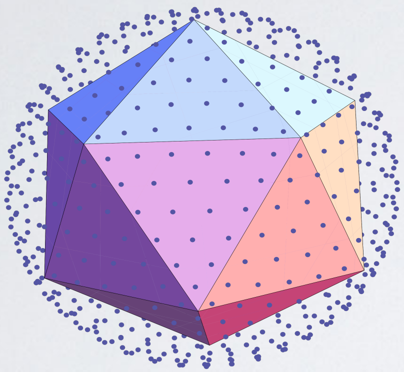
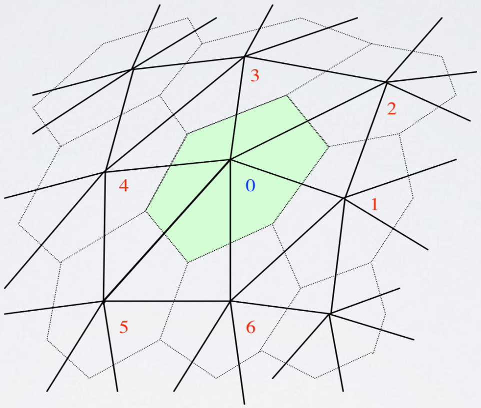
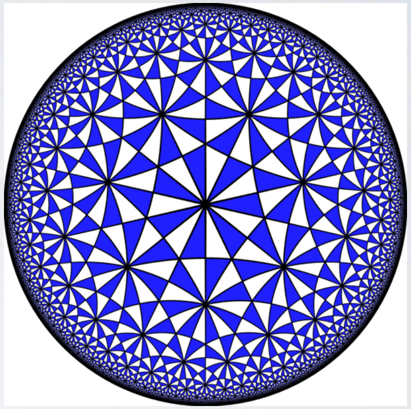
LATTICE FERMIONS FOR CURVED RIEMANNIAN MANIFOLDS







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QCDNA 2016 August 1, 2016

*collaboration with G. Fleming, A. Gabarro, T. Ruben, C-I Tan and E. Weinberg

Some references

- Finite Element Method* (FEM): Discrete Exterior Calculus (de Rahm Complex, Whitney), even ‘t Hooft

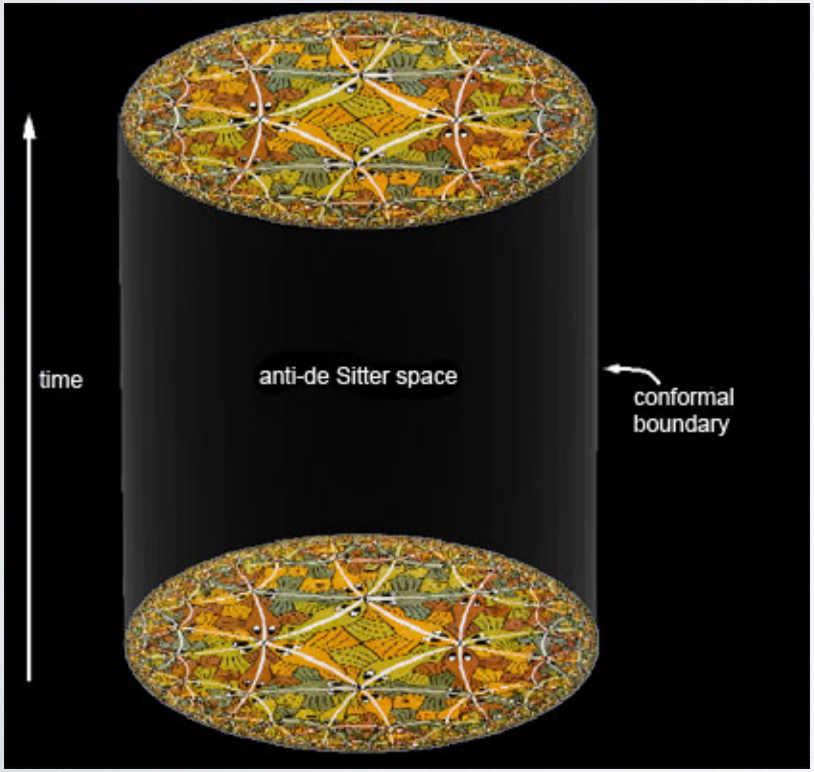
- Regge Calculus T. Regge, Nuovo Cimento 19 (1961) 558.

- Random Lattices: N. H. Christ, R. Friedberg, and T. D. Lee, Nucl. Phys. B 202, 89 (1982).

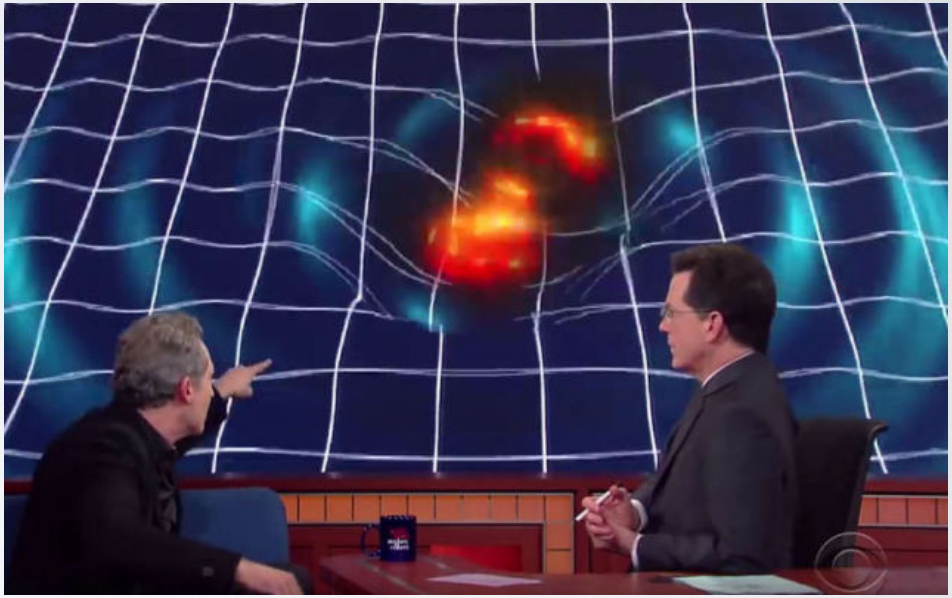
* 2,910,000 GOOGLE RESULTS

NOT ALL QUANTUM FIELDS LIVE ON FLAT RIEMANNIAN MANIFOLDS

Conformal Field Theory lives on cylinder$\mathbb{R} \times \mathbb{S}^4$ or boundary of $dAdS^5$ in Radial Quantization



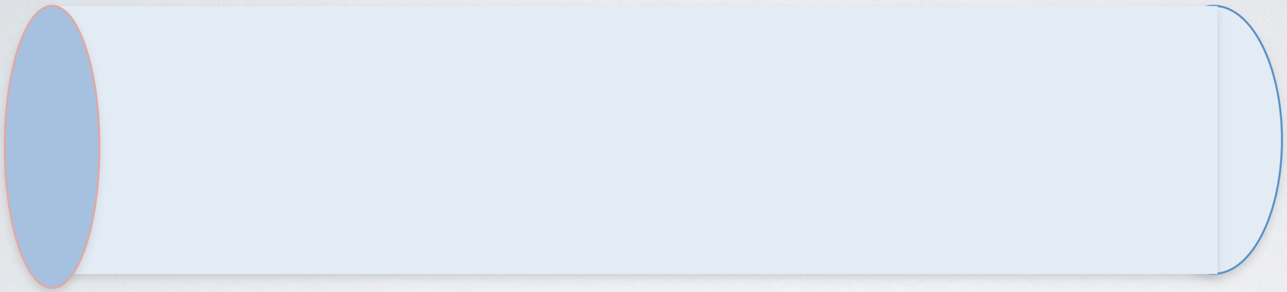
Space-Time:Two Black Holes Merge



RADIAL QUANTIZATION OF CONFORMAL FIELD THEORY

$$\mathbb{R} \times \mathbb{T}^3 \quad \text{vs} \quad \mathbb{R} \times \mathbb{S}^3$$

Boundary of Global AdS5



Isometries are the conformal group O(5,1)

On lattice scales exponentially

$$H = P_0 \text{ in } t \implies D \text{ in } \tau = \log(r)$$

$$1 < t < aL \implies 1 < \tau = log(r) < L$$

Applications:

(1) Near IR conformality — composite Higgs
(2) CFT c-theorems, anomalies on sphere,
(3) Maybe Quantum Field near Blackholes ?
(4) Quantum Phase Transitions Critical Phenomena etc.

Radial Quantization for CFT

$$ds^2 = dx^\mu dx_\mu = e^{2\tau}[d\tau^2 + d\Omega^2]$$

Can drop Weyl factor!

$$\mathbb{R}^d \to \mathbb{R} \times \mathbb{S}^{d-1}$$

Evolution: $H = P_0 \text{ in } t \implies D \text{ in } \tau = \log(r)$

$$\text{"time"} \quad \tau = log(r)\,, \quad \text{"mass"} \quad \Delta = d/2 - 1 + \eta$$

$$D \to x_\mu \partial_\mu = r\partial_r = \frac{\partial}{\partial \tau}$$

CAN WE CONSTRUCT LATTICE FIELD THEORY FOR RIEMANN MANIFOLD?

Lattice Field Theory (LFT) on $\mathbb{R}^D$ Euclidean Manifold provides a rigorous solutions to UV complete (renormalizable) quantum field theory.

Apparently renormalized perturbation theory does exit.* Very large literature starting from the 1980's

Can this be generalized to a smooth Riemann manifold $(\mathcal{M}, g)$?

* Luscher "Dimensional Regularization in the Presence of Large in Presence of Large Background Fields" (1982); Jack and Osborn "Background Field Calculations on Curved Space Time" NP (1984) et al … Not sure what are the best references!

FREE CONTINUUM ACTIONS

$$S_{scalar} = \frac{1}{2}\int_{\mathcal{M}} d^D x \sqrt{g}[g^{\mu\nu}(x)\partial_\mu \phi(x)\partial_\nu \phi(x) + m^2\phi^2(x)$$

$$S_{Dirac} = \frac{1}{2}\int_{\mathcal{M}} d^D x \sqrt{g}\bar{\psi}[\mathbf{e}^\mu(x)(\partial_\mu - \boldsymbol{\omega}_\mu(x)) + m]\psi(x)$$

$$Sgauge = \frac{1}{2}\int_{\mathcal{M}} d^D x \sqrt{g}\; g^{\mu\nu'}(x) g^{\mu'\nu}(x) F^a_{\mu\nu}(x) F^a_{\mu'\nu'}(x)$$

IS THERE A SYSTEMATIC APPROACH ?

- **Topology – Simplicial Complex:**

  Replace D-dimensional manifold $\mathcal{M}$ by a simplicial lattice $\mathcal{M}_\sigma$ composed of elementary simplicies: $\sigma_0, \sigma_1 \cdots \sigma_D$

- **Geometry – Regge Calculus:**

  Riemann manifold with metric represented by edge lengths l_{ij} on lattice: $(\mathcal{M}, g) \implies (\mathcal{M}_\sigma, g_\sigma)$

- **Truncated Hilbert Space – Finite Element Basis:**

  Replace continuum fields $\phi(x)$ in Lagrangian expanding local basis on each cell σ_n: $\phi_\sigma(x) = E^i(x)\phi_i$

- **Quantization – LFT Path Integral:**

  Numerical Monte Carlo Simulation of Path Integral with counter terms to control UV divergence in continuum: $a \to 0$

WARM UP: SCALAR FIELD

$$S = \frac{1}{2}\int_{\mathcal{M}} d^D x \sqrt{g}\, [g^{\mu\nu}\partial_\mu\phi(x)\partial_\nu\phi(x) + m^2\phi^2(x) + \lambda\phi^4(x)]$$

REGGE CALCULUS ON SIMPLIES COMPLEX

WARNING #1:
(Piece-wise linear g(x) is bad for Fermions)

$$I_\sigma = \frac{1}{2}\int_\sigma d^D y [\vec{\nabla}\phi(y)\cdot\vec{\nabla}\phi(y) + m^2\phi^2(y) + \lambda\phi^4(y)]$$

$$= \frac{1}{2}\int_\sigma d^D \xi \sqrt{g}\, [g^{ij}\partial_i\phi(\xi)\partial_j\phi^2(\xi) + m^2\phi^2(\xi) + \lambda\phi^4(\xi)]$$

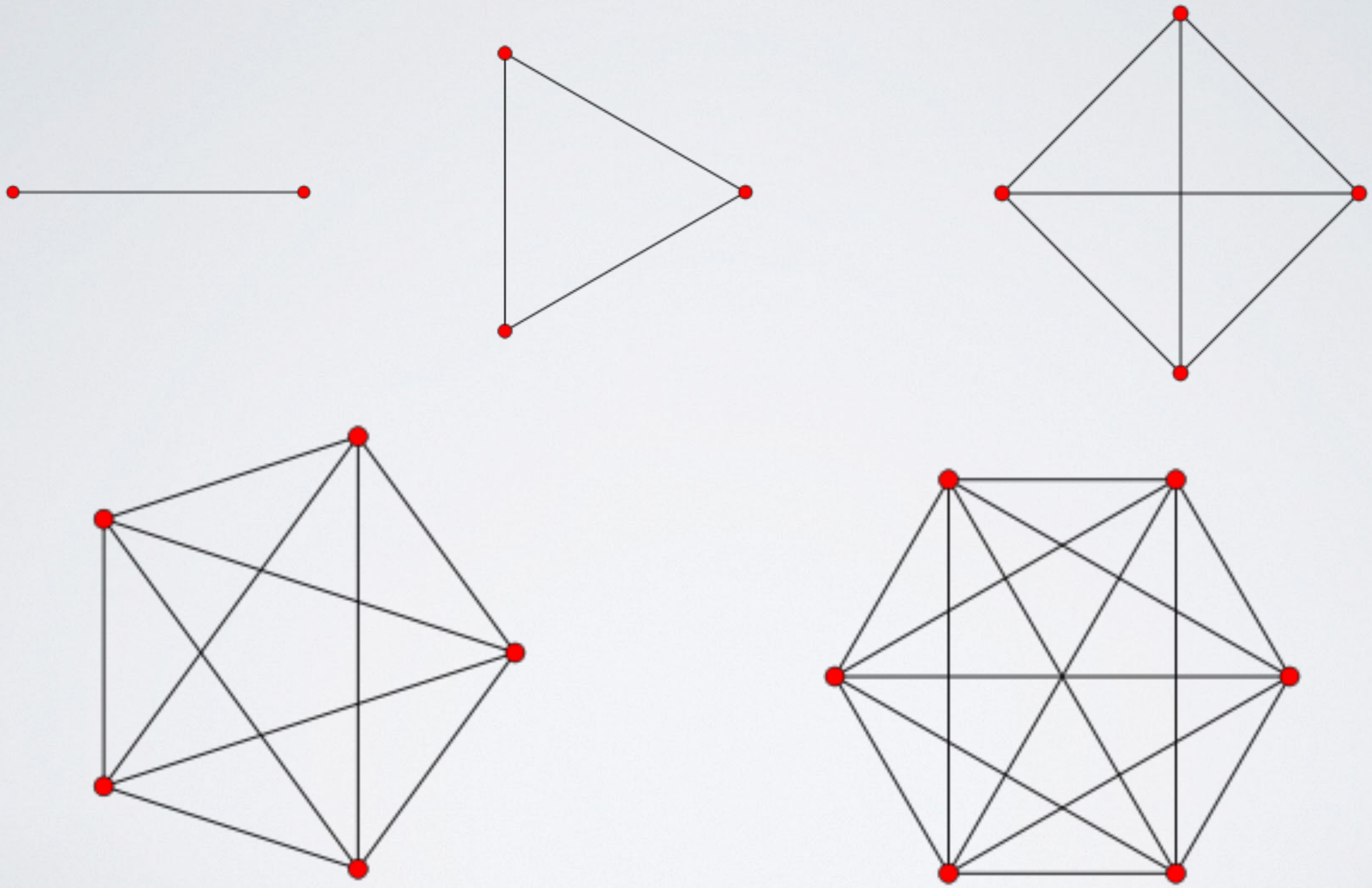
LINEAR FINITE ELEMENTS

WARNING #2.
(Piece-wise linear bad for D > 1 Fermions & D>2 Scalar)

$$I_\sigma \simeq \sqrt{g_0}\left[g_0^{ij}\frac{(\phi_i - \phi_0)(\phi_j - \phi_0)}{l_{i0}\; l_{j0}} + m^2\phi_0^2 + \lambda\phi_0^4\right]$$

