# Direct test of the gauge/gravity duality for the D0-brane system at finite N

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Workshop “Holography, conformal field theories, and lattice”
at Higgs Centre for Theoretical Physics, Edinburgh, UK
June 27-30, 2016

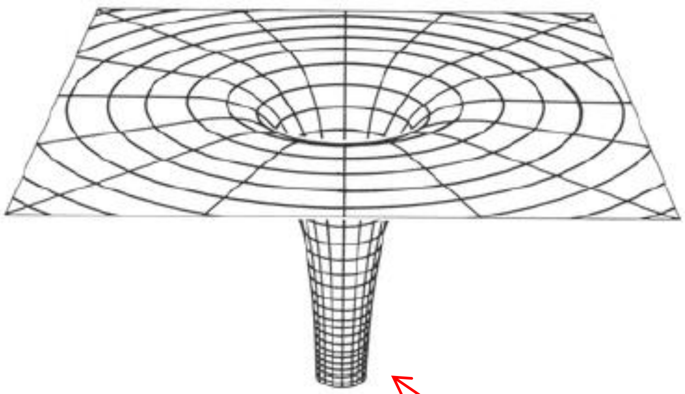
Ref.) M. Hanada, Y. Hyakutake, G. Ishiki, J.N., arXiv:1603.00538 [hep-th]

# 0. Introduction

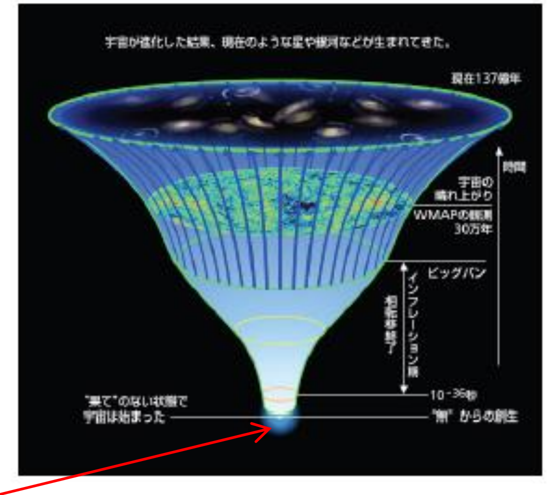
- Why “holography” ?

Two situations where quantum gravity becomes important

Black hole



Big bang

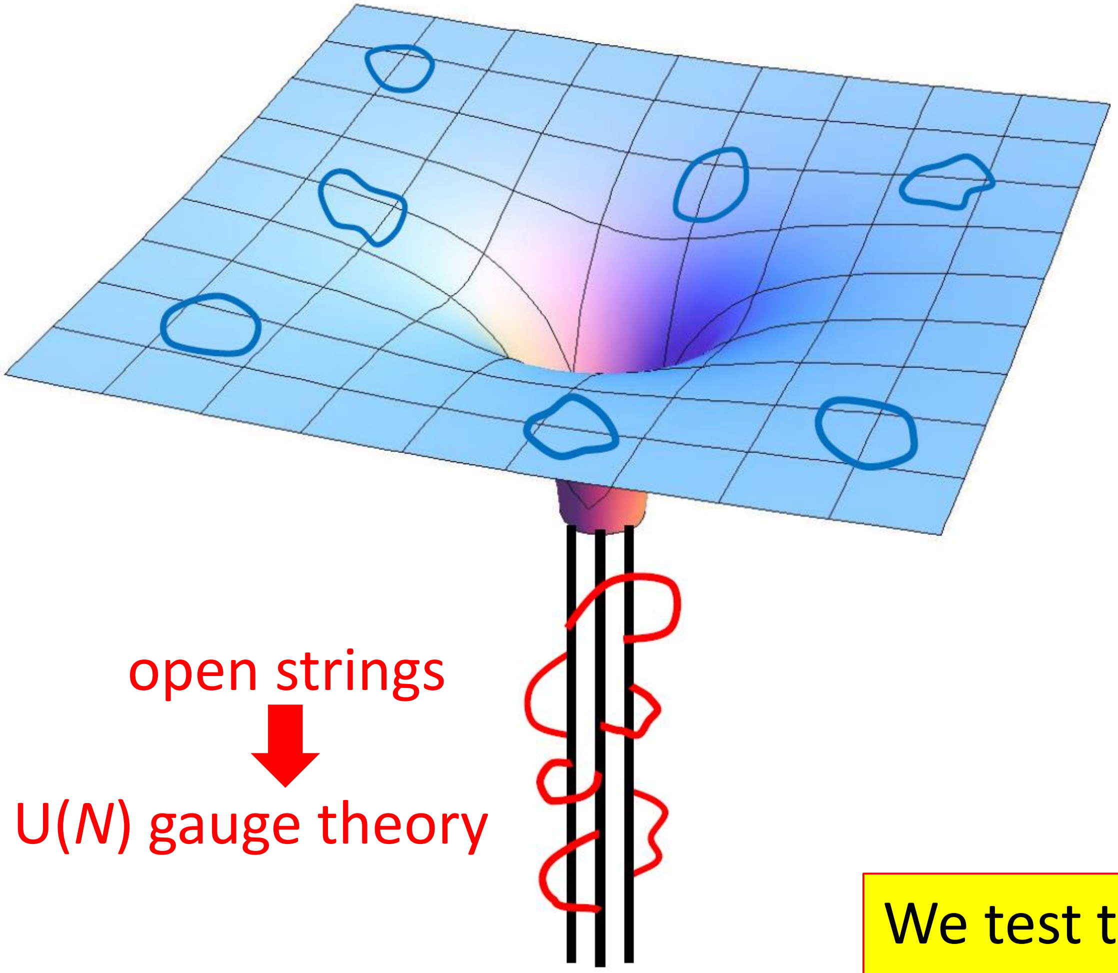


singularity (curvature diverges)

Defining quantum gravity by something else treating space-time as an emergent concept.

# Gauge/gravity duality conjecture

Maldacena ('97)



