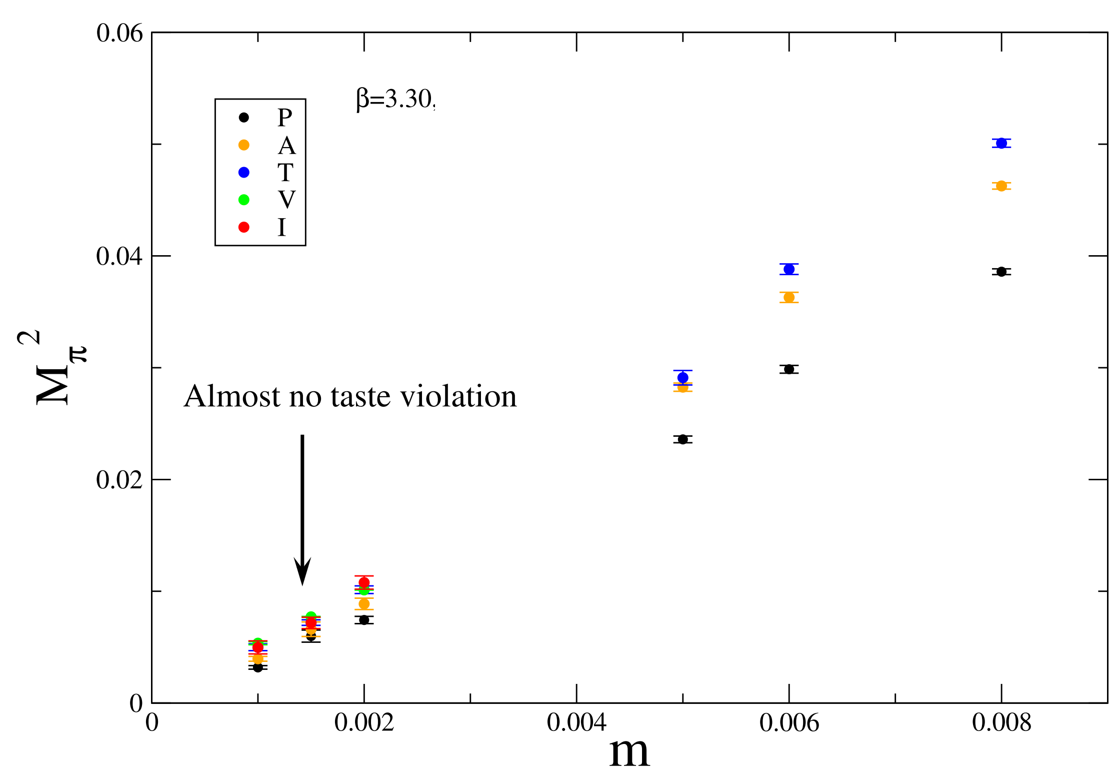
does not fit hyperscaling

Goldstone spectrum, lattice scale, chiPT

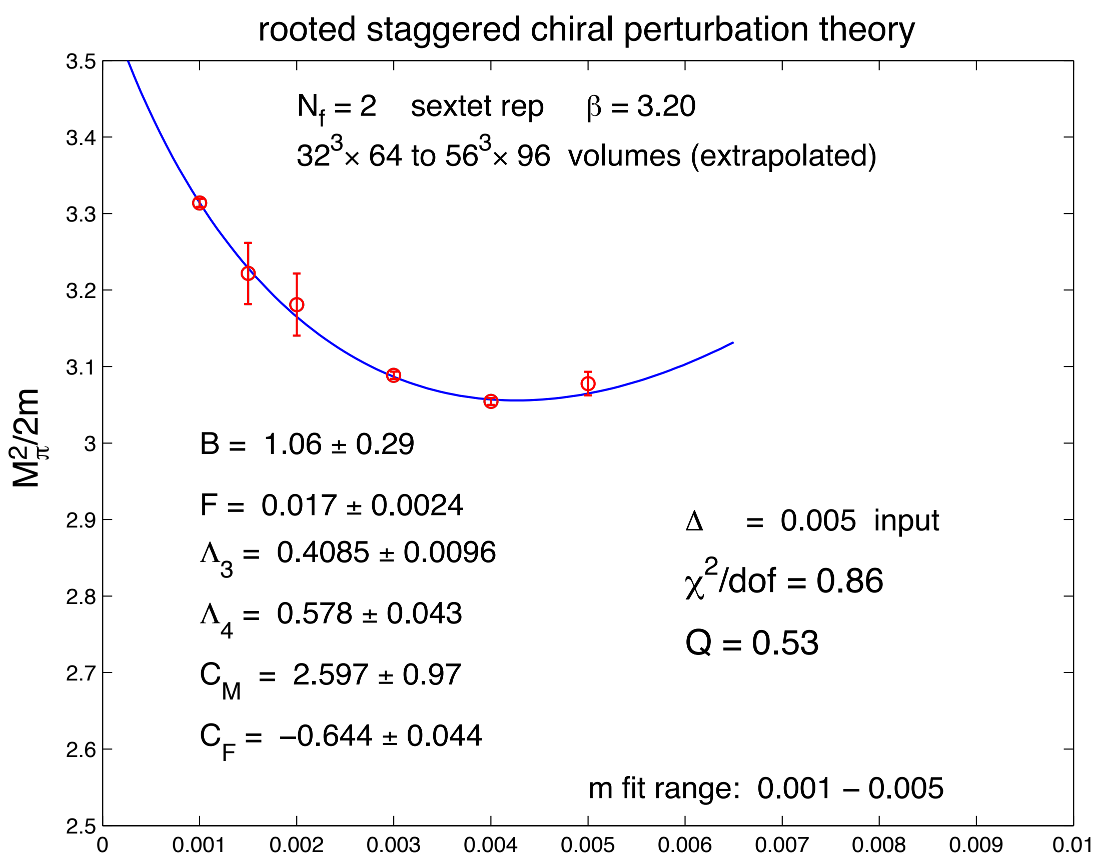


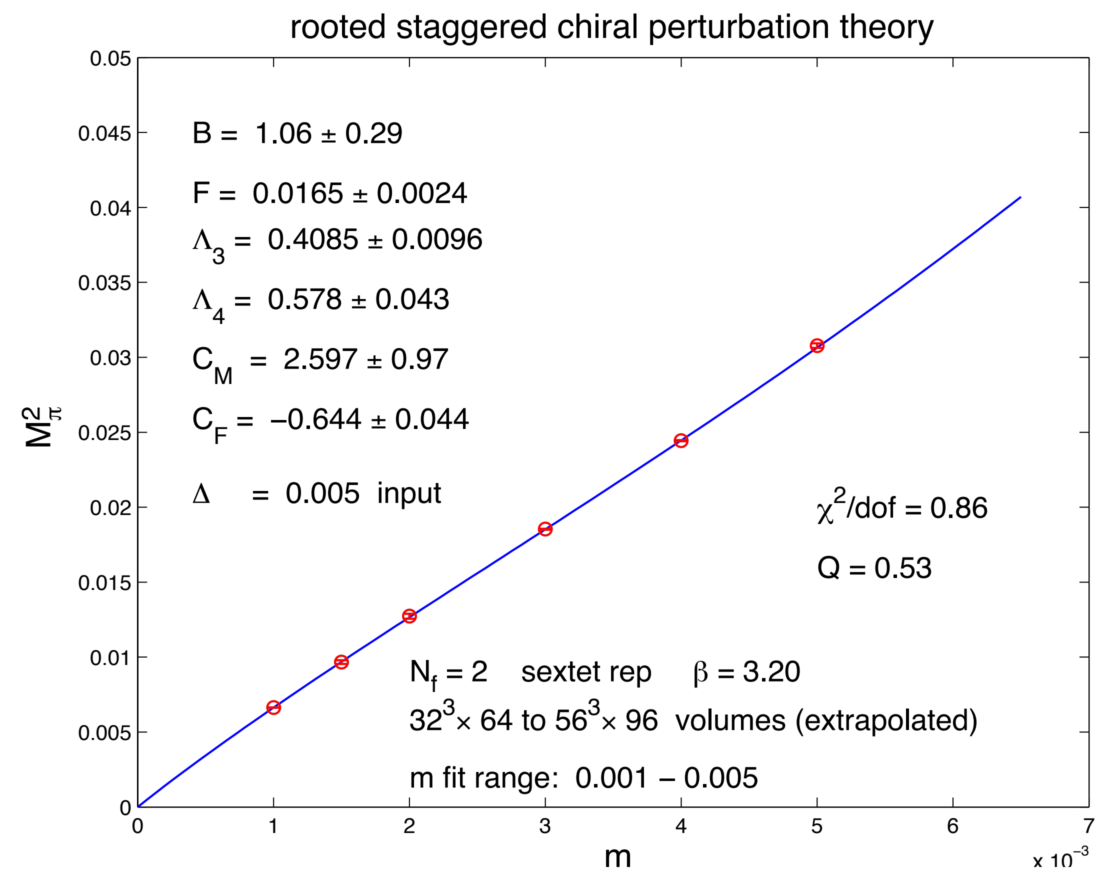


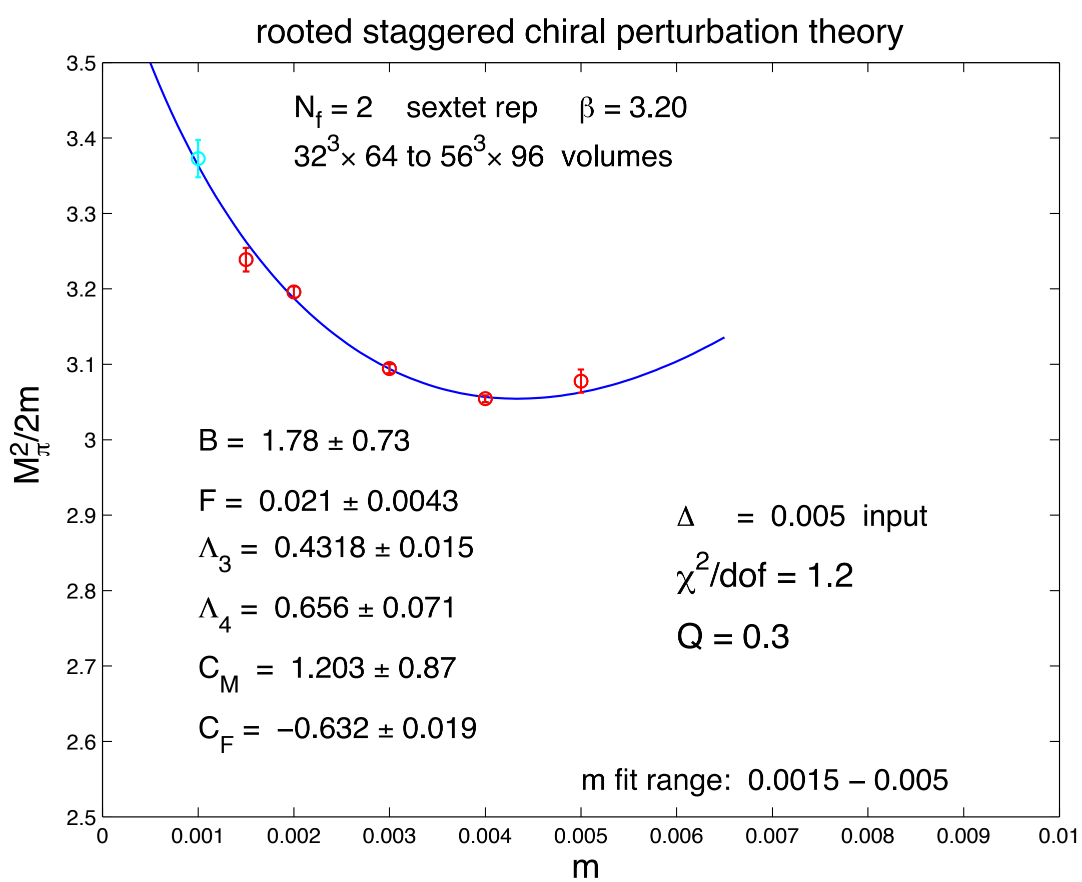


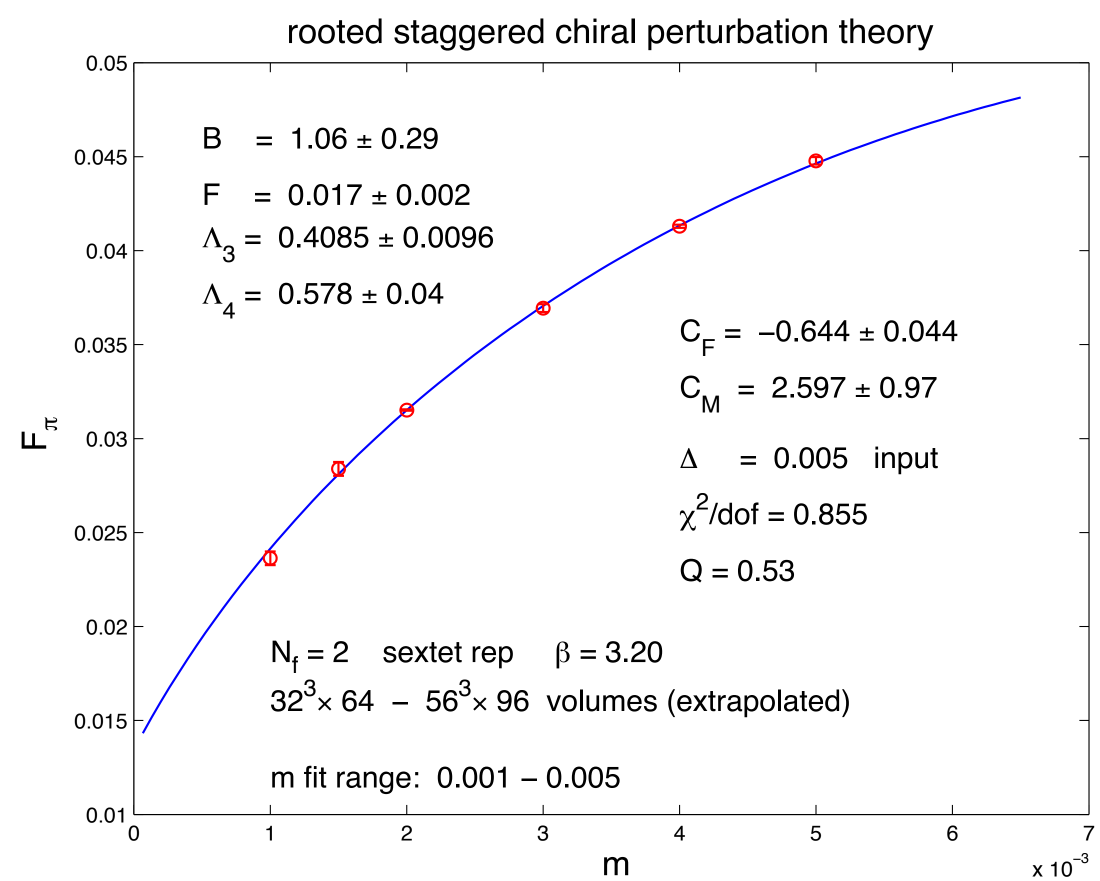


rsChiPT analysis of Mpi and Fpi fitting results

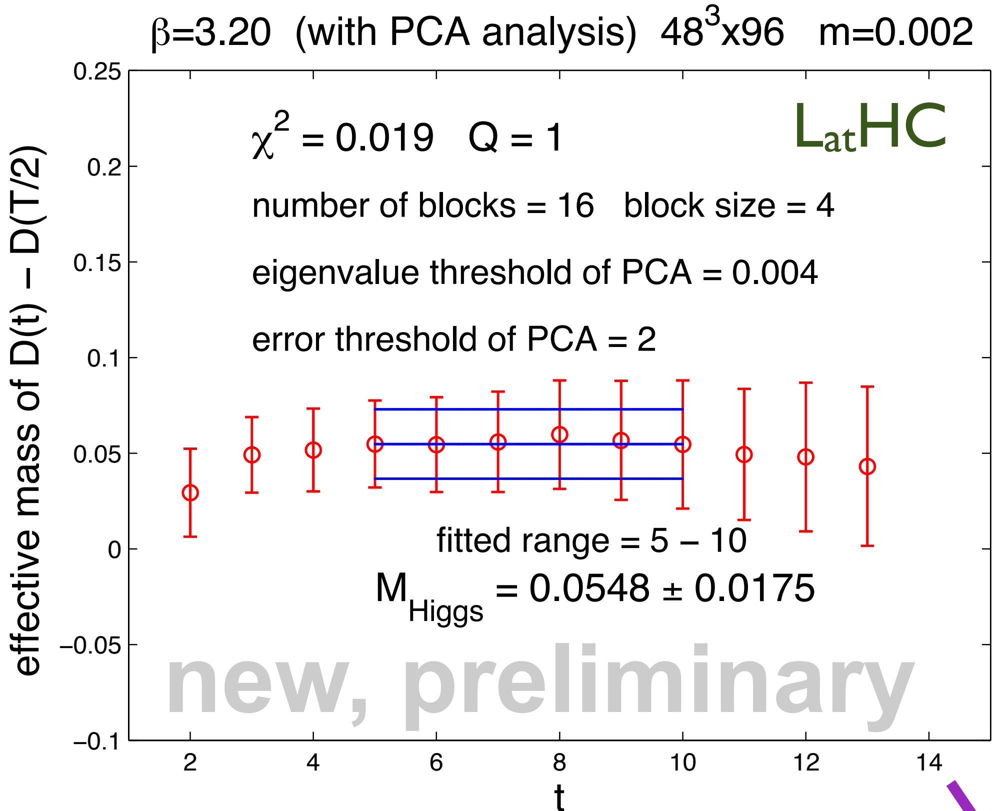


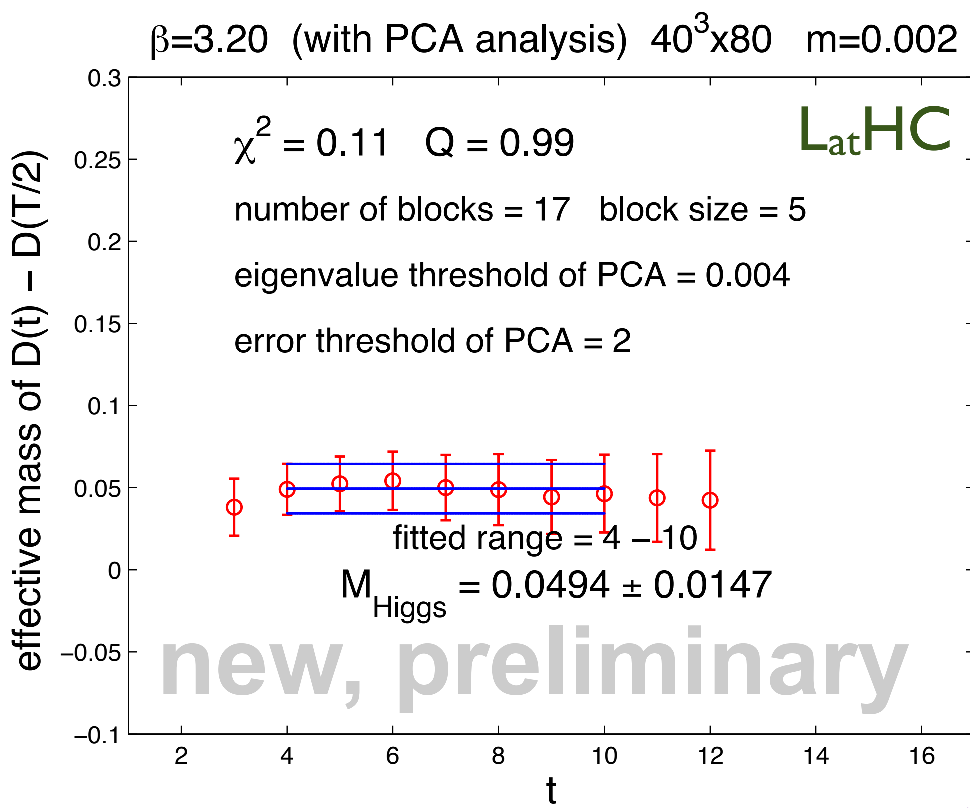


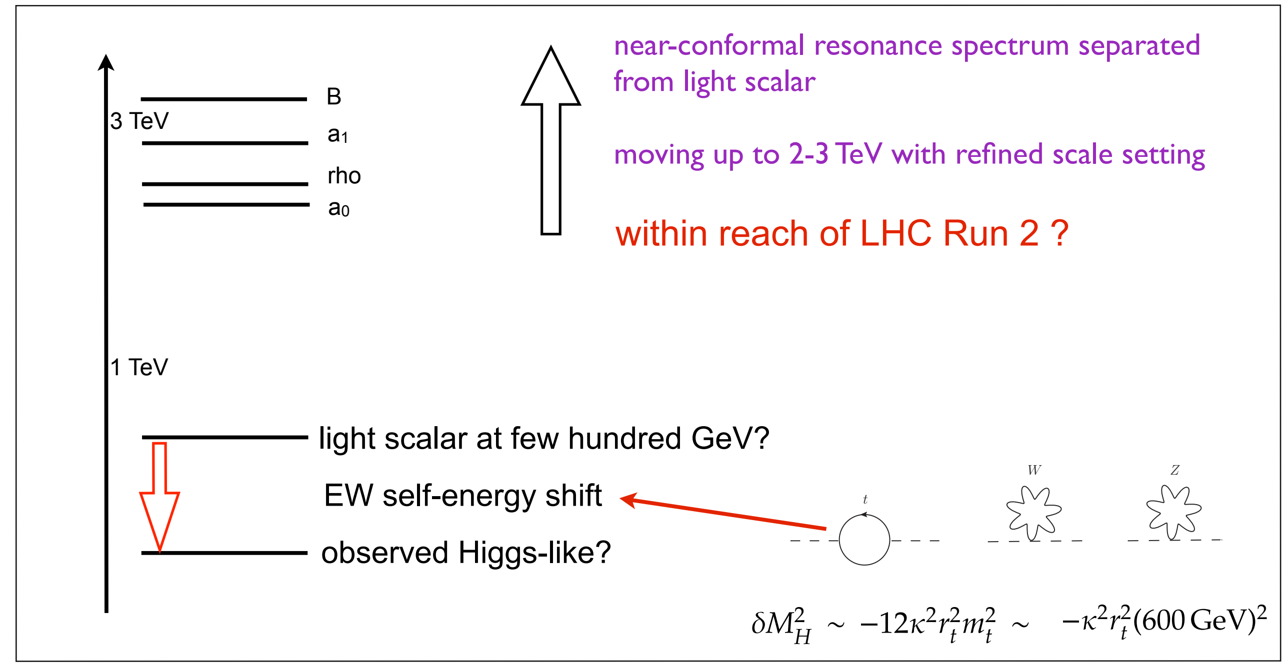


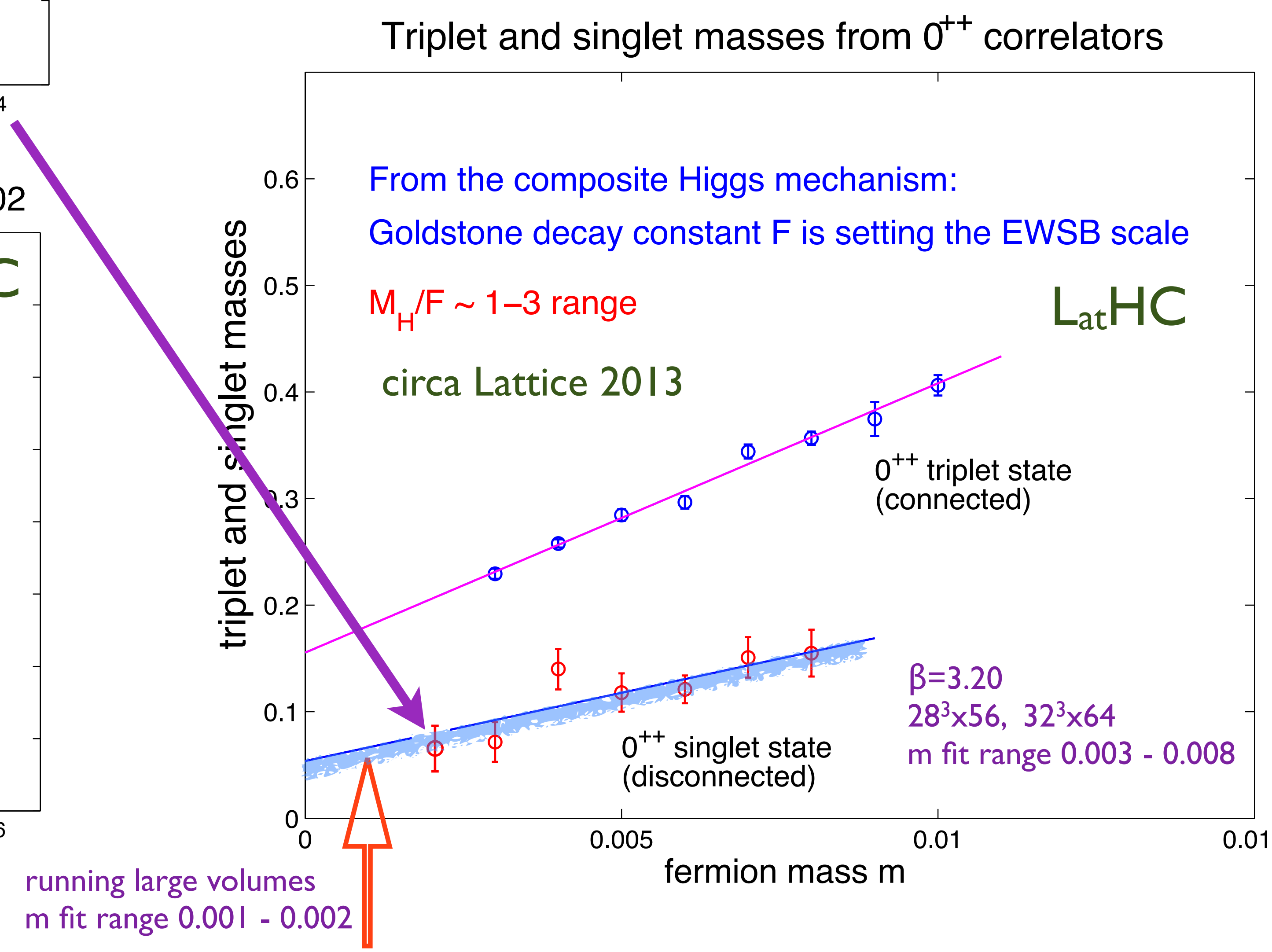


light 0++ scalar and spectrum   sextet model   LatHC









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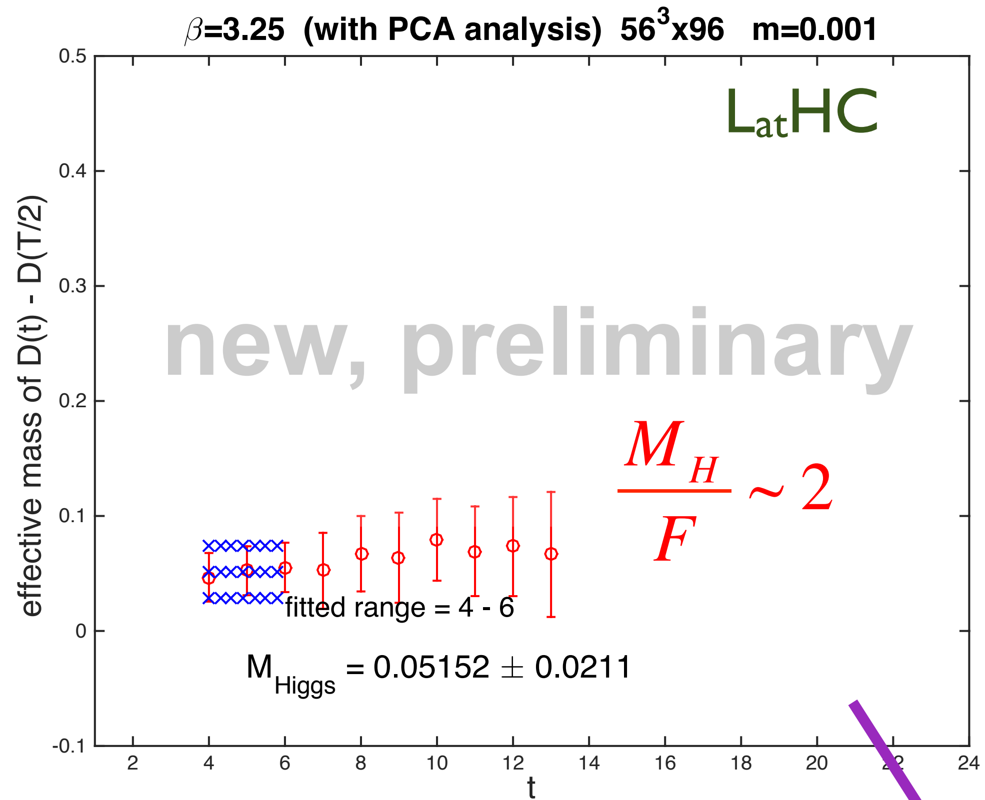
