# Recent results on large N gauge theories on the lattice

A. González-Arroyo

Instituto de Física Teórica UAM/CSIC
Departamento de Física Teórica, UAM

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[1] Talk given in Edinburgh June 2016

# Introduction:Presenting the ingredients

- **Gauge theories** are essential parts of the Standard Model and many of its candidate extensions (QCD-like theories)
- Short-distance properties and quantities can be studied by perturbation theory
- Understanding its properties in the non-perturbative domain remains a challenge

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- ♣ **1/N expansion**: Physical Quantities admit expansion in inv. powers of # of colors $N$, with re-scaled coupling $g^2 \longrightarrow \frac{\lambda}{N}$ *('t Hooft 1974)*
- ♣ In **P.T.** $(1/N^2)^g$ contributions come from diagrams drawn in genus $g$ surfaces. **String Theory?**
- ♣ Qualitatively reproduces the features seen in QCD (N=3)

# The large N limit of QCD

Lowest term in the expansion

| Properties | |
|---|---|
| Asymptotic freedom | |
| Dimensional transmutation | |
| Confinement | |
| Chiral Symmetry breaking | |
| Chiral P.T. | |
| Topological charge | |
| U(1) problem | |
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| Confinement $\surd$ | Factorization |
| Chiral Symmetry breaking $\surd$ | No dynamical quarks (Quenched) |
| Chiral P.T. $\surd$ | No chiral logs |
| Topological charge $\surd$ | No instantons |
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| Difficult To solve $\surd$ | **OOPS!** |

# Large N limit on the lattice

★ It is possible to combine these two non-perturbative approaches

One can use different methodologies:

- Study the theory on the lattice for different values of $N$ and **extrapolate** results to $N = \infty$.
  **Some possible problems:** Those associated with extrapolation. Other related to quenching vs non-quenching (order of limits $m_q \longrightarrow 0$ and $N \longrightarrow \infty$).
- Use **volume independence**: Drastic reduction of degrees of freedom.
  **Possible problems:** Validity of the concept. Survival at continuum limit. Size and type of finite N corrections.

# Short Review of Volume independence

**Volume independence**

Expectation values of Wilson loops become volume independent in the large N limit *(Eguchi-Kawai 1982)*

- Proof based on SD Eqs. asumming
  1. Factorization $\langle \mathrm{Tr}(U)\mathrm{Tr}(U')\rangle = \langle \mathrm{Tr}(U)\rangle\langle \mathrm{Tr}(U')\rangle$
  2. $Z_N^4$ symmetry $W(\mathcal{C}) = 0$ if winding$(\mathcal{C}) \neq 0$
- Volume independence ⇒ **Eguchi-Kawai model: LGT in one point**

$$S = -\frac{1}{\lambda_0} N \sum_{\mu,\nu} \mathrm{Tr}(U_\mu U_\nu U_\mu^\dagger U_\nu^\dagger)$$

- $Z_N^4$ symmetry is broken spontaneously for small $\lambda_0$ (cont. limit):
  $\mathrm{Tr}(U(l)) \neq 0$ (eigenvalues attract) *(Bhanot,Heller, Neuberger)*

# Volume independence

Original Reduction proof breaks down at weak coupling for pure gauge theories.

**Several Possible solutions have been proposed:**

- **Quenched EK model (*Bhanot, Heller, Neuberger*)** The eigenvalues of the link variables are quenched.
- **Twisted EK model (*A.G-A, Okawa*)** Use twisted boundary conditions (discrete chromo-magnetic flux).
- **Double trace deformations** added to the action (*Unsal-Yaffe*)
- **Adding fermions in the adjoint rep.** with cut-off scale masses (*Kovtun-Unsal-Yaffe*)
- **Partial reduction (*Narayanan-Neuberger*)** Reduction only applies beyond a certain length scale $l > l_c$

Reduction could still work for other large N theories. Most notably in the presence of fermions in the adjoint rep.

