N=1 Euler anomaly from dilation effective action

CP^3 Origins
Cosmology & Particle Physics

CENTRE FOR THEORETICAL PHYSICS · Higgs

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based on 1511.03868 V.Prochazka, RZ

Holography, CFT & lattice
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Overview

I. Introduction to RG-flows [5 slides]

RG-flows, trace anomalies, status a,c-thms

II. Komargodski-Schwimmer construction [4 slides]

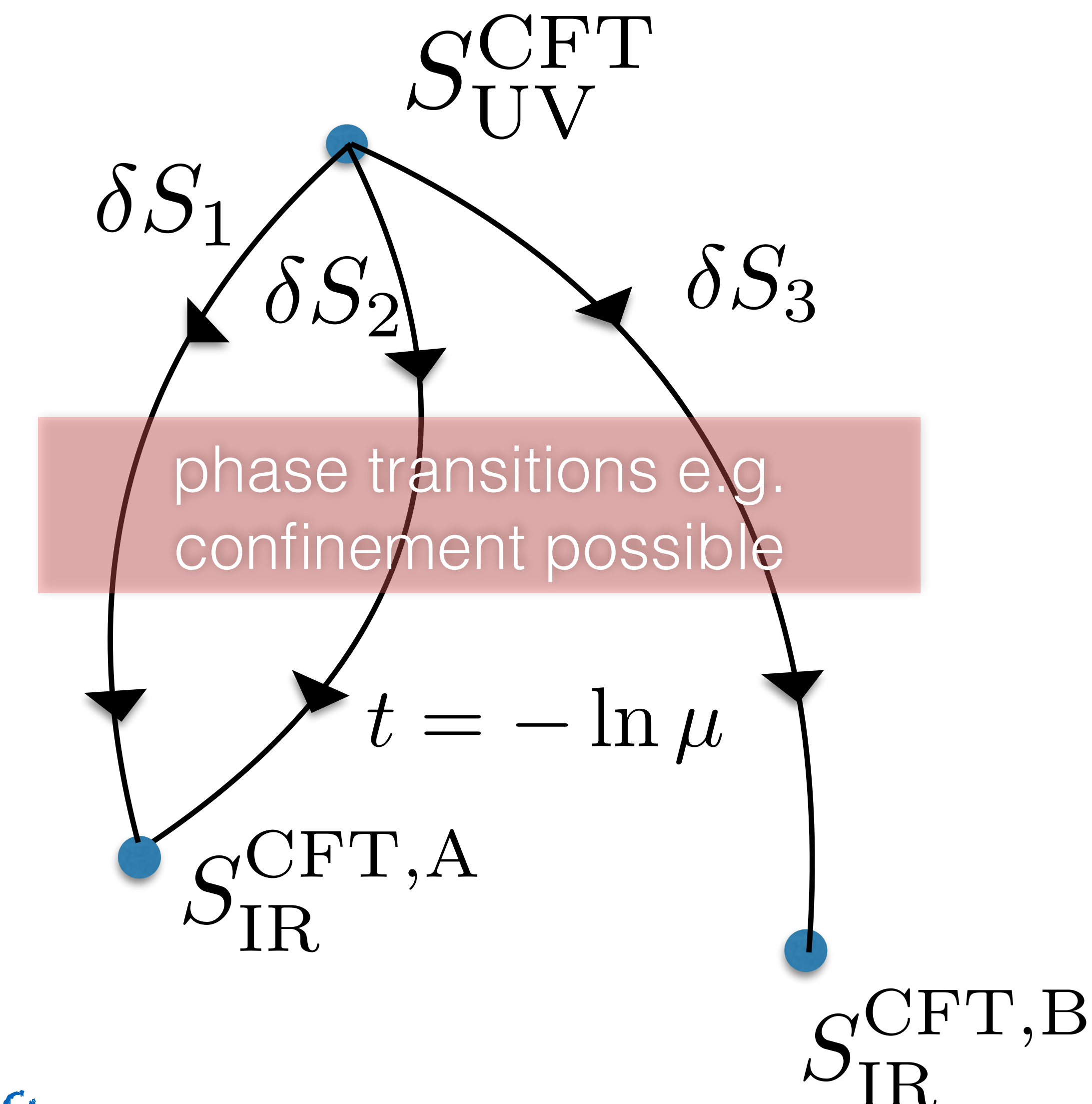
chiral and conformal anomaly matching by
Wess-Zumino-term & dilation scattering

