Theta angle, quark mass, and conformal window in holographic QCD

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Holography, conformal field theories, and lattice

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Outline

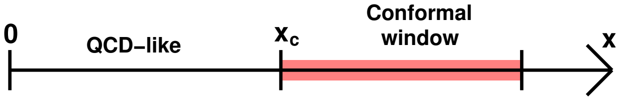
1. Brief introduction and motivation
2. The V-QCD models [Talk by E. Kiritsis]
3. Results at finite quark mass
4. Results at finite θ-angle
5. Outlook and conclusions

Finite T and B $\rightarrow$ Umut Gürsoy's talk

1. Introduction

QCD phases in the Veneziano limit

Veneziano limit: large N_f, N_c with $x = N_f/N_c$ fixed



In the Veneziano limit (discrete) N_f replaced by (continuous) $x = N_f/N_c$

- Transition expected at some $x = x_c$

Computations near the transition difficult

- Schwinger-Dyson approach, ...
- Lattice QCD
- Holography (?)

Our approach: general idea

A holographic bottom-up model for QCD in the Veneziano limit

- Bottom-up, but trying to follow principles from string theory as closely as possible
- Complicated model (because QCD is complicated)

More precisely:

- Derive the model from five dimensional noncritical string theory with certain brane configuration
  $\Rightarrow$ some things do not work (at small coupling)
- Fix model by hand and generalize $\rightarrow$ arbitrary potentials
- Tune model to match QCD physics and data
- Effective description of QCD

Last steps so far incomplete: model not yet tuned to match any QCD data!

2. V-QCD

Holographic V-QCD: the fusion

The fusion:

1. IHQCD: model for glue inspired by string theory (dilaton gravity)

   [Gursoy, Kiritsis, Nitti; Gubser, Nellore]

2. Adding flavor and chiral symmetry breaking via tachyon brane actions

   [Klebanov,Maldacena; Bigazzi,Casero,Cotrone,Iatrakis,Kiritsis,Paredes]

Consider 1. + 2. in the Veneziano limit with full backreaction
$\Rightarrow$ V-QCD models

[MJ, Kiritsis arXiv:1112.1261]

Dictionary

In the flavor/CP-odd sector

1. The tachyon: $T^{ij} \leftrightarrow \bar{\psi}^i_R \psi^j_L$; $(T^\dagger)^{ij} \leftrightarrow \bar{\psi}^i_L \psi^j_R$
   - Source: the (complex) quark mass matrix M^{ij}
     Note: the phase of the tachyon sources the phase of the mass
2. The gauge fields $A^{ij}_{\mu,L/R} \leftrightarrow \bar{\psi}^i_{L/R} \gamma_\mu \psi^j_{L/R} \equiv J^{(L/R)}_\mu$
   - Sources: chemical potentials and background fields (not turned on in this study)
3. The bulk axion $\mathfrak{a} \leftrightarrow \mathbb{T}\mathrm{r}\, G \wedge G$
   - Source: (normalized) theta angle θ/N_c

In the glue sector

1. The dilaton $\lambda \leftrightarrow \mathbb{T}\mathrm{r}\, G^2$
   - Source: the 't Hooft coupling $g^2 N_c$

Full V-QCD action

Concrete, complete V-QCD model

$$\mathcal{S}_{\mathrm{V-QCD}} = \mathcal{S}_g + \mathcal{S}_f + \mathcal{S}_a$$

1. $\mathcal{S}_g$: 5D dilaton-gravity (gluon sector, IHQCD)
2. $\mathcal{S}_f$: Generalized tachyon DBI action for flavor (T, $F_{\mu\nu}$)
3. $\mathcal{S}_a$: The CP-odd action – WZW term

- See talk by E. Kiritsis for more details
- Most results implied by symmetry

Analysis

Numerical solution with full backreaction

1. Choose potentials ($\rightarrow$ talk by E. Kiritsis)
2. Look for numerical saddle points of the action
3. Compute observables using the dictionary
4. Analyze numerically fluctuations around the saddle-points $\rightarrow$ spectrum ...

Analytic expansions

1. Near the UV boundary $\rightarrow$ physics at small quark mass
2. In the IR $\rightarrow$ asymptotic (radial) meson trajectories

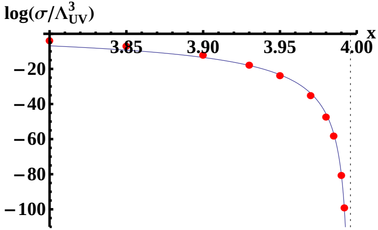
3. Finite quark mass

Energy scales (at zero quark mass)

V-QCD reproduces the picture with Miransky scaling:

1. QCD regime: single energy scale Λ
2. Walking regime ($x_c - x \ll 1$): two scales related by Miransky/BKT scaling law

$$\frac{\Lambda_{\rm UV}}{\Lambda_{\rm IR}} \sim \exp\left(\frac{\kappa}{\sqrt{x_c - x}}\right)$$



3. Conformal window ($x_c \leq x < 11/2$): again one scale Λ, but slow RG flow

"Hyperscaling" relations

In the conformal window all low lying masses obey the "hyperscaling" relations

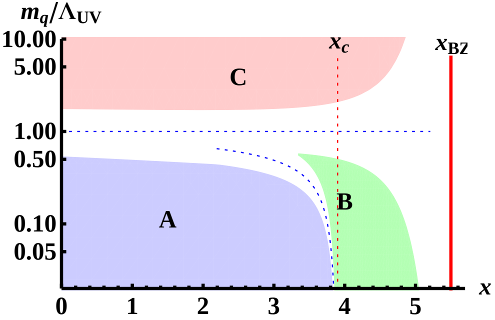
$$m \sim m_q^{\frac{1}{1+\gamma_*}} \qquad (m_q \to 0)$$

$$\langle \bar{q}q \rangle \sim m_q^{\frac{3-\gamma_*}{1+\gamma_*}} \qquad (m_q \to 0)$$

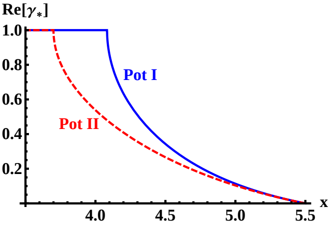
[Kiritsis, MJ arXiv:1112.1261; MJ arXiv:1501.07272]

- Agreement with field theory
  [Del Debbio, Zwicky arXiv:1005.2371]
- Appear independently of the details of the Lagrangian
- Also demonstrated in the "dynamic AdS/QCD" models
  [Evans, Scott arXiv:1405.5373]

"Phase diagram" on the (x, m_q)-plane:

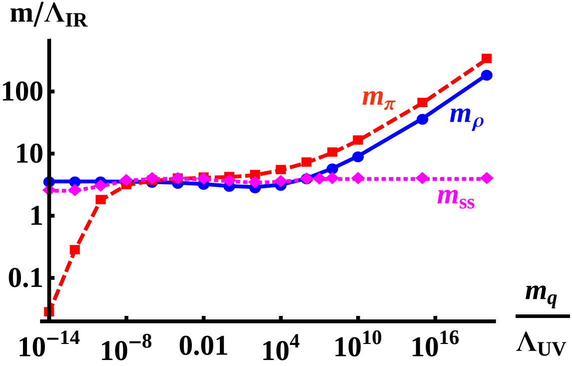


Hyperscaling seen in "regime B":
extends to $x < x_c$ (where $\gamma_* \to 1$)



Example: masses for the walking case

$x_c - x \ll 1$, Masses in units of IR (glueball) scale



- All masses have the same behavior at intermediate m_q (regime B)
- Meson masses enhanced wrt glueballs at large m_q

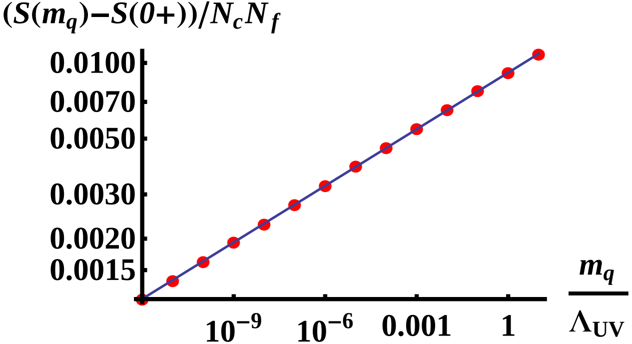
Scaling of the S-parameter

As $m_q \to 0$ in the conformal window,

$$S(m_q) \simeq S(0+) + c \left(\frac{m_q}{\Lambda_{\mathrm{UV}}} \right)^{\frac{\Delta_{FF}-4}{\gamma_*+1}}$$

- Limiting value $S(0+) = \lim_{m_q \to 0+} S(m_q)$ is finite and positive (while $S(0) = 0$)
- Δ_{FF} is the dimension of $\mathrm{tr} F^2$ at the fixed point
- Agreement with FT analysis of RG flows

[Del Debbio, Zwicky, arXiv:1306.4038]



4. Finite θ-angle

The $U(1)_A$ anomaly at large N

- In the 't Hooft (probe) limit ($g^2 N_c$ and N_f fixed, $N_c \to \infty$) anomalous contribution absent at leading order, appears at NLO: anomaly $\sim \mathcal{O}(N_f/N_c)$
- In the Veneziano limit ($g^2 N_c$ and $N_f/N_c \equiv x$ fixed, $N_c \to \infty$ and $N_f \to \infty$) anomaly present at leading order

Witten-Veneziano formula for the mass of η':

$$m_{\eta'}^2 \simeq m_\pi^2 + x \, \frac{\chi}{\bar{f}_\pi^2} = m_\pi^2 + \frac{N_f}{N_c} \, \frac{\chi}{\bar{f}_\pi^2}$$

where $\chi =$ the topological susceptibility and x small

Can the anomaly be implemented in holographic QCD in the Veneziano limit?