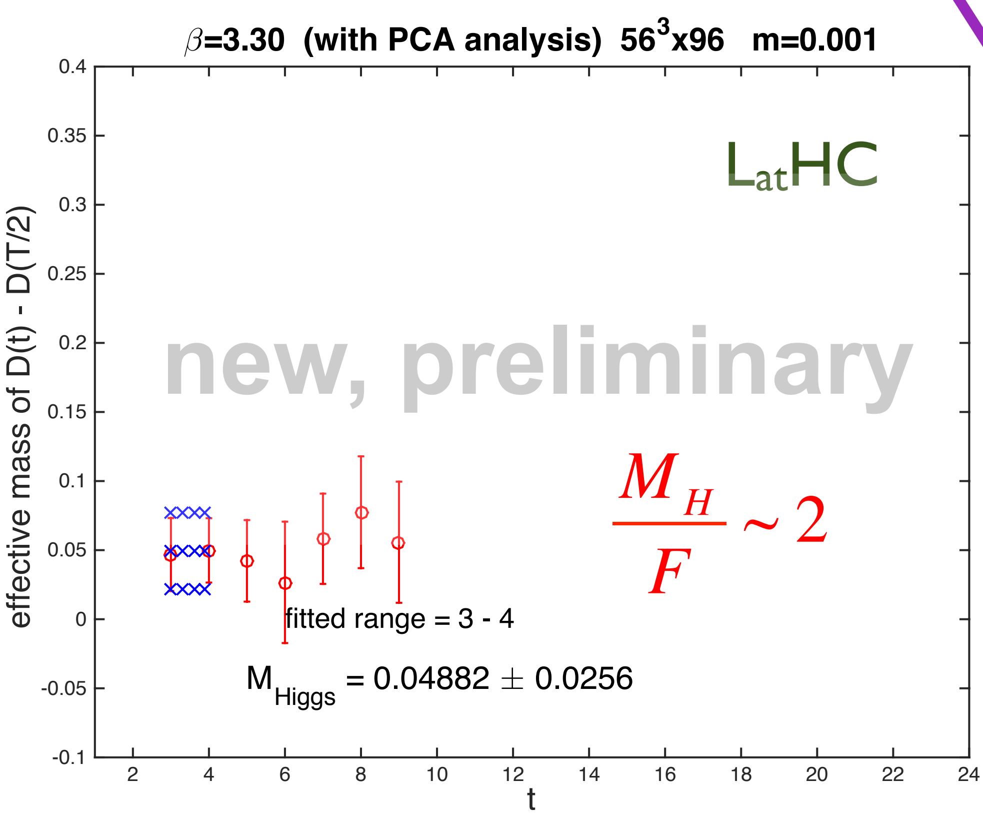
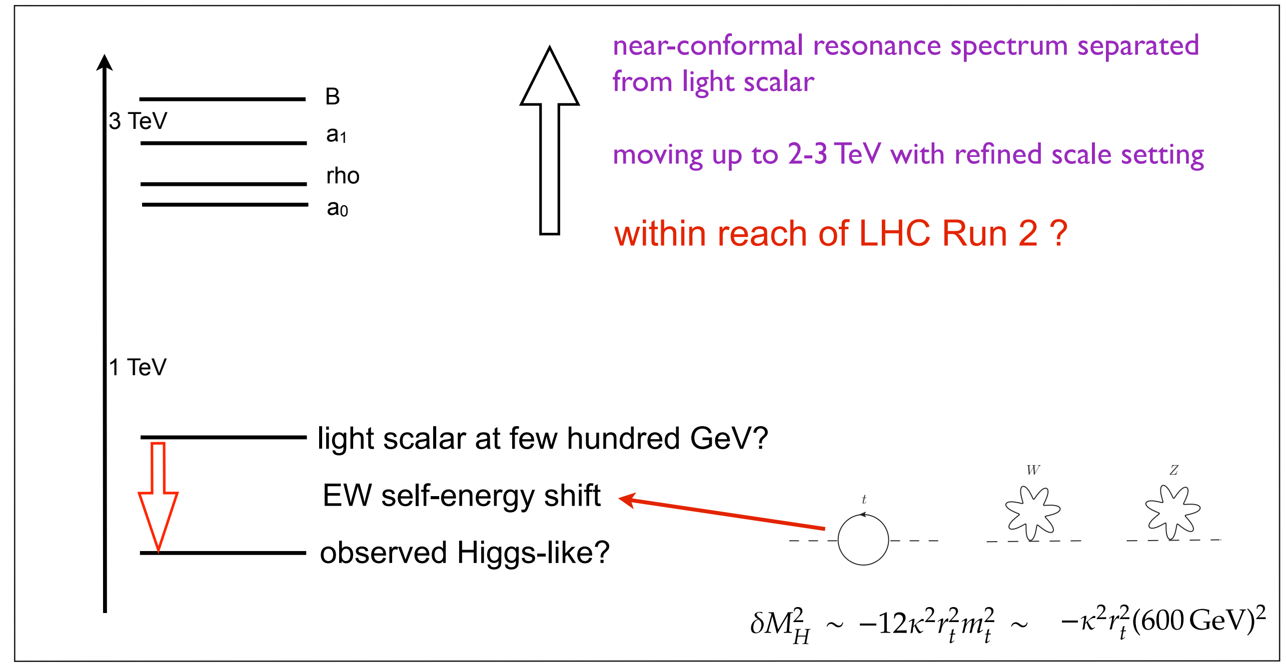
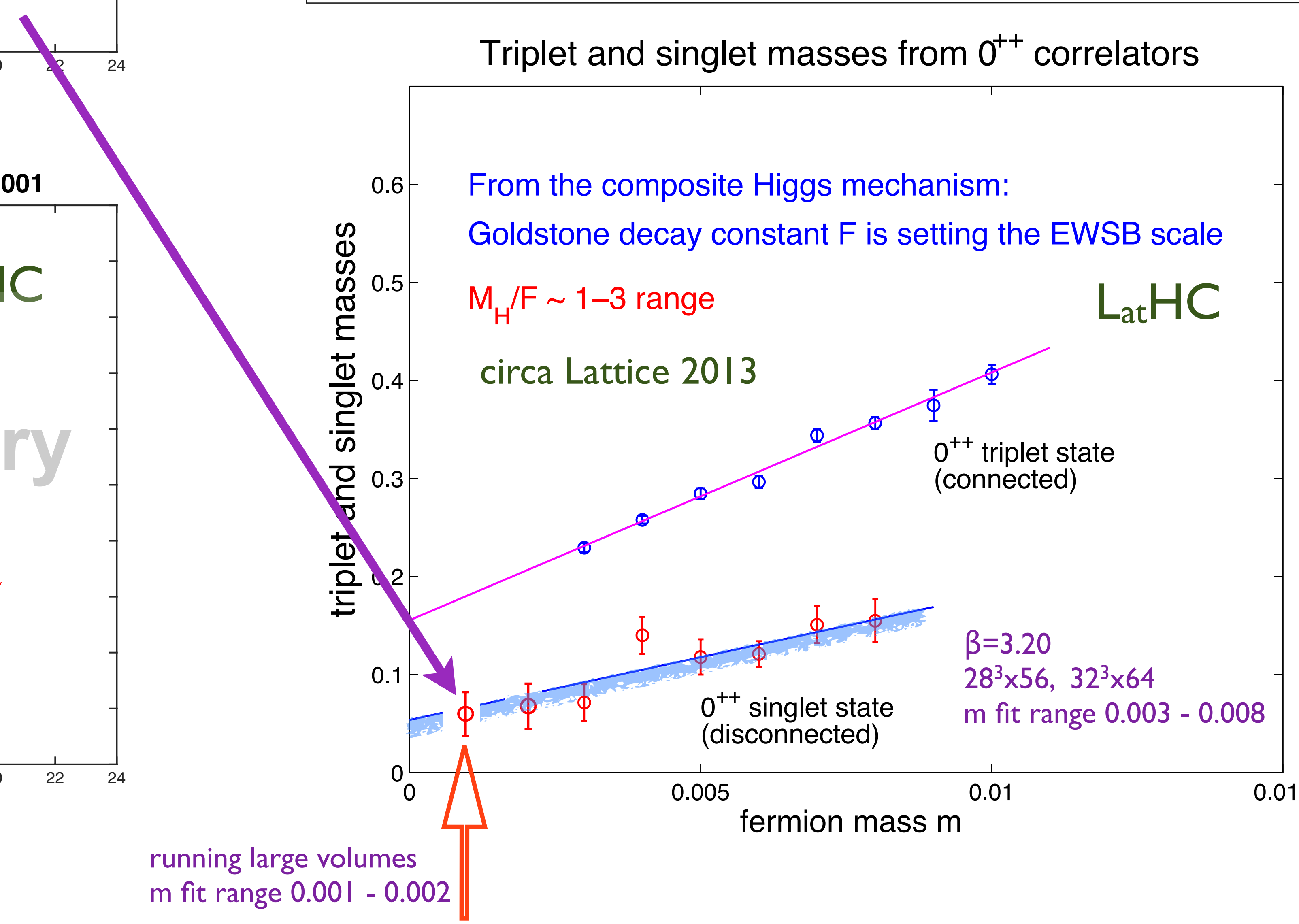








some outstanding spectroscopy problems:

1. effective low energy theory for Goldstone dynamics coupled to the low mass scalar

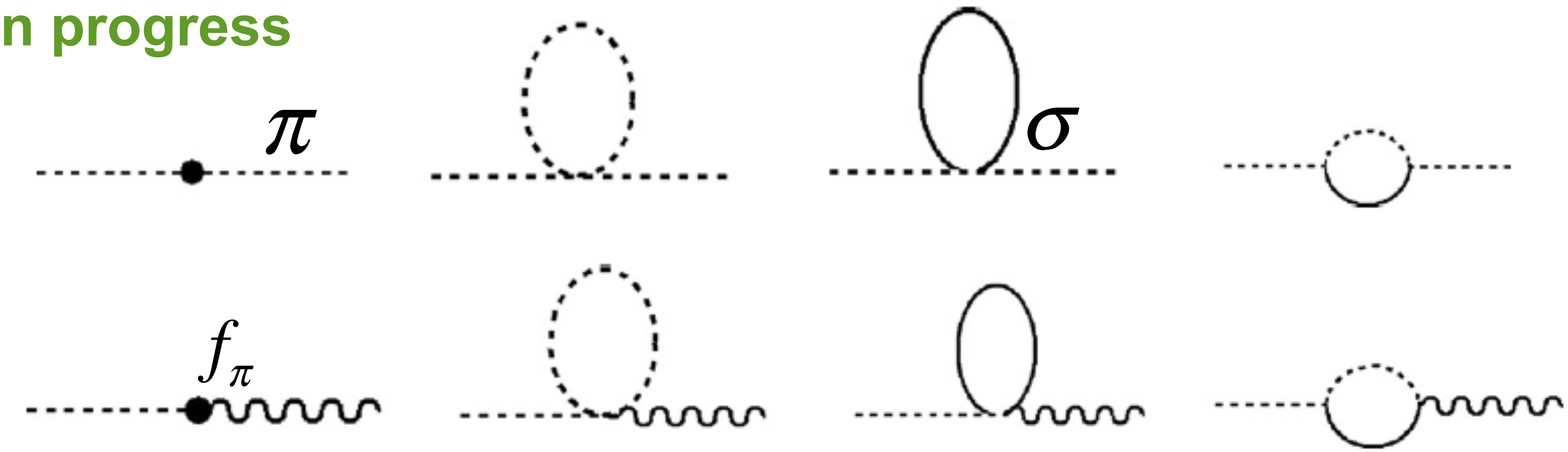
p-regime: nonlinear sigma model or dilaton?

crossover from p-regime to epsilon regime and RMT will be more effective in decoupling the light scalar

2. effect of slow topology on the analysis
ChiPT at fixed topology?

Goldstone dynamics coupled to low mass scalar

work in progress



$$\mathcal{L} = \frac{1}{2}(\partial_\mu h)^2 - V(h) + \frac{v^2}{4}\mathrm{Tr}\left(D_\mu\Sigma^\dagger