closed strings

↓

gravity theory

open strings

↓

U($N$) gauge theory

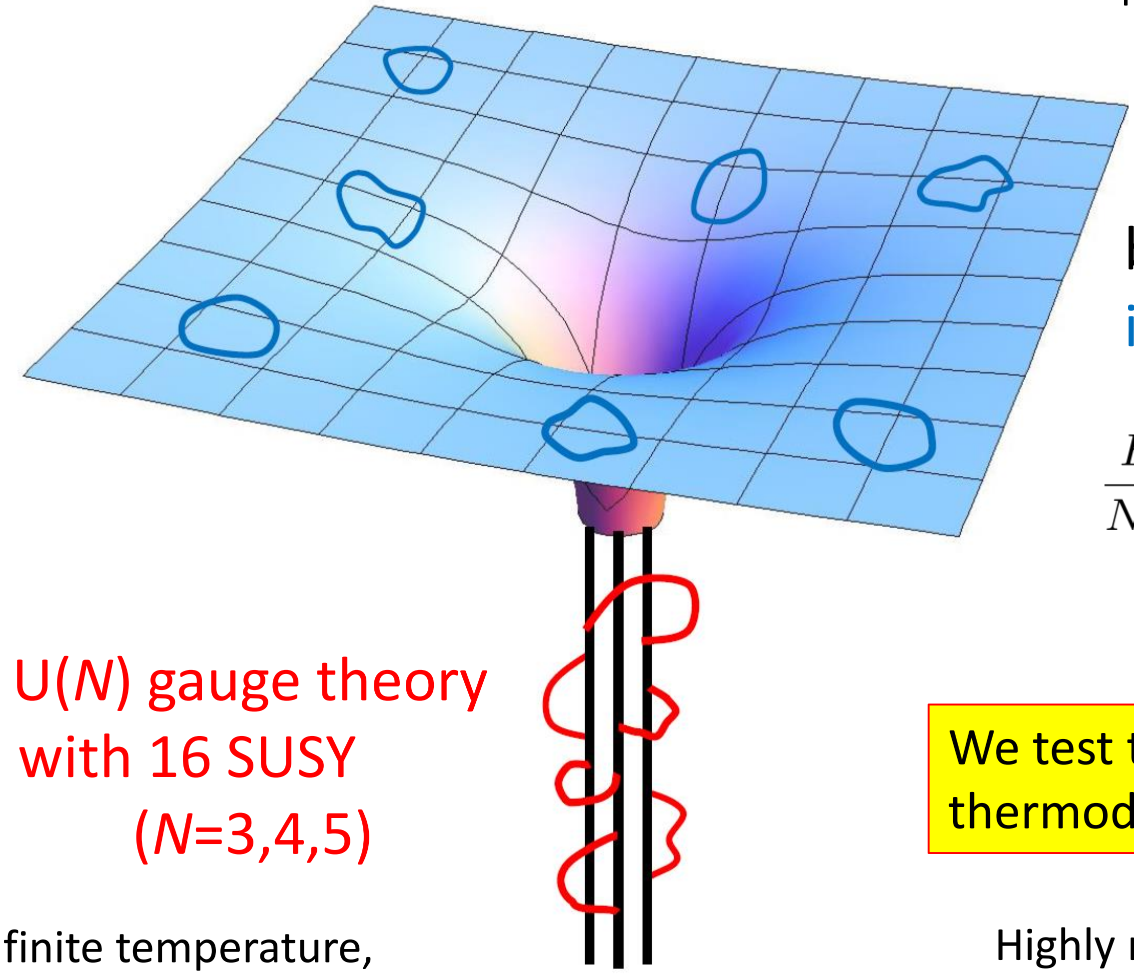
$N$ D-branes

In the large-N limit,
string loop corrections
(incl. QG effects)
can be neglected.

**We test this duality at finite N for the D0-brane case.**

# The duality for the D0-brane system

Itzhaki-Maldacena-Sonnenschein
-Yankielowicz ('98)



black 0-brane solution
in type IIA SUGRA

$$\frac{E}{N^2} = 7.41\, T^{\frac{14}{5}} \boxed{- \frac{5.77}{N^2} T^{\frac{2}{5}}}$$

string loop corrections

1d U(*N*) gauge theory
with 16 SUSY
(*N*=3,4,5)

We test this duality by comparing thermodynamic properties.

finite temperature,
strong coupling

*N* D0-branes

Highly nontrivial, because of the meta-stability due to 1/N effects

# Plan of the talk

0. Introduction
1. Brief review of the gauge/gravity duality
2. The gauge/gravity duality for the D0-brane system
3. Testing the gauge/gravity duality in the large-N limit
4. Testing the gauge/gravity duality at finite N
5. Summary and future prospects

# 1. Brief review of the gauge/gravity duality

# “D-brane”
## a soliton-like object in string theory

a state in which condensation of strings occurs on a $p$-dim hyperplane

$p = 0$ $\quad$ $p = 1$ $\quad$ $p = 2 \cdots$

point-like

string-like

membrane

“*p*-brane” in general

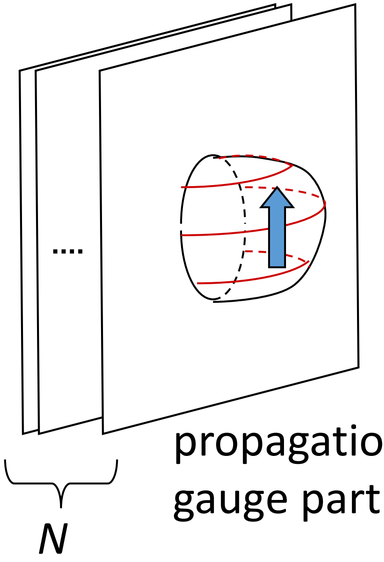
Polchinski (1995)

Dirichlet b.c. are imposed on both ends of the open string

→ D *p*-brane

# The low-energy effective theory of open strings excited on a stack of N D-branes

D $p$-brane



propagation of gauge particle

extended in
$(p+1)$-dimensions

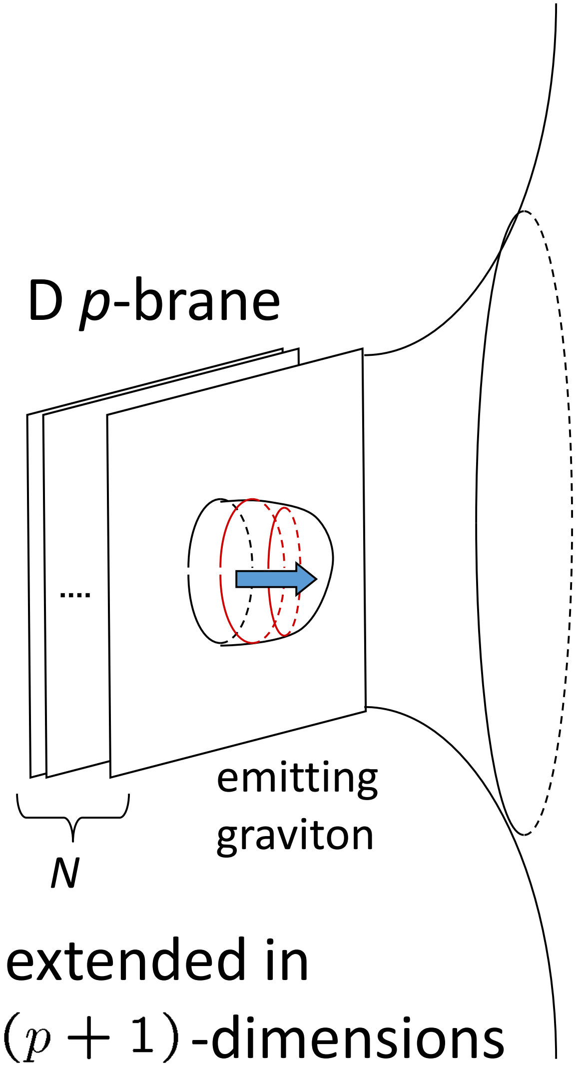
gauge particle appearing from an open string
with two ends on the i-th D-brane and the j-th D-brane

$$A_{\mu}{}^{ij}(x)$$

$$\mu = 0, \cdots, p$$
$$i, j = 1, \cdots, N$$
$$x \in \mathbf{R}^{(p+1)}$$

(p+1)-dim. SUSY U(N) gauge theory

# A stack of N D-branes can emit a closed string, and hence it can source a gravitational force



extended in
$(p+1)$-dimensions

low-energy effective theory
of closed strings
(10d SUGRA)

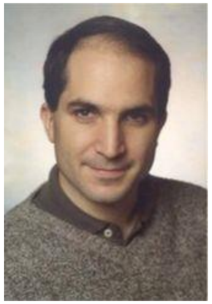
**black p-brane solution**

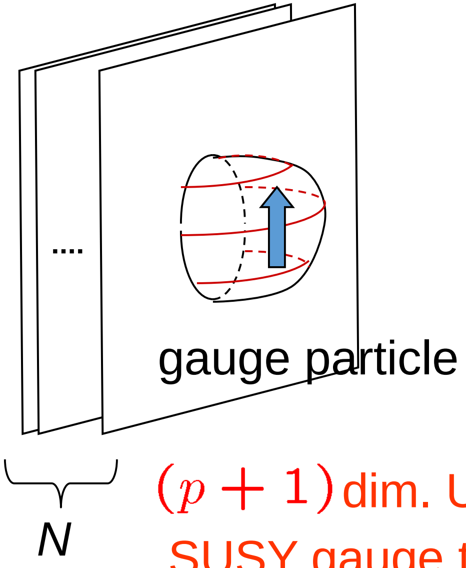
a classical solution to SUGRA
with (p+1)dim. translational inv.

