Large Mass Hierarchies in Strongly-Coupled Dynamics

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Outline

- Introduction: the tensor to scalar mass ratio R.
- Gauge/gravity dualities: a bottom-up toy model.
- Lattice SU(2) with adjoint matter.
- Cutoff effects: lattice vs. gravity.
- Conclusions and Outlook

Statement of the Problem

- Weakly-coupled (Higgs field) models of EWSB are (usually) fine-tuned.
- What about strong-coupling? Replace Higgs sector with new interactions and fields, that UV-complete the theory (technicolor, composite Higgs, little Higgs...)
- Phenomenological problems: QCD-like theory cannot work (precision EW tests, top mass, PNGB proliferation, FCNC, Higgs particle...)
- Calculability problems: NDA shows QCD-like theories not phenomenologically viable, how can we compute/make predictions beyond NDA?
- Goal: find a strongly-coupled model that has dynamical features very different from QCD. In particular, dynamics is multi-scale and large anomalous dimensions emerge (NDA fails/is modified).
- Tools: gauge/gravity dualities and lattice field theory.

Statement of the Problem

- Possibility: theories with (quasi-)conformal behavior (walking TC, conformal TC, holographic TC...).
- Several candidates from lattice studies and gauge/gravity models (both top-down and bottom-up).
- Fundamental question in this talk: how can we tell whether we are onto something useful? In particular: what can we compute that will tell us that the theory of interest has multi-scale dynamics and large anomalous dimensions?
- In principle: build a full model and compute observable quantities to compare to data. In practice this is too ambitious: calculations are ways too hard, and furthermore results are model-dependent, which may hide general lessons.

Statement of the Problem

- Our proposal: focus on observable quantities that exist in very large classes of models, and that do not depend directly on symmetry-breaking pattern. Counter-example: the fact that pions of QCD are light is due to an internal symmetry, QCD is NOT multi-scale.

- Striking example: stress-energy tensor defined in general, and its properties reflected in physics of lightest scalar and tensor states (glueballs, with caveats...).

- What information can we extract from the (universally defined) ratio R?

$$R \equiv \frac{M_T}{M_0}$$

- Does it help us define what is meant by multi-scale?

- Does it know about anomalous dimensions?

- Notice: recent LHC signals of a new particle with mass ~750 GeV (~6 x 125 GeV)...

Gauge/gravity correspondence

- Basic Idea: strong/weak equivalence between gauge theories and higher-dimensional gravity (Maldacena).
- Dictionary exists and observables can be computed and compared:

$$Z_{\mathrm{SUGRA}}[\phi_{(0)}] = \int_{\Phi\sim\phi_{(0)}} D\Phi \exp(-S[\Phi]) = \langle \exp\left(-\int_{\partial AAdS} \phi_{(0)} O\right)\rangle_{QFT}$$

e.g. K. Skenderis, hep-th/0209067

- Mass spectra of glueballs can be computed systematically for large classes of solutions.
- Top-down approach: models built from string theory, fully rigorous. But: very few examples of confining theories, somewhat unrealistic and not flexible.
- Bottom-up models: start from sigma-model coupled to gravity in higher dimensions. Very flexible and easy to work with, but intrinsically incomplete (use caution!).
- Both: large-N and large 't Hooft coupling most useful but unrealistic. Care in extracting general lessons!

Glueball Spectra
5d sigma-model

- Systematic way of constructing sugra backgrounds uses consistent truncation to 5D sigma-model (n scalars) coupled to gravity.

$$
\begin{aligned}
\mathcal{S} \;\equiv\; & \int \mathrm{d}^4x\mathrm{d}r \left\{\sqrt{-g}\Theta\left[\frac{1}{4}R + \mathcal{L}_5(\Phi^a, \partial_M\Phi^a, g)\right]\right. \\
& +\sqrt{-\tilde{g}}\delta(r-r_1)\left[c_K K + \mathcal{L}_1(\Phi^a, \partial_\mu\Phi^a, \tilde{g})\right] \\
& \left. -\sqrt{-\tilde{g}}\delta(r-r_2)\left[c_K K + \mathcal{L}_2(\Phi^a, \partial_\mu\Phi^a, \tilde{g})\right]\right\}
\end{aligned}
$$

$$
\begin{aligned}
\mathrm{d}s^2_{1,4} &\equiv e^{2A}\eta_{\mu\nu}\mathrm{d}x^\mu\mathrm{d}x^\nu + \mathrm{d}r^2 \\
\mathcal{L}_5 &\equiv -\frac{1}{2}G_{ab}g^{MN}\partial_M\Phi^a\partial_N\Phi^b - V(\Phi^a) \\
\mathcal{L}_1 &\equiv -\lambda_{(1)}(\Phi^a)\,, \\
\mathcal{L}_2 &\equiv -\lambda_{(2)}(\Phi^a)\,.
\end{aligned}
$$

- Bulk equations and boundary terms determine 5D background, lift to 10D known.

$$
\begin{aligned}
&\bar{\Phi}''^a + 4A'\bar{\Phi}'^a + \mathcal{G}^a_{\ bc}\bar{\Phi}'^b\bar{\Phi}'^c - V^a = 0 \\
&6A'^2 + 3A'' = -G_{ab}\bar{\Phi}'^a\bar{\Phi}'^b - 2V, \\
&6A'^2 = G_{ab}\bar{\Phi}'^a\bar{\Phi}'^b - 2V.
\end{aligned}
$$

- First-order equations may exist (easier...):

$$
V = \frac{1}{2}G^{ab}W_aW_b - \frac{4}{3}W^2
$$

$$
\begin{aligned}
A' &= -\frac{2}{3}W\,, \\
\bar{\Phi}'^{\,a} &= G^{ab}W_b = W^a
\end{aligned}
$$