D^\mu\Sigma\right)\left[1 + 2a\frac{h}{v} + b\frac{h^2}{v^2} + \ldots\right] \qquad \Sigma = e^{i\sigma_a\pi^a/v}$$

$$V(h) = \frac{1}{2}m_h^2h^2 + d_3\frac{1}{6}\left(\frac{3m_h^2}{v}\right)h^3 + d_4\frac{1}{24}\left(\frac{3m_h^2}{v^2}\right)h^4 + \ldots$$

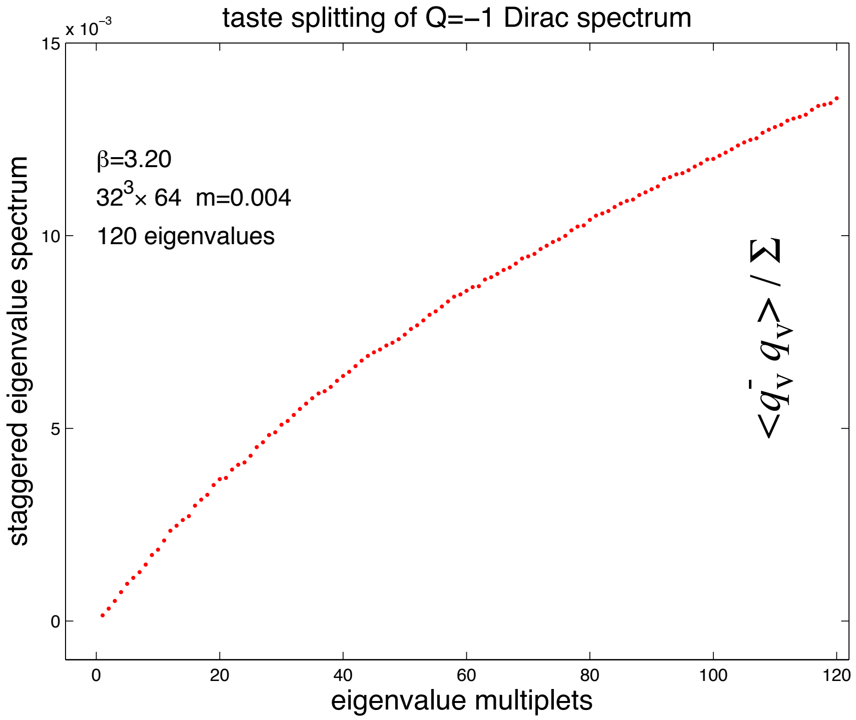
M_π, F_π, M_σ are calculated now to 1-loop: extended chiral SU(2) flavor dynamics

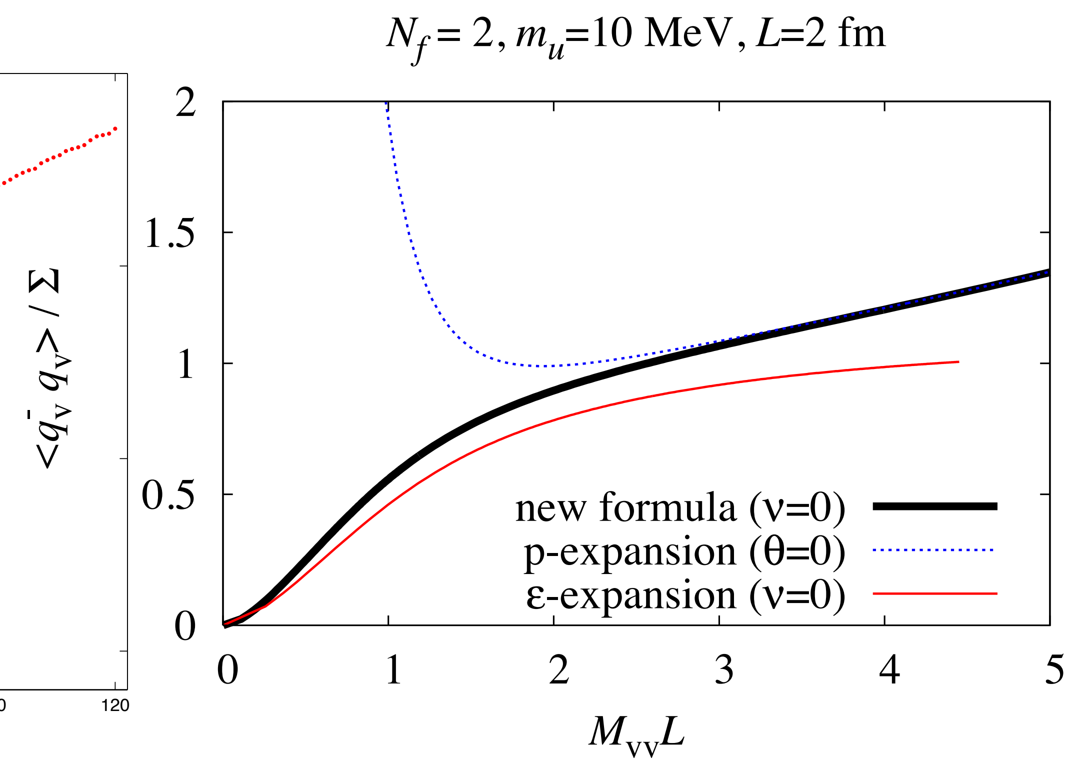
We are analyzing the small pion mass region in the M_π = 0.07- 0.013 range of the p-regime, and lower in the RMT regime

To reach the nonlinear sigma model range requires very small pion masses

how to differentiate from effective dilaton action?

