

Heavy Quarks with Domain Wall fermions

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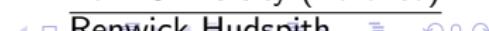
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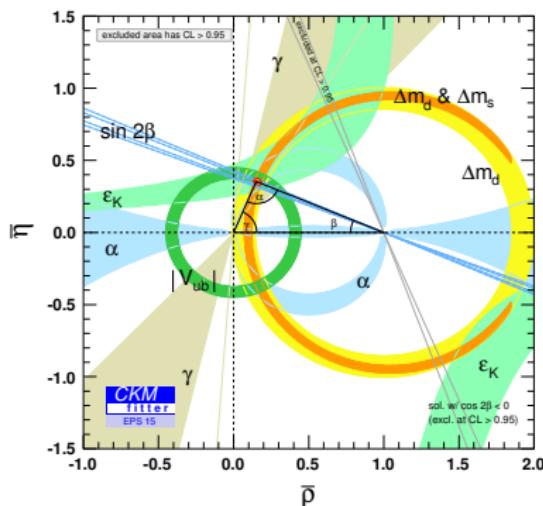
Outline

1 Motivation

2 Heavy Domain Wall Fermions

3 Decay Constant Analysis

Motivation - Flavour Physics



CKMfitter Group (J. Charles et al.), Eur. Phys. J. C41, 1-131 (2005) [hep-ph/0406184], updated results and plots available at: <http://ckmfitter.in2p3.fr>

Experiment

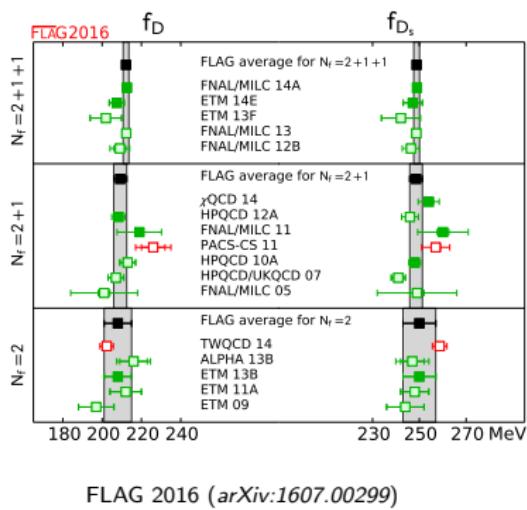
- Belle, BaBar, CLEO-c
- LHCb, Belle II

Theory

- K , D and B physics to test unitarity of the CKM matrix.

⇒ Place tight bounds on SM predictions

D and B physics



Goal: $D_{(s)}$ and $B_{(s)}$ phenomenology

- masses
 - decay constants
 - semi-leptonics

Outcome:

- Test of SM
 - Reduce systematic errors by direct computation

Charm physics in the RBC/UKQCD program

- **Charm phenomenology:**
 f_D , f_{D_s} , semi-leptonics
- **Bottom phenomenology via the ratio method** ([arXiv:0909.3187](#)):
 f_B , f_{B_s} , Bag, ξ , semi-leptonics
- 2+1+1f ensemble generation
- GIM mechanism for Rare Kaons (\Rightarrow need same action for charm and light)
- Charm contribution to the HVP
- Semi-leptonic B decays in RHQ e.g. $B \rightarrow D$

Motivation: Domain Wall Fermions

DWFs provide a method to simulate (approximately) chiral fermions on the lattice

PROS:

- Used for light and strange sea quarks ($N_f = 2 + 1$)
- Automatic $\mathcal{O}(a)$ improvement
- No operator mixing:
⇒ easier renormalisation

CONS:

- More expensive due to fifth dimension
- ⇒ Introduces additional tunable parameters: L_s , M_5

DWF - a (very) brief introduction

- Introduce a finite fifth dimension of length L_s .
- Choose the **Domain Wall Height** M_5
- LH and RH massless modes exponentially localised at boundaries.
- A measure of the **residual chiral symmetry breaking** is given by m_{res} :

$$am_{\text{res}}(t) = \frac{\sum_x \langle J_{5q}(x)P(0) \rangle}{\sum_x \langle P(x)P(0) \rangle}$$