Glueball Spectra
5d sigma-model

- Given a background, one can study the spectrum of scalar and tensor fluctuations (systematic algorithmic procedure exists!), using gauge-invariant variables:

$$\begin{aligned}
\mathfrak{a}^a &= \varphi^a - \frac{\bar{\Phi}'^a}{6A'}h, \\
\mathfrak{b} &= \nu - \frac{\partial_r(h/A')}{6}, \\
\mathfrak{c} &= e^{-2A}\partial_\mu \nu^\mu - \frac{e^{-2A}\Box h}{6A'} - \frac{1}{2}\partial_r H, \\
\mathfrak{d}^\mu &= e^{-2A}\Pi^\mu{}_\nu \nu^\nu - \partial_r \epsilon^\mu, \\
\mathfrak{e}^\mu{}_\nu &= h^{TT\mu}{}_\nu .
\end{aligned}$$

- Bulk equations and boundary terms known in general:

$$\left[\mathcal{D}_r^2 + 4A'\mathcal{D}_r + e^{-2A}\Box\right]\mathfrak{a}^a - \left[V^a{}_{|c} - \mathcal{R}^a{}_{bcd}\bar{\Phi}'^b\bar{\Phi}'^d + \frac{4(\bar{\Phi}'^a V_c + V^a\bar{\Phi}'_c)}{3A'} + \frac{16V\bar{\Phi}'^a\bar{\Phi}'_c}{9A'^2}\right]\mathfrak{a}^c = 0,$$

$$\left[\delta^a{}_b + e^{2A}\Box^{-1}\left(V^a - 4A'\Phi'^a - \lambda^a{}_{|c}\bar{\Phi}'^c\right)\frac{2\bar{\Phi}'_b}{3A'}\right]\mathcal{D}_r\mathfrak{a}^b\Big|_{r_i} =$$
$$\left[\lambda^a{}_{|b} + \frac{2\bar{\Phi}'^a\bar{\Phi}'_b}{3A'} + e^{2A}\Box^{-1}\frac{2}{3A'}\left(V^a - 4A'\bar{\Phi}'^a - \lambda^a{}_{|c}\bar{\Phi}'^c\right)\left(\frac{4V\bar{\Phi}'_b}{3A'} + V_b\right)\right]\mathfrak{a}^b\Big|_{r_i}$$

$$\partial_r \mathfrak{e}^\mu{}_\nu\,\big|_{r_i} = 0$$

- Procedure: take your confining background, introduce UV and IR cutoffs (regulators!), solve bulk equations and apply boundary conditions, repeat by progressively removing the two cutoffs. If IR and UV are healthy, the cutoff effects will decouple.

What model?

- Theory with a mass gap.
- Little or no supersymmetry.
- Large (tunable) anomalous dimensions.
- Known 5d sigma-model description.
- (possibly) simple model.
- No known top-down model: we need a bottom-up toy model to start from.
- Top-down example: GPPZ (dimension 1 deformation and/or dimension 3 VEV)

$$W = -\frac{3}{4}\left(\cosh 2\sigma + \cosh \frac{2m}{\sqrt{3}}\right)$$

- Our toy model: tunable parameter corresponding to the dimension of the deformation:

$$W = -\frac{3}{4}\left(1 + \cosh 2\sqrt{\frac{\Delta}{3}}\Phi\right)$$

Toy Model

- Background known in closed form:

$$\bar{\Phi}(r) = \sqrt{\frac{3}{\Delta}}\mathrm{arctanh}\left(e^{-\Delta(r-c_1)}\right),$$
$$A(r) = A_0 + \frac{1}{2\Delta}\ln\left(-1+e^{2\Delta(r-c_1)}\right)$$

- UV behavior: AdS (CFT) deformed by operator of dimension $4-\Delta$, deformation $m \equiv \sqrt{\frac{3}{\Delta}}e^{\Delta c_1}$

- Integration constants can be set to zero (setting the scale).

- Calculation of spectra of scalars and tensor modes comparatively simple:

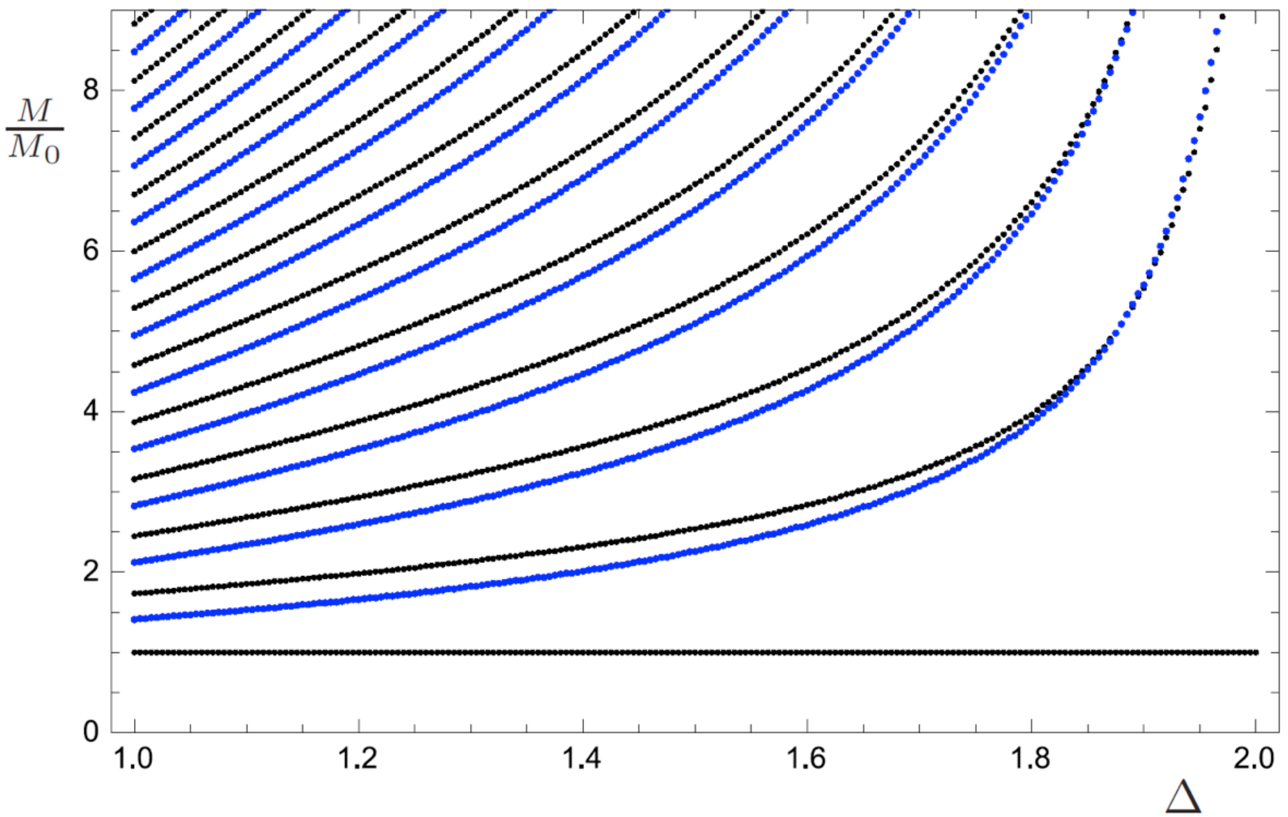
$$\left[\partial_r^2 + 4A'\partial_r + e^{-2A}M^2\right]\mathfrak{a} - \left[V_{\Phi\Phi} + \frac{8\bar{\Phi}'V_\Phi}{3A'} + \frac{16V\bar{\Phi}'^2}{9A'^2}\right]\mathfrak{a} = 0$$
$$\left[\partial_r + \frac{M^2}{e^{2A}}\frac{3A'}{2\bar{\Phi}'^2} - \left(\frac{4V\bar{\Phi}'}{3A'} + V_\Phi\right)\right]\mathfrak{a}\Big|_{r_i} = 0$$

