The meson spectrum in large-N QCD

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Holography, CFT & Lattice

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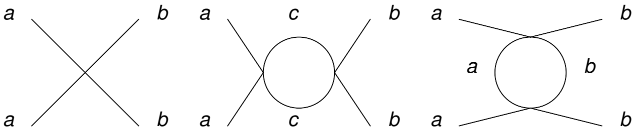
Outline

- Large-N QCD: motivation
- Lattice simulation: details and techniques
- Results
  - Chiral logs and meson masses
  - NP renormalization, chiral condensate, decay constants
  - Spectrum
  - Continuum limit
- N counting for mesons, glueballs, tetraquarks
- Conclusions

Example: scalar field theory with N-component field ϕ^a, $a = 1, \ldots, N$

$$\mathscr{L} = \frac{1}{2}\partial_\mu\phi^a\partial^\mu\phi^a - \frac{1}{2}\mu^2\phi^a\phi^a - g^2(\phi^a\phi^a)^2\,.$$

We define the 't Hooft coupling $\lambda = g^2N$:



$g^2 = \frac{\lambda}{N}$ $\qquad$ $g^4N = \frac{\lambda^2}{N}$ $\qquad$ $g^4 = \frac{\lambda^2}{N^2}$

Now we take the limit $g^2 \to 0$ and $N \to \infty$ at fixed λ ('t Hooft limit).
Obviously, this leads to simplifications!

Large-N QCD

SU(N) Lagrangian (from now on Euclidean spacetime):

$$\mathscr{L} = N\left[\frac{1}{4\lambda}F^a_{\mu\nu}F^a_{\mu\nu} + \bar{\psi}\,(\not{D}+m)\,\psi\right] \qquad (\psi \mapsto \sqrt{N}\,\psi)\,.$$

Counting rules:

$$\left.\begin{array}{ccc} \text{corner} & \text{each vertex} & \propto N \\ \text{edge} & \text{each propagator} & \propto 1/N \\ \text{face} & \text{closed color loop} & \propto N \end{array}\right\} \quad \Rightarrow \quad \langle\cdot\rangle \propto N^{V-E+F} = N^{\chi}$$

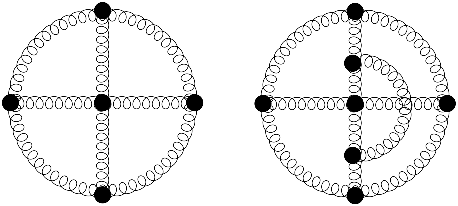
$\chi = V - E + F = 2 - 2h$(andles) $- b$(oundaries, holes) is the

Euler characteristic.

sphere: $h = b = 0 \Rightarrow \chi = 2$, torus: $h = 1, b = 0 \Rightarrow \chi = 0$.

Consequences of counting rules

- Only "planar" diagrams survive at large N.



planar non-planar

- The leading connected vacuum diagrams are of order N^2 (planar graphs made of gluons only).
- The leading connected vacuum diagrams with quark lines are of order N.
- Corrections are suppressed by factors $1/N^2$ in the pure gauge theory and by N_f/N in the theory with N_f fermions.

Some properties of large-N QCD

- Sea quark effects $\propto 1/N \Rightarrow$ The $N = \infty$ limit is "quenched".
- Meson decay widths $\propto 1/N \Rightarrow$ mesons do not decay at $N = \infty$.
- Glueballs, $\bar{q}q$, $(\bar{q}q)^2$ etc. states naively decouple.
- OZI rule exact at $N = \infty$.

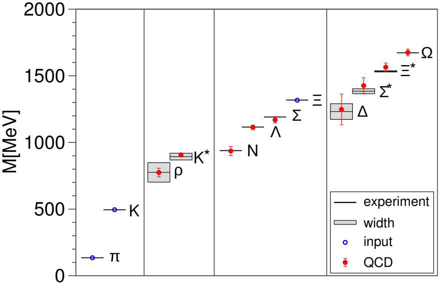
Is $N = \infty$ close to $N = 3$ QCD?