(where J_{5q} is the pseudoscalar density in 5th dimension)

Quenched Pilot Study

IDEA

- Map out parameter space of DWF suitable for heavy quarks
- Interested in cut-off effects \Rightarrow keep $L \approx \text{const}$
- Test continuum scaling of basic observables
- Expect similar behaviour in the dynamical case

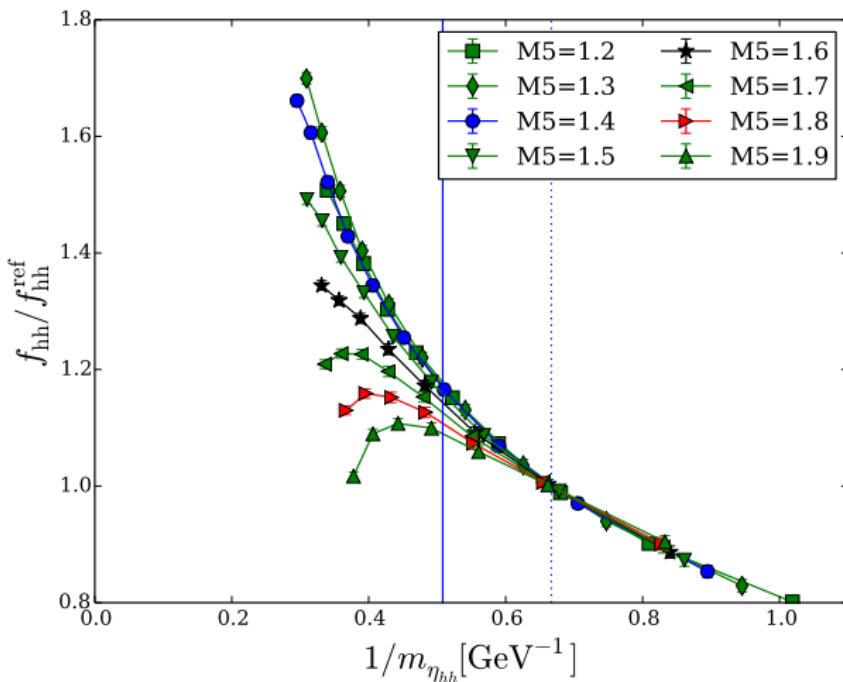
β	L/a	$a^{-1}(\text{GeV})$	$L(\text{fm})$
4.41	16	2.0	1.55
4.66	24	2.9	1.66
4.89	32	3.9	1.63
5.20	48	5.7	1.65

- tree-level Symanzik improved gauge action
- Over-relaxation heat bath

JHEP **05** (2015) 072 (arXiv:1504.01630 [hep-lat])
JHEP **04** (2016) 037 (arXiv:1602.04118 [hep-lat])