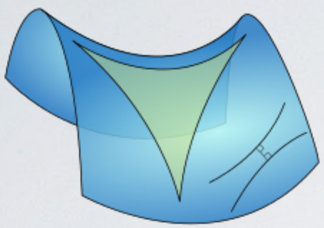
WARNING #3.
(UV divergence require x-dep QFE Counter terms)

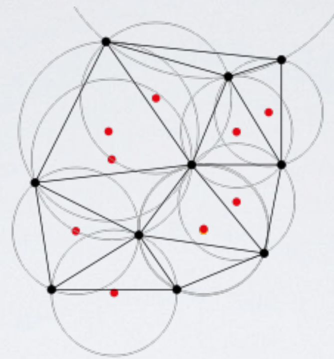
K-SIMPLICIES

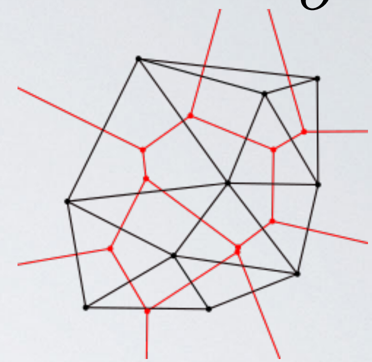


SIMPLICIAL TOPOLOGY

$$\mathcal{M} \quad \rightarrow \quad \mathcal{M}_\sigma \quad \rightarrow \quad \mathcal{M}_{\sigma^*}$$







Set of points, lines, triangles, etc.

$$\sigma_0 \rightarrow \sigma_1 \rightarrow \sigma_2 \rightarrow \cdots \rightarrow \sigma_D$$

Boundary Matrix :

$$\partial\sigma_n(i_0, i_1, \cdots, i_n) = \sum_{k=0}^{n} (-1)^k \sigma_{n-1}(i_0, i_1, \cdots, \widehat{i_k}, \cdots, i_n) \; .$$

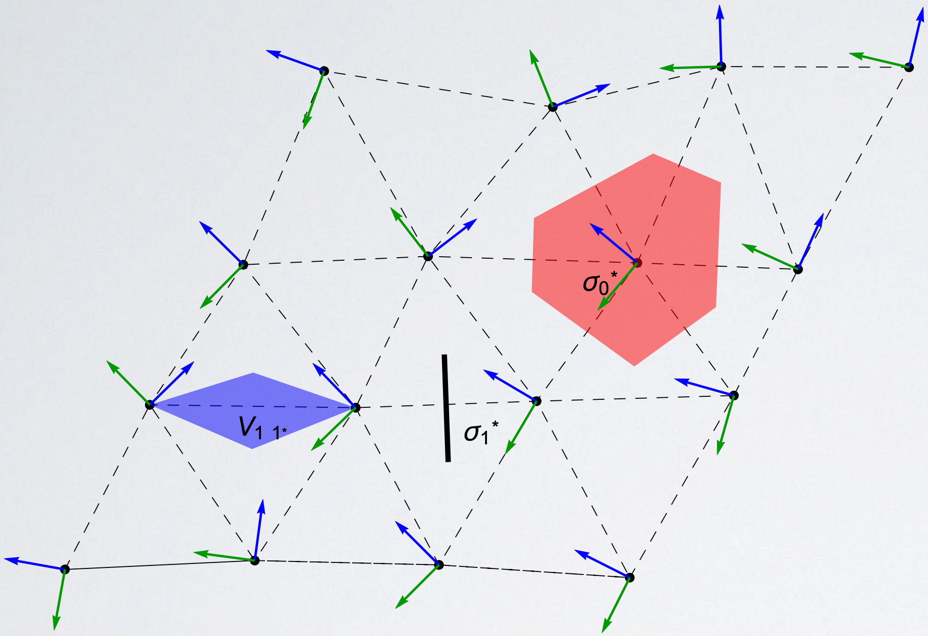
Co- Boundary Matrix : ∂^T Dual Simplex: $\sigma_0^* \leftarrow \sigma_1^* \leftarrow \cdots \leftarrow \sigma_D^*$

Beautiful Discrete Topology with chains, duality, homology sequences etc. NO metric necessary

REGGE CALCULUS MANIFOLD

Discrete Metric Data $\{l_{ij} = |\sigma_1(i,j)|\}$

$$g_{\mu\nu}(x) \implies g_\sigma(y)$$



This is just piece-wise linear FEM of metric

Circumcenter Dual Lattice

$$V_{nn'} = \int \sigma_n \wedge \sigma^*_{n'}$$

$$= \frac{n!(D-n)!}{D!} |\sigma_n||\sigma^*_{n'}|$$

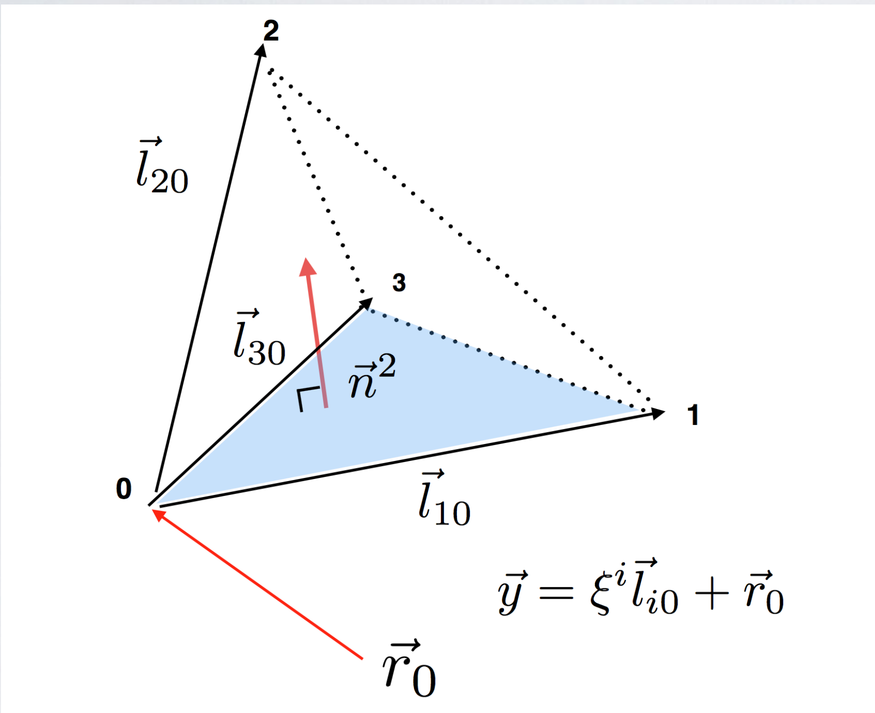
Curvature is concentrated at Vertex in 2D or Hinge D > 2

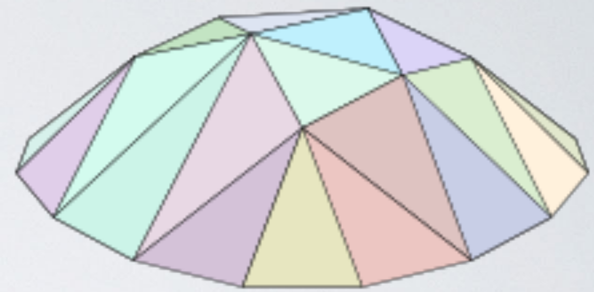
$$\text{Hinge} = \sigma_{D-2}$$

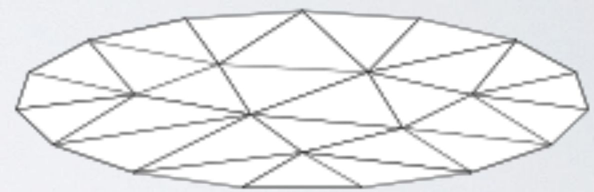
$$\text{Einstein Action} = \sum_{\sigma_{D-2}} (\text{deficit angle of } \sigma_{D-2}) \times |\sigma^*_{D-2}|$$

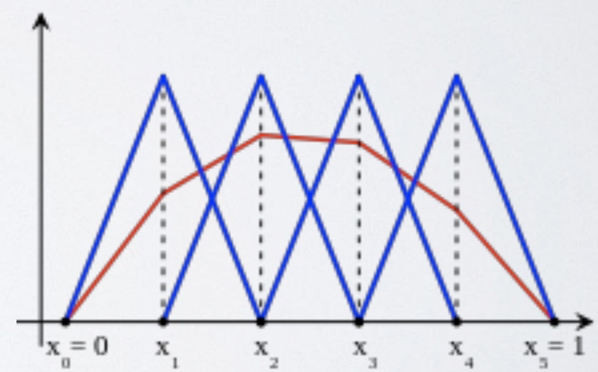
Hilbert Space: Finite Element Interpolates

$$\phi(x) \to \phi_\sigma(x) = \sum_i W^i(x)\phi_i$$



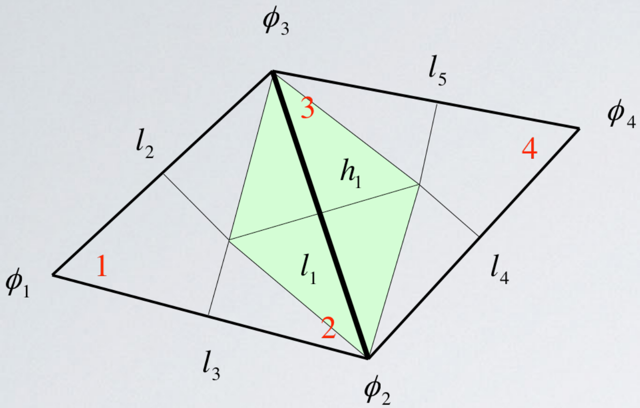






RCB, M. Cheng and G.T. Fleming,
"Improved Lattice Radial Quantization" PoS LATTICE2013 (2013) 335

SCALAR KINETIC TERM IN LINEAR FEM/ REGGE CALCULUS



$$\sum_{\triangle_{kij}} \sqrt{g(k)} g^{ij}(k) \frac{(\phi_k - \phi_i)(\phi_k - \phi_j)}{l_{ki} l_{kj}}$$

$$\downarrow$$

$$A_{23} \frac{(\phi_2 - \phi_3)^2}{l_{23}^2}$$

Only for D = 2

$$A_{23} = \frac{h_1 l_1}{2}$$

ALL D by DEC

BUT DISCRETE EXTERIOR CALCULUS DOES GIVE THIS APPLYING STOKES THEOREM TO DEFINE THE DISCRETE EXTERIOR DERIVATIVE

$$\longrightarrow \quad V_{23} \frac{(\phi_2 - \phi_3)^2}{l_{23}^2}$$

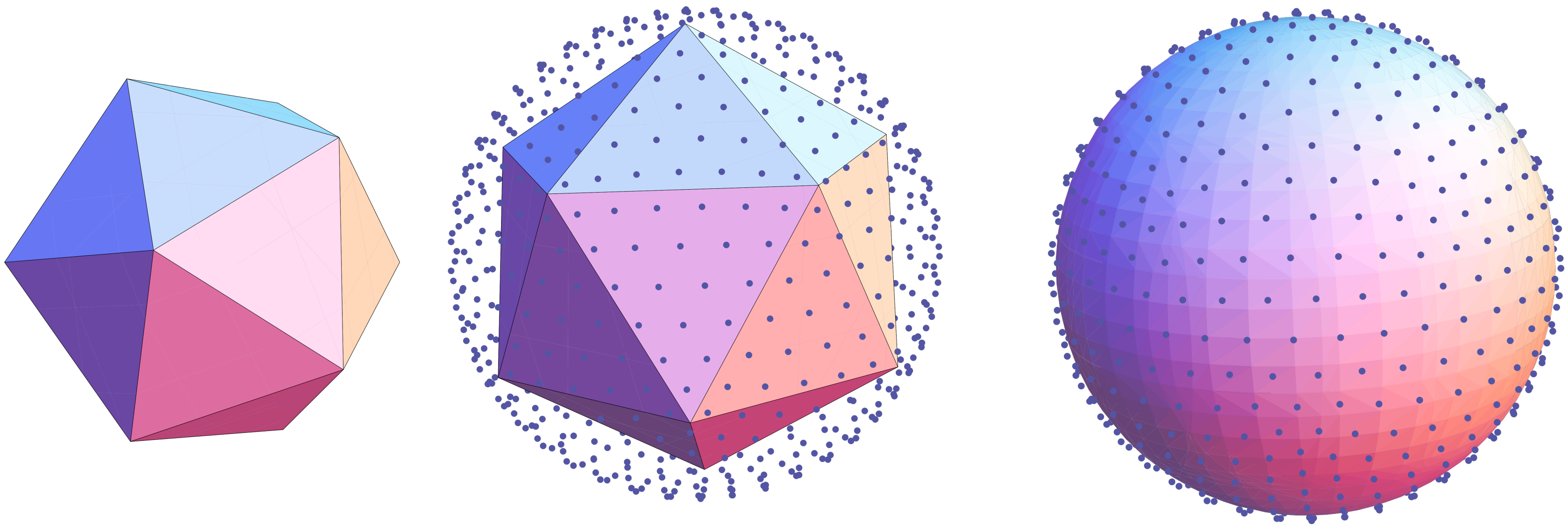
$$\langle \mathcal{M} | d\omega \rangle = \langle \partial \mathcal{M} | \omega \rangle$$

$$V_{23} = \frac{|\sigma_1^*(2,3)| l_1}{D}$$

$$\nabla^2 \phi \to (*d*d + d*d*)\phi$$

2D SphereTest Case

$s = 1$ $\qquad$ $s = 8$

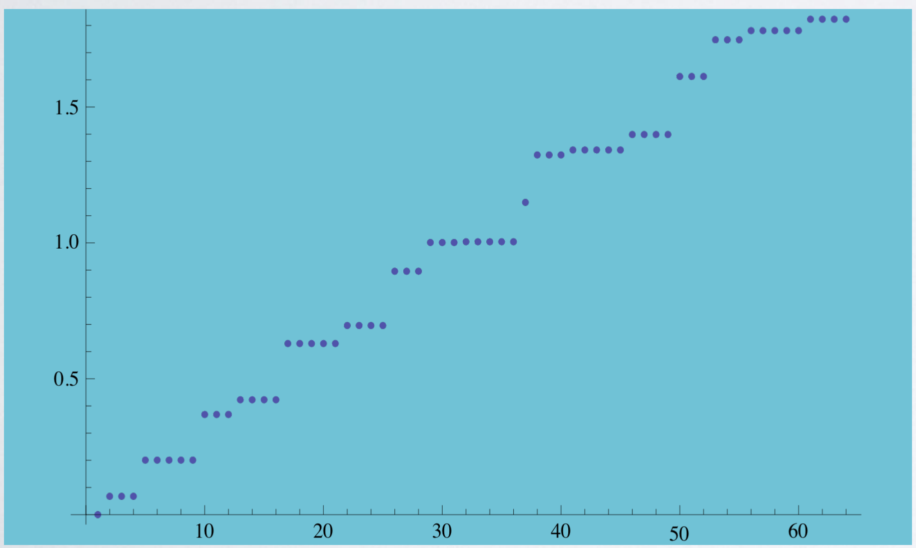


l = 0 (A),1 (T1) , 2 (H) are irreducible 120
Iscosahedral subgroup of O(3)

THE LAPLACIAN ON THE SPHERE

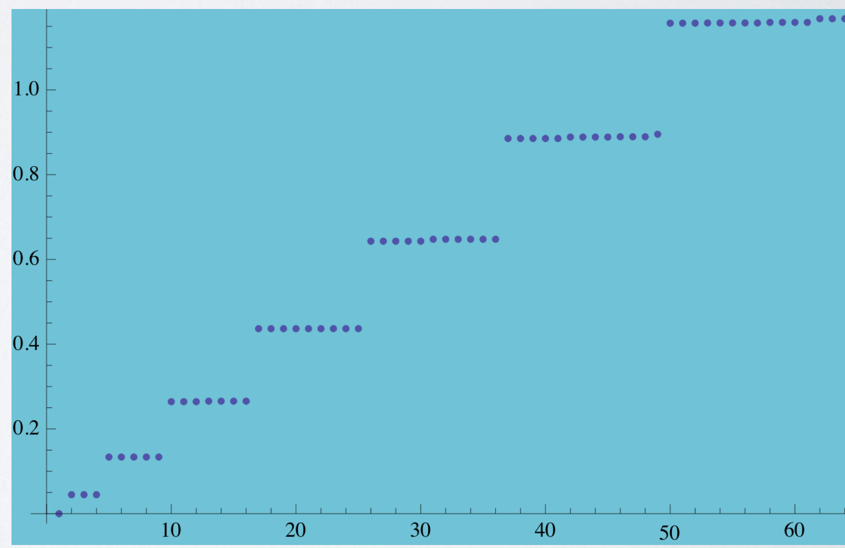
For s = 8 first (l+1)*(l +1) = 64 eigenvalues