III. a-thm @work N=1 SUSY-QCD using KS-ideas [5 slides]

I. Introduction to RG-flows, status of a,c-thms and interpretation

Irreversibility of RG-flows

- consider UV-CFT
- relevant perturbations **δS** induce **RG-flow**
- **a/c-thm**: settle & characterise **irreversibility** of flow



weak $$\exists f(t) \text{ s.t. } \Delta f \equiv f_{\mathrm{UV}} - f_{\mathrm{IR}} > 0$$

strong $$\dot{f} = -\beta^i \partial_{g_i} f \leq 0$$ decreasing along flow

strongest $$\beta^i = G^{ij} \partial_{g_j} f$$ gradient flow (implies strong version)

G^{ij}: POSITIVE DEF METRIC COUPLING-SPACE

Candidates for f-functions

ideally physical quantity which we can compute easily

dimension	even	odd
conformal anomaly	yes	no!
	$\langle T(z)T(0)\rangle_{2D} \sim \frac{c}{z^4}$	Free energy (F-theorem)
	this talk 4D version (later SUSY N=1)	Appelquist,Cohen,Schmaltz '99 Klebanov, Pufu, Safdi '11

Conformal anomaly ...

.. reveals itself under a probe in external gravitational field analogous chiral anomaly and F_{ab}

translation current = EMT: $T_{\mu\nu}$ - **dilatation** current: D_μ

$$\partial \cdot D = T^\rho{}_\rho \equiv \Theta = \begin{cases} 0 & \text{CFT} \\ \partial \cdot V^{\text{irial}} & \text{SFT}^* \end{cases}$$

- **Dilatation/trace-anomaly** in flat space

Adler et al, Collins et al, Nielsen, Minkowski '77

$$T^\rho{}_\rho = \sum_i \beta^i O_i + \text{EOM} + \text{explicit}$$

- **Weyl-anomaly** in curved space Capper, Duff '74

$$\langle T^\rho{}_\rho \rangle = \begin{cases} 0 & \text{odd dim} \\ \beta_c R^{**} & 2D \\ \beta_a E_4 + \beta_b R^2 + b' \Box R + \beta_c W^2 & 4D \end{cases}$$

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* this talk not distinguish CFT and SFT as no interesting counterexamples known

** $\beta_c = c$ depending on literature

Status: a,c-thms

$$\langle T^{\rho}{}_{\rho} \rangle = \begin{cases} \beta_c R \\ \beta_a E_4 + \beta_b R^2 + b' \Box R + \beta_c W^2 \end{cases}$$

c-thm

- 2D: Zamolodchikov'86 $\Delta\beta_c = \beta_c^{UV} - \beta_c^{IR} \geq 0$ using i) transl.-inv. ii) lorentz-decomp. & iii) reflection positivity (unitarity)

a-thm

- 4D: Cardy'88 conjectured $\Delta\beta_a \geq 0$ by exclusion of remaining candidates:
  - i) $\beta_b^{CFT} = 0$ since $\delta_{dil} R^2 \sim \Box R$ - b' -flow dependent
  - ii) β_c^{CFT} counterexamples known

topological quantities

- selected important work:

'91	Capelli, Friedan Latorre	dispersive program (2D proposed 4D)
'98	Forte, Latorre	proof S^4 - controversery contact terms (unresolved)
'04	Intriligator, Wecht , ..	a-maximization shown hold SUSY (R-currents)
'11	Komargodski, Schwimmer, ..	proof dilaton effective action & 4 point amplitude

Interpretation of a,c thms

... $\beta_{a,c}$ measure for dof. & by integrating out dof they ought to decrease

- 2D: - entanglement entropy on strip L: $S \sim \beta_c/3 \ln L$ Affleck, Ludwig'91
  - dispersion relation $\Delta\beta_c = \int \rho_{spec}(s)\, ds$ Capelli, Friedan, Latorre '91

- 4D: consider **QCD**: - **UV**: free quarks & gluons **IR**: free pions

$$\beta_a^{free} = \#(\text{scalar} + \mathbf{11}\ \text{Dirac-fermions} + \mathbf{62}\ \text{Spin-1})$$

$$\beta_a^{UV,QCD} = \#(\mathbf{11}\ N_c N_f + \mathbf{62}\ (N_c^2-1))$$
$$\beta_a^{IR,QCD} = \#(N_f^2-1)$$