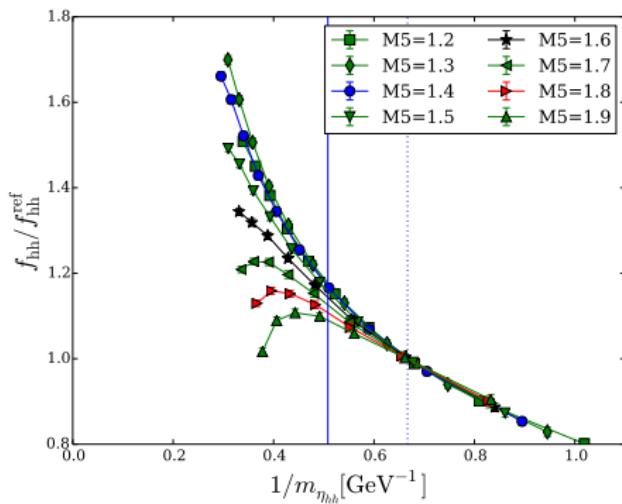
M_5 -Scan

$$\beta = 4.41$$

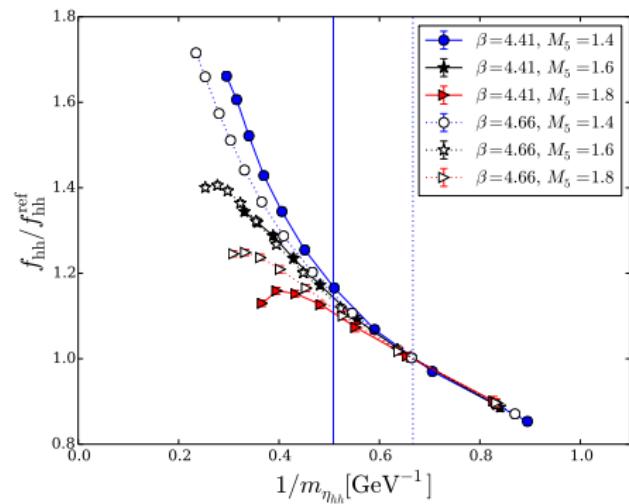


M_5 -Scan

$\beta = 4.41$

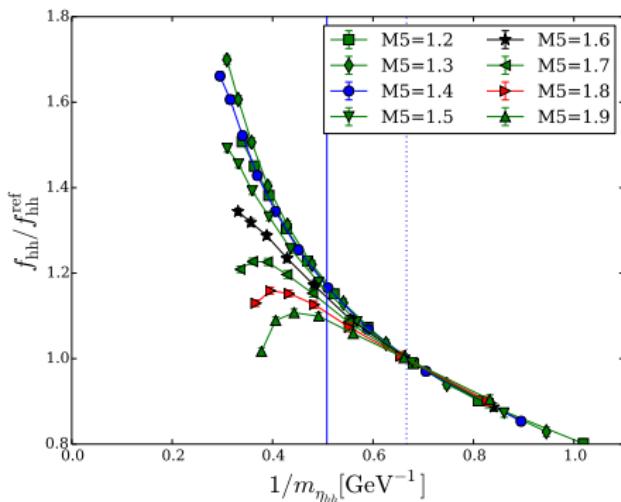


$\beta = 4.41, 4.66$

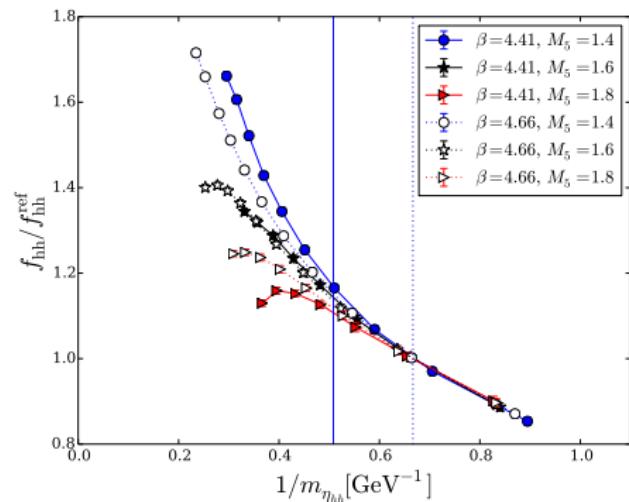


M_5 -Scan

