Holography, Conformal Field Theories, and Lattice

Higgs Centre for Theoretical Physics, Edinburgh, 27 June 2016

Holography, Conformal invariance and quantum phase transitions

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Introduction

- Admittedly, the title is much broader than the talk.

- This is on purpose: to force us to think a bid wider than the specific problems we are typically focus on.

- I would like to offer some thoughts and some recent progress on issues in quantum field theory that are important for understanding extensions of the SM.

- One potential issue is conformal invariance and associated non-trivial CFTs

- It is important as a non-trivial UV completion of the SM and it is one of the possible solutions of the hierarchy problem. Unfortunately no good realization exists so far.

- Most of the physics we study experimentally and theoretically is NOT conformal invariant.

• A key step towards incorporating CFTs to the description of real physics involves the understanding of the breaking of conformal invariance by relevant operators both infinitesimally (Scaling region) and finitely.

• There are many tools to address CFTs.

(a) Perturbation theory of free or near free CFTs has been the tools of choice for 50 years. It is powerful when applicable, but its range of applicability is probably of measure zero in the space of CFTs.

(b) The bootstrap. Proposed long ago, it has been successfully applied until 3-4 years ago to 2d CFTs with success. Recently it generated quite a few breakthroughs in 3, 4 and 6 dimensions, and it is not yet obvious how far it can go.

(c) Integrability. This until 5 years ago was considered useless for 4d CFTs/QFTs. We know now that it will provide soon the full solution of a 4d CFT at large N. How far it can go it is not obvious.

(d) Computational (Lattice) techniques. They have the advantage of a direct attack on the solution. Despite the brute-force power, scale invariance is hard to track or reach on the lattice for reasons obvious to all.

(e) Holography. Its reach is limited (large N, strong coupling), but it is wide enough to give effective theories for infinite numbers of scale invariant of scale-covariant QFTs. Much of the recent progress in understanding CFTs stems or was motivated from investigations in holography.

It is interesting to compare (d) and (e) that are as complementary as they can be:

• Lattice starts with a hard breaking of scale invariance and struggles to find it as a needle in the haystack of numerical data.

• Holography is by construction tuned to describe (near) scale invariant theories but its regime of validity can be seriously restrictive.

• I have argued for several years that a judicious collaboration between the two techniques can bring results that go much further that any of the two techniques alone.

Quantum Phase transitions

• Another issue that may be important for physics beyond the standard model relates to quantum phase transitions.

• They are phase transitions that happens at $T = 0$ while changing some parameter of the theory.

• They are extremely difficult to discern.

• They are rare, as most phase transitions happen at $T \neq 0$.

• In the context of 4d QFT they have been very little understood as most happen at strong coupling.

• A known and studied case is the conformal transition in QCD. Its relevance had been advocated long time ago for BSM physics.

• But there can be many others.

The plan

• My contribution to these question will be modest:

• I will study them in the context of the closest to us theory where they are realized: QCD with N_c colors and N_f flavors.

• I will use holographic techniques to study the theory at strong coupling using a class of holographic theories known as V-QCD.

For the rest:

• The Veneziano limit

• The conformal window and the conformal phase transition

• The holographic realization: V-QCD

• The dynamics and properties of the conformal transition.

• Outlook

The Veneziano limit

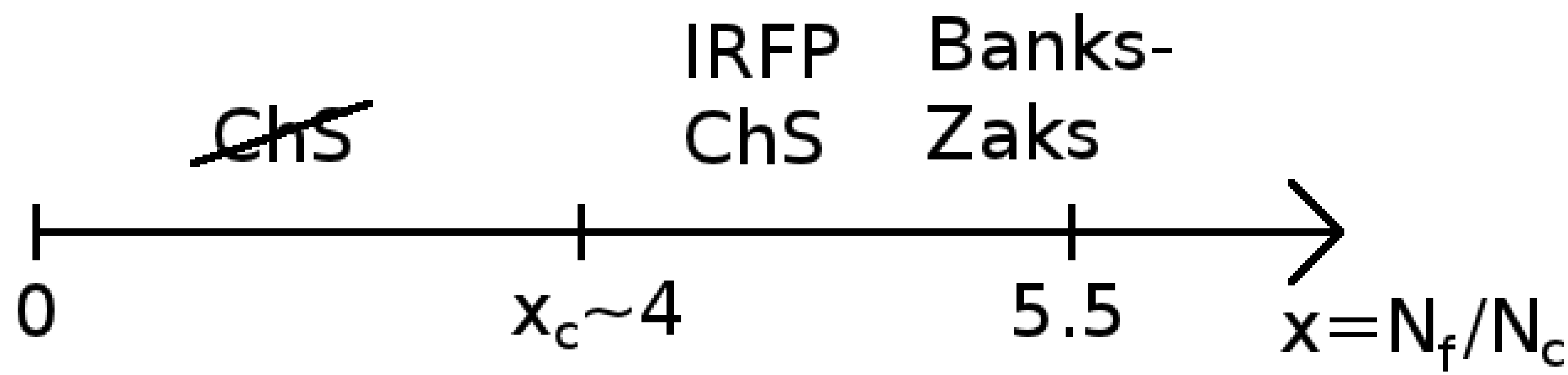
• The proper limit in order to study the previous phenomena in the large N_c approximation is the limit introduced by Veneziano in (1974)

$$N_c \to \infty \quad , \quad N_f \to \infty \quad , \quad \frac{N_f}{N_c} = x \to \text{fixed} \quad , \quad \lambda = g^2_{\text{YM}} N_c \to \text{fixed}$$

• In terms of the dual string theory, the boundaries of string diagrams are not suppressed anymore: $\frac{N_f}{N_c} \sim \mathcal{O}(1)$ and surfaces with an arbitrary number of boundaries contribute at the same order in $1/N_c$.

• The 4d theory provides a formidable (and so far unsolved) problem, much harder than in the 't Hooft limit.

The expected phase diagram



♠ QCD regime $(x \simeq 1,\ x < x_c)$ not close to x_c.

♠ Walking regime: $x \to x_c^-$

♠ Conformal window: $\frac{11}{2} > x > x_c$.

♠ Banks-Zaks regime: $x \to \frac{11}{2}^-$.

Ending the conformal window

How can a conformal window end?

Kaplan+Son+Stephanov (2008)

1) A fixed point moves off to zero coupling. (BZ)

2) A fixed point moves off to infinite coupling (Dual BZ, happens in N=1 super QCD)

3) Two fixed points collide and move off in the complex plane. The only known realization is in the BKT transition in 2d.

• In large-N theories there is a GENERIC mechanism for this to happen: when the BF bound is about to be violated by a scalar operator!

$$\Delta_\pm = \frac{d}{2} \pm \sqrt{\frac{d^2}{4} + m^2\ell^2}$$

• For relevant operators, $m^2 < 0$. For real Δ, $m^2\ell^2 > -\frac{d^2}{4}$ (BF bound).

$$\Delta_\pm = \frac{d}{2} \pm \sqrt{\frac{d^2}{4} + m^2\ell^2}$$

- Case I: When $\frac{d-2}{2} < \Delta < \frac{d}{2}$ the correct branch is Δ_-.

- Case II: When $\frac{d}{2} < \Delta < \frac{d}{2} + 1$ the correct branch is Δ_+.

- In the regime $\mathrm{I} \cup \mathrm{II}$, for each bulk m there are two possible values of Δ.

- In a large N theory where $O(x)$ is in case I: $\frac{d-2}{2} < \Delta < \frac{d}{2}$, then $O(x)^2$ is a relevant operator.
- Perturbing the CFT by O^2, we flow to a new CFT that is isomorphic to the previous one with one difference: O now has dimension $d - \Delta$ (and is in Case II)

Witten

- As $m^2\ell^2 \to -\frac{d^2}{4}$, then $\Delta_\pm \to \frac{d}{2}$ and then they become complex:

$$\Delta_\pm = \frac{d}{2} \pm i\nu \quad , \quad \nu = \sqrt{-m^2\ell^2 - \frac{d^2}{4}}$$

- Exactly at the BF bound, the operator O^2 is marginal classically (and really marginally relevant).

Below the BF bound: BKT scaling

• There is a correlation of the violation of BF bound and the conformal phase transition

• For $\Delta_{\rm IR}(4-\Delta_{\rm IR}) < 4$

$$\phi(r) \sim m_q \ r^{4-\Delta_{\rm IR}} + \sigma \ r^{\Delta_{\rm IR}} + \cdots$$

• For $\Delta_{\rm IR}(4-\Delta_{\rm IR}) > 4$

$$\phi(r) \sim C \ r^2 \sin\left[(\nu)\log r + \chi\right] + \cdots \quad , \quad \nu = {\rm Im}\Delta_{\rm IR}$$

Two possibilities:

• $x > x_c$: BF bound satisfied at the fixed point $\Rightarrow$ only trivial massless solution ($\phi \equiv 0$, fixed point hit)

• $x < x_c$: BF bound violated at the fixed point $\Rightarrow$ a nontrivial solution exists where the bulk field ϕ dual to the operator O, drives the system away from the fixed point.

• The parameter $\nu \geq 0$ above controls the violation of the BF-bound.

• When $\nu=0$, there is no BF-bound violation and everything is "normal".

• When $\nu > 0$, there is BF-bound violation and therefore the structure of the solution radically changes: the scalar, even without a source, starts "running" (the dual operator acquires a vev) and the IR changes.

• When $\nu \ll 1$ we are in the walking region for the coupling dual to ϕ.

Conclusion: There is a *quantum phase transition* at the point where the BF bound is violated

• Above the BF bound, if the source of ϕ is zero, the vev is also usually zero.

$$\phi \simeq \phi_0 \; r^{\Delta_-} + \sigma \; r^{\Delta_+} + \cdots \quad , \quad r \to 0$$

• Below the BF bound, even if $\phi_0 = 0$, $\sigma \neq 0$. There are three regimes:

a) UV regime: $0 < r \ll \Lambda_{UV}^{-1}$

$$\phi \simeq \sigma \; r^{\Delta_+} + \cdots$$

b) Intermediate quasi-conformal regime : $\Lambda_{UV}^{-1} \ll r \ll \Lambda_{IR}^{-1}$

$$\phi \simeq C \; r^2 \sin\left(\nu \log\left(r\right) + \chi\right)$$

but still $\phi \ll 1$.

c) Ultra IR regime: $r \gg \Lambda_{IR}^{-1}$ and $\phi \gtrsim 1$

• When $\nu \to 0$ we can estimate the end points by asking:

• Continuity at $r = \Lambda_{UV}^{-1}$

$$\cot\left[\nu \log(\Lambda_{UV}^{-1}) + \chi\right] = \frac{\Delta^+ - 2}{\nu} \quad \to \quad \nu \log(\Lambda_{UV}^{-1}) + \chi \simeq \pi$$

$$C \simeq \frac{(\Delta_+ - 2)}{\nu} \sigma \Lambda_{UV}^{(\Delta_+ - 2)}$$

- $\phi \simeq 1$ at $r = \Lambda_{IR}^{-1}$

We obtain from these conditions:

$$\frac{\sigma}{\Lambda_{UV}^{\Delta_+}} \quad \sim \quad \frac{\Lambda_{IR}^2}{\Lambda_{UV}^2} \quad \sim \quad e^{-\frac{2\pi}{\nu}}$$

- This is BKT scaling

- This is generic to any violation of the BF bound.

- Even if the scalar operator is irrelevant in the UV, it will still acquire a non-zero vev and will drive the system far away from the fixed point:

This is a concrete and generic example of a UV irrelevant operator becoming relevant due to non-perturbative effects.

- In QCD, the operator that drives the conformal transition is the quark-mass operator. Its dimension in the IR CFT starts at 3 in the BZ limit and ends with 2 at the conformal transition.

The Efimov Spiral

• Consider now in the walking region ($\nu \ll 1$), the two linearly independent solutions associated with source and vev ($\phi_m \simeq m_q r$ and $\phi_\sigma \simeq \sigma r^3$ as $r \to 0$). They solve the linearized equations as $\phi \ll 1$.

$$\phi_m \simeq \frac{m_q}{\Lambda_{UV}} K_m (r\Lambda_{UV})^2 \sin\left[\nu \log(r\Lambda_{UV}) + \phi_m\right]$$

$$\phi_\sigma \simeq \frac{\sigma}{\Lambda_{UV}^3} K_\sigma (r\Lambda_{UV})^2 \sin\left[\nu \log(r\Lambda_{UV}) + \phi_\sigma\right]$$

• In order to reach in the IR the regular solution only a fixed behavior is allowed

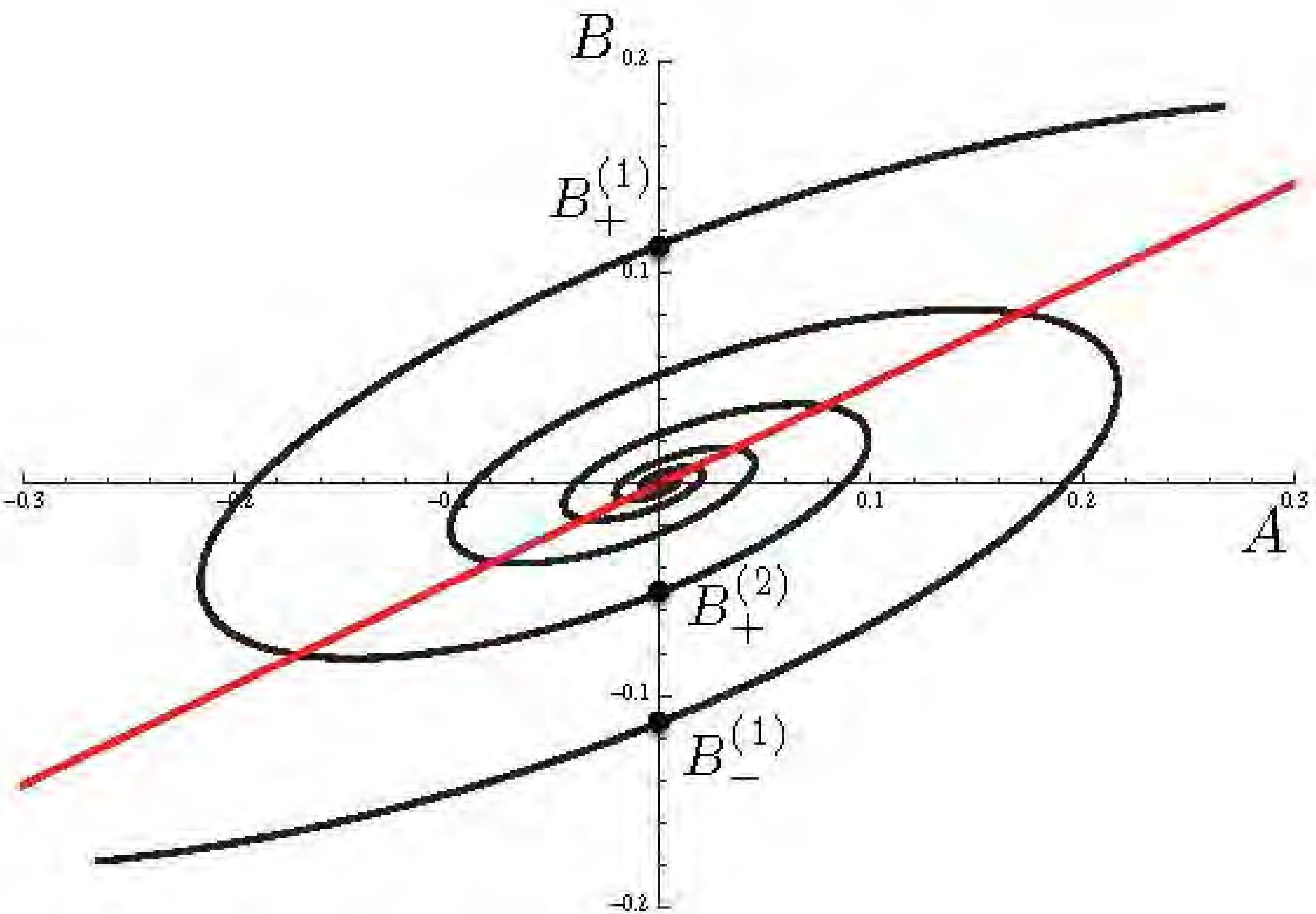
$$\phi_{IR} \simeq K_{IR}(r\Lambda_{IR})^2 \sin\left[\nu \log(r\Lambda_{IR}) + \phi_{IR}\right] \quad , \quad \Lambda_{UV}^{-1} \ll r \ll \Lambda_{IR}^{-1}$$

• In the intermediate region we must have $\tau_m + \tau_\sigma \simeq \tau_{IR}$ from which we obtain

$$\frac{m_q}{\Lambda_{UV}} = \frac{K_{IR}}{K_m} \; \frac{\sin(\phi_{IR} - \phi_\sigma - \nu w)}{\sin(\phi_m - \phi_\sigma)} e^{-2w} \quad , \quad e^w \equiv \frac{\Lambda_{UV}}{\Lambda_{IR}}$$

$$\frac{\sigma}{\Lambda_{UV}^3} = \frac{K_{IR}}{K_\sigma} \; \frac{\sin(\phi_{IR} - \phi_m - \nu w)}{\sin(\phi_\sigma - \phi_m)} e^{-2w}$$

• The only thing that varies in the left hand side is w. If we plot m_q, σ we obtain the Efimov spiral.



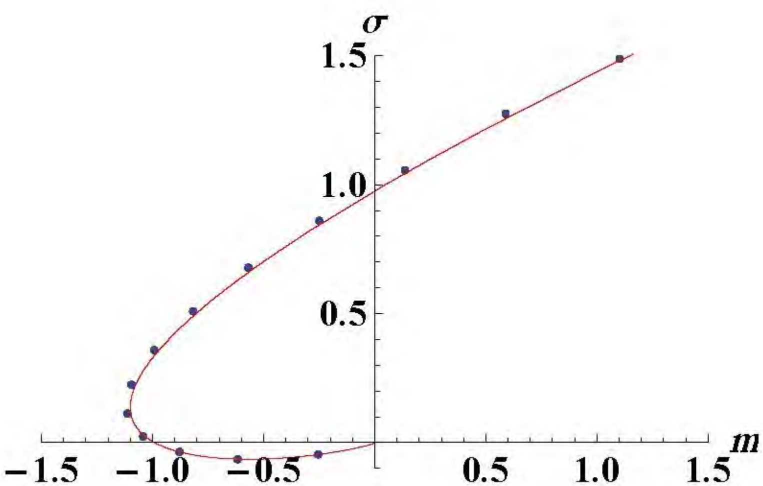
• The straight line is the limit of the spiral as we approach the transition, $\nu \to 0$.

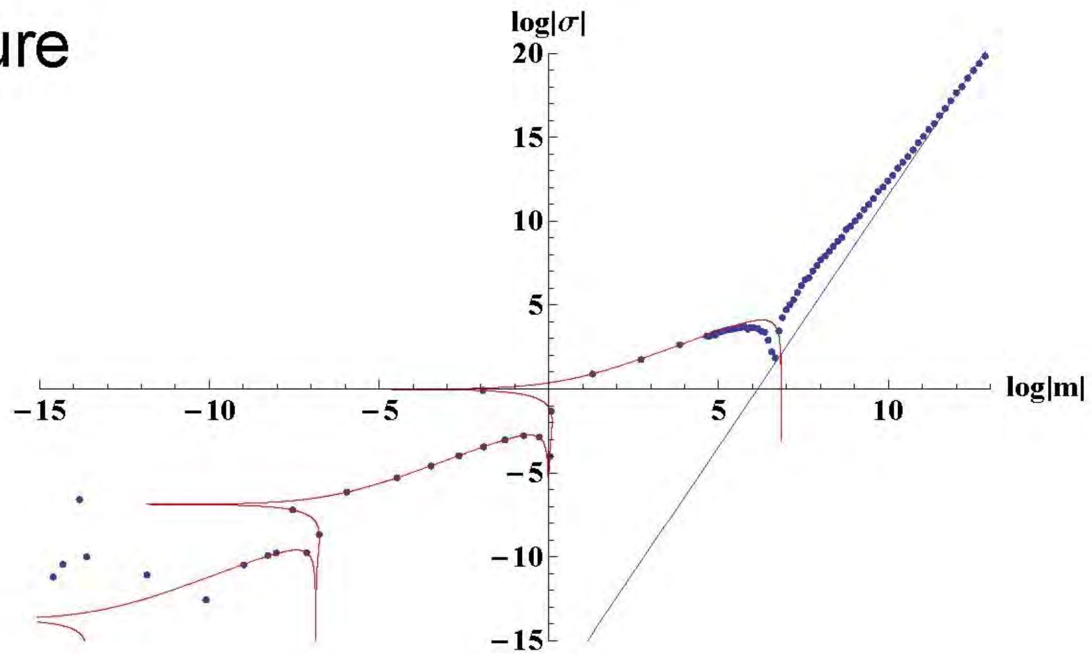
• The susceptibility is discontinuous across the transition.

Efimov spiral

Ongoing work: $\sigma(m)$ dependence

❑ For $x < x_c$ spiral structure





The strategy

• Construct (toy) holographic models resembling QCD in the Veneziano limit. The goal is to investigate the phase diagrams and get input about the (exotic) phenomena that could appear.

• Put together two ingredients: the holographic model for glue developed earlier: IHQCD

Gursoy+E.K+Nitti, Gursoy+E.K.+Mazzanti+Nitti

• and the model for flavor based in Sen's tachyon action in string theory (brane-antibrane pairs).

Casero+E.K.+Paredes, Iatrakis+E.K.+Paredes

Drawbacks:

a) No controlled stringy construction of the background.

b) Quantum effects from light states are unsuppressed (and need to be taken eventually into account).

The holographic models: glue

For YM, ihQCD is a well-tested holographic, string-inspired bottom-up model with action

Gursoy+Kiritsis+Nitti, Gubser+Nelore

$$\mathcal{S}_{\rm g} = M^3 N_c^2 \int d^5x \sqrt{g} \left[R - \frac{4}{3}(\partial\phi)^2 + V_g(\phi) \right]$$

and "vacuum" described by a Poincaré-invariant metric and running dilaton (gauge coupling)

$$ds^2 = e^{2A(r)}(dr^2 + \eta_{\mu\nu}dx^\mu dx^\nu)$$

- The potential V_g $\leftrightarrow$ QCD β-function
- the "scale factor" A $\leftrightarrow$ $\log\mu$ energy scale.
- e^ϕ $\leftrightarrow$ λ 't Hooft coupling

• The UV and IT asymptotics of V_g can be fixed from first principles and the rest of the potential parameterized in terms of two phenomenological parameters that are fit to lattice data.

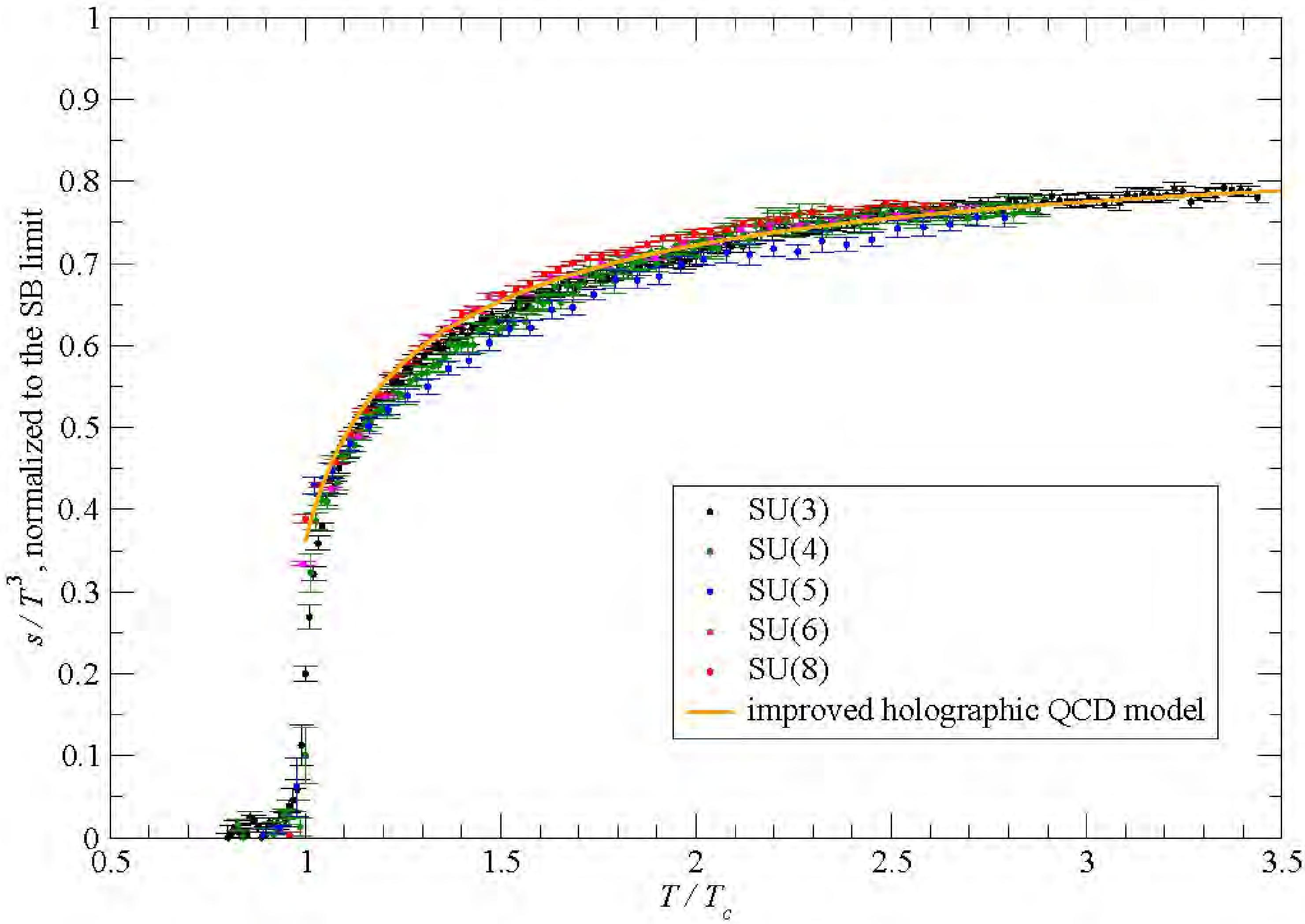


Figure 4: (Color online) Same as in fig. 1, but for the s/T^3 ratio, normalized to the SB limit.

Panero

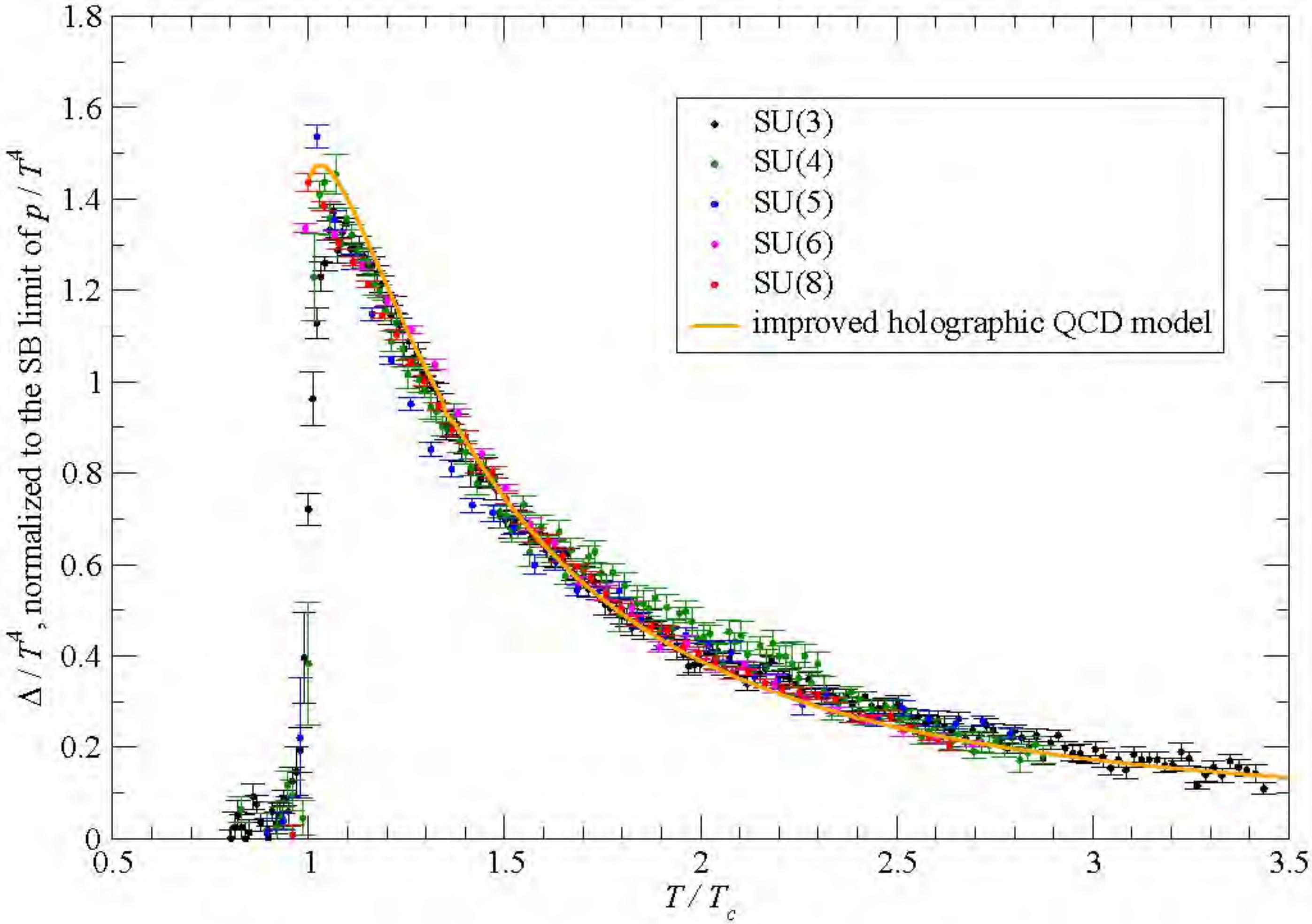
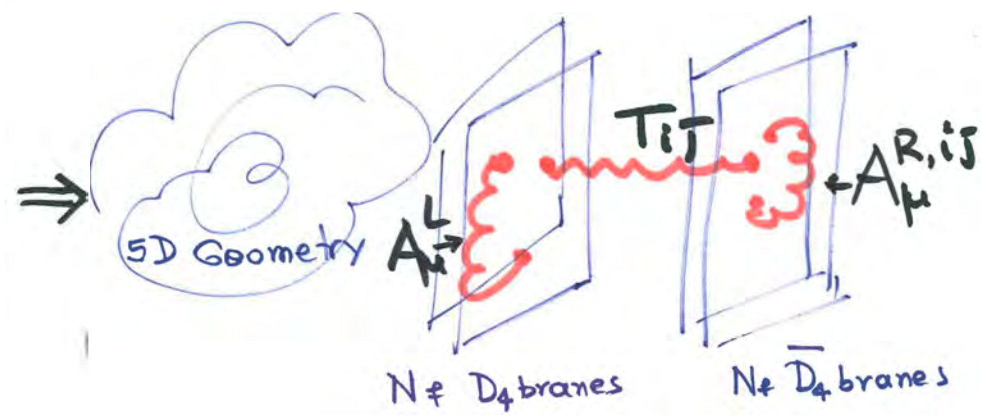


Figure 2: (Color online) Same as in fig. 1, but for the Δ/T^4 ratio, normalized to the SB limit of p/T^4.

Panero

The holographic models: flavor



- There are three important operators in the flavor sector,

$$J_\mu^{L,R} = \bar{q}^{L,R}\ \gamma_\mu\ q^{L,R} \quad , \quad \bar{q}_L q_R$$

and their dual fields: $A_\mu^{L,R}$, T that realize the global $U(N_f)_L \times U(N_f)_R$ symmetry.

- An action for the tachyon was given by Sen and has been advocated as the proper dynamics of the chiral condensate giving in general all the expected features of χSB.

Casero+Kiritsis+Paredes

$$\mathcal{S}_{\mathrm{TDBI}} = -N_f N_c M^3 \int d^5x\ V_f(T)\ e^{-\phi} \sqrt{-\det(g_{ab} + \partial_a T \partial_b T + F_{ab})}$$

$$V(T) = V_0 e^{-aT^2}$$

Kutasov+Marino+Moore
Kraus+Larsen
Takayanagi+Terashima+Uesugi

Fusion

The idea is to put together the two ingredients in order to study the chiral dynamics and its backreaction to glue.

$$\mathcal{S} = N_c^2 \; M^3 \int d^5x \sqrt{g} \left[R - \frac{4}{3} \frac{(\partial\lambda)^2}{\lambda^2} + V_g(\lambda) \right] -$$

$$- N_f N_c \; M^3 \int d^5x V_f(\lambda, T) \sqrt{- \det(g_{ab} + h(\lambda) \partial_a T \partial_b T)}$$

with the "adiabatic ansatz"

$$V_f(\lambda, T) = V_0(\lambda) \; \exp(-a(\lambda) T^2)$$

- We must choose $V_0(\lambda), a(\lambda), h(\lambda)$. How to determine them?

- In the UV ($\lambda \to 0$), they can be adapted to the QCD β function and mass anomalous dimension (two loops).

- Overall the theory has a metric, two $SU(N_f)$ gauge fields a real scalar ϕ and a complex matrix scalar T.

• In the IR, we need to scan all possibilities: We impose

(a) Existence of a fixed point in the potential in the theory without chiral symmetry breaking.

(b) Asymptotically linear trajectories for mesons.

• The surprise: $V_0(\lambda), a(\lambda), h(\lambda)$ have as $\lambda \to \infty$ the values they have in naive flat space string theory corrected by logs.

$$V_0(\lambda) \sim \lambda^{\frac{7}{3}} \quad , \quad a(\lambda) \sim \lambda^0 \quad , \quad h(\lambda) \sim \lambda^{-\frac{4}{3}} \quad , \quad V_g(\lambda) \sim \lambda^{\frac{4}{3}}$$

• Most of the qualitative physics depends very little on the intermediate regime in λ.

• For every x there are two extrema of the potentials:

♠ $T_* = 0$, we have an IR fixed point at $\lambda = \lambda_*(x_f)$.

♠ $T_* = \infty$, $V_{eff} = V_g(\lambda)$ with no fixed points.

Parameters

• A theory with a single relevant (or marginally relevant) coupling like YM has no parameters.

• The same applies to QCD with massless quarks.

• QCD with all quarks having mass m has a single (dimensionless) parameter : $\frac{m}{\Lambda_{QCD}}$.

• After various rescalings this single parameter can be mapped to the parameter T_0 (integration constant) that controls the diverging tachyon in the IR.

• There is also $x = \frac{N_f}{N_c}$ that has become continuous in the large N_c Veneziano limit.

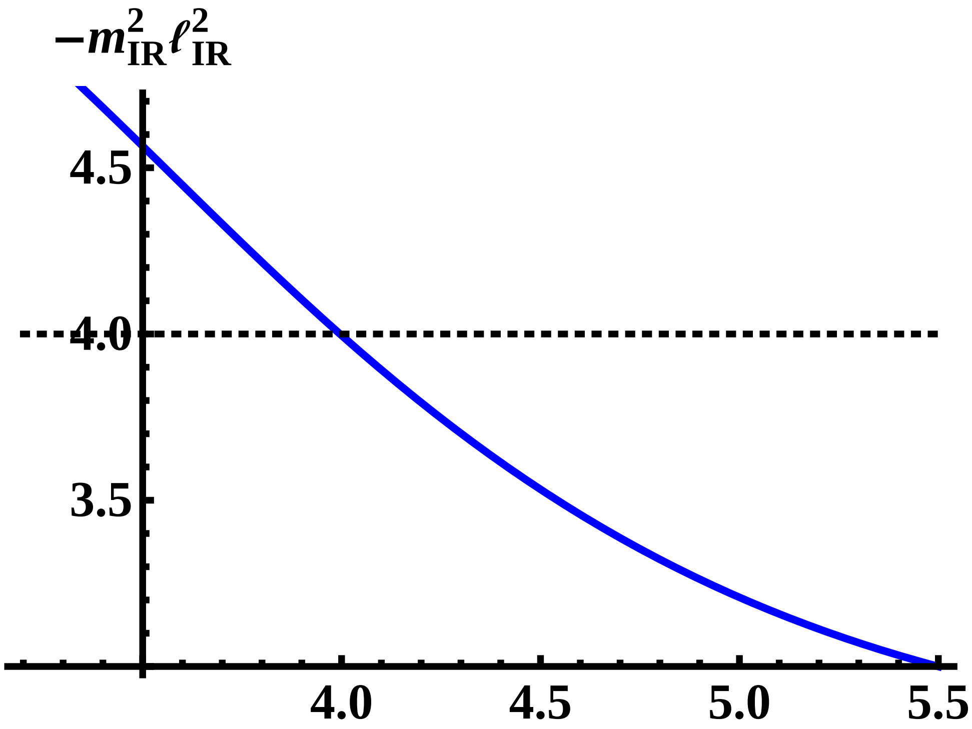
The lower end of the conformal window

• By expanding the DBI action we obtain the IR tachyon mass at the IR fixed point $\lambda = \lambda_*$ which gives the chiral condensate dimension:

$$-m_{\rm IR}^2 \ell_{\rm IR}^2 = \Delta_{\rm IR}(4 - \Delta_{\rm IR})$$

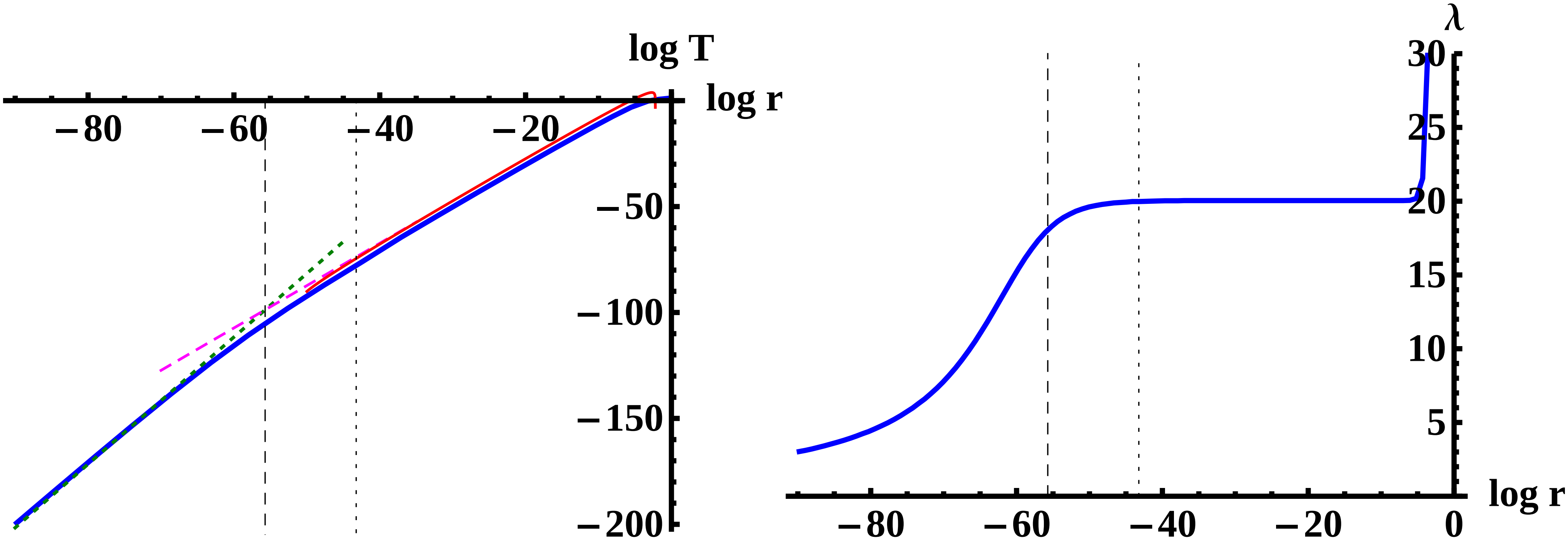
• Must reach the Breitenlohner-Freedman (BF) bound (horizontal line) at some x_c.