$\Delta\beta_a^{QCD} \geq 0$ obeys a-thm (consistent with asymptotic freedom)

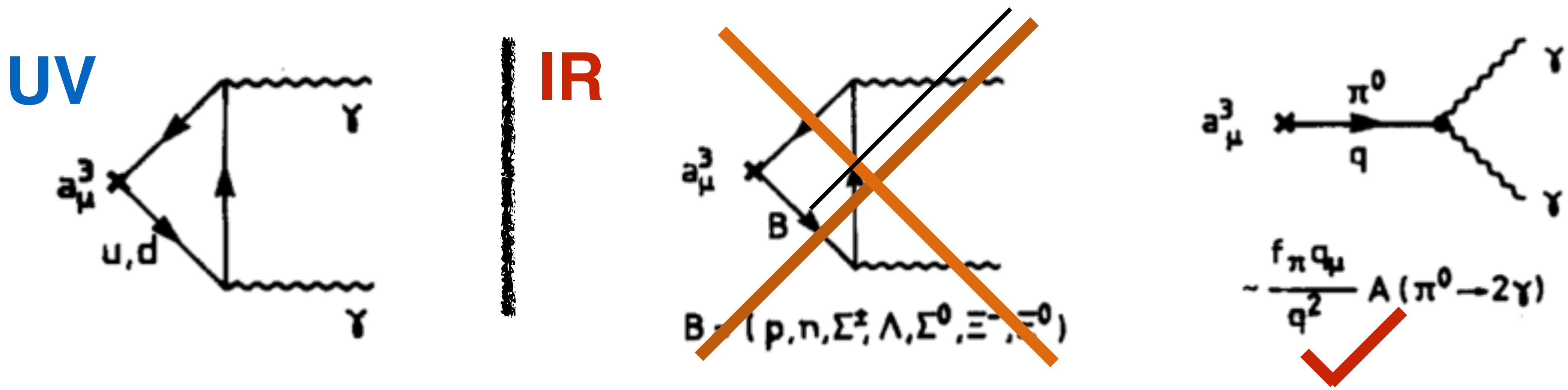
- N.B. **scalars** are "**low cost**", could say that there is tendency for **low-energy chiral symmetry breaking**

2. Komargodski-Schwimmer construction
dilation effective action, $\Delta\beta_a$ & Wess-Zumino-terms

chiral & dilatation anomaly matching

- **'t Hooft'80 anomaly matching - idea**:
  cancellation of chiral anomaly scale-independent (e.g. **UV**,**IR**)

  ⇒ **UV**-dof impose consistency constraint on massless **IR**-dof
  (e.g. QCD exclude massless baryons Banks,Frishman,Schwimmer,Yank. '81)



- **a-thm** - similarities but difference is that **dilatation symmetry**, unlike chiral symmetry, **broken along flow**

  Komargodski & Schwimmer'11-idea: use **dilation** as **spurion** (compensator for breaking of dilatation symmetry) to render conformal anomaly scale independent

conformal anomaly matching in 2D

- **dilatation** implemented **Weyl transformation** $g_{ab} \to e^{-2\alpha} g_{ab}$ **broken**

  1) **mass scales** $M \to Me^{\alpha}$ 2) **conformal anomaly**

- introduce **dilaton** τ (spurion = external BF) to remedy 1)
  i) couple $Me^{-\tau}$ ii) $\tau \to \tau + \alpha \Rightarrow Me^{-\tau}$ Weyl-invariant

⇒ at all scales:

$$\delta_\alpha S_{\text{eff}} = \# \beta_c^{\text{UV}} \int \sqrt{g} R$$

- **conformal anomaly matching:**

$$\delta_\alpha S_{\text{UV}} = \delta_\alpha S_{\text{IR}} \quad \Leftrightarrow \quad \delta_\alpha S_{\text{WZ}} = \# \int \sqrt{g} R$$

$$S^{\text{UV}} = S^{\text{UV}}_{\text{CFT}} \text{ and } S^{\text{IR}}(g,\tau) = S^{\text{IR}}_{\text{CFT}} + S^{\text{Weyl-inv}}(g,\tau) + \mathbf{\Delta}\beta_{\mathbf{c}} S_{\text{WZ}}(g,\tau)$$