CP-odd action

CP-odd action $\mathcal{S}_a$ arises from the WZW term

[hep-th/0702155, 1309.2286]

$$\mathcal{S}_a = \mathcal{S}_{\rm open} + \mathcal{S}_{\rm closed}\,; \qquad \mathcal{S}_{\rm closed} = -\frac{M^3}{2}\int d^5x\sqrt{-\det g}\,\frac{|dC_3|^2}{Z(\lambda)}$$

$$\mathcal{S}_{\rm open} = i\int C_3\wedge d\Omega_1\,; \qquad \Omega_1 = iN_f\left[2V_aA - \xi dV_a\right]$$

Couples the RR axion C_3 to the $U(1)_A$ gauge field
$A = (A_L - A_R)/2$ and the tachyon phase $\xi = \arg\det T/N_f$

Integrating out $C_3 \to$ axion field $\mathfrak{a}$

$$\mathcal{S}_a = -\frac{M^3\,N_c^2}{2}\int d^5x\,\sqrt{-\det g}\,Z(\lambda)\left[d\mathfrak{a} - x\left(2V_a\,A - \xi\,dV_a\right)\right]^2$$

$U(1)_A$ symmetry and the θ-angle

$$\mathcal{S}_a = -\frac{M^3\, N_c^2}{2} \int d^5x \sqrt{-\det g}\, Z(\lambda)\, [d\mathfrak{a} - x\,(2V_a\, A - \xi\, dV_a)]^2$$

The $U(1)_A$ gauge transformation is

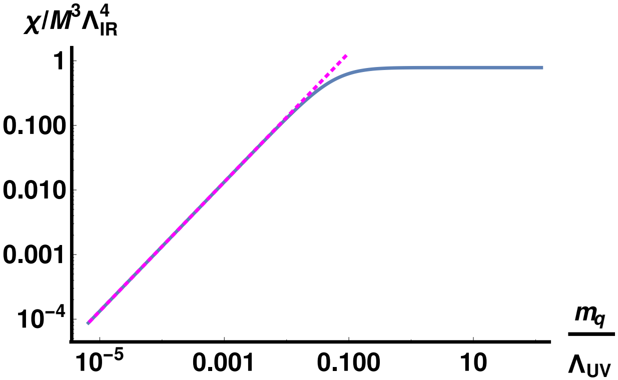
$$A_\mu \to A_\mu + \partial_\mu \epsilon\ , \qquad \xi \to \xi - 2\epsilon\ , \qquad \mathfrak{a} \to \mathfrak{a} + 2x\, V_a \epsilon$$

reflecting the axial anomaly in QCD (with $V_a \to 1$ at boundary)

- Implies $\partial_\mu J^\mu_A = N_f \mathbb{T}\mathrm{r}\, G\, \tilde{G}/16\pi^2 - 2m_q i\bar{\psi}\gamma_5\psi$
- Gauge invariant $\mathfrak{a} + xV_a\xi$ sources $\bar{\theta}/N_c = (\theta + \arg\det M)/N_c$
- Periodicity $\bar{\theta} \mapsto \bar{\theta} + 2\pi$ through branch structure

Topological susceptibility

$$\chi = \frac{d^2\mathcal{E}}{d\bar{\theta}^2} \propto \int d^4x \langle \mathbb{T}\mathrm{r}\, G\tilde{G}(x)\ \mathbb{T}\mathrm{r}\, G\tilde{G}(0)\rangle$$



Small m_q means $m_q\langle\bar{\psi}\psi\rangle \ll x\,\Lambda_{\mathrm{IR}}^4$ – tricky in the probe limit!

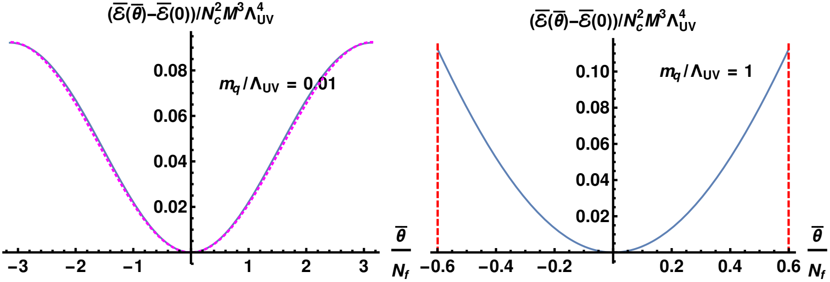
$$\chi \simeq -\frac{m_q\langle\bar{\psi}\psi\rangle}{N_f^2} \sim \frac{m_q}{x}\,; \qquad \left(\chi \simeq \frac{1}{\chi_{\mathrm{YM}}^{-1} - N_f^2/m_q\langle\bar{\psi}\psi\rangle}\right)$$

The free energy

For a single branch of vacua $\mathcal{E} = N_c^2 f(\bar{\theta}/N_c)$

Agreement with chiral Lagrangians as $m_q \to 0$:

- As small quark mass, $\mathcal{E} = -m_q \langle\bar{\psi}\psi\rangle \left(1 - \cos(\bar{\theta}/N_f)\right)$
- When $m_q \sim x \to 0$, so that both pions and η' are light,
  $\mathcal{E} = \min_{\xi_0}[-m_q\langle\bar{\psi}\psi\rangle\,(1 - \cos\xi_0) + \chi_{\rm YM}(N_f\xi_0 - \bar{\theta})^2/2]$



Imposing the 2π periodicity of $\bar{\theta}$:

$$\mathcal{E}(\bar{\theta}) = \frac{1}{2}\chi \min_k \left(\bar{\theta} + 2\pi k\right)^2 = \mathcal{O}\left(N_c^0\right)$$

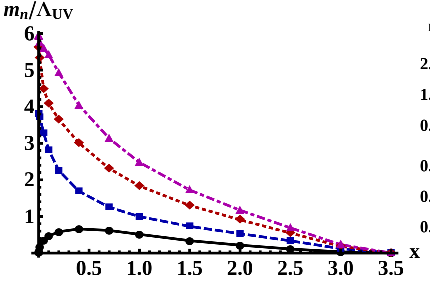
The mass of η' in V-QCD

Analytic "brute force" derivation: perturbative analysis of the coupled flavor singlet (pseudoscalar meson+glueball) fluctuation equations (at $\bar{\theta} = 0$) $\Rightarrow$

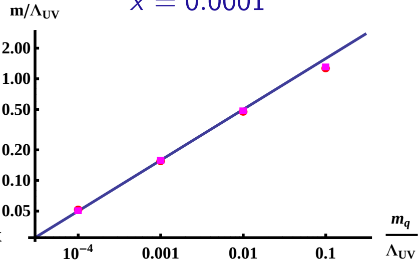
The Witten-Veneziano relation: η' becomes light as $x \to 0$

$$m_{\eta'}^2 \simeq m_\pi^2 + x \, \frac{\chi}{\bar{f}_\pi^2}$$

PS masses at $m_q = 0$

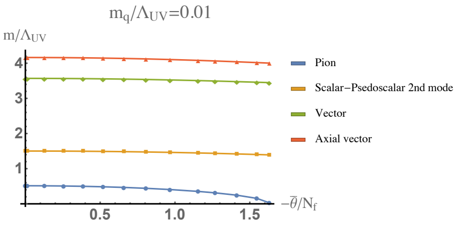


π and η' masses at $x = 0.0001$

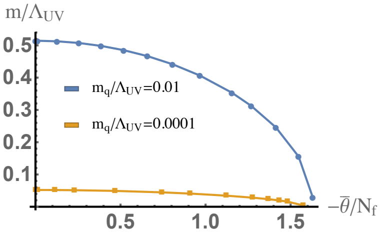


The masses of flavored mesons

Lowest modes in each sector



Pion modes



- Pion mass $m_\pi^2 \propto \cos(\bar{\theta}/N_f)$ at low m_q
- Instability at $\bar{\theta}/N_f \gtrsim \pi/2$
- Other modes show weak $\bar{\theta}$ dependence

5. Outlook and conclusions

Outlook

Ongoing work: tuning the potentials of the model to match QCD data

- First step: match glue sector with equation state + glueballs from lattice YM data
- Lattice results at finite N_f: Anomalous dimensions, equation of state, meson spectra, B dependence ...
- Experiments: meson spectra, decay constants, critical temperature
- Veneziano limit $\leftrightarrow$ finite N_f, N_c, and other approximations $\Rightarrow$ Complications? Can the approximations be effectively absorbed in the potentials? (so far looks good)

Conclusions

- V-QCD agrees with field theory results for QCD at qualitative level for large range of parameters
- Precise agreement with effective field theory at small quark mass
- Most results close to the conformal transition independent of details
- Next step: tuning the model to match quantitatively with experimental/lattice QCD data

Extra slides

First building block: model for glue

"Improved holographic QCD" (IHQCD): well-tested string-inspired bottom-up model for pure Yang-Mills

[Gursoy, Kiritsis, Nitti arXiv:0707.1324, 0707.1349]
[Gubser, Nellore arXiv:0804.0434]

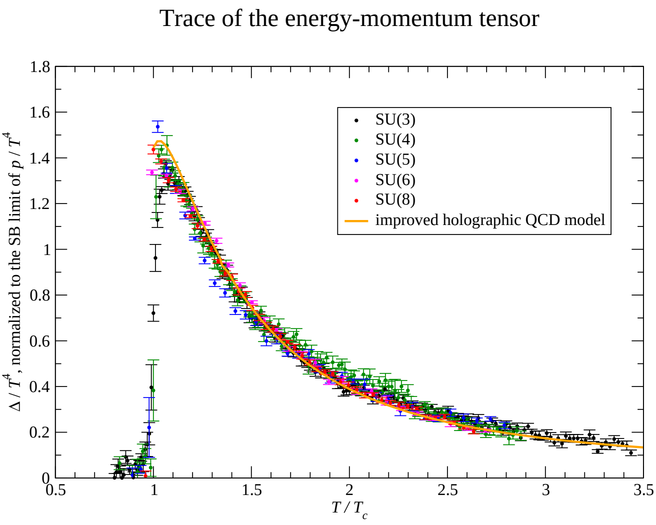
$$\mathcal{S}_{\rm g} = M^3 N_c^2 \int d^5x \sqrt{g} \left[R - \frac{4}{3}\frac{(\partial\lambda)^2}{\lambda^2} + V_g(\lambda) \right]$$

with the metric

$$ds^2 = e^{2A(r)}(dr^2 + \eta_{\mu\nu}x^\mu x^\nu)$$

- $A \leftrightarrow \log\Lambda$ energy scale
- $\lambda = e^\phi \leftrightarrow$ 't Hooft coupling $g^2 N_c$
- Dilaton potential $V_g \leftrightarrow$ YM β-function in the UV ($\lambda \to 0$)