• x_c marks the *conformal phase transition*



We obtain: $3.7 \lesssim x_c \lesssim 4.2$

Walking

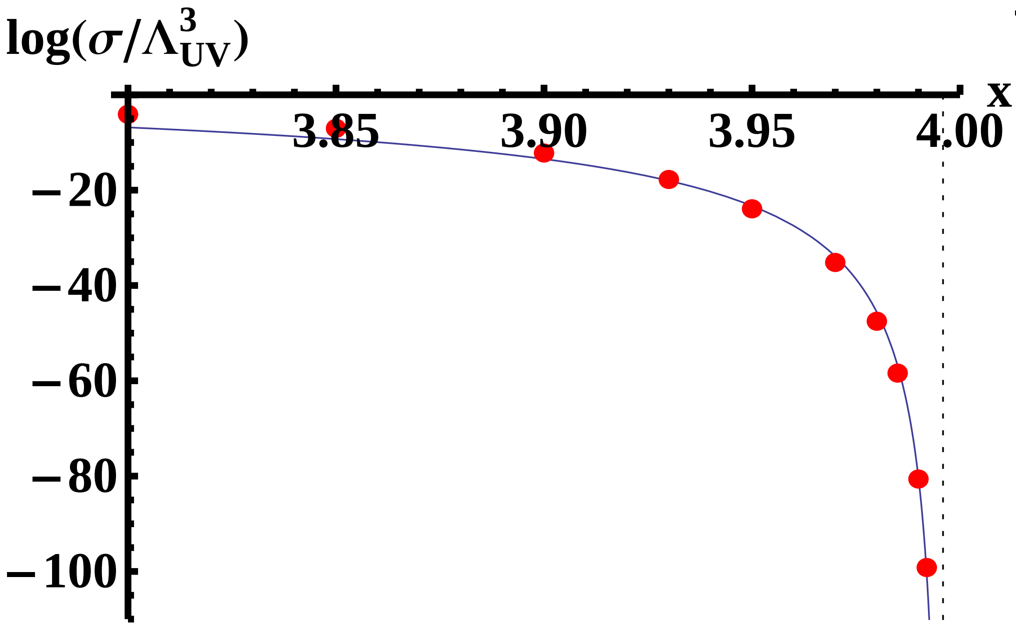


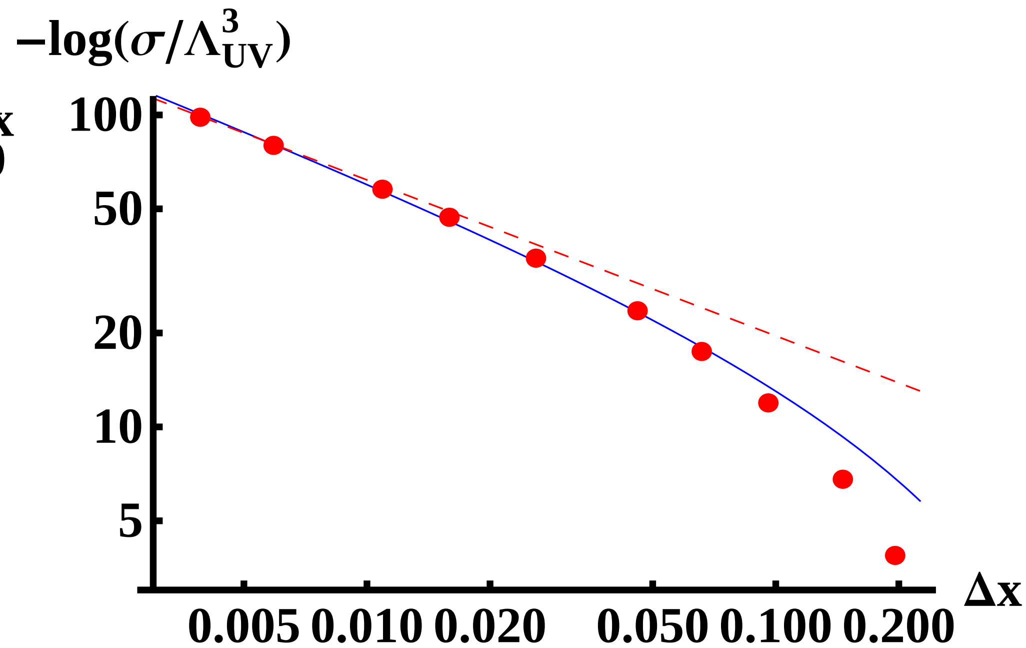
The tachyon $\log T$ (left) and the coupling λ (right) as functions of $\log r$ for an extreme walking background with $x = 3.992$. The thin lines on the left hand plot are the approximations used to derive the BKT scaling.

BKT/Miransky scaling

We obtain BKT-Miransky scaling:

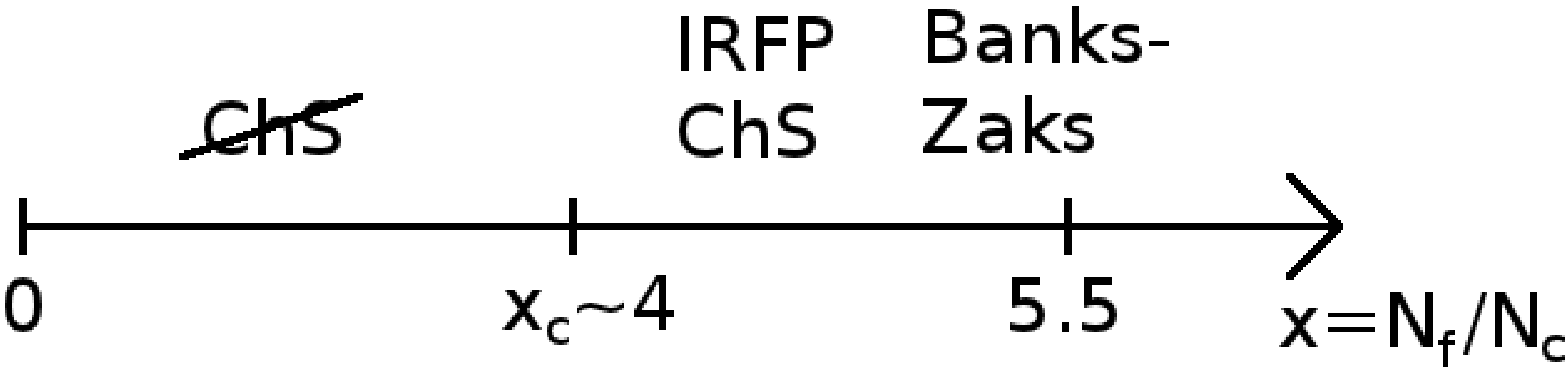
$$\sigma \sim \Lambda_{\text{UV}}^3 \ \exp\left(-\frac{2\hat{K}}{\sqrt{x_c - x}}\right) \ .$$





Left: $\log(\sigma/\Lambda^3)$ as a function of x (dots), compared to a BKT scaling fit (solid line). The vertical dotted line lies at $x = x_c$. Right: the same curve on log-log scale, using $\Delta x = x_c - x$.

Recap



• For $x = 0$, the theory has a mass gap, and confines.

• $0 < x < x_c \simeq 4$ the theory has chiral symmetry breaking, massless pions, and gapped spectrum otherwise.

• $x_c < x < \frac{11}{2}$ the theory is chirally symmetric, and flows to a non-trivial fixed point in the IR.

Spectra

• The main difference from all previous calculations is that here flavor back reacts on color.

• In the singlet sector the glueballs and mesons mix to leading order and the spectral problem becomes complicated.

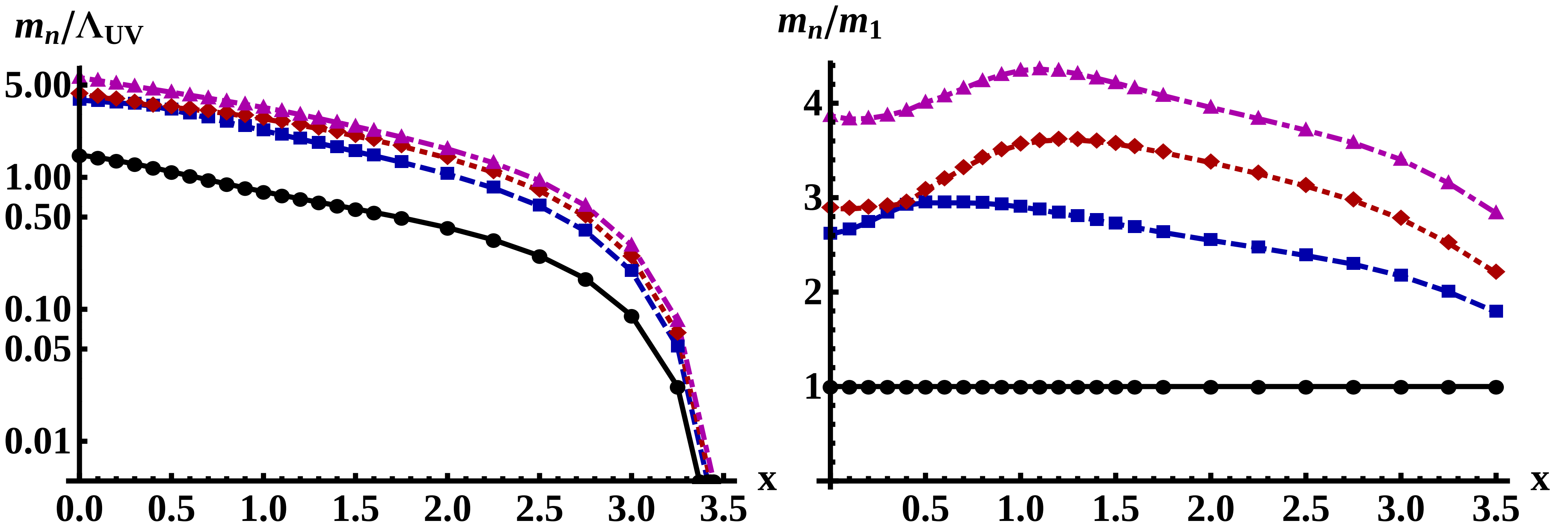
• The conclusions are:

♠ All masses follow Miransky scaling in the walking region.

♠ There is no dilaton.Instead all (bound-state) masses go to zero exponentially fast.

♠ There are several level crossings as x varies but they seem accidental

♠ There is a subtle (and unexpected) discontinuity associated with the S-parameter.



Singlet scalar meson spectra in the potential II class with SB normalization for W_0. They contain the 0^{++} glueballs and the singlet 0^{++} mesons that mix here at leading order. Left: the four lowest masses as a function of x in units of Λ_{UV}. Right: the ratios of masses of up to the fourth massive states as a function of x.

The S parameter: definitions

For non-zero quark mass we have

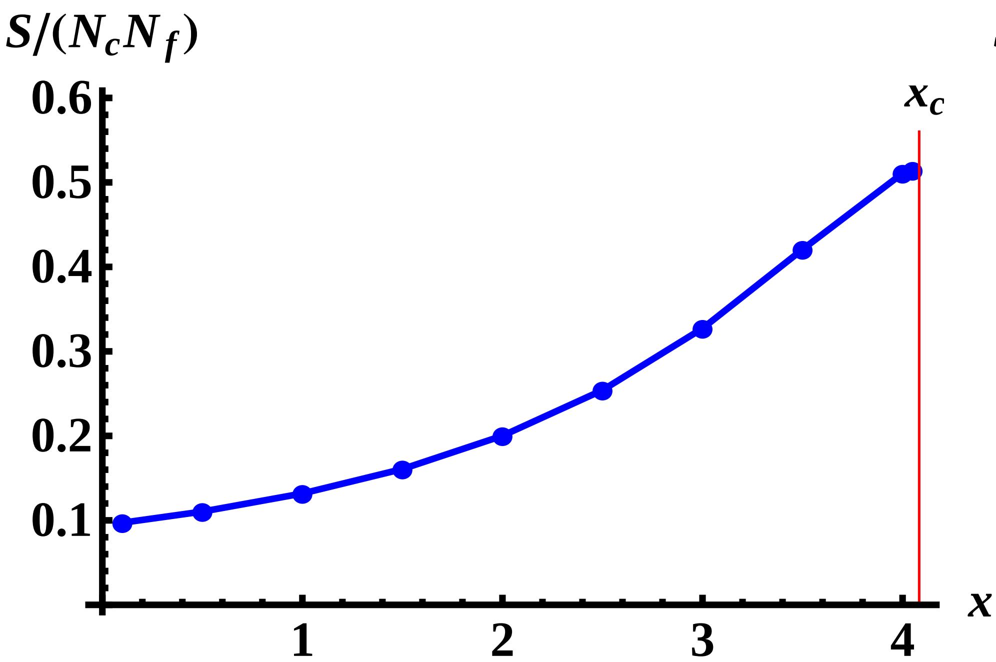
$$i\langle J_\mu^{a\,(V)}(q) J_\nu^{b\,(V)}(p)\rangle = -(2\pi)^4\delta^4(p+q)\frac{2\delta^{ab}}{N_f}\left(q^2\eta_{\mu\nu} - q_\mu q_\nu\right)\Pi_V(q^2)$$

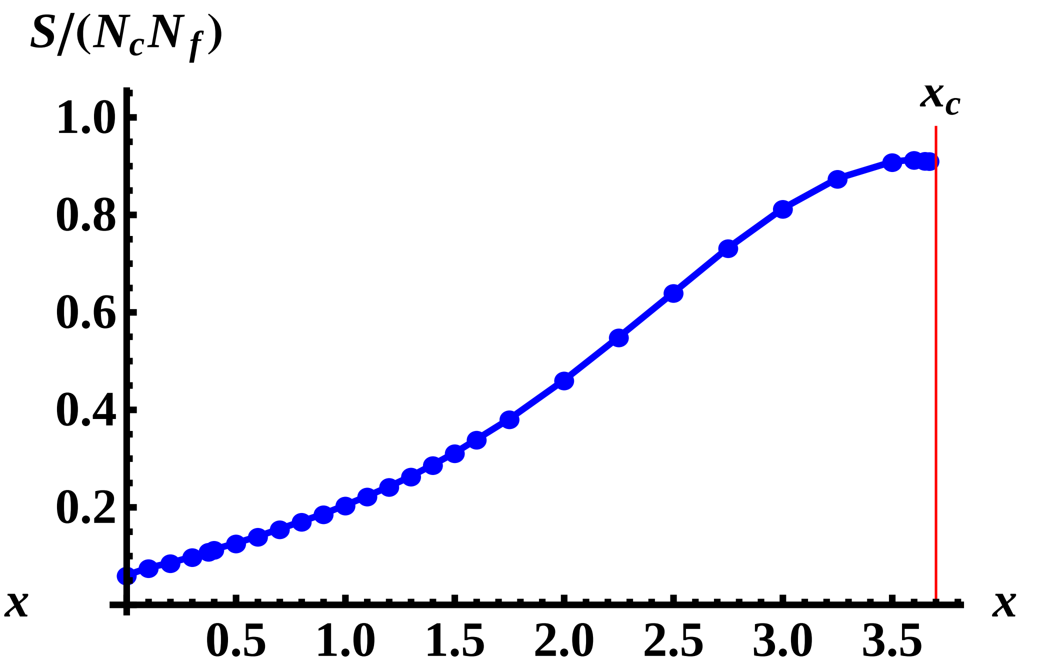
$$i\langle J_\mu^{a\,(A)}(q) J_\nu^{b\,(A)}(p)\rangle = -(2\pi)^4\delta^4(p+q)\frac{2\delta^{ab}}{N_f}\left[\left(q^2\eta_{\mu\nu} - q_\mu q_\nu\right)\Pi_A(q^2) + q_\mu q_\nu \Pi_L(q^2$$

$$D(q^2) = q^2(\Pi_A(q^2) - \Pi_V(q^2)) \simeq C - \frac{S}{4\pi}q^2 + \frac{S'}{4\pi}q^4 + \cdots$$

• The parameter S controls the renormalization of W-bosons if the SM is coupled to this large-N theory.

The S parameter at $m_q = 0$

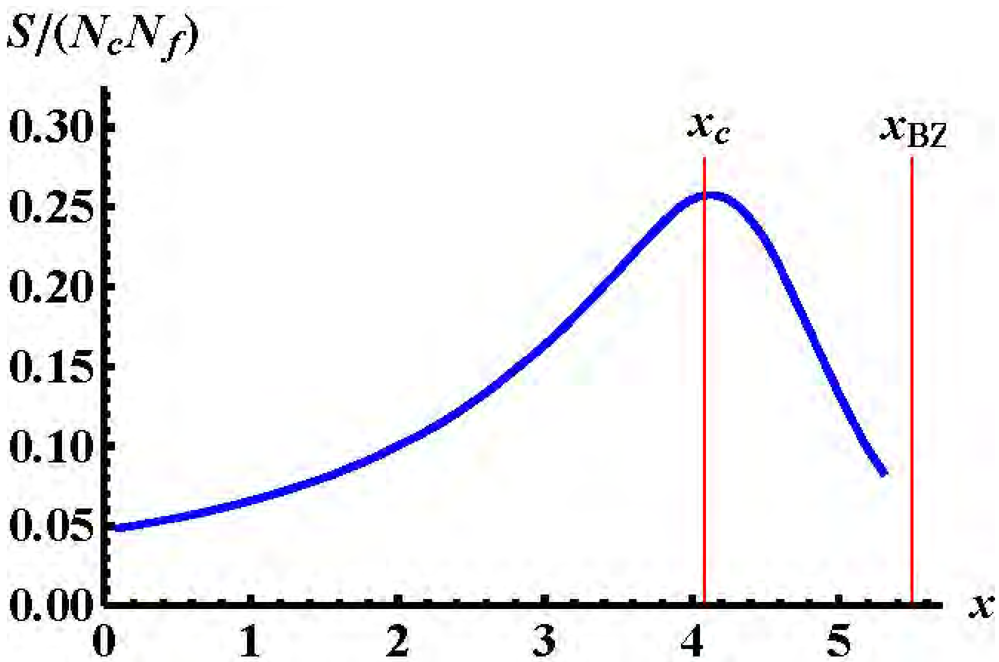


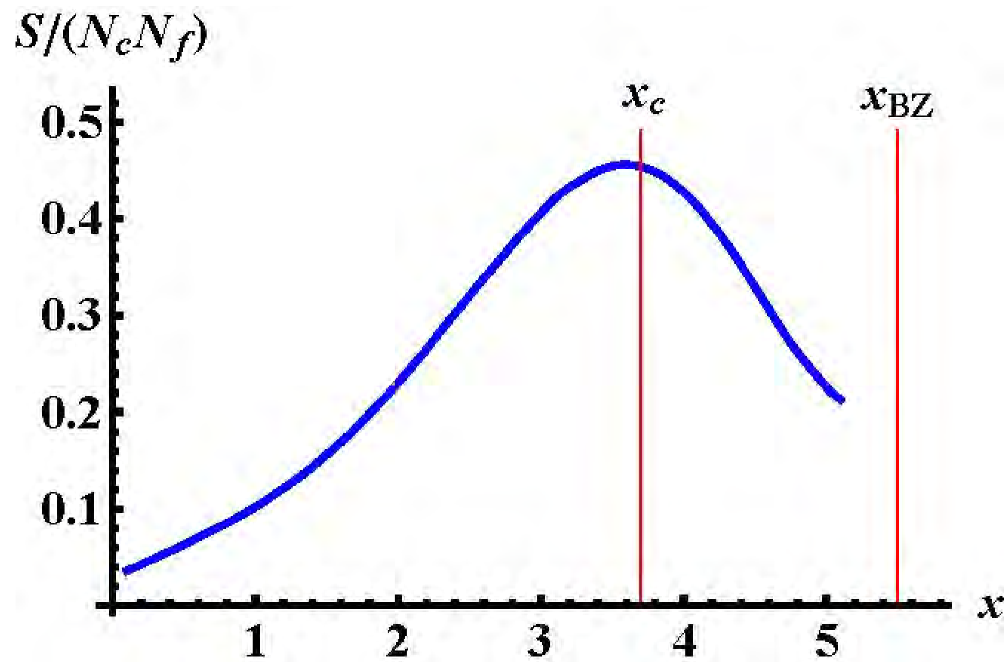


Left: The S-parameter as a function of x for potential class I with $W_0 = \frac{3}{11}$. Right: The S-parameter as a function of x for potential class II with SB normalization for W_0. In both cases S asymptotes to a finite value as $x \to x_c$.

The S parameter at $m_q \neq 0$

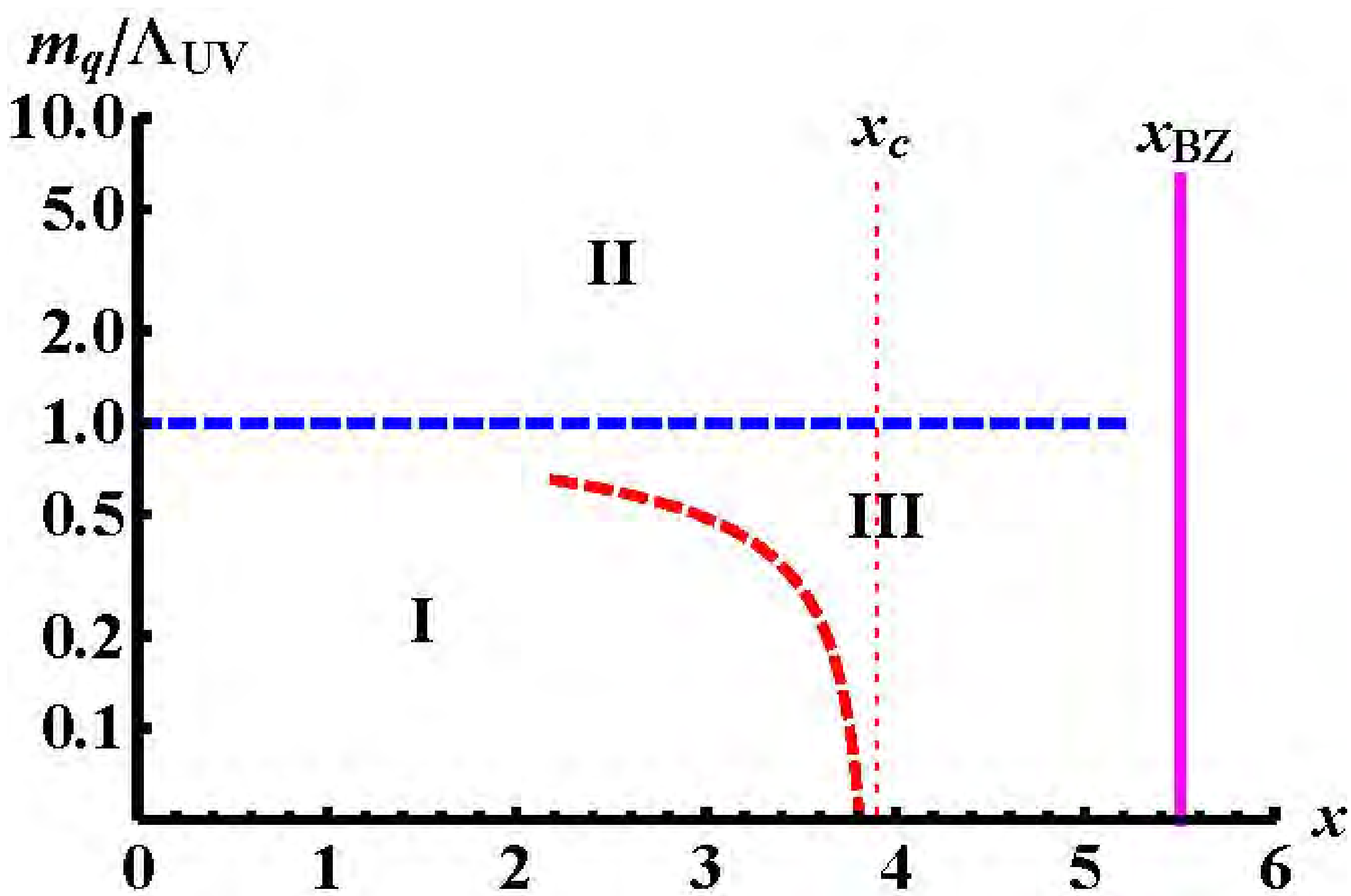
- We turn on a small quark mass $\frac{m_q}{\Lambda_{UV}} = 10^{-6}$ and we calculate C, S, S'





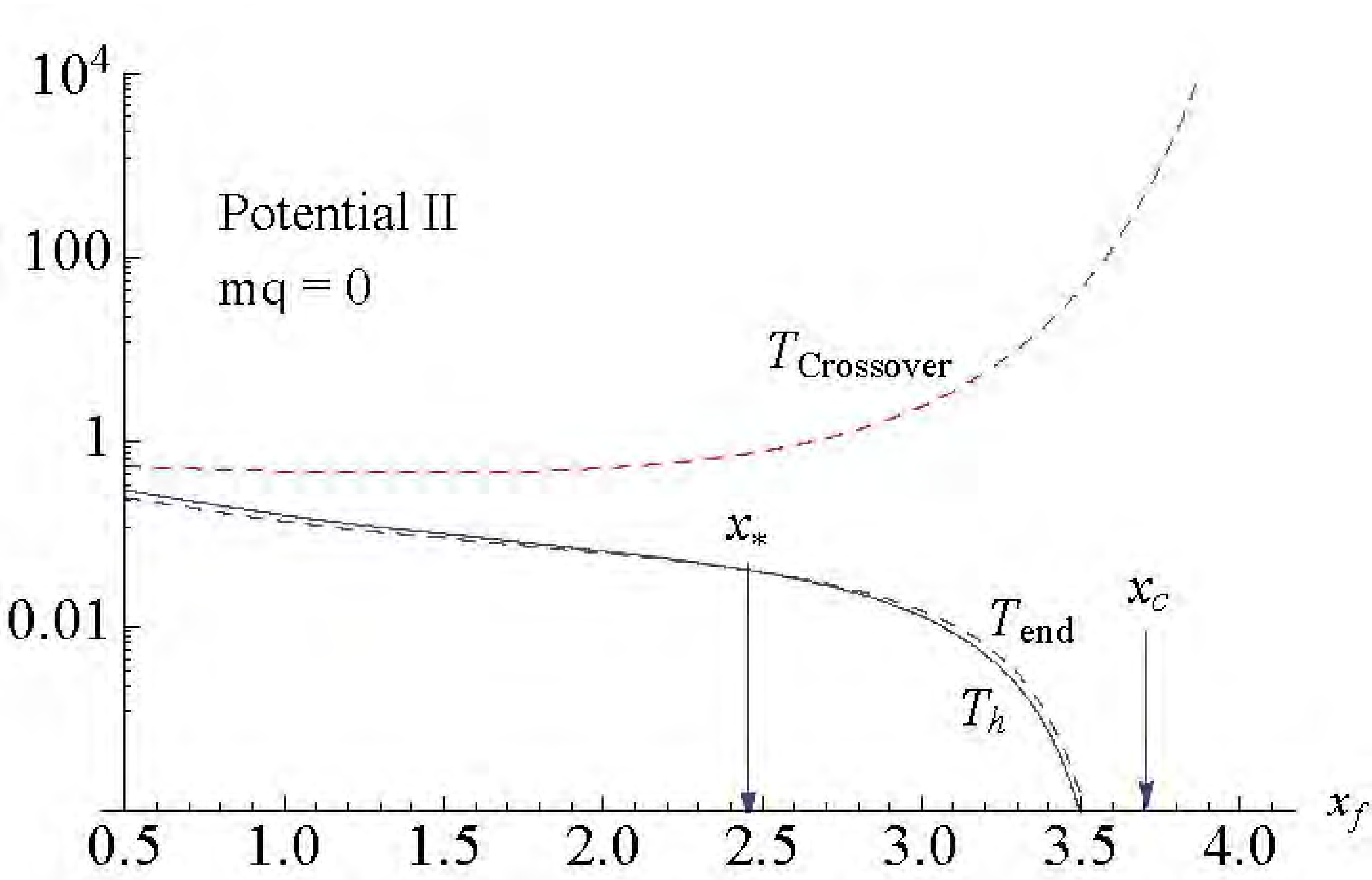
The normalized S-parameter as a function of x for $m_q/\Lambda_{UV} = 10^{-6}$. Left: potentials I with $W_0 = 3/11$. Right: potentials II with SB normalized W_0.

Mass scales in V-QCD: non-zero quark masses



• There are related interesting scaling laws at finite but small mass, that M. Jarvinen will discuss in his presentation. They might be very useful in find a conformal window on the lattice. The analysis of four-fermi couplings is also important.

The phase diagram



Finite density

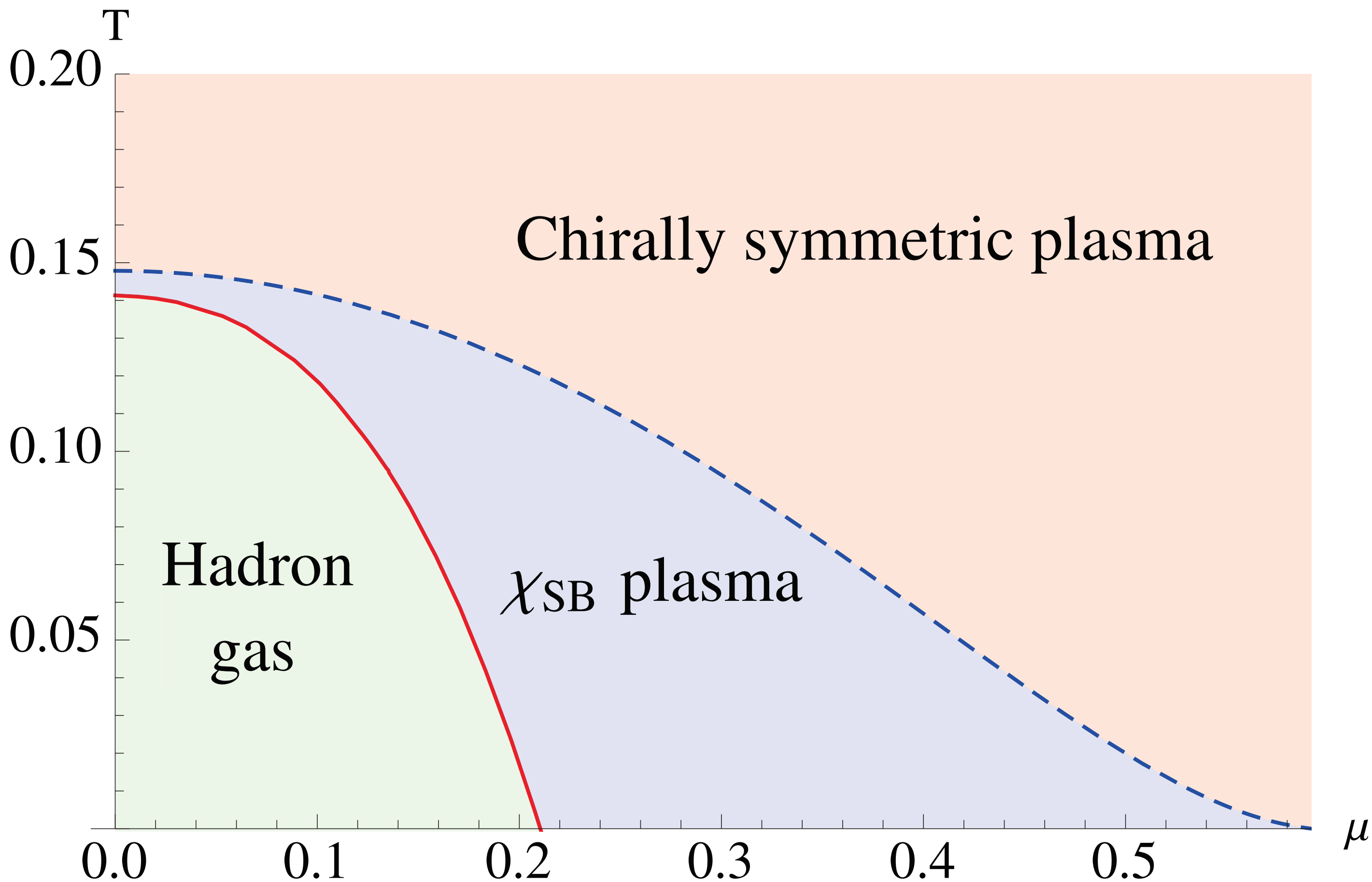
• In the presence of quarks there is no order parameter for deconfinement.

• In the presence of massless quarks there is an order parameter for chiral symmetry breaking (chiral condensate).

• At large N_c, there is a criterion for deconfinement: Whether the free energy is $\mathcal{O}(1)$ or $\mathcal{O}(N_c)^2$. In a black hole phase the theory is "deconfined".

Conclusion: In the Veneziano limit (a) there could be two phase transitions (deconfinement+chiral restoration) (b) the phase diagram may be qualitatively different from finite N_f case.

• For example, for a V-QCD model with $x = 1$ we have found



Alho+Jarvinen+Kajantie+Kiritsis+Rosen+Tuominen

• No baryon backreaction is included

• No gravity loop-corrections are included

• No tuning of parameters to lattice data.

A new quantum critical regime

- The most remarkable new feature is a new quantum critical regime at $T = 0$ and finite μ.

- This is clear in the (cold) quark gluon plasma phase at $\mu/\Lambda > 0.5$.

- It seems also to be present in the chirally-broken plasma phase.

- This is a phase with a $AdS_2 \times R^3$ geometry as in the RN black hole. Spacial points cannot communicate, it is like the speed of light is equal to 0.

- Such critical points are highly unstable, and are expected to give rise to superconducting states.

- Is this an artifact of the large-N_c expansion?

Outlook

• QCD in the Veneziano limit seems to host an interesting collection of exotic phenomena.

• Holographic approaches have given and can give further clues on such phenomena.

• They involve a new frontier in holography: back-reacting branes.

• They also face new and nontrivial problems

(a) Estimating/calculating unsuppressed loop corrections.

(b) Handling explicit (point-like) baryon presence in the vacuum at finite density.

(c) There are many other dynamical problems that need to be addressed (CP-odd physics, chiral magnetic effect, “quantum gravity” effects in the walking region etc)

• There are many other theories with similar or more exotic phenomena awaiting to be discovered

• There is hard work ahead of us.

.

Thank you

Bibliography

Based on recent work with:

Matti Jarvinnen (Crete) arXiv:1112.1261 [hep-ph]

T. Alho (Helsinki), M. Jarvinnen (Crete), K. Kajantie (Helsinki), K. Tuominnen (Helsinki).
arXiv: 1210.4516 [hep-ph]

- D. Arean (Max Planck, Munich) I. Iatrakis, (Stony Brook), M. Jarvinnen (Crete)
arXiv:1211.6125 [hep-ph] arXiv:1309.2286 [hep-ph]

- T. Alho (Helsinki), M. Jarvinnen (Crete), K. Kajantie (Helsinki), C. Rosen (Crete), K. Tuominnen (Helsinki). arXiv:1312.5199 [hep-ph]

and past work with:

- A. Paredes (Barcelona), I. Iatrakis, (Crete)

arXiv:1003.2377 [hep-ph] ; arXiv:1010.1364 [hep-ph]

Bibliography

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The 't Hooft Large N_c limit

- The large N_c limit offers a non-perturbative tool into strong coupling physics.