- the **solution to** $\delta_\alpha S_{\rm WZ} = \# \int \sqrt{g} R$ is given by

$$S_{\rm WZ} = \# \int \sqrt{g}(\tau R + (\partial \tau)^2)$$

- chiral anomaly / chiral perturbation theory dictionary

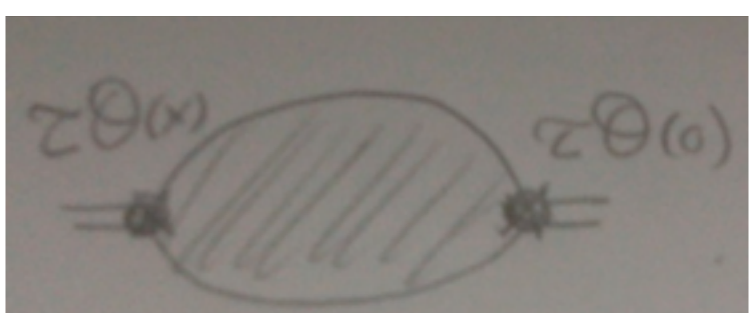
$$S_{\rm WZ} = \# \int (\pi F \tilde{F} + \mathcal{L}_{\rm WZ}(\pi))$$

- moral: **flat space WZ-term** IR-theory carries **relevant information**

$$S^{\rm IR} = \boldsymbol{\Delta} \beta_{\mathbf{c}} \# \int d^2x (\partial \tau)^2 + \ldots$$

- how to compute $S_{WZ}(\tau)$?, τ = source trace EMT

Komargodski'11



$$\Delta \beta_c = |\#| \int d^2 z z^2 \langle \Theta(z) \Theta(0) \rangle \geq 0$$

reflection positivity

rederivation of elegant formula by Cardy'88

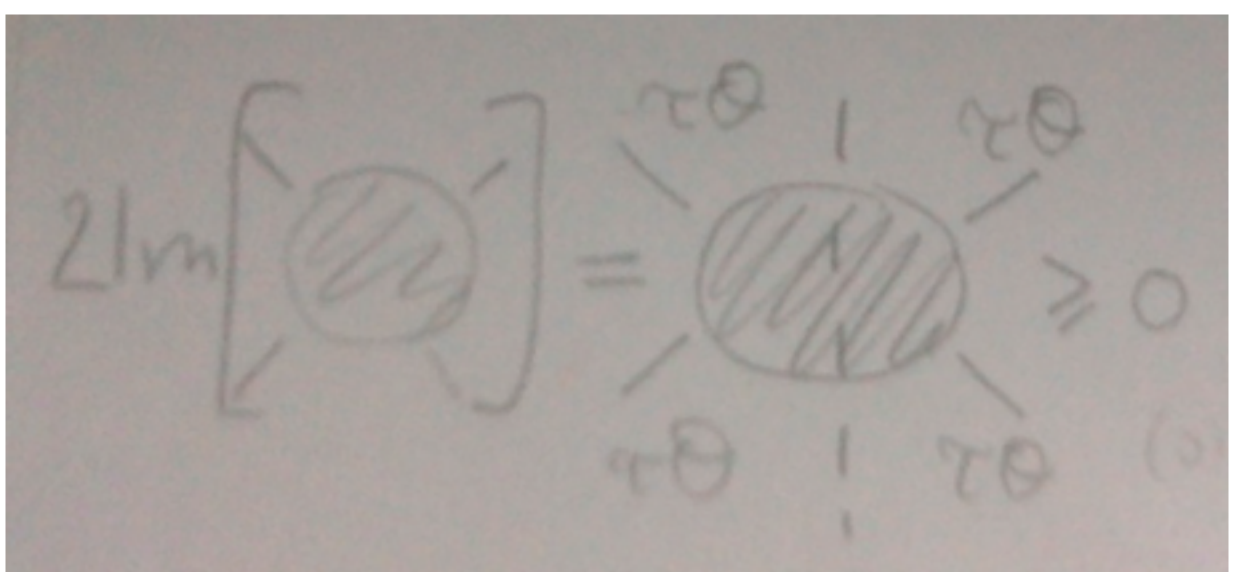
Brief summary of extension to **4D**

- more involved but analogous Schwimmer, Theisen'10, KS'11

$$S_{\rm WZ}^{\rm 4D,flat} = \int 2(\partial\tau)^2\Box\tau - (\partial\tau)^4$$

- major complication as involves 3-4 pt functions (impose constraint $\Box\tau = 0$ → 4 point function)