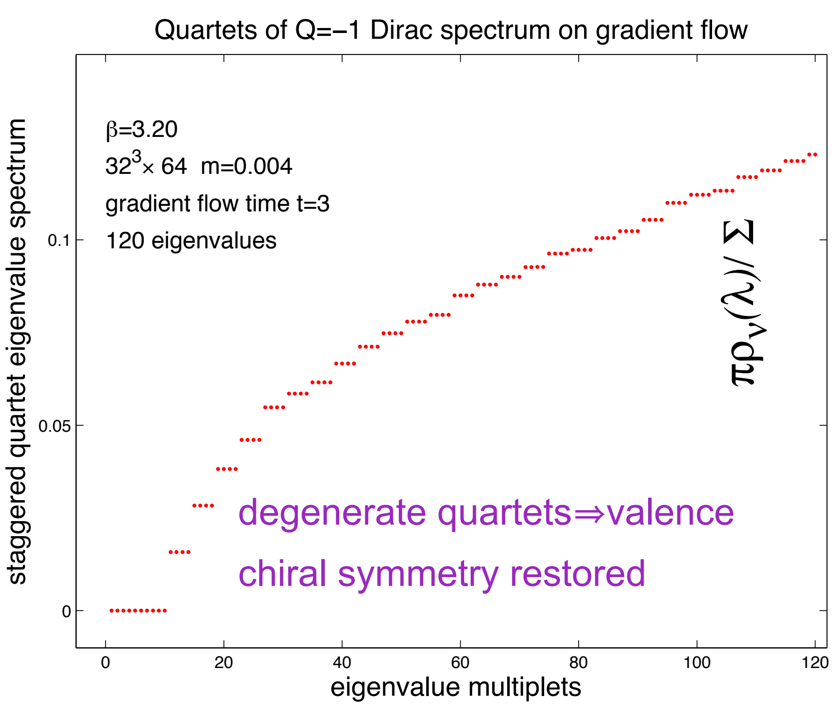
mixed action

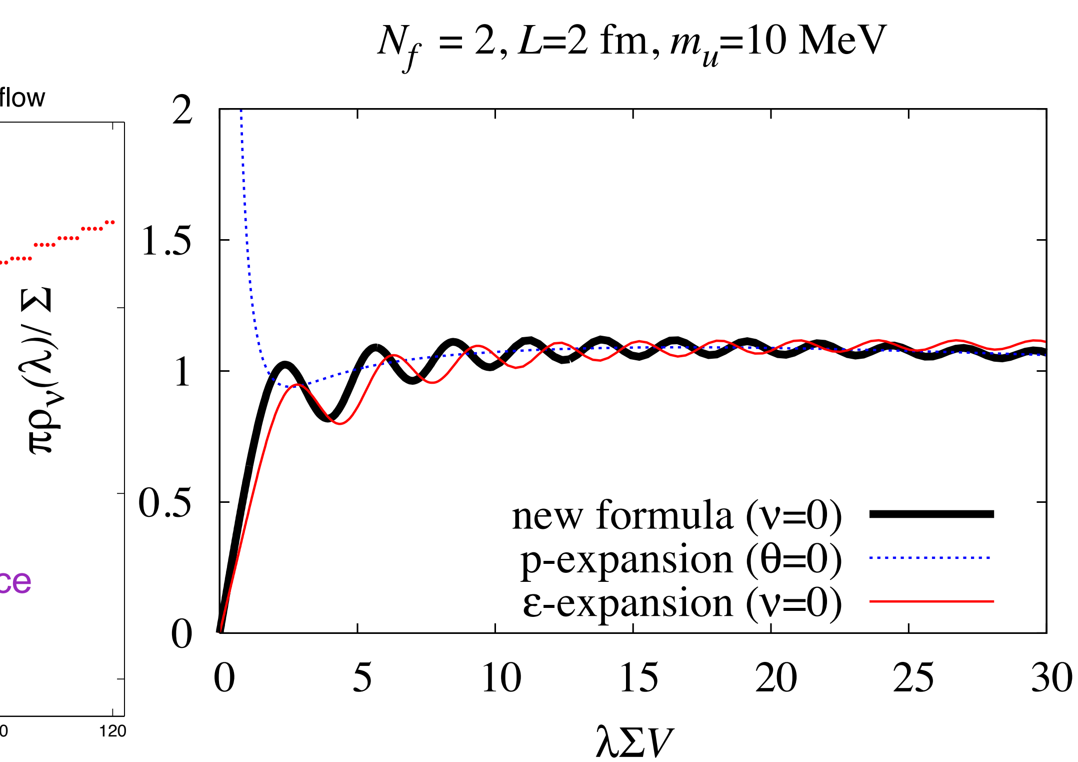


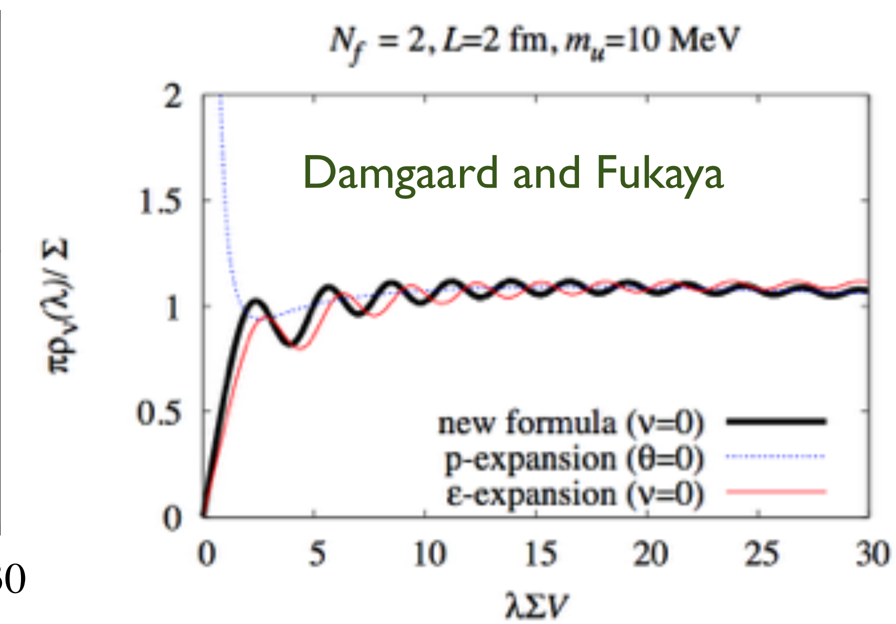


idea for improvement:

- use the gauge configurations generated with sea fermions
- taste breaking makes chiPT analysis complicated
- in the analysis use valence Dirac operator with gauge links on the gradient flow
- taste symmetry is restored in valence spectrum
- Mixed Action analysis should agree with original standard analysis when cutoff is removed: this is OK!

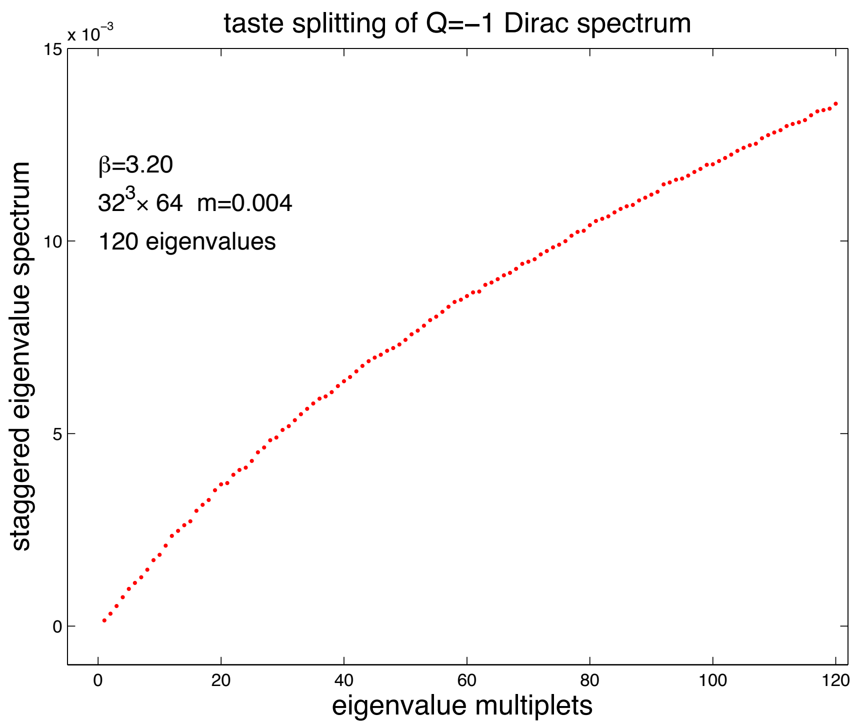


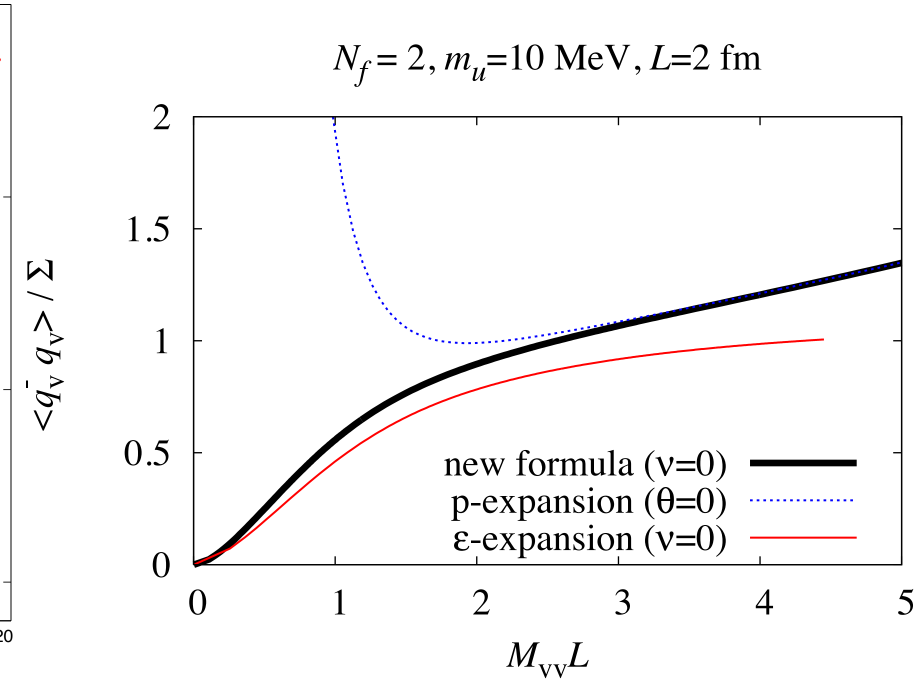


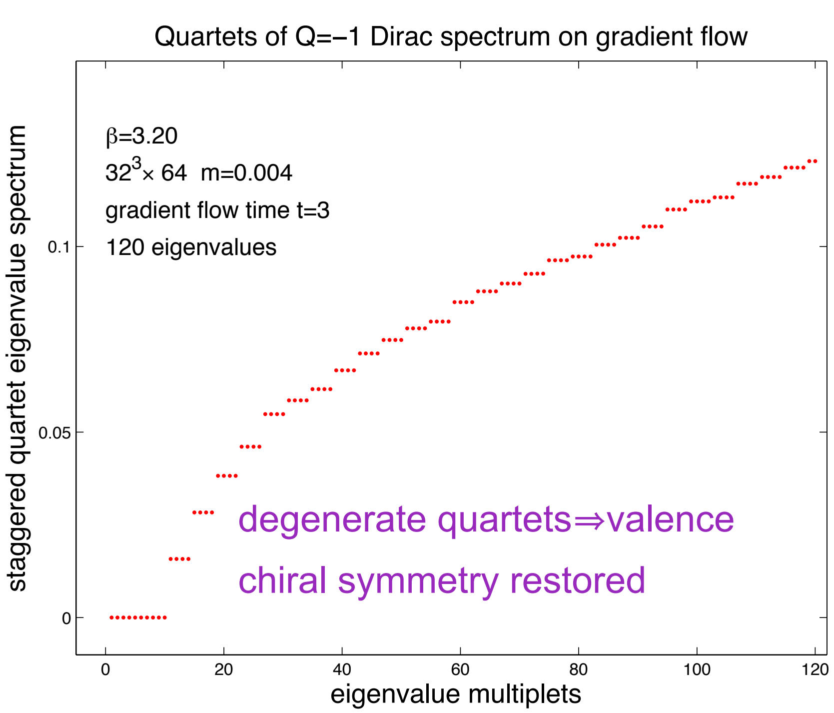


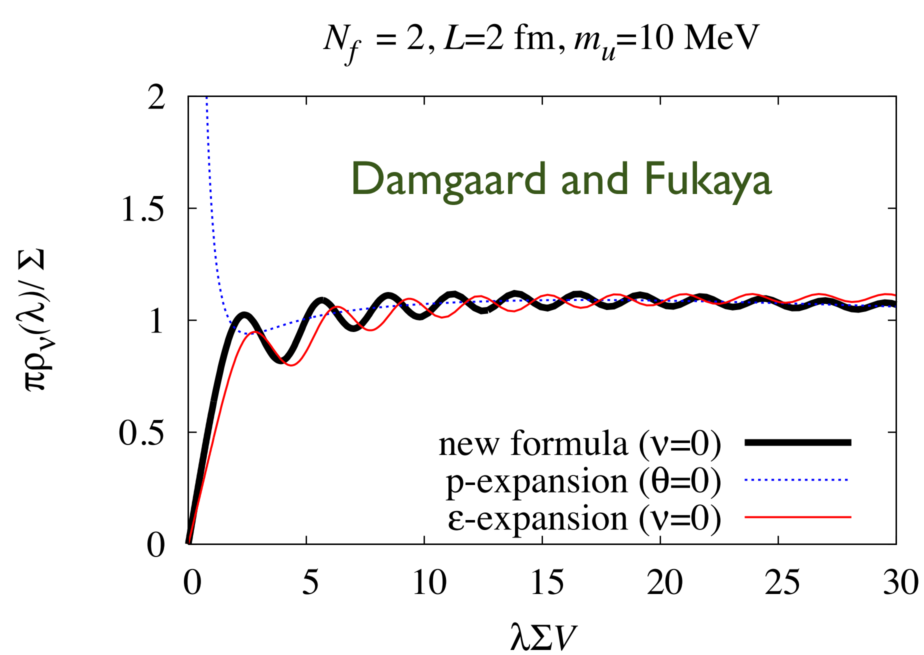
new analysis in crossover and RMT regime opens up with mixed action on gradient flow