# The twisted Eguchi-Kawai model

- ♠ E-K proof valid for **PBC** and **TBC**
- ♠ For small $\lambda_0$ (Weak coupling), behaviour is very different

$$U(\mathcal{C} + \hat{l}_\nu) = e^{i2\pi\omega_\mu(\mathcal{C}) n_{\mu\nu}/N} U(\mathcal{C})$$

Frustrates constant Polyakov lines ($\omega_\mu(\mathcal{C}) \neq 0$).

- ♣ **TEK model** *(GA, Okawa 1983)*

Put TBC for $\Omega_\mu(n) = \Gamma_\mu$, change variables to $V_\mu = U_\mu \Gamma_\mu$ and shrink the lattice to one point:

$$S = -\frac{1}{\lambda_0} N \sum_{\mu,\nu} z_{\mu\nu} \mathrm{Tr}(V_\mu V_\nu V_\mu^\dagger V_\nu^\dagger)$$

with $z_{\mu\nu} = e^{2\pi i n_{\mu\nu}/N}$.

A nice compatible choice (**symmetric twist**):

$N = \hat{L}^2 \qquad z_{\mu\nu} = \exp\{i2\pi k\epsilon_{\mu\nu}/\hat{L}\} \qquad k, \hat{L}$ coprime

**Is the TEK matrix model equivalent to Yang-Mills theory?**

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- **Tests at weak coupling:** One can study this regime in perturbation theory.

# Weak coupling behaviour of the TEK model

At large $b = 1/\lambda_0$ the Path integral is dominated by the minima of the action: $V_\mu = \Gamma_\mu$ with

$$\Gamma_\mu \Gamma_\nu = z_{\nu\mu} \Gamma_\nu \Gamma_\mu$$

The set of solutions: $\Gamma_\mu \longrightarrow z_\mu \Omega \Gamma_\mu \Omega^\dagger$.
A choice of classical vacuum breaks the $Z_N^4$ symmetry down to $Z_{\hat{L}}^4$:
**Enough to preserve volume independence at large N**
Wilson loops $W(C)$ are given by

$$W(\mathcal{C}) = \frac{1}{N} \mathrm{Tr}(U(\mathcal{C})) = Z(\mathcal{C}) \frac{1}{N} \mathrm{Tr}(V(\mathcal{C}))$$

Plugging the minimum action sol. $\Rightarrow W(C) = 1$
$( \mathrm{Tr}(V(\mathcal{C})) = Z^*(\mathcal{C}))$

# Weak coupling behaviour of the TEK model

Beyond lowest order: $V_\mu = e^{-i\lambda_0 A_\mu}\Gamma_\mu$

**A nice basis of the Lie algebra**

$$\lambda^a \longrightarrow \lambda(\vec{p}) \quad / \qquad \delta_\mu \lambda(\vec{p}) = e^{ip_\mu}\lambda(\vec{p})$$

where $\delta_\mu$ is translation by 1 period ($\vec{p} = 2\pi n/\hat{L}$)

**Propagator is the same as in an $\hat{L}^4$ lattice**

Finite N corrections look like finite volume corrections.

**Vertices** are like in ordinary theory with

$$f_{abc} \longrightarrow f(\vec{p},\vec{q},\vec{l}) = \delta(\vec{p}+\vec{q}+\vec{l})\exp\{i\hat{L}\bar{k}\theta_{\mu\nu}p_\mu q_\nu/(2\pi)\}$$

---

- ★ Phases cancel for planar diagrams: **PT** same as ordinary theory on $\hat{L}^4$ box
- ★ An overall phase remains for non-planar diagrams: at large $N$ oscillatory cancelations kill contributions

Continuum version (A.G-A-Korthals Altes) ⇒ QFT on non-commutative space

# Perturbation theory with Periodic or Twisted BC

Rectangular ($R \times T$) Wilson loops $W$ can be computed in P.T.