# Gauge/gravity duality (conjecture)

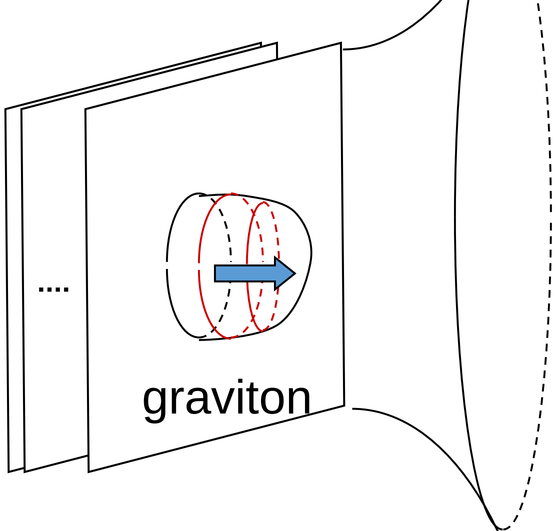
D $p$-branes extended in $(p+1)$-dimensions

Maldacena (1997)





two points of view for D-branes



$(p+1)$ dim. U(N) SUSY gauge theory = black p-brane solutoin

a classical solution in 10d SUGRA

't Hooft limit

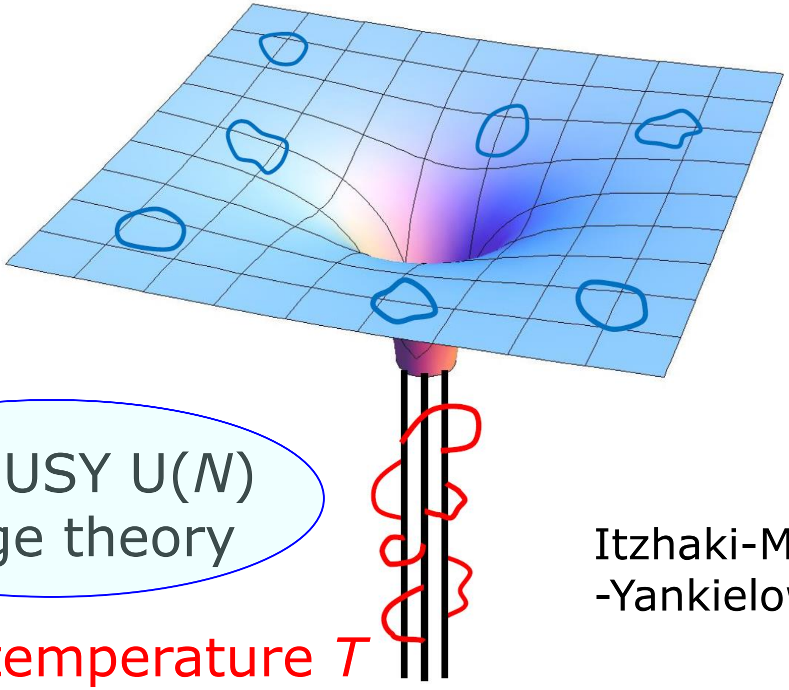
$\left( \begin{array}{l} N \to \infty \text{ limit} \\ \text{with } \lambda = g_{\mathrm{YM}}^2 N \text{ fixed} \end{array} \right)$ → string loop effects : negligible

strong coupling limit $\lambda \to \infty$ → the extent of strings: negligible

( $\alpha'$ corrections)

# 2. The gauge/gravity duality for the D0-brane system

# Testing gauge/gravity duality for D0-branes



1d SUSY U(*N*)
gauge theory

finite temperature *T*

0-brane solution in
10d SUGRA

Hawking temperature *T*

Itzhaki-Maldacena-Sonnenschein
-Yankielowicz ('98)

the low-energy limit is taken in such a way that the open string d.o.f. attached to the branes are decoupled from the closed string d.o.f. outside the horizon (decoupling limit)

Can they reproduce the thermodynamic properties of the black 0-brane solution ?

# 0-brane solution in 10d SUGRA (gravity side)

after taking the decoupling limit :

$$U \equiv \frac{r}{\alpha'} \quad , \quad \lambda \equiv g_s N \alpha'^{-3/2} \quad \text{(fixed)}$$

$$ds^2 = \alpha' \left\{ f(U) dt^2 + \frac{1}{f(U)} dU^2 + \sqrt{\lambda} U^{-3/2} d\Omega^2_{(8)} \right\}$$

$$f(U) \equiv \frac{U^{7/2}}{\sqrt{\lambda}} \left\{ 1 - \left( \frac{U_0}{U} \right)^7 \right\}$$

the validity region of SUGRA

$$N^{-10/21} \ll \left( \frac{U_0}{\lambda^{1/3}} \right)^{5/2} \ll 1$$

string loop effects negligible

the extent of strings (α’ corrections) negligible

# The validity region of the SUGRA

- curvature radius of space-time near horizon $\rho^2 \sim \left(\frac{\lambda^{1/3}}{U_0}\right)^{3/2} \alpha'$

In order for the $\alpha'$ corrections to be neglected :

$$\frac{\alpha'}{\rho^2} \sim \left(\frac{U_0}{\lambda^{1/3}}\right)^{3/2} \ll 1$$

- string coupling constant $g_{\text{str}} \sim \frac{1}{N}\left(\frac{U_0}{\lambda^{1/3}}\right)^{-21/4}$

In order for the string loop corrections to be neglected :

$$g_{\text{str}} \sim \frac{1}{N}\left(\frac{U_0}{\lambda^{1/3}}\right)^{-21/4} \ll 1$$

# Thermodynamic properties of the black 0-brane solution (gravity side)

$$\left\{\begin{array}{ll}\text{Hawking temperature :} & \dfrac{T}{\lambda^{1/3}} = \dfrac{7}{16\sqrt{15}\pi^{7/2}}\left(\dfrac{U_0}{\lambda^{1/3}}\right)^{5/2} \\[2ex] \text{Bekenstein-Hawking entropy :} & \mathcal{S} = \dfrac{1}{28\sqrt{15}\pi^{7/2}}N^2\left(\dfrac{U_0}{\lambda^{1/3}}\right)^{9/2}\end{array}\right.$$

$$\Rightarrow \quad \frac{1}{N^2}\frac{E}{\lambda^{1/3}} = \underbrace{\frac{9}{14}\left\{4^{13}15^2\left(\frac{\pi}{7}\right)^{14}\right\}^{1/5}}_{7.41}\left(\frac{T}{\lambda^{1/3}}\right)^{14/5}$$

Klebanov-Tseytlin (1996)

the validity region of SUGRA : $$N^{-10/21} \ll \frac{T}{\lambda^{1/3}} \ll 1$$

string loop effects negligible

the extent of strings (α’ corrections) negligible

# Low-energy effective theory of D0-branes (gauge theory side)

1d SUSY U(N) gauge theory

$$D = \partial_t - i\,[A(t),\ \cdot\ ]$$

$$S_{\rm b} = \frac{1}{g^2}\int_0^\beta dt\ {\rm tr}\left\{\frac{1}{2}(DX_i(t))^2 - \frac{1}{4}[X_i(t),X_j(t)]^2\right\}$$

$$S_{\rm f} = \frac{1}{g^2}\int_0^\beta dt\ {\rm tr}\left\{\frac{1}{2}\Psi_\alpha D\Psi_\alpha - \frac{1}{2}\Psi_\alpha(\gamma_i)_{\alpha\beta}[X_i,\Psi_\beta]\right\}$$

N × N Hermitian matrix

$$\begin{cases} X_j(t) & (j=1,\cdots,9) \\ \Psi_\alpha(t) & (\alpha=1,\cdots,16) \end{cases}$$

periodic b.c.

anti periodic b.c.