Example of fit to lattice data: interaction measure of Yang-Mills



[Panero]

Second building block: Adding flavor

A recipe for adding quarks (in the fundamental of $SU(N_c)$ and in the probe approximation)

- Space-filling probe $D4 - \bar{D}4$ branes in 5D $\longrightarrow$
  - Tachyon $T \leftrightarrow \bar{q}q$
  - Gauge fields $A^{\mu}_{L/R} \leftrightarrow \bar{q}\gamma^{\mu}(1 \pm \gamma_5)q$
- For the vacuum structure only the tachyon is relevant
- Sen-like tachyon DBI action with $V_T \sim \exp(-|T|^2)$
  - Confining IR asymptotics of the geometry triggers ChSB
  - Gell-Mann-Oakes-Renner relation
  - Linear radial "Regge" trajectories for mesons
  - A very good fit of the light meson masses

[Klebanov,Maldacena]

[Bigazzi,Casero,Cotrone,Iatrakis,Kiritsis,Paredes hep-th/0505140,0702155; arXiv:1003.2377,1010.1364]

Defining V-QCD

Degrees of freedom

- The tachyon τ , and the dilaton λ
- $\lambda = e^{\phi}$ is identified as the 't Hooft coupling $g^2 N_c$
- τ is dual to the $\bar{q}q$ operator

$$\mathcal{S}_{\mathrm{V-QCD}} = N_c^2 M^3 \int d^5x \sqrt{g} \left[R - \frac{4}{3} \frac{(\partial\lambda)^2}{\lambda^2} + V_g(\lambda) \right]$$
$$- N_f N_c M^3 \int d^5x \, V_f(\lambda,\tau) \sqrt{-\det(g_{ab} + \kappa(\lambda)\partial_a\tau\partial_b\tau)}$$

$$V_f(\lambda,\tau) = V_{f0}(\lambda) \exp(-a(\lambda)\tau^2) \, ; \qquad ds^2 = e^{2A(r)}(dr^2 + \eta_{\mu\nu}x^\mu x^\nu)$$

Need to choose V_{f0}, a, and κ ...

A simple strategy works (!):

- Match to perturbative QCD in the UV (asymptotic AdS_5)
- Logarithmically modified string theory predictions in the IR

V-QCD literature

An ongoing program for studying V-QCD

Exploring the model at qualitative level (good match with QCD!):

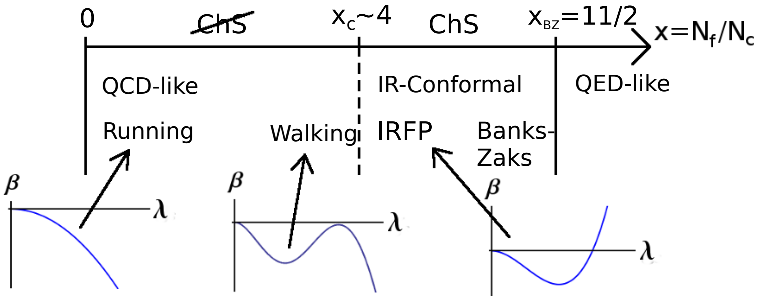
- Phase diagram at finite T and μ
  [Alho, MJ, Kajantie, Kiritsis, Tuominen arXiv:1210.4516, 1501.06379]
  [Alho, MJ, Kajantie, Kiritsis, Rosen, Tuominen arXiv:1312.5199]
- Fluctuation analysis: meson spectra, S-parameter, quasi normal modes...
  [Arean, Iatrakis, MJ, Kiritsis arXiv:1211.6125, 1309.2286]
  [Iatrakis, Zahed arXiv:1410.8540]
- Phase diagram at finite quark mass (this talk)
  [MJ, arXiv:1501.07272]
- CP-odd terms: axial anomaly (this talk)
  [To appear, with Arean, Iatrakis, Kiritsis]
- Physics at finite magnetic field (talk by U. Gürsoy)
  [Drwenski, Gürsoy, Iatrakis arXiv:1506.01350]
  [Work in progress with Gürsoy, Iatrakis, Nijs]

Also started: quantitative fit to QCD data

Phase diagram of V-QCD

- Choose reasonable potentials
- Ansatz $\tau(r)$, $\lambda(r)$, $A(r)$ in equations of motion
- Construct numerically all vacua (various IR geometries)

Desired phase diagram obtained:



- Matching to QCD perturbation theory $\rightarrow$ Banks-Zaks
- Conformal transition (BKT) at $x = x_c \simeq 4$

[Kaplan,Son,Stephanov;Kutasov,Lin,Parnachev]

(With tuned potentials, the phase diagram may change)

How does the phase structure arise?

Turning on a tiny tachyon in the conformal window

$$\tau(r) \sim m_q r^{\gamma_*+1} + \sigma r^{3-\gamma_*} \qquad (IR, \quad r \to \infty)$$

Breitenlohner-Freedman (BF) bound for γ_* at the IRFP

$$(\gamma_* + 1)(3 - \gamma_*) = \Delta_*(4 - \Delta_*) = -m_\tau^2 \ell_*^2 \leq 4$$

Violation of BF bound $\Rightarrow$ instability $\Rightarrow$ tachyon/chiral condensate

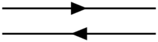
- $\Rightarrow$ bound saturated at the conformal phase transition ($x = x_c$)
- $\gamma_* = 1$ at the transition
- Predictions near the transition to large extent independent of model details: consequences a BKT transition triggered by the violation of the BF bound at the IR fixed point
- Similar results obtained in other models within the same class
  [Kutasov,Lin,Parnachev; Alvares,Alho,Erdmenger,Evans,Kim,Scott,Tuominen]

The QCD string in the Veneziano limit

Quarks:

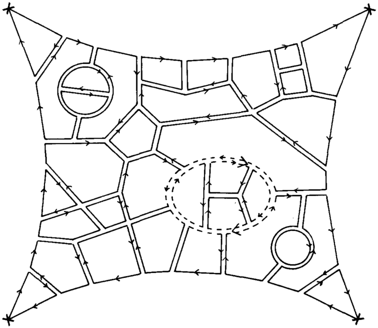
N_f 

Gluons:

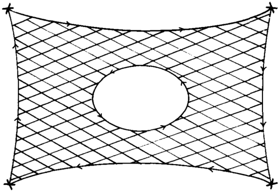


Leading diagrams in $1/N_c$:
gluonic with quark boundaries

['t Hooft]



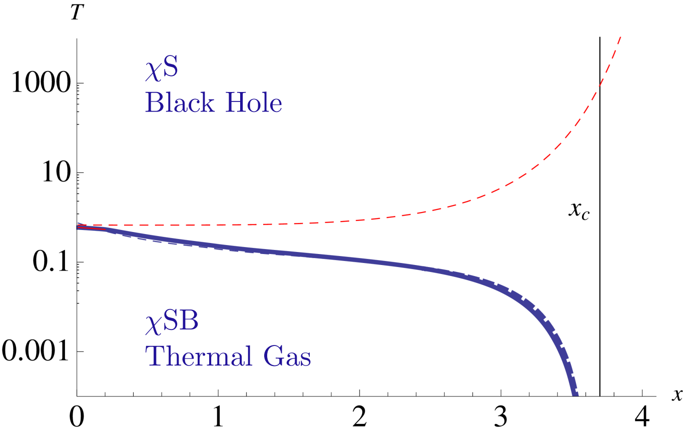
Veneziano limit $\Rightarrow$ boundaries not suppressed $\Rightarrow$ open string loops!



$= \mathcal{O}\left(N_f/N_c\right)$

Phase diagram: example at finite T

Phases on the (x, T)-plane



Loop effects may affect the order of the transition

[Alho,MJ,Kajantie,Kiritsis,Tuominen, arXiv:1210.4516, 1501.06379]

Vector correlators and S-parameter

1. Introduce bulk gauge fields dual to vector operators

$$A_\mu^{L/R} \leftrightarrow \bar{q}\gamma_\mu(1 \pm \gamma_5)q$$

2. Fluctuate full flavor action of V-QCD

$$S_f = -\frac{1}{2}M^3 N_c \mathbb{T}r \int d^4x\, dr\, \left(V_f(\lambda, T^\dagger T)\sqrt{-\det \mathbf{A}_L} + (L \to R)\right)$$

$$\mathbf{A}_{L/R\,MN} = g_{MN} + w(\lambda, T) F_{MN}^{(L/R)} + \\ + \frac{\kappa(\lambda, T)}{2}\left[(D_M T)^\dagger (D_N T) + (D_N T)^\dagger (D_M T)\right]$$