$$W(b, N, L, k) = \hat{W}_0 - \sum_{j=1} \hat{W}_j(N, L) \frac{1}{b^j}$$

We have studied $\hat{W}_i$, with $i = 0, 1, 2$ (*Garcia Perez, AGA, Okawa 2016*)

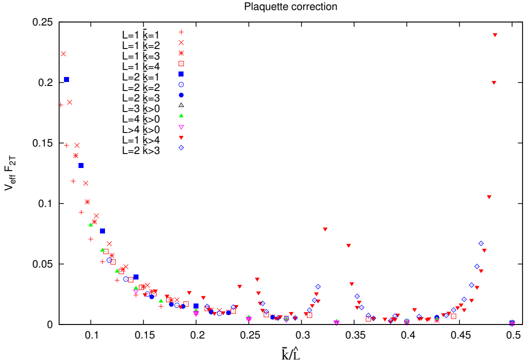
- Leading order dominated by flat connections: $\hat{W}_0 = 1$ (For P.B.C. :**TORONS**)
- First order:
  $\hat{W}_1^{\mathrm{TBC}}(N, L) = \hat{W}_1^{\mathrm{PBC}}(\infty, L\hat{L}) - \frac{1}{N^2} \hat{W}_1^{\mathrm{PBC}}(\infty, L) \sim \hat{W}_1(N, \infty) + \mathcal{O}(\frac{1}{L^6 N^2})$
- Second order:
  The general expression for TBC is:

$$\hat{W}_2^{\mathrm{TBC}}(N, L) = \hat{W}_2^{\mathrm{PBC}}(\infty, L\hat{L}) + \frac{1}{N^2} \delta\hat{W}_P + \delta\hat{W}_{NP}$$

where $\delta\hat{W}_P / \delta\hat{W}_{NP}$ comes from planar/non-planar diags.

# Non Planar contribution

The non-planar contribution $\delta \hat{W}_{NP}$ goes to zero as $1/(L\hat{L})^4$ with a coefficient depending on $\bar{k}/\hat{L}$:



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# Tests at strong coupling

The most ubiquous and well measured lattice quantity is the **expectation value of the plaquette**

$$E = \frac{1}{N}\langle \mathrm{Tr}(U(P))\rangle$$

We measured this quantity at large N by extrapolation (with a second degree polynomial in $1/N^2$) of results in a $L^4 = 16^4$ lattice and $N = 8 - 16$ at various couplings (*AGA, Okawa*).

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**Do we need to extrapolate?**

For $L = 16$ $N = 8$: **2.5 %** ; For $L = 16$ $N = 16$: **0.2 %**
$L = 8$ $N = 16$: **0.7 %** (**volume matters for PBC**)

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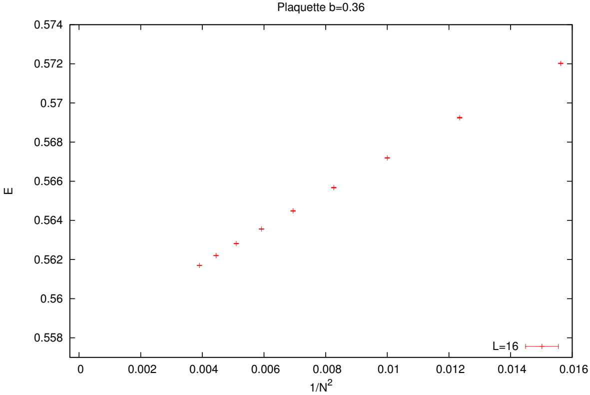