BEFORE (K = 1)



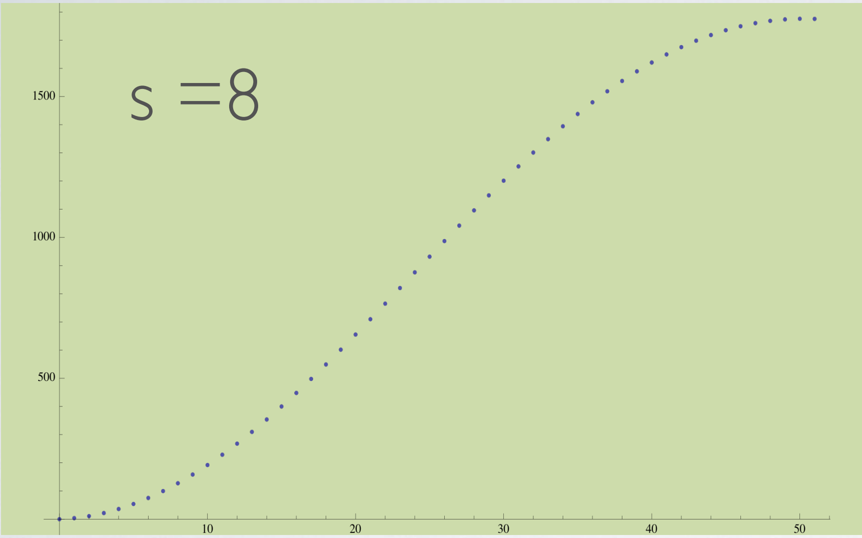
l, m

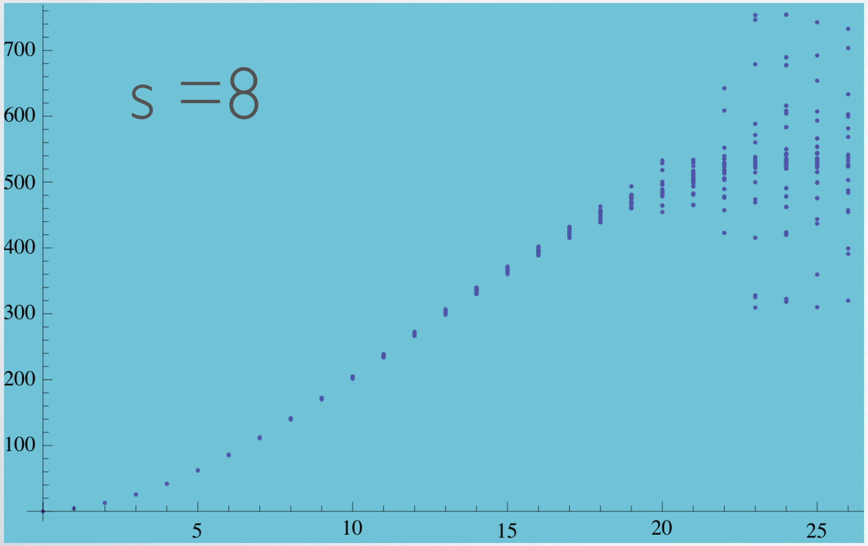
AFTER (FEM K's)

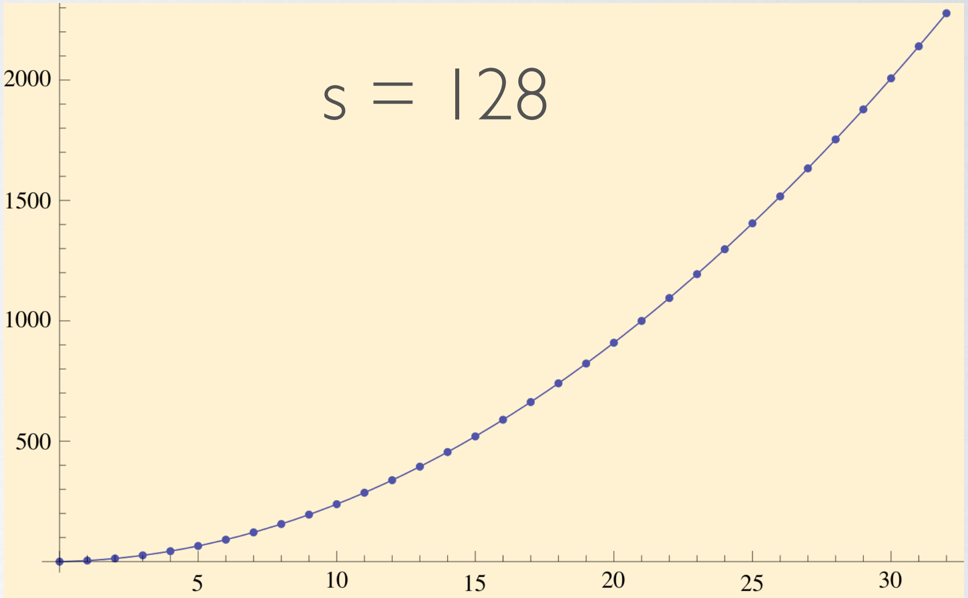


l, m

SPECTRUM OF FE LAPLACIAN ON A SPHERE







Fit

$$l + 1.00012\, l^2 - 13.428110^{-6}\, l^3 - 5.5724410^{-6}\, l^4$$

FREE DIRAC EQUATION

$$S = \frac{1}{2}\int d^D x \sqrt{g}\bar{\psi}[\mathbf{e}^\mu(\partial_\mu - \frac{i}{4}\boldsymbol{\omega}_\mu(x)) + m]\psi(x)$$

$\mathbf{e}^\mu(x) \equiv e^\mu_a(x)\gamma^a$ Vierbein & Spin connection*

$\boldsymbol{\omega}_\mu(x) \equiv \omega^{ab}_\mu(x)\sigma_{ab}$, $\sigma_{ab} = i[\gamma_a, \gamma_a]/2$

Tetrad Postulate $\partial_\mu \mathbf{e}^\nu + \Gamma^\nu_{\mu,\lambda}\mathbf{e}^\lambda = i[\boldsymbol{\omega}_\mu, \mathbf{e}^\nu]\,.$

(1) New spin structure "knows" about intrinsic geometry

(2) Need to avoid simplex curvature singularities at sites.

(3) Spinors rotations: Spin(D) is double of Lorentz O(D).

e.g. D = 2 as $\theta \to 2\pi$ $e^{i(\theta/2)\sigma_3/2} \to -1$

FLAT SPACE DIRAC IN GENERAL GAUGE

$$S_{naive} \simeq \frac{1}{2}\sum_{\langle\, i,j \rangle} \frac{V_{ij}}{l_{ij}^2}[\bar{\psi}_i \vec{l}_{ij}\cdot\vec{\gamma}\psi_j - \bar{\psi}_j \vec{l}_{ij}\cdot\vec{\gamma}\psi_i] + \frac{1}{2} m V_i \bar{\psi}_i \psi_i$$

Λ_i Rotates to a general gauge

$$S_{naive} = \frac{1}{2}\sum_{\langle\, i,j \rangle} \frac{V_{ij}}{l_{ij}}[\bar{\psi}_i \vec{e}^{\;(i)j}\cdot\vec{\gamma}\Omega_{ij}\psi_j - \bar{\psi}_j \Omega_{ji}\vec{e}^{\;(i)j}\cdot\vec{\gamma}\psi_i] + \frac{1}{2} m V_i \bar{\psi}_i \psi_i \;,$$

Where $\Omega_{ij} = \Lambda_i^\dagger \Lambda_j = \Omega_{ji}^\dagger$

$$\mathbf{e}^{(i)j} = e_a^{(i)j}\gamma^a \equiv \vec{e}^{\;(i)j}\cdot\vec{\gamma} = \Lambda_i^\dagger \hat{l}_{ij}\cdot\vec{\gamma}\Lambda_i$$

$$e_a^{(j)i}\gamma^a = -\Omega_{ji} e_a^{(i)j}\gamma^a \Omega_{ij} \,.$$

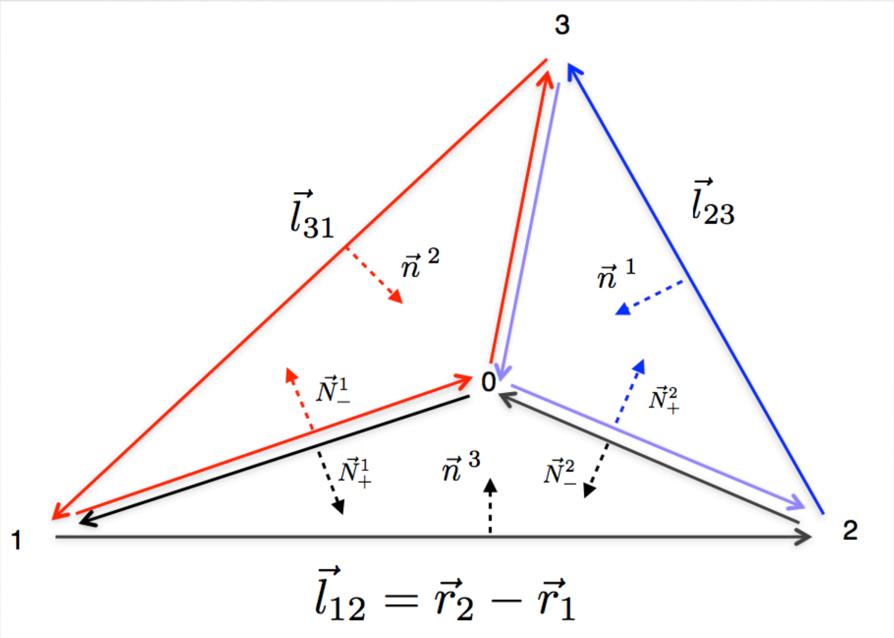
EVEN 2D FLAT LINEAR FEM IS STRANGE

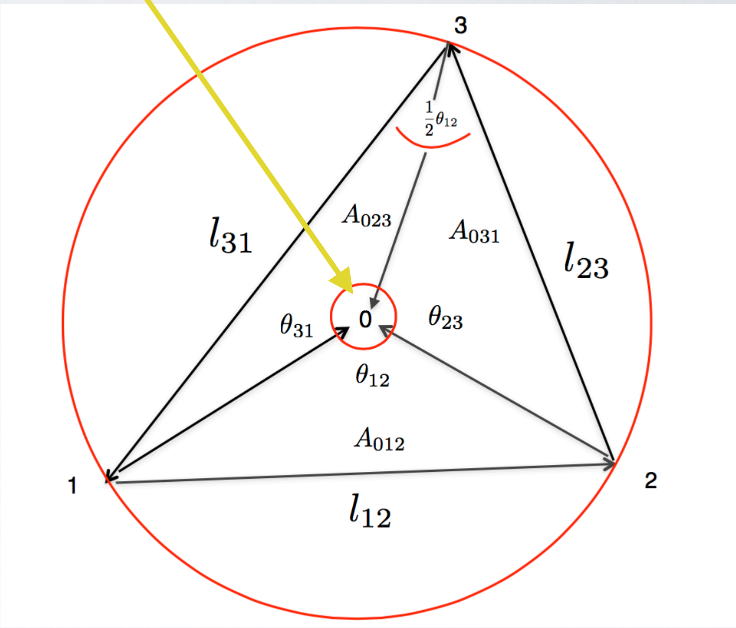
$$S_{linear} = \frac{A_{123}}{6} \sum_{\langle i,,j \rangle} \bar{\psi}_i (\vec{n}^{\,j} - \vec{n}^{\,i}) \cdot \vec{\sigma} \psi_j$$

BAD ALLIGNMENT EXCET FOR ISOCILLES TRANGLE!

New Dirac Element is 3 linear elements meeting ghost sites at Circumcenter

$$\phi_0 = c_1\phi_1 + c_2\phi_2 + c_3\phi_3$$



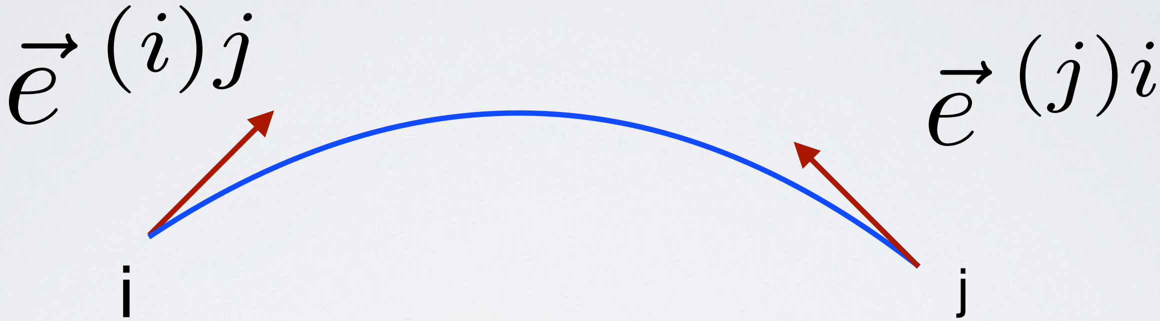


$$c_k = \frac{4A_{0ij}}{l_{ij}^2}\frac{4A_{0ik}}{l_{ik}^2} = \cot(\theta_{ik}/2)\cot(\theta_{jk}/2)$$

Sort of ? $\sqrt{\delta d + d\delta} = \delta + d$

CONSTUCTING THE DIRAC ACTION

$$S_{naive} = \frac{1}{2}\sum_{\langle i,j \rangle} \frac{V_{ij}}{l_{ij}}[\bar{\psi}_i \vec{e}^{\,(i)j} \cdot \vec{\gamma}\Omega_{ij}\psi_j - \bar{\psi}_j \Omega_{ji}\vec{e}^{\,(i)j} \cdot \vec{\gamma}\psi_i] + \frac{1}{2} m V_i \bar{\psi}_i \psi_i$$



DO NOT USE PIECEWISE LINEAR REGGE CALCULUS MANIFOLD!