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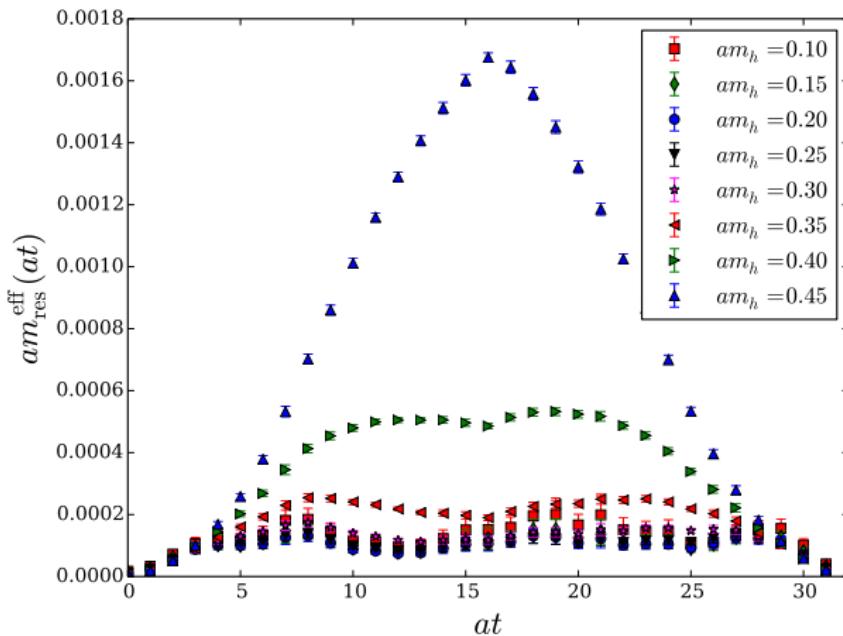
$\beta = 4.41, 4.66$



\Rightarrow Expect flat continuum limit for $M_5 = 1.6$

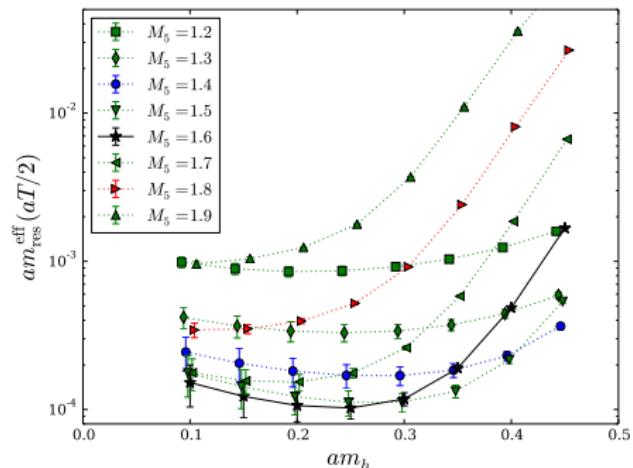
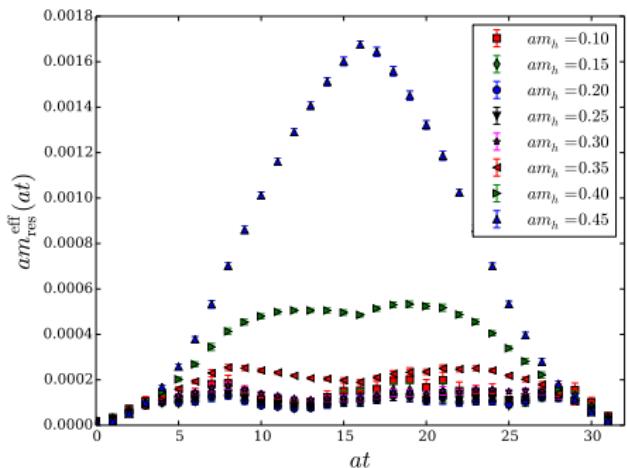
Residual mass behaviour