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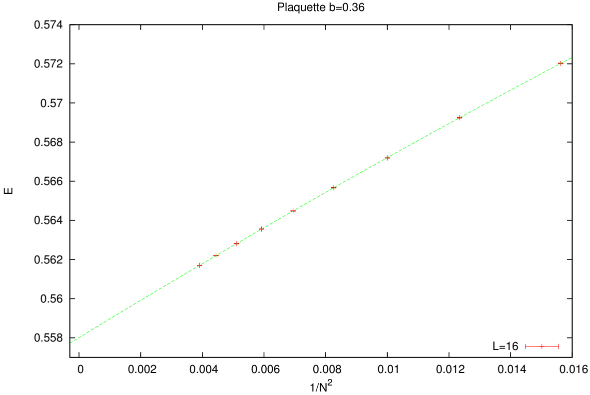

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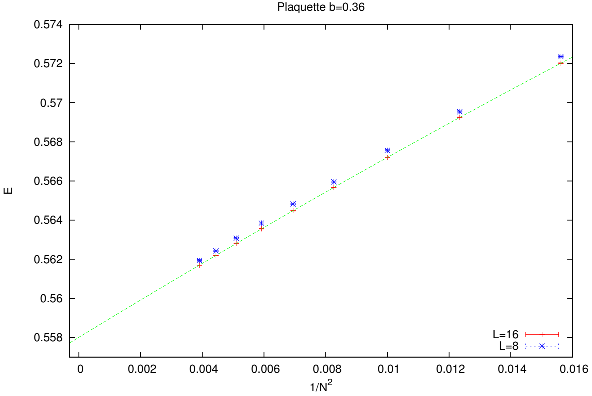

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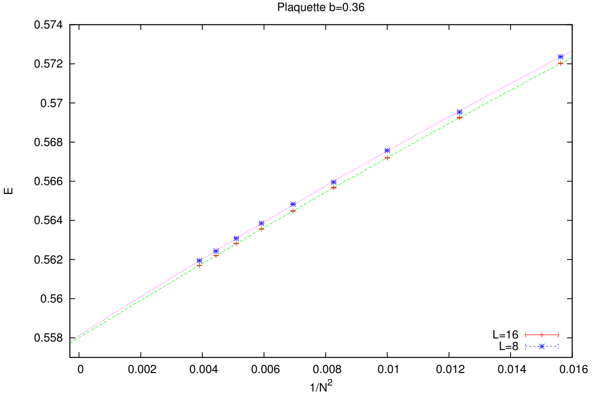

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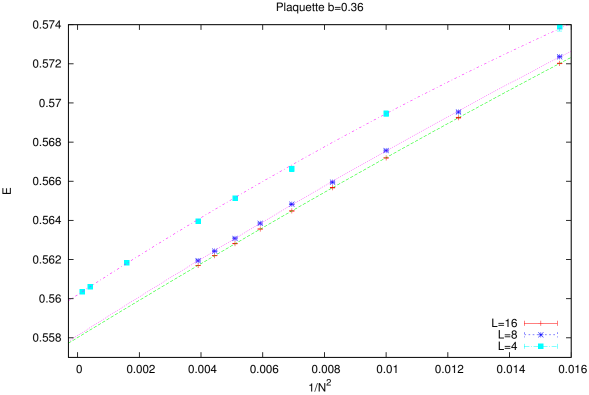

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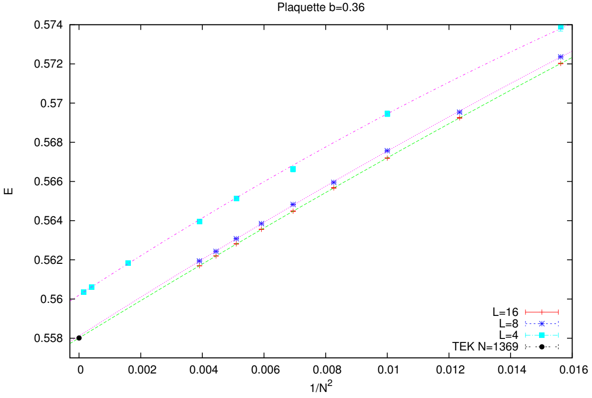
And twisted boundary conditions $k \neq 0$ (TEK $L = 1$)