mixed action









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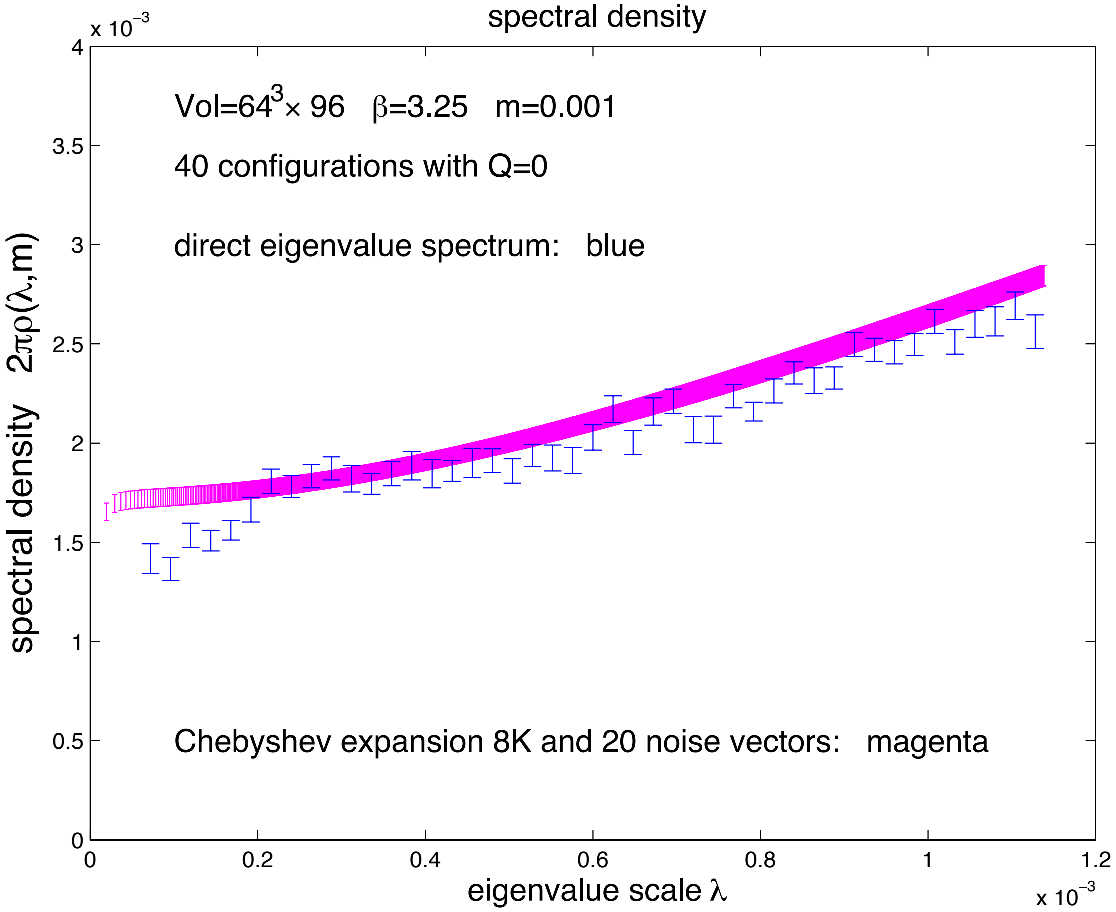
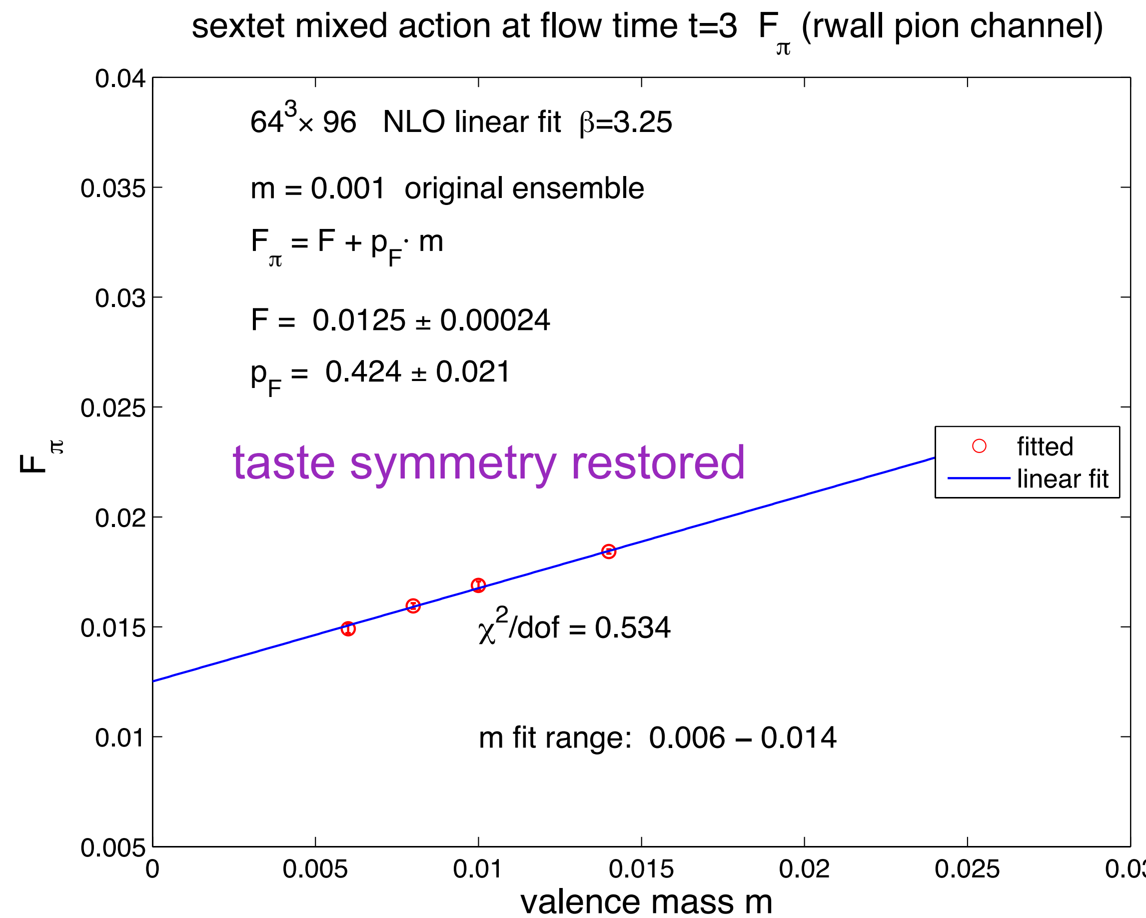
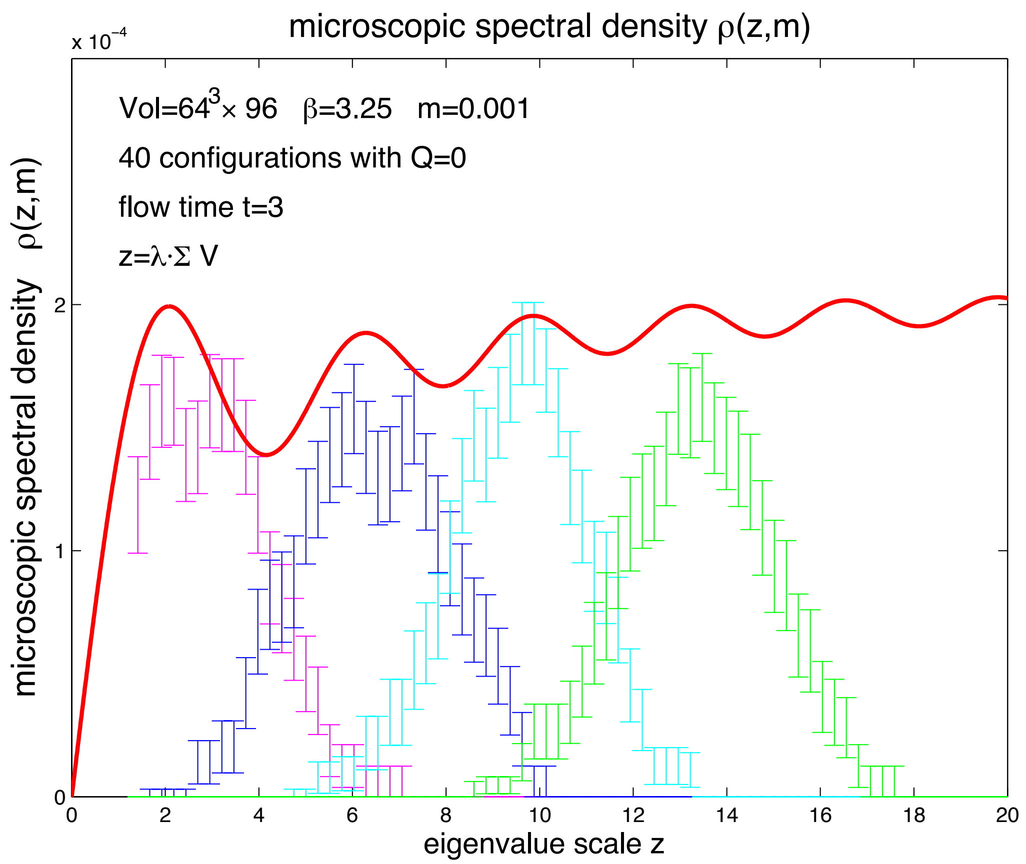
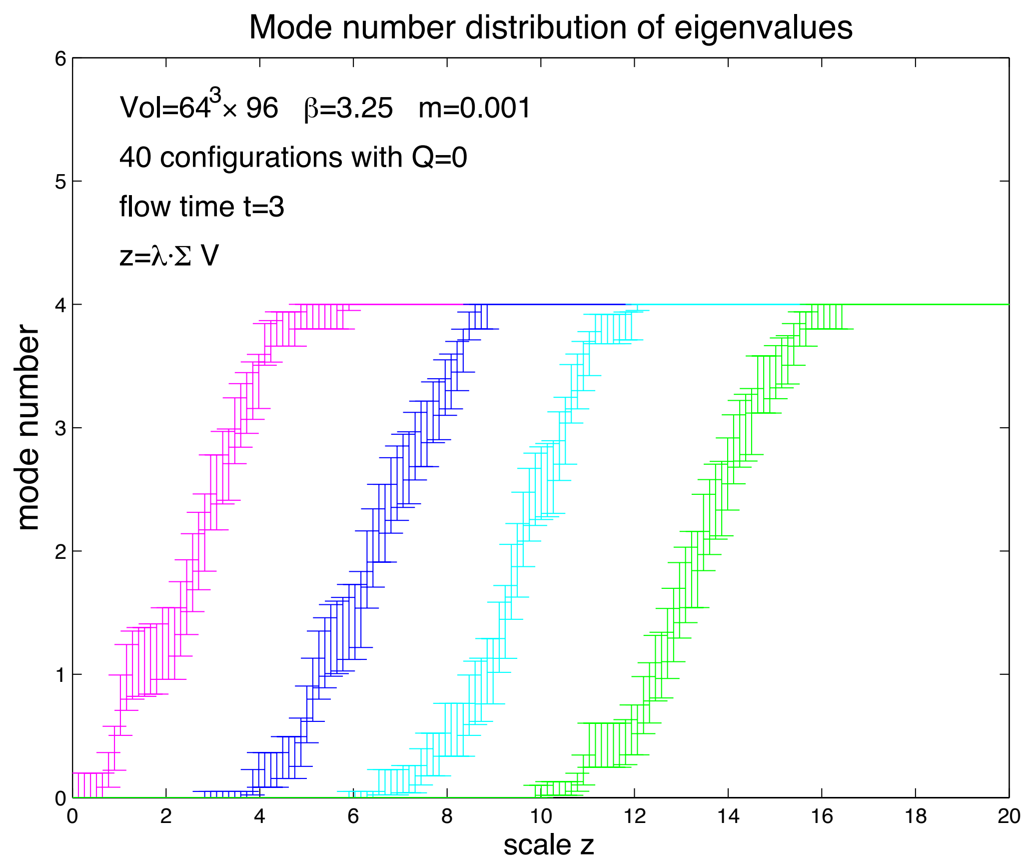
mixed action RMT regime