$$\left[\partial_r^2 + 4A'\partial_r + e^{-2A}M^2\right]\mathfrak{e}^\mu{}_\nu = 0$$
$$\partial_r\mathfrak{e}^\mu{}_\nu\big|_{r_i} = 0$$

- Well behaved dependence on IR and UR cutoffs (see later).

Gravity Toy Model

- Scalar (black) and tensor (blue) masses.
- Normalized to lightest scalar.
- Vary $\Delta = 1 + \gamma^*$
- For small anomalous dimension, $R \to \sqrt{2}$
- For large anomalous dimension, $R \to +\infty$
- Calculation reliable for all values of anomalous dimensions (numerics deteriorates towards $R \to +\infty$)
- Final calculations for $r_{\rm UV} = 25$ and $r_{\rm IR} = 10^{-6}$.



Gravity Toy Model

- The ratio R depends monotonically on the anomalous dimension.
- It diverges when the anomalous dimension reaches ~1.
- In the model, it can be computed.
- Ratio R can be as large as 5-10: relevant to multi-scale dynamics, to 750 GeV particle...
- Are we learning anything useful? Is this exercise just model-dependent?
- Look at the lattice for examples where comparison is meaningful.

SU(2) with Adjoint fermions on the Lattice

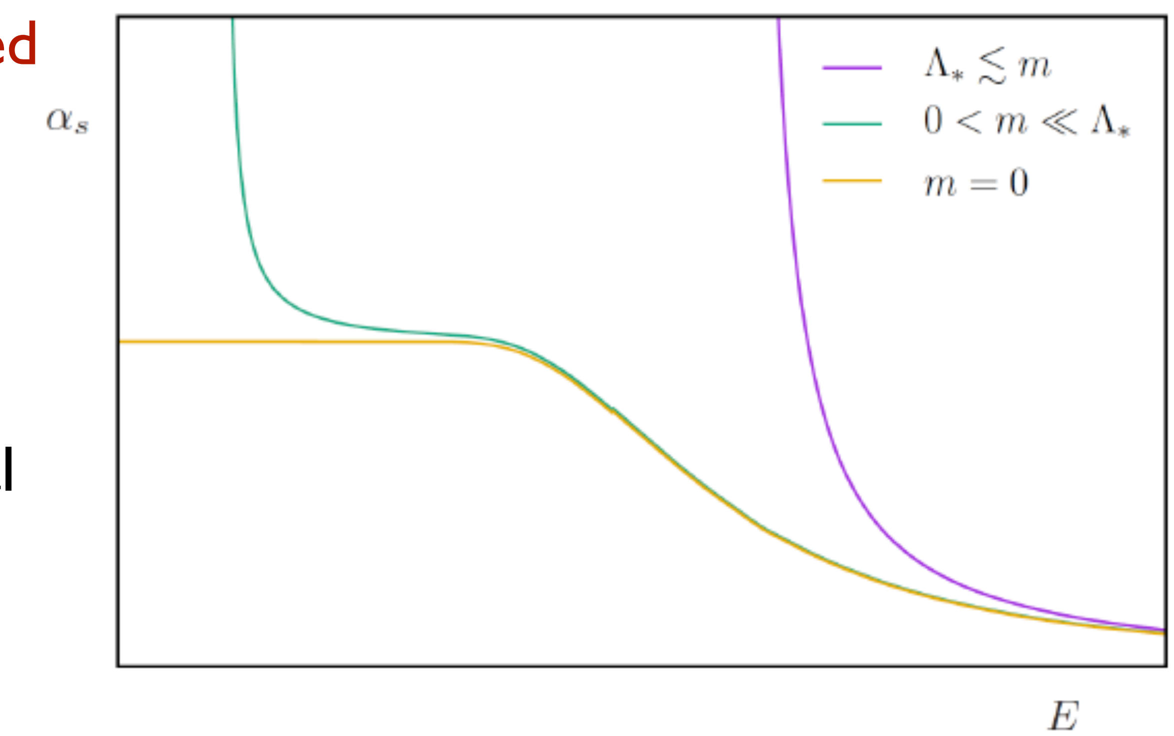
- Consider SU(2) gauge theory, with N=0, 1 or 2 Dirac fermions on the Adjoint.
- Spectra of mesons and glueballs (mixing...) known.
- N=0 theory is Yang-Mills, R=1.44(4).
- N=1,2 (quasi-)conformal in the IR.
- Lattice studies with Wilson fermions: mass term explicitly present.
- Anomalous dimensions can be computed: for N=2 $\gamma^* = 0.371(20)$, for N=1 $\gamma^* = 0.925(25$
- Spectra of glueballs can be computed.
- Lattice artefacts subtle: see later.
- We can use the toy model to compute R, and compare.