$$\beta = 4.41, \quad M_5 = 1.6$$



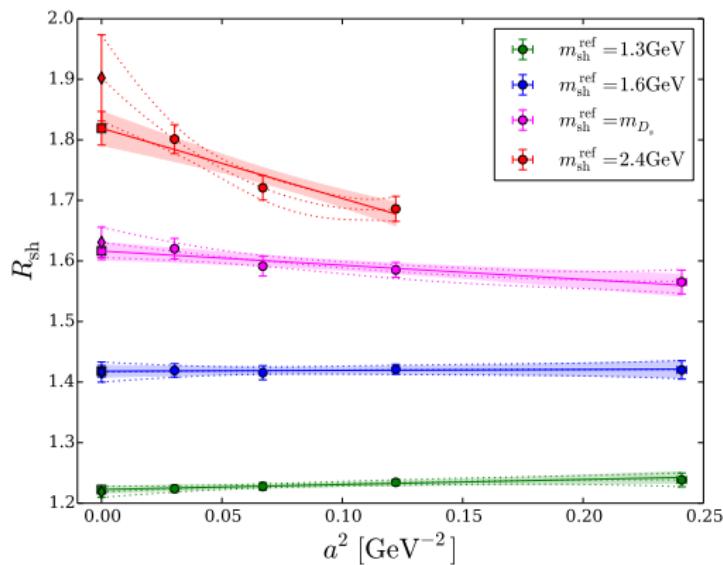
Residual mass behaviour

$$\beta = 4.41$$



\Rightarrow Limitation of $am_h \lesssim 0.4$

Continuum Limit of decay constants



- Consider

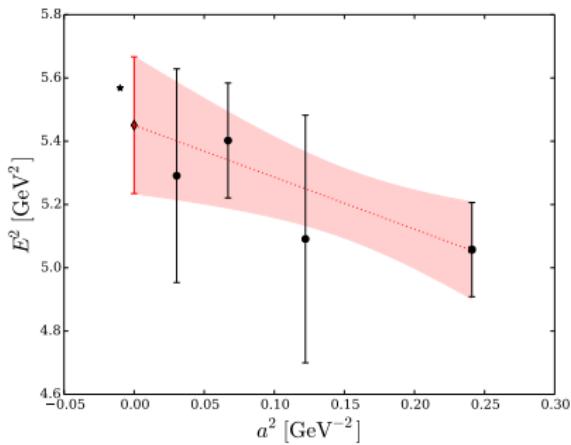
$$R_{sh} = \frac{f_{sh} \sqrt{m_{sh}}}{f_{sh}^{\text{norm}} \sqrt{m_{sh}^{\text{norm}}}},$$

with $m_{sh}^{\text{norm}} = 1.0 \text{ GeV}$

- Very flat continuum limit
- can simulate charm even on the coarsest ensemble

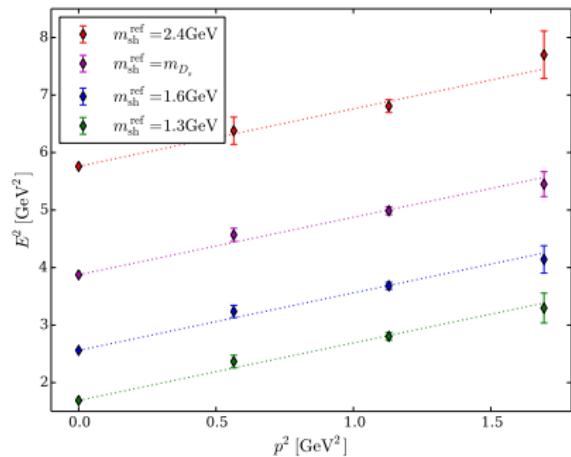
Continuum Limit of dispersion relation

Example Continuum Limit



$\mathbf{p} = \frac{2\pi}{L}(1, 1, 1)$ and $m_{sh} = m_{D_s}$

Comparison to Continuum

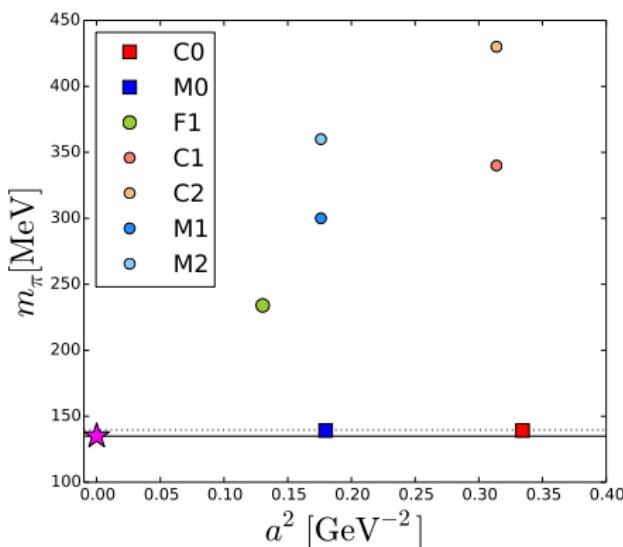


Summary of quenched pilot study

- Can simulate charm quarks with Moebius DWFs:
- Found sweet-spot in DWF parameter space to be $\underline{M_5 = 1.6}$
- Limitation: $\underline{am_h \leq 0.4}$
 \Rightarrow Exponentially localised
- Flat CL for decay constants and dispersion relation
- Expect same features in dynamical case:
 \Rightarrow Set up charm program on RBC/UKQCD's dynamical 2+1f ensembles