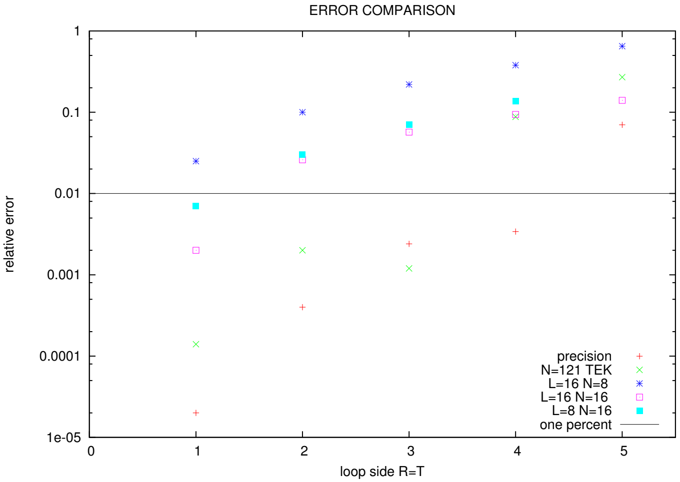


# Precision comparison

## Who needs such a small error?

# Precision comparison

**Who needs such a small error?** For other square loops one has:



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- **Tests in the deep non-perturbative region:** Test volume independence of observables by comparing with values obtained by extrapolation. Gives an idea of corrections.
- **Intermediate coupling region:** We did not find any problems provided one scales the fluxes $k$ and $\bar{k}$ as N goes to $\infty$. Tests show $k/N > 0.1$, $\bar{k}/N > \epsilon$ *(AGA-Okawa 2010)*. No symmetry breaking up to $N = 1369 = 37^2$

# Volume independence in the continuum limit

**Does reduction survive the continuum limit?**

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**How do errors compare?**

Wilson loops of size $R \times T$ at different $b$ values:

**Scaling** $\Rightarrow$ they depend on $r = a(b)R$ and $t = a(b)T$

To be precise we use the *Force*:

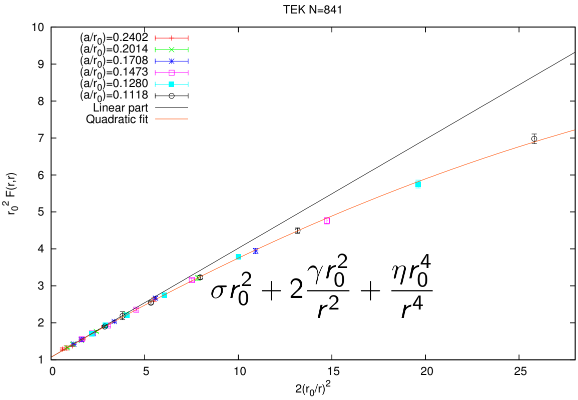
$$F(r,t) = -\frac{\partial \log W(r/a(b), t/a(b))}{\partial r \partial t}$$

We fix the scale $r_0$ (a la Sommer) as

$$r_0^2 F(r_0, r_0) = 1.65$$

# Scaling Plot

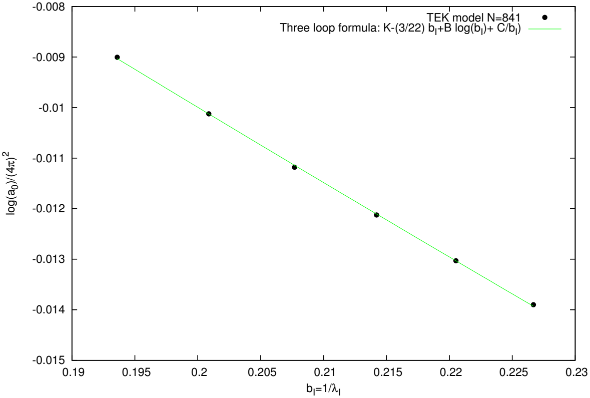
## Good Scaling behaviour:



# Beta function

The scale parameter follows the two loop formula with $\lambda_I$

$$\log(a/r_0) = K - \frac{1}{2\beta_0\lambda_0} - \frac{\beta_1}{2\beta_0^2}\log(\lambda_0) - \frac{(\beta_2\beta_0 - \beta_1^2)}{2\beta_0^3}\lambda_0 + \dots$$



# Large N physical quantities: The string tension

Determination using extrapolation and volume independence.

1. Extract String tension from Wilson loops for $N = 3, 4, 5, 6$ and 8 with $L^4 = 32^4$ and the TEK Model $N = 841$.
2. Scale results towards the continuum limit.
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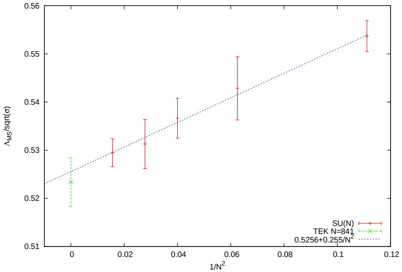
   Linear fit in $1/N^2$

$$\Lambda_{\overline{\rm MS}}/\sqrt{\sigma} = 0.525(2) \quad \text{for TEK} \quad 0.523(5)$$



# Meson spectrum

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- TEK is a pure gauge theory: In the large N limit: The meson spectrum $\equiv$ pure gauge observable (no dynamical fermions)
- Masses are computed from correlators at different points; How can you think of space-time correlators in a 1-point box?
- Twist is topological property of the pure gauge theory (gauge group $SU(N)/Z(N)$). How can you introduce quarks in the fundamental representation?

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**CLUE**: Quark fields propagate in space in a background gauge field periodic modulo twist. Consistent choice for fermion box size $= \hat{L}^4$.

# The formula

Meson operators: $\mathbf{O}_\Gamma(x) = \bar{\Psi}(x)\Gamma\Psi(x)$

$$C_{\Gamma\Gamma'}(t) = \int d^3\vec{x}\langle \mathbf{O}_\Gamma(0)\mathbf{O}_\Gamma(t,\vec{x})\rangle$$

After some derivation

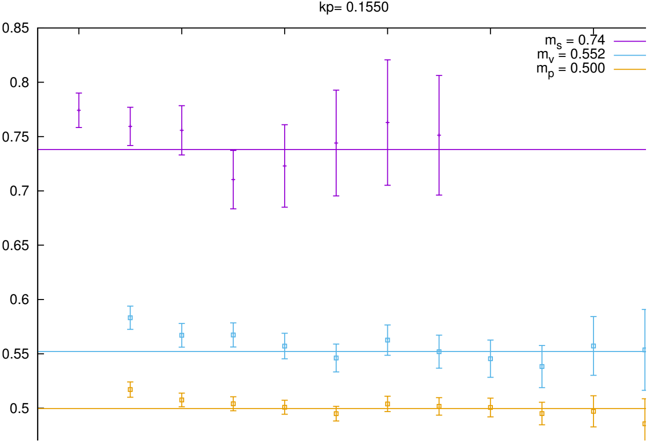
$$C_{\Gamma\Gamma'}(t) = \sum_{p_0} e^{i\rho_0 t}\,\mathrm{Tr}(\Gamma D^{-1}(p_0)\Gamma' D^{-1}(0))$$

$D(p_\mu) =$ Wilson-Dirac operator of a single-site with $U_\mu \longrightarrow U_\mu \otimes \Gamma^*_\mu e^{ip_\mu}$. $D$ is an $N^2 \times N^2$ matrix.