- The 't Hooft large N_c limit is defined as

$$N_c \to \infty, \qquad \lambda = g^2_{\text{YM}} N_c \to \text{fixed} \quad , \quad N_f \quad \text{fixed}$$

- As N_f is kept fixed while $N_c \to \infty$, $N_f \ll N_c$, $x \to 0$ and it always samples the "quenched" approximation.

- The planar (sphere) diagrams give the dominant contribution, $O(N_c^2)$.

- Fermion loops (boundaries of the Riemann surface in the string theory picture) give subleading contributions in $1/N_c$ (each loop costs a factor of $x = \frac{N_f}{N_c} \ll 1$).

- Because $x \equiv \frac{N_f}{N_c} \to 0$ in the 't Hooft limit, it is difficult to capture any of the effects for which the presence of flavor is important.

Topics

- Conformal transitions, the BF bound and BKT scaling

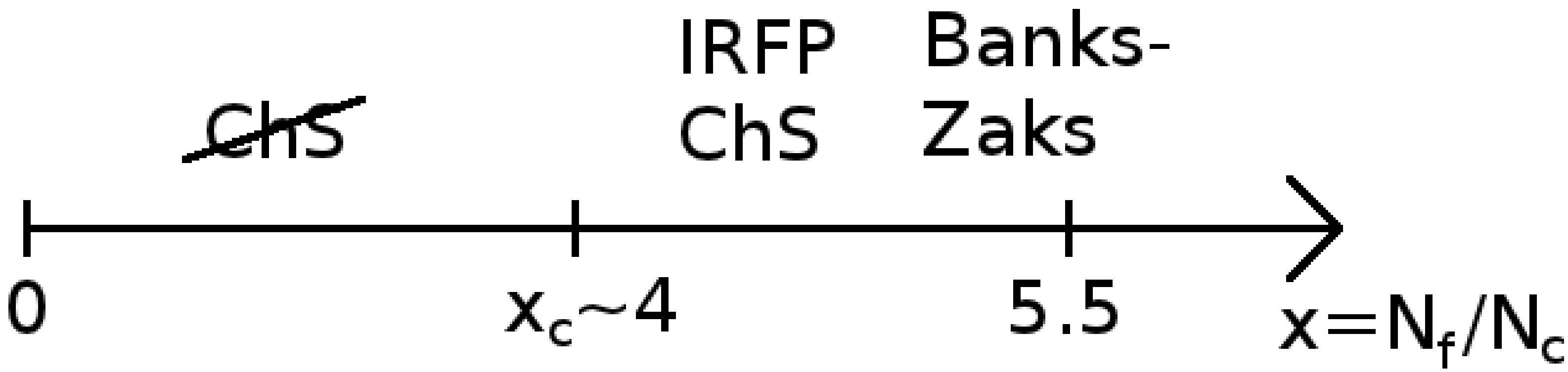
- QCD$_*$

- Comparison to N=1 sQCD

- Massive QCD

- On anomaly matching (?)

Review



♠ QCD regime $(x \simeq 1,\ x < x_c)$ not close to x_c.

♠ Walking regime: $x \to x_c^-$

♠ Conformal window: $\frac{11}{2} > x > x_c$.

♠ Banks-Zaks regime: $x \to \frac{11}{2}^-$.

The Banks-Zaks region

- The (massless) QCD β function in the Veneziano limit is

$$\dot{\lambda} = \beta(\lambda) = -b_0\lambda^2 + b_1\lambda^3 + \mathcal{O}(\lambda^4) \ , \ \ b_0 = \frac{2}{3}\frac{(11-2x)}{(4\pi)^2} \ , \ \ b_1 = -\frac{2}{3}\frac{(34-13x)}{(4\pi)^4}$$

- For $x > \frac{11}{2}$ the theory is IR free. This means it makes only sense as a low energy effective theory (non-abelian weakly coupled phase).

- Notice that at $x = \frac{11}{2}$, $b_0 = 0$, $b_1 > 0$.

- The Banks-Zaks region is

$$x = \frac{11}{2} - \epsilon \quad \text{with} \quad 0 < \epsilon \ll 1$$

- We obtain a fixed point of the β-function at

$$\lambda_* \simeq \frac{(8\pi)^2}{75}\epsilon + \mathcal{O}(\epsilon^2)$$

which is trustworthy in perturbation theory, as λ_* can be made arbitrarily small.

• The mass operator, $\bar{\psi}_L \psi_R$ has now dimension smaller than three, from the perturbative anomalous dimension (in the V-limit)

$$-\frac{d\log m}{d\log\mu} \equiv \gamma = \frac{3}{(4\pi)^2}\lambda + \frac{(203-10x)}{12\,(4\pi)^4}\lambda^2 + \mathcal{O}(\lambda^3, N_c^{-2})$$

• It is the only relevant operator of the theory. If it is turned-on the theory misses the non-trivial fixed point and flows to (massive) pure YM in the IR.

• There are two extreme limits:

(a) $m_q \gg \Lambda_{UV}$. In this case the theory never goes close to the IR fixed point, but for energy scales $E \ll m_q$ is pure YM as the fermions have decoupled.

The IR YM scale is

$$\Lambda_{IR} \simeq m_q \gg \Lambda_{UV}$$

(b) $m_q \ll \Lambda_{UV}$. In this case the theory first flows near the nontrivial fixed point and then flows to YM in the IR.

The IR YM scale is

$$m_q \ll \Lambda_{UV} \quad , \quad \beta = -\epsilon\lambda^2 + b_1\lambda^3 + \cdots \quad , \quad \lambda_* = \frac{\epsilon}{b_1}$$

$$\Lambda_{IR} \simeq m_q \; e^{-\frac{1}{b_0\lambda_*}} \quad ,$$

and we have the large hierarchy of scales

$$\Lambda_{UV} \gg m_q \gg \Lambda_{IR}$$

which is controlled by ϵ. For this to happen the large N_c limit is crucial.

• This is a good example of walking theory, albeit a weakly coupled one.

• It demonstrates a general feature of the conformal window: A "small" perturbation from conformality gives a "walking" theory.

• Extrapolating to lower x we expect

(a) the fixed point to move towards strong coupling.

(b) The quark mass operator to become more relevant.

• The naive extrapolation of these two observations gives the phase diagram:

Interesting question: What marks the transition from Conformality to QCD IR physics?

Ending the conformal window

How can a conformal window end?

Kaplan+Son+Stephanov (2008)

1) A fixed point moves off to zero coupling. (BZ)

2) A fixed point moves off to infinite coupling (Dual BZ, happens in N=1 super QCD)

3) Two fixed points collide and move off in the complex plane. The only known realization is in the BKT transition in 2d.

• At large N theories this happens when the BF bound is about to be violated!

$$\Delta_\pm = \frac{d}{2} \pm \sqrt{\frac{d^2}{4} + m^2\ell^2}$$

• For relevant operators, $m^2 < 0$. For real Δ, $m^2 > -\frac{d^2}{4}$ (BF bound).

$$\Delta_\pm = \frac{d}{2} \pm \sqrt{\frac{d^2}{4} + m^2\ell^2}$$

- Case I: When $\frac{d-2}{2} < \Delta < \frac{d}{2}$ the correct branch is Δ_-.

- Case II: When $\frac{d}{2} < \Delta < \frac{d}{2} + 1$ the correct branch is Δ_+.

- In the regime I+II, for each bulk m there are two possible values of Δ.

- In a large N theory where $O(x)$ is in case I: $\frac{d-2}{2} < \Delta < \frac{d}{2}$, then $O(x)^2$ is a relevant operator. Perturbing the CFT by O^2, we flow to a new CFT that is isomorphic to the previous one with one difference: O now has dimension $d - \Delta$ (and is in Case II)

Witten

- As $m^2 \to -\frac{d^2}{4}$, then $\Delta_\pm \to \frac{d}{2}$ and then they become complex:

$$\Delta_\pm = \frac{d}{2} \pm i\nu \quad , \quad \nu = \sqrt{-m^2\ell^2 - \frac{d^2}{4}}$$

- Exactly at the BF bound, the operator O^2 is marginal classically (and really marginally relevant).

Below the BF bound

• Correlation of the violation of BF bound and the conformal phase transition

• For $\Delta_{\rm IR}(4-\Delta_{\rm IR}) < 4$

$$T(r) \sim m_q r^{4-\Delta_{\rm IR}} + \sigma r^{\Delta_{\rm IR}}$$

• For $\Delta_{\rm IR}(4-\Delta_{\rm IR}) > 4$

$$T(r) \sim C r^2 \sin\left[(\nu)\log r + \phi\right] \quad , \quad \nu = {\rm Im}\Delta_{\rm IR}$$

Two possibilities:

• $x > x_c$: BF bound satisfied at the fixed point ⇒ only trivial massless solution ($T \equiv 0$, ChS intact, fixed point hit)

• $x < x_c$: BF bound violated at the fixed point ⇒ a nontrivial solution exists where the bulk field ϕ dual to the operator O, drives the system away from the fixed point.

Conclusion: There is a *phase transition* at the point where the BF bound is violated

• Above the BF bound, if the source of ϕ is zero, the vev is also usually zero.

$$\phi \simeq \phi_0 \; r^{\Delta_-} + \sigma \; r^{\Delta_+} + \cdots \quad , \quad r \to 0$$

• Below the BF bound, even if $\phi_0 = 0$, $\sigma \neq 0$. There are three regimes:
a) UV regime: $0 < r \ll \Lambda_{UV}^{-1}$

$$\phi \simeq \sigma \; r^{\Delta_+} + \cdots$$

b) Intermediate quasi-conformal regime $\Lambda_{UV}^{-1} \ll r \ll \Lambda_{IR}^{-1}$

$$\phi \simeq C \; r^2 \sin\left(\nu \log\left(r\right) + \phi\right)$$

c) Ultra IR regime $r \gg \Lambda_{IR}^{-1}$

• When $\nu \to 0$ we can estimate the end points by asking:

• Continuity at $r = \Lambda_{UV}^{-1}$

$$\cot\left[\nu \log(\Lambda_{UV}^{-1}) + \phi\right] = \frac{\Delta^+ - 2}{\nu} \quad \to \quad \nu \log(\Lambda_{UV}^{-1}) + \phi \simeq \pi$$

$$C \simeq \frac{(\Delta_+ - 2)}{\nu} \sigma \Lambda_{UV}^{(\Delta_+ - 2)}$$

• $\phi \simeq 1$ at $r = \Lambda_{IR}^{-1}$

We obtain from these conditions:

$$\frac{\sigma}{\Lambda_{UV}^{\Delta_+}} \sim \frac{\Lambda_{IR}^2}{\Lambda_{UV}^2} \sim e^{-\frac{2\pi}{\nu}}$$

• This is BKT scaling

The Efimov Spiral

• Consider now in the walking region the two linearly independent solutions associated with source and vev ($\tau_m \simeq m_q r$ and $\tau_\sigma \simeq \sigma r^3$ as $r \to 0$). They solve the lineraized equations as $t \ll 1$.

$$\tau_m \simeq \frac{m_q}{\Lambda_{UV}} K_m (r\Lambda_{UV})^2 \sin\left[\nu \log(r\Lambda_{UV}) + \phi_m\right]$$

$$\tau_\sigma \simeq \frac{\sigma}{\Lambda_{UV}^3} K_\sigma (r\Lambda_{UV})^2 \sin\left[\nu \log(r\Lambda_{UV}) + \phi_\sigma\right]$$

• In order to reach in the IR the regular solution only a fixed behavior is allowed

$$\tau_{IR} \simeq K_{IR} (r\Lambda_{IR})^2 \sin\left[\nu \log(r\Lambda_{IR}) + \phi_{IR}\right] \quad , \quad \Lambda_{UV}^{-1} \ll r \ll \Lambda_{IR}^{-1}$$

• In the intermediate region we must have $\tau_m + \tau_\sigma \simeq \tau_{IR}$ from which we obtain

$$\frac{m_q}{\Lambda_{UV}} = \frac{K_{IR}}{K_m} \; \frac{\sin\left(\phi_{IR} - \phi_\sigma - \nu w\right)}{\sin\left(\phi_m - \phi_\sigma\right)} e^{-2w} \quad , \quad e^w \equiv \frac{\Lambda_{UV}}{\Lambda_{IR}}$$

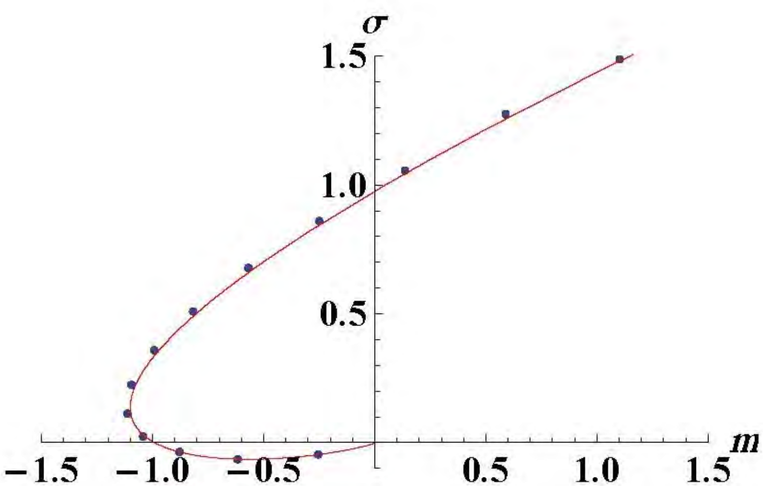
$$\frac{\sigma}{\Lambda_{UV}^3} = \frac{K_{IR}}{K_\sigma} \; \frac{\sin\left(\phi_{IR} - \phi_m - \nu w\right)}{\sin\left(\phi_\sigma - \phi_m\right)} e^{-2w}$$

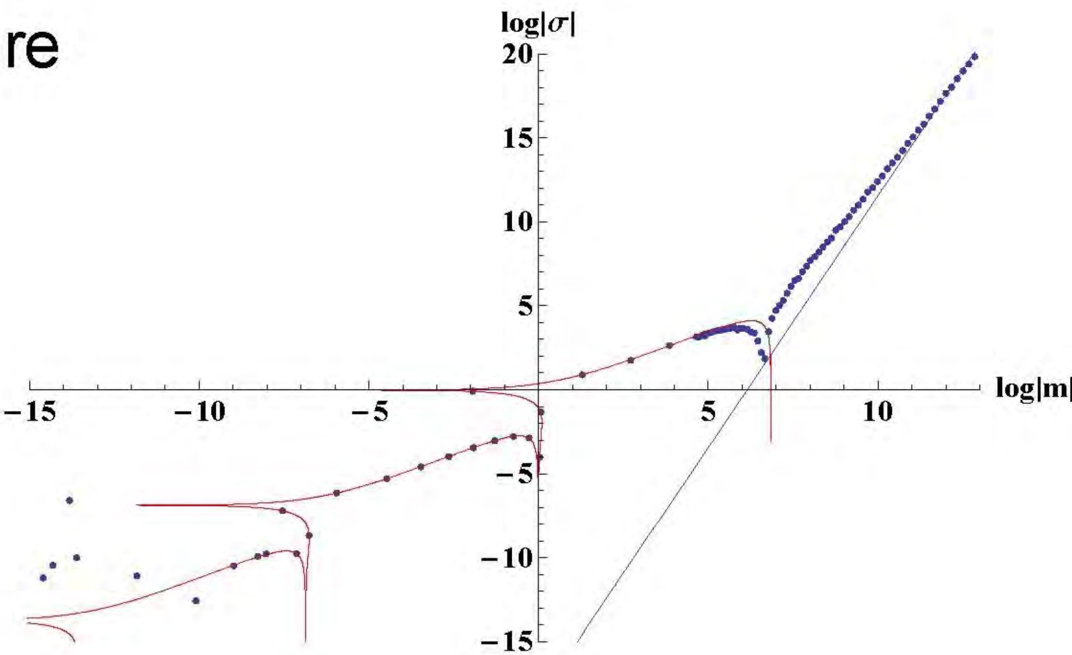
• The only thing that varies in the left hand side is w. If we plot m_q, σ we obtain the Efimov spiral.

Efimov spiral

Ongoing work: $\sigma(m)$ dependence

❑ For $x < x_c$ spiral structure





QCD$_*$

• The same bulk (holographic) theory with a scalar ϕ of mass m, is dual to two distinct CFTs:

a) With one quantization, (CFT$_a$) ϕ is dual to an operator O of dimension Δ_-.

b) With the other quantization (CFT$_b$) ϕ is dual to an operator O of dimension Δ_+.

• CFT$_a$ and CFT$_b$ differ very little and are very closely connected. A O^2 perturbation of CFT$_a$ produces CFT$_b$ in the IR.

• As m^2 approaches the BF bound, the two theories, CFT$_a$ and CFT$_b$ "collide" exactly at the BF bound, and then they move off the real axis into the complex plane.

• If $QCD(x)$ in the IR of the conformal window is CFT_b, which theory is CFT$_a$ (this theory has been called QCD$_*(x)$).

• The special scalar operator is $O = \bar{\psi}\psi$ in QCD(x) with dimension $2 \leq \Delta(x) \leq 3$. $\Delta(x_c) = 2$, $\Delta(11/2) = 3$.

• The defining properties of QCD$_*$:

1. At $QCD(x_c) = QCD_*(x_c)$.

2. The operator $\tilde{O}$ in QCD$_*$ satisfies $2 \geq \tilde{\Delta}(x) \leq 1$. $\tilde{\Delta}(x_c) = 2$, $\tilde{\Delta}(11/2) = 1$.

3. A perturbation of QCD$_*(x)$ by $\tilde{O}^2$ should drive the theory to QCD(x) in the IR.

Is there a QCD$_*$?

• My answer: YES!

I claim that the following theory is QCD$_*(x)$: The non-trivial IR fixed point of a gauge theory with

- $SU(N_c)$ gauge fields,

- N_f quarks and antiquarks,

- and an $N_f \times N_f$ complex matrix scalar, that is a color singlet but transforms as a bifundamental under the $U(N_f) \times U(N_f)$ chiral symmetry.

The action is

$$S = \int d^4x \left(\mathcal{L}_g + \mathcal{L}_f + \mathcal{L}_M\right) \quad , \quad S_g = -\frac{1}{2g^2} Tr[F^2]$$

$$S_f = (q^\dagger)^i_a i \not{D}_{ab} q^i_b + (\tilde{q}^\dagger)^i_a i \not{D}_{ab} \tilde{q}^i_b$$

$$S_M = Tr[\partial_\mu M \partial^\mu M^\dagger] - (Y M_{ij} T^{ij} + cc) - \frac{\lambda_1}{4!} Tr[M M^\dagger]^2 - \frac{\lambda_2}{4!} Tr[M M^\dagger M M^\dagger]$$

• The operator $\tilde{O}$ is M_{ij}.

• At $x = 11/2$, it has $\Delta = 1$.

• If we add $\tilde{O}^2$ this amounts to mass term for M, and the IR theory (at weak coupling) is the theory with M which is QCD.

• There is a non-trivial BZ-like IR fixed point in that theory! (some two loop scalar β functions have not been checked yet).

N=1 sQCD

The case of $\mathcal{N} = 1$ $SU(N_c)$ superQCD with N_f quark multiplets is known and provides an interesting (although much more complex) example for the non-supersymmetric case.

Seiberg

- The theory contains an adjoint $SU(N_c)$ N=1 vector multiplet and N_f chiral multiplets for quarks and antiquarks.

- $x = 0$ the theory has confinement, a mass gap and N_c distinct vacua associated with a spontaneous breaking of the leftover R symmetry Z_{N_c}.

- At $0 < x < 1$, the theory has a runaway ground state. Most probably it breaks supersymmetry spontaneously.

- At $x = 1$, the theory has a quantum moduli space with no singularity. This reflects confinement with χSB.

- At $x = 1 + \frac{1}{N_c}$, the moduli space is classical (and singular). The theory confines, but there is no χSB.

• At $1 + \frac{2}{N_c} < x < \frac{3}{2}$ the theory is in the non-abelian magnetic IR-free phase, with the magnetic gauge group $SU(N_f - N_c)$ IR free.

• At $\frac{3}{2} < x < 3$, the theory flows to a CFT in the IR. (conformal window)

Near $x = 3$ this is the Banks-Zaks region where the original theory has an IR fixed point at weak coupling. Moving to lower values, the coupling of the IR $SU(N_c)$ gauge theory grows.

However near $x = \frac{3}{2}$ the dual magnetic $SU(N_f - N_c)$ is in its Banks-Zaks region, and provides a weakly coupled description of the IR fixed point theory.

• At $x > 3$, the theory is IR free.

• The magnetic theory has elementary fields that are the meson composites of the electric theory. In particular the magnetic gauge bosons are the ρ-mesons, the magnetic quarks are the mesinos, and the adjoint scalars, are the scalar mesons.

Komargodski

• Why there is no conformal transition like it happens in QCD?

• The dimension of the quark mass operator, $2 < \Delta_{\bar{\psi}\psi} < 3$ in the conformal window. So we might think we are in the same situation.

• However supersymmetry relates this to the quarkino mass operator that has $1 < \Delta_{\bar{\psi}\psi} < 2$ in the conformal window.

• At the end of the conformal window it is a free boson, reflecting the BZ property of the magnetic theory.

There is a long way to go!

THANK YOU!

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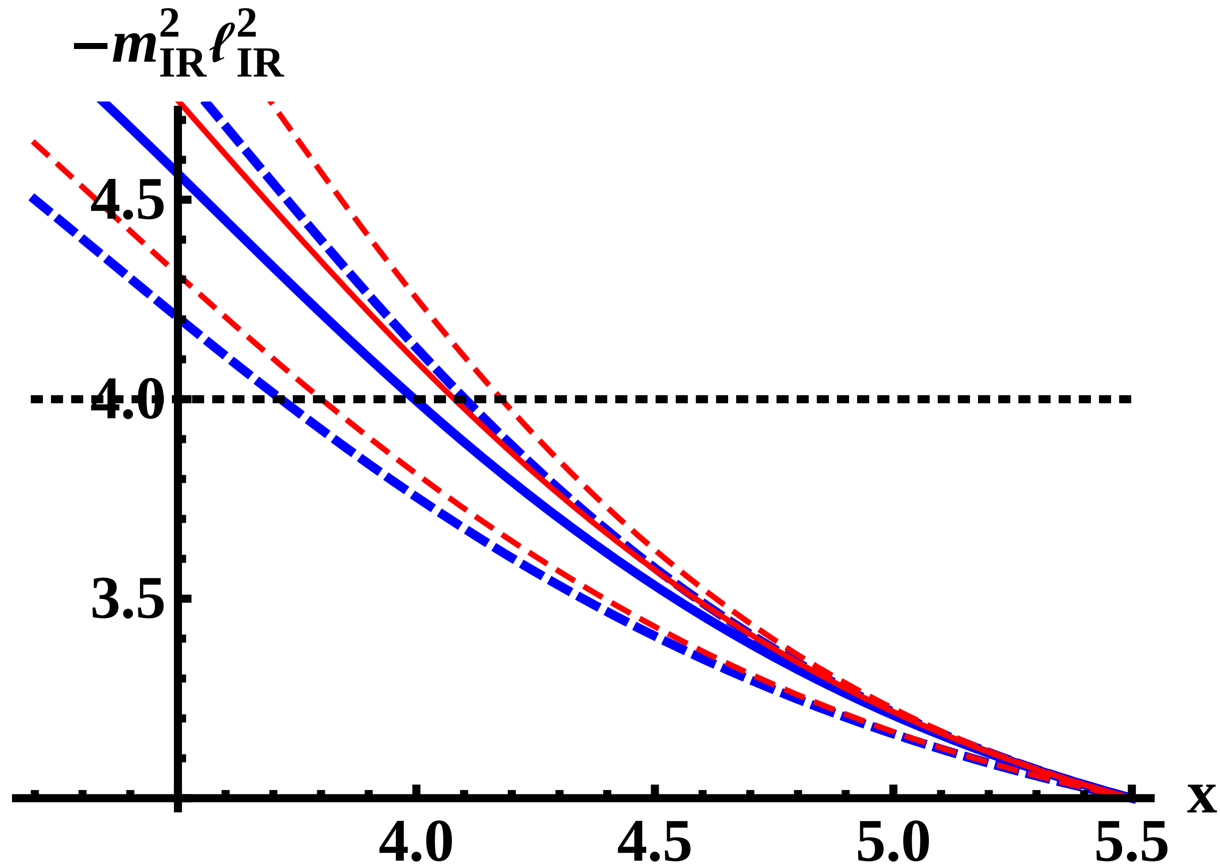
Varying the model

"prediction" for x_c

After fixing UV coefficients from QCD, there is still freedom in choosing the leading coefficient of V_0 at $\lambda \to 0$ and the IR asymptotics of the potentials



Thick blue $\to V_I$

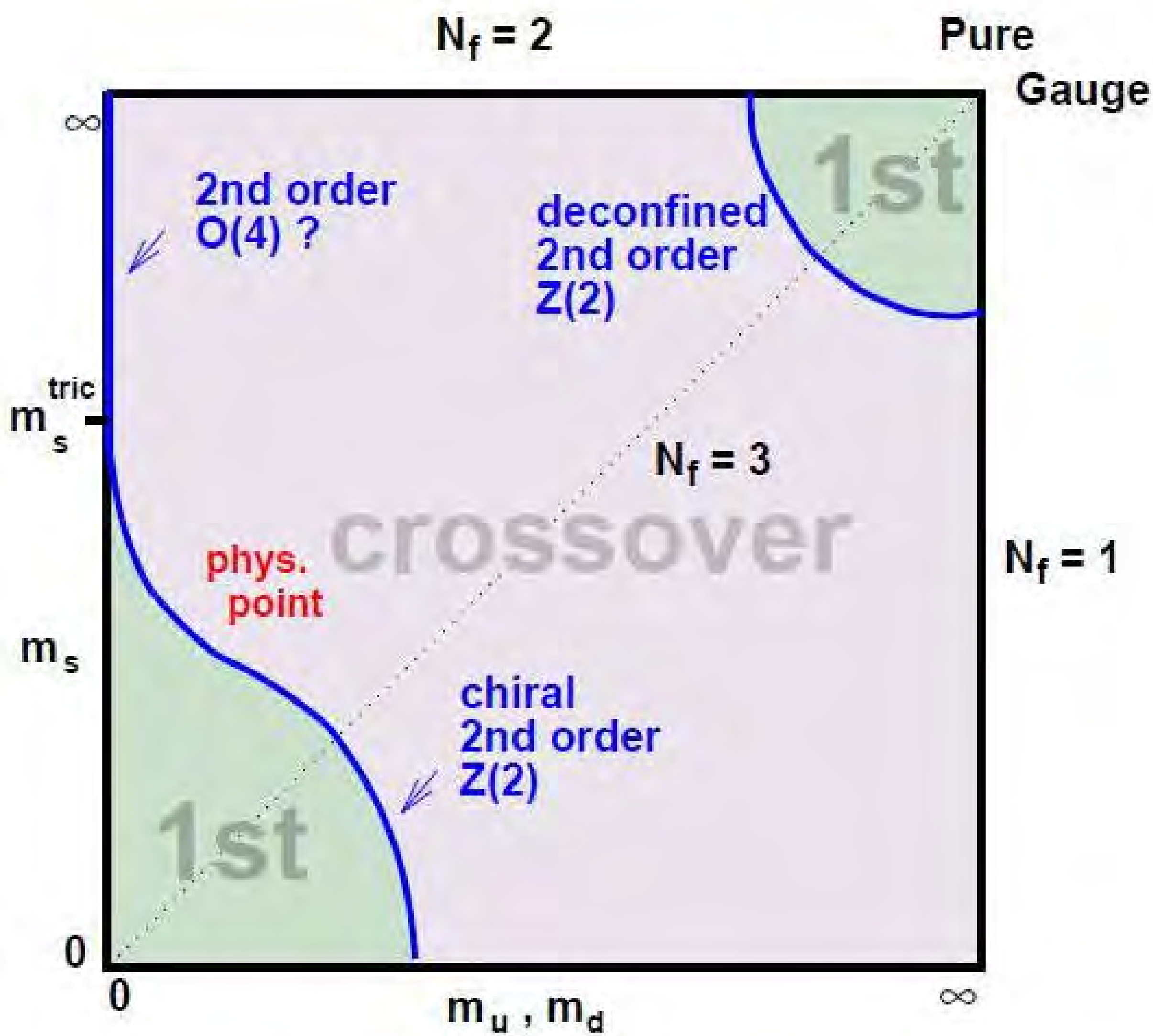
Thin red $\to V_{II}$

Resulting variation of the edge of conformal window

$$3.7 \lesssim x_c \lesssim 4.2$$

RETURN

Phase diagram as a function of quark masses



Matching to QCD

- $V_g(\lambda)$ is fixed from glue.

- The UV is adjusted to perturbative QCD.

$$V_g \sim V_0 + \mathcal{O}(\lambda) \quad , \quad V_0 \sim W_0 + \mathcal{O}(\lambda)$$

$$V_0 - xW_0 = \frac{12}{\ell_{UV}^2}$$

- W_0 is one of the most important parameters of the models.

• There are two classes of tachyon potentials:

♠ Type I: $T \sim e^{Cr}$ as $r \to \infty$.

♠ Type II $T \sim \sqrt{r}$ as $r \to \infty$.

• In all cases the "regular" IR solution depends on a single undetermined constant (instead on two).

• The phase structure is essentially independent of IR choices.

Matching to QCD: UV

• As $\lambda \to 0$ we can match:

♠ $V_g(\lambda)$ with (two-loop) Yang-Mills β-function.

♠ $V_g(\lambda) - xV_0(\lambda)$ with QCD β-function.

♠ $a(\lambda)/h(\lambda)$ with anomalous dimension of the quark mass/chiral condensate

• The matching allows to mark the BZ point, that we normalize at $x = \frac{11}{2}$.

• After the matching above we are left with a single undetermined parameter in the UV:

$$V_g \sim V_0 + \mathcal{O}(\lambda) \quad , \quad V_0 \sim W_0 + \mathcal{O}(\lambda)$$

$$V_0 - xW_0 = \frac{12}{\ell_{UV}^2}$$

Matching to QCD: IR

• In the IR, the tachyon has to diverge $\Rightarrow$ the tachyon action $\propto e^{-T^2}$ becomes small

♠ $V_g(\lambda) \simeq \lambda^{\frac{4}{3}}\sqrt{\lambda}$ chosen as for Yang-Mills, so that a "good" IR singularity exists etc.

♠ $V_0(\lambda)$, $a(\lambda)$, and $h(\lambda)$ chosen to produce tachyon divergence: there are several possibilities.

♠ The phase structure is essentially independent of IR choices.

Choice I, for which in the IR

$$T(r) \sim T_0 \exp\left[\frac{81\ 3^{5/6}(115-16x)^{4/3}(11-x)}{812944\ 2^{1/6}}\frac{r}{R}\right] \quad , \quad r \to \infty$$

R is the IR scale of the solution. T_0 is the control parameter of the UV mass.

Choice II: for which in the IR

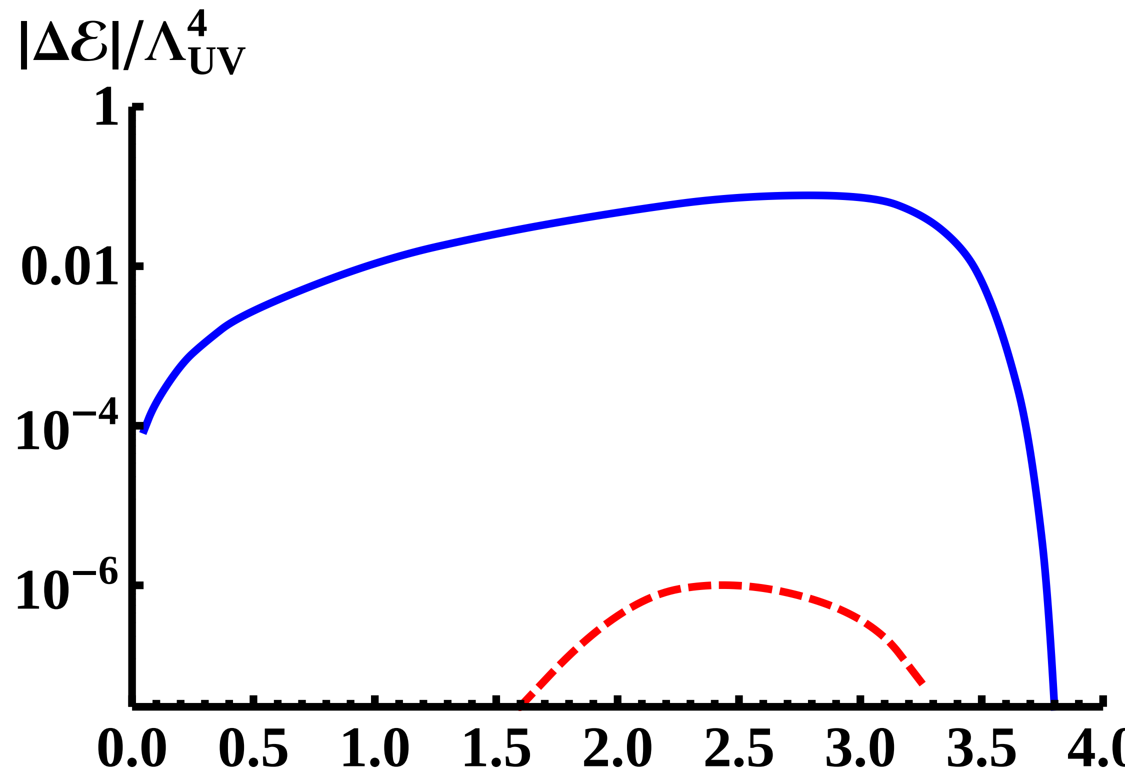
$$T(r) \sim \frac{27\ 2^{3/4} 3^{1/4}}{\sqrt{4619}}\sqrt{\frac{r-r_1}{R}} \quad , \quad r \to \infty$$

R is the IR scale of the solution. r_1 is the control parameter of the UV mass.