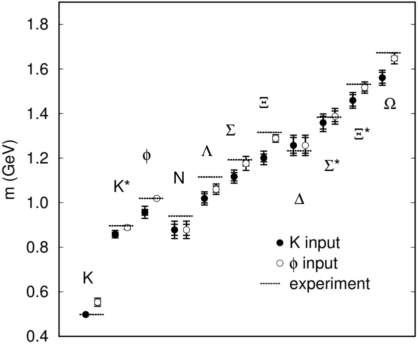
AdS/CFT starts from $N = \infty$. Also many simplifications in chiral EFT!

But even $N = \infty$ QCD is far from being solved!

Light hadrons: $1/N^2 = 1/9 \overset{?}{\ll} 1$, $N_f/N = 3/3 \overset{?}{\ll} 1$

If $1/9 \approx 0$ then $\exists$ evidence that $1 \approx 0$:



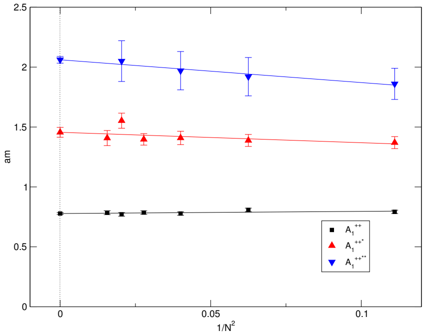


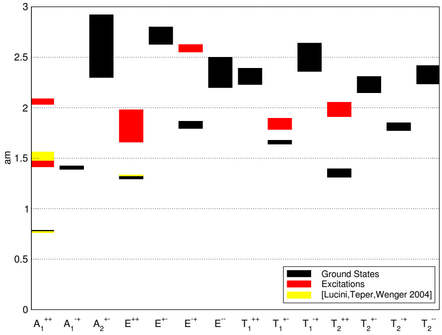
Full SU(3) QCD BMW-c: S Dürr et al 08

Quenched SU(3) PACS-CS: S Aoki et al 02

Obviously cannot work for flavour singlets $(f_0(500), \eta', \omega)$ but still ...

Glueballs at large-N





from B Lucini, A Rago, E Rinaldi 10 $a^{-1} \approx 1.5$ GeV

What about mesons?

Lattice parameters

- Volumes:

N	vol
2,3	$16^3 \times 32$, $24^3 \times 48$, $32^3 \times 64$
4,5,6,7	$24^3 \times 48$
17	$12^3 \times 24$

- 200 configs for each N and volume (80 configs for $N = 17$)
- lattice spacing $a \approx 0.093\,\text{fm}$
- pion mass as low as $m_\pi \approx 230\,\text{MeV}$
- Wilson gluon and quark actions

GB, F Bursa, L Castagnini, S Collins, L Del Debbio, B Lucini, M Panero, JHEP 1306 071

Recently added

- 3 additional lattice spacings
- Non-perturbative renormalization

Matching the scale

- Inverse coupling $\beta = 2N/g^2 = 2N^2/\lambda$ is fixed by imposing $a\sqrt{\sigma} \approx 0.2093$ for all $\mathrm{SU}(N)$. (Lattice spacing $a \approx 0.093$ fm is kept constant in units of the string tension $\sigma \approx 1\,\mathrm{GeV/fm}$).
- Other possible choices include $aT_c =$ const, $aF/\sqrt{N} =$ const, etc.
- The κ-parameter ($2am_q = \kappa^{-1} - \kappa_c^{-1}$) is adjusted so that our set of pseudoscalar masses matches between different N (achieved by exploratory simulations).

Plan:

- Vary κ to study $m_A(m_q, N), f_A(m_q, N)$ for each meson A.
- Extrapolate to $N = \infty$ and study $1/N^2$ corrections.
- Repeat at different lattice spacings a and perform a combined $a \to 0$, $N \to \infty$ extrapolation.

Couplings used in main set of configs

N	2	3	4	5	6	7	17
β	2.4645	6.0175	11.028	17.535	25.452	34.8343	208.45
λ	3.246	2.991	2.901	2.851	2.829	2.813	2.773

$\lambda = Ng^2 = 2N^2/\beta$.