Simplicial Tetrad Hypothesis $\quad e_a^{(i)j}\gamma^a\Omega_{ij} + \Omega_{ij}e_a^{(j)i}\gamma^a = 0$

Gauge Invariance under Spin(D) transformations

$$\psi_i \to \Lambda_i \psi \quad , \quad \bar{\psi}_j \to \bar{\psi}_j \Lambda_j^\dagger \quad , \quad \mathbf{e}^{(i)j} \to \Lambda_i \mathbf{e}^{(i)j} \Lambda_i^\dagger \quad , \quad \Omega_{ij} \to \Lambda_i \Omega_{ij} \Lambda_j^\dagger$$

Construction Procedure for Discrete Spin connection

(1) Assume Elements with Spherical Triangles (i,j,k) or boundaries given by geodesics on an 2D manifold

(Angles at each vertex add to 2 pi exactly)

(2) Calculate discrete "curl" around the triangle

$$\Omega_{ij}\Omega_{jk}\Omega_{ki} = e^{i(2\pi - \delta_\triangle)\sigma_3/2}$$

(3) Fix $\Omega_{ij} \to \pm\Omega_{ij}$ so $\delta_\triangle \sim A_{ijk}/4\pi R$

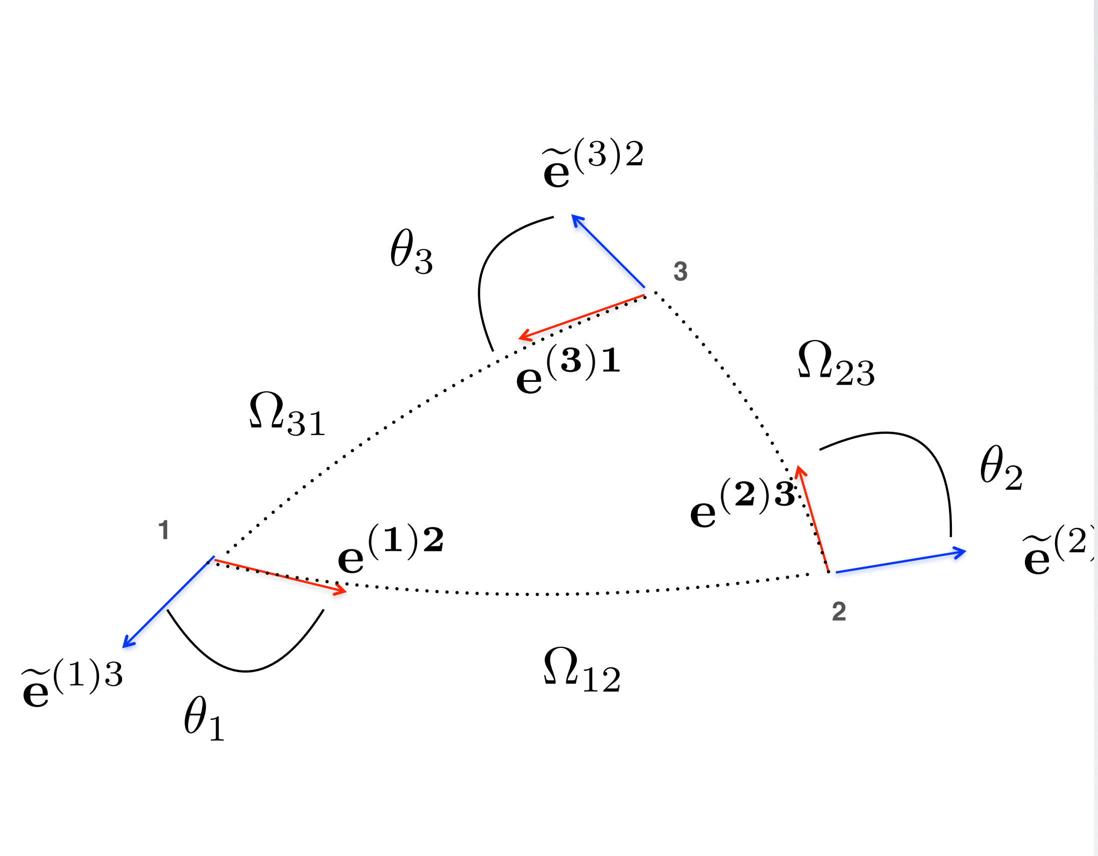
Sphere: or any manifold with this topology has a unique lattice spin connection up to gauge Lorentz transformation on spinors

Cylindar: There are 2 solutions (periodic or anti periodic)

Torus: There are 4 solutions: (periodic/anti-periodic): Non-contractible loops.

Category Theory: A spin structure is a property shared between any simplicial complex and Riemann manifolds to which they correspond.

Fixing the Spin Connection: $\Lambda_{ij} \in O(D) \;\rightarrow\; \Omega_{ij} \in \texttt{Spin(D)}$



$$\Lambda_{ij} = e^{i\theta_{\mu\nu} J^{\mu\nu}} \quad \rightarrow \quad \Omega^{(+)}_{ij} = e^{i\theta_{\mu\nu}\sigma^{\mu\nu}/2}$$

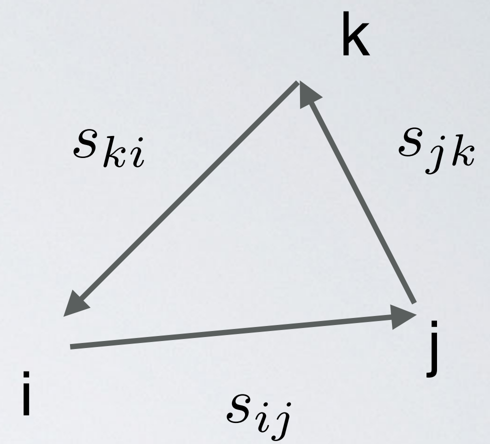
Pick a Square Root

$$\Omega_{ij} = s_{ij}\Omega^{(+)}_{ij} \quad \text{where} \quad s_{ij} = \pm 1$$

FIXING THE "SQUARE ROOT"

Can we find signs $s_{ij} = \pm 1$ so that

$$Tr[\Omega_{\triangle(ijk)}] = Tr[\Omega_{ij}\Omega_{jk}\Omega_{ki}] > 1$$



- TWO SOLUTIONS:

1. Iteration: Fix all signs on each D-plex starting with a seed and extending the D-1 surface. No solution implies there is no spin connection. Multiplicity depend on the holonomy. (I believe)

2. Relaxation: Compare curvature for each triangle at each site i Lattice vs Continuum

$$\left[\Omega_{\triangle(ijk)}\right]^{\alpha\beta} \simeq \left[e^{i\boldsymbol{R}_{\mu\nu}(i)A^{\mu\nu}_{\triangle_{ijk}}}\right]^{\alpha\beta}$$

$$A^{\mu\nu}_{\triangle_{ijk}} = \frac{1}{2}[(r^\mu_i - r^\mu_j)(r^\nu_k - r^\nu_i) - (r^\nu_i - r^\nu_j)(r^\mu_k - r^\mu_i)]$$

WILSON/CLOVER TERM

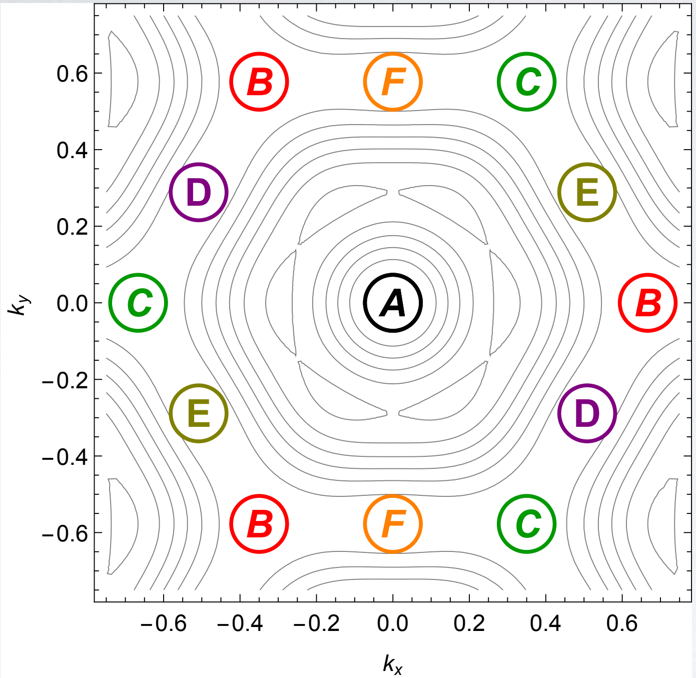
$$[\gamma_\mu(\partial_\mu - iA_\mu)]^2 = (\partial_\mu - iA_\mu)^2 - \frac{1}{2}\sigma^{\mu\nu}F_{\mu\nu}\ ,$$

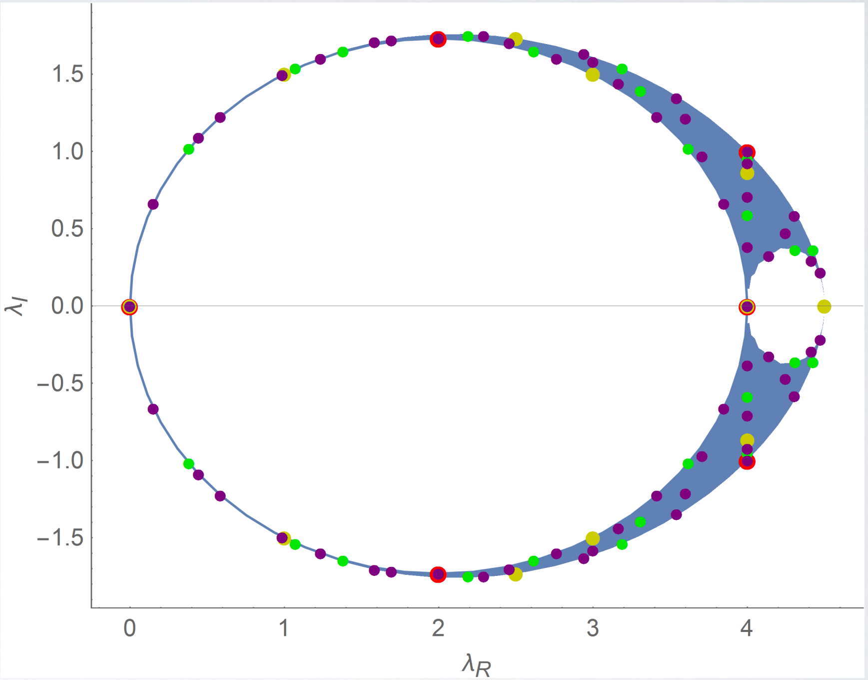
$$\boldsymbol{D}_\mu = \partial_\mu - i\boldsymbol{\omega}_\mu$$

$$[\mathbf{e}_a^\mu(\partial_\mu - i\boldsymbol{\omega}_\mu)]^2 = \frac{1}{\sqrt{g}}\boldsymbol{D}_\mu\sqrt{g}g^{\mu\nu}\boldsymbol{D}_\nu - \frac{1}{2}\sigma^{ab}e_a^\mu e_b^\nu \boldsymbol{R}_{\mu\nu}$$

$$S_{Wilson} = \frac{r}{2}\sum_{\langle i,j\rangle}\frac{aV_{ij}}{l_{ij}^2}(\bar{\psi}_i - \bar{\psi}_j\Omega_{ji})(\psi_i - \Omega_{ij}\psi_j)$$

2D DIRAC SPECTRA REGULAR TRIANGLES



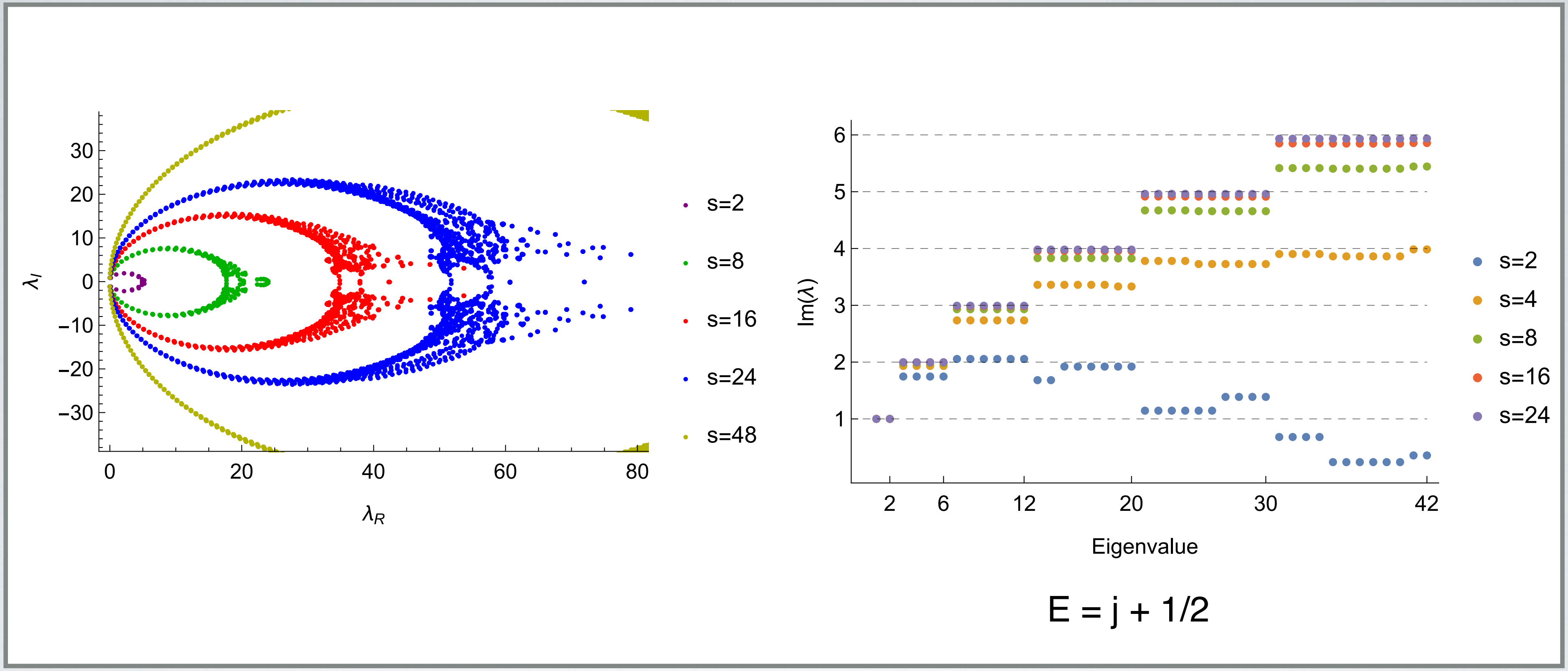


"Doublers": No Wilson Term

With Wilson Term

9 pts (orange) 16 pts (red) 25 pts(green) 100 pts (yellow)

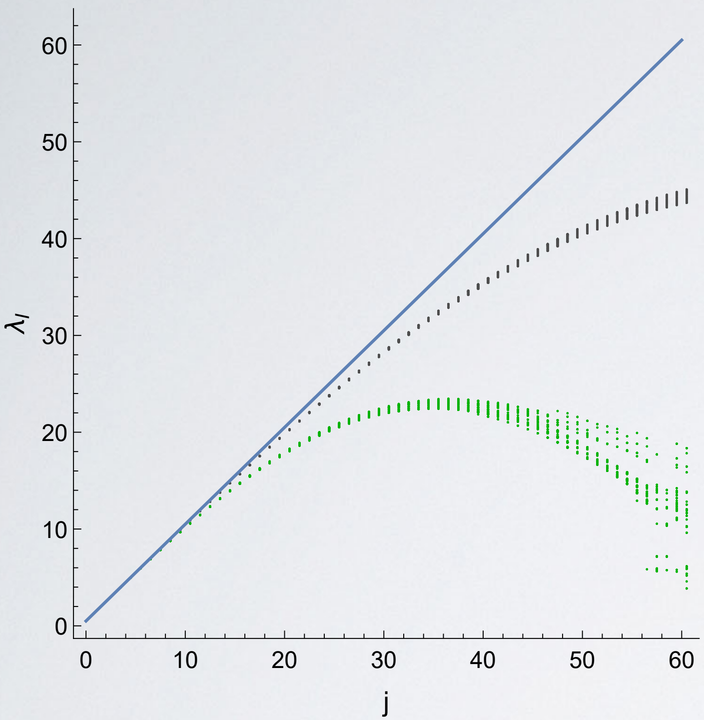
2D DIRAC SPECTRA ON SPHERE

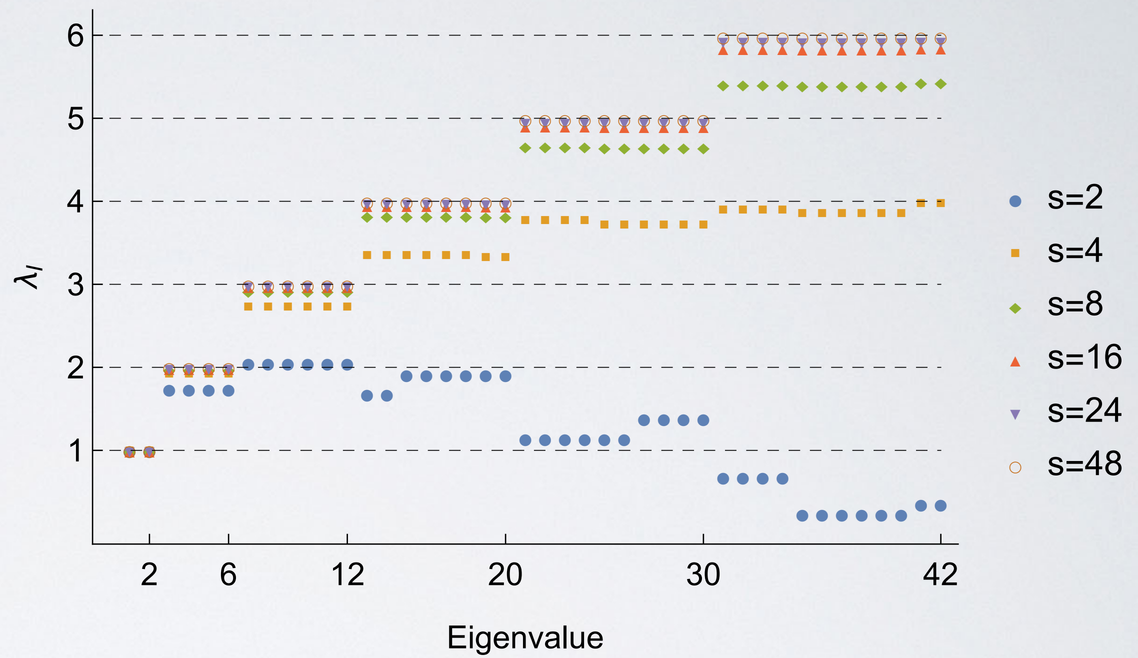


Exact is integer spacing for j = 1/2, 3/2, 5/2 ...