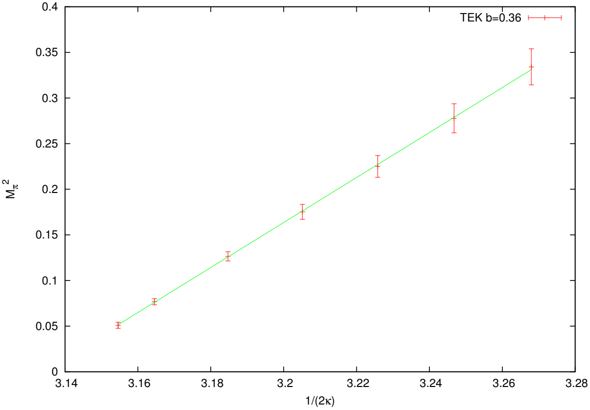
Good Initial results *(AGA, Okawa 2015)*. Precise measurement currently under study *(Garcia Perez, AGA, Okawa 2015)*

1. Good exponential fall of correlators
2. Pion mass square vanishes linearly with quark mass
3. Edinburgh plot (scale invariant) compatible with previous results.
4. Approximate scaling

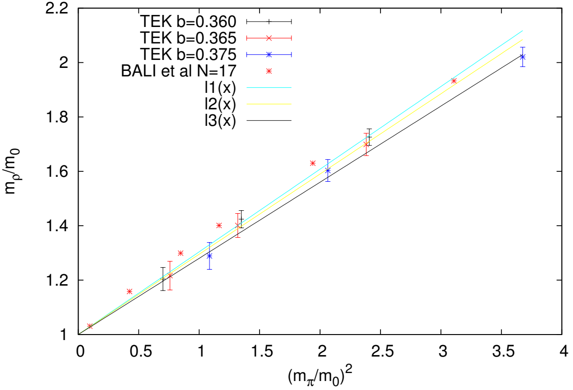
# Correlators



# Pion Mass


TEK b=0.36
$M_\pi^2$
0.4
0.35
0.3
0.25
0.2
0.15
0.1
0.05
0
3.14
3.16
3.18
3.2
3.22
3.24
3.26
3.28
1/(2κ)

# Masses: Edinburgh Plot



# Pure gauge theory at large N in 1+1 dim

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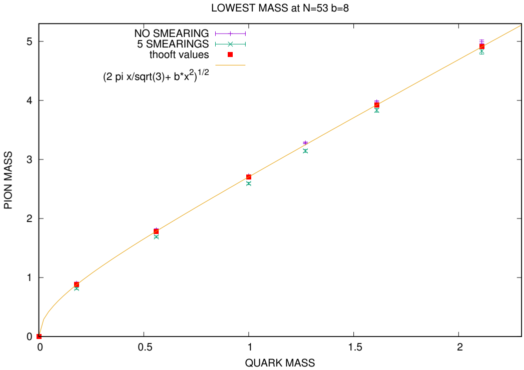
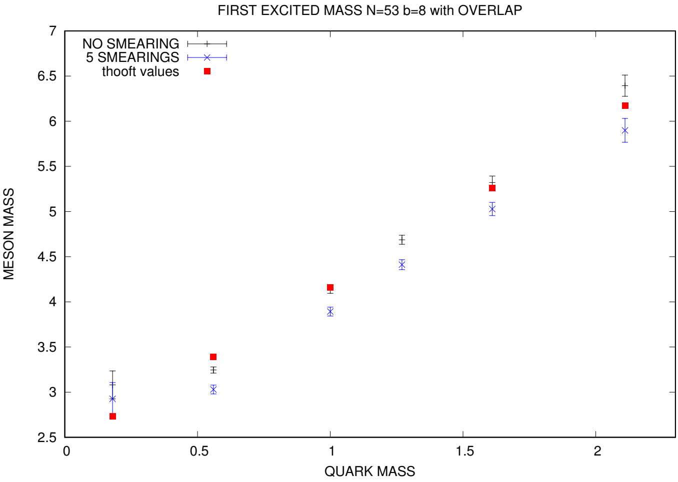