`A Hietanen et al, PLB 674(09)80`:
SU(17) at $\beta = 208.08$ (We have slightly smaller a).

Strong/weak coupling transition at coarse $\sqrt{\sigma}a \gtrsim 1.2 \gg 0.2093$.

Deconfinement "transition" (similar to finite-T) at $\sqrt{\sigma}N_s a \lesssim 2$.

In principle one could take $N \to \infty$ at $\lambda =$ const. (rather than keeping $a\sqrt{\sigma} =$ const.) but:

- SU(3) at $\lambda = 2.773$ ($\beta \approx 6.47$) requires $N_s \gtrsim 20$.
- SU(17) is very coarse at $\lambda = 2.991$.
- It is always nicer to work at "constant" physics.

Axial and vector Takahashi-Ward identity masses

Partially conserved axial current (AWI):

$$\sum_{\mathbf{x}} \partial_4 \langle 0|A_4(\mathbf{x},t)|\pi\rangle = 2m_{\mathrm{AWI}} \sum_{\mathbf{x}} \langle 0|P(\mathbf{x},t)|\pi\rangle \quad \text{where} \quad \begin{cases} A_\mu(x) &= \bar{u}(x)\gamma_\mu\gamma_5 d(x) \\ P(x) &= \bar{u}(x)\gamma_5 d(x) \end{cases}$$

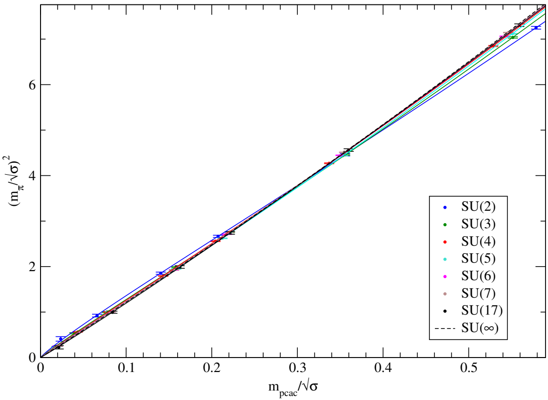
Fit:

$$am_{\mathrm{AWI}} = \frac{Z_P}{Z_A Z_S}(1 + bam_q)\underbrace{\frac{1}{2}\left(\frac{1}{\kappa} - \frac{1}{\kappa_c}\right)}_{am_q}$$

Fit parameters (for each N): $Z = Z_P/(Z_A Z_S), b, \kappa_c$.

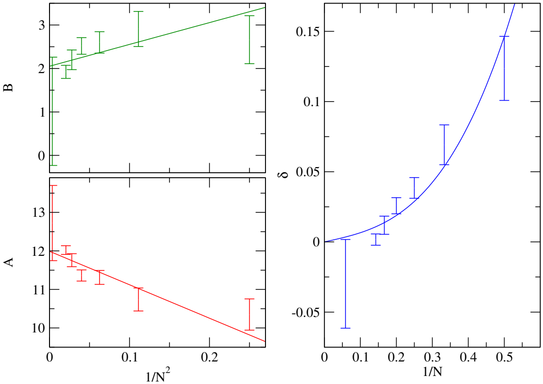
SU(3): $Z \approx 0.75$ ($\beta = 6.0175$)
[agrees with independent determination 0.81(7) at $\beta = 6$
V Giménez et al NPB531 (98) 429]

Pion mass squared vs. AWI quark mass



$$\frac{m_\pi^2}{\sigma} = A\left(\frac{m_{\rm AWI}}{\sqrt{\sigma}}\right)^{\frac{1}{1+\delta}} + B\,\frac{m_{\rm AWI}^2}{\sigma}$$