Dynamical Ensembles ($N_f = 2 + 1$)

2 ensembles with physical pion masses



$L^3 \times T/a^4$	a^{-1}/GeV	m_π/MeV
$48^3 \times 96$	1.73	139
$24^3 \times 64$	1.78	340
$24^3 \times 64$	1.78	430
$64^3 \times 128$	2.36	139
$32^3 \times 64$	2.38	300
$32^3 \times 64$	2.38	360
$48^3 \times 96$	2.77	230

(C0 + M0: arXiv:1411.7017)

Set up of Simulation

Valence Quarks:

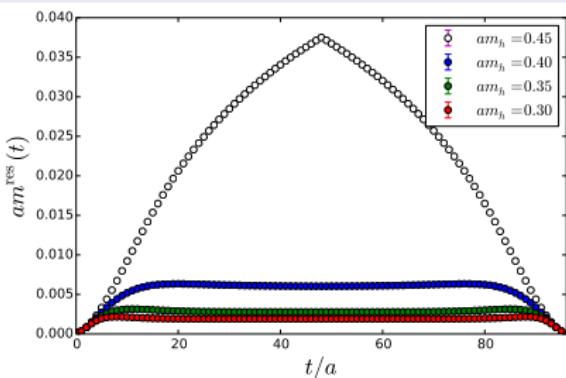
- light and strange: unitary ($M_5 = 1.8$)
 - heavy from quenched study: $am_h \leq 0.4$, $M_5 = 1.6$, $L_s = 12$
 - Expect same qualitative features in $N_f = 2 + 1$
- ⇒ **Mixed action** between the (light+strange) and the heavy quark sector.

Computational Details:

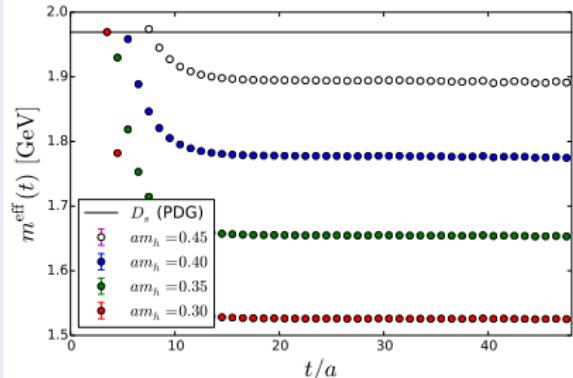
- Volume averaging by using \mathbb{Z}_2 -Wall sources.
- HDCG ([arXiv:1402.2585](#)) for light and strange. CG for heavy
- Monitored time-slice residual ([arXiv:0508023](#)):
$$r_t = \max_t \frac{|D\psi - \eta|_t}{|\psi|_t}$$
- Many source position per configuration

Check bound on am_h in dynamical case (C0)

Residual mass ($N_f = 2 + 1$)