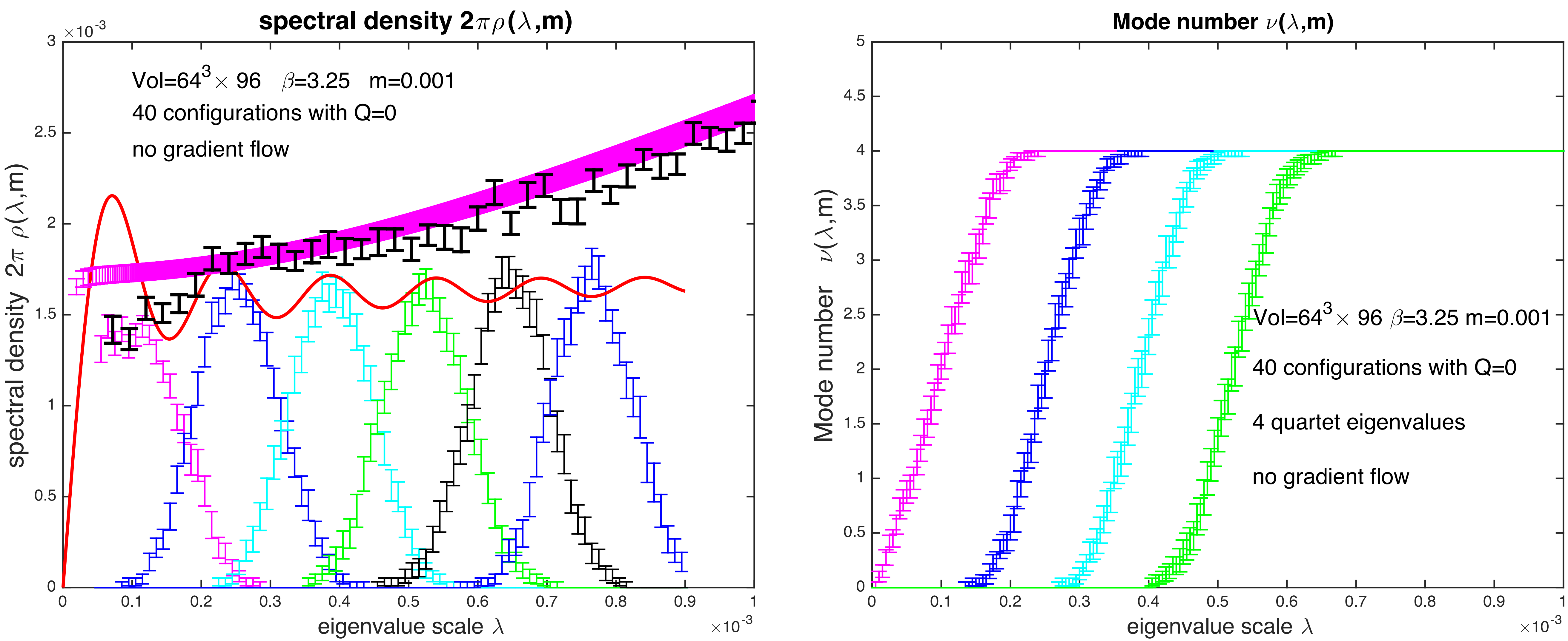






The chiral condensate RMT spectrum t=0

reached on original configurations without flow, or MA:



The chiral condensate Chebyshev expansion

chiral condensate and RG: mode number distribution of Dirac spectrum

$$\rho(\lambda,m) = \frac{1}{V}\sum_{k=1}^{\infty}\langle\delta(\lambda-\lambda_k)\rangle \qquad \lim_{\lambda\to 0}\lim_{m\to 0}\lim_{V\to\infty}\rho(\lambda,m) = \frac{\Sigma}{\pi}$$

spectral density (Banks-Casher)

$$\nu(M,m) = V\int_{-\Lambda}^{\Lambda} \mathrm{d}\lambda\,\rho(\lambda,m), \qquad \Lambda = \sqrt{M^2-m^2}$$

mode number function

$$\nu_{\rm R}(M_{\rm R},m_{\rm R}) = \nu(M,m\;\;)$$

renormalized and RG invariant

(Giusti and Luscher)

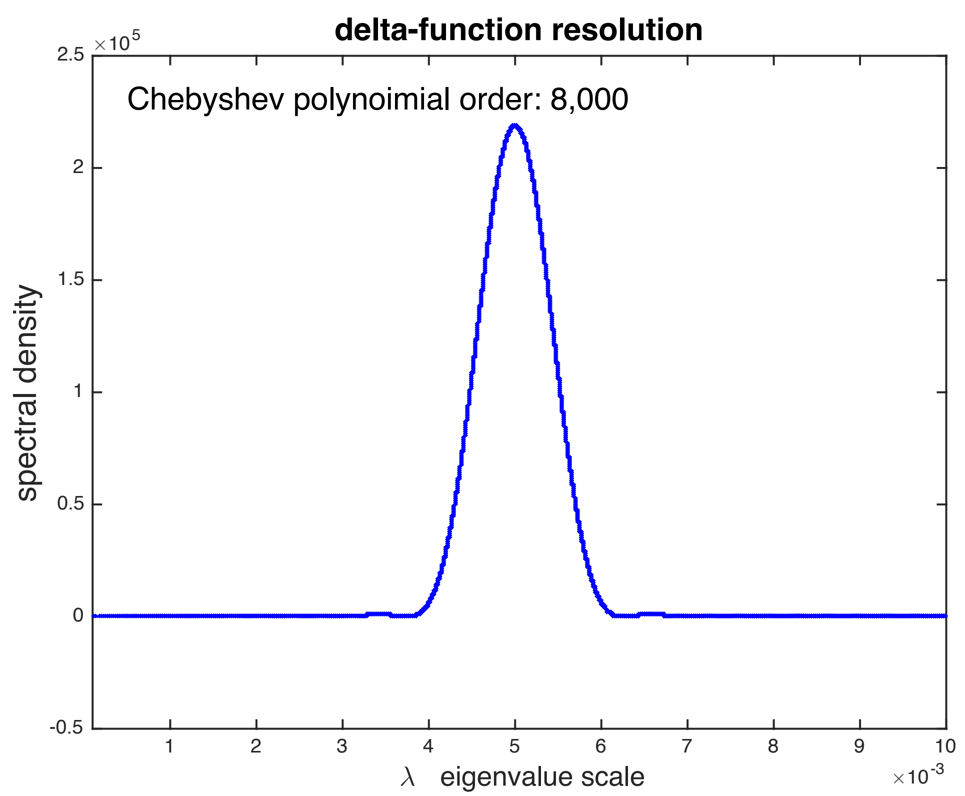
spectral density $\rho(t)$ from ensemble averages over the $D^{\dagger}D$ matrix with dimension N

$$\rho(t) = \left\langle \frac{1}{N}\sum_{i=1}^{N}\delta(t-\lambda_i)\right\rangle_{gauge\ ensemble}$$

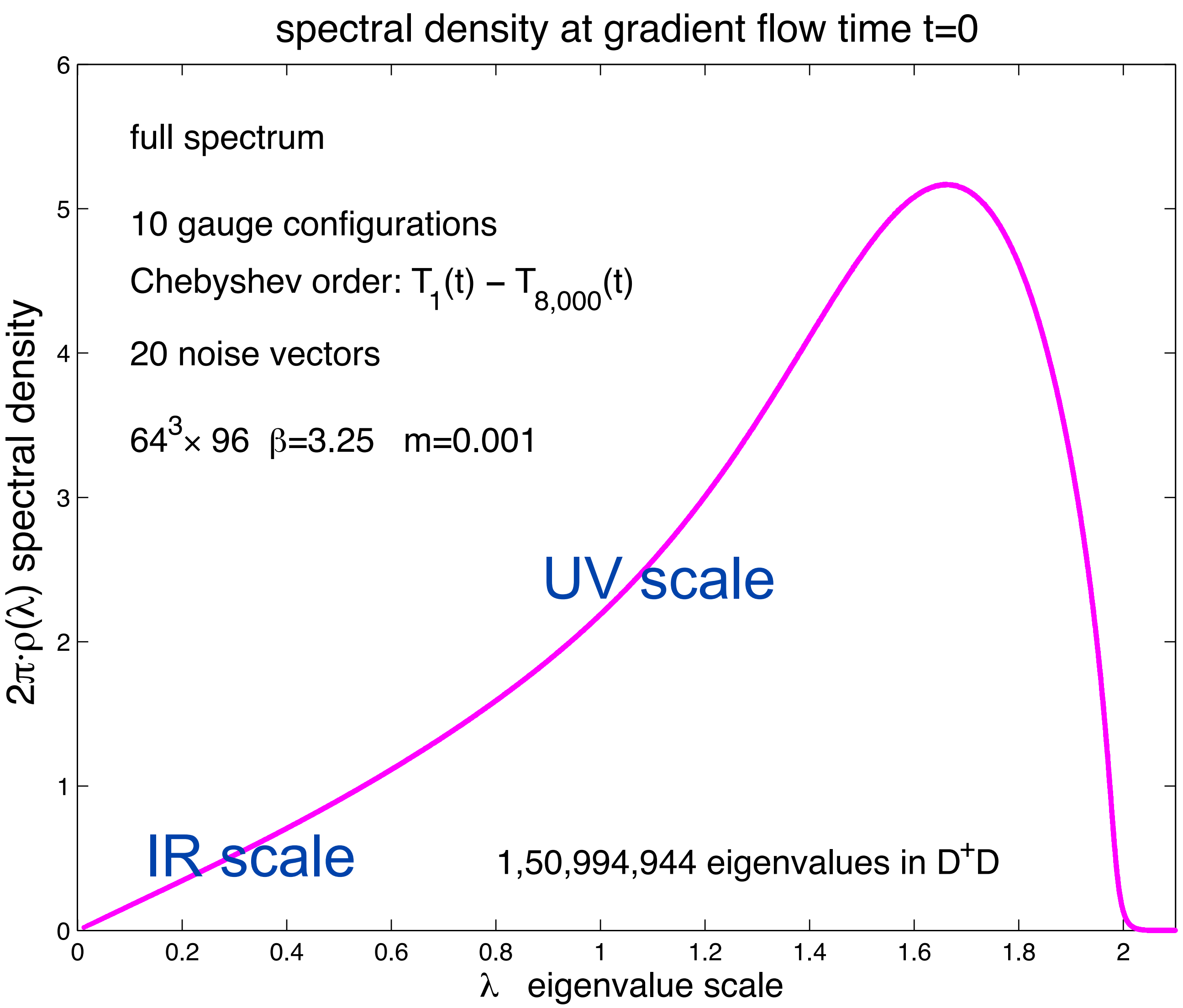
$$\rho(t) = \frac{1}{\sqrt{1-t^2}}\sum_{k=0}^{\infty}c_k T_k(t)$$ expansion in Cebyshev polynomials

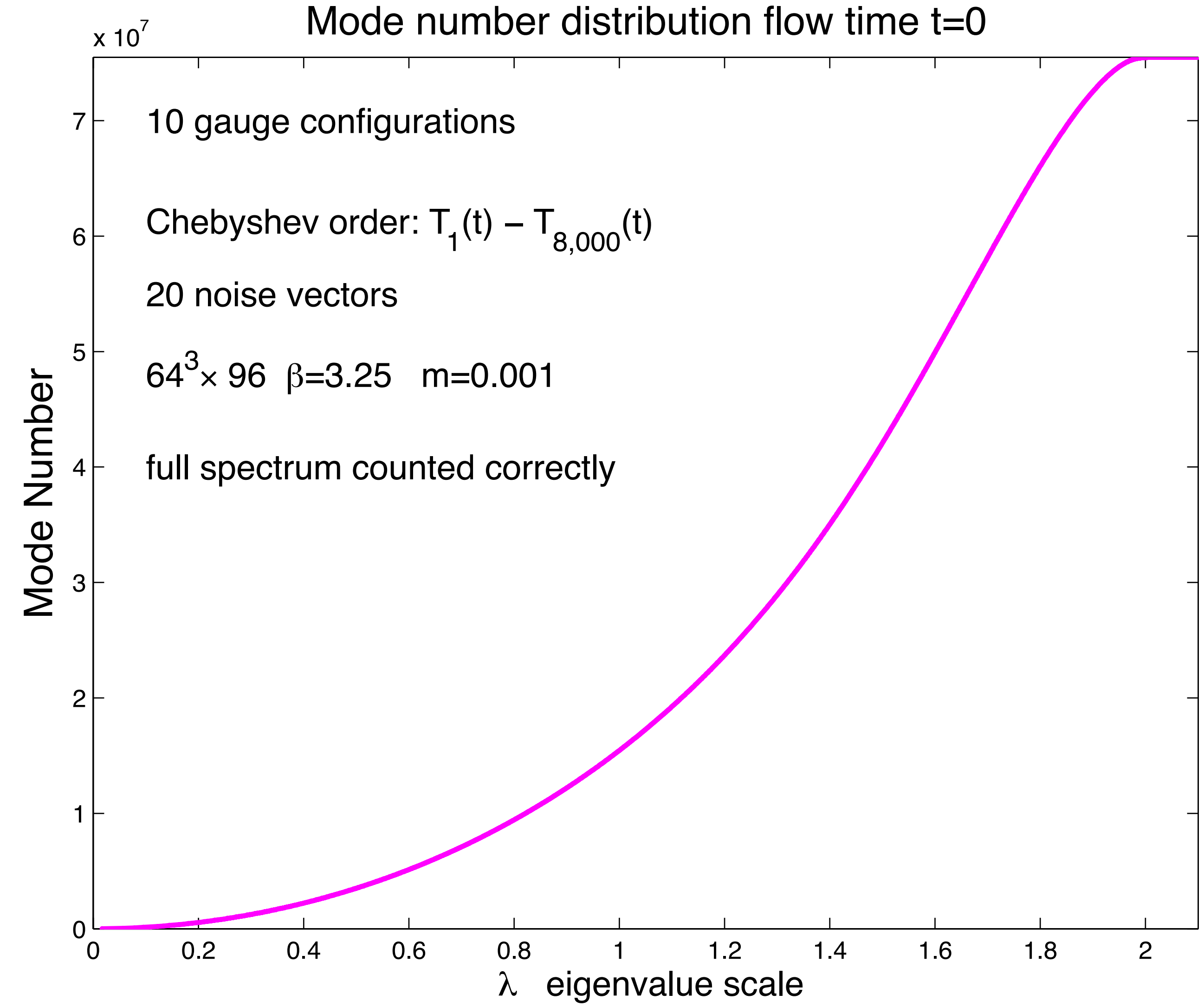
$$c_k = \begin{cases} \frac{2}{\pi}\int_{-1}^{1} T_k(t)\rho(t) & \mathrm{k}=0 \\ \frac{1}{\pi}\int_{-1}^{1} T_k(t)\rho(t) & \mathrm{k}\neq 0 \end{cases} \Rightarrow c_k = \begin{cases} \frac{2}{N\pi}\sum_{i=1}^{N} T_k(\lambda^2{}_i) & \mathrm{k}=0 \\ \frac{1}{N\pi}\sum_{i=1}^{N} T_k(\lambda^2{}_i) & \mathrm{k}\neq 0 \end{cases}$$