Pion mass: $1/N^2$ fit of the parameters



$$A = 11.99(0.10) - \frac{8.7(1.6)}{N^2}$$

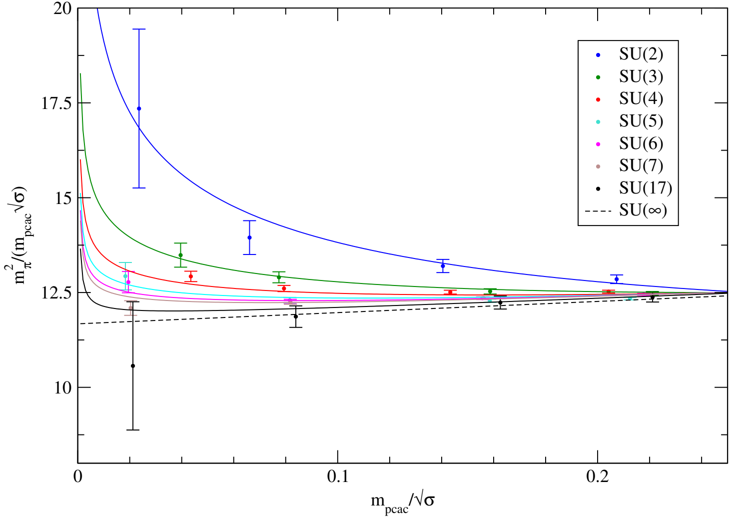
$$B = 2.05(0.13) + \frac{5.0(2.2)}{N^2}$$

$$\delta = \frac{0.056(19)}{N} + \frac{0.94(21)}{N^3}$$

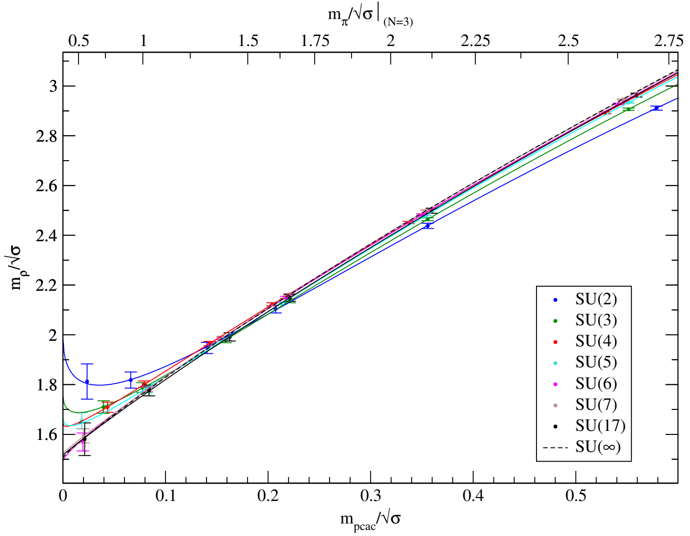
Expectation (S Sharpe PRD 46 (92) 3146):

$\delta = c_1/N + c_2/N^3 + \cdots$.