The free energy

The free energy difference between the ChS and ChSB $m_q = 0$ solutions

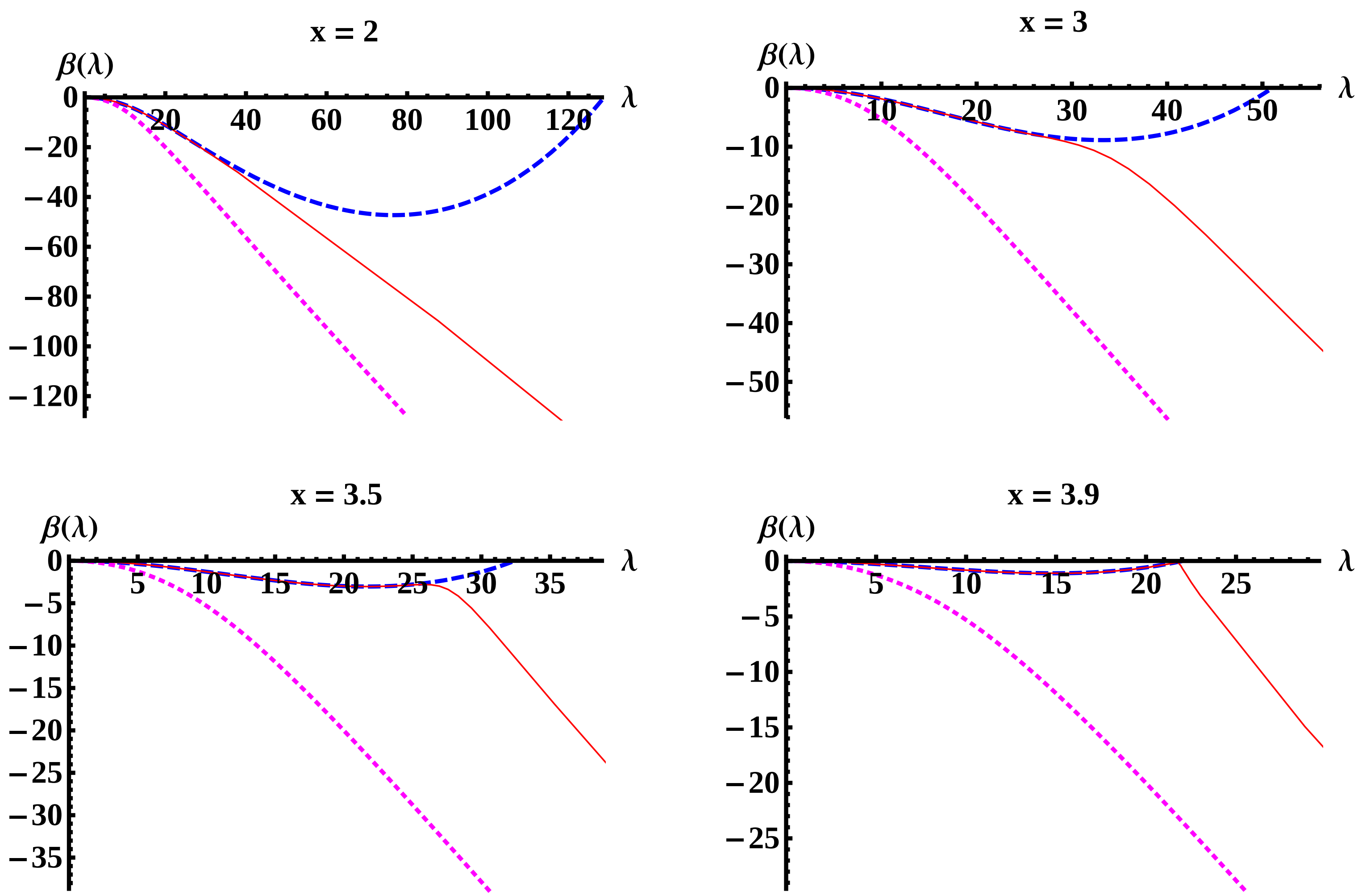
Chiral symmetry breaking solution favored whenever it exists ($x < x_c$)



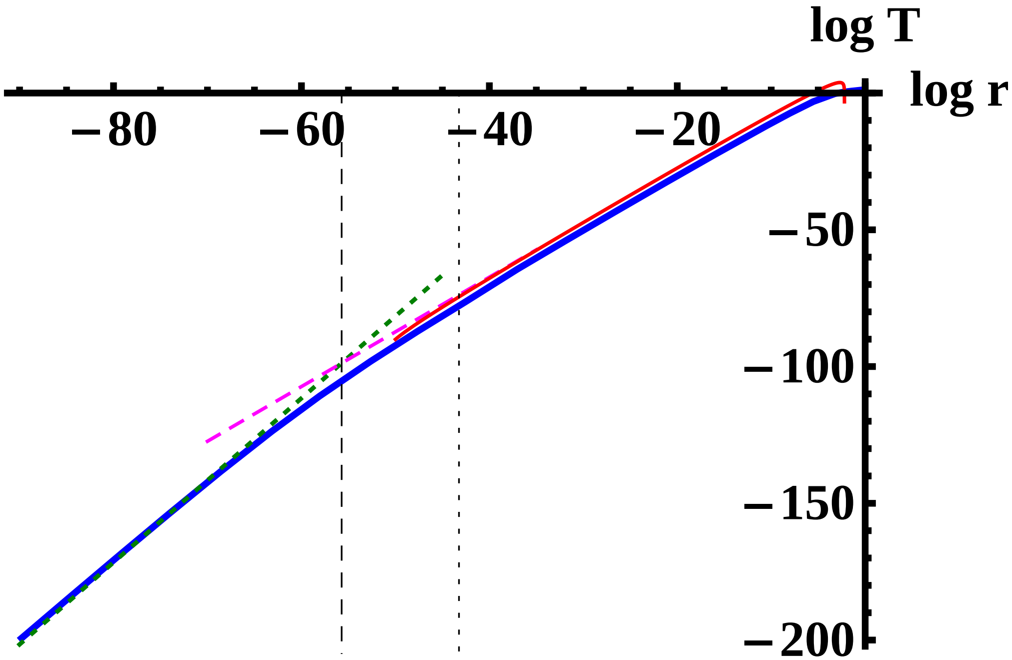
- The Efimov minima have free energies ΔE_n with

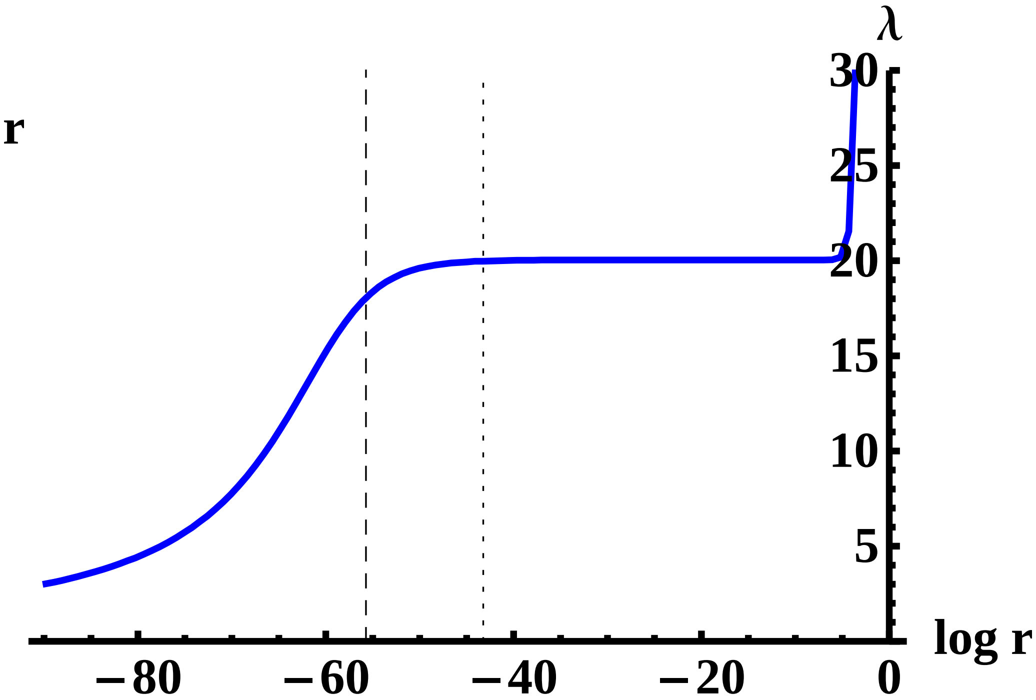
$$\Delta E_0 > \Delta E_1 > \Delta E_2 > \cdots$$

Walking



The β-functions for vanishing quark mass for various values of x. The red solid, blue dashed, and magenta dotted curves are the β-functions corresponding to the full numerical solution ($d\lambda/dA$) along the RG flow, the potential $V_{\text{eff}} = V_g - xV_{f0}$, and the potential V_g, respectively.





The tachyon $\log T$ (left) and the coupling λ (right) as functions of $\log r$ for an extreme walking background with $x = 3.992$. The thin lines on the left hand plot are the approximations used to derive the BKT scaling.

Holographic β-functions

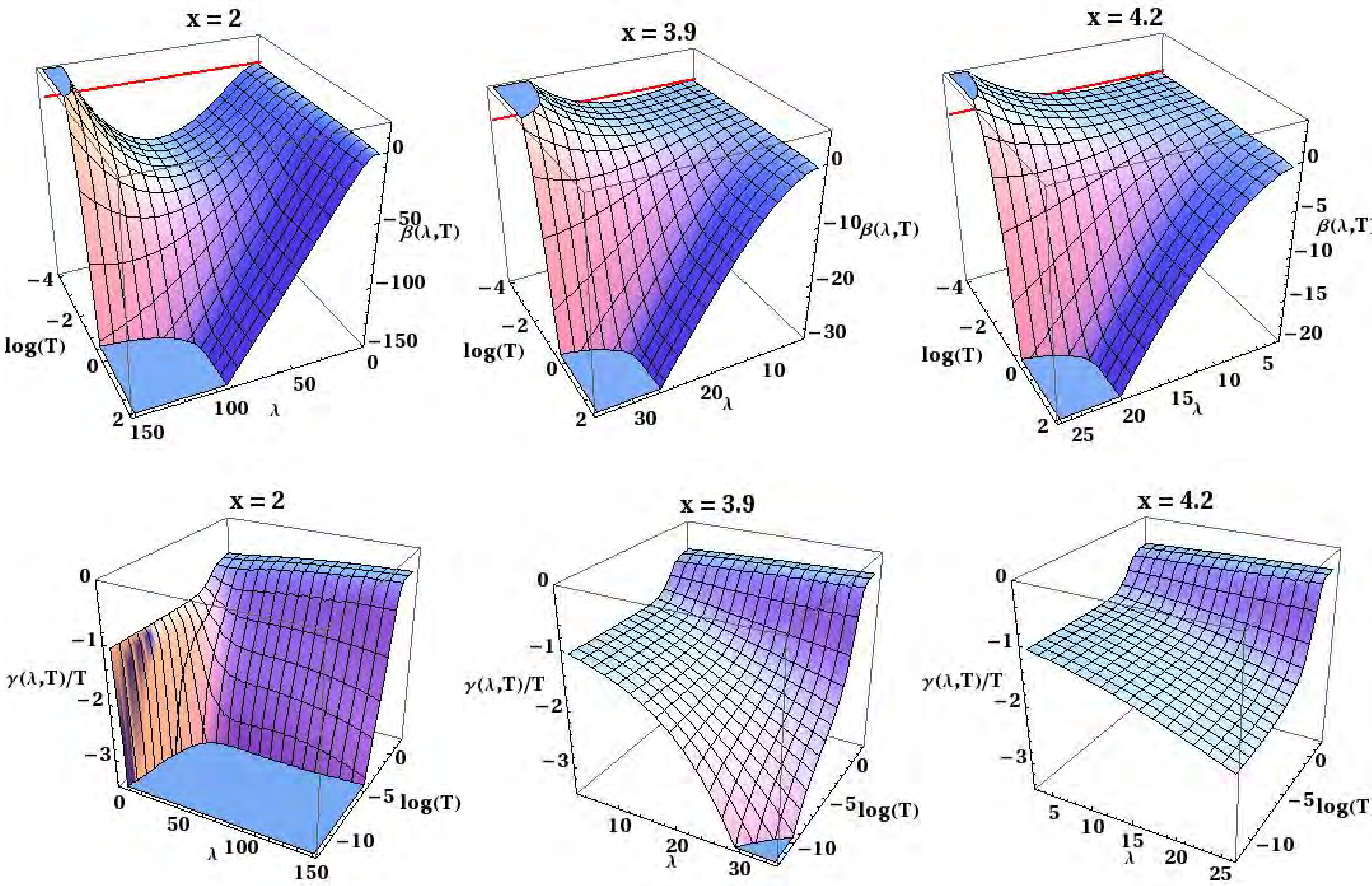
The second order equations for the system of two scalars plus metric can be written as first order equations for the β-functions

Gursoy+Kiritsis+Nitti

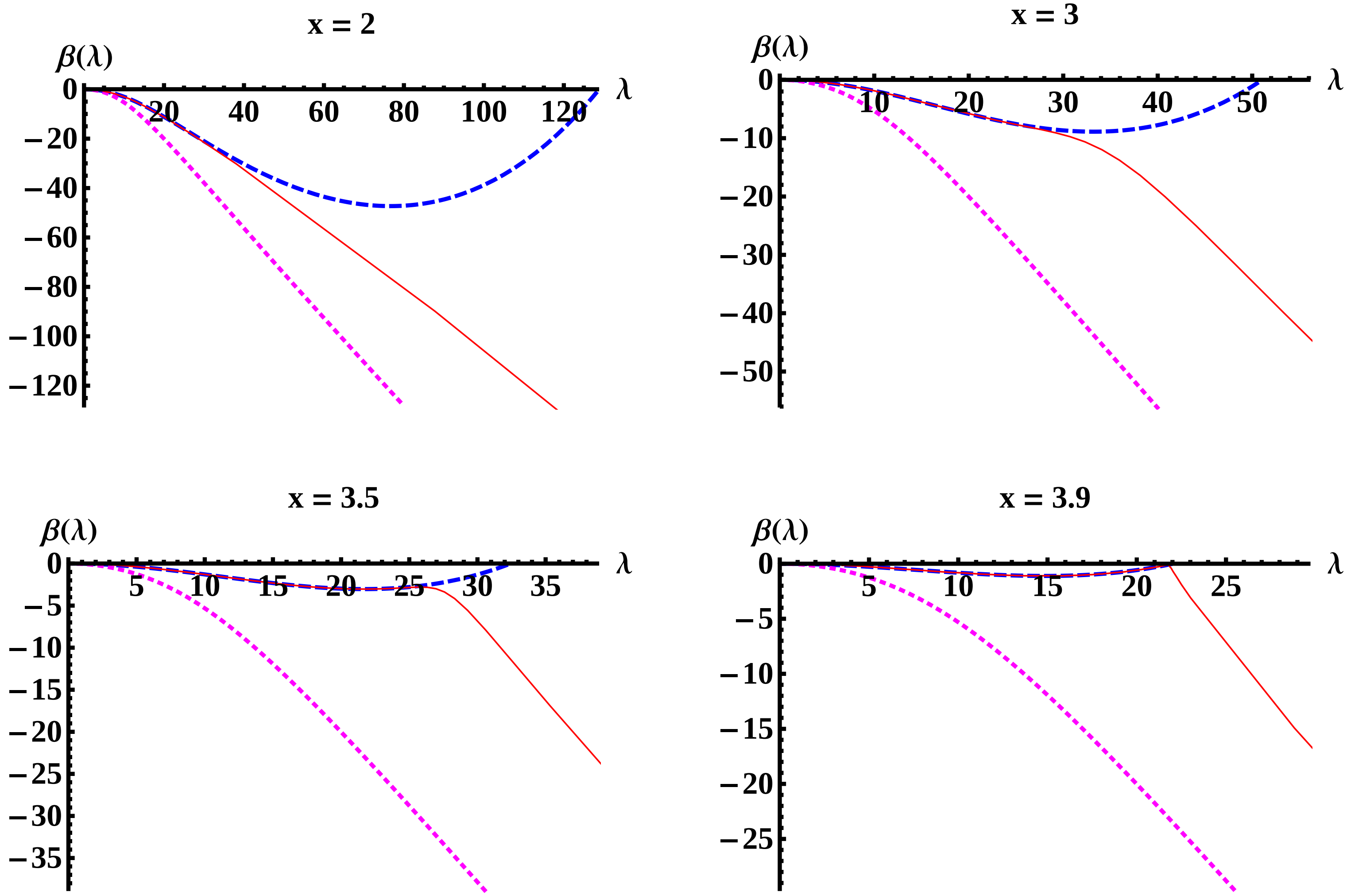
$$\frac{d\lambda}{dA} = \beta(\lambda, T) \quad , \quad \frac{dT}{dA} = \gamma(\lambda, T)$$

The equations of motion boil down to two partial non-linear differential equations for β, γ.

Such equations have also branches as for DBI and non-linear scalar actions the relation of $e^{-A}A'$ with the potentials is a polynomial equation of degree higher than two.

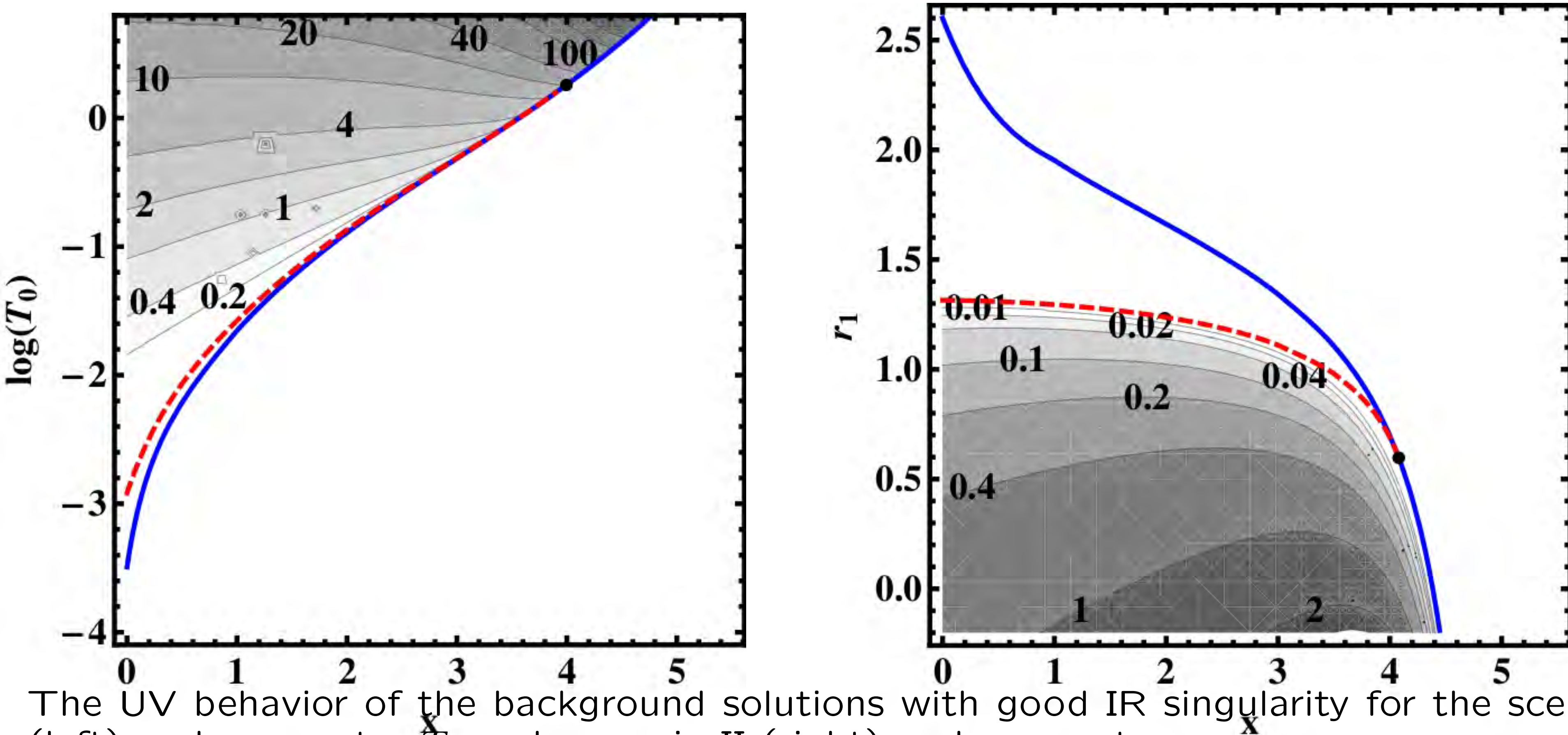


The red lines are added on the top row at $\beta = 0$ in order to show the location of the fixed point.



The β-functions for vanishing quark mass for various values of x. The red solid, blue dashed, and magenta dotted curves are the β-functions corresponding to the full numerical solution $(d\lambda/dA)$ along the RG flow, the potential $V_{\rm eff} = V_g - xV_{f0}$, and the potential V_g, respectively.

UV mass vs T_0 and r_1

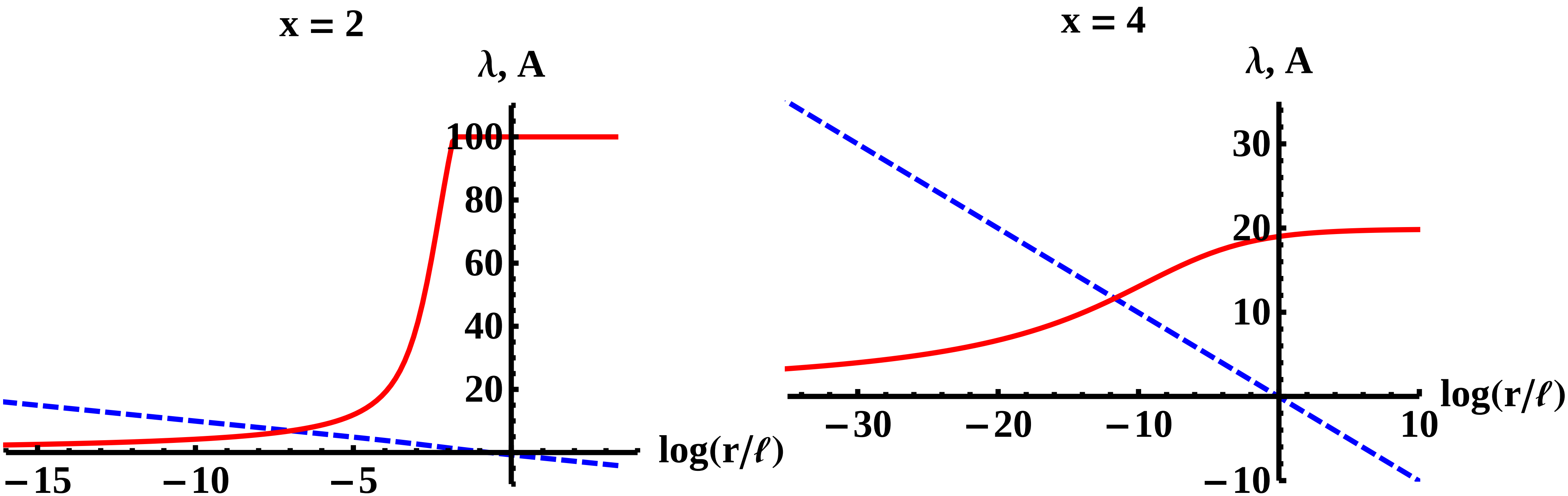


The UV behavior of the background solutions with good IR singularity for the scenario I (left) and parameter T_0 and scenario II (right) and parameter r_1.

The thick blue curve represents a change in the UV behavior, the red dashed curve has zero quark mass, and the contours give the quark mass. The black dot where the zero mass curve terminates lies at the critical value $x = x_c$. For scenario I (II) we have $x_c \simeq 3.9959$ ($x_c \simeq 4.0797$).

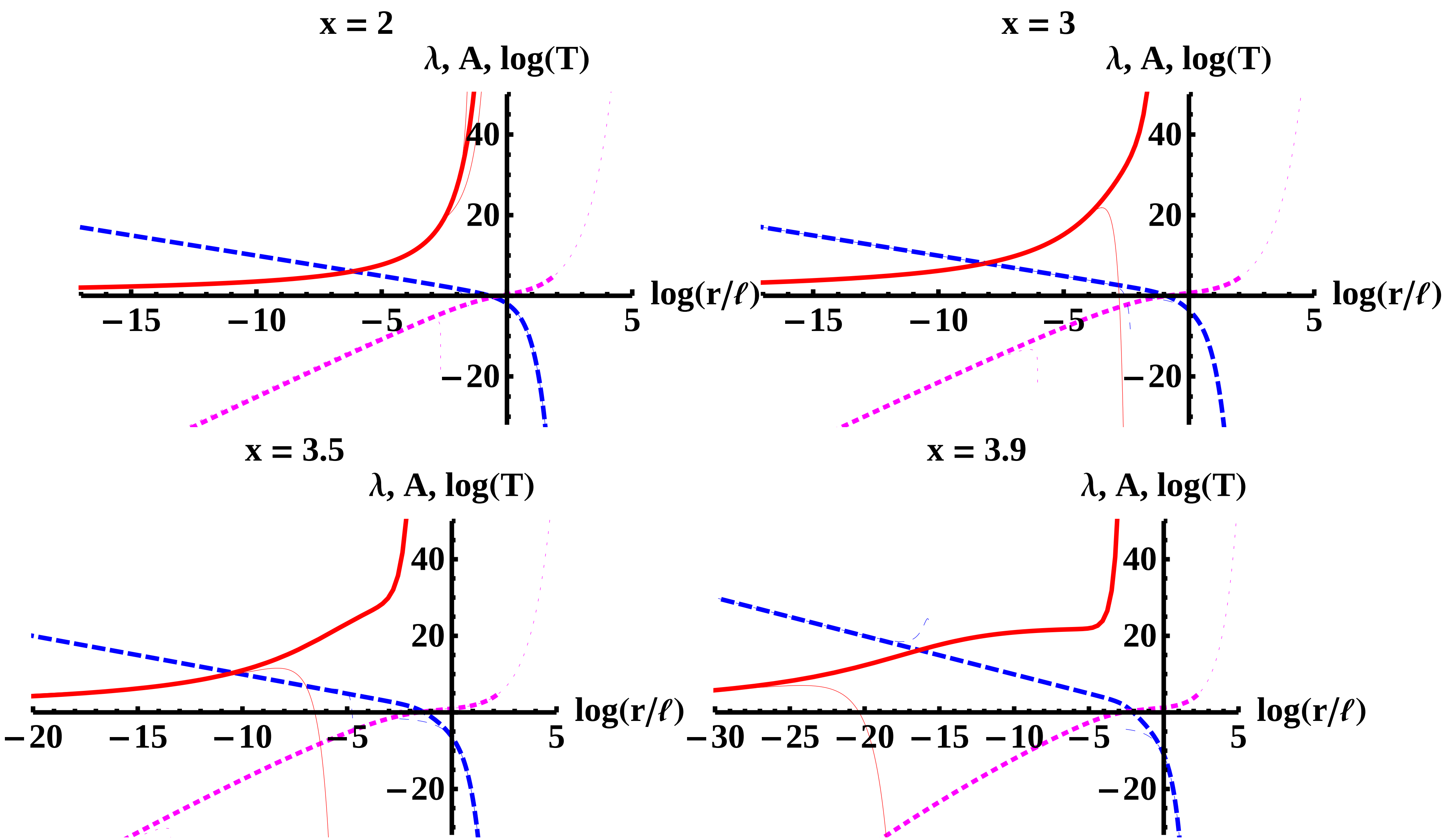
Numerical solutions: $T = 0$

$T \equiv 0$ backgrounds (color codes λ, A)

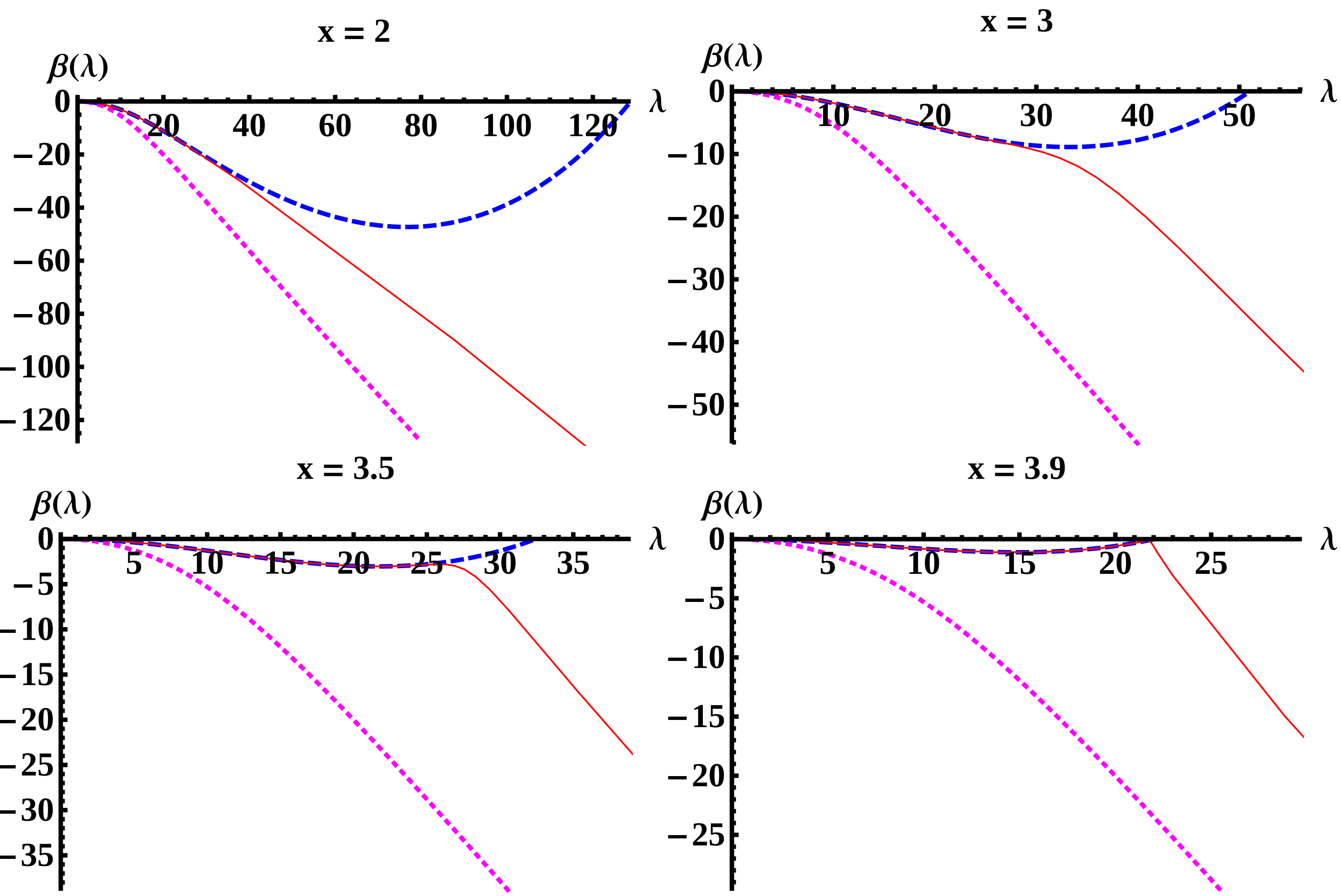


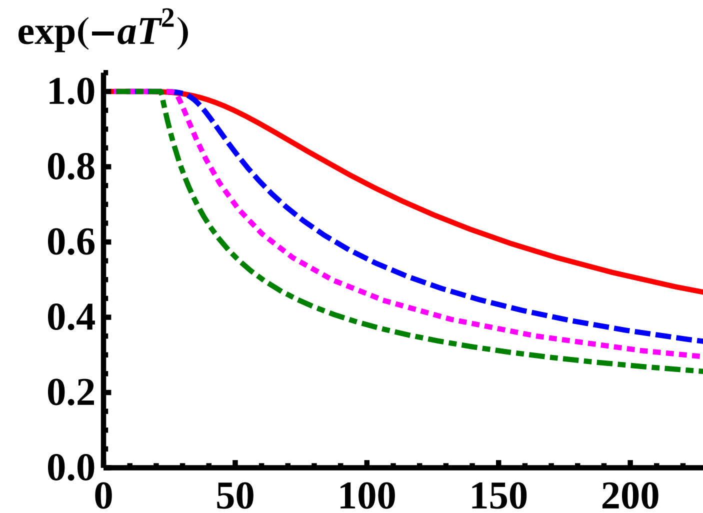
Numerical solutions: Massless with $x < x_c$

Massless backgrounds with $x < x_c \simeq 3.9959$ (λ, A, T)



Massless backgrounds: beta functions $\beta = \frac{d\lambda}{dA}$, ($x_c \simeq 3.9959$)

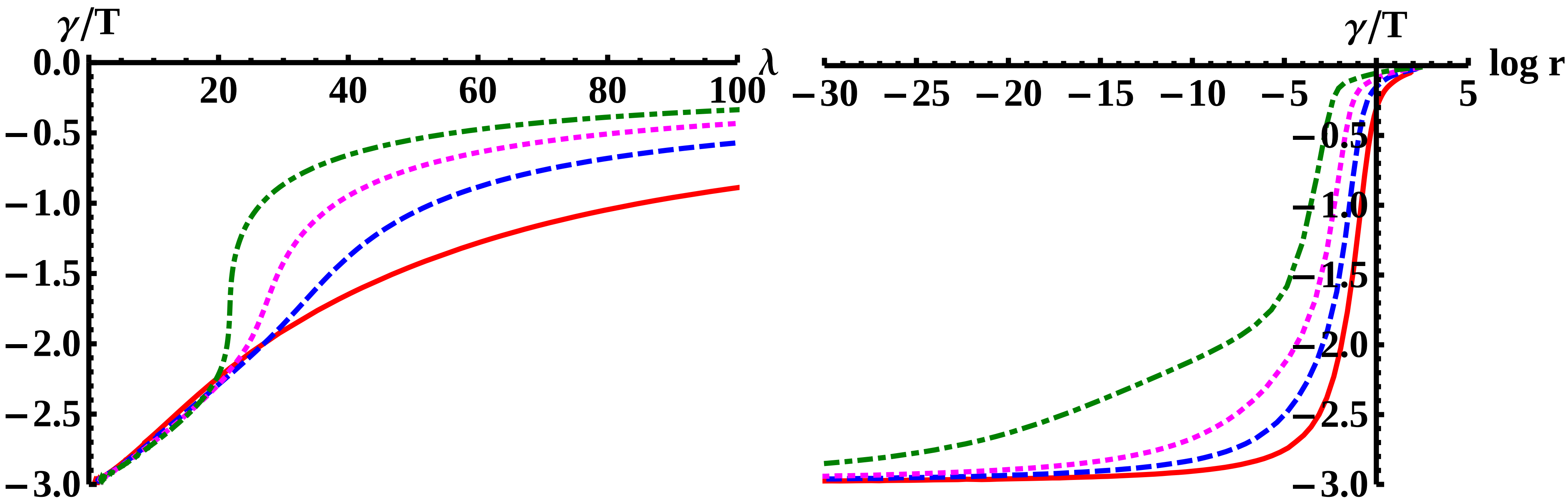




$x = 2, 3, 3.5, 3.9$

.

Massless backgrounds: gamma functions $\frac{\gamma}{T} = \frac{d \log T}{dA}$



$x = 2, 3, 3.5, 3.9$

Matching to QCD: IR

• In the IR, the tachyon has to diverge $\Rightarrow$ the tachyon action $\propto e^{-T^2}$ becomes small

♠ $V_g(\lambda) \simeq \lambda^{\frac{4}{3}}\sqrt{\lambda}$ chosen as for Yang-Mills, so that a "good" IR singularity exists etc.

♠ $V_0(\lambda)$, $a(\lambda)$, and $h(\lambda)$ chosen to produce tachyon divergence: there are several possibilities.

♠ The phase structure is essentially independent of IR choices.

Choice I:

$$V_g(\lambda) = 12 + \frac{44}{9\pi^2}\lambda + \frac{4619}{3888\pi^4}\frac{\lambda^2}{(1+\lambda/(8\pi^2))^{2/3}}\sqrt{1+\log(1+\lambda/(8\pi^2))}$$

$$V_f(\lambda, T) = V_0(\lambda) e^{-a(\lambda)T^2}$$

$$V_0(\lambda) = \frac{12}{11} + \frac{4(33-2x)}{99\pi^2}\lambda + \frac{23473 - 2726x + 92x^2}{42768\pi^4}\lambda^2$$

$$a(\lambda) = \frac{3}{22}(11-x)$$

$$h(\lambda) = \frac{1}{\left(1 + \frac{115-16x}{288\pi^2}\lambda\right)^{4/3}}$$

For which in the IR

$$T(r) \sim T_0 \exp\left[\frac{81\ 3^{5/6}(115-16x)^{4/3}(11-x)}{812944\ 2^{1/6}}\frac{r}{R}\right] \quad , \quad r \to \infty$$

R is the IR scale of the solution. T_0 is the control parameter of the UV mass.

Choice II:

$$a(\lambda) = \frac{3}{22}(11-x)\frac{1+\frac{115-16x}{216\pi^2}\lambda+\frac{\lambda^2}{\lambda_0^2}}{(1+\lambda/\lambda_0)^{4/3}}$$

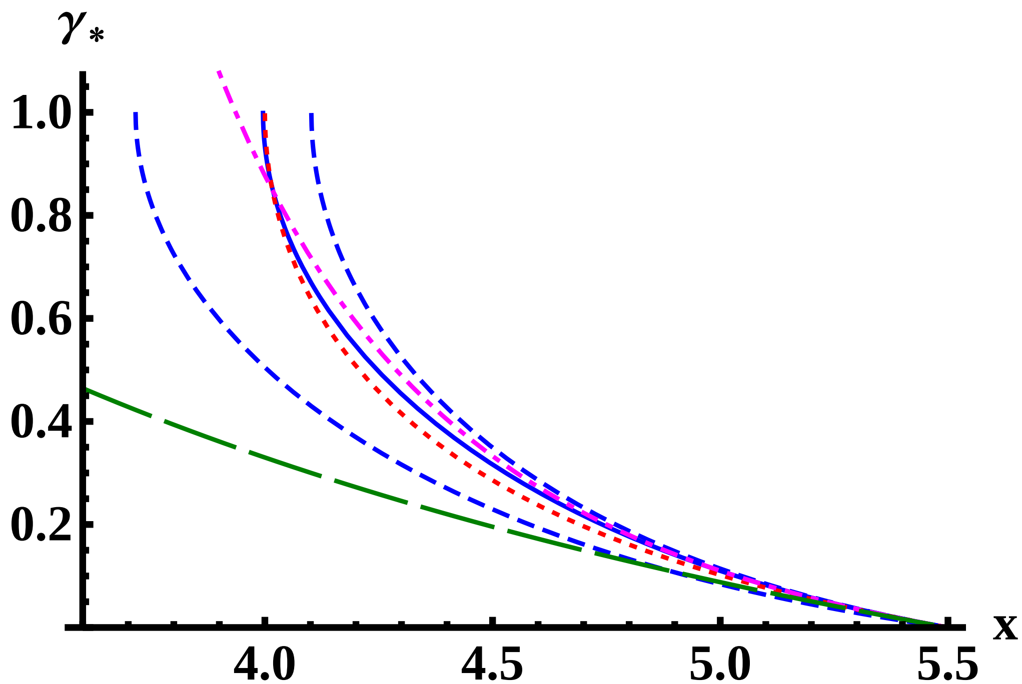
$$h(\lambda) = \frac{1}{(1+\lambda/\lambda_0)^{4/3}}$$

for which in the IR

$$T(r) \sim \frac{27\ 2^{3/4}3^{1/4}}{\sqrt{4619}}\sqrt{\frac{r-r_1}{R}} \quad , \quad r\to\infty$$

R is the IR scale of the solution. r_1 is the control parameter of the UV mass.