Here T and $A^{(L/R)}$ matrices in flavor space

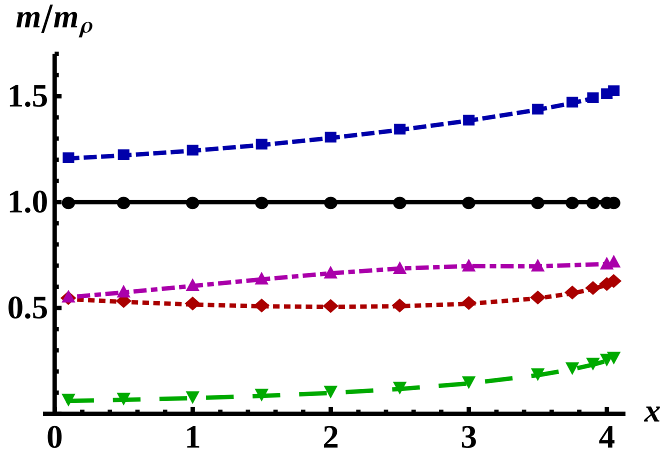
3. Compute vector-vector correlators using standard recipes

$$-i\langle J_\mu^{a\,(V)} J_\nu^{b\,(V)}\rangle \propto \delta^{ab}\left(q^2\eta_{\mu\nu} - q_\mu q_\nu\right)\Pi_V(q^2)$$

$$-i\langle J_\mu^{a\,(A)} J_\nu^{b\,(A)}\rangle \propto \delta^{ab}\left[\left(q^2\eta_{\mu\nu} - q_\mu q_\nu\right)\Pi_A(q^2) + q_\mu q_\nu \Pi_L(q^2)\right]$$

Meson mass ratios as a function of x

Lowest states of various sectors, normalized to m_ρ



All ratios tend to constants as $x \to x_c$: no technidilaton mode

[Arean,Iatrakis,MJ,Kiritsis arXiv:1211.6125, 1309.2286]

Interpreting the absence of the dilaton

What have we shown?

- Violation of BF bound does not automatically yield a light dilaton ..
- .. while Miransky scaling and hyperscaling relations are reproduced

However . . .

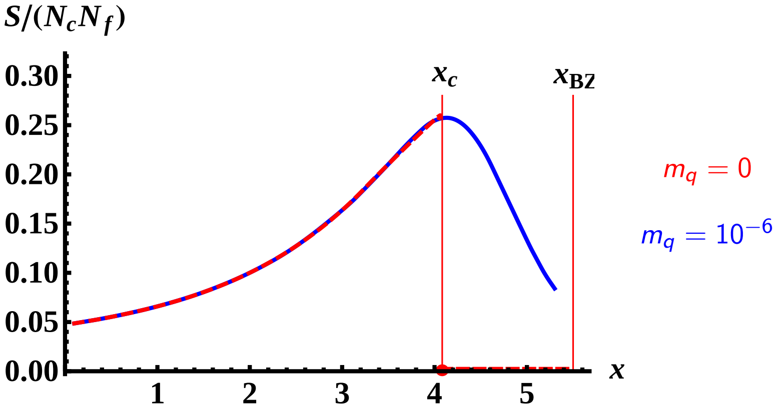
- Analytic analysis: scalar fluctuations "critical" in the walking region, suggesting a light state
- But criticality not enough: presence of such a light state is sensitive to IR

Could this be a computational error or numerical issue?

- Scalar singlet fluctuations are a real mess ..
- .. but we did nontrivial checks and all results look reasonable

Notice: easy to obtain light (but not parametrically light) scalars

S-parameter



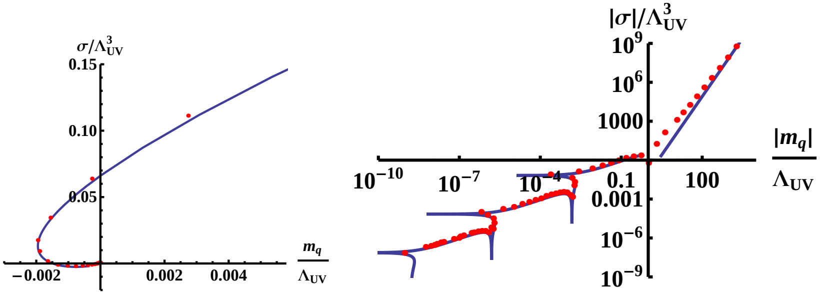
- Discontinuity at $m_q = 0$ in the conformal window
- Qualitative agreement with field theory expectations

[Sannino]

The dependence of $\sigma \propto \langle \bar{q} q \rangle$ on the quark mass

- For $x < x_c$ spiral structure

[MJ arXiv:1501.07272]



- Dots: numerical data
- Continuous line: (semi-)analytic prediction

Allows to study the effect of four-fermion operators

Four-fermion operators

Witten's recipe: modified UV boundary conditions for the tachyon

$$m_q \to \alpha \qquad \sigma \sim \langle \bar{q} q \rangle \to \beta$$

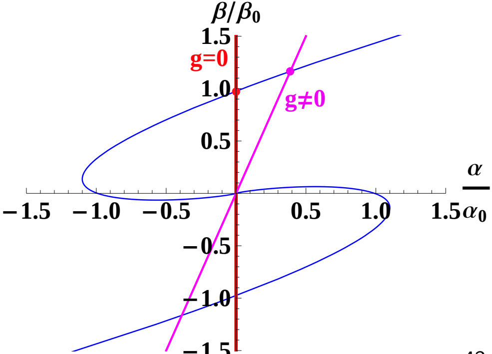
For interaction term in field theory ($\mathcal{O} = \bar{q}q$)

$$W = -m_q \int d^4x \mathcal{O}(x) + \frac{g_2}{2} \int d^4x \mathcal{O}(x)^2$$

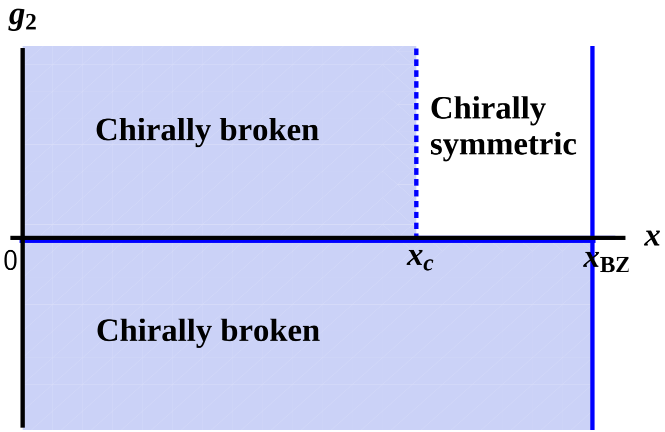
Example: $x < x_c$
and $m_q = 0$

Efimov spiral:
all sols from holography

Straight lines:
Witten's boundary condition $\alpha = g_2 \beta$

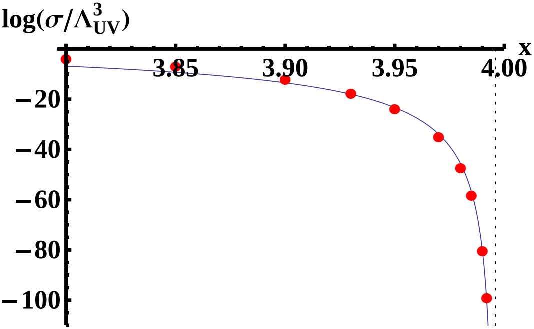


Find all intersection points, check free energy ⇒ phase diagram



- Instability at any negative g_2
- No change in phase diagram for positive g_2

Consequences of the BKT transition



$$\langle \bar{q}q \rangle \sim \sigma \sim \exp\left(-\frac{\kappa}{\sqrt{x_c - x}}\right)$$

1. Miransky/BKT scaling as $x \to x_c$ from below
   - E.g., The chiral condensate $\langle \bar{q}q \rangle \propto \sigma$
2. Unstable Efimov vacua observed for $x < x_c$
3. Turning on the quark mass possible

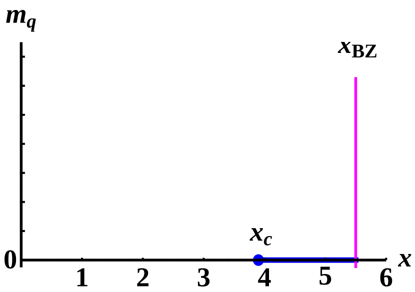
Turning on finite m_q

Quark mass defined through the tachyon boundary conditions in the UV:

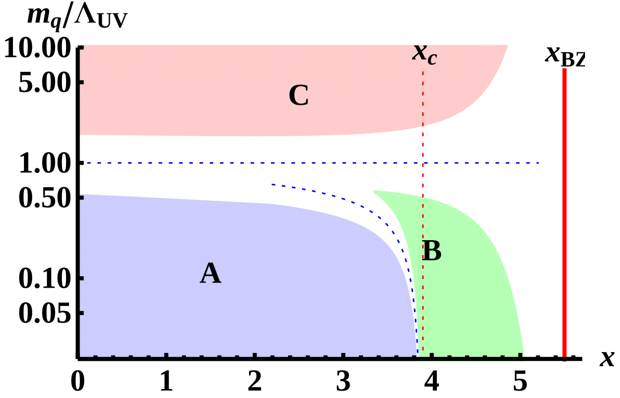
$$\tau(r) \simeq m_q(-\log r)^{-\gamma_0/\beta_0} r + \sigma(-\log r)^{\gamma_0/\beta_0} r^3$$

with $\sigma \sim \langle \bar{q}q \rangle$

- Finite (flavor independent) m_q implies nonzero tachyon and chiral symmetry breaking
- Conformal transition becomes a crossover
- Discontinuous change of IR geometry in the conformal window at $m_q = 0$



Analysis of the tachyon solution $\Rightarrow$ separate different regimes:



Crossover between A and B: $m_q \sim \exp\left[-\frac{2K}{\sqrt{x_c - x}}\right] \sim \langle\bar{q}q\rangle$

- Regimes A and B "model independent"

Axial anomaly at large N_c

$U(1)_A$ anomalously broken in QCD

However: axial anomaly is suppressed at large N_c (in the 't Hooft limit)

- "Solved" in the Veneziano limit, where axial anomaly appears at LO
- η' meson (flavor-singlet pseudoscalar) is the corresponding "Goldstone mode"