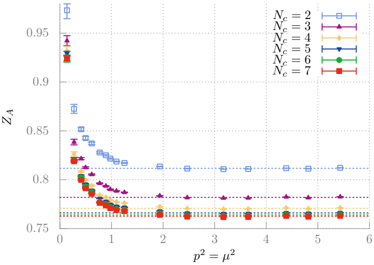
Quenched chiral logs

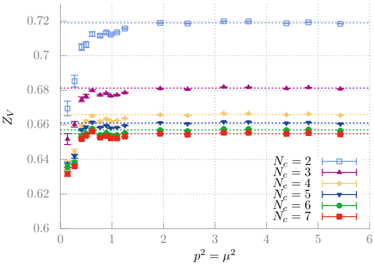


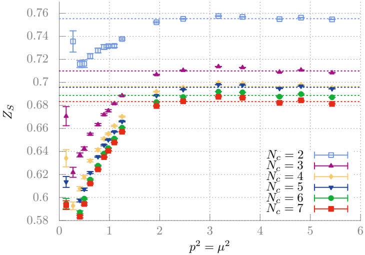
ρ vs. AWI mass



Non-perturbative renormalization I





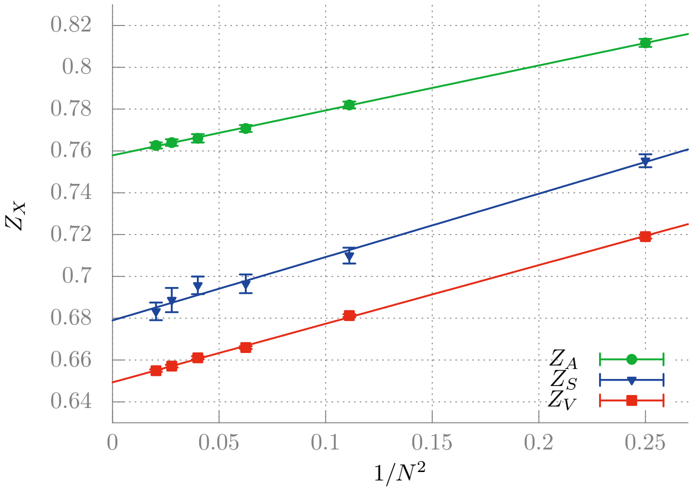


Matching to RI'MOM scheme

$$Z_S = Z_S^{\overline{\mathrm{MS}}}(2\,\mathrm{GeV})$$

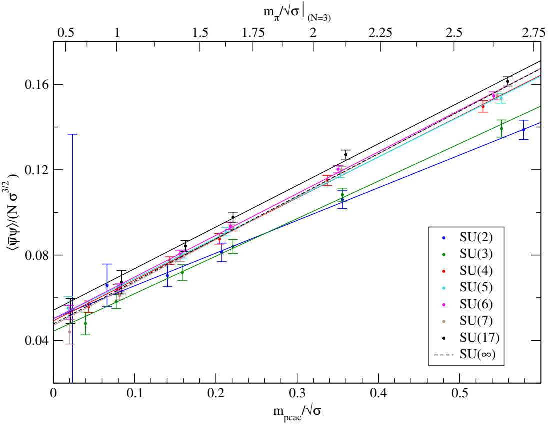
$$Z_P(\mu) = Z_A Z_S(\mu) Z$$

Non-perturbative renormalization II

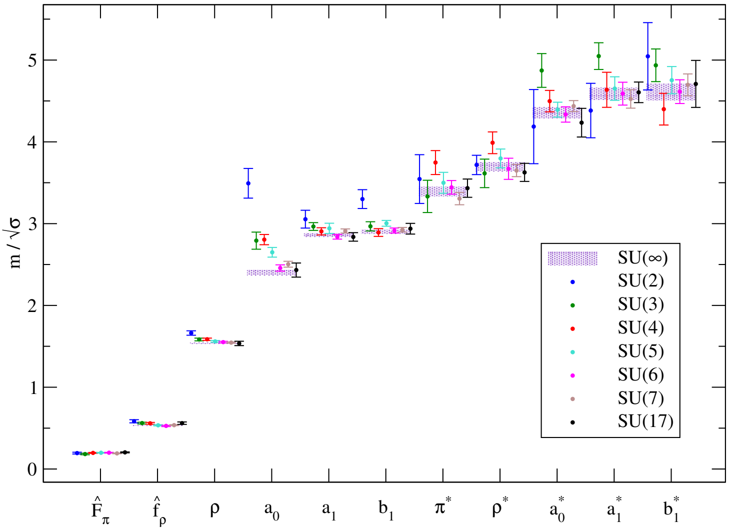


Chiral condensate

$$\left|\langle\bar{\psi}\psi\rangle^{\overline{\rm MS}}(\mu = 2\,\text{GeV})\right| = Z_S^{\overline{\rm MS}}(\mu, a) \lim_{m_q\to 0} \frac{F_\pi^2(m_q)m_\pi^2(m_q)}{2m_q} \propto N$$



Meson spectrum at $m_q = 0$, $a \approx 0.093$ fm



Scale setting and fixing quark masses

Definitions:

$$f_X = \sqrt{2}\,F_X\,, \quad F = F_\pi(m_q = 0)\,, \quad \hat{F} = \sqrt{\frac{3}{N}}F$$