Mass-deformed IR-Conformal Theories

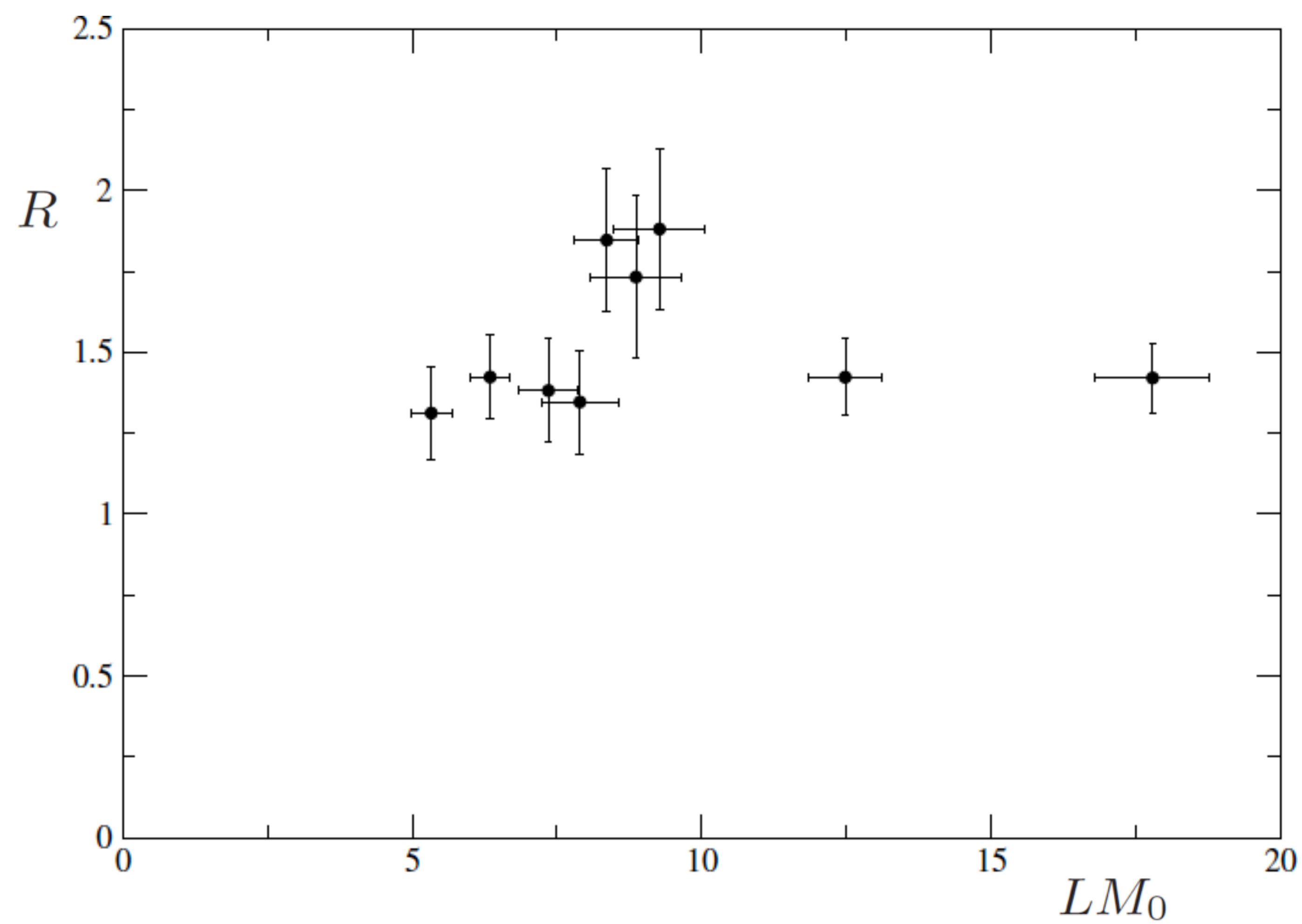
- Cartoon of running coupling with IR fixed point.
- Mass term m introduces second scale, that can be parametrically separated.
- Expected universal scaling of the spectral masses, barring lattice artefacts:

$$M_i \propto m^{\frac{1}{\Delta}} \qquad a \ll \frac{1}{M} \ll L$$

- Is it possible that the ratio R also shows some universal behavior? What does it mean?