$T = \beta^{-1}$ temperature

$\lambda = g^2 N$ 't Hooft coupling const.

$$\lambda_{\rm eff} = \frac{\lambda}{T^3}$$

If we fix $\lambda$

- low temperature ⇨ strong coupling α' corrections negligible
- high temperature ⇨ weak coupling high T expansion

Kawahara-J.N.-Takeuchi ('07)

# 3. Testing the gauge/gravity duality in the large-N limit

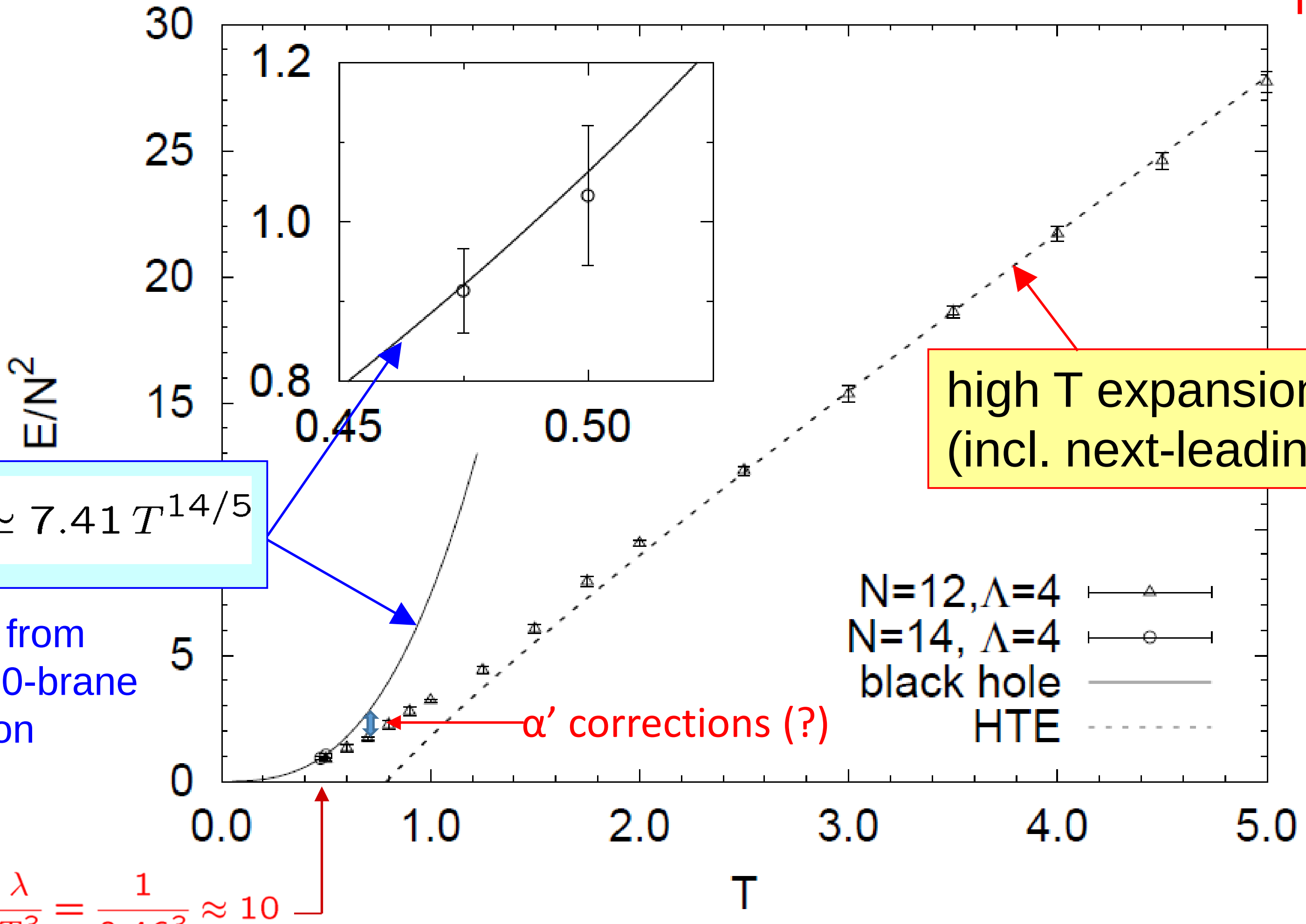
# Internal energy (gauge theory side)

$$E = \frac{\partial}{\partial \beta}(\beta \boxed{\mathcal{F}})$$

free energy

We set $\lambda = 1$

Anagnostopoulos-Hanada- J.N.-Takeuchi,
PRL 100 ('08) 021601 [arXiv:0707.4454]



$$\frac{E}{N^2} \simeq 7.41\, T^{14/5}$$

result from
black 0-brane
solution

high T expansion
(incl. next-leading order)

α' corrections (?)

$$\lambda_{\rm eff} = \frac{\lambda}{T^3} = \frac{1}{0.46^3} \approx 10$$

# $\alpha'$ corrections to the black 0-brane thermodynamics

black 0-brane solution : classical solution to SUGRA

$$\mathcal{S}_{(0)} = \frac{1}{16\pi G_\mathrm{N}} \int d^{10}x\sqrt{-g}\,\left\{\mathrm{e}^{-2\phi}(R + 4\partial_\mu\phi\partial^\mu\phi) - \frac{1}{4}G_{\mu\nu}G^{\mu\nu}\right\}$$

$$G_\mathrm{N} \sim \alpha'^4 g_s^2$$

the $O(\alpha'^0)$ terms in the low-energy effective action of superstring theory

(calculation of scattering amplitudes (at tree level) for the massless modes)

- 2pt and 3pt scattering amplitudes ⇨ $\mathcal{S}_{(1)} = \mathcal{S}_{(2)} = 0$
- 4pt amplitudes ⇨ $\mathcal{S}_{(3)} = \frac{\alpha'^3}{16\pi G_\mathrm{N}} \int d^{10}x\sqrt{-g}\,\left\{\mathrm{e}^{-2\phi}\mathcal{R}^4 + \cdots\right\}$

not know completely

Need to calculate the $\alpha'$ corrections to the black 0-brane solution and investigate its thermodynamics.

We can make a dimensional analysis using the fact that the corrections starts from $\alpha'^3$ .

# $\alpha'$ corrections to the black 0-brane thermodynamics (continued)

- curvature radius of space-time near horizon $$\rho^2 \sim \left(\frac{\lambda^{1/3}}{U_0}\right)^{3/2} \alpha'$$

- $\alpha'$ corrections appear as: $$\frac{\alpha'}{\rho^2} \sim \left(\frac{U_0}{\lambda^{1/3}}\right)^{3/2} \sim \left(\frac{T}{\lambda^{1/3}}\right)^{3/5}$$ $$\frac{T}{\lambda^{1/3}} \sim \left(\frac{U_0}{\lambda^{1/3}}\right)^{5/2}$$

- Corrections at the order of $\alpha'^3$

$$\begin{aligned} \frac{1}{N^2}\frac{E}{\lambda^{1/3}} &= 7.41 \left(\frac{T}{\lambda^{1/3}}\right)^{14/5} \left\{1 + c\left(\frac{T}{\lambda^{1/3}}\right)^{9/5}\right\} \\ &= 7.41 \left(\frac{T}{\lambda^{1/3}}\right)^{14/5} - C\left(\frac{T}{\lambda^{1/3}}\right)^{23/5} \end{aligned}$$

Hanada-Hyakutake-J.N.-Takeuchi, PRL 102 ('09) 191602

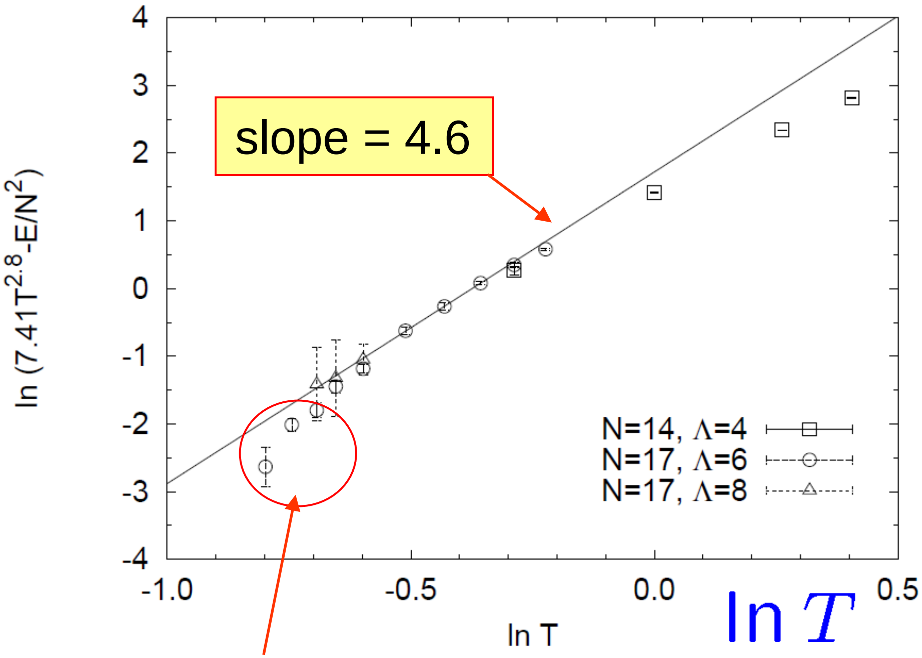
# First results on $\alpha'$ corrections