- choose **forward kinematics** t=0, s=-u, **optical theorem** & assumption on **analyticity** KS'11 arrive at **a-thm**:



$$\Delta\beta_a = \frac{1}{4\pi}\int_{s>0} ds \frac{{\rm Im}[A^{\tau\tau\to\tau\tau}(s)]}{s^3} \geq 0$$

ingenious construction whereby KS relate $\Delta\beta_a$ to physical quantity avoiding problems of def. & subtractions

Can the abstract construction be used for computations ?

3. Use of KS-idea to compute $\Delta\beta_a$ N=1 4D QCD-SUSY

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mostly need to recall:

$$\delta S^{\mathrm{IR}} = \boldsymbol{\Delta}\beta_{\mathbf{a}} S^{\mathrm{4D,flat}}_{\mathrm{WZ}}$$

basic setup

- consider RG-flow & **introduce dilation g(μ/M) → g(μ/Meτ)**

$$e^{W_\tau} = \int [D\phi]_\mu e^{-S_W(g(\mu e^\tau),\mu e^\tau,\phi)} \; .$$

- From KS'11 we learn:

$$\Delta W_\tau \equiv -\int_{-\infty}^{\infty} d\ln\mu\, \partial_{\ln\mu} W_\tau = -\Delta\beta_a \; S_{\rm WZ} + \ldots$$

- From **RG-equation** $\partial_{\ln\mu} W_\tau = \int d^4x \sqrt{\tilde{\rm g}} \; \langle\Theta\rangle_\tau$

- Obtain **master formula**:

$$\Delta\beta_a = \int_{-\infty}^{\infty} d\ln\mu \; \int d^4x \sqrt{\tilde{\rm g}} \langle\Theta\rangle_\tau |_{S_{\rm WZ}}$$

compute $\Delta\beta_a$ ⇔ compute TEMT in dilation background

… how to do it (efficiently)?

A seemingly absurd toy-example

- free scalar field with flow(!?!): $S_{\mathrm{W}}(\mu) = \int d^4x\, Z(\mu)\delta^{\rho\lambda}\partial_\rho\phi\partial_\lambda\phi\ .$
- **key idea:** Weyl-transformation*:

$$\tilde{g}_{\rho\lambda} = e^{-2s(\tau)}\delta_{\rho\lambda} \qquad s(\mu e^{\tau}) = -\tfrac{1}{2}\ln Z(\mu e^{\tau(x)})$$

⇒ RG-flow absorbed into metric - **vacuum sector** equivalent to **free** field theory in **curved background** $\tilde{g}(Z(\tau))$ encoding RG-flow

geometrization of dynamics

$$S_W(\mu) = \int d^4x\sqrt{\tilde{g}}\,\tilde{g}^{\rho\lambda}D_\rho\phi D_\lambda\phi$$

dynamics in geometry ....

- result known 70's: $\langle\Theta\rangle_\tau = \beta_a^{\mathrm{free}}(\tilde{E}_4 - 2\tilde{\Box}\tilde{R})$

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* experts: differs from KS where g = $e^{-2\tau}$ δ

Unfolding the dynamics from geometry

- anomalous dimension γ in geometry

$$\begin{aligned}\sqrt{\tilde{g}}\tilde{E}_4 &= -8(\frac{1}{2}\Box(\partial s)^2 - \partial\cdot(\partial s(\Box s d - (\partial s)^2))) \\ &= -[\gamma^2(\Box(\partial\tau)^2 - 2\partial^\lambda(\partial_\lambda\tau\Box\tau)) - \gamma^3\partial^\lambda(\partial_\lambda\tau(\partial\tau)^2) + \\ &\quad 2\gamma\dot{\gamma}(\partial_\lambda\tau\partial^\lambda(\partial\tau)^2 - 2(\partial\tau)^2\Box\tau) - 3\gamma^2\dot{\gamma}(\partial\tau)^4]\end{aligned}$$

used: $\partial_\rho s = -\frac{1}{2}\frac{\partial \ln Z(\mu e^\tau)}{\partial(\mu e^\tau)}\partial_\rho(\mu e^\tau) = -\frac{1}{2}\gamma\,\partial_\rho\tau$