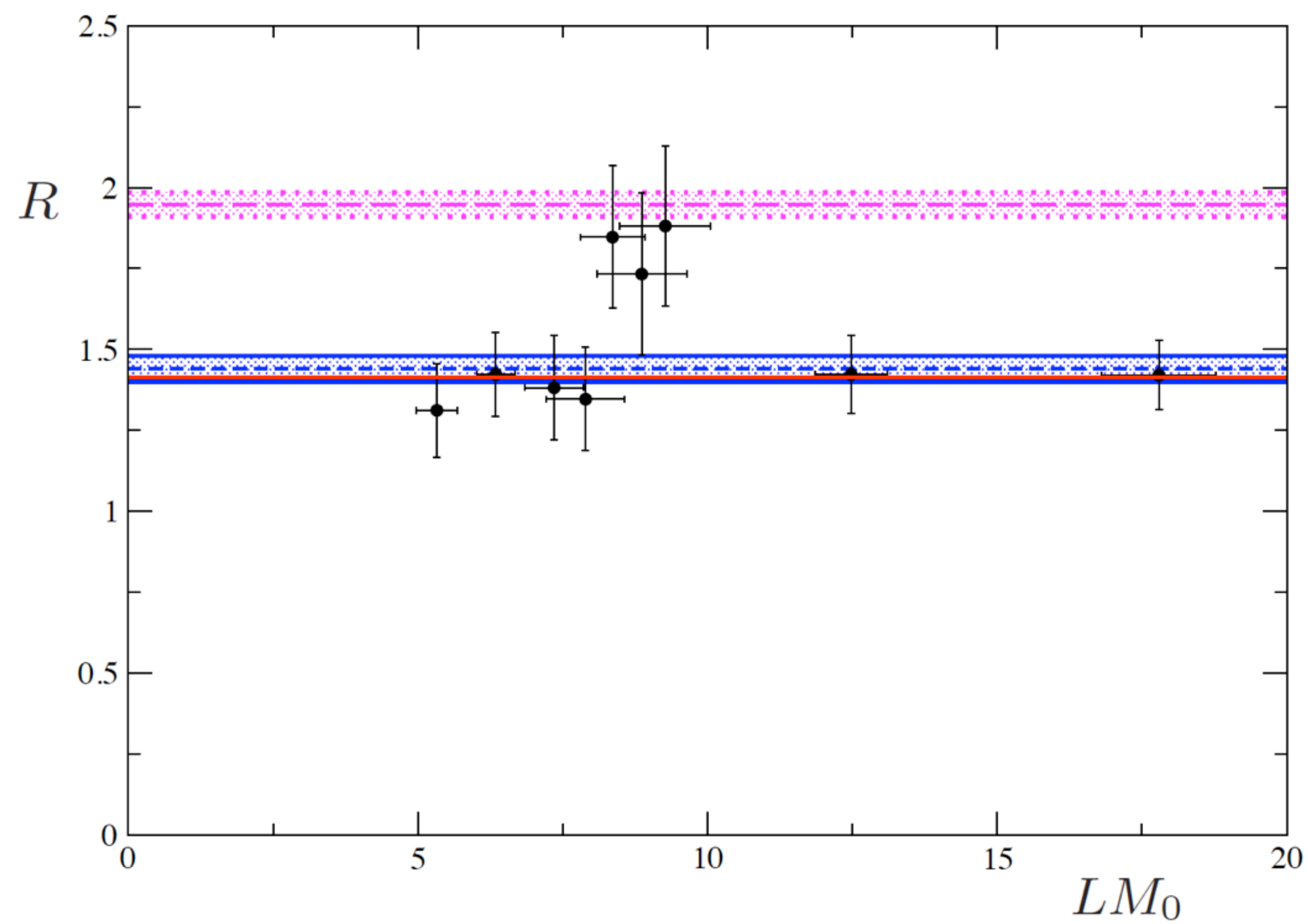


SU(2) + 2 Adjoints



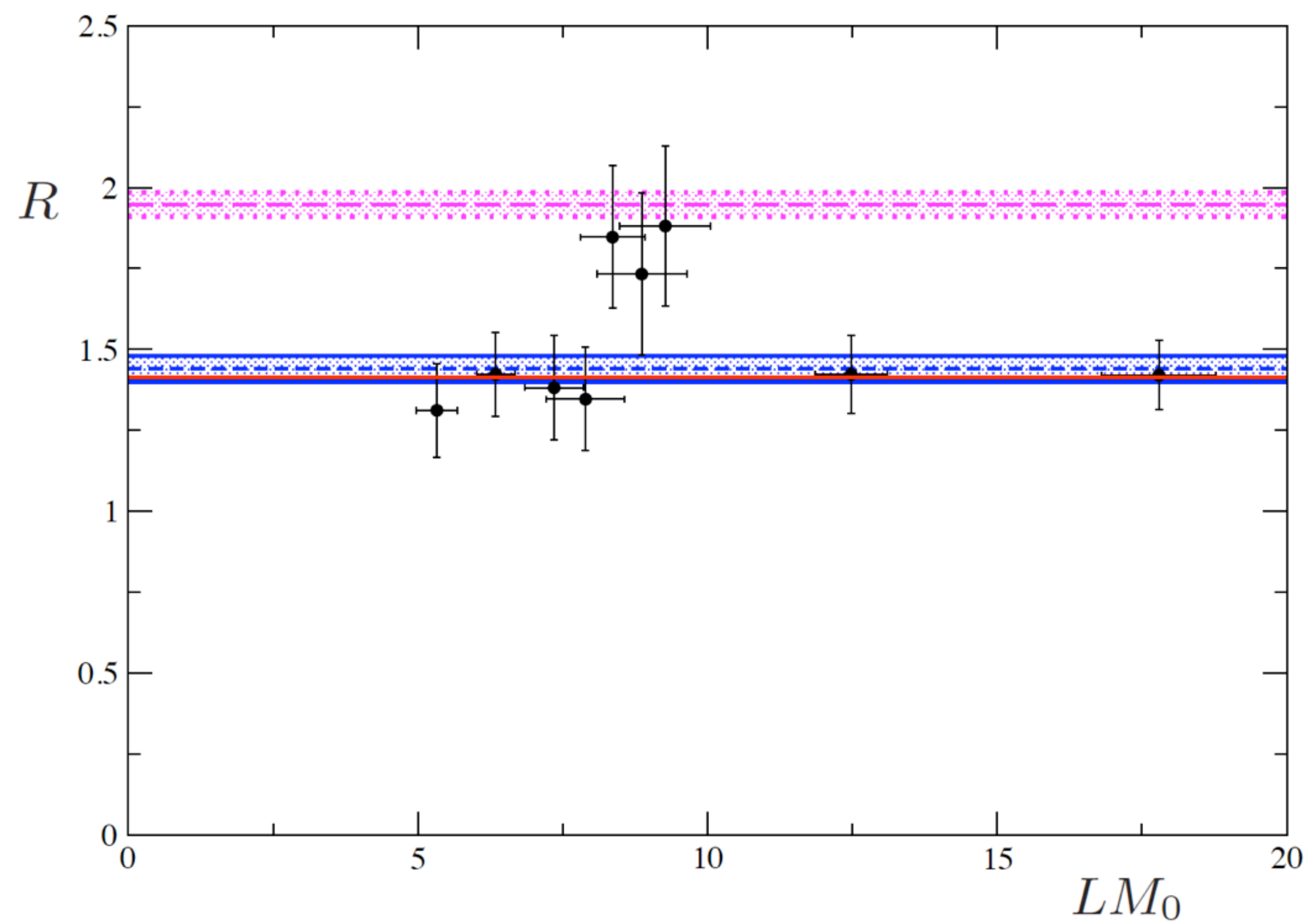
- Lattice calculation: fixed coupling, vary number of lattice sites and bare mass.
- Compute ratio tensor to scalar mass R, as a function of dimensionless scalar mass.