Exact degeneracy 2j + 1: No zero mode in chiral limit!.

DIRAC DISPERSION RELATION



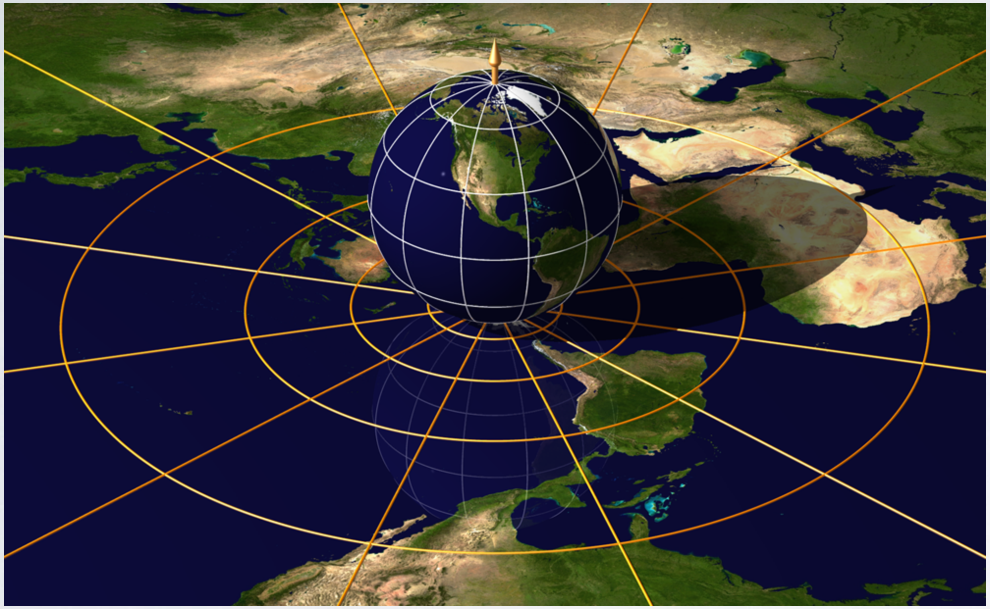


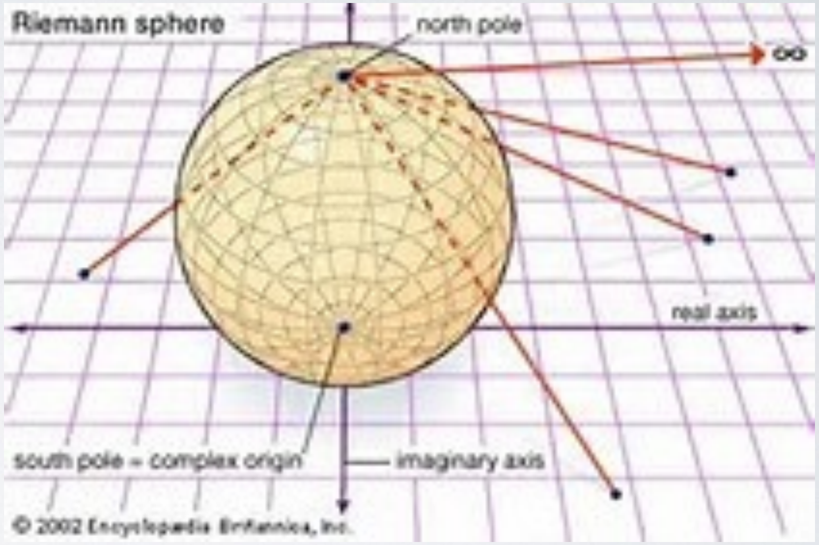
$$m + i\lambda = m \pm i\left(j + 1/2\right)$$

No Zero Mode in Chiral limit

$$j = 1/2, 3/2, 5/2, \cdots \quad \text{Digeneracy} \quad 2j+1$$

TEST CASE: CONFORMAL FIELDS THEORY ON THE 2D RIEMANN SPHERE





Projection is Weyl Rescaling to the Sphere

projection

$$w = u + iv = \frac{x + iy}{1 + z} = \frac{\sin\theta}{1 + \cos\theta} e^{i\phi}$$

$$\vec{r} \cdot \vec{r} = x^2 + y^2 + z^2 = 1$$

WEYL MAP FOR CFT

$$\langle \mathcal{O}_1(x_1)\mathcal{O}_2(x_2)\cdots\rangle_{g_{\mu\nu}} = [\frac{1}{\Omega(x_1)^{\Delta_1}}\frac{1}{\Omega(x_2)^{\Delta_2}}\cdots]\langle \mathcal{O}_1(w_1)\mathcal{O}_2(w_2)\cdots\rangle_{flat}$$

Where $$g_{\mu\nu}(x) = \frac{\partial w^\alpha}{\partial x^\mu}\frac{\partial w^\alpha}{\partial x^\nu} = \Omega^2(x)\delta_{\mu\nu}$$

Apply to Riemann 2D Sphere

$$ds^2_{\mathbb{S}^2} = \sin^2\theta d\phi^2 + d\theta^2 = 4\frac{dwd\bar{w}}{(1+w\bar{w})^2} \equiv \cos^2(\theta)dwd\bar{w}$$

$$\langle \mathcal{O}_1(x_1)\mathcal{O}_2(x_2)\rangle_{\mathbb{S}^2} = \frac{1}{[\Omega(x_1)|w_1-w_2|^2\Omega(x_2)]^\Delta} = \frac{1}{(2-2\cos\theta_{12})^\Delta},$$

Where $\Omega(\theta) = \cos(\theta/2)$

$$u = \frac{x_{12}^2 x_{34}^2}{x_{13}^2 x_{24}^2} = \frac{r_{12}^2 r_{34}^2}{r_{13}^2 r_{24}^2} \qquad v = \frac{x_{14}^2 x_{32}^2}{x_{13}^2 x_{24}^2} = \frac{r_{14}^2 r_{32}^2}{r_{13}^2 r_{24}^2}$$

PROJECTING DIRAC TO MAJORANA

$$M_{z_1,z_2} = \begin{bmatrix} m_D & \partial \\ \bar{\partial} & m_D \end{bmatrix}_{z_1,z_2} \to \begin{bmatrix} W & \nabla \\ -\nabla^\dagger & W \end{bmatrix}_{z_1,z_2}$$

$$G(z_1, z_2; m_D) = \begin{bmatrix} m_D^{-1} + m_D^{-1}\partial(m_D^2 - \bar{\partial}\partial)^{-1}\bar{\partial} & -\partial(m_D^2 - \bar{\partial}\partial)^{-1} \\ -(m_D^2 - \bar{\partial}\partial)^{-1}\bar{\partial} & m_D(m_D^2 - \bar{\partial}\partial)^{-1} \end{bmatrix}_{zw}$$

$$G(z_1, z_2) = \begin{bmatrix} W^{-1} + W^{-1}\nabla\boldsymbol{\Delta}_s^{-1}\nabla^\dagger W^{-1} & -W^{-1}\nabla\boldsymbol{\Delta}_s^{-1} \\ \boldsymbol{\Delta}_s^{-1}\nabla^\dagger W^{-1} & \boldsymbol{\Delta}_s^{-1} \end{bmatrix}_{zw}$$

ISING: FREE MAJORANA FERMIONS

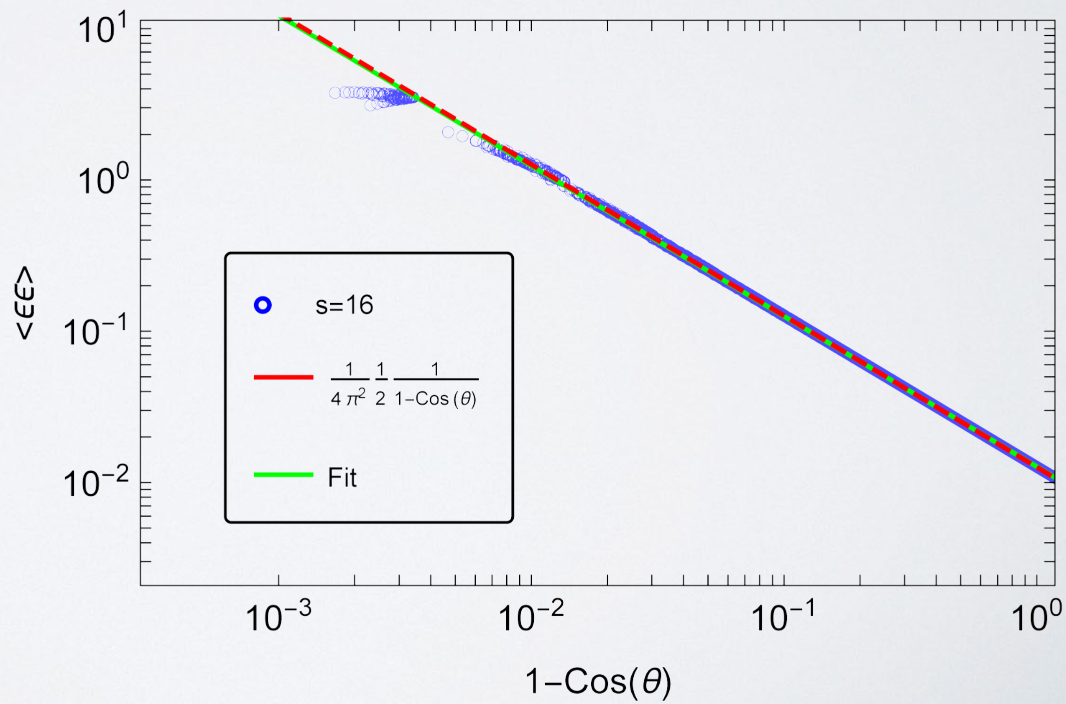
c =1/2 Minimal Model OPE: $\sigma \times \sigma = \mathbf{1} + \epsilon \quad , \quad \epsilon \times \sigma = \epsilon \quad , \quad \epsilon \times \epsilon = \mathbf{1}$

Even Ope $\epsilon(z) = i\bar{\psi}(z)\psi(z)$ Odd operator is twist $\sigma(z)$

$$S_{Dirac} = \int d^2x[\psi\partial_{\bar{z}}\psi + \bar{\psi}\partial_z\bar{\psi}]$$

$$\langle \psi(z_1)\bar{\psi}(z_1)\bar{\psi}(z_1)\psi(z_2)\rangle = \left[\frac{1}{\partial}\right]_{z_1,z_2}\left[\frac{1}{\bar{\partial}}\right]_{z_1,z_2} = \frac{1}{4\pi^2}\frac{1}{|z_1 - z_2|^2}$$

$$\langle \epsilon(z_1)\epsilon(z_2)\rangle$$



ADD TWISTS AT N/S POLES

Subtract Cos term

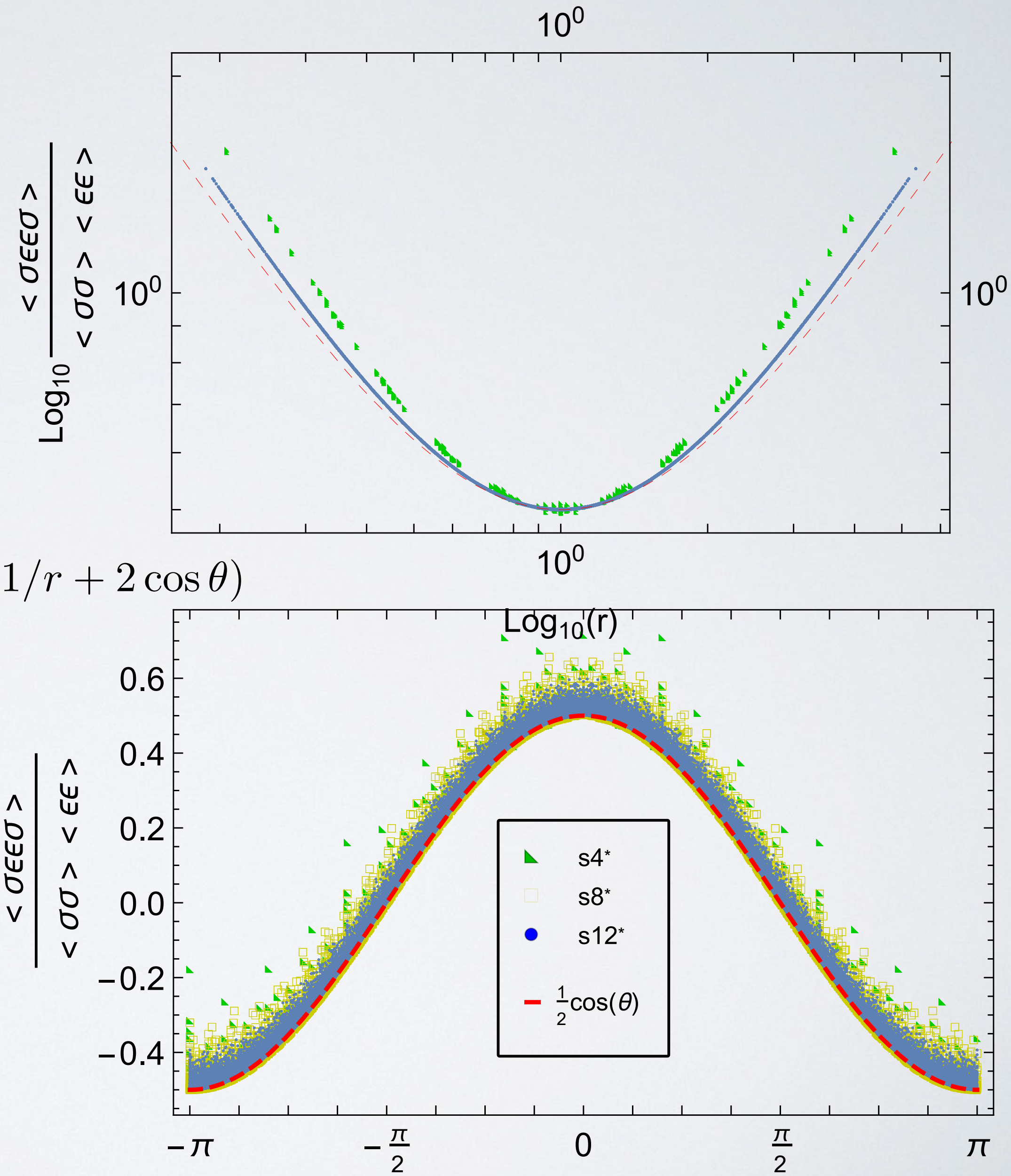
$$z_1/z_2 = re^{i\theta}$$

$$\frac{\langle\sigma(0)\epsilon(z_2)\epsilon(z_3)\sigma(\infty)\rangle}{\langle\epsilon(z_2)\epsilon(z_3)\rangle}$$
$$= \frac{1}{4}|\sqrt{z_1/z_2} + \sqrt{z_2/z_1}|^2 = \frac{1}{4}(r + 1/r + 2\cos\theta)$$