Hanada-Hyakutake-J.N.-Takeuchi,
PRL 102 ('09) 191602 [arXiv:0811.3102]

$$\boxed{\frac{E}{N^2} = 7.41\, T^{14/5} - C\, T^{23/5}}$$

$\alpha'$ corrections

$\ln \left( 7.41\, T^{14/5} - \frac{E}{N^2} \right)$

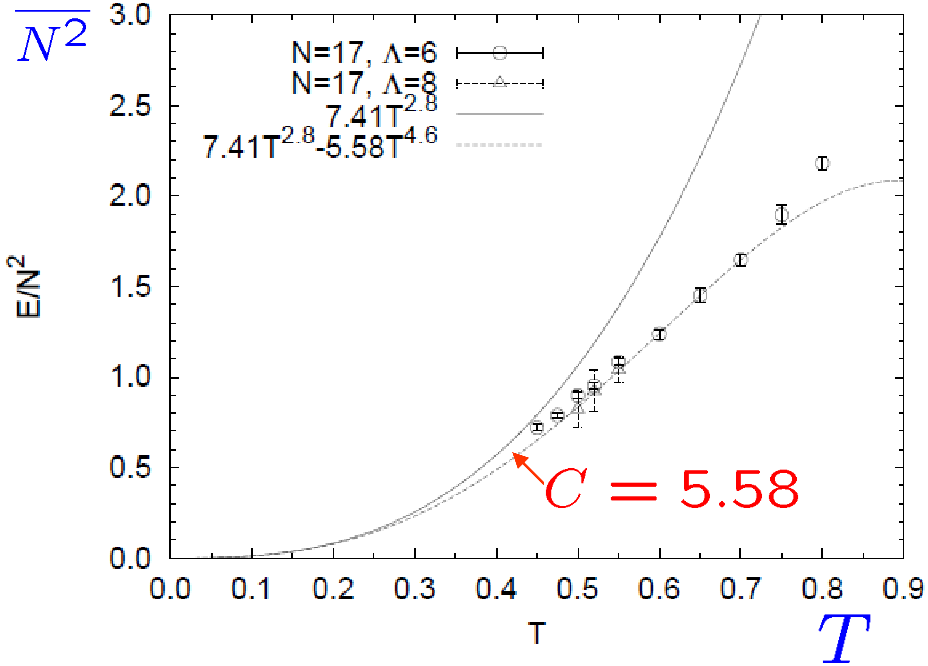


slope = 4.6

$\ln T$

finite UV cutoff effects

$\frac{E}{N^2}$



$C = 5.58$

$T$

Data points in $T \lesssim 0.7$
can be fitted well with $C = 5.58$

# Other groups working on this model

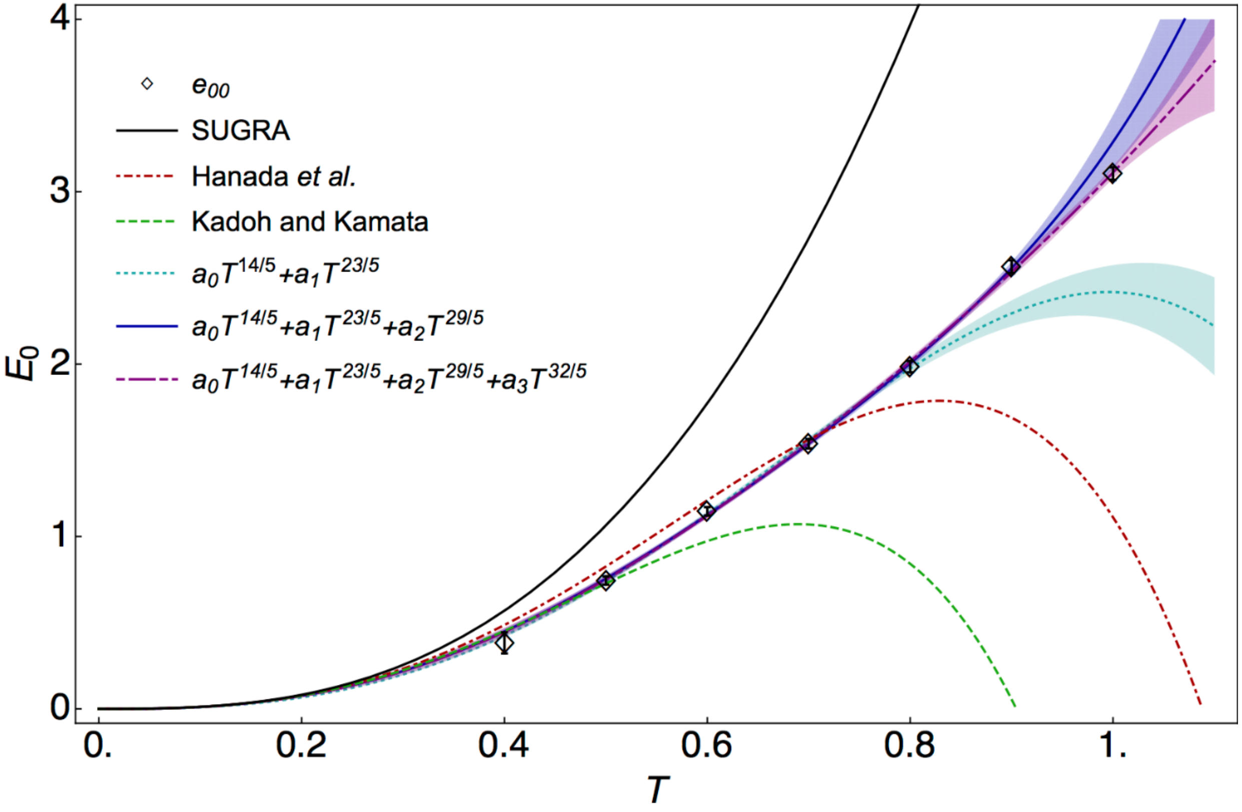
- Simon Catterall, Toby Wiseman,
  "Extracting black hole physics from the lattice",
  JHEP 1004 (2010) 077

- Daisuke Kadoh, Syo Kamata,
  "Gauge/gravity duality and lattice simulations of one dimensional SYM with sixteen supercharges"
  arXiv:1503.08499 [hep-lat]

- Vaselin G.Filev, Denjoe O'Connor,
  "The BFSS model on the lattice"
  JHEP 1605 (2016) 167

# Recent results on $\alpha'$ corrections

Berkowitz, Rinaldi, Hanada, Ishiki, Shimasaki, Vranas, 1606.04951 [hep-lat]



lattice ＋ Fourier accelaration

continuum limit, large-N limit

Fitting range : 0.5 ≦T ≦0.9

$$\frac{E}{N^2} = a_0 T^{14/5} - a_1 T^{4.6} + a_2 T^{5.8}$$

$$a_0 = 7.4(5),\ a_1 = 9.7(2.2),\ a_2 = 5.6(1.8)$$

# 4. Testing the gauge/gravity duality at finite N

Ref.)