$\sum_{i=1}^{N} T_k(\lambda^2{}_i)$ is given by trace of $T_k(D^{+}D)$ operator



The chiral condensate full spectrum t=0





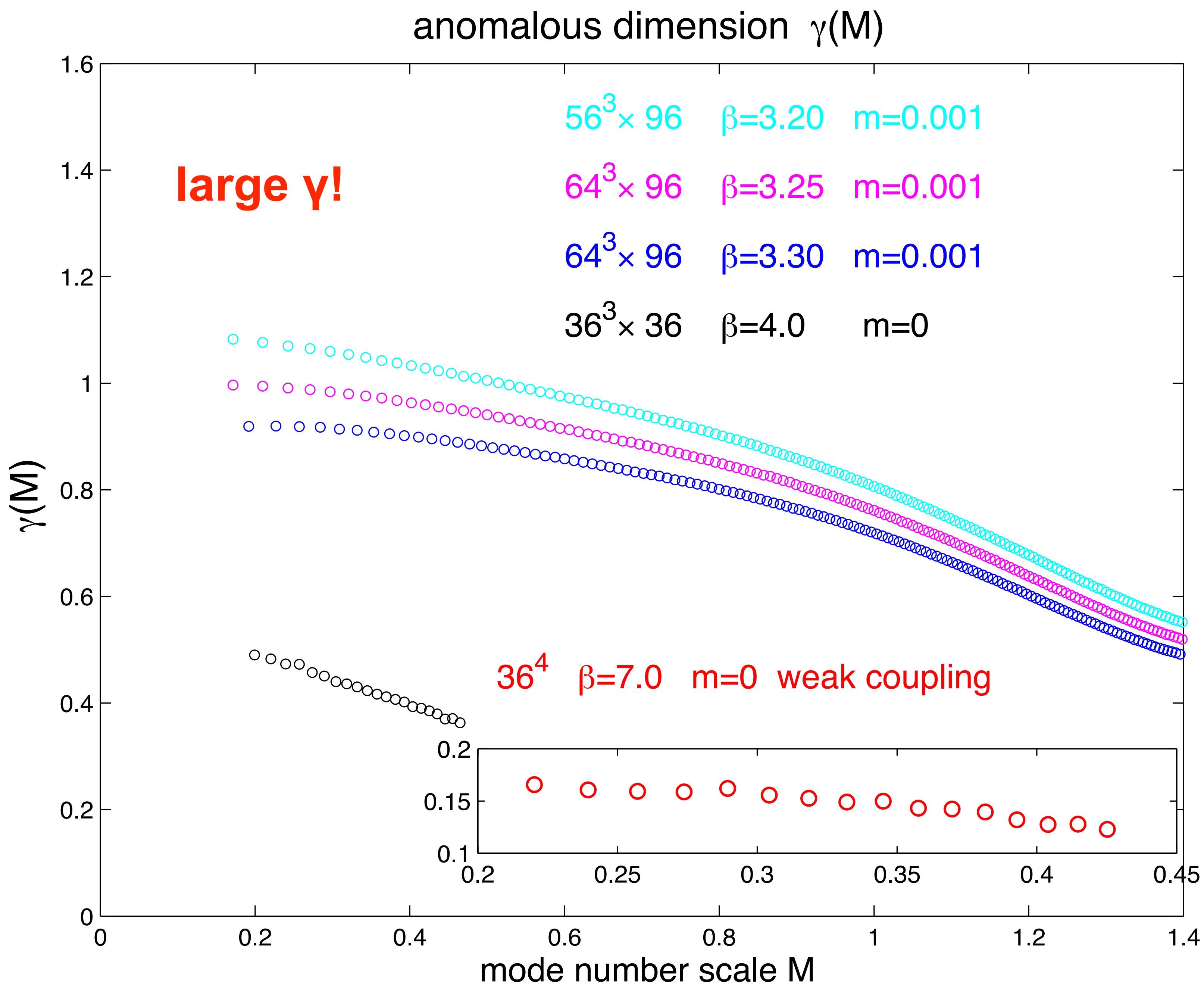
- nf=2 sextet example illustrates results from the Chebyshev expansion
- full spectrum with 8,000 Chebyshev polynomials in the expansion
- the integrated spectral density counts the sum of all eigenmodes correctly
- Jackknife errors are so small that they are not visible in the plots.

The chiral condensate   mass anomalous dimension

Del Debbio-Zwicky and collaborators, Patella,
Boulder group with lead from Anna Hasenfratz

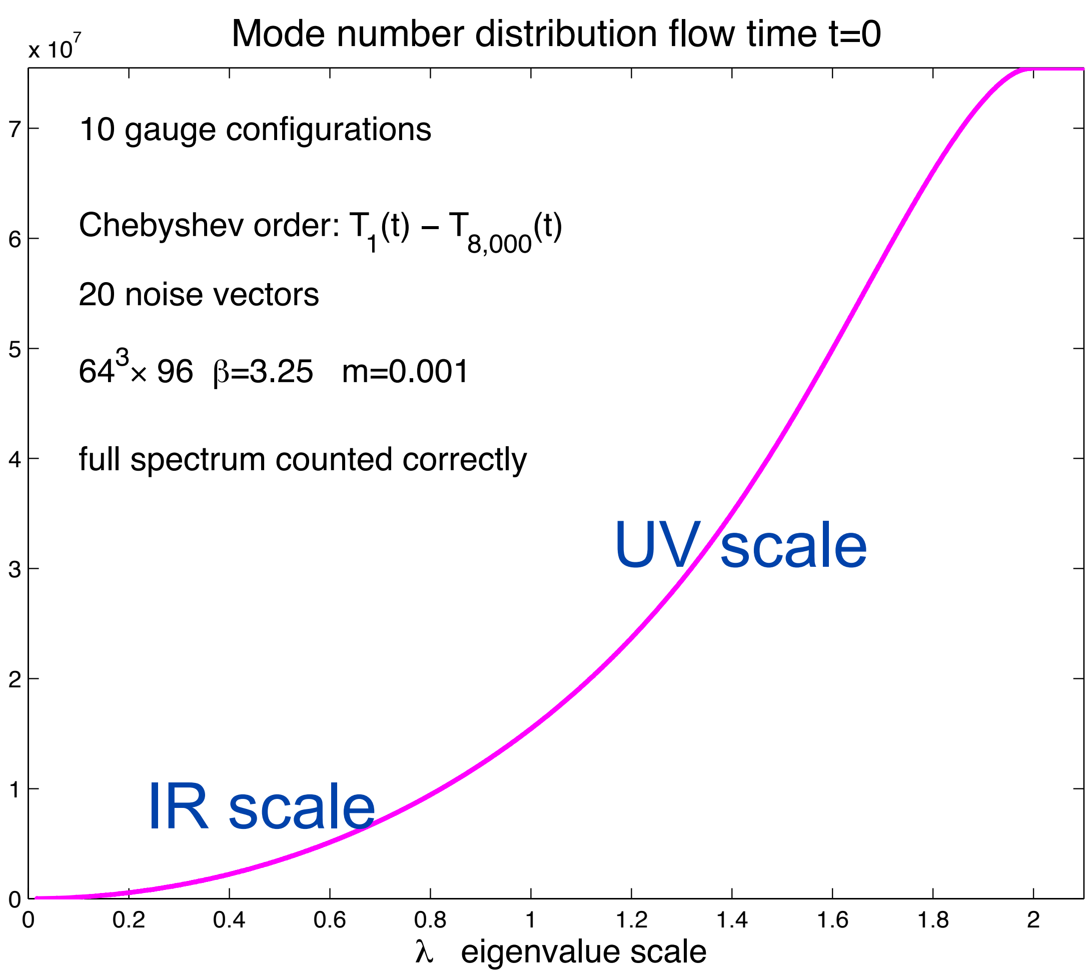
$$\nu_R(M_R, m_R) = \nu(M, m) \approx const \cdot M^{\frac{4}{1+\gamma_m(M)}},$$

or equivalently, $\nu(M,m) \approx const \cdot \lambda^{\frac{4}{1+\gamma_m(\lambda)}}$, with $\gamma_m(\lambda)$ fitted



also working on the alternate method using the pseudoscalar correlator and stepped Zp important cross-check

new results also for Nf=12

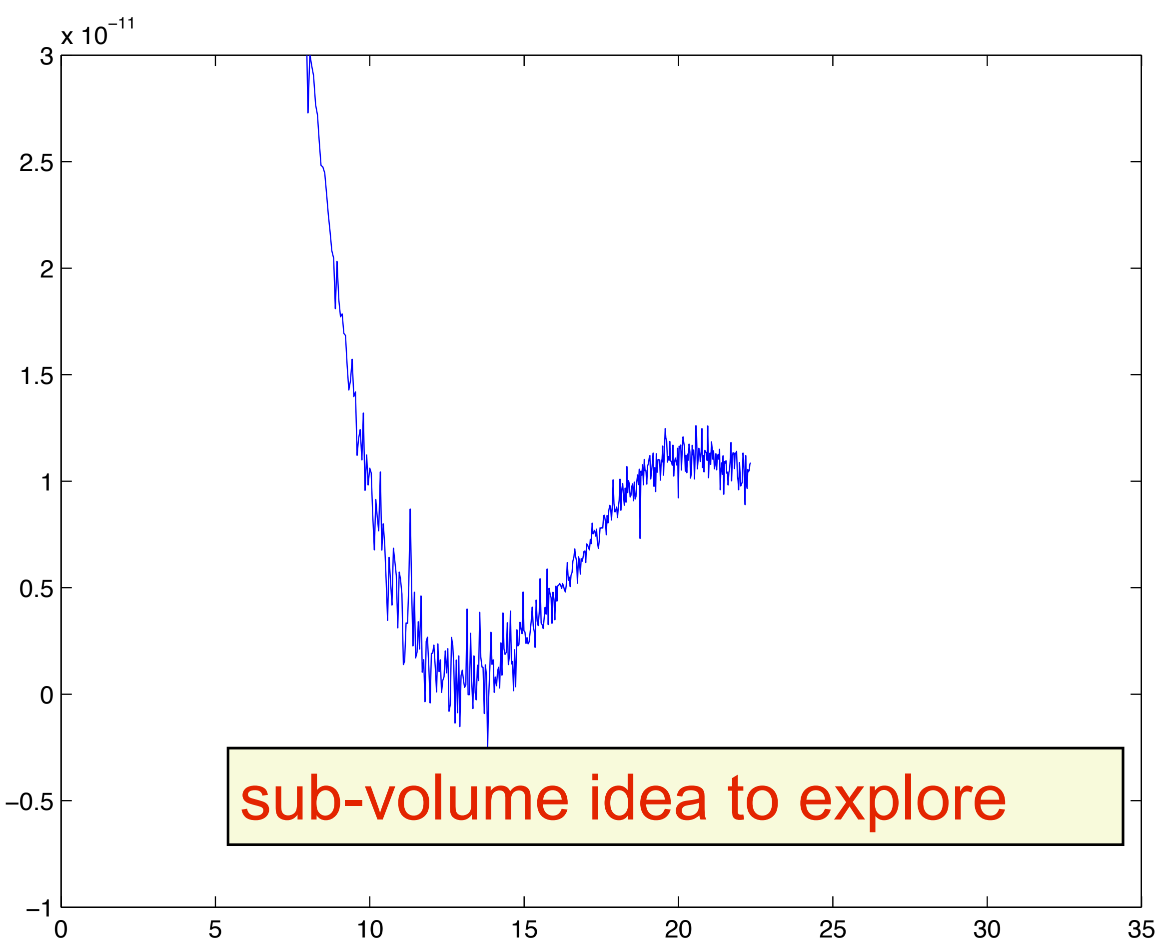


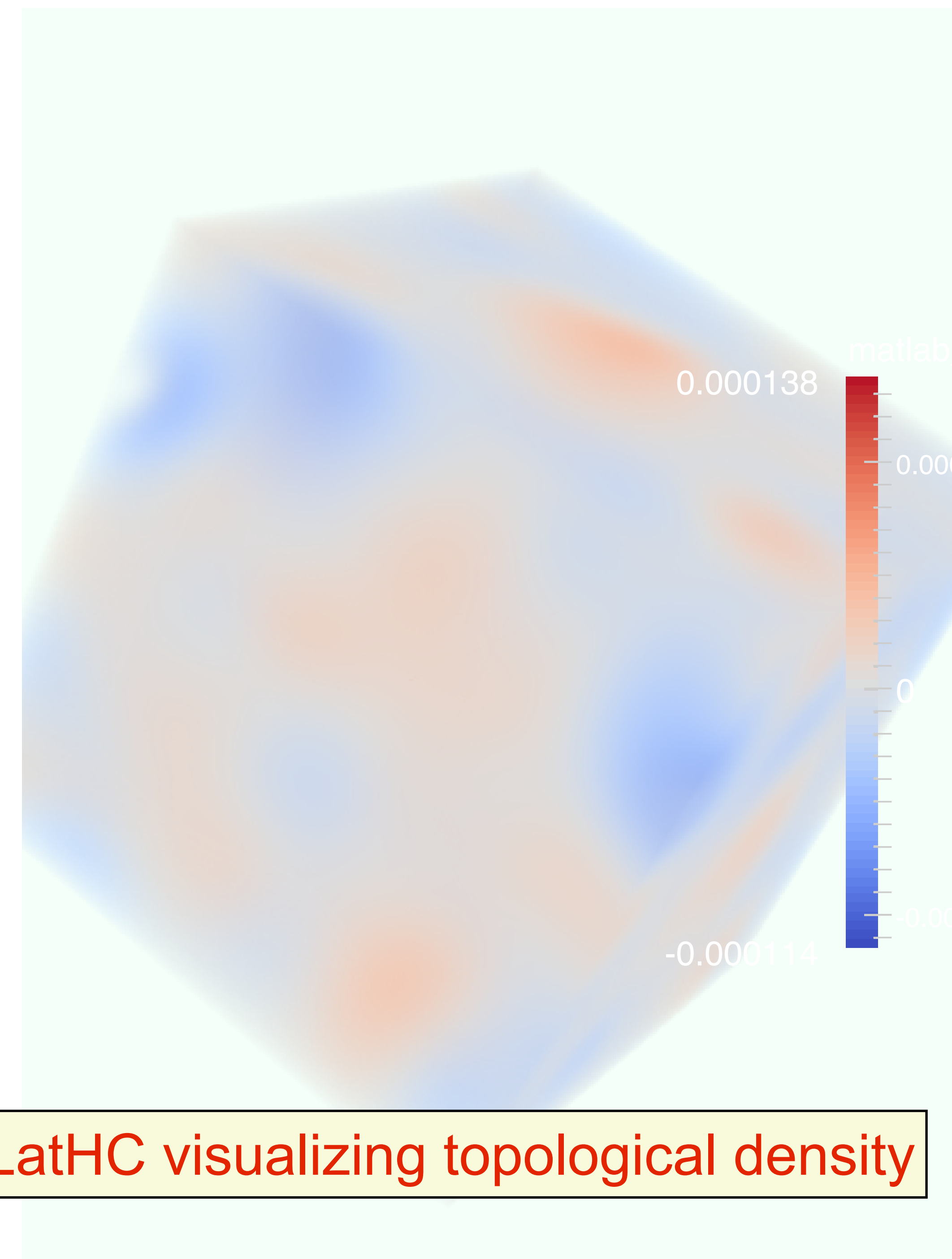
eta' ? diphoton bump?