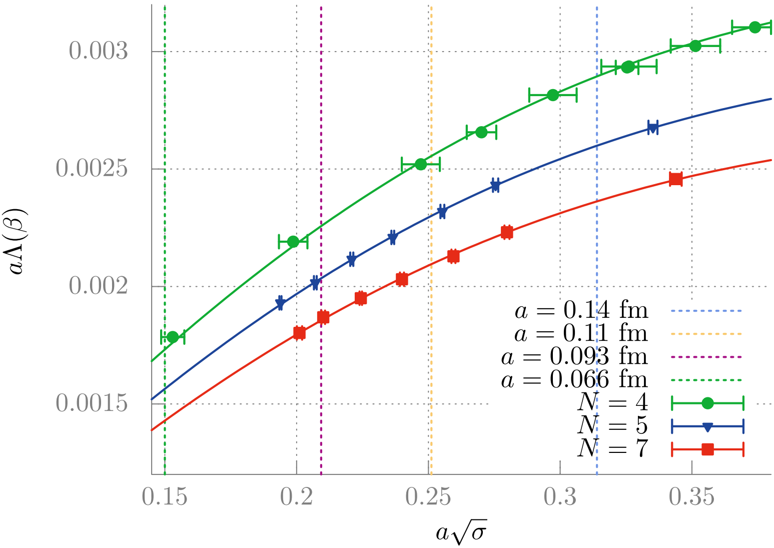
Real world value: 85.8(1.2) MeV.

Set scale using $F = 85.8$ MeV instead of $\sqrt{\sigma} = 1$ GeV/fm ≈ 444 MeV.
(We will see that for $a \to 0$ and $N \to \infty$ this gives $\sqrt{\sigma} \approx 436$ MeV)

Then set m_{ud}, m_s at $N = \infty$, requiring

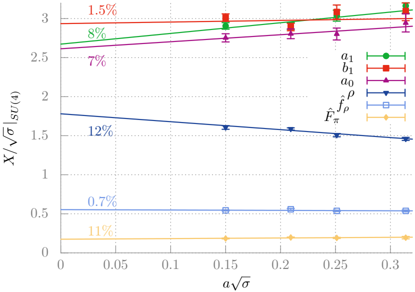
$m_\pi(m_{ud}) = m_{\pi^0} \approx 135$ MeV
$m_\pi(m_s) = (m_{K^\pm}^2 + m_{K^0}^2 - m_{\pi^\pm}^2)^{1/2} \approx 686.9$ MeV.

Continuum limit taken for SU(4), SU(5), SU(7)

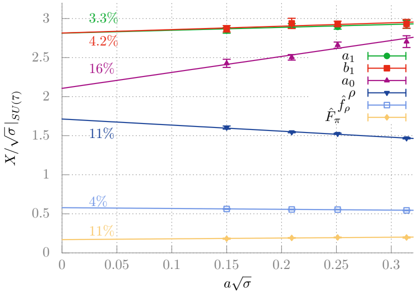


Continuum limit II

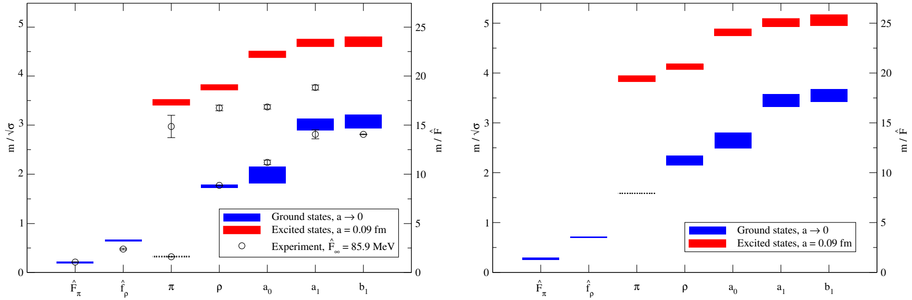
SU(4)



SU(7)