SU(2) + 2 Adjoints



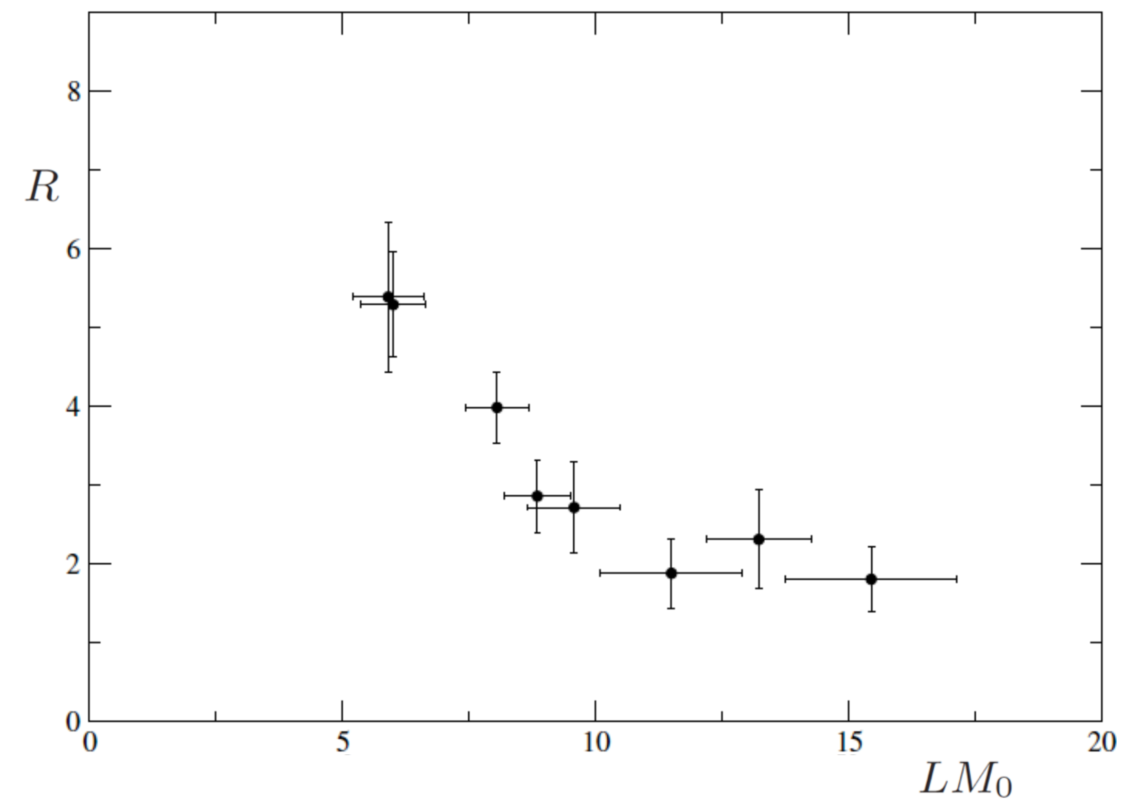
- Comparison with gravity toy model.
- Pink: use anomalous dimension for N=2, and find $R = 1.95(4)$.
- Blue: compare to Yang-Mills $R = 1.44(4)$
- Red: GPPZ for $\Delta = 1\,, R = \sqrt{2}$

SU(2) + 2 Adjoints



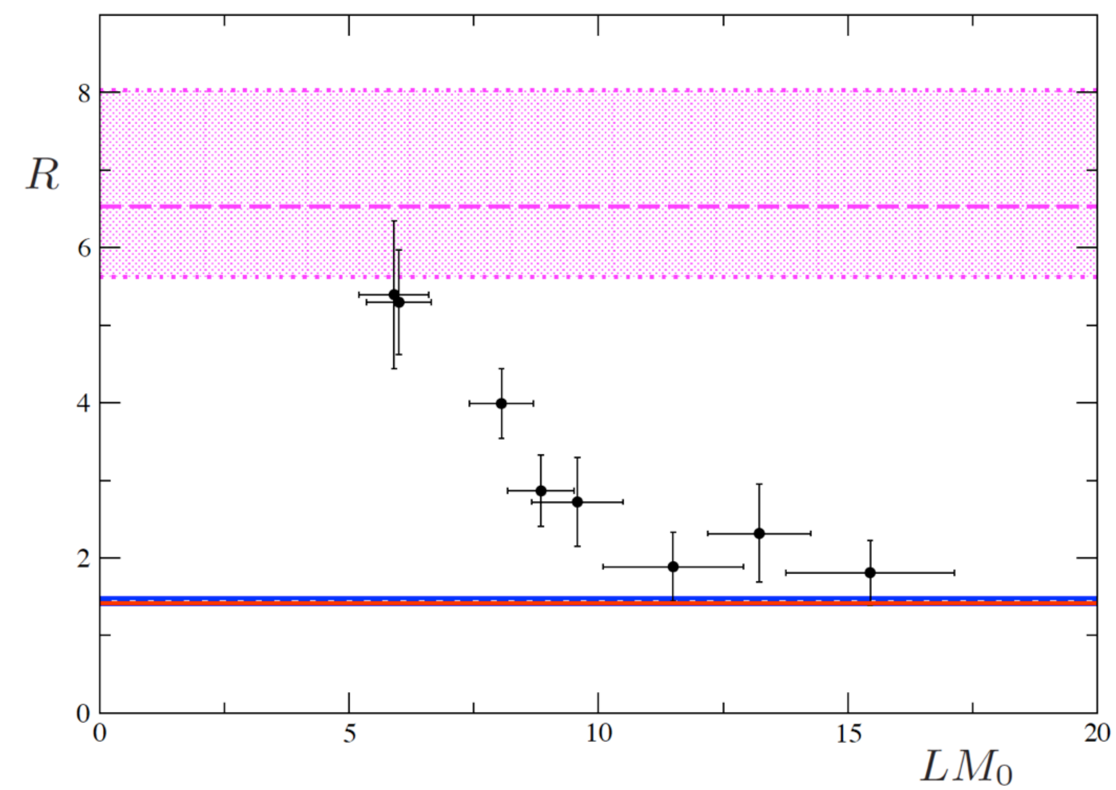
- Three plateaux.
- At very large mass agreement with Yang-Mills (lattice artefact).
- At very low mass agreement with femto-universe (lattice artefact).
- Intermediate region: agreement with gravity toy model (with caveats).