[Witten, Veneziano]

$$m_{\eta'}^2 \simeq m_\pi^2 + x \, \frac{\chi}{\bar{f}_\pi^2}$$

- χ is the topological susceptibility (constant term in $F \wedge F$ correlator)
- $\bar{f}_\pi$ is the pion decay constant with $N_{c,f}$ factors divided out
- Good agreement with experimental+lattice values for QCD

Vacua at finite θ-angle

V-QCD action has extrema with nontrivial axion $\mathfrak{a}$ and complex tachyon $T = \tau(r)e^{\xi(r)}\mathbb{I}$

1. Perturbative UV analysis
   - Nontrivial solutions only at finite quark mass m_q as expected: for $m_q = 0$ the θ-angle can be gauged away
   - Exact results at small quark mass and/or small $x = N_f/N_c$
2. Numerical approach:
   - Choose reasonable values for the parameters in $\mathcal{S}_{\mathrm{V-QCD}} = \mathcal{S}_g + \mathcal{S}_f + \mathcal{S}_a$
   - Analyze saddle points + fluctuations numerically
   - Full backreaction between all sectors

The CP-odd term in V-QCD

Bulk axion a

- dual to $\mathrm{tr} F \wedge F$
- background value identified as θ/N_c, where θ is the theta angle of QCD

Tachyon Ansatz $T = \tau e^{i\xi}\, \mathbb{I}$

String motivated CP-odd term added in the action

$$S_a = -\frac{M^3 N_c^2}{2} \int d^5x \sqrt{-\det g}\, Z(\lambda) \times \left[da - x\left(2V_a(\lambda,\tau)\, A - \xi\, dV_a(\lambda,\tau)\right)\right]^2$$

[Casero, Kiritsis, Paredes]

Symmetry

$$A_\mu \to A_\mu + \partial_\mu \epsilon \,, \qquad \xi \to \xi - 2\epsilon \,, \qquad a \to a + 2x\, V_a \epsilon$$

reflects the axial anomaly in QCD (with $\epsilon = \epsilon(x_\mu)$)

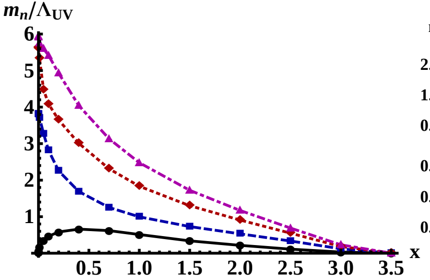
The mass of η' in V-QCD

Analytic derivation by perturbative analysis of the coupled flavor singlet (pseudoscalar meson+glueball) fluctuation equations $\Rightarrow$

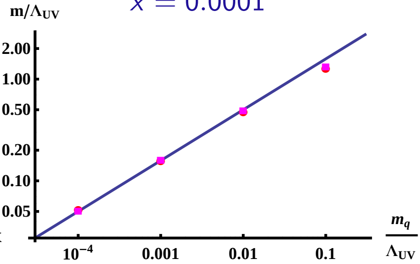
The Witten-Veneziano relation: η' becomes light as $x \to 0$

$$m_{\eta'}^2 \simeq m_\pi^2 + x\ \frac{\chi}{f_\pi^2}$$

PS masses at $m_q = 0$

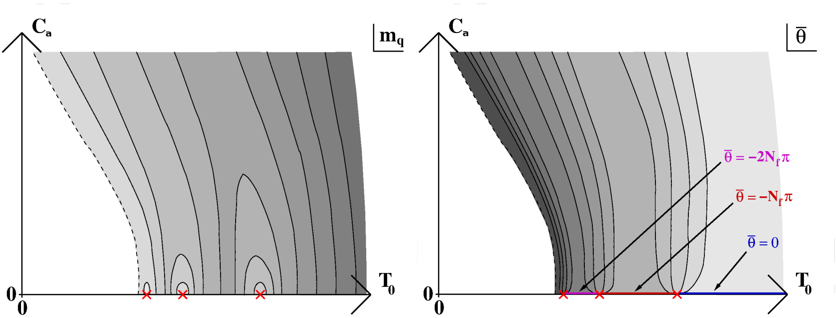


π and η' masses at $x = 0.0001$



Vacua at finite θ-angle: more details

All vacua characterized by two parameters, T_0 and C_a



- Regular solutions in the shaded region
- IR fixed point reached on the dotted curve
- Crosses: subdominant Efimov vacua at $m_q = 0$
- $\bar{\theta}$-dependence irregular as $m_q \to 0$

Real tachyon $\leftrightarrow$ $C_a = 0$

Finite T and μ – definitions

Add gauge field

$$\mathcal{S}_{\mathrm{V-QCD}} = N_c^2 M^3 \int d^5x \sqrt{g} \left[R - \frac{4}{3} \frac{(\partial\lambda)^2}{\lambda^2} + V_g(\lambda) \right]$$
$$- N_f N_c M^3 \int d^5x V_f(\lambda, \tau)$$
$$\times \sqrt{-\det(g_{ab} + \kappa(\lambda)\partial_a\tau\partial_b\tau + w(\lambda)F_{ab})}$$

$$F_{r0} = \partial_r \Phi \qquad \Phi = \mu - nr^2 + \cdots$$

A more general metric (A and f solved from EoMs)

$$ds^2 = e^{2A(r)} \left(\frac{dr^2}{f(r)} - f(r)dt^2 + d\mathbf{x}^2 \right)$$

Nontrivial blackening factor f: black hole solutions possible

Various solutions

Two classes of IR geometries:

1. Black hole solutions $\to$ temperature and entropy through BH thermodynamics
   - $f'(r_h) = -4\pi T$; $\quad s = 4\pi M^3 N_c^2 e^{3A(r_h)}$
2. Thermal gas solutions ($f \equiv 1$)
   - Any T and μ, zero s

Two types of tachyon behavior ($\tau \leftrightarrow \bar{q}q$, quark mass and condensate from UV boundary behavior):

1. Vanishing tachyon – chirally symmetric
2. Nontrivial tachyon – chirally broken

$\Rightarrow$ four possible types of background solutions

Computation of pressure

Three phases turn out to be relevant (at small x)

- Tachyonic Thermal gas (chirally broken)
- Tachyonic BH (chirally broken)
- Tachyonless BH (chirally symmetric)

Nontrivial numerical analysis:

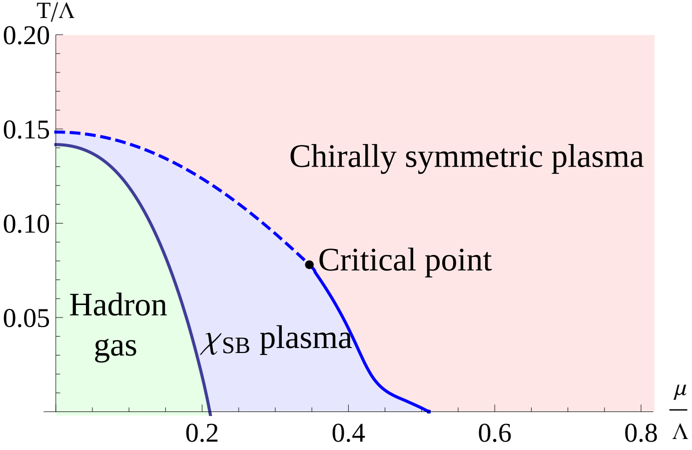
1. T, μ not input parameters, they need to be calculated first
2. Integrate numerically for each phase

$$dp = s\,dT + n\,d\mu$$

3. Phase with highest p dominates

Phase diagram at finite μ (example at fixed x)

First attempt: $x = N_f/N_c = 1$, Veneziano limit, zero quark mass



- $\mathrm{AdS}_2 \times \mathbb{R}^3$ IR geometry as $T \to 0$
- Finite entropy at zero temperature $\Rightarrow$ instability?

Fluctuation analysis

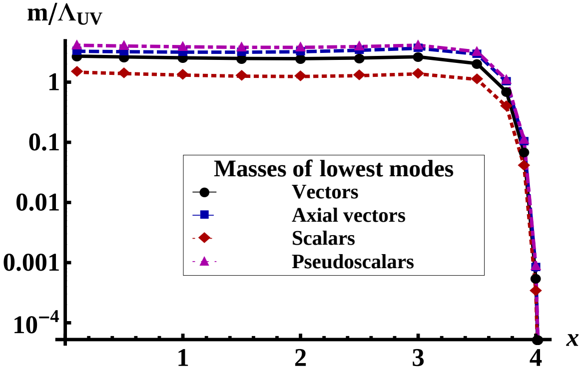
1. Meson spectra (at zero temperature and quark mass)
   - Implement (left and right handed) gauge fields in $\mathcal{S}_{\mathrm{V-QCD}}$
   - Four towers: scalars, pseudoscalars, vectors, and axial vectors
   - Flavor singlet and nonsinglet ($SU(N_f)$) states

In the region relevant for “walking” technicolor ($x \to x_c$ from below):

- Possibly a light “dilaton” (flavor singlet scalar): Goldstone mode due to almost unbroken conformal symmetry.
  Could the dilaton be the 125 GeV Higgs?

Meson masses

Flavor nonsinglet masses (Example: PotI)



- Miransky scaling:

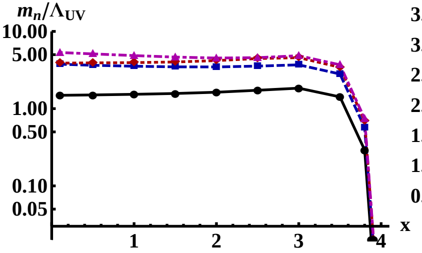
$$m_n \sim \exp\left(-\frac{\kappa}{\sqrt{x_c - x}}\right)$$

- Radial trajectories $m_n^2 \sim n$ or $m_n^2 \sim n^2$ depending on potentials

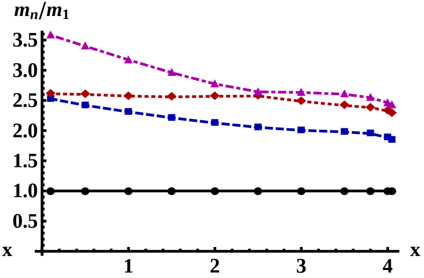
Scalar singlet masses

Scalar singlet (0^{++}) spectrum (PotI):

In log scale



Normalized to the lowest state



- No light dilaton state as $x \to x_c$?