M. Hanada, Y. Hyakutake, G. Ishiki, J.N., Science 344 (2014) 882-885

M. Hanada, Y. Hyakutake, G. Ishiki, J.N., arXiv:1603.00538 [hep-th]

# String loop corrections to black 0-brane thermodynamics

obtained form type IIA superstring theory

1-loop

$$g_{\mathrm{str}}{}^{2}\left(\frac{\alpha'}{\rho^{2}}\right)^{\textcircled{3}} \sim \frac{1}{N^{2}}\left(\frac{T}{\lambda^{1/3}}\right)^{-12/5}$$

Bern, Rozowsky, Yan

Bern, Dixon, Dunbar, Perelstein, Rozowsky

2-loop

$$g_{\mathrm{str}}{}^{4}\left(\frac{\alpha'}{\rho^{2}}\right)^{\textcircled{5}} \sim \frac{1}{N^{4}}\left(\frac{T}{\lambda^{1/3}}\right)^{-27/5}$$

Kawai-Lewellen-Tye relation to SYM amplitudes used

$$g_{\mathrm{str}} \sim \frac{1}{N}\left(\frac{T}{\lambda^{1/3}}\right)^{-21/10} \qquad \frac{\alpha'}{\rho^{2}} \sim \left(\frac{T}{\lambda^{1/3}}\right)^{3/5}$$

$$\frac{1}{N^{2}}\frac{E}{\lambda^{1/3}} = 7.41\left(\frac{T}{\lambda^{1/3}}\right)^{14/5}\left\{1+\frac{a}{N^{2}}\left(\frac{T}{\lambda^{1/3}}\right)^{-12/5}+\frac{b}{N^{4}}\left(\frac{T}{\lambda^{1/3}}\right)^{-27/5}\right\}$$

$$\sim 7.41\left(\frac{T}{\lambda^{1/3}}\right)^{14/5}+\frac{A}{N^{2}}\left(\frac{T}{\lambda^{1/3}}\right)^{2/5}+\frac{B}{N^{4}}\left(\frac{T}{\lambda^{1/3}}\right)^{-13/5}$$

$$A = -5.77$$

(Hyakutake, PTEP (2014) 033B04)

This may be tested by studying small *N* and low *T* region.

# Meta-stability at finite N

- Low energy effective theory of N D0-branes

$$S_{\mathrm{b}} = \frac{1}{g^2} \int_0^\beta dt \ \mathrm{tr} \left\{ \frac{1}{2} (DX_i(t))^2 - \frac{1}{4} [X_i(t), X_j(t)]^2 \right\}$$

$$S_{\mathrm{f}} = \frac{1}{g^2} \int_0^\beta dt \ \mathrm{tr} \left\{ \frac{1}{2} \Psi_\alpha D \Psi_\alpha - \frac{1}{2} \Psi_\alpha (\gamma_i)_{\alpha\beta} [X_i, \Psi_\beta] \right\}$$

Potential for $X_i(t)$

Vanishes for configurations satisfying $[X_i(t), X_i(t)] = 0$

The potential has flat directions.

- Viewed as a quantum mechanical system,
  the D0-brane bound state stabilizes only at large N.

⇨ At finite N, the bound state is only meta-stable.

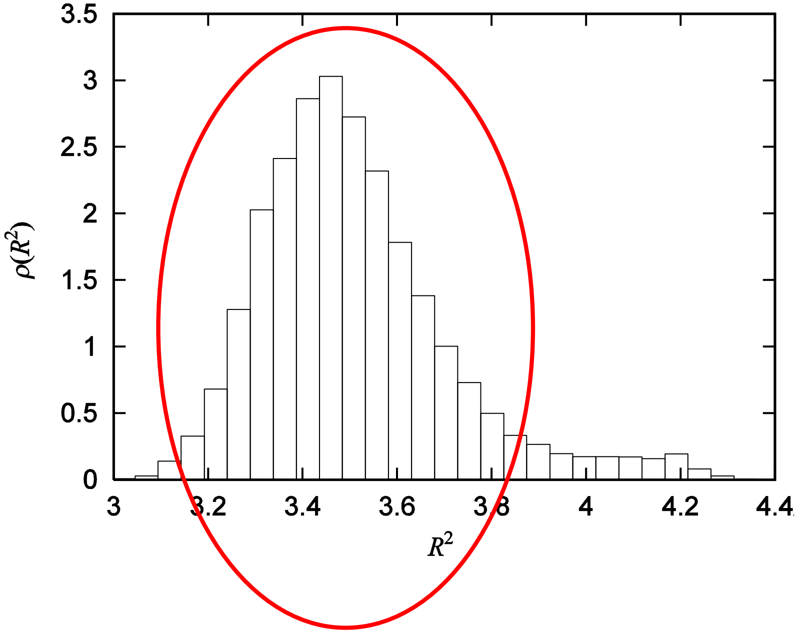
(suggested by numerical simulation)

⇨ Calculate the internal energy with a cutoff on the extent of the D0-branes, and look for a region in which the results do not depend on the cutoff.

# Identification of the meta-stable bound state and the measurement of internal energy

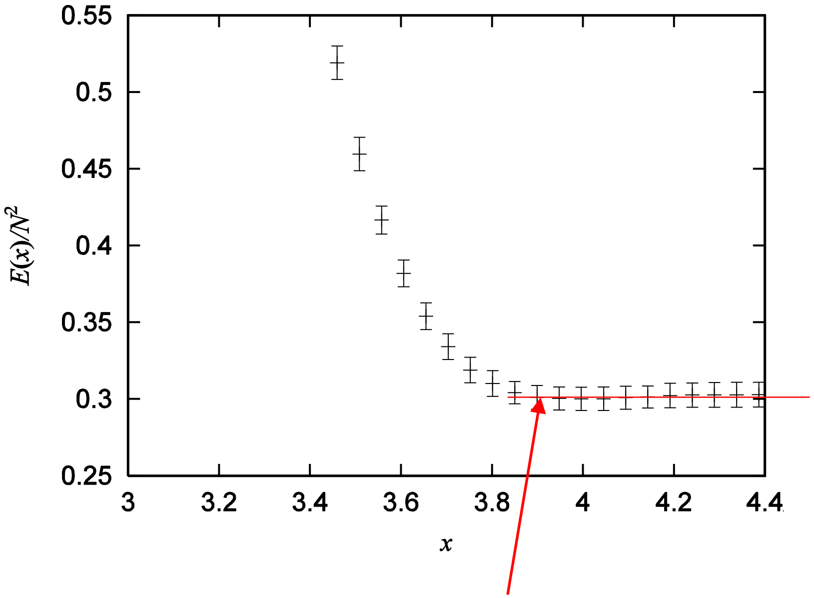
The extent of D0-branes : $R^2 = \frac{1}{N\beta}\int_0^\beta dt\ \mathrm{tr}X_i(t)^2$

histogram of $R^2$



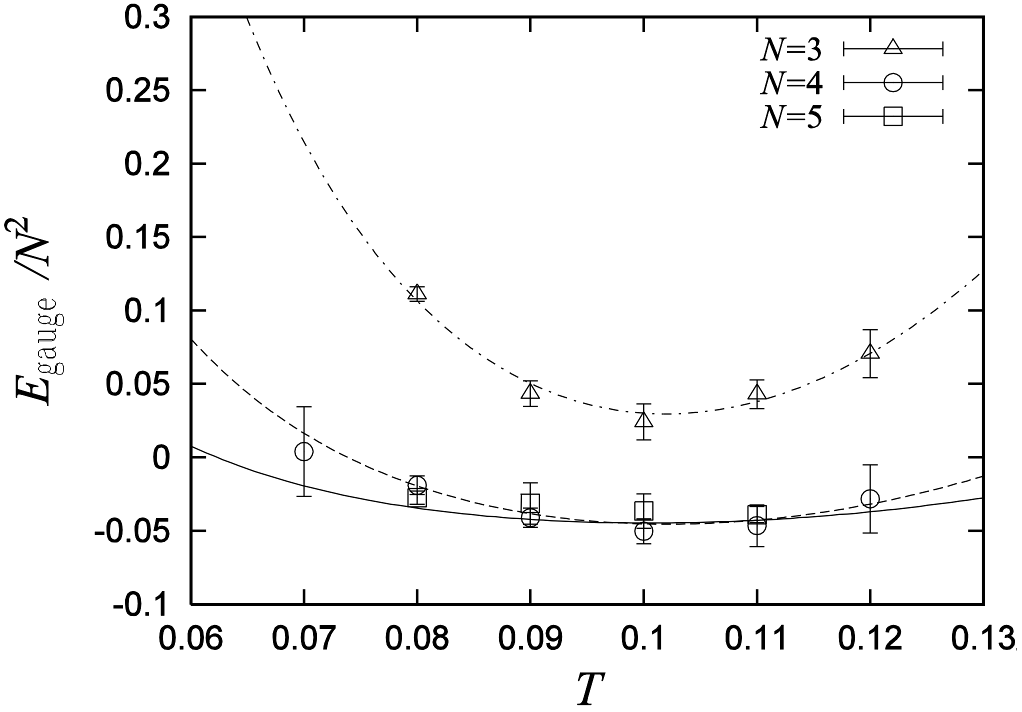
suggests the existence of a meta-stable bound state

internal energy obtained by restricting to $R^2 < x$



Evaluate the internal energy in the region where $x$ dependence disappears.