$N = \infty$ spectrum at $m_q = m_{ud}, m_s$



$\sqrt{\sigma} \approx 5.08\,\hat{F} = 436\,\mathrm{MeV}$, $\hat{F}_\pi/\hat{F} = 1.018(17)$, FLAG: 1.0744(67)

$$\hat{\Sigma} := \frac{3}{N}|\langle\bar{\psi}\psi\rangle^{\overline{\mathrm{MS}}}(2\,\mathrm{GeV})| = [2.70(13)\hat{F}]^3 = [232(11)\,\mathrm{MeV}]^3$$

real world QCD ($N = 3$ with sea quarks): FLAG: $[271(15)\,\mathrm{MeV}]^3$
$m_{\rho,\phi} = 753(14), 981(44)\,\mathrm{MeV}$ vs. experimental values $775, 1019\,\mathrm{MeV}$

Comparison with AdS/CFT models

AdS/CFT

J Babington et al
PRD 69 (04) 066007

$$\frac{M_\rho(M_\pi)}{M_\rho(0)} \simeq 1 + 0.307 \left[\frac{M_\pi}{M_\rho(0)}\right]^2$$

Holographic model

T Sakai, S Sugimoto
hep-th/0412141

$$\frac{M^2_{a_1(1260)}}{M^2_\rho} \simeq 2.4$$

$$\frac{M^2_{\rho(1450)}}{M^2_\rho} \simeq 4.3$$

$$\frac{M^2_{a_0(1450)}}{M^2_\rho} \simeq 4.9$$

Our lattice results

$$\frac{M_\rho(M_\pi)}{M_\rho(0)} \simeq 1 + 0.360(64) \left[\frac{M_\pi}{M_\rho(0)}\right]^2$$

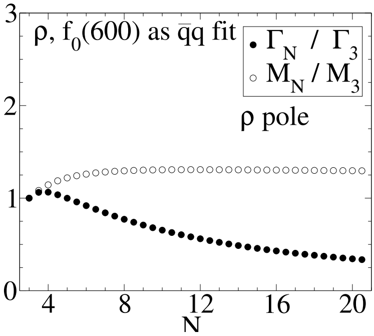
$$\frac{M^2_{a_1(1260)}}{M^2_\rho} = 3.0(2)$$

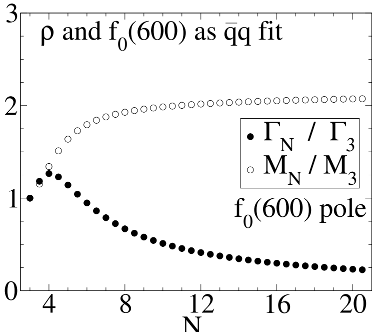
$$\frac{M^2_{\rho(1450)}}{M^2_\rho} \simeq 4.8(2)$$

$$\frac{M^2_{a_0(1450)}}{M^2_\rho} \simeq 7.7(3)$$

Comparison with phenomenology

$N = \infty$ is a good starting point for studies of strong decays and mixing between different sectors: glueballs, mesons, (meson)^2 etc.





Prediction for full $N \geq 3$ QCD from unitarized χPT from
J Peláez, G Ríos PRL 97 (06) 242002
See also J Nieves et al, PRD 84 (11) 096002

Glueballs, mesons, molecules, tetraquarks

Standard N counting conventions

- Each closed colour loop: N.
- Vacuum state $|0\rangle$: 1.
- (Non-flavour singlet) mesons $\hat{\pi}^\dagger|0\rangle$: $1/\sqrt{N}$
  (so that $\langle\pi|\pi\rangle = \langle 0|\hat{\pi}\hat{\pi}^\dagger|0\rangle = \mathcal{O}(1)$).
- Glueballs $|G\rangle = \hat{G}^\dagger|0\rangle$: $1/N$ (then $\langle GG\rangle = \mathcal{O}(1)$).
- Molecules/tetraquarks $\widehat{\bar{q}q\bar{q}q}|0\rangle$: naively $1/N$ but...
- Pion decay constant $\langle 0|\bar{d}\gamma_\mu\gamma_5 u|\pi^+\rangle = \sqrt{2}F_\pi p_\mu = \mathcal{O}\left(\sqrt{N}\right)$.
- Chiral condensate $\langle 0|\bar{\psi}\psi|0\rangle = \mathcal{O}(N)$.