Comparison to previous "guesses"



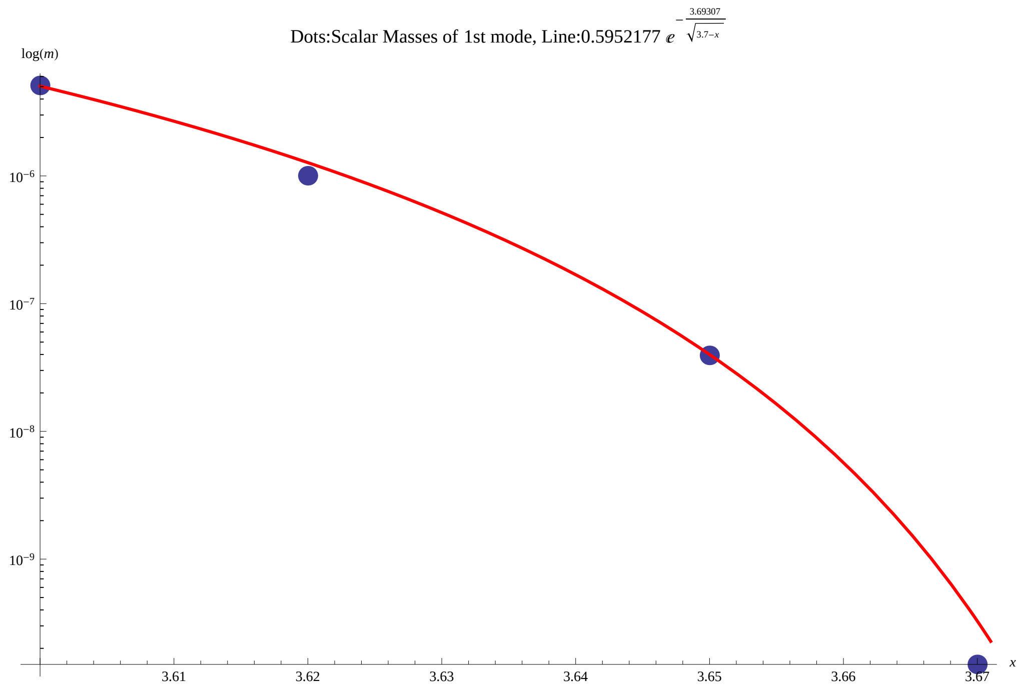
The anomalous dimension of the quark mass at the IR fixed point as a function of x within the conformal window in various approaches.

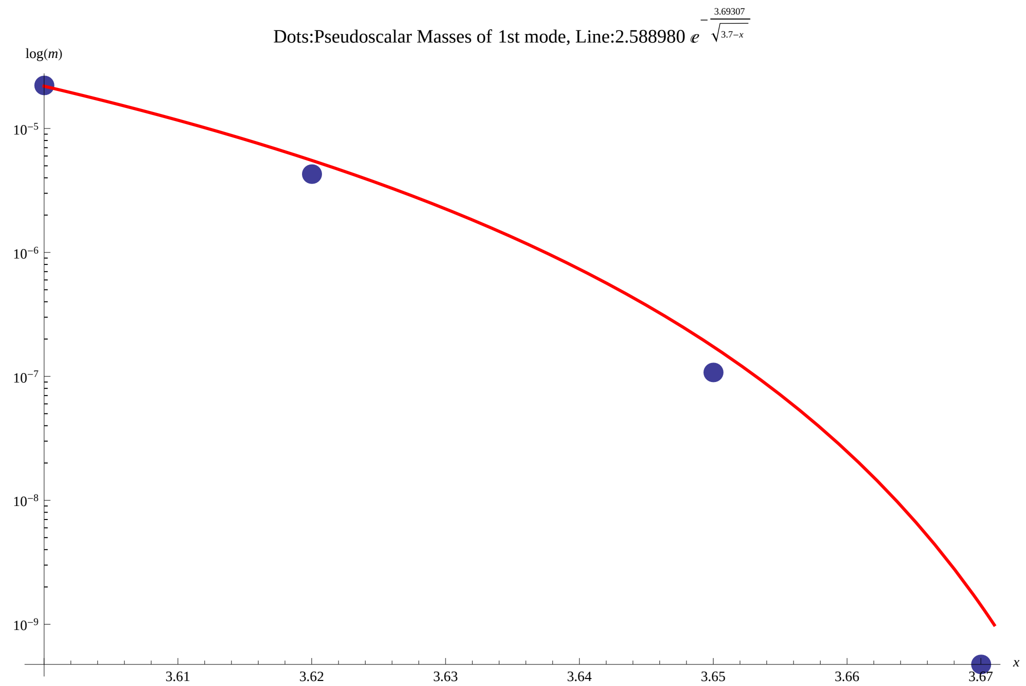
The solid blue curve is our result for the potential I.

The dashed blue lines show the maximal change as W_0 is varied from 0 (upper curve) to 24/11 (lower curve).

The dotted red curve is the result from a Dyson-Schwinger analysis, the dot-dashed magenta curve is the prediction of two-loop perturbative QCD, and the long-dashed green curve is based on an all-orders β-function.

Miransky scaling for the masses





The plots depict the scalar and pseudoscalar masses of the first mode close to x_c fit to the Miransky exponential factor.

RETURN

The holographic models: flavor

- Fundamental quarks arise from $D4$-$\bar{D}4$ branes in 5-dimensions.

$$D4 - D4 \quad \text{strings} \quad \to \quad A_\mu^L \quad \leftrightarrow \quad J_\mu^L = \bar{\psi}_L \sigma_\mu \psi_L$$

$$\overline{D4} - \overline{D4} \quad \text{strings} \quad \to \quad A_\mu^R \quad \leftrightarrow \quad J_\mu^R = \bar{\psi}_R \bar{\sigma}_\mu \psi_R$$

$$D4 - \overline{D4} \quad \text{strings} \quad \to \quad T \quad \leftrightarrow \quad \bar{\psi}_L \psi_R$$

- For the vacuum structure only the tachyon is relevant.

- An action for the tachyon motivated by the Sen action has been advocated as the proper dynamics of the chiral condensate, giving in general all the expected features of χSB.

Casero+Kiritsis+Paredes

$$\mathcal{S}_{\rm TDBI} = -N_f N_c M^3 \int d^5x \; V_f(T) \; e^{-\phi} \sqrt{-\det(g_{ab} + \partial_a T \partial_b T)}$$

- It has been tested in a 6d asymptotically-AdS confining background (with constant dilaton) due to Kuperstein+Sonneschein.

Iatrakis+Kiritsis+Paredes

It was shown to have the following properties:

• Confining asymptotics of the geometry trigger chiral symmetry breaking.

• A Gell-Mann-Oakes-Renner relation is generically satisfied.

• The Sen DBI tachyon action with $V \sim e^{-T^2}$ asymptotics induces linear Regge trajectories for mesons.

• The Wess-Zumino (WZ) terms of the tachyon action, computed in string theory, produce the appropriate flavor anomalies, include the axial $U(1)$ anomaly and η'-mixing, and implement a holographic version of the Coleman-Witten theorem.

• The dynamics determines the chiral condensate uniquely a s function of the bare quark mass.

• The mass of the ρ-meson grows with increasing quark mass.

• By adjusting the same parameters as in QCD ($\Lambda_{\rm QCD}$, m_{ud}) a good fit can be obtained of the light meson masses.

The chiral vacuum structure

• We take the potential to be the flat space one

$$V = V_0 \, e^{-T^2}$$

with a maximum at $T = 0$ and a minimum at $T = \infty$.

• Near the boundary $z = 0$, the solution can be expanded in terms of two integration constants as:

$$\tau = c_1 z + \frac{\pi}{6} c_1^3 z^3 \log z + c_3 z^3 + \mathcal{O}(z^5)$$

• c_1, c_3 are related to the quark mass and condensate.

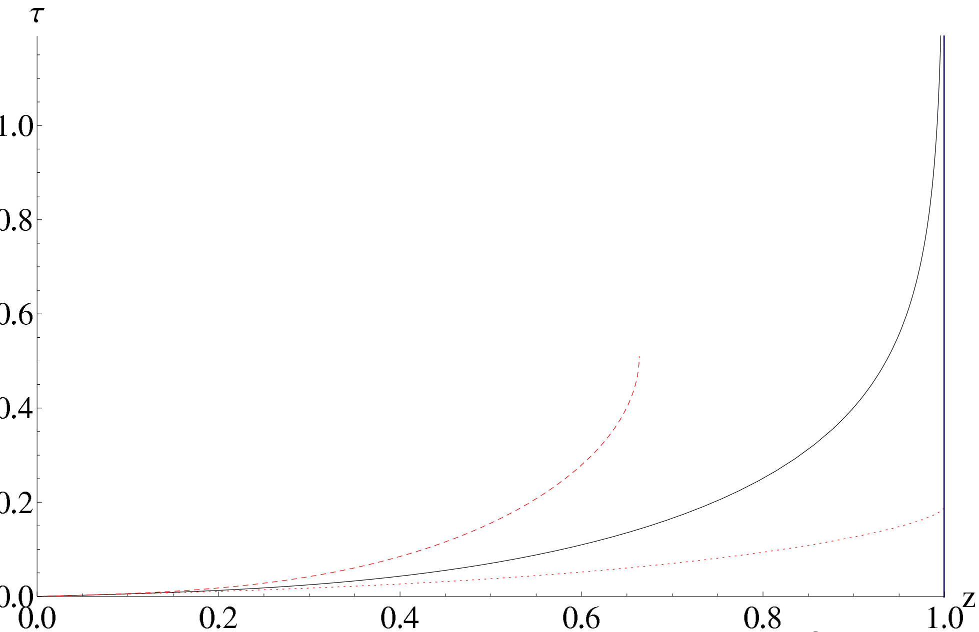
• At the tip of the cigar, the generic behavior of solutions is

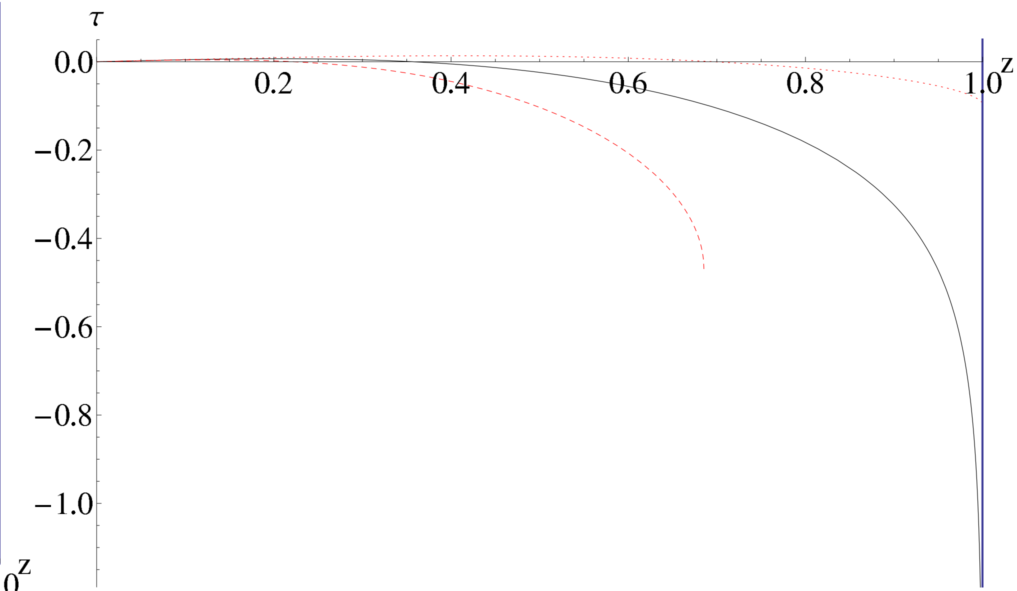
$$\tau \sim constant_1 + constant_2 \sqrt{z - z_\Lambda}$$

• With special tuned condition there is a one-parameter family of diverging solutions in the IR depending on a single parameter:

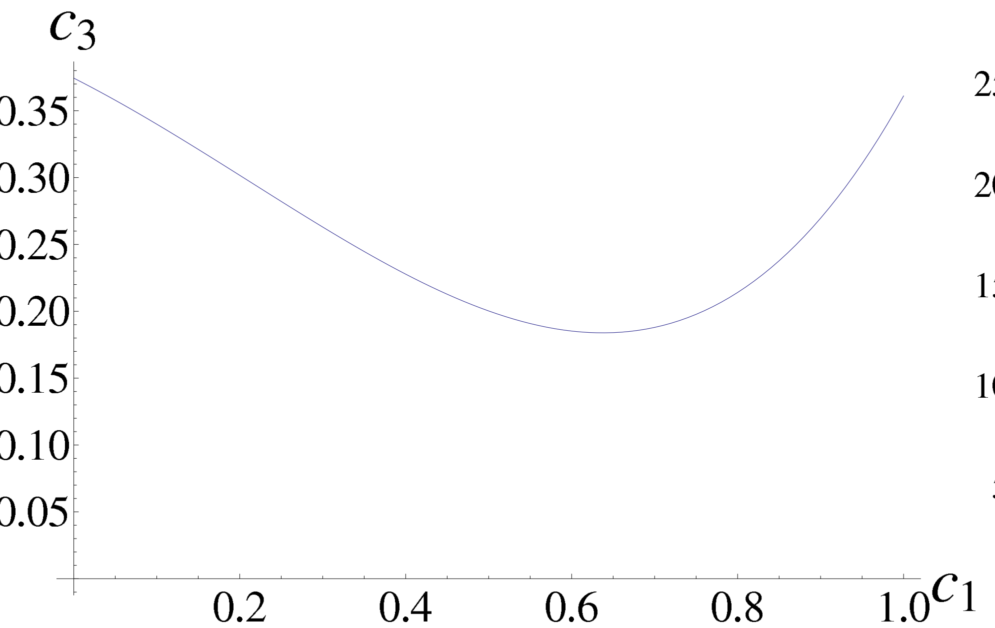
$$\tau = \frac{C}{(z_\Lambda - z)^{\frac{3}{20}}} - \frac{13}{6\pi C}(z_\Lambda - z)^{\frac{3}{20}} + \ldots$$

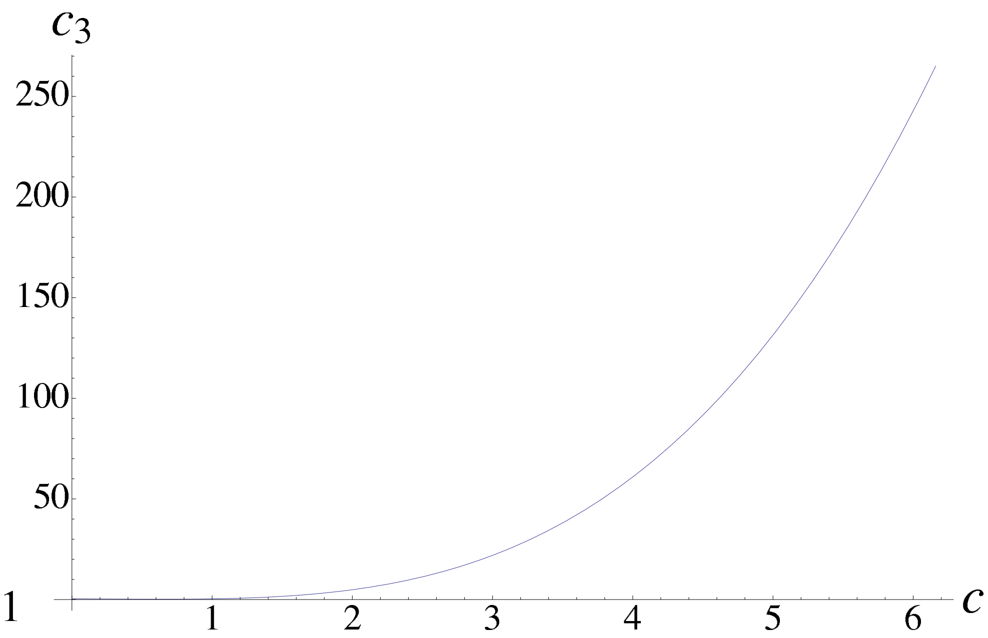
• This is the correct "regularity condition" in the IR as τ is allowed to diverge only at the tip.





All the graphs are plotted using $z_\Lambda = 1$, $\mu^2 = \pi$ and $c_1 = 0.05$. The tip of the cigar is at $z = z_\Lambda = 1$. On the left, the solid black line represents a solution with $c_3 \approx 0.3579$ for which τ diverges at z_Λ. The red dashed line has a too large c_3 ($c_3 = 1$) - such that there is a singularity at $z = z_s$ where $\partial_z \tau$ diverges while τ stays finite. This is unacceptable since the solution stops at $z = z_s$ where the energy density of the flavor branes diverges. The red dotted line corresponds to $c_3 = 0.1$; this kind of solution is discarded because the tachyon stays finite everywhere. The plot in the right is done with the same conventions but with negative values of $c_3 = -0.1, -0.3893, -1$. For $c_3 \approx -0.3893$ there is a solution of the differential equation such that τ diverges to $-\infty$. This solution is unstable.





- Chiral symmetry breaking is manifest.

Chiral restauration at deconfinement

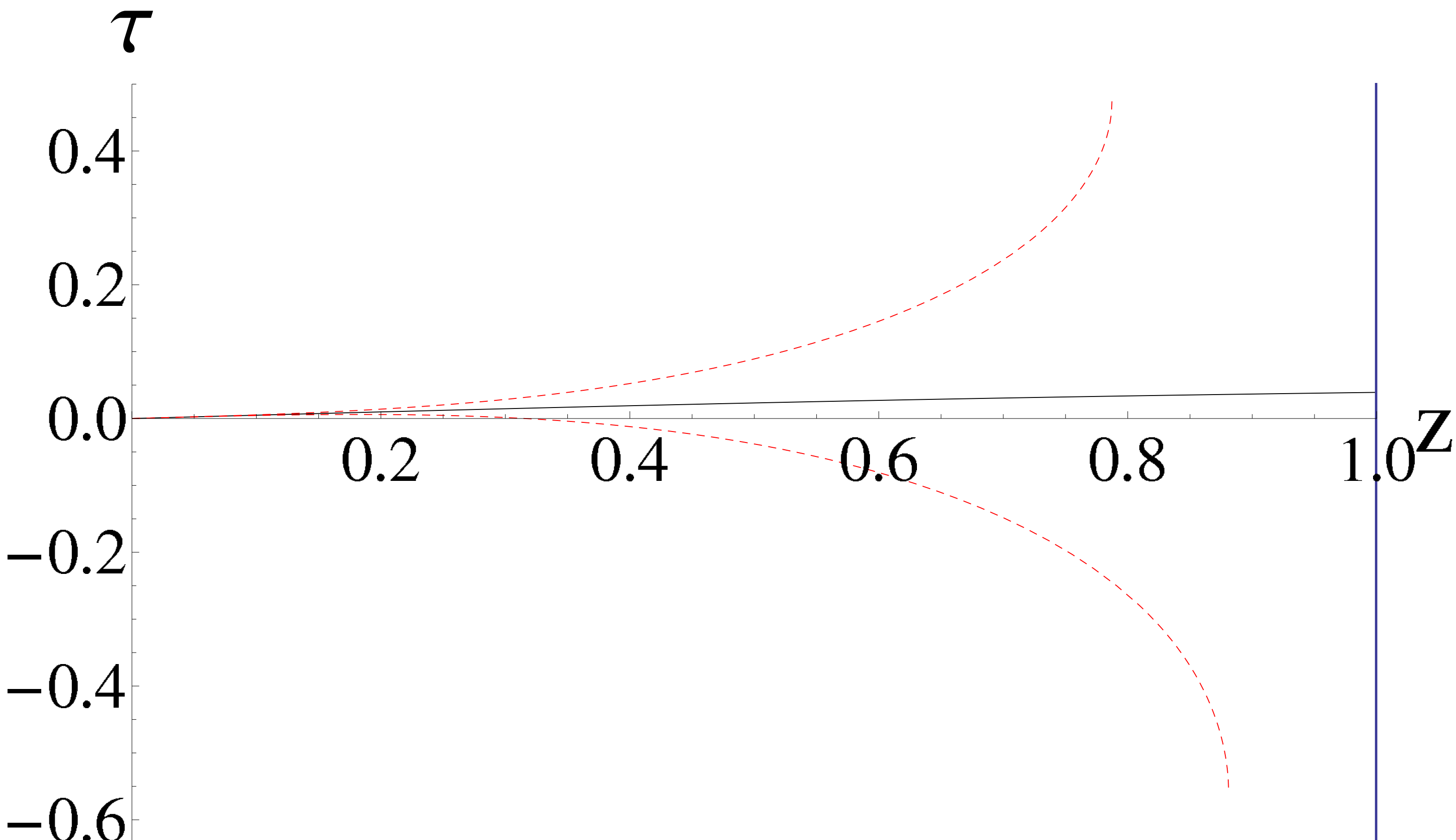
- In the deconfined phase, the bulk metric is that of a bh.

- The branes now are allowed to enter the horizon without recombining.

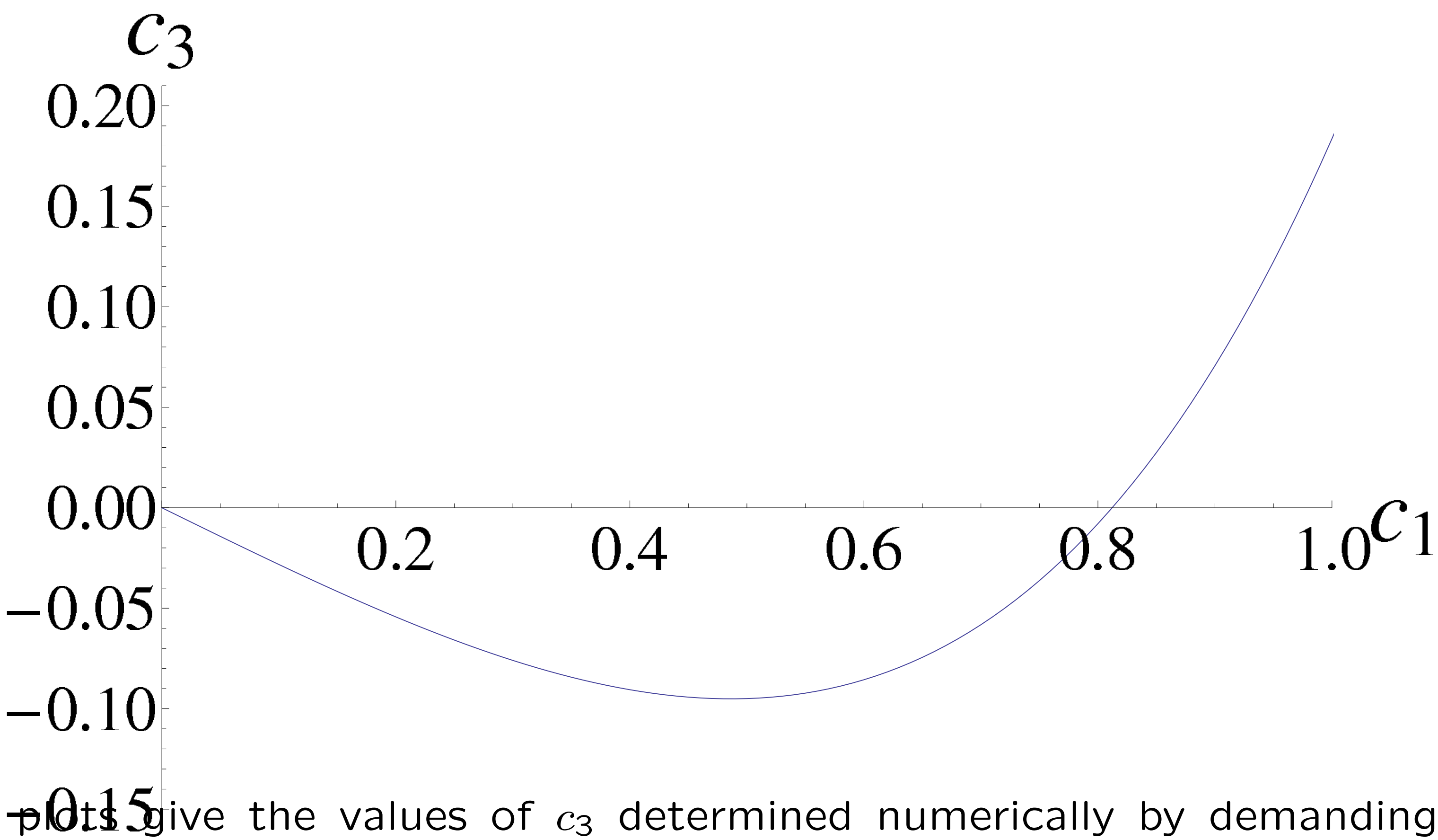
- To avoid intermediate singularities of the solution the boundary conditions must be tuned so that tachyon is finite at the horizon.

- Near the horizon the correct solution behaves as a one-parameter family

$$\tau = c_T - \frac{3c_T}{5z_T}(z_T - z) - \frac{9c_T}{200z_T}(8 + \mu^2 {c_T}^2)(z_T - z)^2 + \ldots$$



Plots corresponding to the deconfined phase. We have taken $c_1 = 0.05$. The solid line displays the physical solution $c_3 = -0.0143$ whereas the dashed lines ($c_3 = -0.5$ and $c_3 = 0.5$) are unphysical and end with a behavior of the type $\tau = k_1 - k_2\sqrt{z_s - z}$.



These plots give the values of c_3 determined numerically by demanding the correct IR behavior of the solution, as a function of c_1.

Jump of the condensate at the phase transition

- From holographic renormalization we obtain

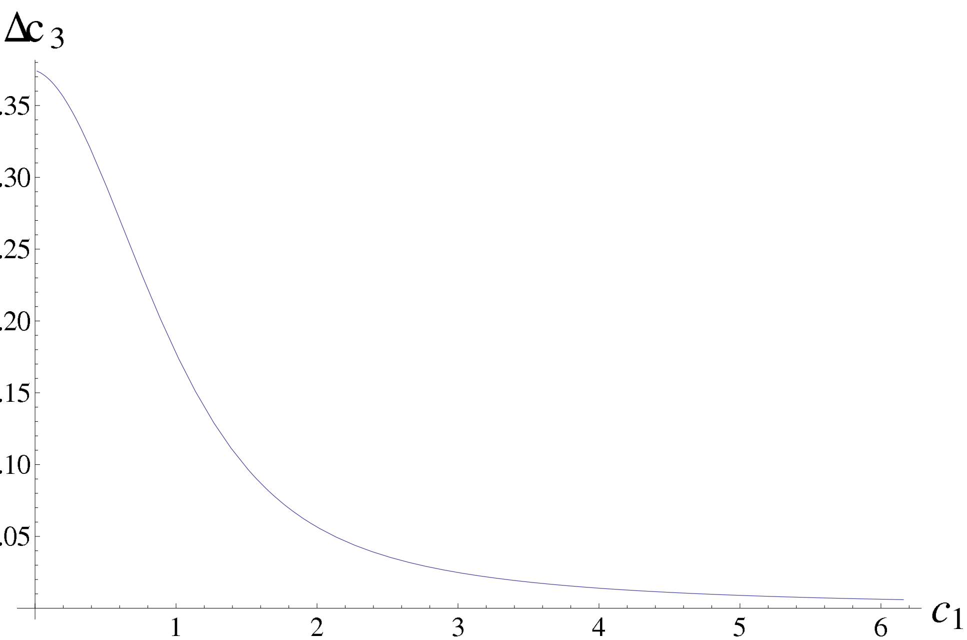
$$\langle \bar{q}q \rangle = \frac{1}{\beta}(2\pi\alpha' \mathcal{K} R^3 \lambda) \left(-4c_3 + \left(\frac{m_q}{\beta} \right)^3 \mu^2 (1 + \alpha) \right) \quad , \quad m_q = \beta \; c_1$$

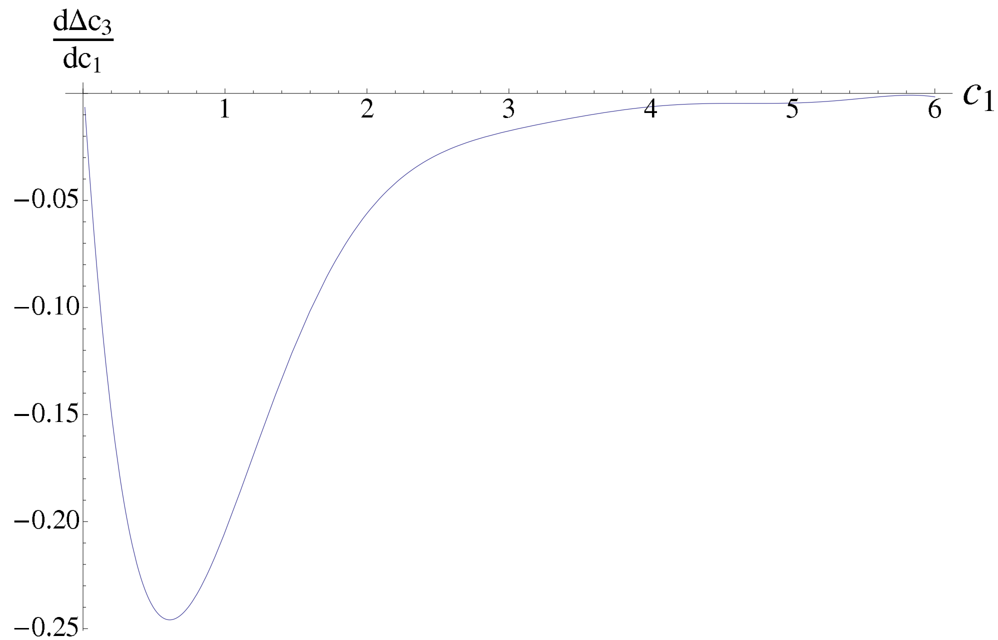
- We calculate the jump at the phase transition that is scheme independent for a fixed quark mass.

$$\Delta \langle \bar{q}q \rangle \equiv \langle \bar{q}q \rangle_{conf} - \langle \bar{q}q \rangle_{deconf} = -4\frac{1}{\beta}(2\pi\alpha' \mathcal{K} R^3 \lambda) \Delta c_3$$

- This is equivalent to Δc_3

• We plot it as a function of the quark mass.





The finite jump of the quark condensate and its derivative with respect to c_1 when the confinement-deconfinement transition takes place. The important features appear when $m_q \sim \Lambda_{QCD}$

Meson spectra

For the vectors

$$z_\Lambda m_V^{(1)} = 1.45 + 0.718 c_1 \,, \quad z_\Lambda m_V^{(2)} = 2.64 + 0.594 c_1 \,, \quad z_\Lambda m_V^{(3)} = 3.45 + 0.581 c_1 \,,$$
$$z_\Lambda m_V^{(4)} = 4.13 + 0.578 c_1 \,, \quad z_\Lambda m_V^{(5)} = 4.72 + 0.577 c_1 \,, \quad z_\Lambda m_V^{(6)} = 5.25 + 0.576 c_1 \,.$$

For the axial vectors:

$$z_\Lambda m_A^{(1)} \approx 2.05 + 1.46 c_1 \,, \quad z_\Lambda m_A^{(2)} \approx 3.47 + 1.24 c_1 \,, \quad z_\Lambda m_A^{(3)} \approx 4.54 + 1.17 c_1 \,,$$
$$z_\Lambda m_A^{(4)} \approx 5.44 + 1.13 c_1 \,, \quad z_\Lambda m_A^{(5)} \approx 6.23 + 1.11 c_1 \,, \quad z_\Lambda m_A^{(6)} \approx 6.95 + 1.10 c_1 \,.$$

For the pseudoscalars:

$$z_\Lambda m_P^{(1)} \approx \sqrt{3.53 c_1^2 + 6.33 c_1} \,, \quad z_\Lambda m_P^{(2)} \approx 2.91 + 1.40 c_1 \,, \quad z_\Lambda m_P^{(3)} \approx 4.07 + 1.27 c_1 \,,$$
$$z_\Lambda m_P^{(4)} \approx 5.04 + 1.21 c_1 \,, \quad z_\Lambda m_P^{(5)} \approx 5.87 + 1.17 c_1 \,, \quad z_\Lambda m_P^{(6)} \approx 6.62 + 1.15 c_1 \,.$$

For the scalars:

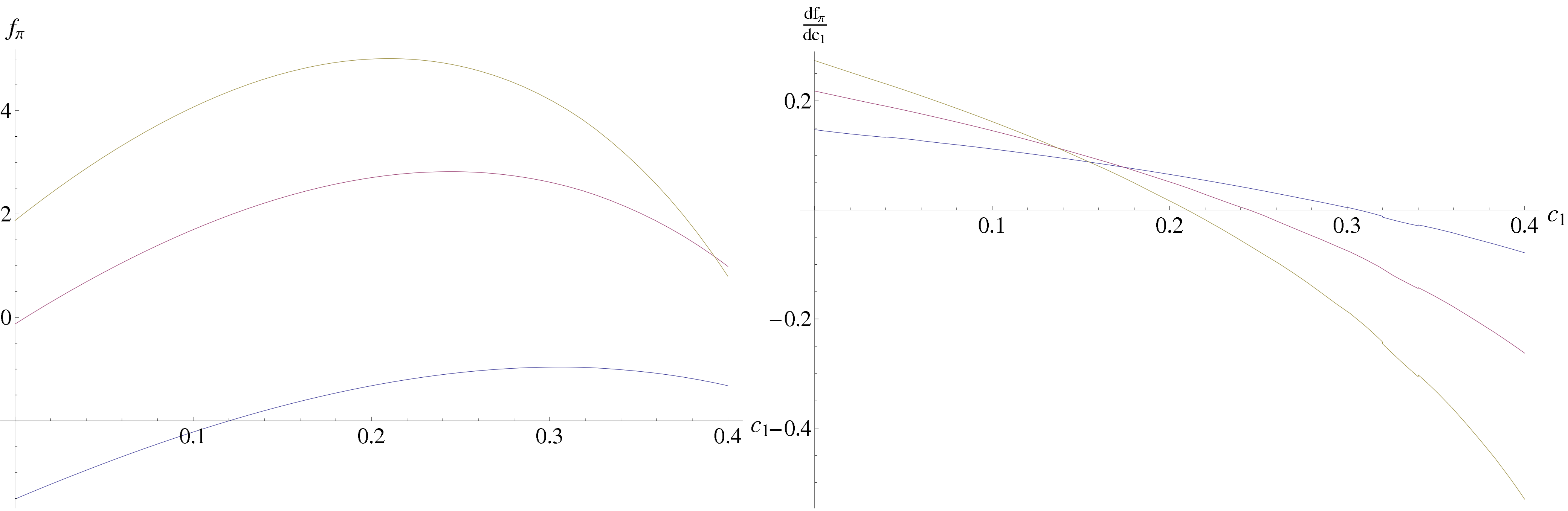
$$z_\Lambda m_S^{(1)} = 2.47 + 0.683 c_1 \,, \quad z_\Lambda m_S^{(2)} = 3.73 + 0.488 c_1 \,, \quad z_\Lambda m_S^{(3)} = 4.41 + 0.507 c_1 \,,$$
$$z_\Lambda m_S^{(4)} = 4.99 + 0.519 c_1 \,, \quad z_\Lambda m_S^{(5)} = 5.50 + 0.536 c_1 \,, \quad z_\Lambda m_S^{(6)} = 5.98 + 0.543 c_1 \,.$$

- Valid up to $c_1 \sim 1$.

- In qualitative agreement with lattice results

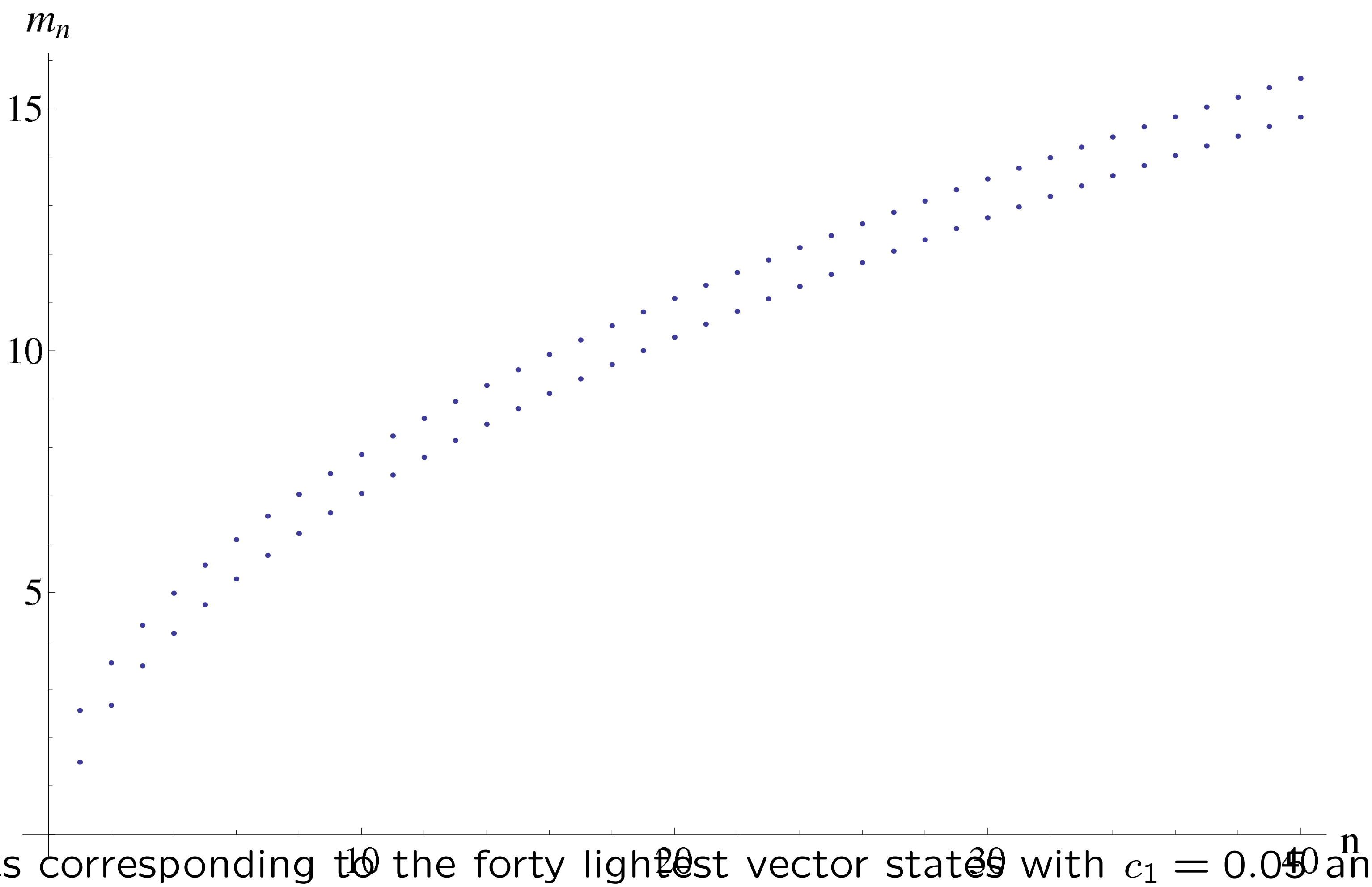
Laerman+Schmidt., Del Debbio+Lucini+Patela+Pica, Bali+Bursa

Mass dependence of f_π



The pion decay constant and its derivative as a function of c_1 - the quark mass. The different lines correspond to different values of k. From bottom to top (on the right plot, from bottom to top in the vertical axis) $k = \frac{12}{\pi^2}, \frac{24}{\pi^2}, \frac{36}{\pi^2}$. The pion decay constant comes in units of z_Λ^{-1}.