SU(2)+1 Adjoint



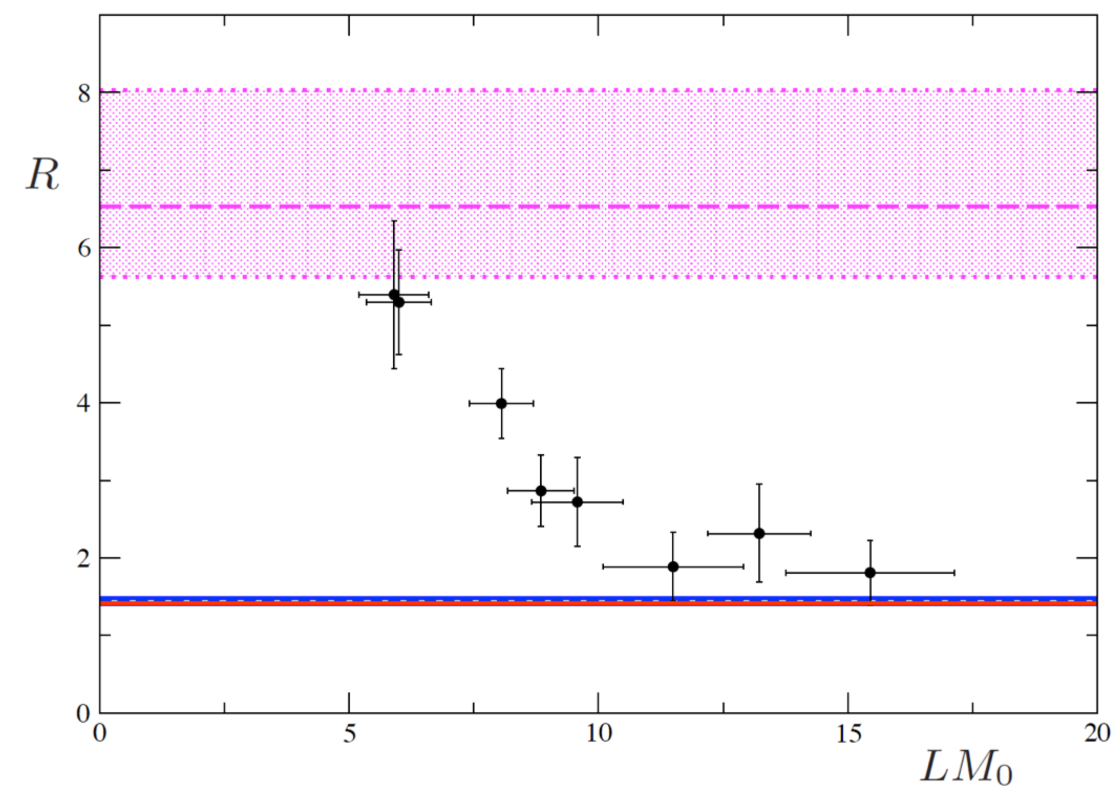
- Lattice calculation: fixed coupling, vary number of lattice sites and bare mass.
- Compute ratio tensor to scalar mass R, as a function of dimensionless scalar mass.
- Calculation for N=1 much more expensive: fewer configurations at small mass and large number of lattice sites. PRELIMINARY STUDY

SU(2)+1 Adjoint



- Comparison with gravity toy model.
- Pink: use anomalous dimension for N=1, and find $R = 6.53^{+1.50}_{-0.91}$
- Blue: compare to Yang-Mills $R = 1.44(4)$
- Red: GPPZ for $\Delta = 1\,, R = \sqrt{2}$