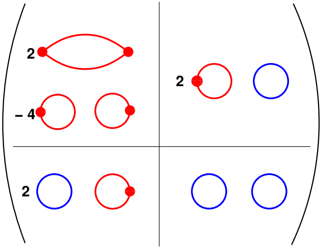
This is why we defined above

$$\hat{F} = \lim_{N\to\infty}\sqrt{\frac{3}{N}}F(N) = 85.8\,\mathrm{MeV}\,,$$

$$\hat{\Sigma} = \lim_{N\to\infty}\frac{3}{N}|\langle\bar{\psi}\psi\rangle^{\overline{\mathrm{MS}}}(2\,\mathrm{GeV})| = [232(11)\,\mathrm{MeV}]^3.$$

Counting subtleties

Mixing of singlet $\bar{q}q$ with glueball ($N_f = 2$): Wick contractions



$$= \begin{pmatrix} \# + \# N & \# \sqrt{N} \\ \# \sqrt{N} & \# \end{pmatrix}$$

Mixing between flavour-singlet mesons and glueballs is governed by

$$\frac{C_{12}}{\sqrt{C_{11} C_{22}}} = \frac{\# \sqrt{N}}{\sqrt{\# + \# N}} \stackrel{N \to \infty}{\longrightarrow} \mathcal{O}(1)$$

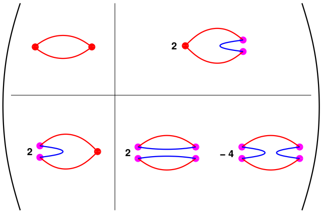
Glueballs and singlets become the same! This is not surprising:

$$\propto \left[\cancel{e^{-m_\pi t}} + \left(N e^{-m_{\eta_1} t} - \cancel{e^{-m_\pi t}} \right) \right]$$

Cancellation since $m_\pi < m_{\eta_1}$. Therefore we should normalize $\hat{\eta}_1 |0\rangle \propto 1/N$.

But what component dominates other singlet mesons at $N = 3$?

Mixing of $\bar{u}d$ with $\bar{u}q\bar{q}d$ ($N_f = 2$):



$$= \begin{pmatrix} \# & \#/\sqrt{N} \\ \#/\sqrt{N} & \# + \#/N \end{pmatrix}$$

Now

$$\frac{C_{12}}{\sqrt{C_{11} C_{22}}} = \mathcal{O}(1/\sqrt{N}) \quad \text{or} \quad \mathcal{O}(1),$$

depending on whether quark line disconnected diagram (with mass $m_{\pi\pi} = 2m_\pi + \mathcal{O}(1/N)$) or connected diagram dominates (Weinberg).
Assume state with $m_{4q} < 2m_\pi + \mathcal{O}(1/N)$ exists. Then connected diagram

$$\propto e^{-m_{4q} t}$$

will dominate. Is this the case for $a_0 \leftrightarrow K\overline{K}$, $D_{s0} \leftrightarrow DK$ etc?
$\Rightarrow$ Calculation of $N = \infty$ tetraquark states should be interesting.
NB: for singlet states $\bar{q}q$/tetraquark/glueball all mix at $\mathcal{O}(1)$ and the leading diagram topologically resembles the glueball propagator.

Conclusions

- $N = \infty$ is a good starting point to study strong decays and mixing between different quark model sectors: glueballs, mesons, "mesons^2". Phenomenology of light scalars?
- At $N = \infty$ a connection can be made to AdS/QCD models.
- We computed the quenched meson spectrum of SU(N) for degenerate quark masses and extrapolated the results to $N = \infty$.
  This limit is the same for the theory with sea quarks!
- Isovector SU(3) masses, decay constants and chiral condensate are close to the $N = \infty$ limit.
- $1/N^2$ corrections are small for $N = 3$.
- The fact that many mass ratios also differ by less than 10 % between the $N_f = 2 + 1$ theory and the quenched approximation indicates that N_f/N corrections may often be small too.
- Nonperturbative renormalization enabled the determination of the chiral condensate and decay constants.