Linear Regge Trajectories



Results corresponding to the forty lightest vector states with $c_1 = 0.05$ and $c_1 = 1.5$.

Spectra

• The main difference from all previous calculations is that here flavor back reacts on color.

• In the singlet sector the glueballs and mesons mix to leading order and the spectral problem becomes complicated.

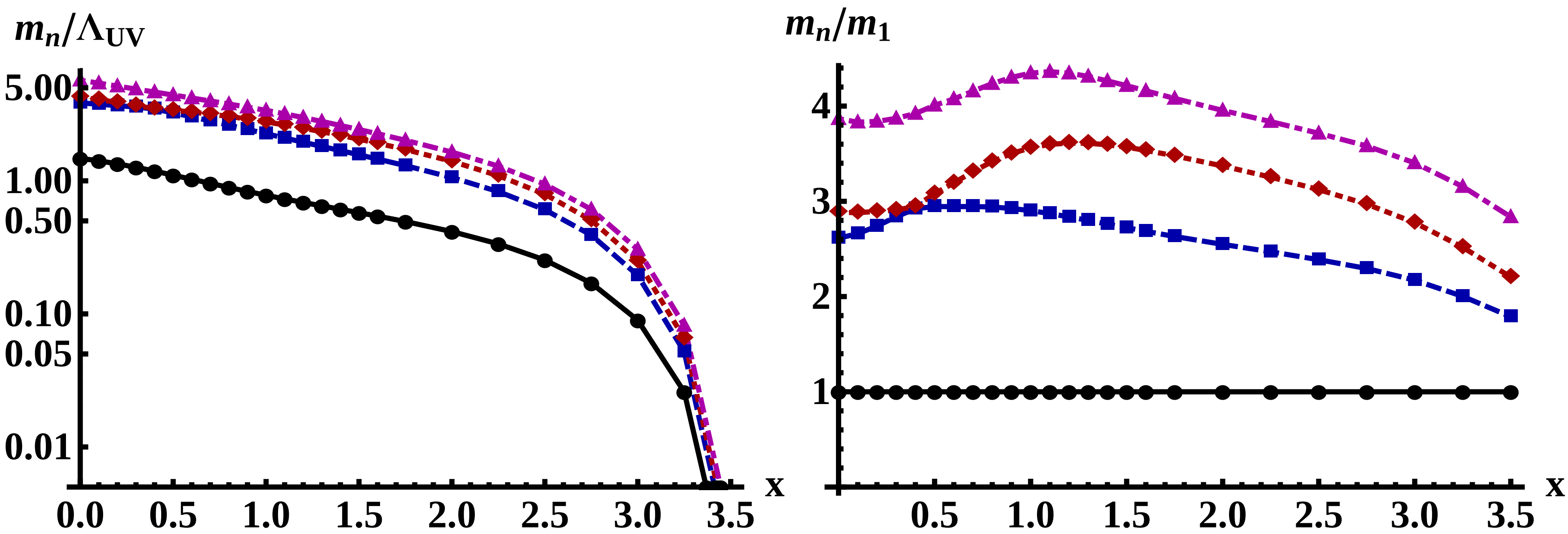
• The conclusions are:

♠ All masses follow Miransky scaling in the walking region.

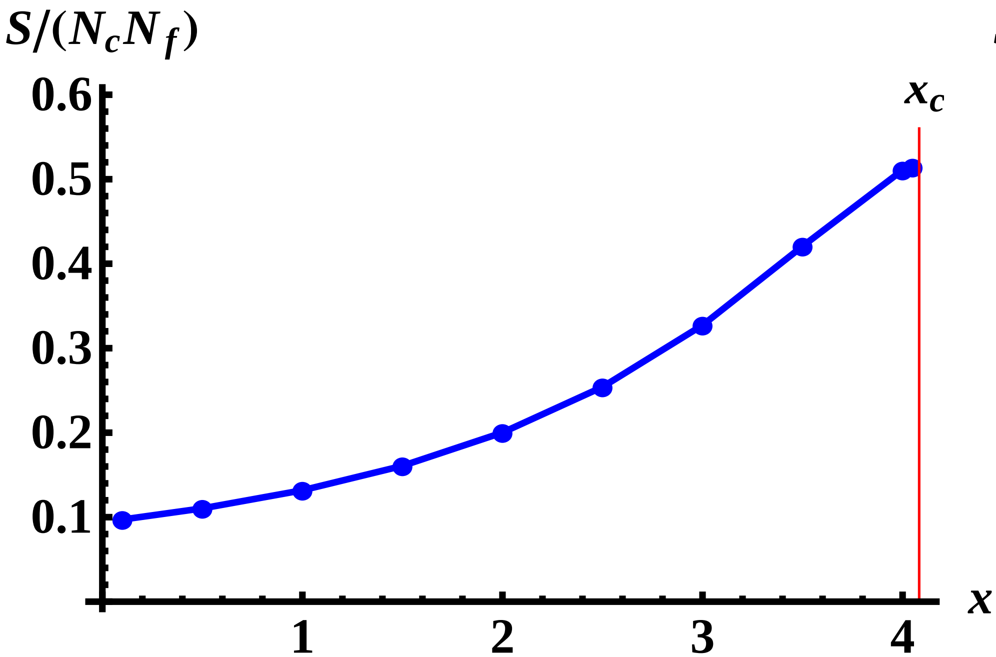
♠ There is no dilaton.Instead all (bound-state) masses go to zero exponentially fast.

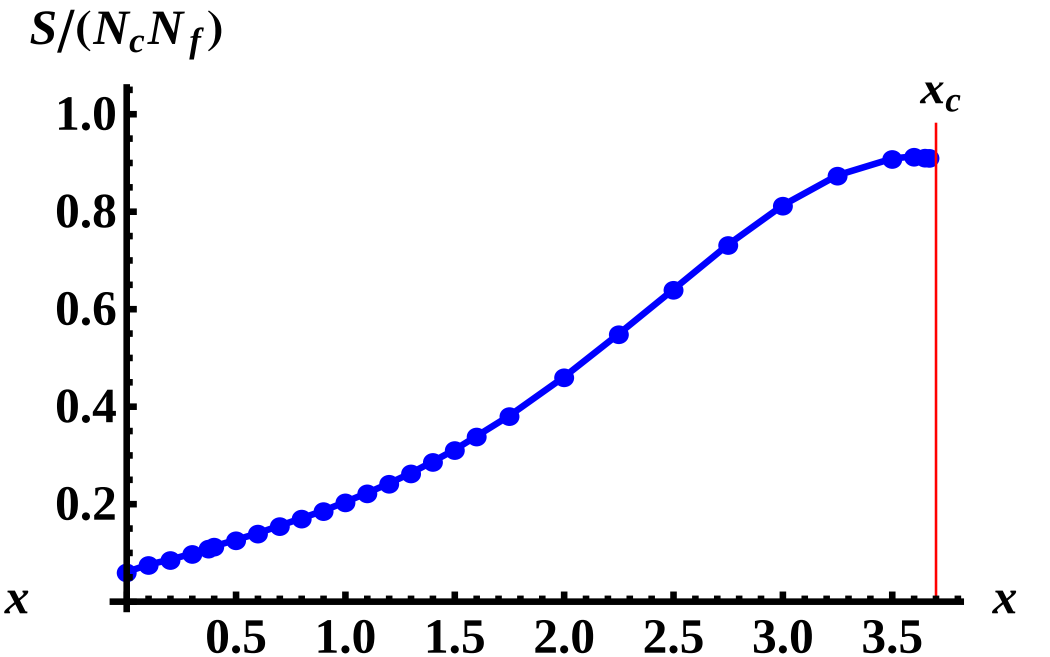
♠ There are several level crossings as x varies but they seem accidental

♠ There is a subtle (and unexpected) discontinuity associated with the S-parameter.



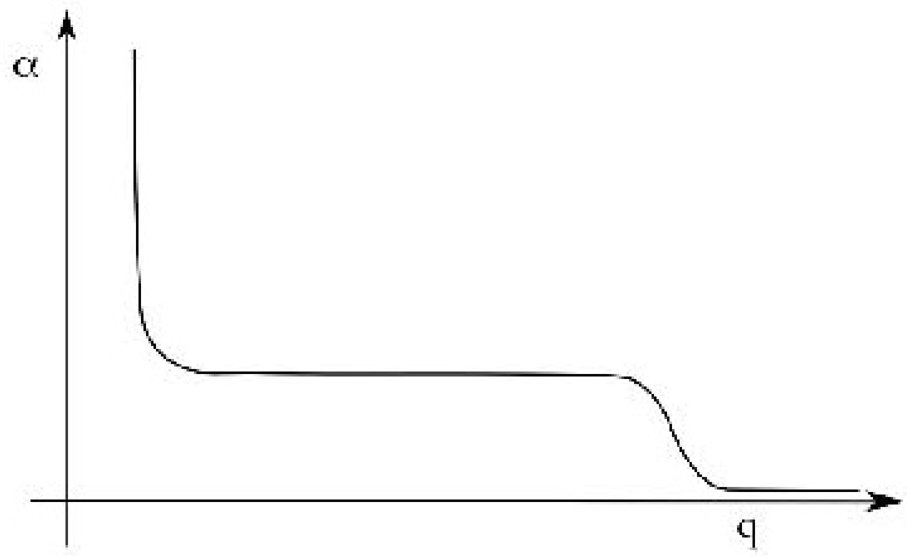
Singlet scalar meson spectra in the potential II class with SB normalization for W_0. They contain the 0^{++} glueballs and the singlet 0^{++} mesons that mix here at leading order. Left: the four lowest masses as a function of x in units of Λ_{UV}. Right: the ratios of masses of up to the fourth massive states as a function of x.

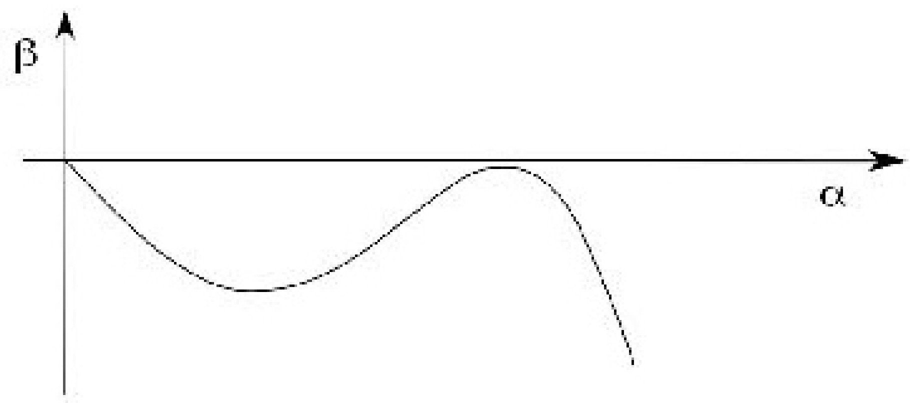




Left: The S-parameter as a function of x for potential class I with $W_0 = \frac{3}{11}$. Right: The S-parameter as a function of x for potential class II with SB normalization for W_0. In both cases S asymptotes to a finite value as $x \to x_c$.

Walking





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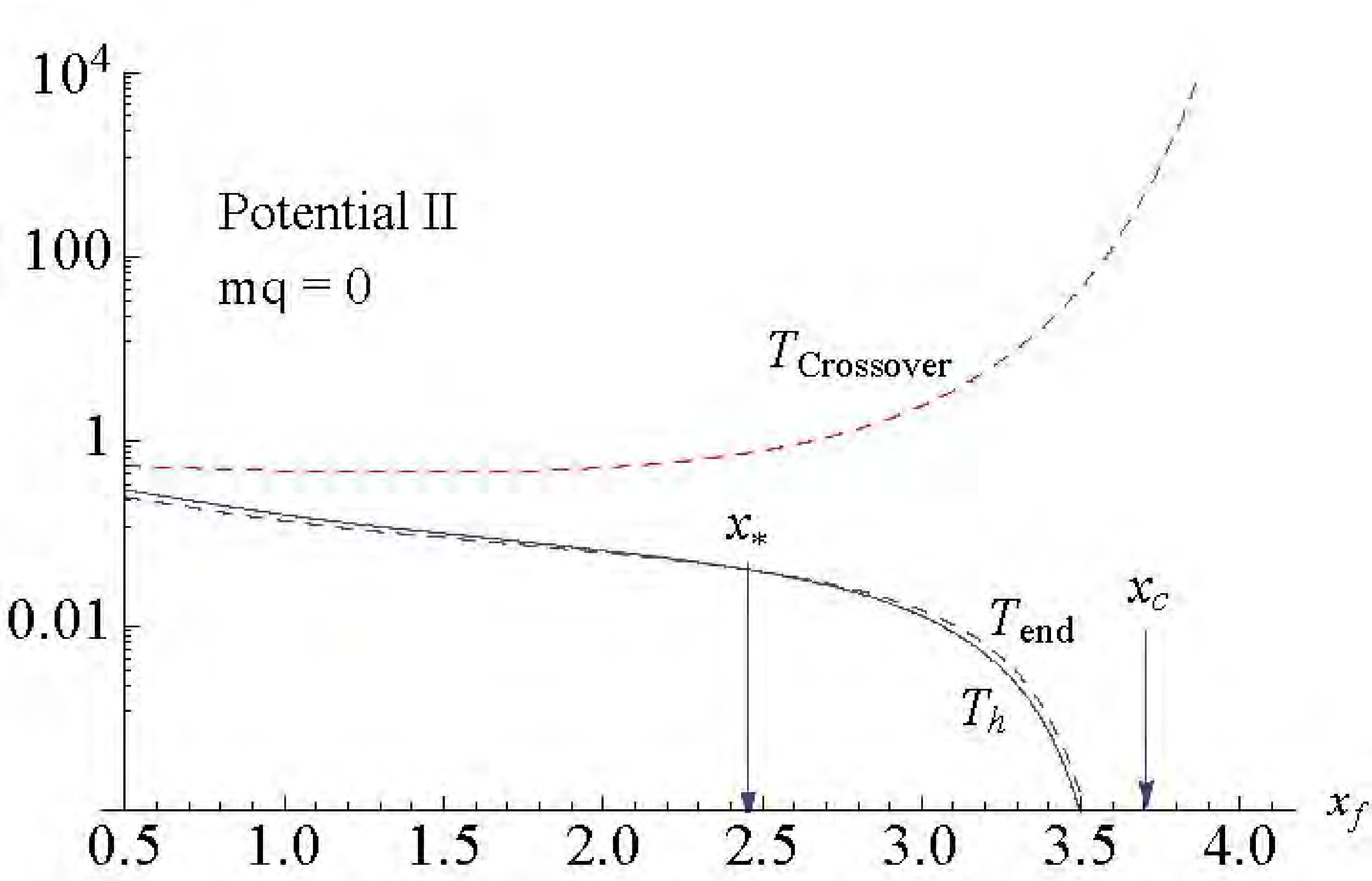
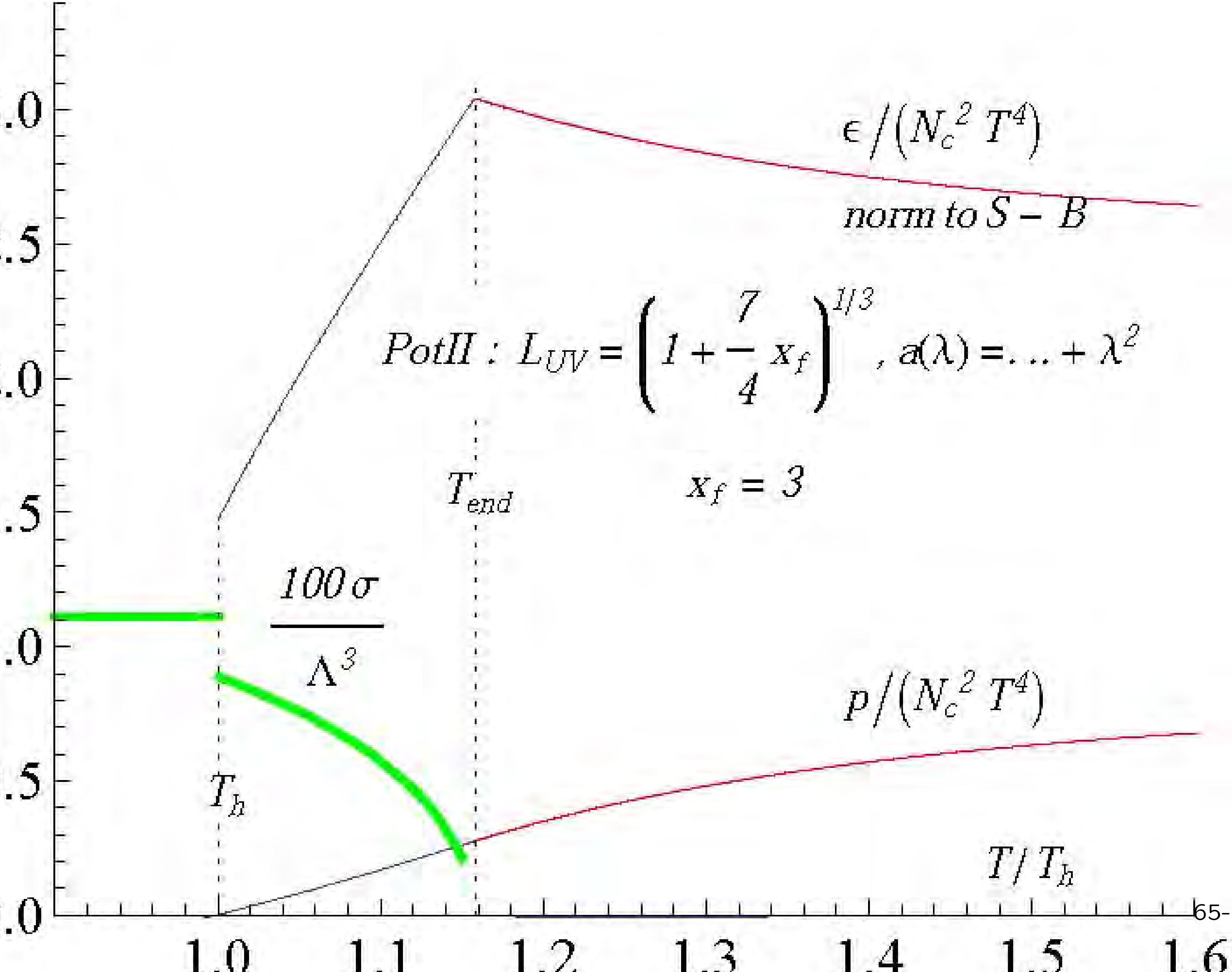
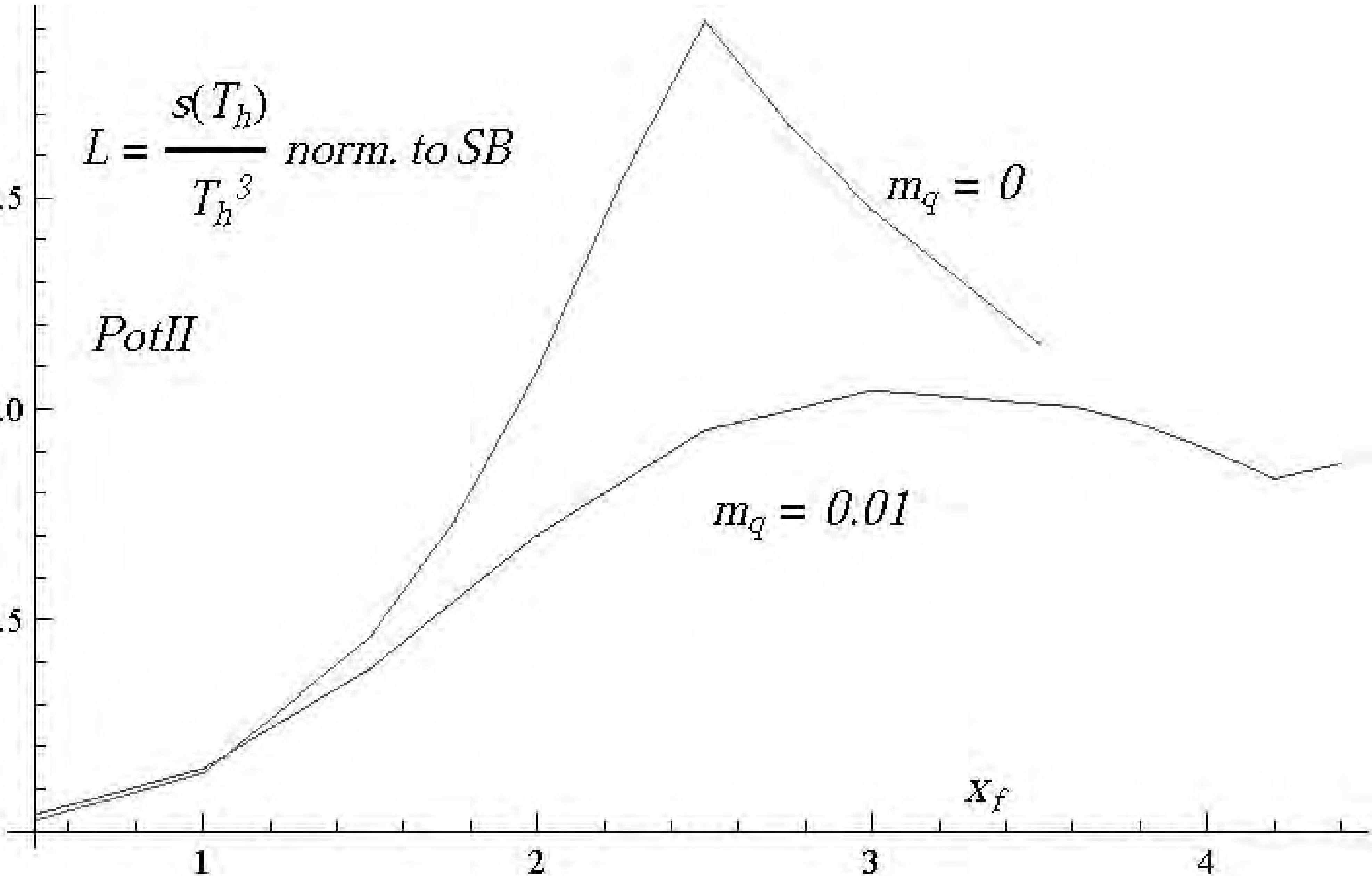
The phase diagram

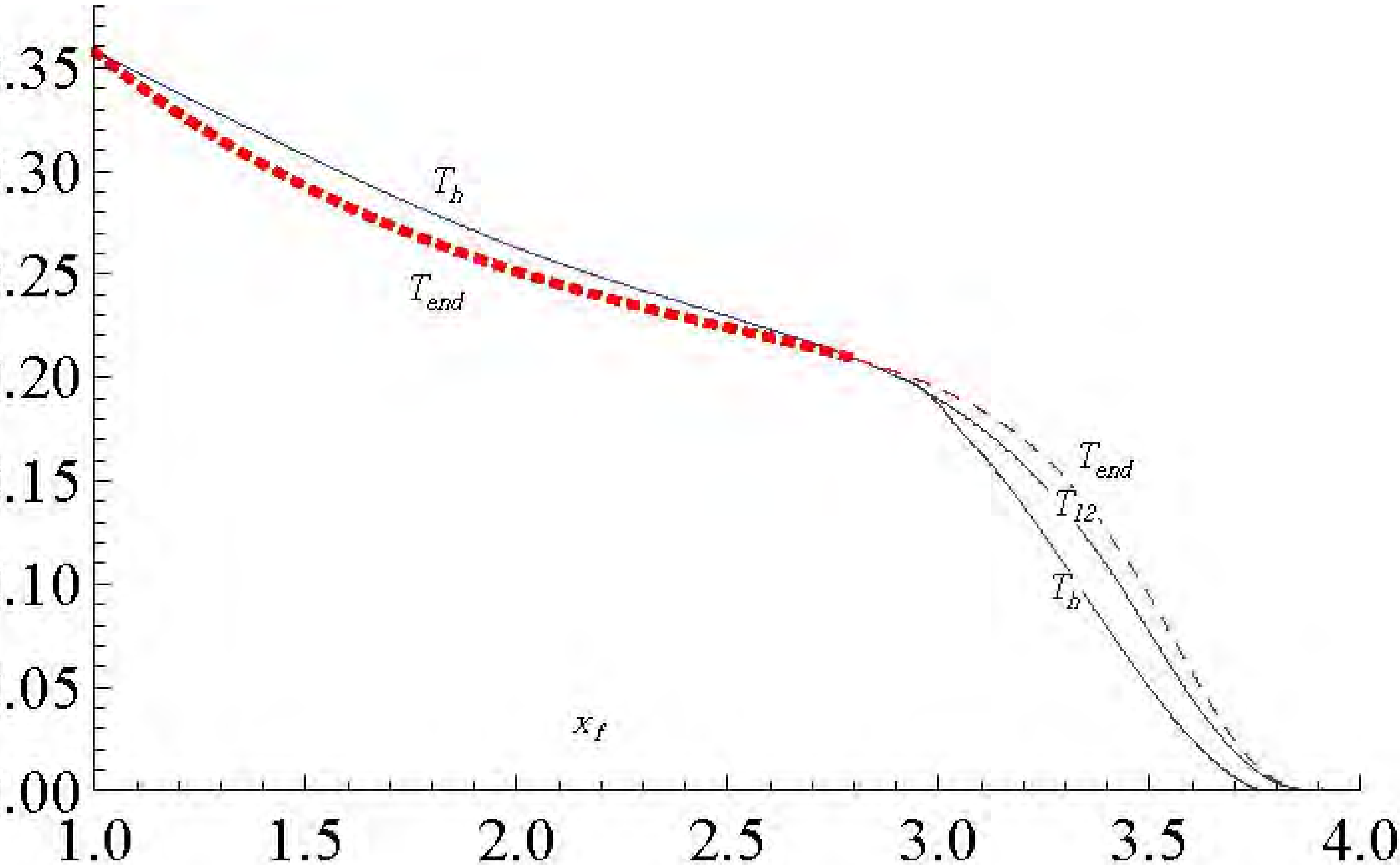




$\epsilon/(N_c^2 T^4)$
norm to S - B
$PotII : L_{UV} = \left(1+\frac{7}{4}x_f\right)^{1/3}, a(\lambda) = \ldots + \lambda^2$
$x_f = 3$
T_{end}
$\frac{100\sigma}{\Lambda^3}$
$p/(N_c^2 T^4)$
T_h
T/T_h
.0
.5
.0
.5
.0
.5
.0
1.0
1.1
1.2
1.3
1.4
1.5
1.6



$L = \frac{s(T_h)}{T_h^3}$ norm. to SB
PotII
$m_q = 0$
$m_q = 0.01$
x_f
.5
.0
.5
1
2
3
4


.35
.30
.25
.20
.15
.10
.05
.00
1.0
1.5
2.0
2.5
3.0
3.5
4.0
T_h
T_{end}
T_{end}
T_{12}
T_h
x_f

The improved Sen tachyon action

• The Sen action is conjectured for unstable branes and $D\bar{D}$ pairs in flat space. It passes several constraints on its dynamics.

• There are background independent arguments that $V_f \to 0$ when D-branes annihilate.

Sen

• The IhQCD background for glue, becomes flat in the string frame in the IR. The dilaton runs however (quadratically).

• The Sen action is expected to have open string corrections: these are not expected to change it basic asymptotics: $V_f \to 0$.

• The $\sqrt{|DT|^2}$ at large T is subleading and does not affect qualitatively the dynamics.

RETURN

The effective potential

For solutions $T = T_* = constant$ the relevant non-linear action simplifies

$$\mathcal{S} = M^3 N_c^2 \int d^5x \sqrt{g} \left[R - \frac{4}{3}\frac{(\partial\lambda)^2}{\lambda^2} + V_g(\lambda) - x V_f(\lambda, T) \right]$$

$$V_f(\lambda, T) = V_0(\lambda)\ e^{-a(\lambda) T_*^2}$$

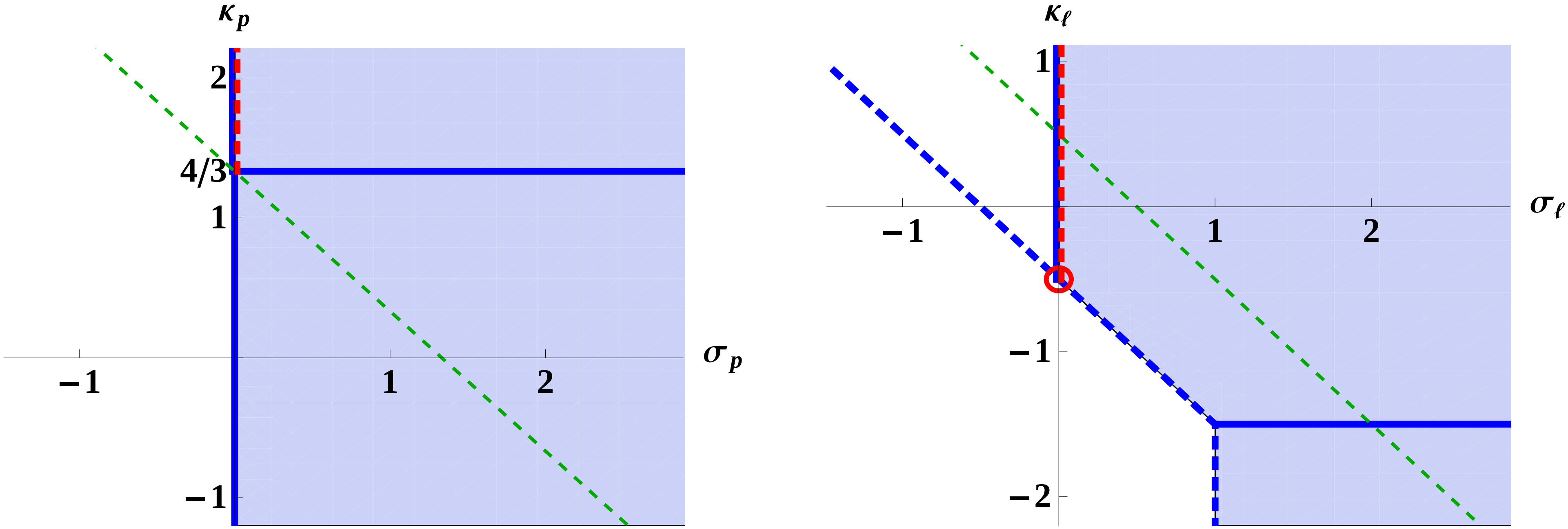
• Minimizing for T_* we obtain $T_* = 0$ and $T_* = \infty$. The effective potential for λ is

♠ $T_* = 0$, $V_{eff} = V_g(\lambda) - x V_0(\lambda)$ with a IR fixed point at $\lambda = \lambda_*(x_f)$.

♠ $T_* = \infty$, $V_{eff} = V_g(\lambda)$ with no fixed points.

• From that point on, according to holography rules, we should find regular solutions for the metric, T and λ, that start with their sources in the UV (UV 't Hooft coupling and quark mass) and compare their free energies.

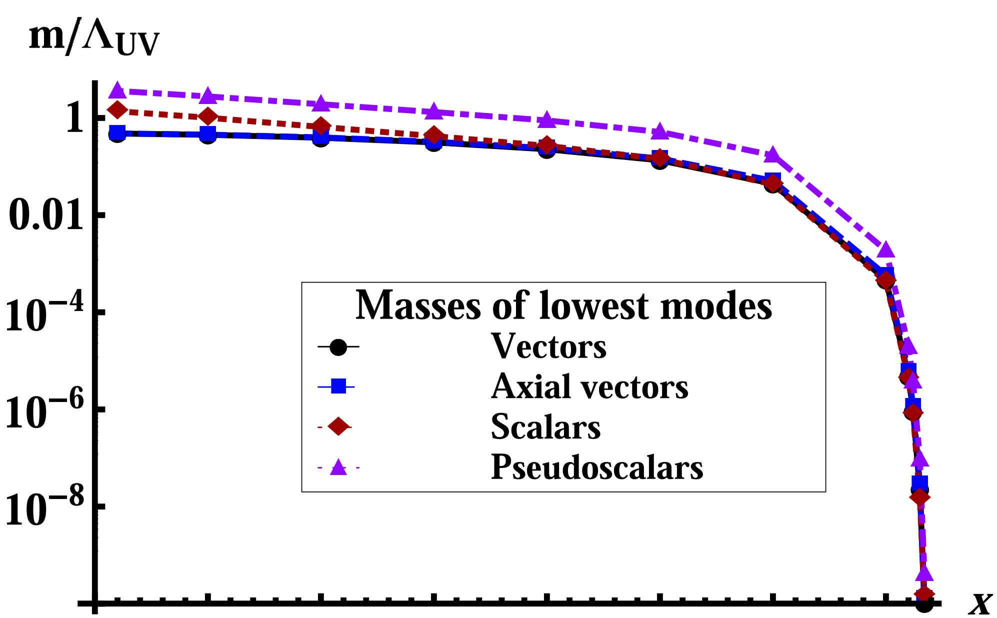
Characterizing IR asymptotics

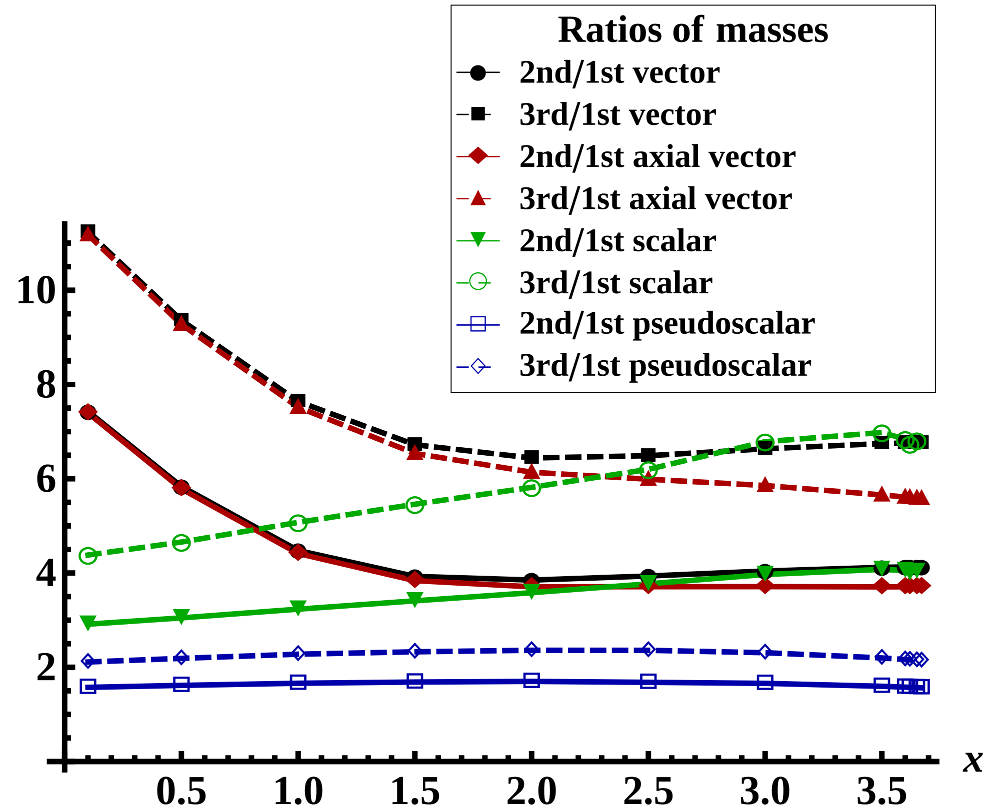


Map of the acceptable IR asymptotics of the functions $\kappa(\lambda) \sim \lambda^{-\kappa_p}(\log\lambda)^{-\kappa_\ell}$ and $a(\lambda) \sim \lambda^{\sigma_p}(\log\lambda)^{\sigma_\ell}$. Left: qualitatively different regions of tachyon asymptotics as a function of the parameters κ_p and a_p characterizing the power-law asymptotics of the functions. Right: regions of tachyon asymptotics at the critical point $\kappa_p = 4/3$, $a_p = 0$ as a function of the parameters κ_ℓ and a_ℓ characterizing the logarithmic corrections to the functions. In each plot, the shaded regions have acceptable IR behavior, and the thick blue lines denote changes in the qualitative IR behavior of the tachyon background. On the solid blue lines good asymptotics can be found, whereas on the dashed lines such asymptotics is absent. The thin dashed green line shows the critical behavior where the BF bound is saturated as $x \to 0$. Potentials above this line are guaranteed to have broken chiral symmetry at small x.