# First excited state



# Gauge Theory with fermions in the adjoint

One can add $N_f$ flavours of Dirac fermions in the adjoint representation. The corresponding large N gauge theory is very interesting for the following reasons

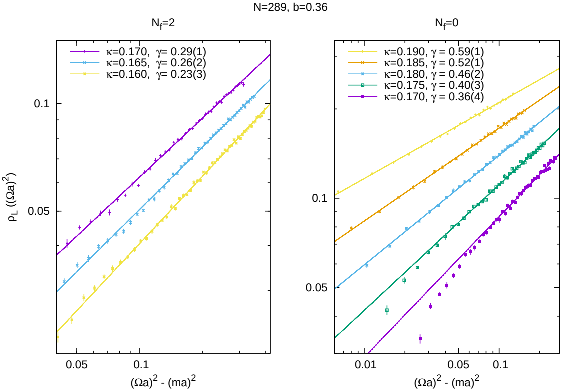
- The fermions survive the large N limit. But if they are very massive they have little influence in the long-distance dynamics
- The fermion loops help to achieve $Z_N$ symmetry $\Rightarrow$ Volume independence: The k=0 (EK model) theory might respect the symmetry for a range of masses (Kovtun, Unsal, Jaffe).
- Connected through orientifold planar equivalence to a different large N limit of QCD (Armoni-Shifman-Veneziano)
- For $N_f = 1/2$ (Majorana fermion)= SUSY Yang-Mills.
- For $N_f = 2$ This model is very interesting because it is a candidate for being conformal or walking.
- It can be studied with (Rational-)Hybrid Monte Carlo techniques.

# Lattice model at $N = \infty$

- Some initial tests done with a reduced model with Wilson fermions with PBC (*Koren-Sharpe*). Window of quark masses where symmetry is not broken.
- If we use twist together with adjoint fermions (*AGA-Okawa 2013*): center symmetry valid at all quark masses. $N$ dependence of lattice quantities strongly reduced.
- String tension seems to vanish when quark mass goes to zero. Anomalous dimension $\gamma^*$ has large systematic errors.
- We tried to determine $\gamma^*$ using (Patella) mode number method (*Garcia Perez, AGA, Keegan, Okawa 2015*):

$$\rho((a\Omega)^2) \propto ((a\Omega)^2 - (am)^2)^{(1-\gamma^*)/(1+\gamma^*)}$$

We found $\gamma^* = 0.269(2)(50)$


N=289, b=0.36
$N_f$=2
κ=0.170, γ= 0.29(1)
κ=0.165, γ= 0.26(2)
κ=0.160, γ= 0.23(3)
$\rho_L$ ((Ωa)$^2$)
0.1
0.05
0.05
0.1
(Ωa)$^2$ - (ma)$^2$
$N_f$=0
κ=0.190, γ = 0.59(1)
κ=0.185, γ = 0.52(1)
κ=0.180, γ = 0.46(2)
κ=0.175, γ = 0.40(3)
κ=0.170, γ = 0.36(4)
0.1
0.05
0.01
0.05
0.1
(Ωa)$^2$ - (ma)$^2$

# Conclusions and Outlook

- ★ Lattice calculations of large N quantities are feasible using various strategies
- ★ Volume independence seems to work well even for pure gauge theories.
- ★ Use of twisted BC is advantageous even in combination with other strategies and costs nothing: (*AGA, Narayanan, Neuberger 2005*, *Azeyanagi-Hanada-Unsalc 2010*, . . . )
- ★ Large N theories with dynamical fermions in the adjoint rep are feasible with twisted reduction methods.

**Questions for vol. independence approach**

**What other observables can be computed and how?**
(Condensate, $T_c$, glueball spectrum, etc)
**Other extensions like Veneziano limit at specific values of $(N_f/N)$**