S-parameter

$$S \sim \frac{d}{dq^2} q^2 \left[\Pi_V(q^2) - \Pi_A(q^2)\right]_{q^2=0}$$

where (at zero quark mass)

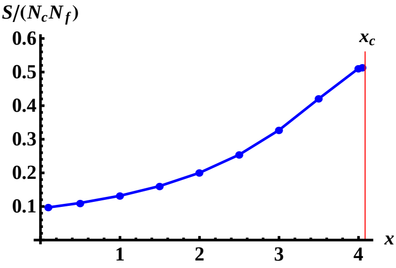
$$\Pi_{V/A}(q^2)\left(q^2 g^{\mu\nu} - q^\mu q^\nu\right)\delta^{ab} \propto \langle J^{\mu\, a}_{V/A} J^{\nu\, b}_{V/A}\rangle$$

in terms of the vector-vector and axial-axial correlators

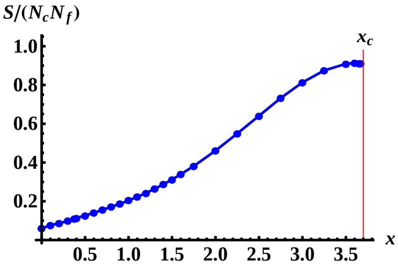
- The S-parameter might be reduced in the walking regime

Results:

PotI



PotII

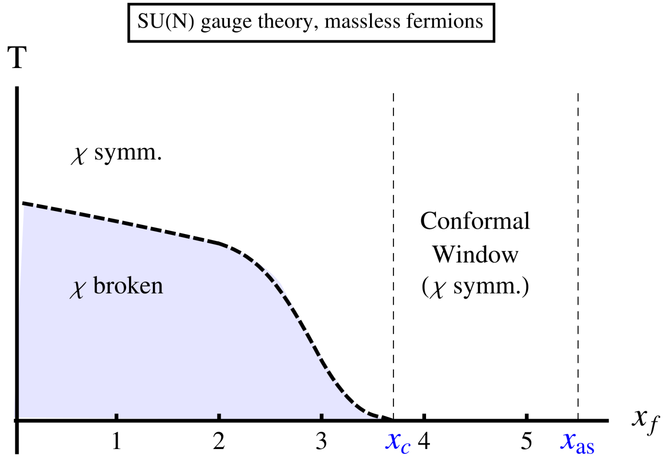


The S-parameter increases with x: expected suppression absent

Jumps discontinuously to zero at $x = x_c$

QCD at finite T (and x)

Expected phase structure at finite temperature (and x)



Potentials I

$$
\begin{aligned}
V_g(\lambda) &= 12 + \frac{44}{9\pi^2}\lambda + \frac{4619}{3888\pi^4}\frac{\lambda^2}{(1+\lambda/(8\pi^2))^{2/3}}\sqrt{1+\log(1+\lambda/(8\pi^2))} \\
V_f(\lambda,\tau) &= V_{f0}(\lambda)e^{-a(\lambda)\tau^2} \\
V_{f0}(\lambda) &= \frac{12}{11} + \frac{4(33-2x)}{99\pi^2}\lambda + \frac{23473-2726x+92x^2}{42768\pi^4}\lambda^2 \\
a(\lambda) &= \frac{3}{22}(11-x) \\
\kappa(\lambda) &= \frac{1}{\left(1+\frac{115-16x}{288\pi^2}\lambda\right)^{4/3}}
\end{aligned}
$$

In this case the tachyon diverges exponentially:

$$
\tau(r) \sim \tau_0 \exp\left[\frac{81\;3^{5/6}(115-16x)^{4/3}(11-x)}{812944\;2^{1/6}}\frac{r}{R}\right]
$$

Potentials II

$$
\begin{aligned}
V_g(\lambda) &= 12 + \frac{44}{9\pi^2}\lambda + \frac{4619}{3888\pi^4}\frac{\lambda^2}{(1+\lambda/(8\pi^2))^{2/3}}\sqrt{1+\log(1+\lambda/(8\pi^2))} \\
V_f(\lambda,\tau) &= V_{f0}(\lambda)e^{-a(\lambda)\tau^2} \\
V_{f0}(\lambda) &= \frac{12}{11} + \frac{4(33-2x)}{99\pi^2}\lambda + \frac{23473-2726x+92x^2}{42768\pi^4}\lambda^2 \\
a(\lambda) &= \frac{3}{22}(11-x)\frac{1+\frac{115-16x}{216\pi^2}\lambda+\lambda^2/(8\pi^2)^2}{(1+\lambda/(8\pi^2))^{4/3}} \\
\kappa(\lambda) &= \frac{1}{(1+\lambda/(8\pi^2))^{4/3}}
\end{aligned}
$$

In this case the tachyon diverges as

$$
\tau(r) \sim \frac{27\ 2^{3/4}3^{1/4}}{\sqrt{4619}}\sqrt{\frac{r-r_1}{R}}
$$

Effective potential

For solutions with $\tau = \tau_* = \text{const}$

$$\mathcal{S} = M^3 N_c^2 \int d^5x \sqrt{g} \left[R - \frac{4}{3} \frac{(\partial\lambda)^2}{\lambda^2} + V_g(\lambda) - xV_f(\lambda, \tau_*) \right]$$

IHQCD with an effective potential

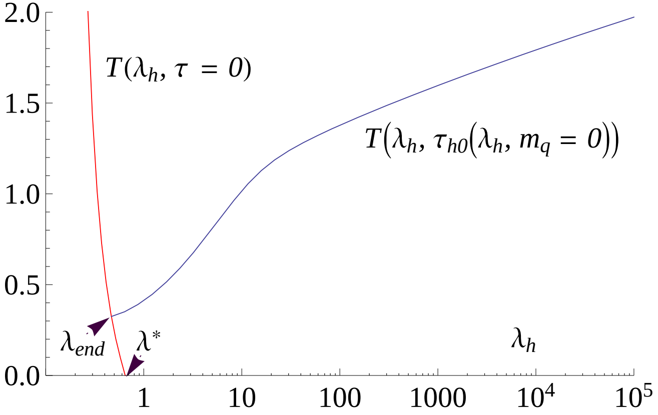
$$V_{\text{eff}}(\lambda) = V_g(\lambda) - xV_f(\lambda, \tau_*) = V_g(\lambda) - xV_{f0}(\lambda) \exp(-a(\lambda)\tau_*^2)$$

Minimizing for τ_* we obtain $\tau_* = 0$ and $\tau_* = \infty$

- $\tau_* = 0$: $V_{\text{eff}}(\lambda) = V_g(\lambda) - xV_{f0}(\lambda)$;
  fixed point with $V'_{\text{eff}}(\lambda_*) = 0$
- $\tau_* \to \infty$: $V_{\text{eff}}(\lambda) = V_g(\lambda)$ (like YM, no fixed points)

Black hole branches

Example: PotII at $x = 3$, $W_0 = 12/11$

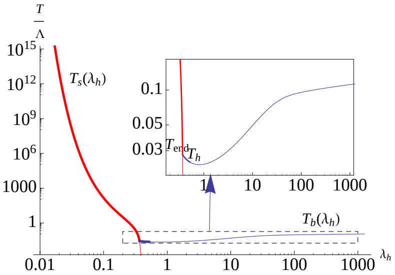


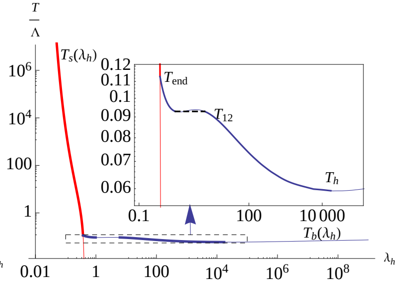
Simple phase structure: 1st order transition at $T = T_h$ from thermal gas to (chirally symmetric) BH

More complicated cases:

PotII at $x = 3$, W_0 SB

PotI at $x = 3.5$, $W_0 = 12/11$



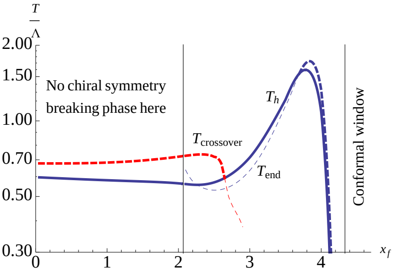


- Left: chiral symmetry restored at 2nd order transition with $T = T_{\text{end}} > T_h$
- Right: Additional first order transition between BH phases with broken chiral symmetry

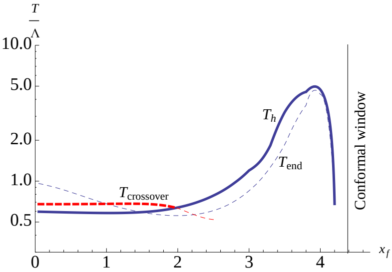
Also other cases . . .