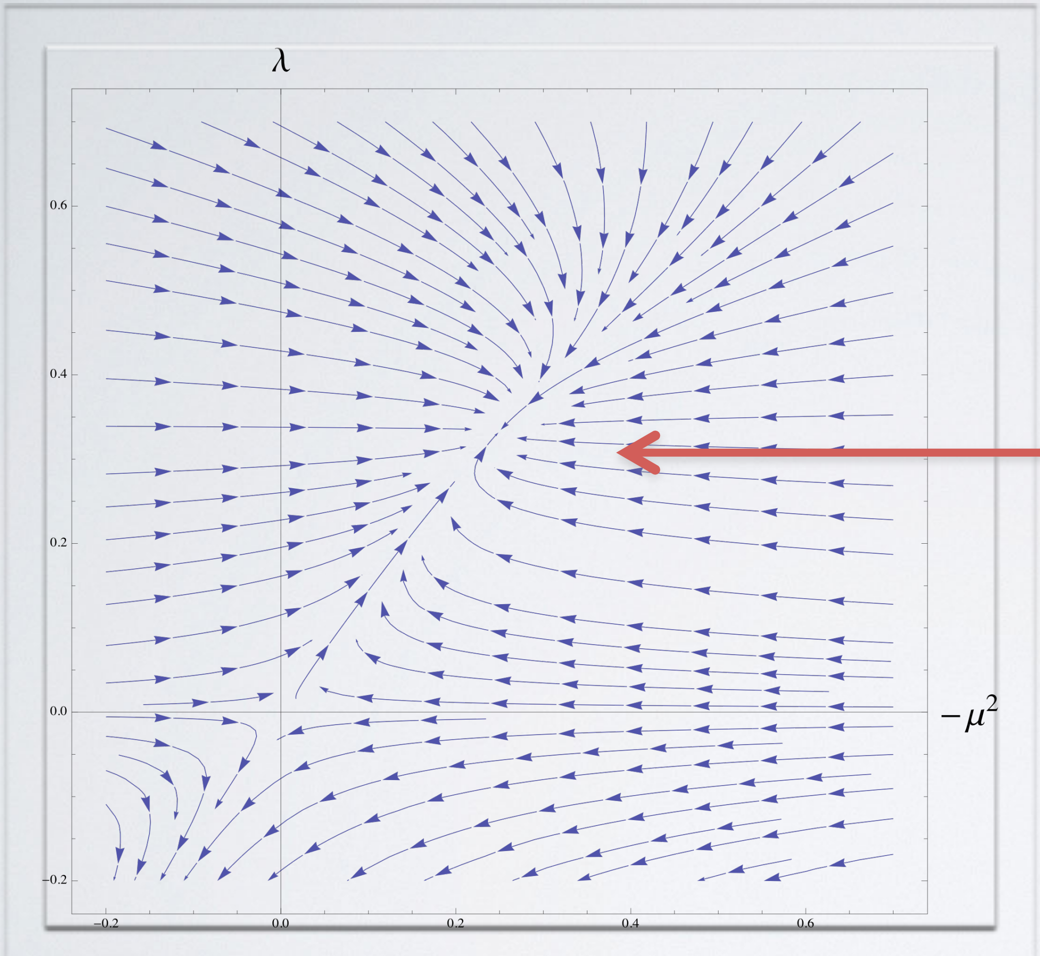
$$u = |z|^2$$

$$v = |1 - z|^2$$

Substract r term



Wilson Fisher CFT Fixed PointModel

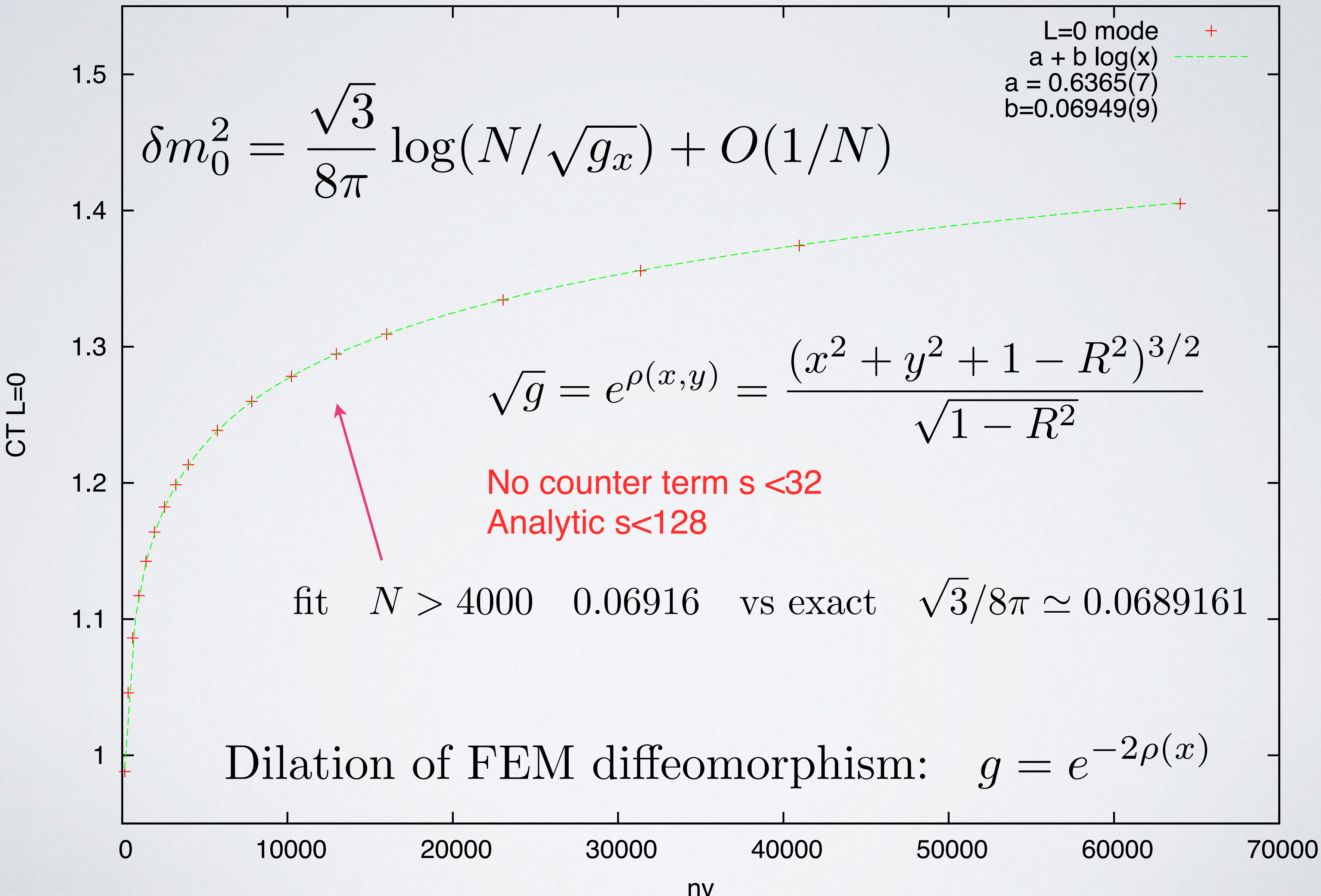


Wilson-Fisher FP

Gaussian FP

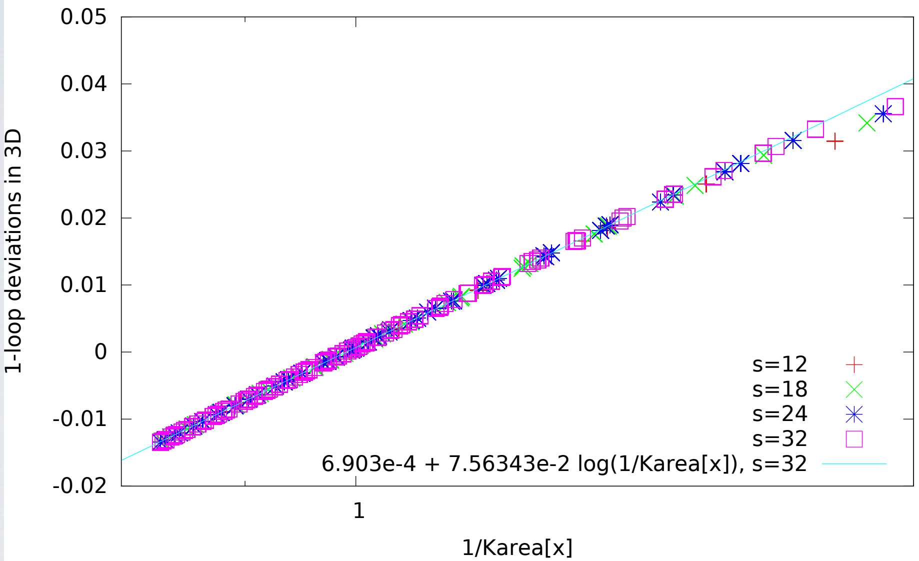
$$\mathcal{L}(x) = -\frac{1}{2}(\nabla\phi)^2 + \lambda(\phi^2 - \mu^2/2\lambda)^2$$

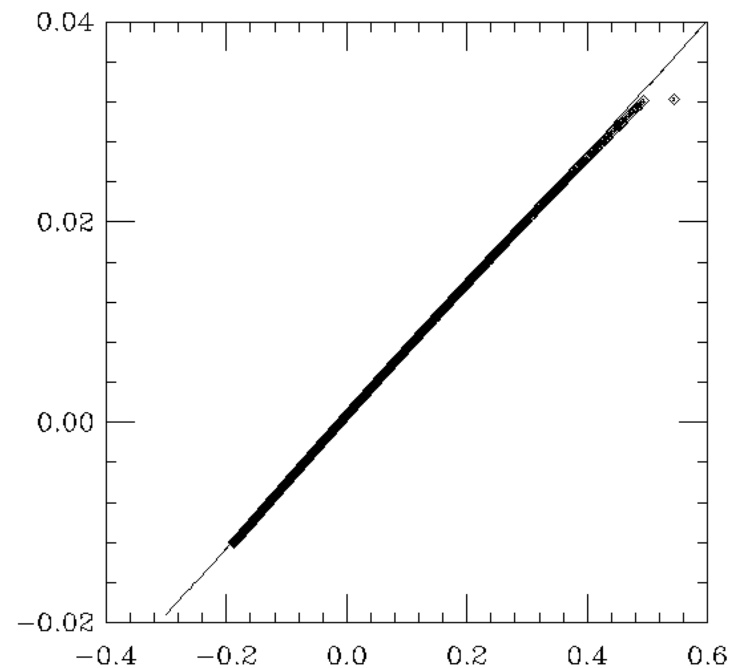
MODEL OF COUNTER TERM



NUMERICAL ONE LOOP CT

One loop counter term agains 1/Log(Kare[x])





s = 267 Counter term

NOW BINDER CUMULANT CONVERGES

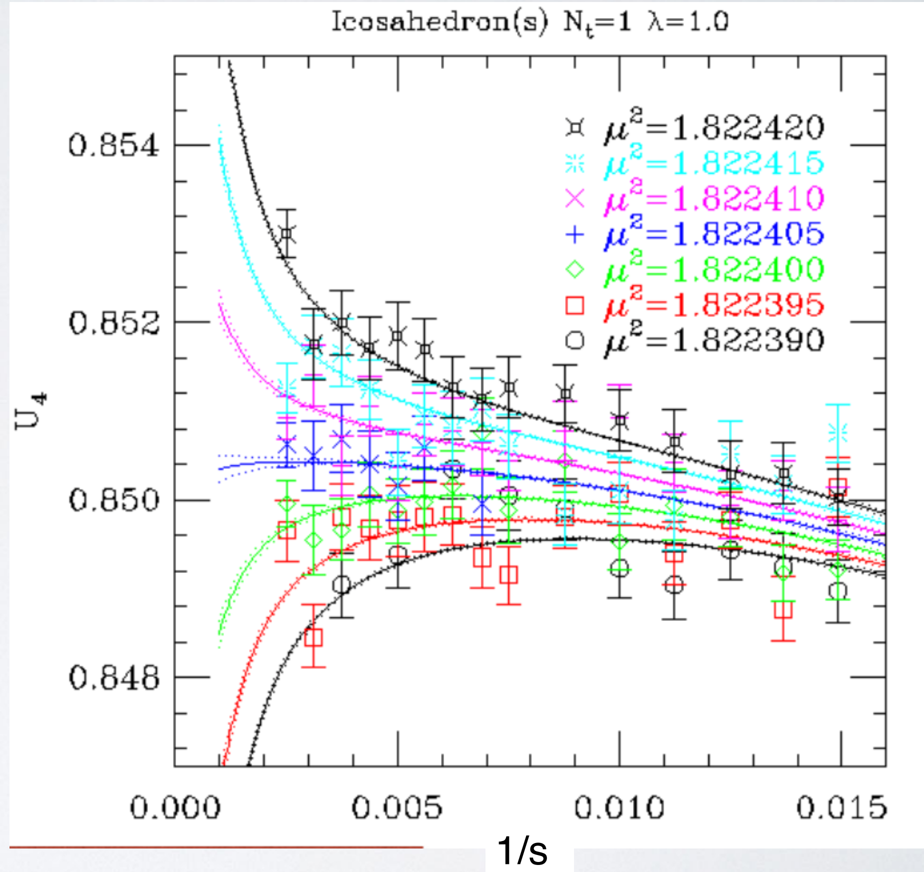
$$U_{2n}(\mu^2, \lambda, s) = U_{2n,cr} + a_{2n}(\lambda)[\mu^2 - \mu^2_{cr}]s^{1/\nu} + b_{2n}(\lambda)s^{-\omega}$$

FIT: $$U_4 = 0.85081(10)$$

EXACT: $$U_4^{exact} = 0.851021(5)$$

HIGHER MOMENT 2n = 4,6,8,10,12

$$U_6 = 0.77280(13)$$



$$U_4 = \frac{3}{2}[1 - \frac{\langle M^4 \rangle}{3\langle M^2 \rangle \langle M^2 \rangle}]$$

$$\mu^2_{cr} = 1.82240070(34)$$

Simultaneous fit for s up 800: E.G. 6,400,002 Sites on Sphere

$$dof = 1701 \quad , \quad \chi^2/dof = 1.026$$

EXACT SOLUTION TO CFT

Exact Two point function

$$\langle\phi(x_1)\phi(x_2)\rangle = \frac{1}{|x_1 - x_2|^{2\Delta}} \to \frac{1}{|1 - cos\theta_{12}|^{\Delta}}$$

$$\Delta = \eta/2 = 1/8$$

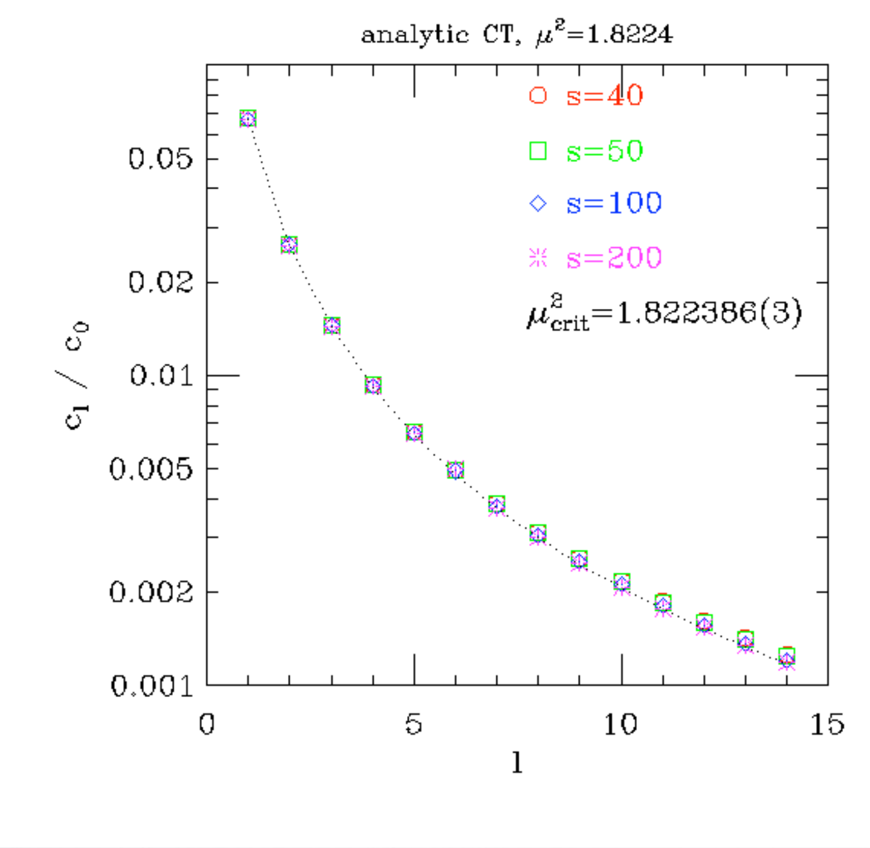
$$x^2 + y^2 + z^2 = 1$$

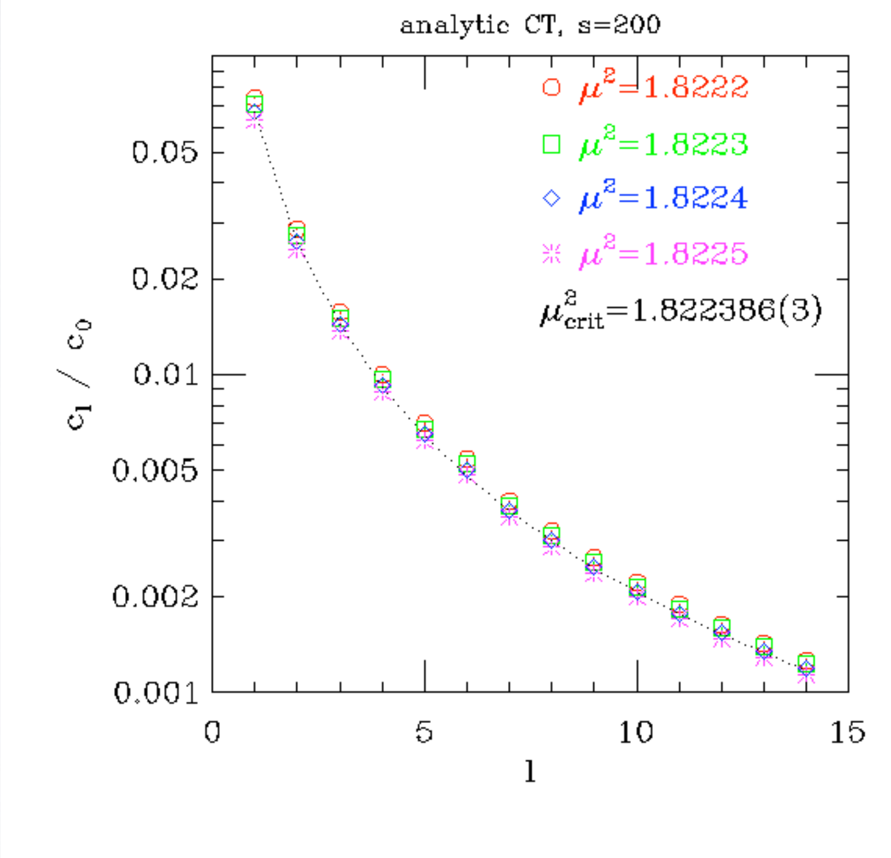
4 pt function $(x_1, x_2, x_3, x_4) = (0, z, 1, \infty)$

$$g(0, z, 1, \infty) = \frac{1}{2|z|^{1/4}|1 - z|^{1/4}}[|1 + \sqrt{1 - z}| + |1 - \sqrt{1 - z}|]$$