SU(2)+1 Adjoint

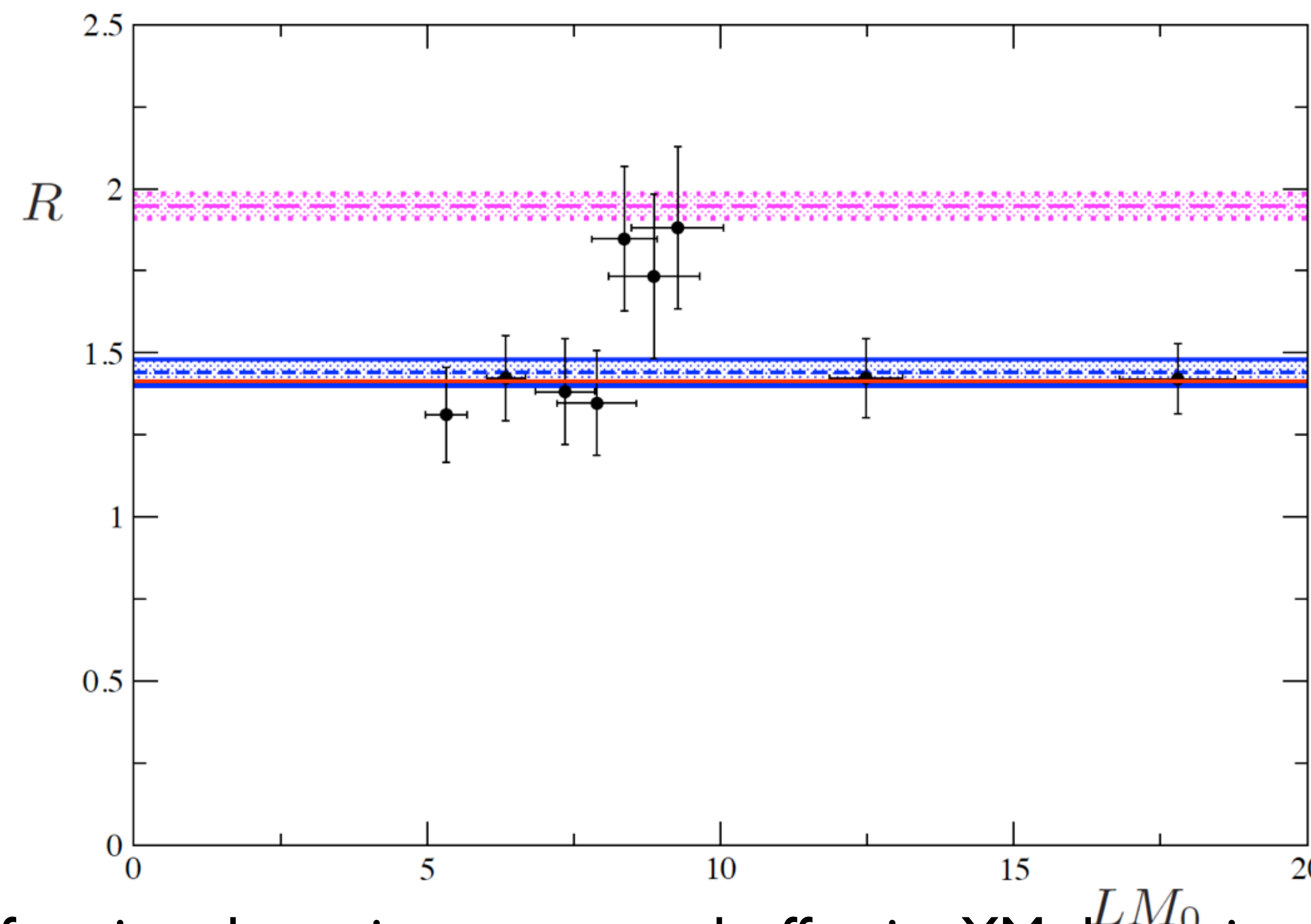


- High mass plateau compatible with Yang-Mills (lattice artefact).
- R grows at smaller mass.
- Lowest mass values compatible with gravity toy model.
- No clear plateau reached (Yet?)
- No Femto-universe yet at small masses (far left, lattice artefact).

Comparison Summary (incomplete)

- The ratio R can be computed in the gravity model, and grows with anomalous dimension.
- Where available, lattice data on SU(2) with N=0,1,2 adjoints agree to surprising degree with gravity toy model.
- Lattice artefacts non-trivial, to be discussed shortly.
- More data on SU(2) with N=1 Dirac adjoint needed.

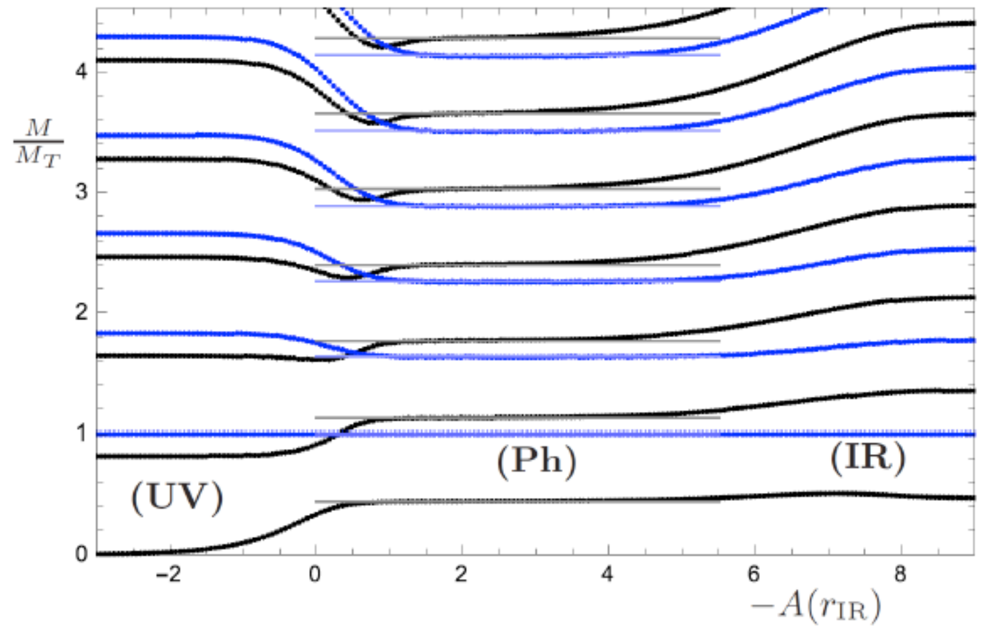
SU(2) + 2 Adjoints
Lattice Artefacts



- Large mass: fermion dynamics suppressed, effective YM dynamics. See far right of plot.

- Small number of site at fixed mass and coupling: femto-universe. The spectrum is discretized not by the dynamics, but by small box size effects. Ratio of masses O(1), and depend on trivial physics of the box (boundary conditions). See far left of plot.

Cutoff Effects in Gravity



- Mass spectra are extracted by varying UV and IR cutoffs independently, and extrapolating to physical region.

- Exercise: fix ratio of UV and IR cutoffs and vary them together.

$$A(r_{\mathrm{UV}}) - A(r_{\mathrm{IR}}) = 8$$

- Physical plateau appears in center.

- For small values of cutoffs, all physics controlled by confinement dynamics (far right).

- For large values of the cutoff, spectrum controlled by IR-cutoff effects (far left). Notice arising of unphysical light scalar, and ratio of masses O(1).

Conclusions

- The ratio R: possibly large in quasi-conformal theories with large anomalous dimensions. Circumstantial evidence from from lattice as well as gravity dualities.
- Remarkable fact: the gravity and lattice calculations are performed with completely different theories: some form of universality at play?
- Interesting curiosity: cutoff effects in gravity resemble lattice artefacts.

Outlook

- Lattice: extensive study of SU(2) with one adjoint at small mass, different coupling, and with larger lattices. Compare to other quasi-conformal theories.
- Field Theory: is there universality? In what sense?
- Gravity: more models? Top-down?
- Phenomenology and Model Building: realistic models? Comparison to LHC data? Do we have something to say about the 750 GeV excess?