Phase diagrams on the (x, T)-plane

PotI$_*$ W_0 SB

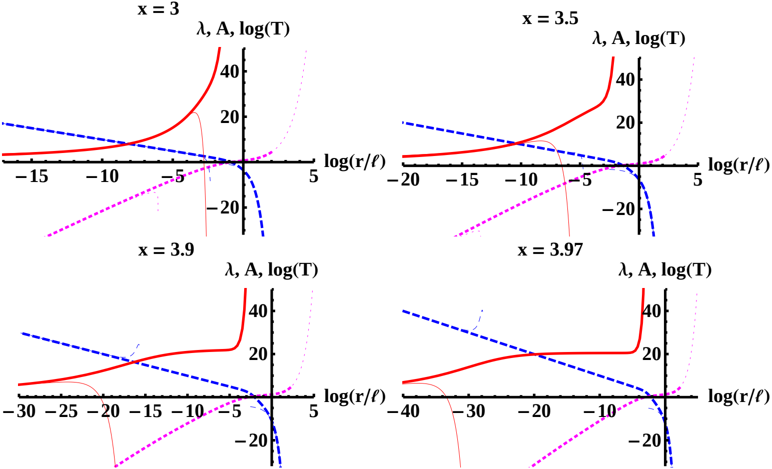


PotII$_*$ W_0 SB

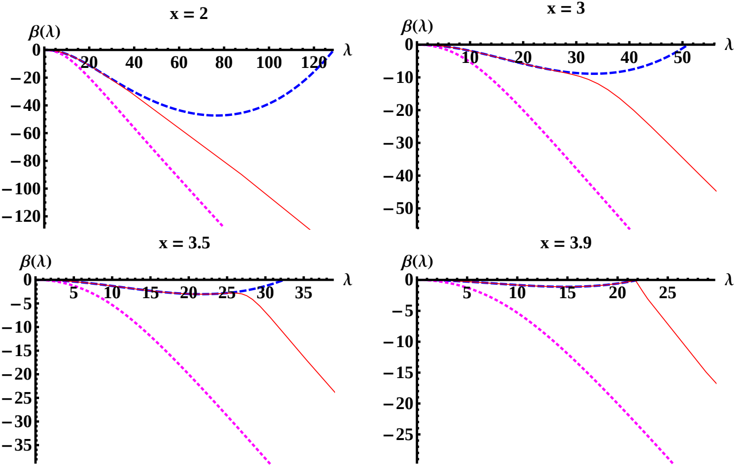


Backgrounds in the walking region

Backgrounds with zero quark mass, $x < x_c \simeq 3.9959$ (λ, A, τ)



Beta functions along the RG flow (evaluated on the background), zero tachyon, YM $x_c \simeq 3.9959$



Holographic beta functions

Generalization of the holographic RG flow of IHQCD

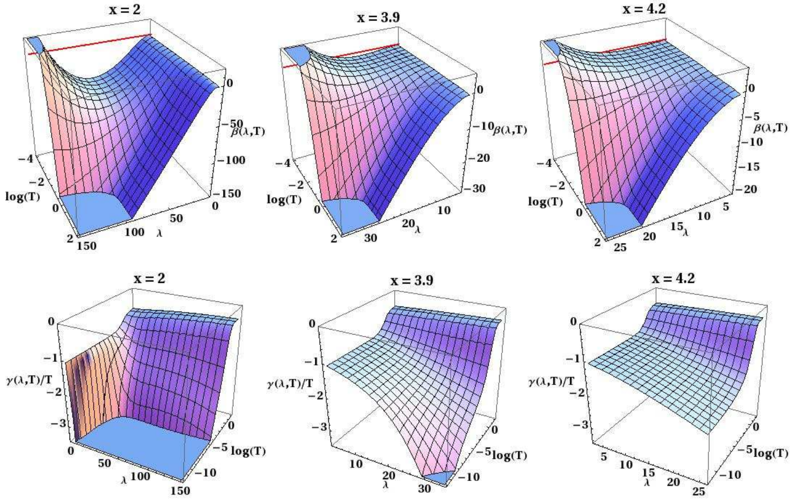
$$\beta(\lambda, \tau) \equiv \frac{d\lambda}{dA} \;; \qquad \gamma(\lambda, \tau) \equiv \frac{d\tau}{dA}$$

linked to

$$\frac{dg_{\text{QCD}}}{d \log \mu} \;; \qquad \frac{dm}{d \log \mu}$$

The full equations of motion boil down to two first order partial non-linear differential equations for β and γ

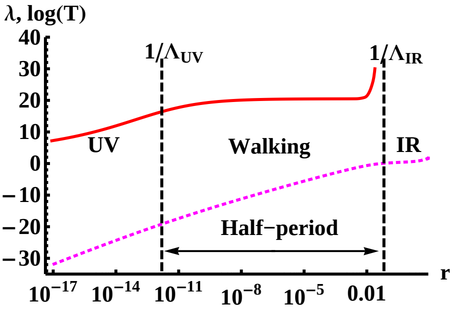
“Good” solutions numerically (unique)



Miransky/BKT scaling

As $x \to x_c$ from below: walking, dominant solution

- BF-bound for the tachyon violated at the IRFP
- x_c fixed by the BF bound: $\Delta = 2$ & $\gamma_* = 1$ at the edge of the conformal window



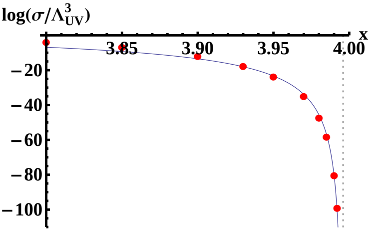
- $\tau(r) \sim r^2 \sin(\kappa\sqrt{x_c - x}\log r + \phi)$ in the walking region
- "0.5 oscillations" $\Rightarrow$ Miransky/BKT scaling, amount of walking $\Lambda_{UV}/\Lambda_{IR} \sim \exp(\pi/(\kappa\sqrt{x_c - x}))$

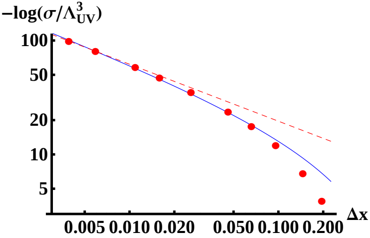
As $x \to x_c$

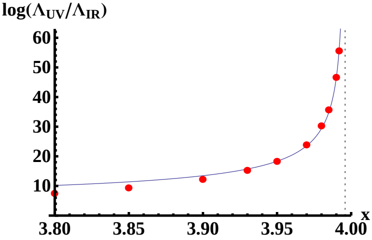
with known κ

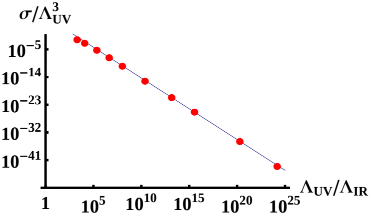
$$\langle \bar{q} q \rangle \sim \sigma \sim \exp(-2\pi/(\kappa\sqrt{x_c - x}))$$

$$\Lambda_{UV}/\Lambda_{IR} \sim \exp(\pi/(\kappa\sqrt{x_c - x}))$$





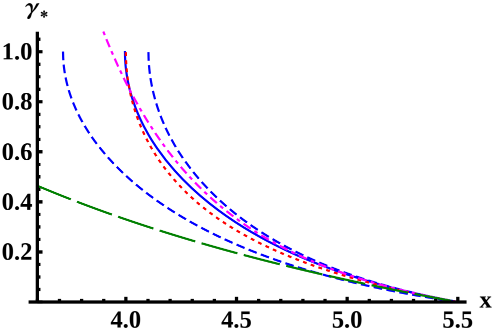




Comparison to other guesses

V-QCD (dashed: variation due to W_0)
Dyson-Schwinger
2-loop PQCD
All-orders β

[Pica, Sannino arXiv:1011.3832]



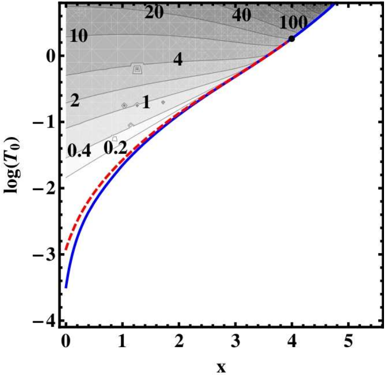
Parameters

Understanding the solutions for generic quark masses requires discussing parameters

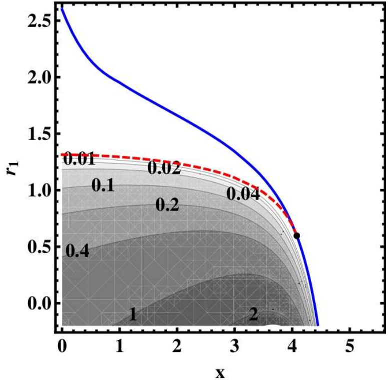
- YM or QCD with massless quarks: no parameters
- QCD with flavor-independent mass m: a single (dimensionless) parameter m/Λ_{QCD}
- In this model, after rescalings, this parameter can be mapped to a parameter (τ_0 or r_1) that controls the diverging tachyon in the IR
- x has become continuous in the Veneziano limit

Map of all solutions

All "good" solutions ($\tau \neq 0$) obtained varying x and τ_0 or r_1
Contouring: quark mass (zero mass is the red contour)

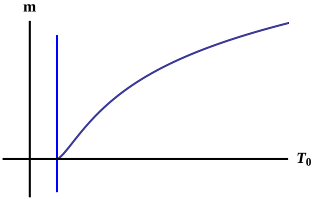


"Potentials I" $\leftrightarrow$ T_0



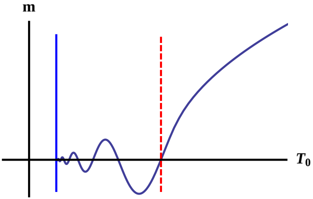
"Potentials II" $\leftrightarrow$ r_1

Mass dependence and Efimov vacua



Conformal window ($x > x_c$)

- For $m = 0$, unique solution with $\tau \equiv 0$
- For $m > 0$, unique "standard" solution with $\tau \neq 0$