# Results for the internal energy of the meta-stable bound state



- N dependence is clearly visible.
- The internal energy starts growing at low T

negative specific heat

# Testing gauge/gravity duality including the string loop corrections

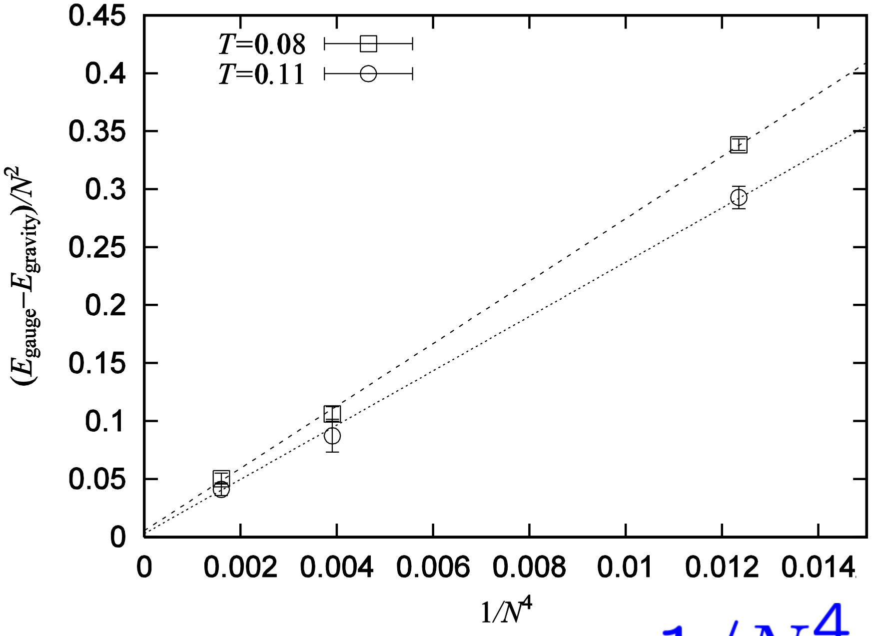
Hanada-Hyakutake-Ishiki-J.N., Science 344 (2014) 882

arXiv:1603.00538 [hep-th]

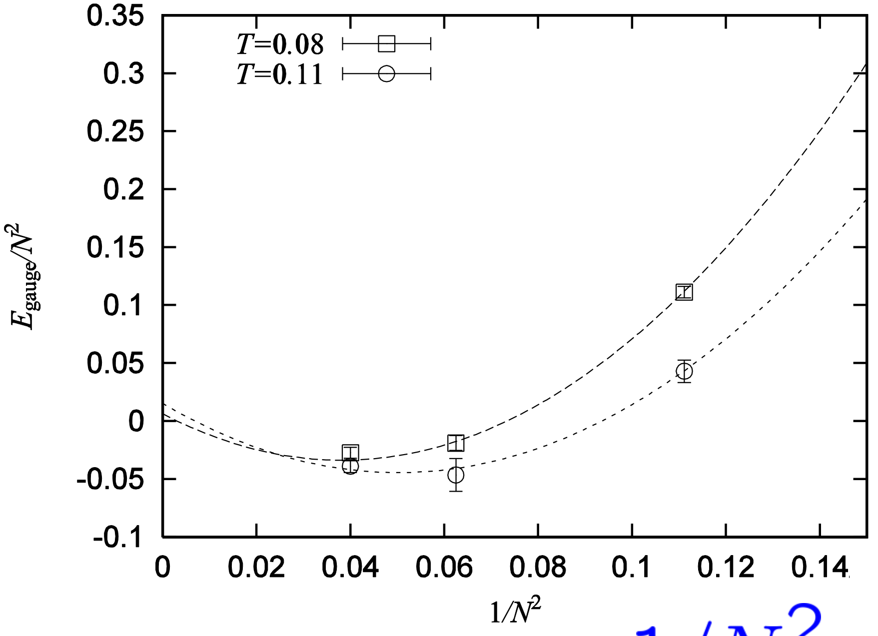
$$\frac{1}{N^2}E_{\text{gravity}} = 7.41\,T^{\frac{14}{5}} - \frac{5.77}{N^2}T^{\frac{2}{5}}$$

$$\frac{1}{N^2}(E_{\text{gauge}} - E_{\text{gravity}})$$

$$\frac{1}{N^2}E_{\text{gauge}}$$



$1/N^4$



$1/N^2$

Fitted to

$$\frac{1}{N^2}E_{\text{gauge}} = 7.41\,T^{\frac{14}{5}} - \frac{5.77T^{\frac{2}{5}}}{N^2} + \frac{\text{const.}}{N^4}$$

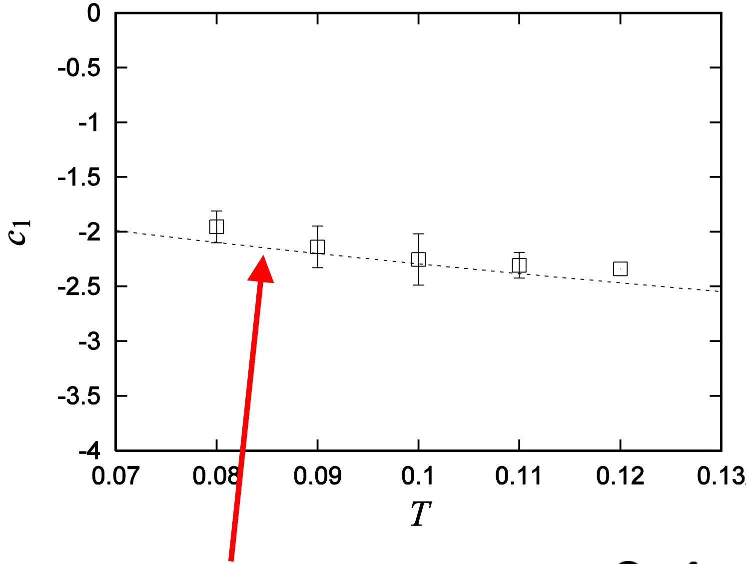
# Testing gauge/gravity duality including the string loop corrections (continued)

Hanada-Hyakutake-Ishiki-J.N., Science 344 (2014) 882

arXiv:1603.00538 [hep-th]

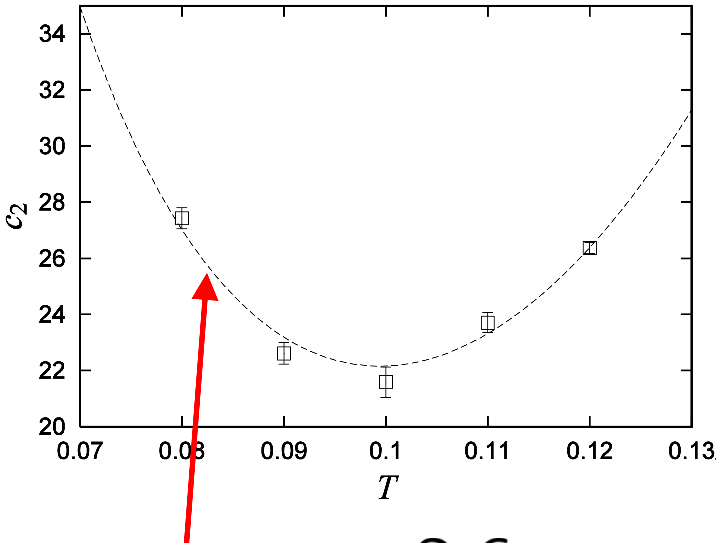
$$\frac{1}{N^2}E_{\text{gauge}} = 7.41\,T^{\frac{14}{5}} + \frac{c_1}{N^2} + \frac{c_2}{N^4}$$