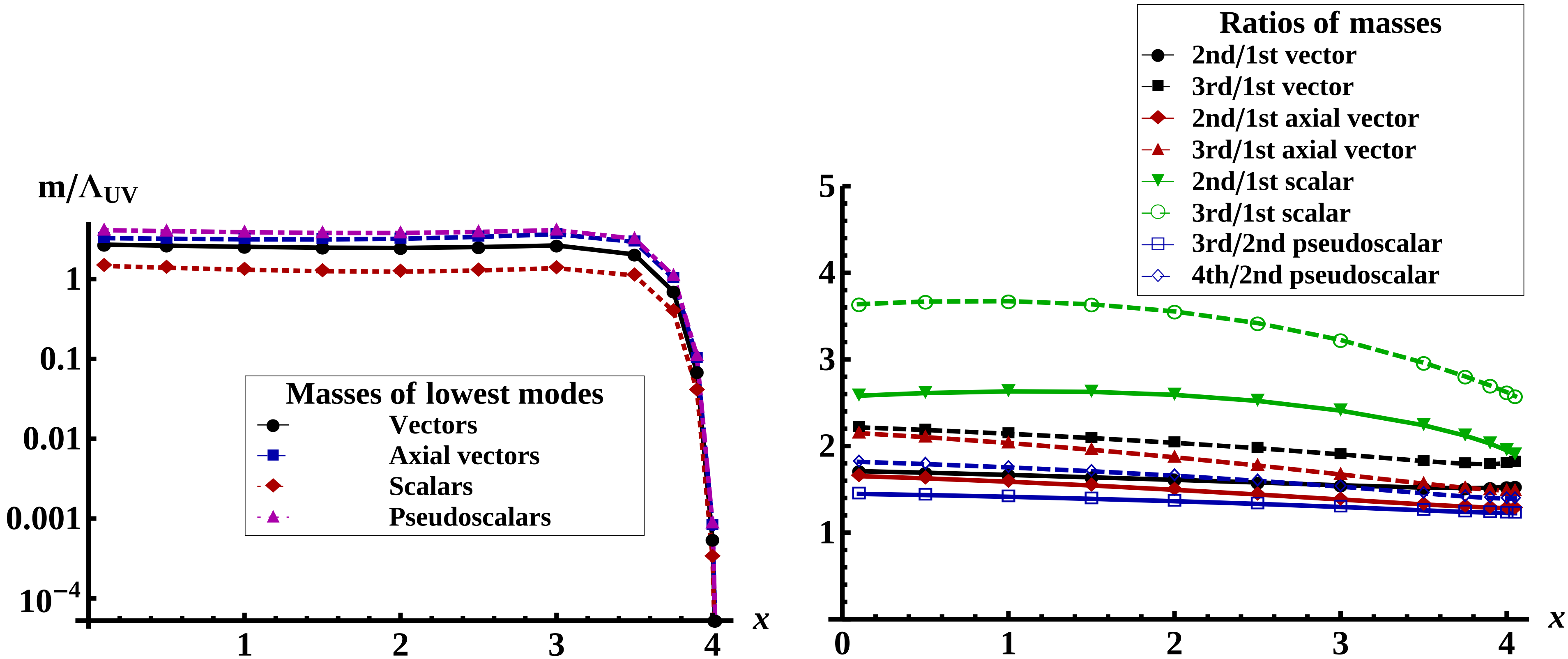
Finally, on the red dashed lines the asymptotic meson mass trajectories are linear with subleading logarithmic corrections. The red circle shows the single choice of parameters where the logarithmic corrections are absent.

Spectra

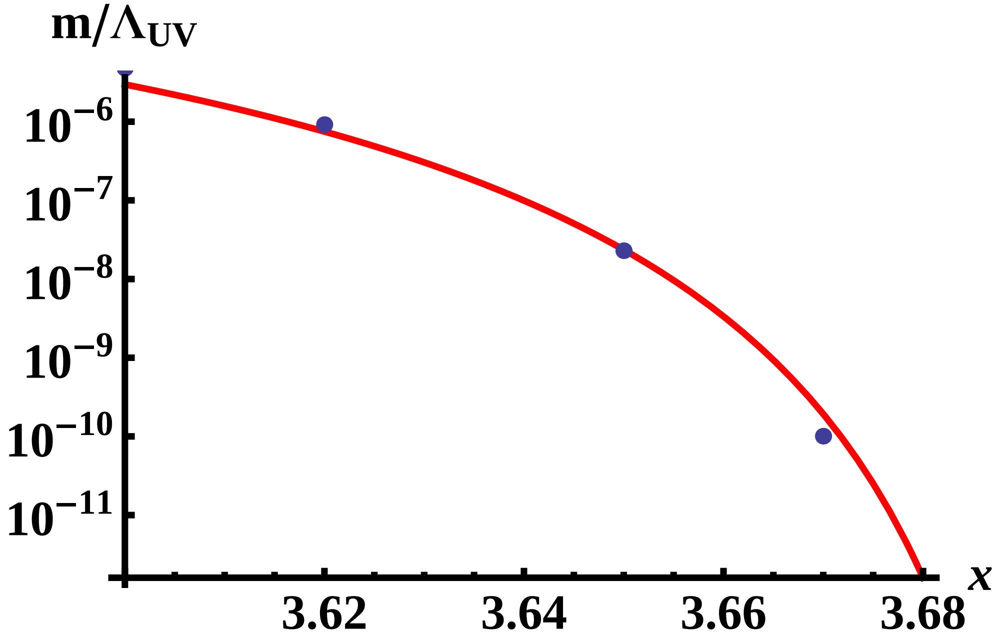


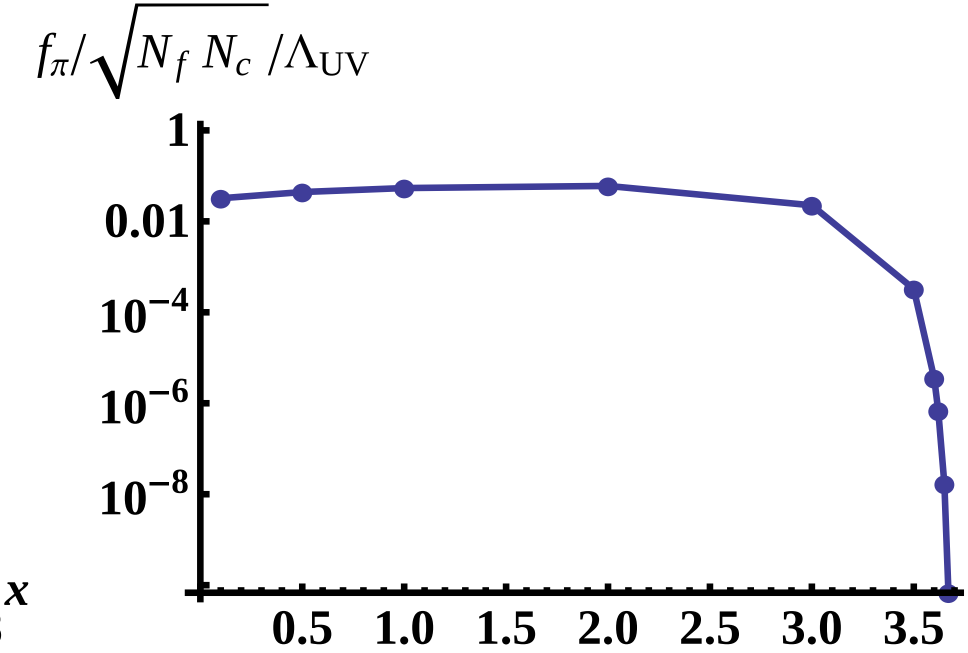


Non-singlet meson spectra in the potential II class with Stefan-Boltzmann (SB) normalization for W_0, with $x_c \simeq 3.7001$. Left: the lowest non-zero masses of all four towers of mesons, as a function of x, in units of $\Lambda_{\rm UV}$, below the conformal window. Right, the ratios of masses of up to the fourth massive states in the same theory as a function of x.



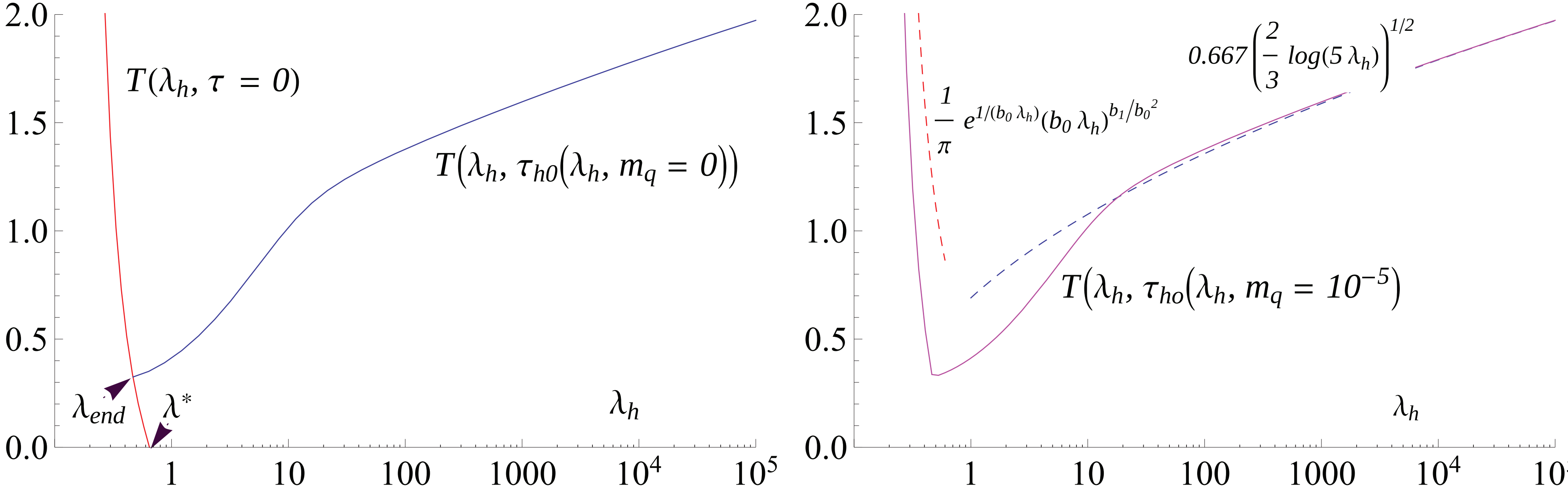
Non-singlet meson spectra in the potential I class ($W_0 = \frac{3}{11}$), with $x_c \simeq 4.0830$. Left: the lowest non-zero masses of all four towers of mesons, as a function of x, in units of $\Lambda_{\rm UV}$, below the conformal window. Right, the ratios of masses of up to the fourth massive states in the same theory as a function of x.



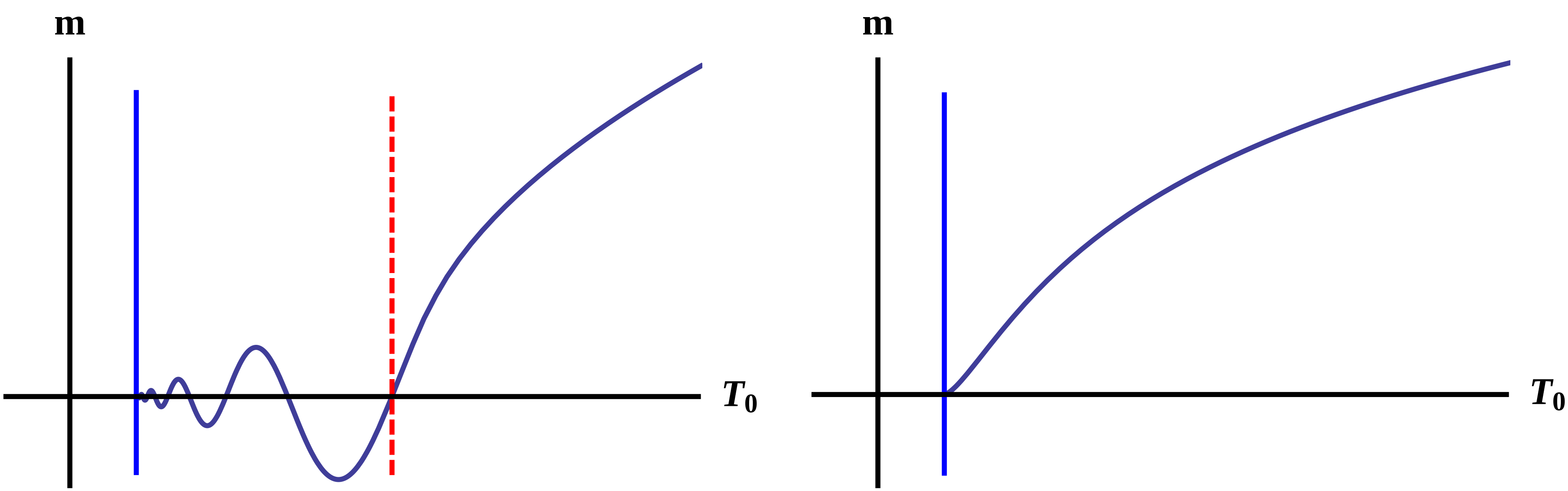


Potential II with SB normalization for W_0. Left: A fit of the ρ mass to the Miransky scaling factor, showing that it displays Miransky scaling in the walking region. Right, f_π as a function of x in units of Λ_{UV}. It vanishes near x_c following again Miransky scaling.

Finite small mass



UV mass vs IR parameter



• Left figure: Plot of the UV Mass parameter m, as a function of the IR T_0 scale, for $x < x_c$. Right figure: Similar plot for $x \geq x_c$.

• At $m = 0$ there is an ∞ number of saddle point solutions (Efimov-like minima)

• The Efimov minima have free energies ΔE_n with

$$\Delta E_0 > \Delta E_1 > \Delta E_2 > \cdots$$

Parameters

• A theory with a single relevant (or marginally relevant) coupling like YM has no parameters.

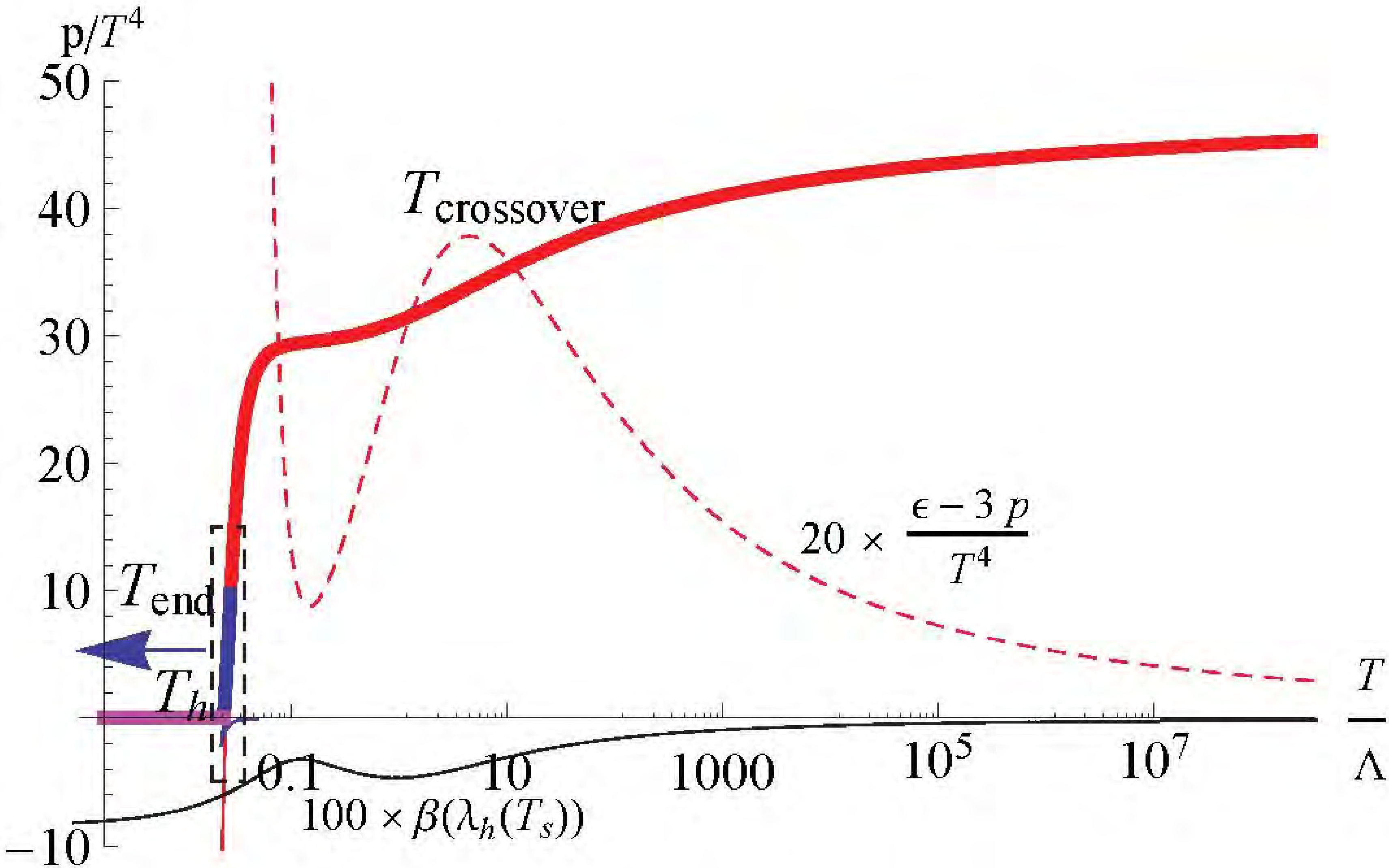
• The same applies to QCD with massless quarks.

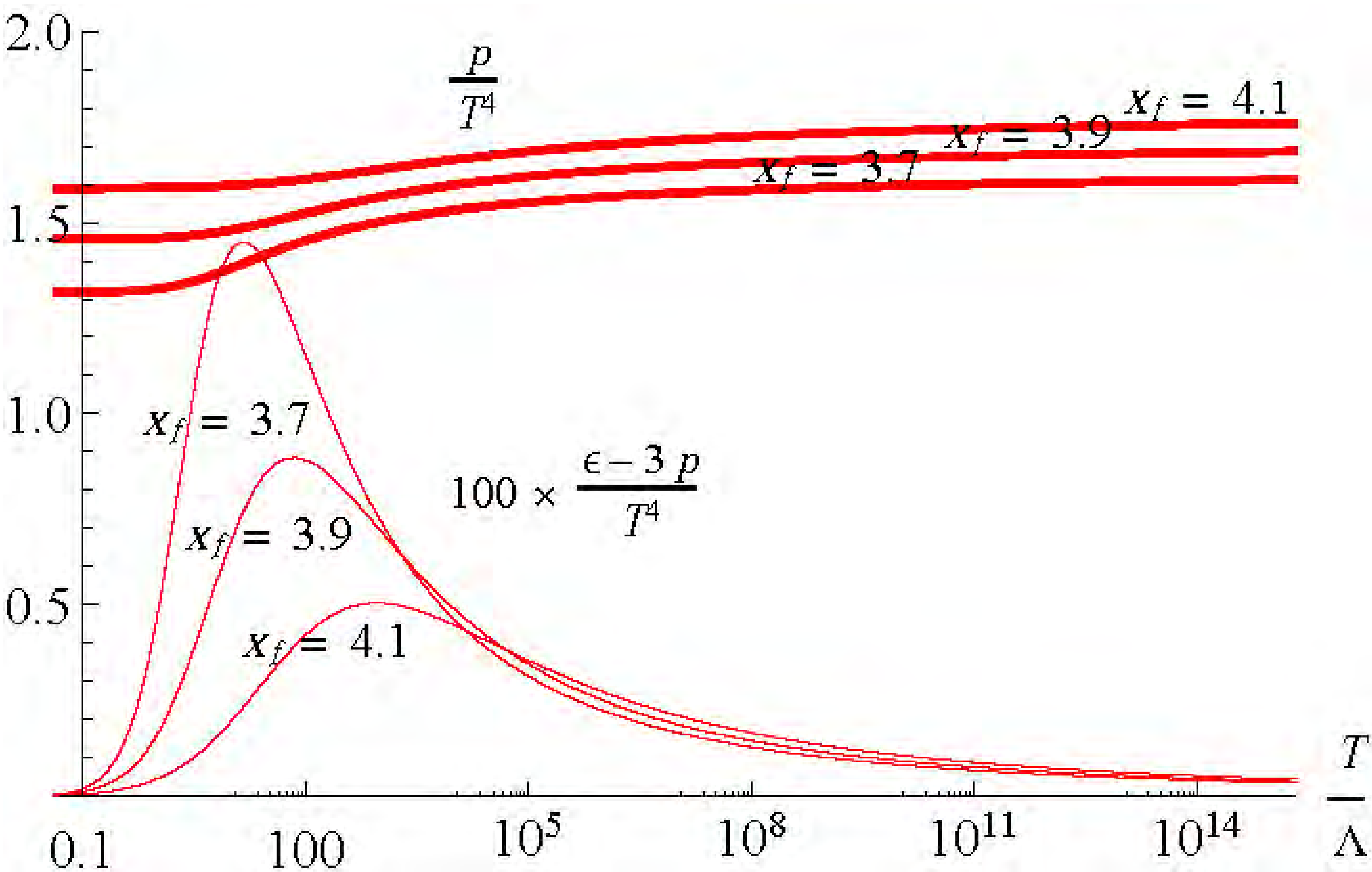
• QCD with all quarks having mass m has a single (dimensionless) parameter : $\frac{m}{\Lambda_{QCD}}$.

• After various rescalings this single parameter can be mapped to the parameter T_0 (integration constant) that controls the diverging tachyon in the IR.

• There is also $x = \frac{N_f}{N_c}$ that has become continuous in the large N_c Veneziano limit.

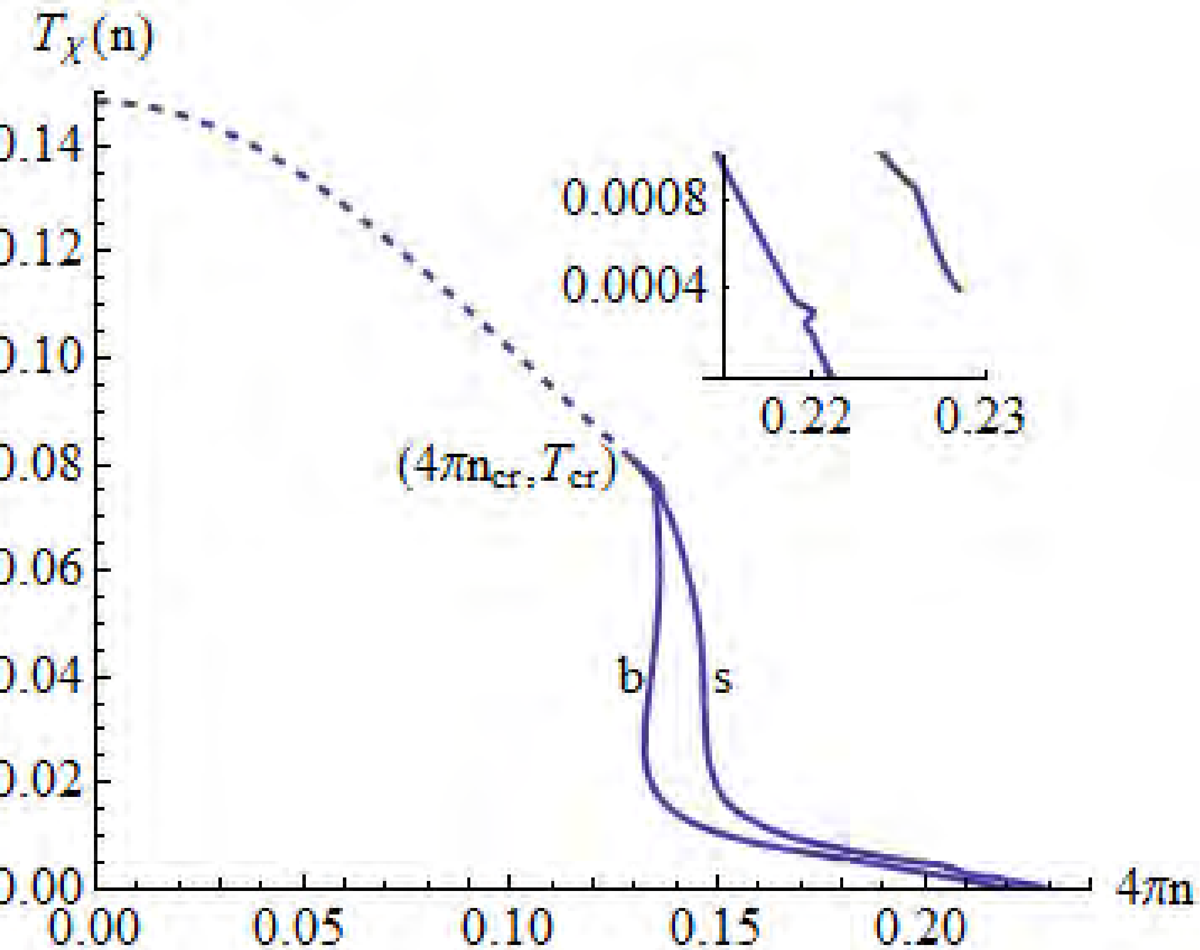
The finite T phase diagram





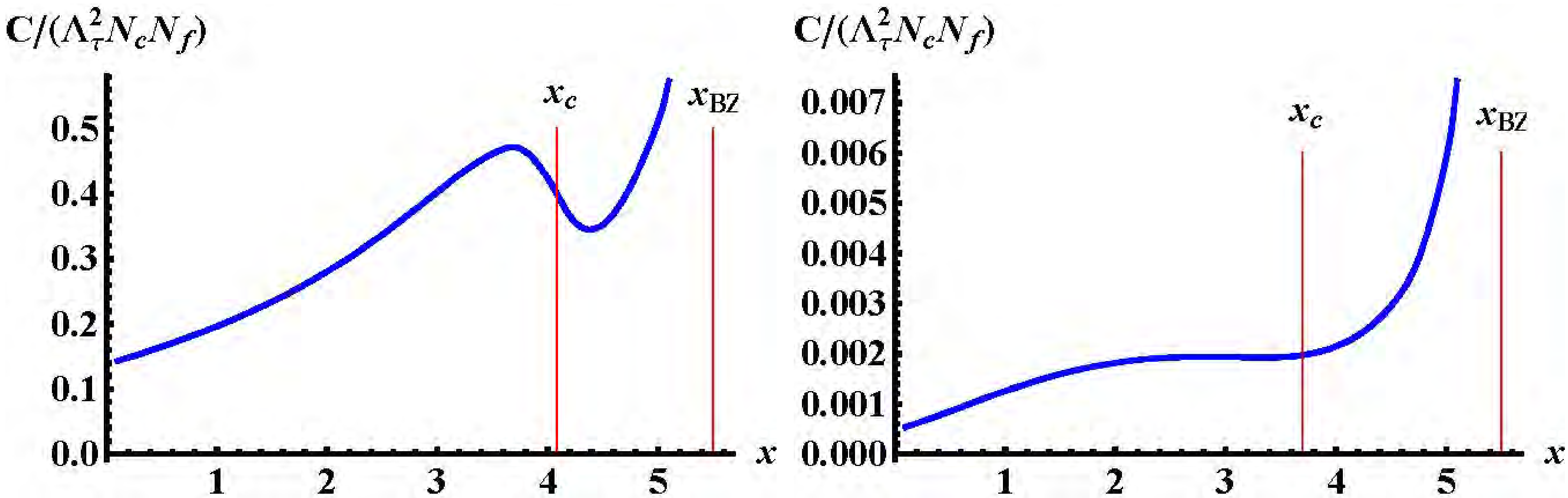
2.0
1.5
1.0
0.5
$\frac{p}{T^4}$
$x_f = 4.1$
$x_f = 3.9$
$x_f = 3.7$
$x_f = 3.7$
$x_f = 3.9$
$x_f = 4.1$
$100 \times \frac{\epsilon - 3p}{T^4}$
0.1
100
10^5
10^8
10^{11}
10^{14}
$\frac{T}{\Lambda}$

The finite density phase diagram

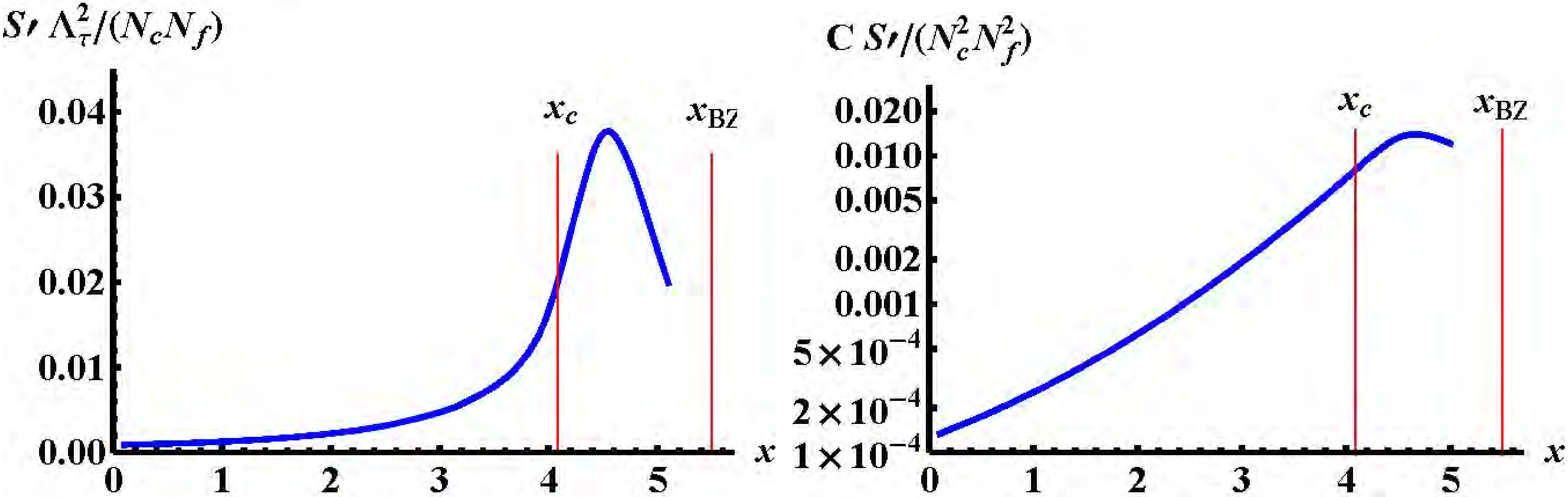


Dependence of the chiral transition temperature on chemical potential or on quark number density. Along the 1st order line there is a jump in n. The inset shows a closeup of the $T = 0$ region with density jump. The hadronic phase has been "squeezed on the $n = 0$ axis.

C, S' parameters

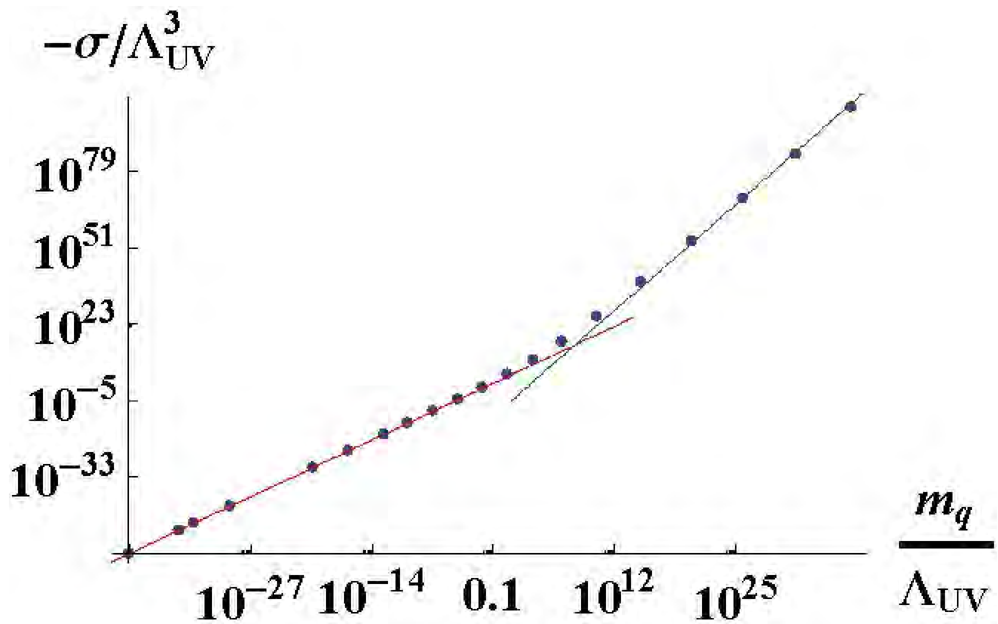


The constant term in the difference of the vector-vector and axial-axial correlators as a function of x for $m_q/\Lambda_{UV} = 10^{-6}$. Left: potentials I with $W_0 = 3/11$. Right: potentials II with SB normalized W_0.



The x dependence of the higher order coefficient S' for $m_q/\Lambda_{UV} = 10^{-6}$ and for potentials I with $W_0 = 3/11$. Left: S' in units of Λ_τ. Right: The dimensionless product CS'.

The chiral condensate



The dependence of σ on the quarks mass for potentials II with SB normalized W_0 at $x = 4$. The blue dots are numerical data while the red and blue curves are analytic fits.

Small quark mass: $\frac{m_q}{\Lambda_{UV}} \ll 1$.

$$\frac{\sigma}{\Lambda^3_{UV}} \sim \left(\frac{m_q}{\Lambda_{UV}}\right)^{\frac{4-\Delta_*}{\Delta_*}}$$

Large quark mass: $\frac{m_q}{\Lambda_{UV}} \gg 1$.

$$\sigma \sim m_q^3$$

Mass scales in V-QCD: non-zero quark masses

- We must recall the tachyon solution in different regimes

♠ UV: $\tau \simeq m_q\, r + \sigma\, r^3 + \cdots \qquad r \ll \Lambda_{UV}^{-1}$

♠ Walking regime: $\tau \simeq C_w\, r^2 + \cdots$, $\Lambda_{UV}^{-1} \ll r \ll \Lambda_\tau^{-1}$.

♠ IR CFT regime: $\tau \simeq C_1 r^{\Delta_*} + C_2\, r^{4-\Delta_*} + \cdots$, $\Lambda_{UV}^{-1} \ll r \ll \Lambda_\tau^{-1}$.

- QCD regime:

a) $m_q \ll \Lambda_{UV}$. We obtain, $\Lambda_{UV} \sim \Lambda_{IR} \sim \Lambda_\tau$ and $\sigma \sim \Lambda_{UV}^3$.

b) $m_q \gg \Lambda_{UV}$. $\Lambda_\tau \sim m_q$,

$$\frac{\Lambda_{UV}}{\Lambda_{IR}} \sim \left(\frac{m_q}{\Lambda_{UV}}\right)^{\frac{b_0}{b_0^{YM}}-1} \quad , \quad \frac{m_q}{\Lambda_{IR}} \sim \left(\frac{m_q}{\Lambda_{UV}}\right)^{\frac{b_0}{b_0^{YM}}} \quad , \quad \sigma \sim m_q^3$$

• <u>walking regime</u>:

a) Small quark mass: from continuity $C_w \sim \Lambda_{IR}^2$ and $\sigma \sim \Lambda_{IR}^2 \Lambda_{UV}$. Then when

$$\frac{m_q}{\Lambda_{UV}} \ll \frac{\Lambda_{IR}^2}{\Lambda_{UV}^2} \sim e^{-\frac{2\pi}{\nu}}$$

Then we have the same situation as $m_q = 0$.

b) Intermediate quark mass: $1 \gg \frac{m_q}{\Lambda_{UV}} \gg e^{-\frac{2\pi}{\nu}}$ the amount of walking is controlled by the quark mass. We still have $\Lambda_\tau \sim \Lambda_{IR}$ and $C_w \sim \Lambda_{IR}^2$ but continuity at $r = \Lambda_{UV}^{-1}$ gives

$$\frac{m_q}{\Lambda_{UV}} \sim \frac{\Lambda_{IR}^2}{\Lambda_{UV}^2} \sim \frac{\sigma}{\Lambda_{UV}^3}$$

and there is no Miransky scaling.

c) Large quark mass $\frac{m_q}{\Lambda_{UV}} \gg 1$. In this regime there is no walking and we get the same results as for heavy quarks in the QCD regime.

• Conformal window regime.

a) Small quark mass: $\frac{m_q}{\Lambda_{UV}} \ll 1$. We have $\Lambda_{IR} \sim \Lambda_\tau$ and $C_1\ \Lambda_{IR}^{\Delta_*} \sim C_2\ \Lambda_{IR}^{4-\Delta_*} \sim 1$. Matching

$$\frac{m_q}{\Lambda_{UV}} \sim \left(\frac{\Lambda_{UV}}{\Lambda_{IR}}\right)^{\Delta_*} \quad , \quad \frac{\sigma}{\Lambda_{UV}^3} \sim \left(\frac{\Lambda_{UV}}{\Lambda_{IR}}\right)^{4-\Delta_*} \sim \left(\frac{m_q}{\Lambda_{UV}}\right)^{\frac{4-\Delta_*}{\Delta_*}}$$

b) Large quark mass: $\frac{m_q}{\Lambda_{UV}} \gg 1$. In this regime the theory never gets close to the IR fixed point and we obtain the same results as for heavy quarks in the QCD regime.