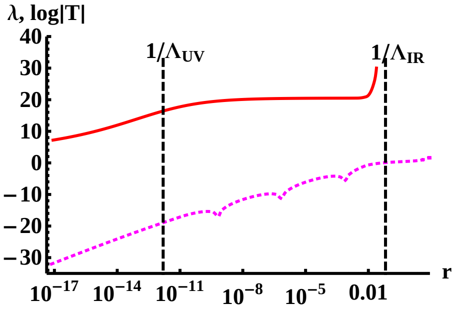


Low $0 < x < x_c$: Efimov vacua

- Unstable solution with $\tau \equiv 0$ and $m = 0$
- "Standard" stable solution, with $\tau \neq 0$, for all $m \geq 0$
- Tower of unstable Efimov vacua (small $|m|$)

Efimov solutions

- Tachyon oscillates over the walking regime
- $\Lambda_{UV}/\Lambda_{IR}$ increased wrt. "standard" solution

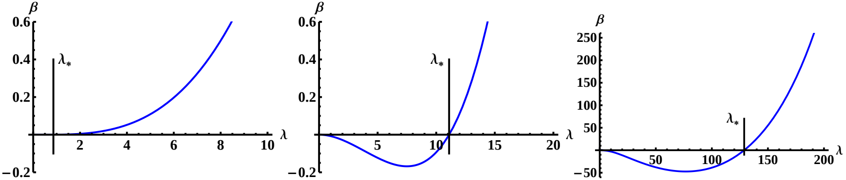


Effective potential: zero tachyon

Start from Banks-Zaks region, $\tau_* = 0$, chiral symmetry conserved $(\tau \leftrightarrow \bar{q}q)$, $V_{\text{eff}}(\lambda) = V_g(\lambda) - xV_{f0}(\lambda)$

- V_{eff} defines a β-function as in IHQCD – Fixed point guaranteed in the BZ region, moves to higher λ with decreasing x
- Fixed point λ_* runs to ∞ either at finite $x(< x_c)$ or as $x \to 0$

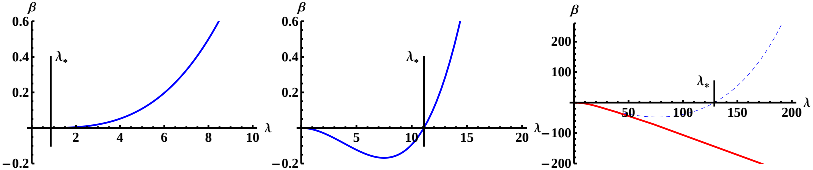
Banks-Zaks	Conformal Window	
$x \to 11/2$	$x > x_c$	$x < x_c$??



Effective potential: what actually happens

Banks-Zaks   Conformal Window

$x \to 11/2$ $x > x_c$ $x < x_c$



$\tau \equiv 0$ $\tau \equiv 0$ $\tau \neq 0$

- For $x < x_c$ vacuum has nonzero tachyon (checked by calculating free energies)
- The tachyon screens the fixed point
- In the deep IR τ diverges, $V_{\text{eff}} \to V_g \Rightarrow$ dynamics is YM-like

Where is x_c?

How is the edge of the conformal window stabilized?
Tachyon IR mass at $\lambda = \lambda_* \leftrightarrow$ quark mass dimension

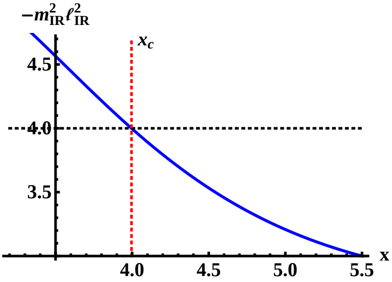
$$-m_{\rm IR}^2 \ell_{\rm IR}^2 = \Delta_{\rm IR}(4 - \Delta_{\rm IR}) = \frac{24 a(\lambda_*)}{\kappa(\lambda_*)(V_g(\lambda_*) - x V_0(\lambda_*))}$$

$$\gamma_* = \Delta_{\rm IR} - 1$$

Breitenlohner-Freedman (BF) bound (horizontal line)

$$-m_{\rm IR}^2 \ell_{\rm IR}^2 = 4 \;\Leftrightarrow\; \gamma_* = 1$$

defines x_c



Why $\gamma_* = 1$ at $x = x_c$?

No time to go into details . . . the question boils down to the linearized tachyon solution at the fixed point

- For $\Delta_{\rm IR}(4 - \Delta_{\rm IR}) < 4 \qquad (x > x_c)$:

$$\tau(r) \sim m_q r^{\Delta_{\rm IR}} + \sigma r^{4-\Delta_{\rm IR}}$$

- For $\Delta_{\rm IR}(4 - \Delta_{\rm IR}) > 4 \qquad (x < x_c)$:

$$\tau(r) \sim C r^2 \sin\left[(\mathrm{Im}\Delta_{\rm IR}) \log r + \phi\right]$$

Rough analogy:
Tachyon EoM $\leftrightarrow$ Gap equation in Dyson-Schwinger approach
Similar observations have been made in other holographic frameworks

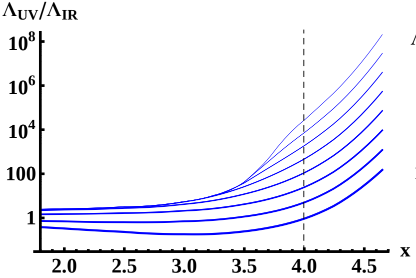
[Kutasov, Lin, Parnachev arXiv:1107.2324, 1201.4123]

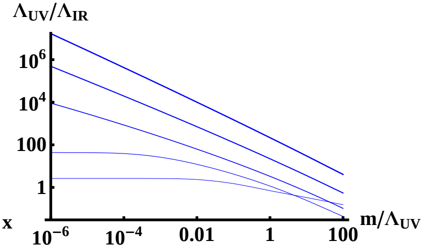
Mass dependence

For $m > 0$ the conformal transition disappears

The ratio of typical UV/IR scales $\Lambda_{UV}/\Lambda_{IR}$ varies in a natural way

$m/\Lambda_{UV} = 10^{-6}, 10^{-5}, \ldots, 10 \qquad x = 2, 3.5, 3.9, 4.25, 4.5$





sQCD phases

The case of $\mathcal{N} = 1$ $SU(N_c)$ superQCD with N_f quark multiplets is known and provides an interesting (and more complex) example for the nonsupersymmetric case. From Seiberg we have learned that:

- $x = 0$ the theory has confinement, a mass gap and N_c distinct vacua associated with a spontaneous breaking of the leftover R symmetry Z_{N_c}.
- At $0 < x < 1$, the theory has a runaway ground state.
- At $x = 1$, the theory has a quantum moduli space with no singularity. This reflects confinement with ChSB.
- At $x = 1 + 1/N_c$, the moduli space is classical (and singular). The theory confines, but there is no ChSB.
- At $1 + 2/N_c < x < 3/2$ the theory is in the non-abelian magnetic IR-free phase, with the magnetic gauge group $SU(N_f - N_c)$ IR free.
- At $3/2 < x < 3$, the theory flows to a CFT in the IR. Near $x = 3$ this is the Banks-Zaks region where the original theory has an IR fixed point at weak coupling. Moving to lower values, the coupling of the IR $SU(N_c)$ gauge theory grows. However near $x = 3/2$ the dual magnetic $SU(N_f - N_c)$ is in its Banks-Zaks region, and provides a weakly coupled description of the IR fixed point theory.
- At $x > 3$, the theory is IR free.

Saturating the BF bound (sketch)

Why is the BF bound saturated at the phase transition (massless quarks)??

$$\Delta_{\mathrm{IR}}(4-\Delta_{\mathrm{IR}}) = \frac{24a(\lambda_*)}{\kappa(\lambda_*)(V_g(\lambda_*) - xV_0(\lambda_*))}$$

- For $\Delta_{\mathrm{IR}}(4-\Delta_{\mathrm{IR}}) < 4$:
  $\tau(r) \sim m_q r^{4-\Delta_{\mathrm{IR}}} + \sigma r^{\Delta_{\mathrm{IR}}}$
- For $\Delta_{\mathrm{IR}}(4-\Delta_{\mathrm{IR}}) > 4$:
  $\tau(r) \sim C r^2 \sin\left[(\mathrm{Im}\Delta_{\mathrm{IR}}) \log r + \phi\right]$
- Saturating the BF bound, the tachyon solutions will engtangle $\rightarrow$ required to satisfy boundary conditions
- Nodes in the solution appear trough UV $\rightarrow$ massless solution

Saturating the BF bound (sketch)

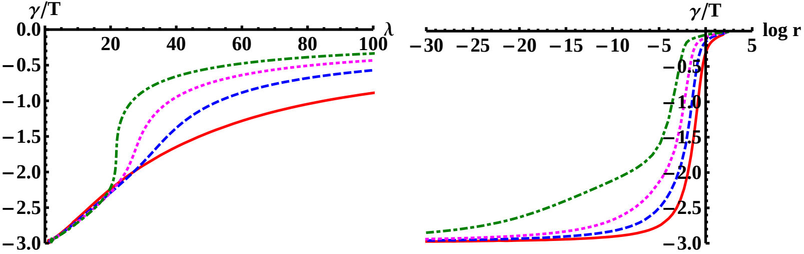
Does the nontrivial (ChSB) massless tachyon solution exist?
Two possibilities:

- $x > x_c$: BF bound satisfied at the fixed point $\Rightarrow$ only trivial massless solution ($\tau \equiv 0$, ChS intact, fixed point hit)
- $x < x_c$: BF bound violated at the fixed point $\Rightarrow$ a nontrivial massless solution exist, which drives the system away from the fixed point

Conclusion: phase transition at $x = x_c$
As $x \to x_c$ from below, need to approach the fixed point to satisfy the boundary conditions $\Rightarrow$ nearly conformal, "walking" dynamics

Gamma functions

Massless backgrounds: gamma functions $\frac{\gamma}{\tau} = \frac{d \log \tau}{dA}$



$x = 2, 3, 3.5, 3.9$