Critical Binder Cumulant $U_B^* = 1 - \frac{\langle M^4\rangle}{3\langle M^2\rangle\langle M^2\rangle} = 0.567336$

Dual to Free Fermion $u = \frac{r_{12}^2 r_{34}^2}{r_{13}^2 x_{24}^2}$, $v = \frac{r_{14}^2 r_{32}^2}{r_{13}^2 r_{24}^2}$ where $r_{ij}^2 = (\vec{r}_i - \vec{r}_j)^2 = 2(1 - \cos\theta_{ij})$





$$\int_{-1}^{1} dz \left(\frac{2}{1-z}\right)^{1/8} P_l(z) \qquad \Delta_\sigma = \eta/2 = 1/8 \simeq 0.128$$

$$\Longrightarrow \frac{16}{7}, \frac{16}{35}, \frac{48}{161}, \frac{816}{3565}, \frac{12240}{64883}, \frac{493680}{3049501}, \frac{33456}{234577}, \frac{55760}{435643}, \frac{3602096}{30930653}, \frac{20129360}{187963199}, \frac{541373840}{5450932771} \cdots$$

Very fast cluster algorithm:
Brower,Tamayo 'Embedded Dynamics for phi 4th Theory" PRL 1989. Wolff single cluster + plus Improved Estimators etc

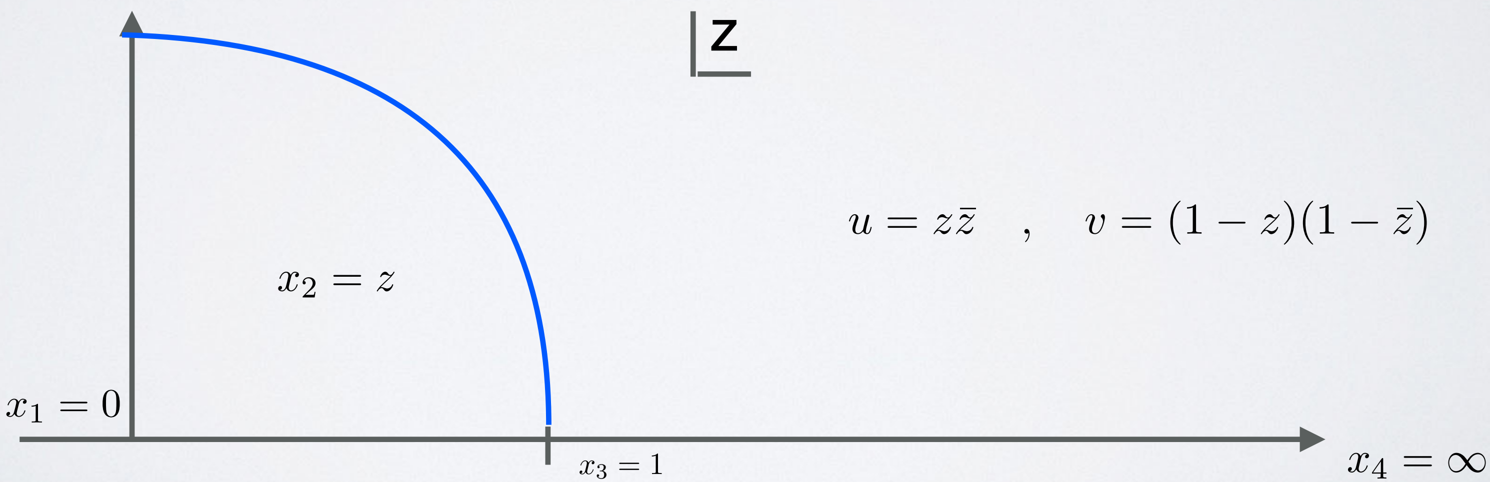
EXACT FOUR POINT FUNCTION

OPE Expansion: $\phi \times \phi = \mathbf{1} + \phi^2$ or $\sigma \times \sigma = \mathbf{1} + \epsilon$

$$g(u,v) = \frac{\langle \phi_1 \phi_2 \phi_3 \phi_4 \rangle}{\langle \phi_1 \phi_3 \rangle \, \langle \phi_2 \phi_4 \rangle}$$

$$= \frac{1}{\sqrt{2}|z|^{1/4}|1-z|^{1/4}} \left[|1+\sqrt{1-z}| + |1-\sqrt{1-z}| \right]$$

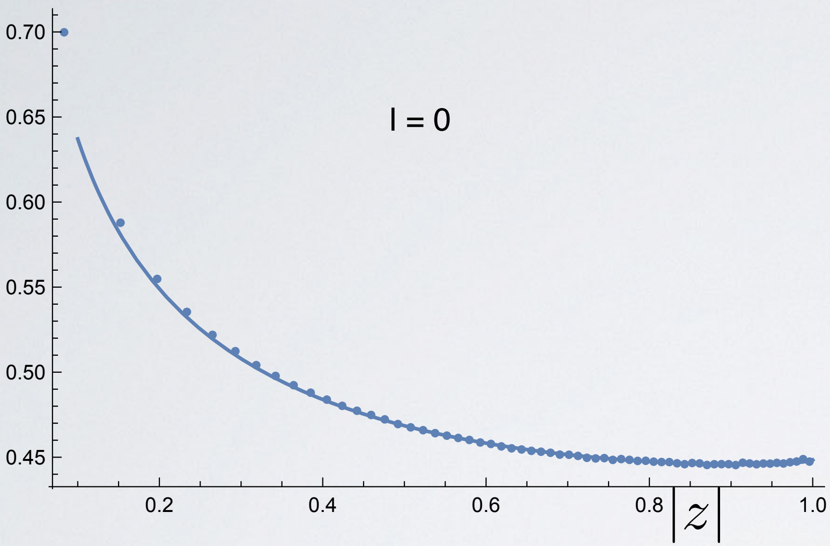
Crossing Sym: $|1+\sqrt{1-z}| + |1-\sqrt{1-z}| = \sqrt{2 + 2\sqrt{(1-z)(1-\bar{z})} + 2\sqrt{z\bar{z}}}$



$u = z\bar{z} \quad , \quad v = (1-z)(1-\bar{z})$

2 TO 2 SCATTERING DATA

$g_0(|z|)$



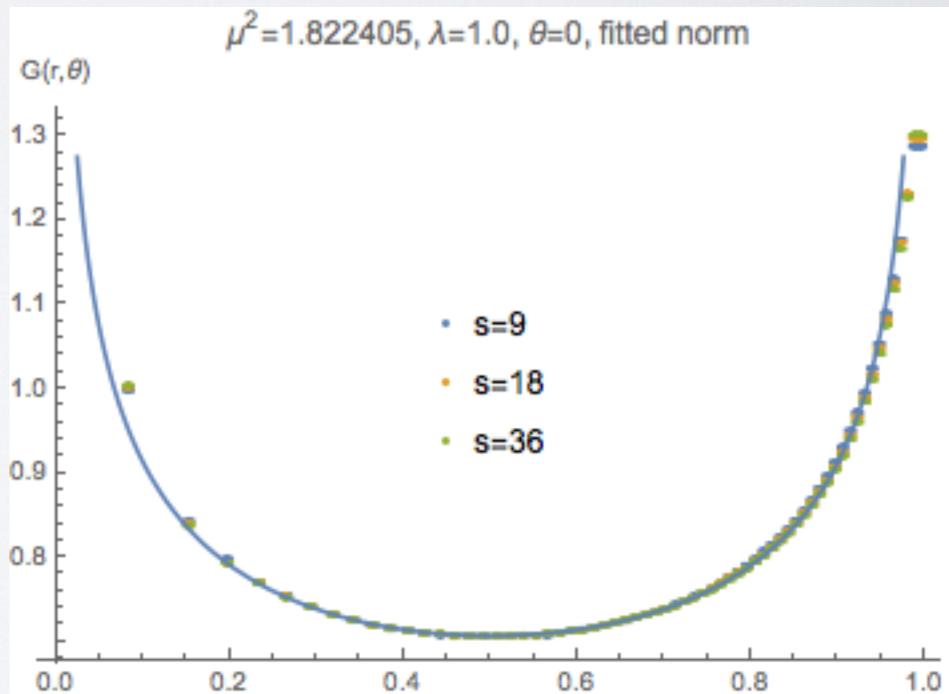
$g_l(|z|)$

ZERO PARAMETER FIT

s= 10 Run for 1/2 hour

$$g(u,v) = \sum_l g_l(|z|)\cos(l\theta)$$

$$z = |z|e^{il\theta}$$



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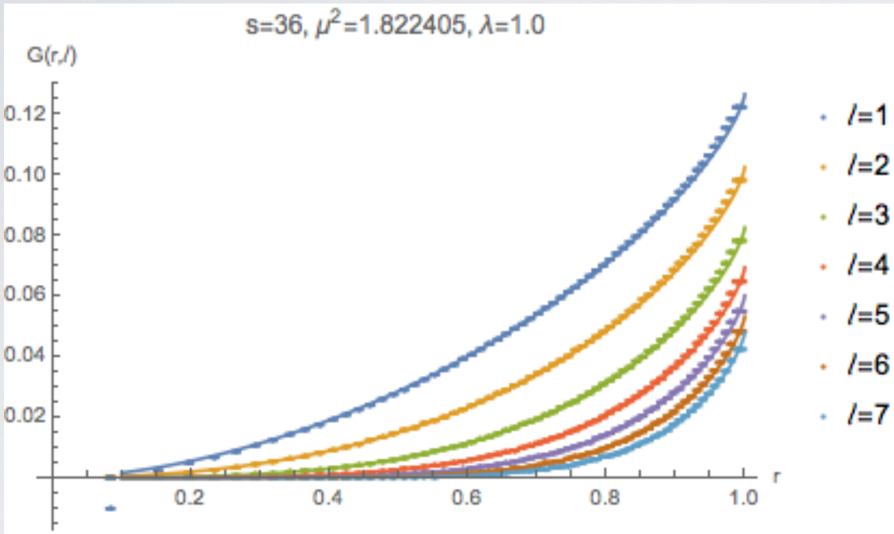

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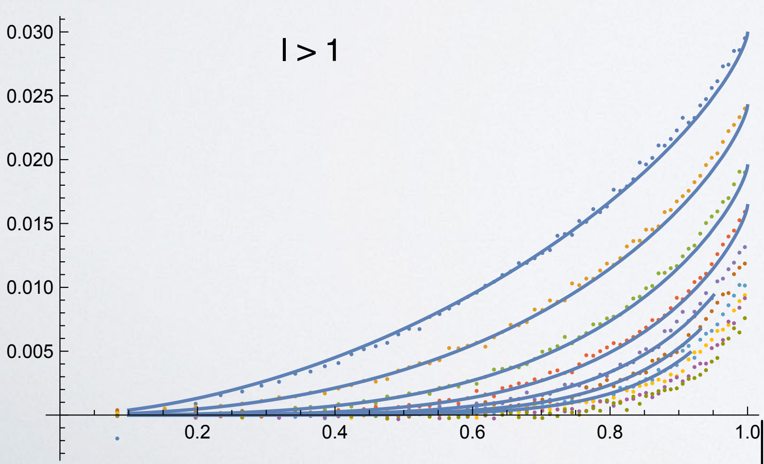
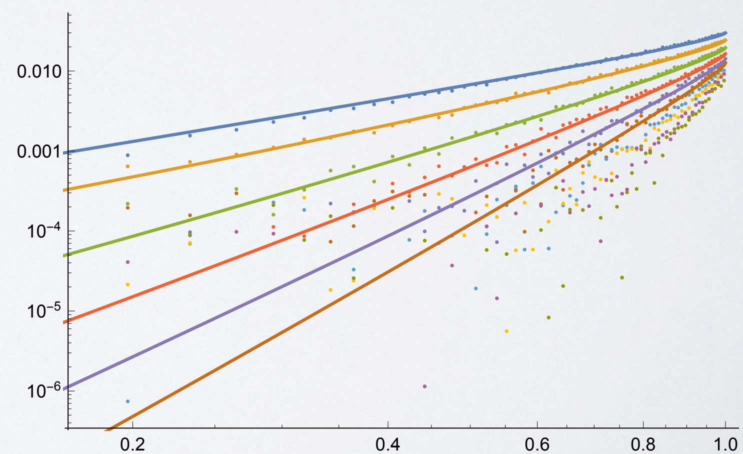
$g_l(|z|)$



$|z|$

$\log(g_l)$



$\log(|z|)$

CONCLUSIONS: QFE PLANS & DREAMS

» CT for all 2D and 3D Super Renormalizable Theories looks promising (to me). Can you prove this? You can use Pauli-Villars as well.

» Interesting problems and tests: 2+1 Radial Phi 4th/3D Ising CFT (with cluster algorithm), 2+1 Scalar/Fermionic QED. Maybe 2D SUSY ?

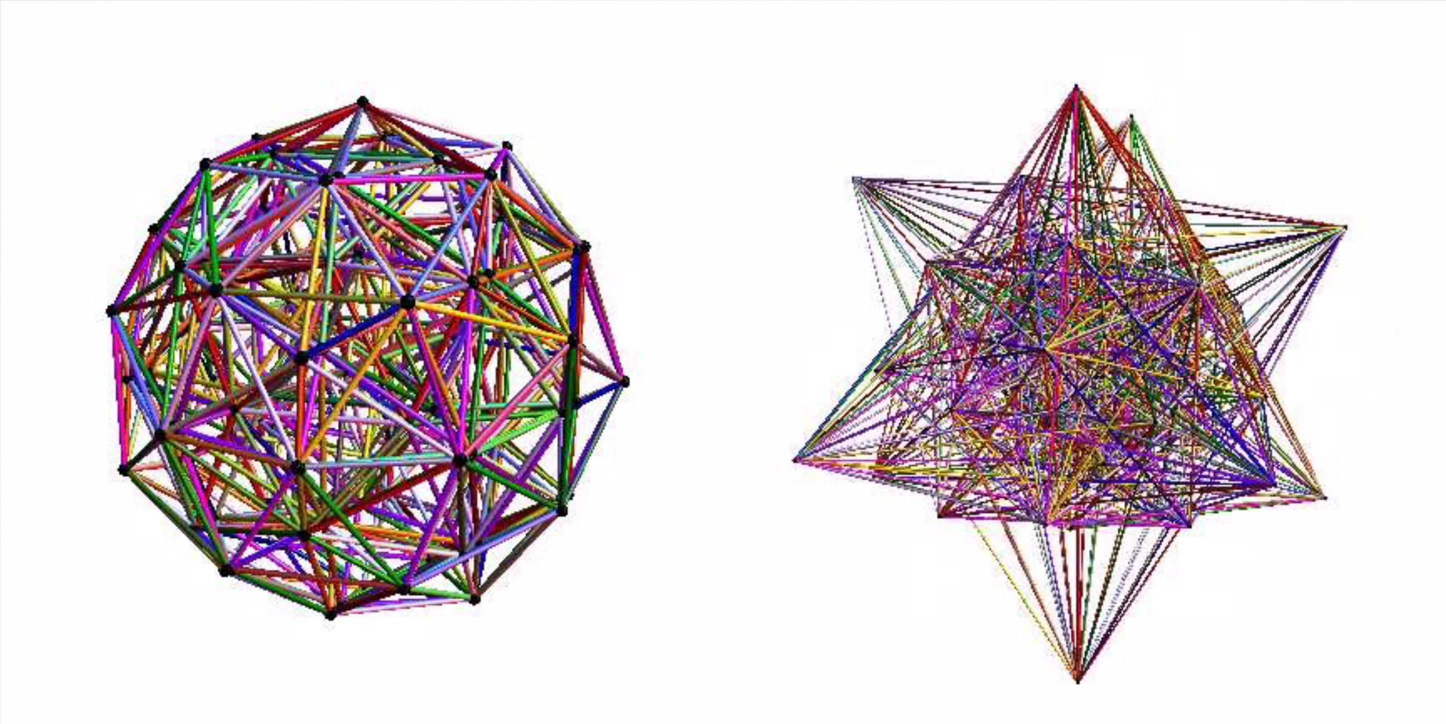
» 3D Sphere starting with 600 cell: 3D sphere or 1 + 3 Radial QFT.