$$\lim_{|x|\to\infty}\langle\rho(x)\rho(0)\rangle_Q = \frac{1}{\Omega}\left(\frac{Q^2}{\Omega} - \chi_t - \frac{c_4}{2\chi_t\Omega}\right) + \mathcal{O}(\Omega^{-3})$$

$$c_4 = -(\langle Q^4\rangle - 3\langle Q^2\rangle^2)/\Omega.$$

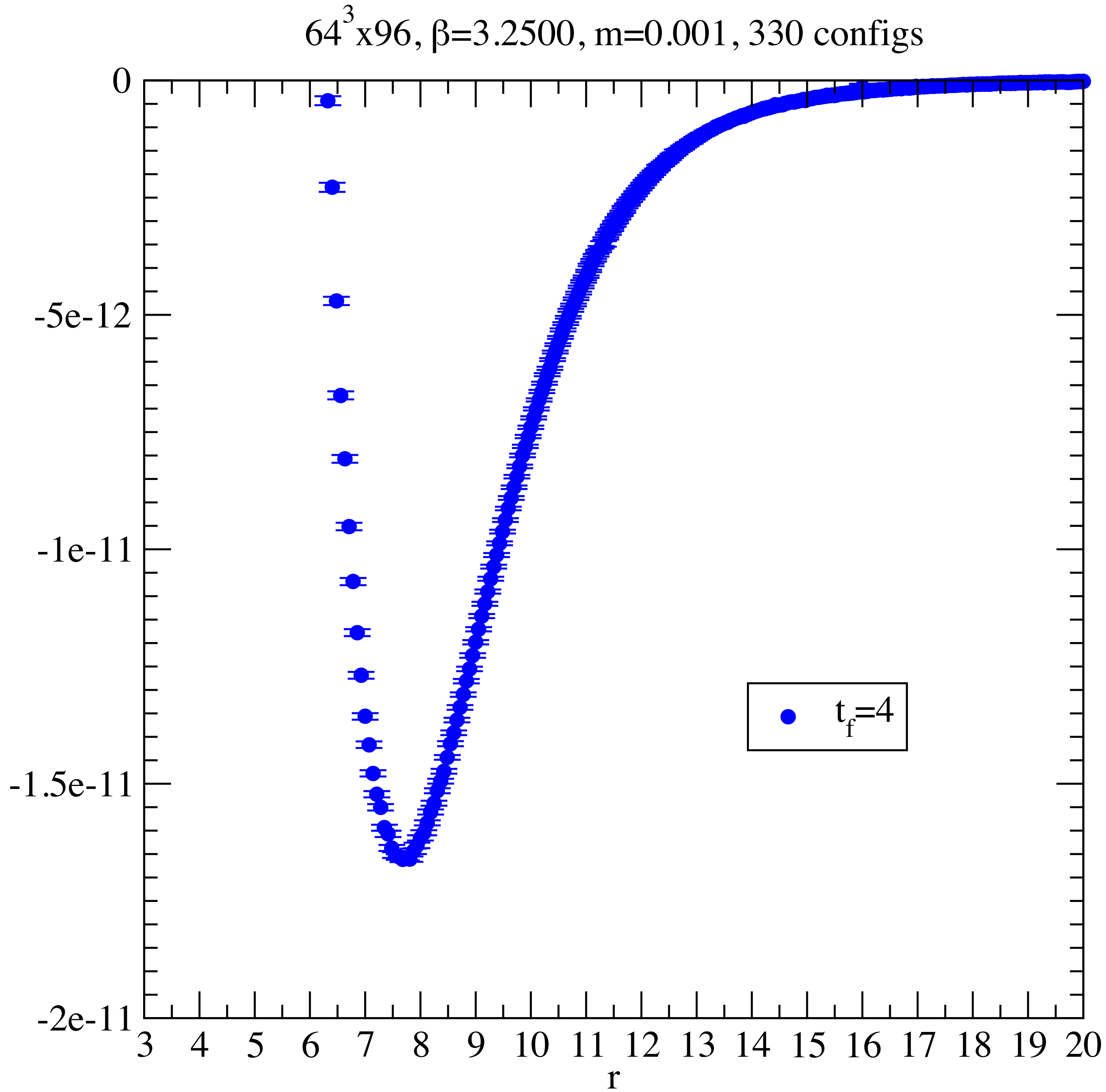
$$C(t_1 - t_2) \equiv \langle Q(t_1)Q(t_2)\rangle = \sum_{\vec{x}_1,\vec{x}_2}\langle\rho(x_1)\rho(x_2)\rangle$$

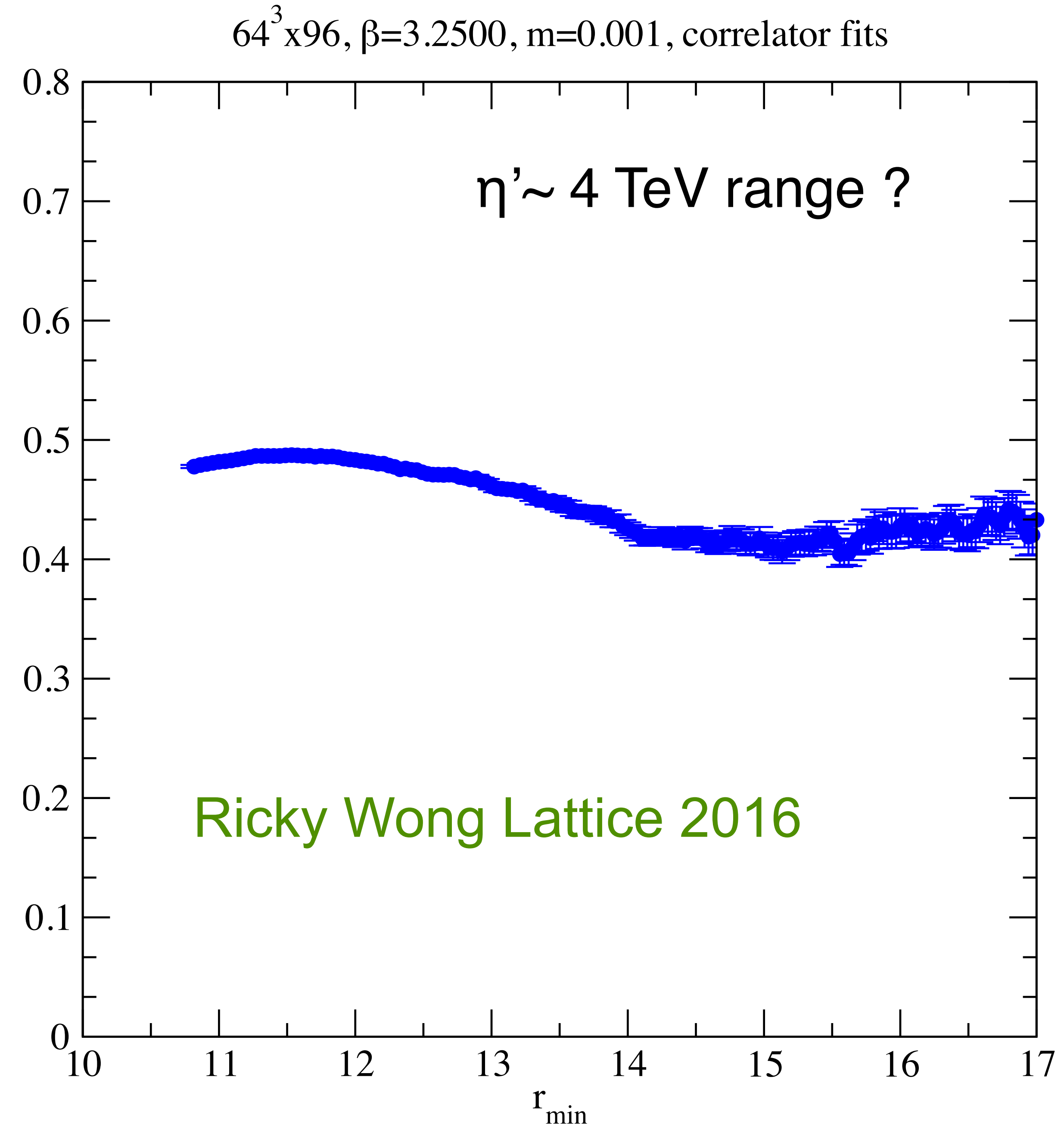




LatHC visualizing topological density

eta' ? diphoton bump?   it is not





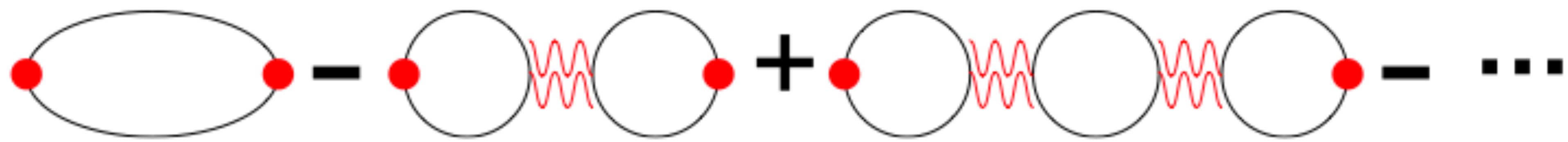
$\partial_\mu J_5^\mu \sim 2\mathrm{N}_f(N \pm 2)q(x)$ flavor singlet current

+ sign in 2-index symmetric rep

- sign in 2-index antisymmetric rep

factor 5 enhancement in WV formula

eta' ? diphoton bump?



$$\int d^3x \langle \eta'(\vec{x},t)\eta'^{\dagger}(\vec{0},0)\rangle = C_{\gamma_5}(t) - N_f D_{\gamma_5}(t) = A_{\eta'} e^{-m_{\eta'} t} + \cdots ,$$

$$m_0^2 = m_{\eta'}^2 - m_\pi^2 = \frac{2N_f}{f_\pi^2}\chi_{\rm top} \qquad \chi_t = \frac{m_q \Sigma}{N_f} + \mathcal{O}(m_q^2)$$

$$\chi_{\rm top} = \frac{\left\langle Q_{\rm top}^2 \right\rangle}{VT}, \quad Q_{\rm top} = \int \rho_{\rm top}(x)\, d^4x$$

$$\lim_{|x|\to \text{large}} \langle mP(x) mP(0)\rangle_Q = \frac{1}{V}\left(\frac{Q^2}{V} - \chi_t - \frac{c_4}{2\chi_t V}\right) + O(e^{-m_\eta |x|})$$

Early universe

Kogut-Sinclair EW phase transition
Relevance in early cosmology (order of the phase transition?)
L$_{at}$HC is doing a new analysis using different methods

- Nf=2 Qu=2/3 Qd = -1/3 fundamental rep
  udd neutral dark matter candidate
- dark matter candidate sextet Nf=2
  electroweak active in the application
- 1/2 unit of electric charge (anomalies)
- rather subtle sextet baryon
  construction (symmetric in color)
- charged relics not expected?

Three $SU(3)$ sextet fermions can give rise to a color singlet. The tensor product $6 \otimes 6 \otimes 6$ can be decomposed into irreducible representations of $SU(3)$ as,

$$6 \otimes 6 \otimes 6 = 1 \oplus 2 \times 8 \oplus 10 \oplus \overline{10} \oplus 3 \times 27 \oplus 28 \oplus 2 \times 35$$

where irreps are denoted by their dimensions and $\overline{10}$ is the complex conjugate of 10.

Fermions in the 6-representation carry 2 indices, ψ_{ab}, and transform as

$$\psi_{aa'} \longrightarrow U_{ab}\ U_{a'b'}\ \psi_{bb'}$$

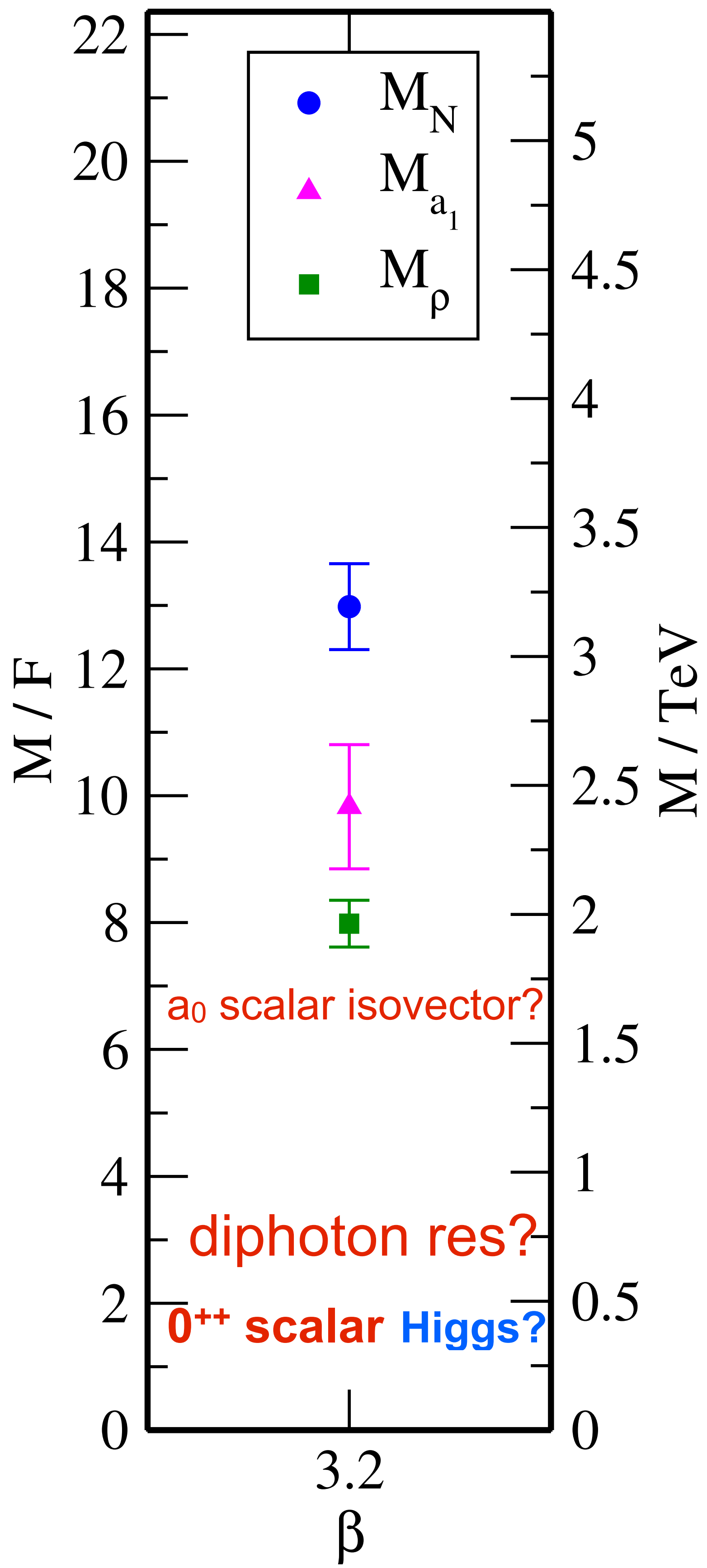
and the singlet can be constructed explicitly as

$$\varepsilon_{abc}\ \varepsilon_{a'b'c'}\ \psi_{aa'}\ \psi_{bb'}\ \psi_{cc'}.$$

topic: challenges of baryon spectroscopy and dark matter implications?

summary

BSM lattice challenges



We want to understand:

light scalar separated from 2-3 TeV resonance spectrum✓

multiple scalars in models close to CW?

Resonance spectrum✓

what is the eta'? ✓

entangled scalar-goldstone dynamics sigma model or dilaton?
how to decouple and isolate the light scalar?

bridge between UV and IR scale?✓
scale-dependent gauge coupling - high precision✓

what list of predictions independent of mass generation?
related phenomenology

consistent EW embedding → dark matter✓

BSM needs new lattice tools → RMT and delta-regime✓

scaled up QCD cannot do the job✓