$c_1$



$$c_1 = -5.76\,T^{0.4}$$

$c_2$



$$c_2 = c\,T^{-2.6} + \tilde{c}\,T^{p}$$

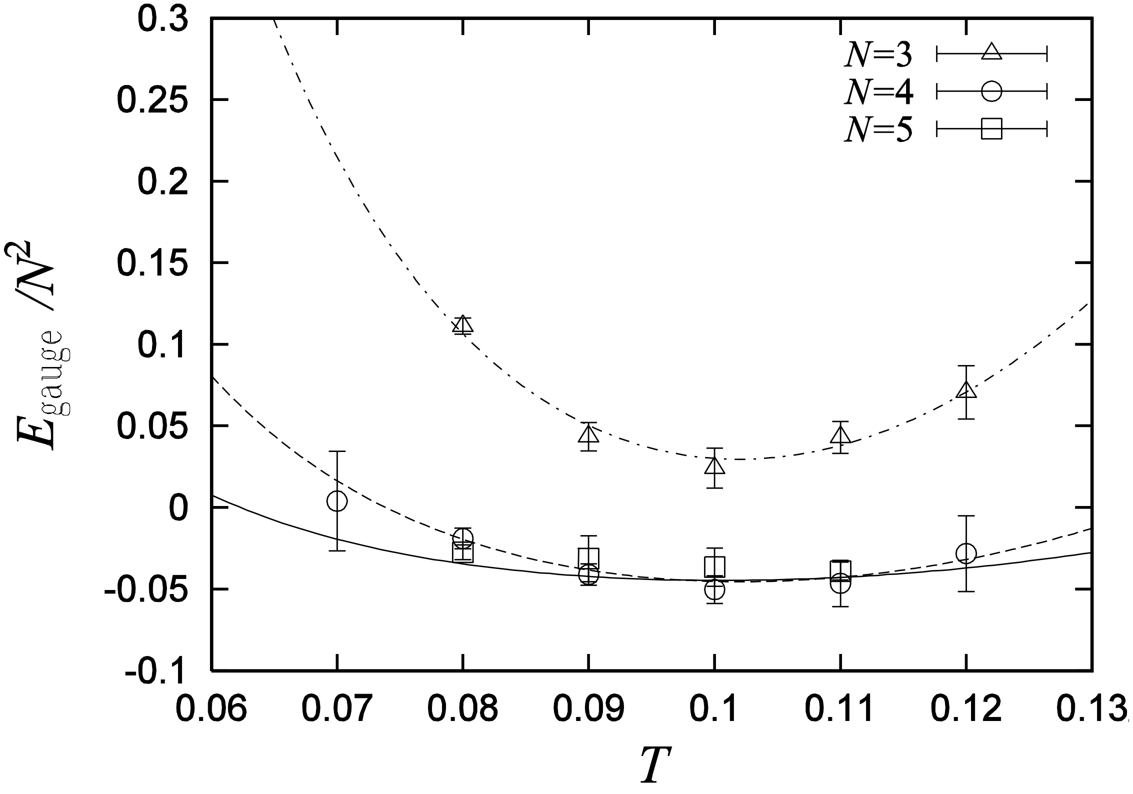
$c = 0.032(2),$

$\tilde{c} = 0.51(61) \times 10^5,\ p = 3.7(6)$

# Testing gauge/gravity duality including the string loop corrections (continued)

Hanada-Hyakutake-Ishiki-J.N., Science 344 (2014) 882
arXiv:1603.00538 [hep-th]

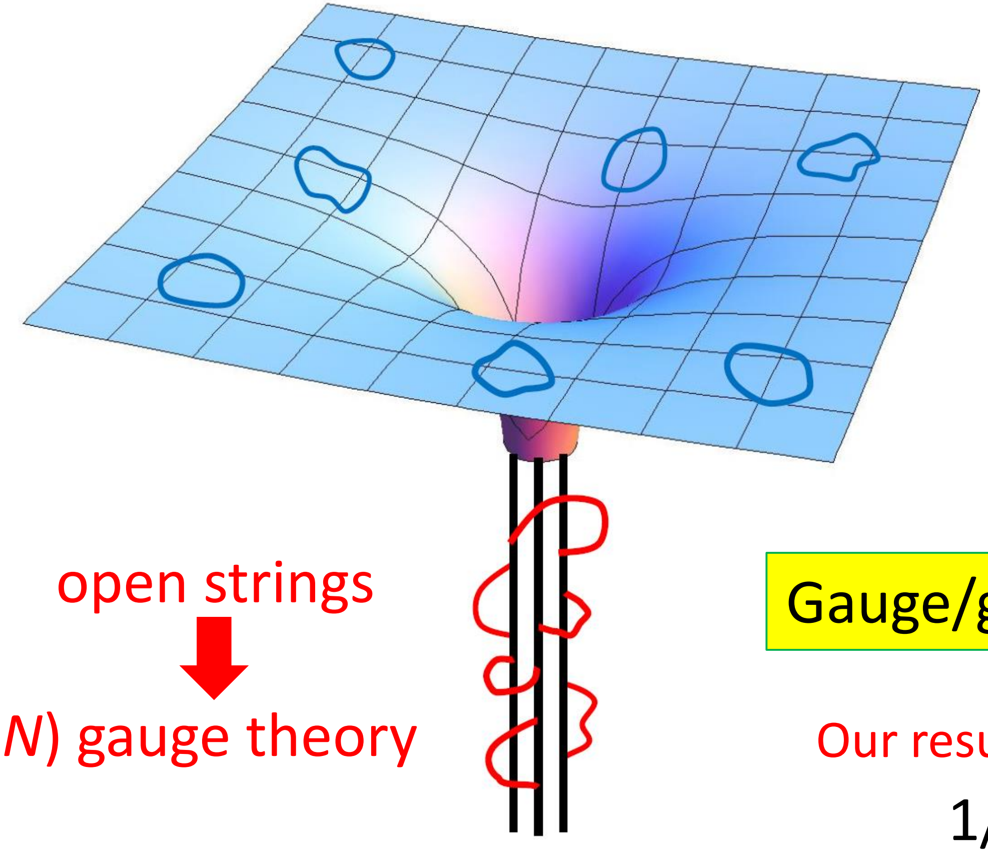
$$\frac{1}{N^2}E_{\text{gauge}} = \frac{1}{N^2}E_{\text{gravity}} + \frac{c\,T^{-2.6} + \tilde{c}\,T^p}{N^4}$$



consistent with string loop corrections.

# 5. Summary and future prospects

# Summary



closed strings
↓
gravity theory

open strings
↓
U(*N*) gauge theory

*N* D-branes

**Gauge/gravity duality (conjecture)**

Our results for *N*=3,4,5 confirm :

1/*N* corrections
= string loop corrections

Gauge theory at finite *N* may be
a nonperturbative formulation of
string theory (quantum gravity)

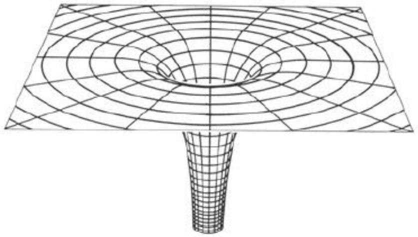
# Future prospects

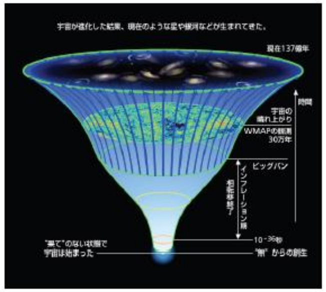
- At $T < T_{\mathrm{c}} = 0.574\, N^{-5/9}$, on the gravity side the Gregory-Laflamme transition occurs and 11d Schwarzschild black hole appears.

  Can this be reproduced on the gauge side ?
- BFSS conjecture (Banks-Fischler-Shenker-Susskind ’96)

  The same 1d SUSY gauge theory describes M theory nonperturbatively.

  Is this true ?
- Is it also possible to describe the beginning of the universe holographically ?





IKKT matrix model
(Ishibashi-Kawai-Kitazawa-Tsuchiya ’97)

Emergence of
(3+1)d expanding Universe

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