» Want to study CFTR at 4D IR fixed point in BSM models. BUT 4D "counter terms" are hard/impossible for calculate? Ideas needed: Wilson Flow and NSPT?

» Classify all CT that break diffeomorphism breaking in RC Geometry. Do Fermions measure the breaking and give new RC formulation.

600 CELL ON S3

HTTPS://EN.WIKIPEDIA.ORG/WIKI/600-CELL



16 vertices of the form:[3] (±½, ±½, ±½, ±½),

8 vertices obtained from (0, 0, 0, ±1) by permuting coordinates.

96 vertices are obtained by taking even permutations of ½ (±ϕ, ±1, ±1/ϕ, 0).

https://en.wikipedia.org/wiki/List_of_regular_polytopes_and_compounds#Five-dimensional_regular_polytopes_and_higher

EXTRA SLIDES!

NEED TO IMPROVE QUANTUM LAGRANGIAN
3 POSSIBLE SOLUTIONS?

(i) Pauli-Villars* 1949

$$\frac{1}{p^2} - \frac{1}{p^2 + M_{PV}^2} \equiv \frac{1}{p^2 + p^4/M_{PV}^2}$$

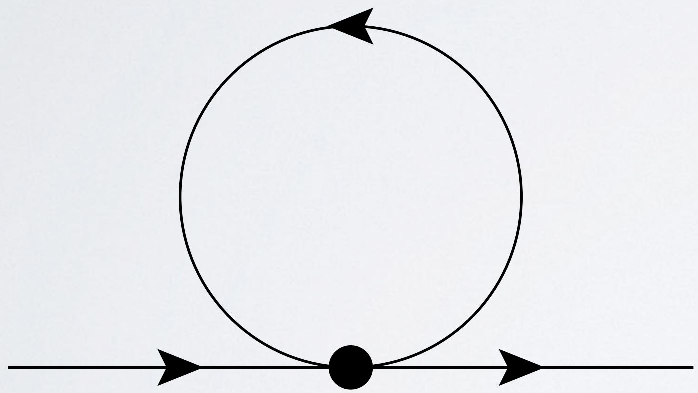
(ii) Subtract x-dependent mass Counter term

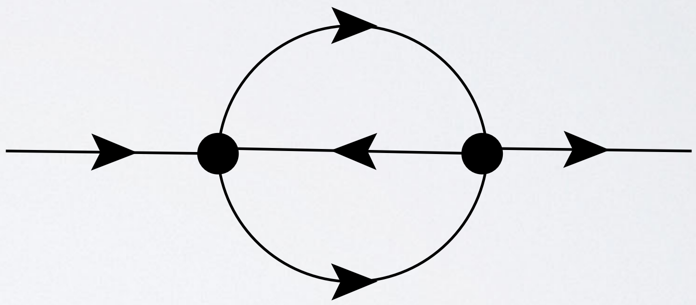
$$1/\xi \ll M_{PV} \ll \pi/a$$

(iii) New methods need for D = 4 & Non-Abelian Gauge Theory

(*Richard Feynman, Ernst Stueckelberg)

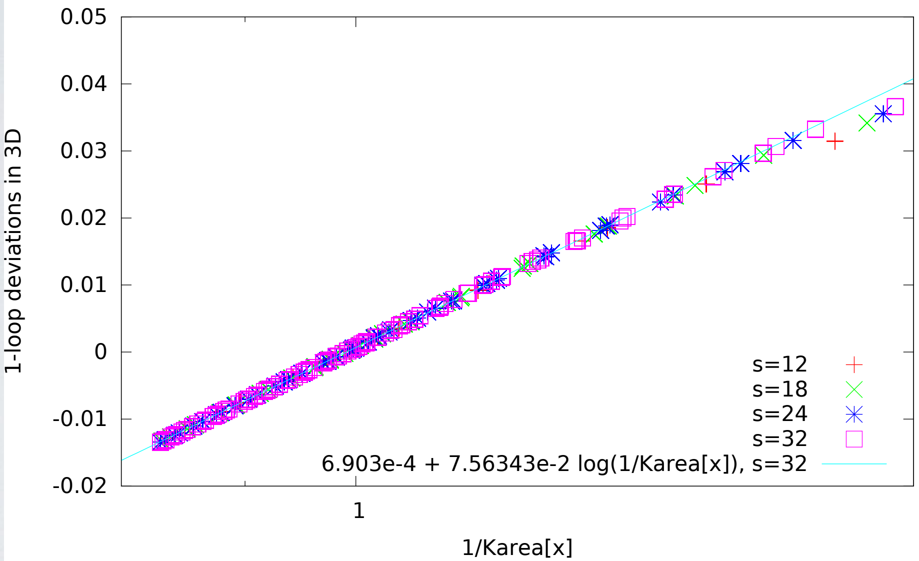
COUNTER TERM IN 3D

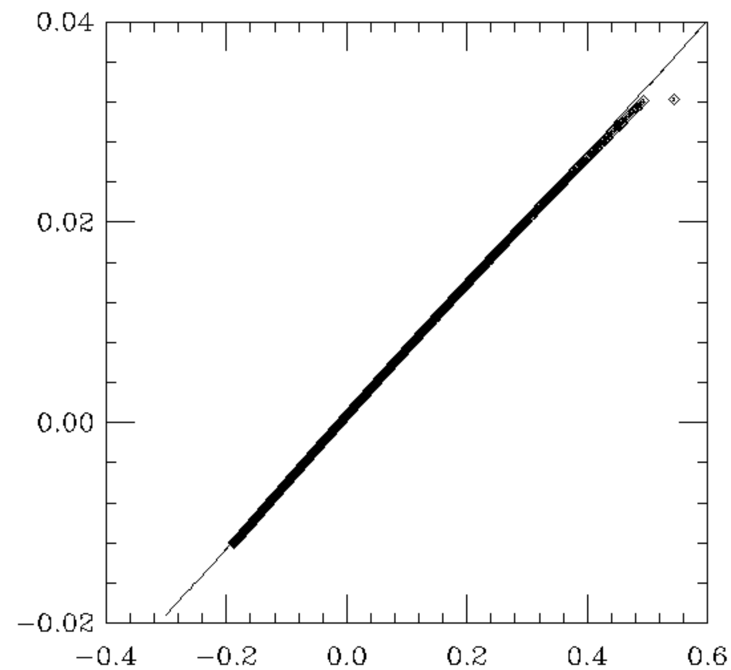




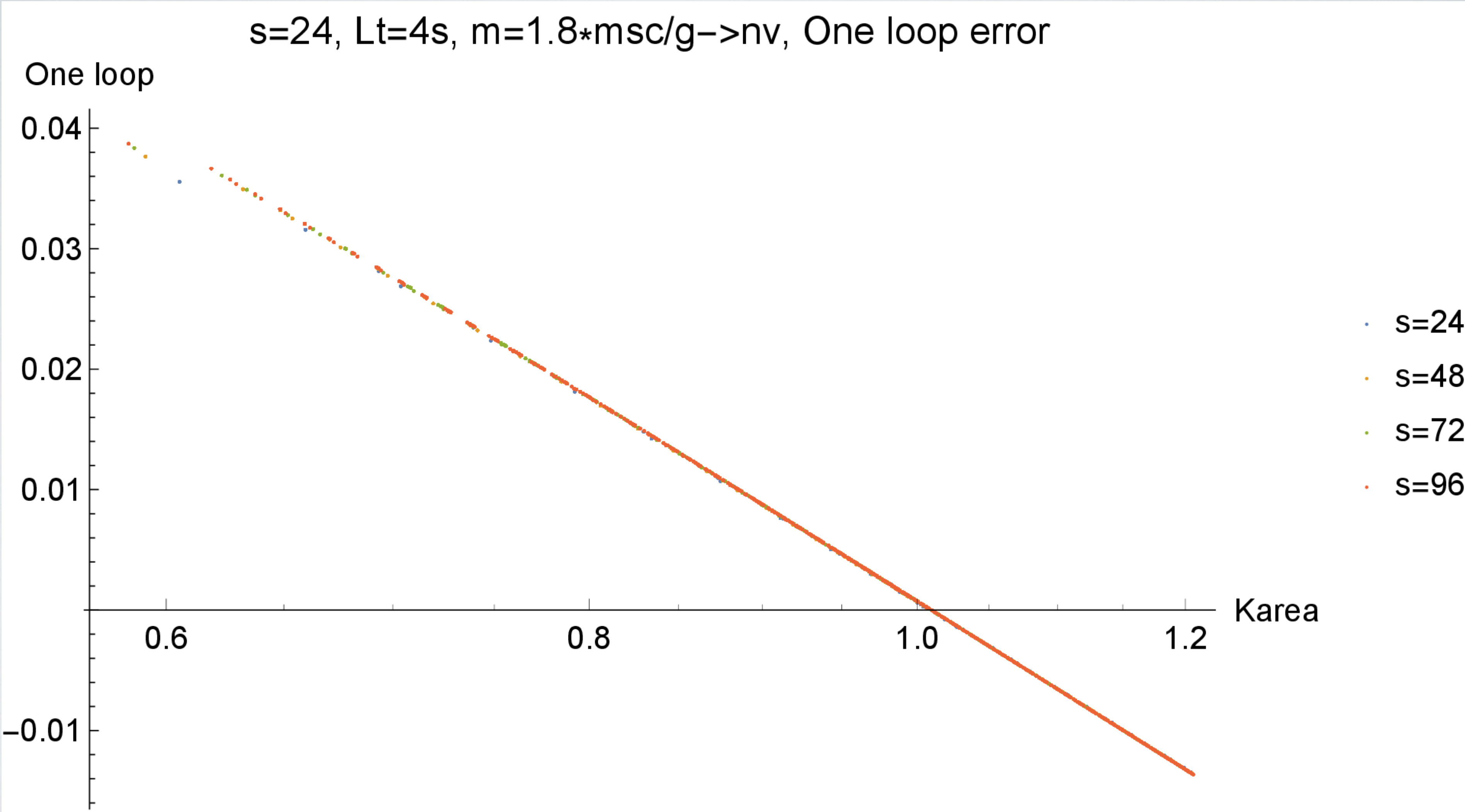
NUMERICAL ONE LOOP CT

One loop counter term agains 1/Log(Kare[x])

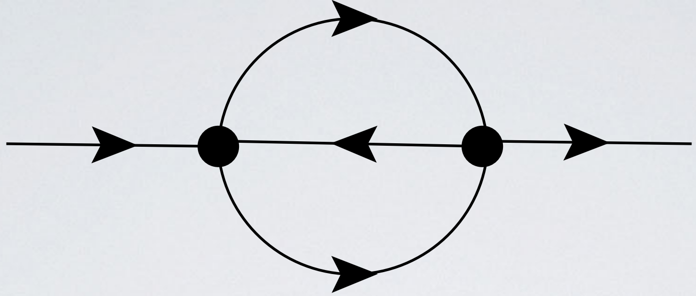


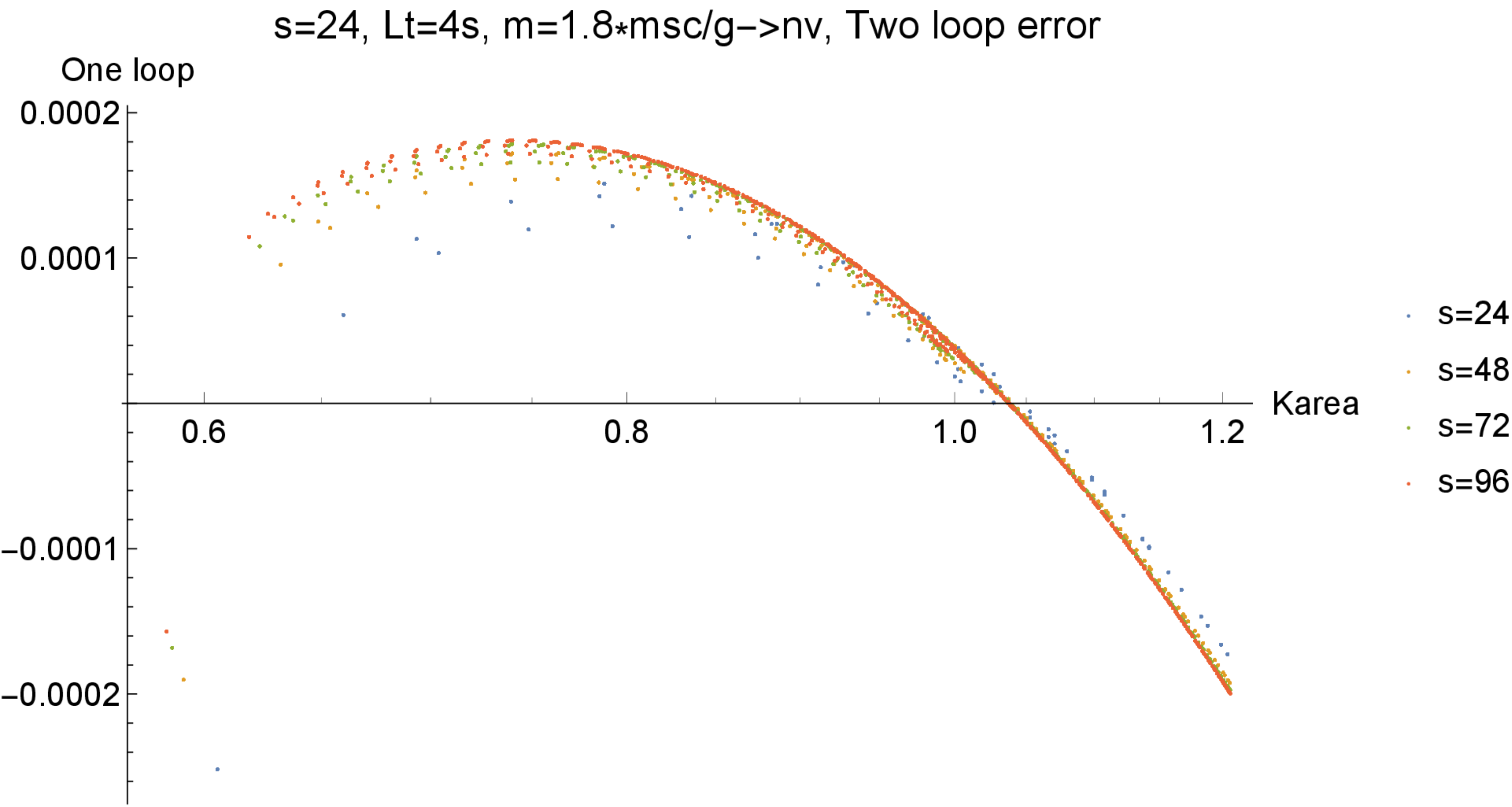


s = 267 Counter term



s=24, Lt=4s, m=1.8*msc/g->nv, One loop error
One loop
0.04
0.03
0.02
0.01
−0.01
0.6
0.8
1.0
1.2
Karea
s=24
s=48
s=72
s=96




s=24, Lt=4s, m=1.8*msc/g->nv, Two loop error
One loop
0.0002
0.0001
-0.0001
-0.0002
0.6
0.8
1.0
1.2
Karea
s=24
s=48
s=72
s=96