Four fermion couplings

• We may introduce a new set of interactions and associated couplings in the theory in the Veneziano limit:

$$\delta S = \int d^4x V(\bar{\psi}\psi) \simeq m_q \ \bar{\psi}\psi + \frac{g_2}{2} \ (\bar{\psi}\psi)^2 + \frac{g_3}{3} \ (\bar{\psi}\psi)^3 + \cdots$$

• As $N_c \to \infty$ they are renormalizable (and easy to accommodate)

• They are generated by a higher gauge theory (ETC) at finite N_c.

• In the holographic description, they amount to introducing a "boundary potential" for the "tachyon"

$$\delta S_{boundary} = \int d^4x V(\tau) \simeq m_q \ \tau + \frac{g_2}{2} \ \tau^2 + \frac{g_3}{3} \ \tau^3 + \cdots$$

$$\to Tr[M_q \ T] + Tr[g_2 \ T^2] + Tr[g_3 \ T^3] + cc + \cdots$$

• The only thing that changes in the holographic system is the identification of source and vev in the tachyon solution.

Witten

$$source = \frac{dV(x)}{dx}\Big|_{x=\sigma} = m_q + g_2\sigma + g_3\sigma^2 + \cdots$$

• Therefore we can "solve" also in this case, if we know the $\sigma(m_q)$ curve in the absence of multitrace couplings.

• We now set $g_{n\geq 3} = 0$. When $g_2 > 0$, there are no qualitative changes: the dominant vacuum for $x < x_c$ is the one with zero nodes, and for $x > x_c$, $\tau = 0$.

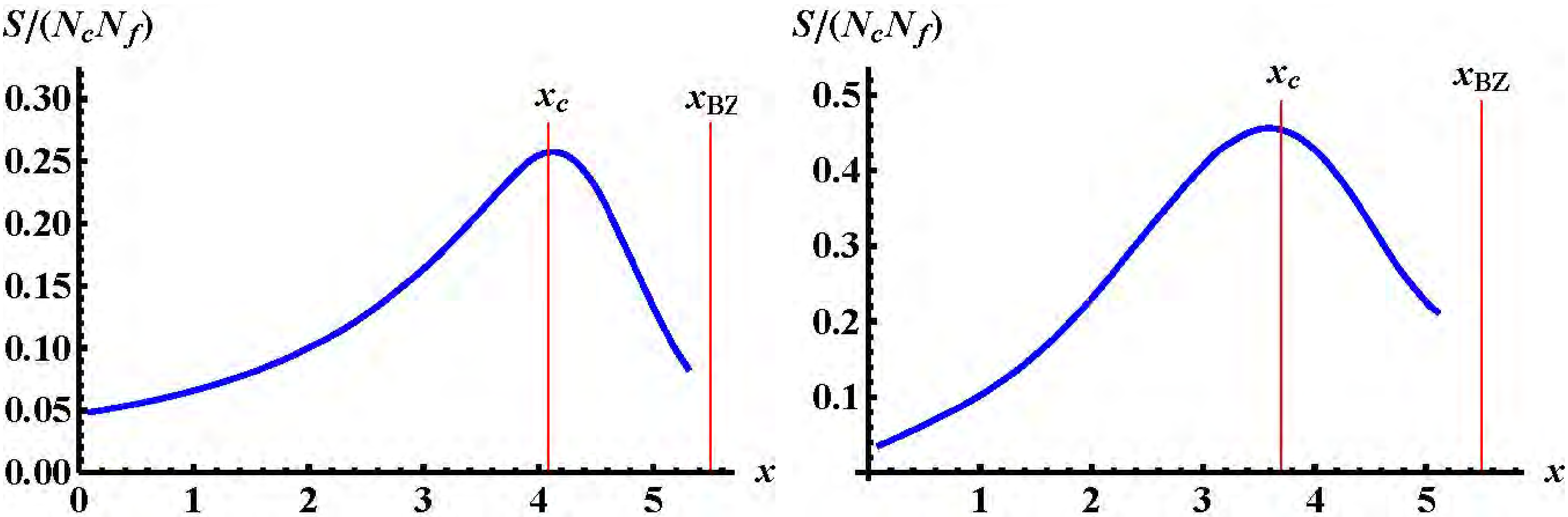
• When $g_2 < 0$ $\tau = 0$ for $x > X_c$, but for $x < x_c$ there are non-trivial changes. At some critical value, chiral symmetry is restored.

• For higher values, there is chiral symmetry breaking again.

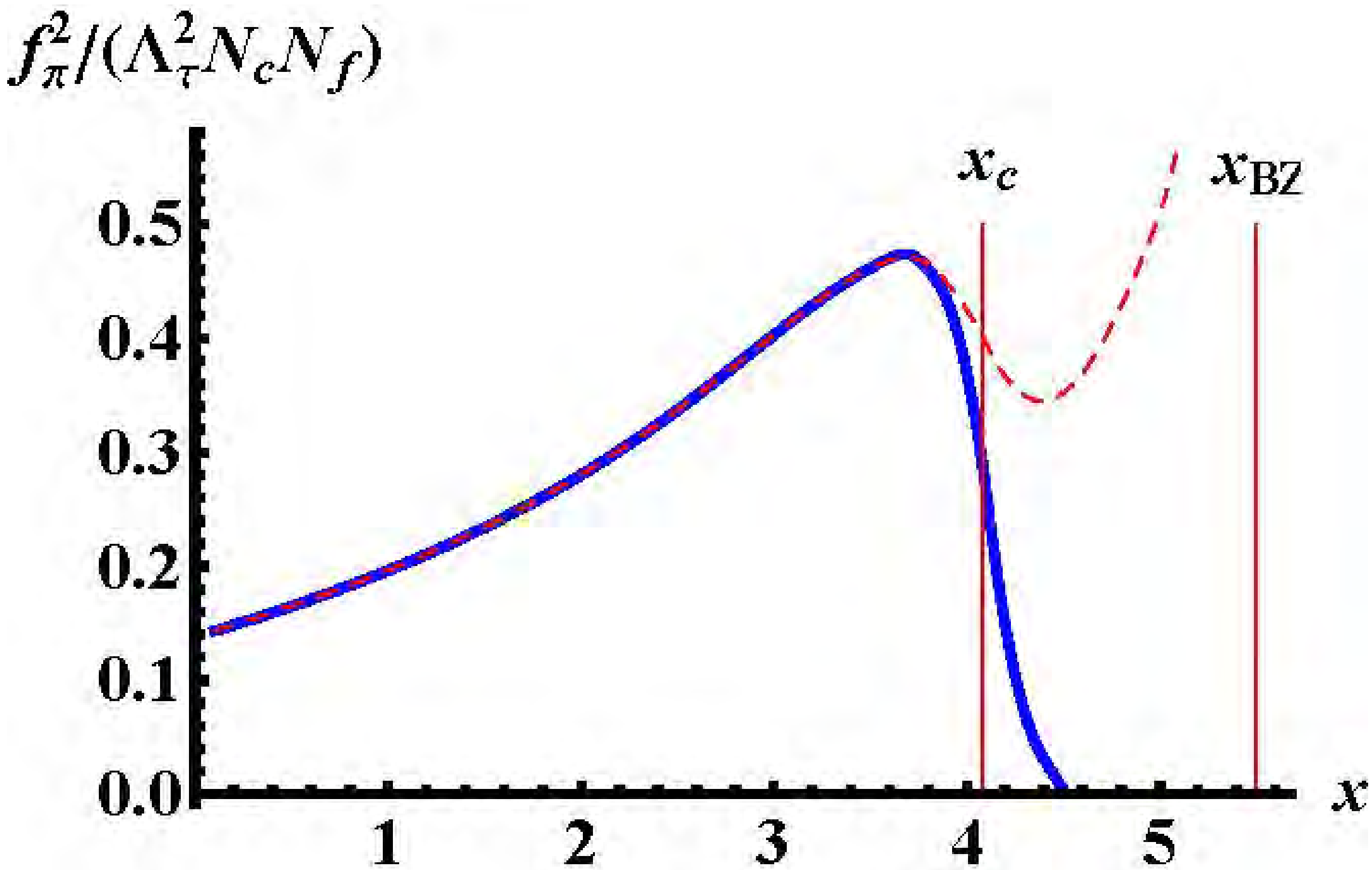
• We believe that there is another critical negative value where the theory is destabilized completely (the dominant saddle-point has a particle with negative-mass-square).

The S parameter at $m_q \neq 0$

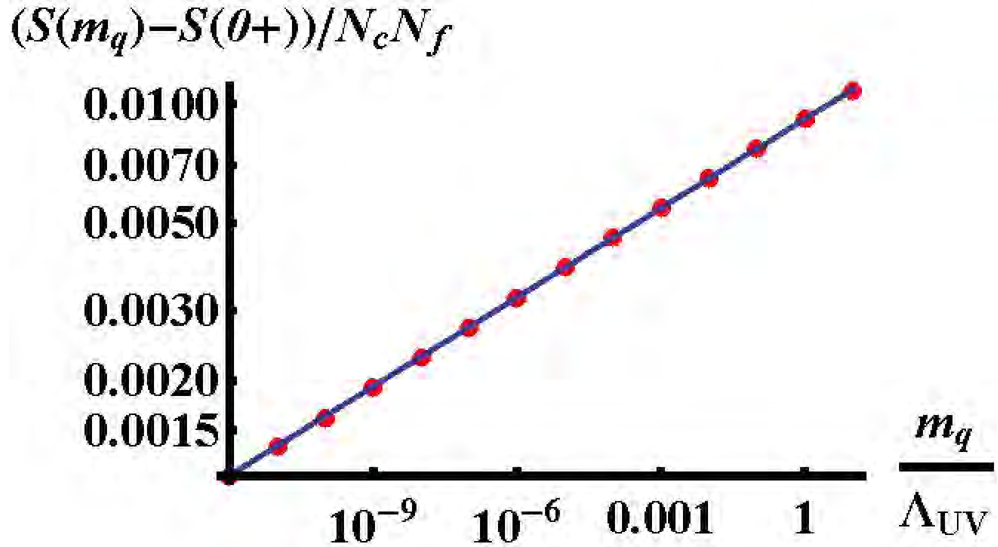
- We turn on a small quark mass $\frac{m_q}{\Lambda_{UV}} = 10^{-6}$ and we calculate C, S, S'



The normalized S-parameter as a function of x for $m_q/\Lambda_{UV} = 10^{-6}$. Left: potentials I with $W_0 = 3/11$. Right: potentials II with SB normalized W_0.



The pion decay constant as a function of x for $m_q/\Lambda_{UV} = 10^{-6}$. (potentials I with $W_0 = 3/11$).



The mass dependence of the S-parameter for $x = 4.5$ in log-log scale. The red dots are the data and the blue line is a power-law fit. (potentials I with $W_0 = 3/11$.)

$$\frac{S(m_q) - S(0+)}{N_c N_f} = \beta_1 m_q^{\beta_2} \quad , \quad S(0+) \equiv \lim_{m_q \to 0+} S(m_q)$$

Thus there is a discontinuity at $m_q = 0$. We find that $\beta_2 \simeq 0.08$.

$$\lambda_* - \lambda_{IR} = \lambda_* - \lambda(r = \Lambda_{IR}^{-1}) \sim \left(\frac{\Lambda_{IR}}{\Lambda_{UV}}\right)^{\delta} \sim \left(\frac{m_q}{\Lambda_{UV}}\right)^{\frac{\delta}{\Delta_*}}$$

$$\frac{\delta}{\Delta_*} \simeq 0.0780 \simeq 0.08 \quad at \quad x = 4.5$$

Mass scales in V-QCD

- There are two UV scales and two IR scales.

- Λ_{UV} is defined in the UV from the running of the 't Hooft coupling

$$\lambda \simeq \frac{1}{b_0 \log(r\Lambda_{UV})} + \cdots$$

- m_q is determined from the tachyon near the boundary

$$\tau \simeq m_q\, r \quad (-\log(r\Lambda_{UV}))^{-\rho} [1 + \cdots] \quad , \quad \rho = \frac{\gamma_0}{b_0}$$

- Λ_{IR} is defined from the IR YM geometry. It is characteristic of the YM IR Phase.

$$e^A \simeq e^{-r^2 \Lambda_{IR}{}^2} \quad , \quad r \to \infty$$

- Λ_τ is defined as the scale at which the tachyon becomes of order one and affects the running of λ.

$$\tau(r = \Lambda_\tau{}^{-1}) \simeq 1$$

- These scales are related, but their relation depends on x.

- At $m_q = 0$, the following four distinct regimes must be considered:

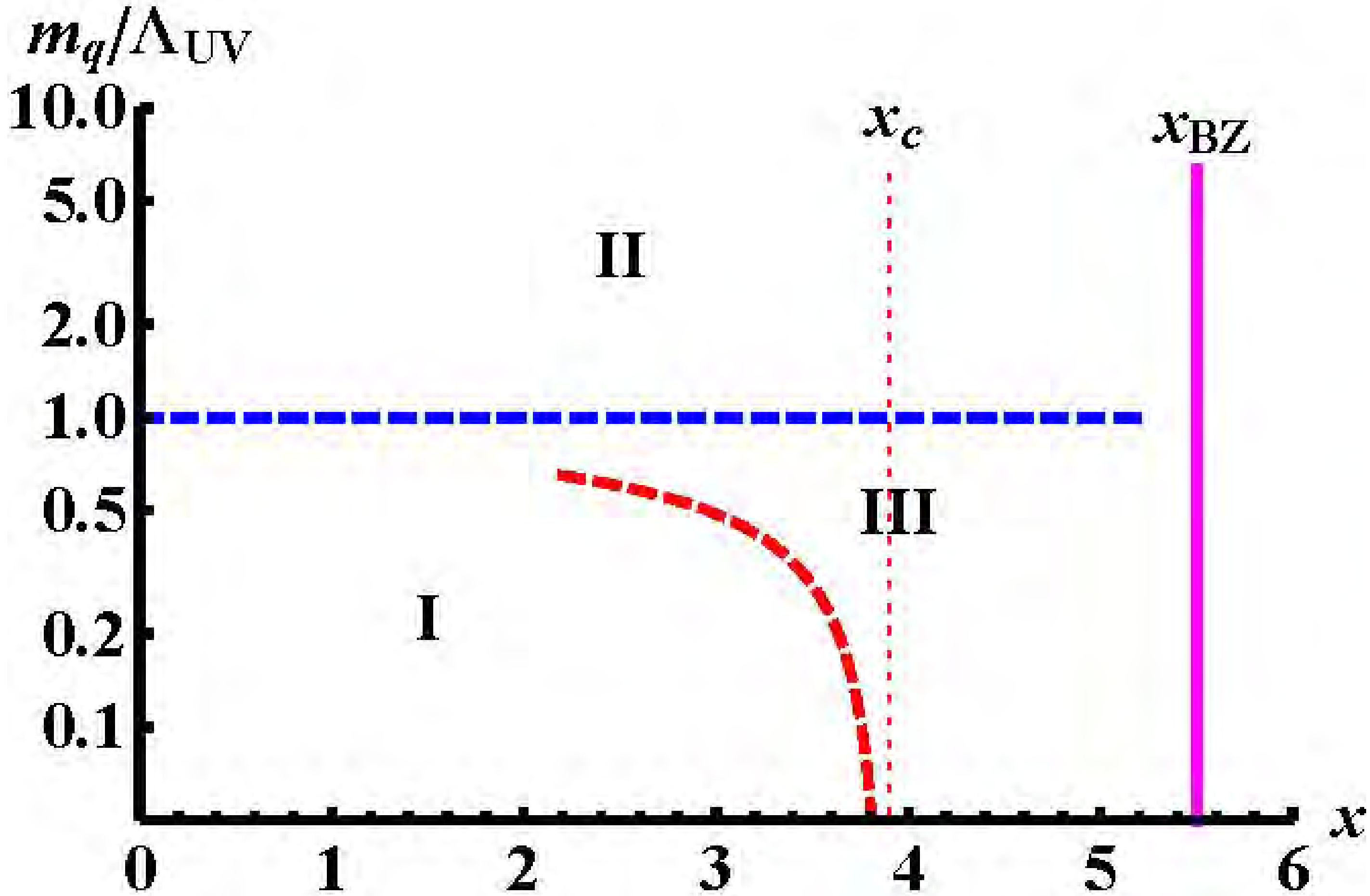
♠ QCD regime $(x \simeq 1,\ x < x_c)$ not close to x_c. Here $\Lambda_{UV} \sim \Lambda_{IR} \sim \Lambda_\tau$.

♠ Walking regime: $x \to x_c^-$: Here $\Lambda_{IR} \sim \Lambda_\tau$ and $\frac{\Lambda_{UV}}{\Lambda_{IR}} \sim e^{\frac{\pi}{\nu}}$, $\nu \sim \sqrt{x_c - x}$

♠ Conformal window: $\frac{11}{2} > x > x_c$. Λ_τ, Λ_{IR} are not defined. (But we can define an alternative Λ_{IR}).

♠ Banks-Zaks regime: $x \to \frac{11}{2}^-$. Similarly.

Mass scales in V-QCD: non-zero quark masses



• QCD regime:

a) $m_q \ll \Lambda_{UV}$. We obtain, $\Lambda_{UV} \sim \Lambda_{IR} \sim \Lambda_\tau$ and $\sigma \sim \Lambda_{UV}^3$.

b) $m_q \gg \Lambda_{UV}$. We obtain $\Lambda_\tau \sim m_q$, $\sigma \sim m_q^3$,

• walking regime:

a) Small quark mass:

$$\frac{m_q}{\Lambda_{UV}} \ll \frac{\Lambda_{IR}^2}{\Lambda_{UV}^2} \sim e^{-\frac{2\pi}{\nu}}$$

Then we have the same situation as $m_q = 0$.

b) Intermediate quark mass: $1 \gg \frac{m_q}{\Lambda_{UV}} \gg e^{-\frac{2\pi}{\nu}}$ the amount of walking is controlled by the quark mass. There is no Miransky scaling.

c) Large quark mass $\frac{m_q}{\Lambda_{UV}} \gg 1$. In this regime there is no walking and we get the same results as for heavy quarks in the QCD regime.

- Conformal window regime.

a) Small quark mass: $\frac{m_q}{\Lambda_{UV}} \ll 1$.

$$\frac{\sigma}{\Lambda_{UV}^3} \sim \left(\frac{m_q}{\Lambda_{UV}}\right)^{\frac{4-\Delta_*}{\Delta_*}}$$

b) Large quark mass: $\frac{m_q}{\Lambda_{UV}} \gg 1$. In this regime the theory never gets close to the IR fixed point and we obtain the same results as for heavy quarks in the QCD regime.

Walking region+Technicolor (TC)

- Technicolor: EW symmetry breaking is due to a new strong gauge interaction with $\Lambda_{TC} \sim 1 TeV$.

- The EW Higgs is a scalar TC meson and the vev is due to a condensate of TC fermions $\langle H \rangle \sim \langle \bar{\psi}_{TC} \psi_{TC} \rangle$ from TC chiral symmetry breaking.

- The composite Higgs couplings to the SM fermions χ are now four-fermi terms,

$$H \bar{\chi} \chi \sim \bar{\psi}_{TC} \psi_{TC} \; \bar{\chi} \chi$$

and should be generated by a new (ETC) interaction at a higher scale, Λ_{ETC}.

- There are some important problems with this idea:

♠ There can be important flavor changing processes (that are suppressed in the SM)

♠ To get the correct size for all masses, the dimension of operators

$$\bar{\psi}_{TC}\psi_{TC}$$

must be close to two (instead of 3 in perturbation theory).

♠ The dimensionless quantity S controls the low-energy corrections to EW gauge boson kinetic terms

Peskin+Takeuchi

$$S = \frac{d}{dq^2}(\Pi_V(q^2) - \Pi_A(q^2))\Big|_{q^2=0} \quad , \quad \left(\delta_{\mu\nu} - \frac{q_\mu q_\nu}{q^2}\right)\Pi_i(q^2) \equiv \langle J^i_\mu(q) J^i_\nu(0)\rangle$$

is $\mathcal{O}(1)$ in generic theories from the spectral decomposition+sum rules, but EW data imply that additional contributions should be $\mathcal{O}(10^{-2})$.

♠ It has been argued by many scientists that a way out of the above is a TC theory that is near conformal (”walking”) and strongly coupled in the TC regime,

Holdom, Appelquist+Karabali+Wijewardhana

♠ Because $S = 0$ in the conformal window it was argued by continuity that $S \to 0$ in the walking region.

Appelquist+Sannino

♠ Because of approximate scale invariance, the theory was expected to have a light scalar, ”the dilaton”, namely the singlet scalar meson (σ-meson).

Yamawaki+Bando+Matumoto

♠ Despite a lot of work in the last 15 years, whether such a theory exists, and whether it has the required properties has remained elusive till now.

♠ There has been an important lattice effort to clear this issue but this problem is hard because this is an almost massless setup and therefore computationally very “costly”.

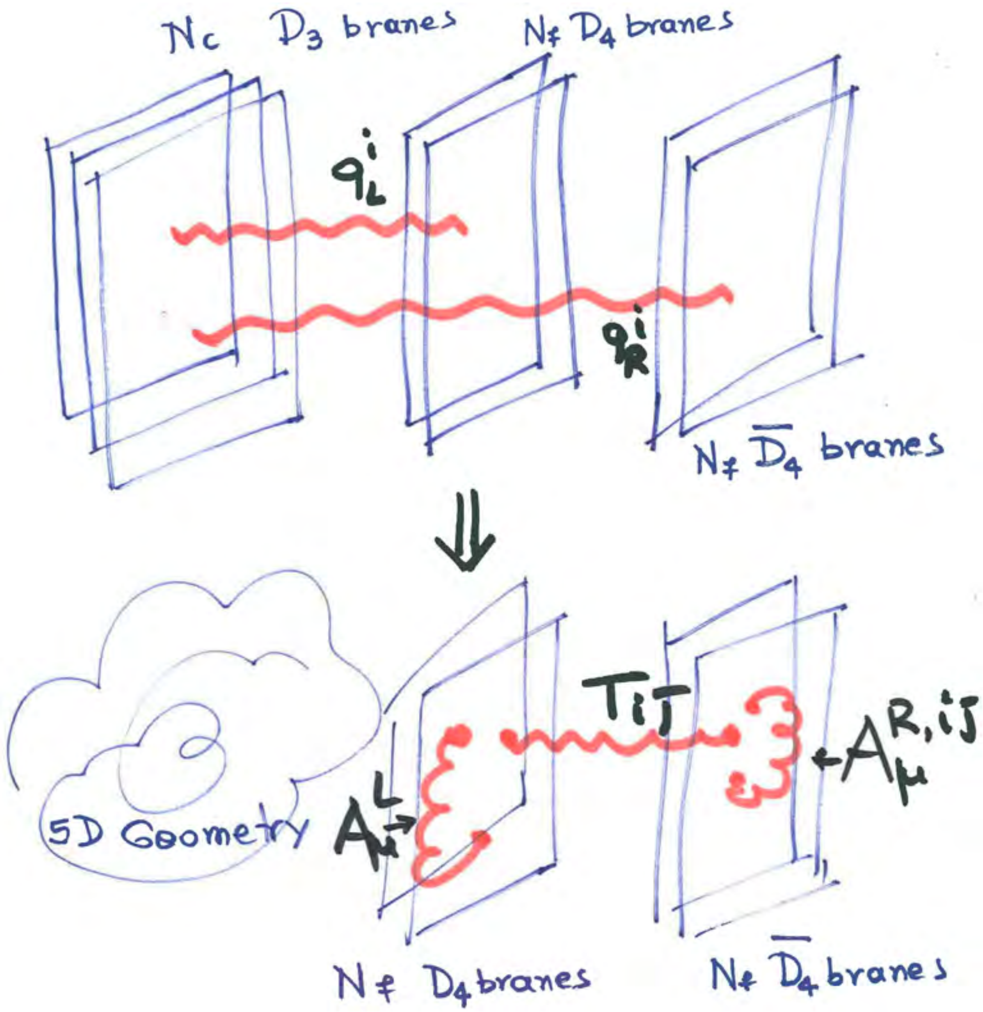
see Del Debbio (2011), Miura+Lombardo (2012), deForcrand+Kim+Unger (2012)

• A recent lattice work has been able to to identify at least the chiral restoration phase in the theory far away from the continuum limit.

de Forcrand+Kim+Unger (2012)

The Veneziano limit in string theory

- How to implement quarks in string theory/holography.



• Implementing the Veneziano limit in string theory is, so far, technically a very difficult problem, with little progress achieved.

• The main reason is that the backreaction of probe branes (carrying the flavor degrees of freedom) is very difficult to handle.

• Techniques where the flavor branes were "smeared" in transverse space have provided some string models for addressing the Veneziano limit.

Bigazzi+Casero+Cotrone+Kiritsis+Paredes, '05

Casero+Nunez+Paredes, '06

• Some progress has been achieved in the controlled construction of the string theory configurations.

Review: Nunez+Paredes+Ramallo

• For QCD, to add N_f quarks q_L^I and antiquarks $q_R^{\bar{I}}$ we must add (in 5d) space-filling N_f D_4 and N_f $\bar{D}_4$ branes.
(tadpole cancellation=gauge anomaly cancellation)

• The q_L^I should be the "zero modes" of the $D_3 - D_4$ strings while $q_R^{\bar{I}}$ are the "zero modes" of the $D_3 - \bar{D}_4$

• The low-lying fields on the D_4 branes ($D_4 - D_4$ strings) are U$(N_f)_L$ gauge fields A^L_μ. The low-lying fields on the $\bar{D}_4$ branes ($\bar{D}_4 - \bar{D}_4$ strings) are U$(N_f)_R$ gauge fields A^R_μ. They are dual to the J^μ_L and J^R_μ

$$\delta S_A \sim \bar{q}^I_L \ \gamma^\mu \ (A^L_\mu)^{IJ} \ q^J_L + \bar{q}^{\bar{I}}_R \ \gamma^\mu \ (A^R_\mu)^{\bar{I}\bar{J}} \ q^{\bar{J}}_R = Tr[J^\mu_L \ A^L_\mu + J^\mu_R \ A^R_\mu]$$

• There are also the low lying fields of the ($D_4 - \bar{D}_4$ strings), essentially the string-theory "tachyon" $T_{I\bar{J}}$ transforming as $(N_f, \bar{N}_f)$ under the chiral symmetry $U(N_f)_L \times U(N_f)_R$. It is dual to the quark mass terms

$$\delta S_T \sim \bar{q}^I_L \ T_{I\bar{J}} \ q^{\bar{J}}_R + \text{complex} \ \ \text{congugate}$$

• The interactions on the flavor branes are weak, so that $A_\mu^{L,R}, T$ are as sources for the quarks.

• Integrating out the quarks, generates an effective action $S_{flavor}(A_\mu^{L,R}, T)$, so that $A_\mu^{L,R}, T$ can be thought as effective $q\bar{q}$ composites, that is : mesons

• On the string theory side: integrating out $D_3 - D_4$ and $D_3 - \bar{D}_4$ strings gives rise to the DBI action for the $D_4 - \bar{D}_4$ branes in the D_3 background:

$$S_{flavor}(A_\mu^{L,R}, T) \quad \longleftrightarrow \quad S_{DBI}(A_\mu^{L,R}, T) \quad \text{holographically}$$

• In the "vacuum" only T can have a non-trivial profile: $T^{I\bar{J}}(r)$. Near the AdS_5 boundary ($r \to 0$)

$$T^{I\bar{J}}(r) = M_{I\bar{J}}\, r + \cdots + \langle \bar{q}_L^I \; q_R^{\bar{J}} \rangle r^3 + \cdots$$

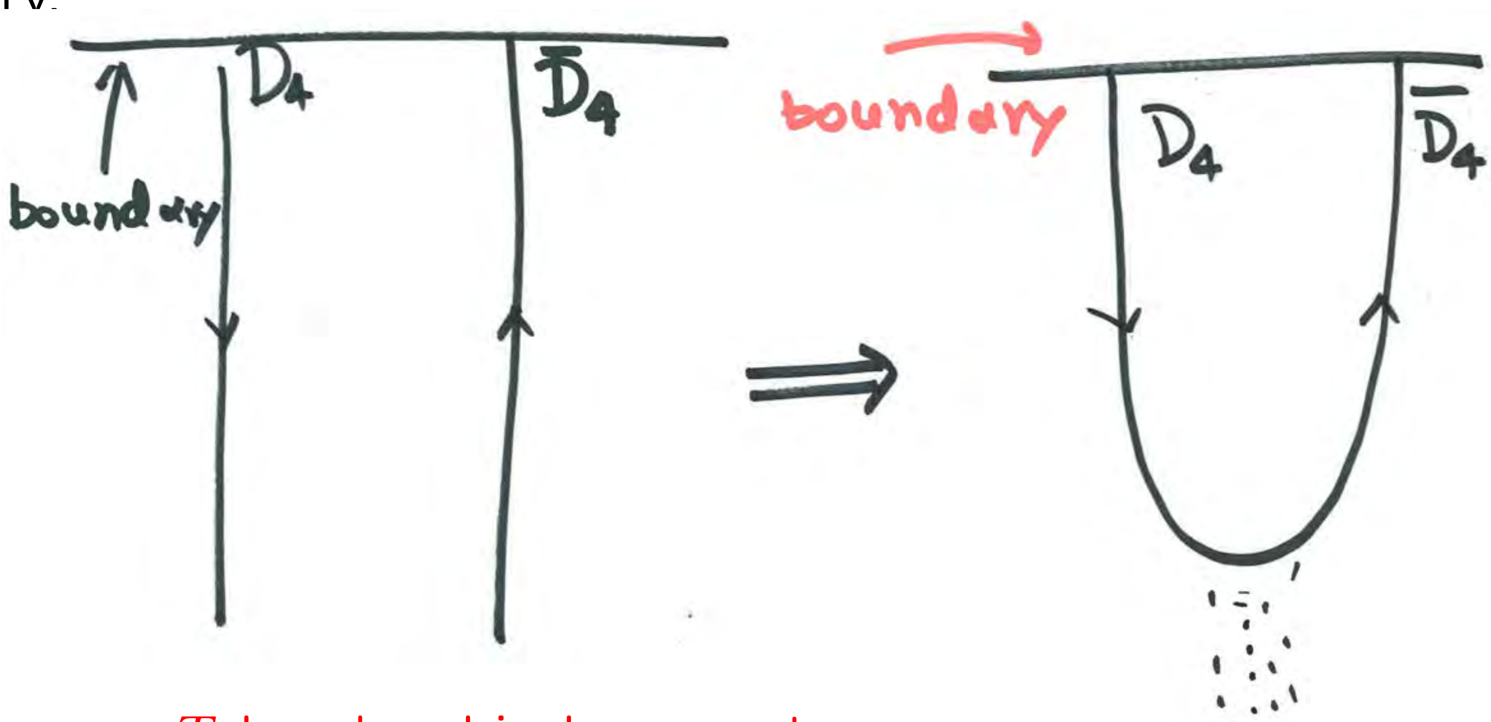
Casero+Kiritsis+Paredes

• For other gauge theories the situation is different, as this depends on the extra dimensions and the type of branes used.

General properties

• A typical solution has (a) T vanishing in the UV and (b) $T \to \infty$ in the IR.

• At the point $r = r_*$ where $T = \infty$, the D_4 and $\bar{D}_4$ branes "fuse". The true vacuum is a brane that enters folds on itself and goes back to the boundary.



• A non-zero T breaks chiral symmetry.

• When $m_q = 0$, the meson spectrum contains N_f^2 massless pseudoscalars, the $U(N_f)_A$ Goldstone bosons.

• The WZ part of the flavor brane action gives the $U(1)_A$ axial anomaly and an associated Stuckelberg mechanism gives an $O\left(\frac{N_f}{N_c}\right)$ mass to the would-be Goldstone boson η', (agreeing with Veneziano-Witten).

• We can derive formulae for the anomalous divergences of flavor currents, when they are coupled to an external source.

• T=0 is a solution. In case of confinement, it is excluded from the absence of IR boundary for the branes: Holographic Coleman-Witten theorem.

• Fluctuations around the $T(r)$ solution for $T, A_\mu^{L,R}$ give the spectra (and interactions) of various meson trajectories.

• A GellMann-Oaks-Renner relation is always satisfied (for an asymptotic AdS_5 space)

$$m_\pi^2 = -2\frac{m_q}{f_\pi^2}\,\langle\bar{q}q\rangle \quad , \quad m_q \to 0$$

The CP-odd sector

• An important ingredient in QCD is the $U(1)_A$ anomaly.

$$\partial_\mu J_5^\mu = \frac{N_f}{32\pi^2} Tr[F \wedge F] + m\bar{\psi}\psi$$

• It implies that the θ-angle can be absorbed into the phases of quark masses. If one quark is massless, then the θ-angle is unobservable.

• Using D_4 branes we can describe this starting from the dual three form of the axion $C_{\mu\nu\rho}$, with field strength, $H_4 = dC_3$

Casero+Kiritsis+Paredes

$$S_a = S_{\text{closed}} + S_{\text{open}} \quad , \quad S_{\text{closed}} = -\frac{M^3}{2}\int d^5x\,\sqrt{g}\,\frac{|H_4|^2}{Z(\lambda)} \quad , \quad H_4 = dC_3$$

$$S_{\text{open}} = i\int C_3 \wedge \Omega_2 = i\int C_3 \wedge d\Omega_1\,, \qquad A_M = \frac{A_M^L - A_M^R}{2}$$

$$\Omega_1 = i\,N_f\,[2V_a(\lambda,T)\,A - \theta\,dV_a(\lambda,T)]$$

Here θ is the overall phase of the tachyon, $T = \tau e^{i\theta} \cdot \mathbb{I}_{N_f}$.

We may dualize the three-form to a pseudo-scalar axion field a

$$\frac{H_4}{Z(\lambda)} = {}^*\left(d\tilde{a} + i\,\Omega_1\right)\,.$$

The dual action takes the form

$$S_a = -\frac{M^3\,N_c^2}{2}\int d^5x\,\sqrt{g}\,Z(\lambda)\left[da - x\left(2V_a(\lambda,T)\,A - \theta\,dV_a(\lambda,T)\right)\right]^2$$

in terms of the QCD axion $a = \tilde{a}/N_c$.

• This is normalized so that a is dual to θ/N_c with θ being the standard θ-angle of QCD.

• The coupling to the axial vector, A, reflects the axial anomaly in QCD

$$A_\mu \to A_\mu + \partial_\mu \epsilon \quad , \quad \theta \to \theta - 2\epsilon \quad , \quad a \to a + 2x\,V\epsilon$$

with $V_a(\lambda, T = 0) = 1$, which gives the correct $U(1)_A$ anomaly.

• Out of the three degrees of freedom, two are independent: 0^{+-} and η' that mix, except as $x \to 0$. In this limit one of the states (η') goes to zero mass as demanded by the anomaly in agreement with Veneziano-Witten.

Color Superconductivity

• It is known since BCS, that if you have almost free charges at finite density (ie a FERMI surface), with a weak attractive interaction, then there is a superconducting instability.

• At high energy (weak force) and finite baryon density, such an effect will break color.

• A one-loop attraction+ a phenomenological form factor (to implement asymptotic freedom) indicate that there might be a color-superconducting phase for $N_f = 2, N_c = 3$.

Alford+Rajagopal+Wilczek

• The calculation involves the phenomenological lagrangian and the solution of a gap equation.

• A similar calculation shows another instability when $N_f = N_c = 3$: color-flavor locking.

Alford+Rajagopal+Wilczek

• Both phenomena involve non-gauge invariant (colored) order parameters:

$$\Phi^{ij}_{\alpha\beta} \sim \langle\ q^i_\alpha q^j_\beta\ \rangle$$

• In holography however we must work with gauge-invariant fields. The way to do this is to consider

$$X^{ij,kl} = \left(\Phi^{ij}_{\alpha\beta}\right)^\dagger \Phi^{kl}_{\alpha\beta} = \Psi^{ik}\Psi^{jl}$$

where Ψ is a single-trace gauge invariant (chiral condensate) operator

$$\Psi^{ij} = \left(q^i_\alpha\right)^\dagger q^j_\alpha$$

• The order parameter is a double "trace operator" that should get an expectation value without Ψ^{ij} getting an expectation value.

• This issue is subtle and there are two possibilities

(a) There is no color superconductivity in the large N limit

(b) There is, but one needs the one-loop gravitational corrections.

Detailed plan of the presentation

- The lower end of the conformal window 36 minutes
- Walking 37 minutes
- BKT scaling 38 minutes
- Recap 39 minutes
- Spectra 41 minutes
- The S-parameter: definitions 42 minutes
- The S-parameter: $m_q = 0$ 43 minutes
- The S-parameter: $m_q \neq 0$ 45 minutes
- Finite masses 46 minutes
- The phase diagram 47 minutes
- Finite density 51 minutes
- A new quantum critical regime 52 minutes
- Outlook 55 minutes

- The 't Hooft Large N_c limit 2 minutes
- Bibliography 2 minutes
- Topics 3 minutes
- Review 4 minutes
- Ending the conformal window 7 minutes
- Below the BF bound 12 minutes
- Effimov Spiral 16 minutes
- QCD$_*$ 23 minutes
- N=1 sQCD 27 minutes
- Outlook 29 minutes

- The Banks-Zaks region 34 minutes
- N=1 sQCD 38 minutes
- The IR fixed point 39 minutes
- Varying the model 41 minutes
- Phase transitions as a function of quark masses 42 minutes
- Matching to QCD 45 minutes
- Matching to QCD : UV 46 minutes
- Matching to QCD : IR 49 minutes
- The free energy 51 minutes
- Walking 56 minutes
- Holographic β-functions 59 minutes
- UV mass vs T_0 and r_1 65 minutes
- Numerical solutions :$T = 0$ 67 minutes
- Numerical solutions: Massless with $x < x_c$ 72 minutes
- Matching to QCD 73 minutes