- From master formula $\Delta\beta_a = \int_{-\infty}^{\infty} d\ln\mu \int d^4x\sqrt{\tilde{g}}\langle\Theta\rangle_\tau|_{S_{\rm WZ}}$

$$\Delta\beta_a = \tfrac{1}{2}\left((\gamma_{\rm UV}^3 - \gamma_{\rm IR}^3) + 3(\gamma_{\rm UV}^2 - \gamma_{\rm IR}^2)\right)\beta_a^{\rm free} \qquad (\star)$$

obtain intermediate result a-anomaly in terms of $\gamma_{\rm UV}$, $\gamma_{\rm IR}$.

How is this of any use?

⇒ manipulate N=1 effective action by use of Konishi anomaly

N=1 QCD-like effective action superfield-formalism

$$S_{\mathrm{W}}(\mu) = \int d^6 z \frac{1}{g^2(\mu)} \mathrm{tr} W^2 + \frac{1}{8} Z(\mu) \int d^8 z \Phi_f^\dagger e^{-2V} \Phi_f + \mathrm{h.c.}$$

gauge-field $\frac{1}{g^2} F^2$ matter

- using **holomorphicity** NSVZ, Arkani-Hamed & Murayama'97

$$\frac{1}{g^2(\mu)} = \frac{1}{g^2(\Lambda_{\mathrm{UV}})} - \frac{b_0}{8\pi^2} \ln \frac{\Lambda_{\mathrm{UV}}}{\mu} \,, \qquad b_0 \equiv 3N_c - N_f$$

- using **rescaling/Konishi-anomaly** (exact in N=1 SUSY!)

$\Phi \to (\Lambda_{\mathrm{UV}}/\mu)^{\gamma_*/2} \Phi$ $\qquad$ $\Phi_f \to Z^{-1/2} \Phi_f$

$$\gamma_* \equiv -b_0/N_f = 1 - 3N_c/N_f$$

"dynamics" as matter prefactor $\qquad$ Eqn ⇒ NSVZ β-fct

$$S_{\mathrm{W}}(\mu) = \int d^6 z \frac{1}{g(\Lambda_{\mathrm{UV}})^2} \mathrm{tr} W^2 + \frac{1}{8} \int d^8 z \hat{Z}(\mu) \Phi_f^\dagger e^{-2V} \Phi_f + + \mathrm{h.c.}$$

$$\hat{Z}(\mu) \equiv Z(\mu) \left(\frac{\Lambda_{\mathrm{UV}}}{\mu} \right)^{\gamma_*}$$

Δβ_a N=1 from dilation effective action

$$SU(N_c)\text{-conformal window: } 3/2N_c < N_f < 3N_c$$

- ⇒ apply **Weyl transformation**: $s(\mu e^{\tau}) = -\frac{1}{2}\ln \hat{Z}(\mu e^{\tau(x)})$

  N.B. gauge term Weyl-invariant and matter Weyl-variant

⇒ **vacuum sector**: **free** field theory in **curved backgd carrying dyn.**

- count free matter-dof:
  and use previous result (☆) $\nu \equiv 2\Big|_{\mathbb{C}\text{-scalar}} + \frac{11}{2}\Big|_{\text{Weyl-fermion}} = \frac{15}{2}$

$$\Delta\beta_a|_{\mathcal{N}=1} = \frac{15}{2} N_c N_f (-\gamma_*^3 + 3\gamma_*^2)\beta_a^{\text{free}}$$

- matches old result Anselmi, Freedman, Grisaru, Johanson'98
  Δβ_a > 0 consistent with unitarity bound (0 > γ_* > -1)

discussion & conclusions

- a,c-theorem & 4D-CFT active field of research
  open problems: - 3D "a,c-thm" (free-energy theorem)
  - does scale imply conformal invariance?
  - euclidian proof of a-thm (no analyticity assumptions)
  - gradient flow? (beyond perturbation theory)
  - .....

- showed how to make us of **KS-construction** to **compute $\Delta\beta_a$ for N=1**
  alternatives: - effective action gravitational background a la Jack & Osborn'90
  - assume or show gradient flow equation and solve

- **extension**: RG-flows of several couplings difficult ..
  beyond supersymmetry (in progress…)

Thanks for you attention !