reach in m_h

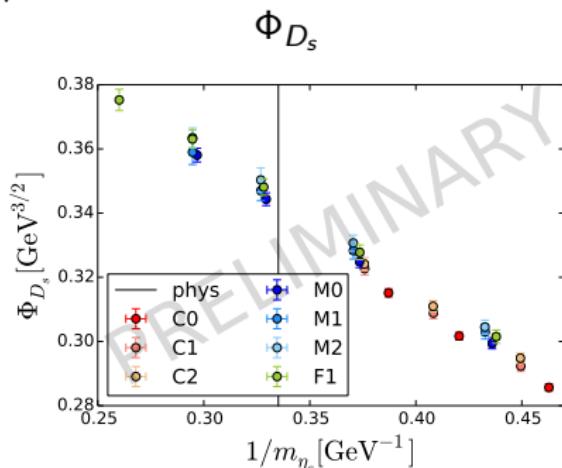
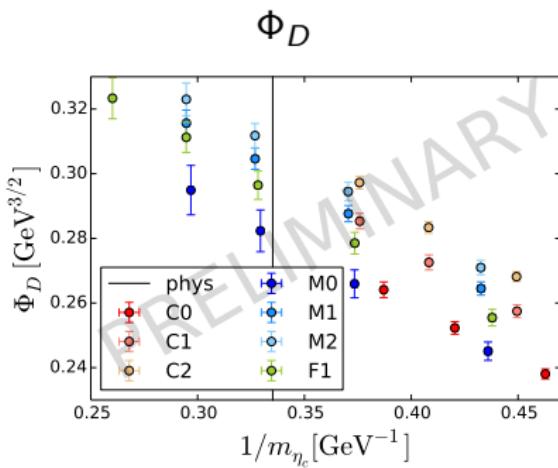


Restriction $am_h \lesssim 0.4$ in agreement with quenched
 \Rightarrow **Cannot reach physical charm on the coarsest ensembles**

Decay Constant Data

Masses and MEs from combined fit to $\langle AP \rangle$ and $\langle PP \rangle$ including 1st excited states: $\mathcal{O}(a, m_l, m_s, m_h)$.¹

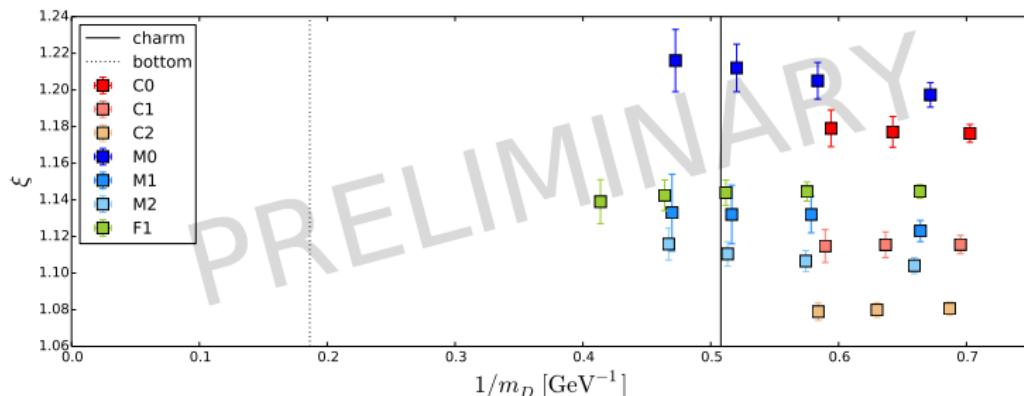
$$\Phi_P = f_P \sqrt{m_P}$$



¹so far: Z_A from conserved light-light current

Ratio of Bag parameters ξ

$$B_P = \frac{\langle P^0 | O_{VV+AA} | \bar{P}^0 \rangle}{\frac{8}{3} f_P^2 m_P^2}, \quad \xi = \frac{f_{hs} \sqrt{B_{hs}}}{f_{hl} \sqrt{B_{hl}}}$$



PLAN:

- Renormalisation of mixed action for Bag parameters.
- Extrapolate to B via ratio method. arXiv:0909.3187

Analysis

$$\mathcal{O}(a, m_I, m_s, m_h) \rightarrow \mathcal{O}(a = 0, m_I^{\text{phys}}, m_s^{\text{phys}}, m_c^{\text{phys}})$$

- Extrapolate to m_s^{phys} (from global fit arXiv:1411.7017).
- Global fit: CL+heavy quark mass + light quark mass

$$\begin{aligned} \mathcal{O}(a, m_I, m_h) &= \mathcal{O}(0, m_I^{\text{phys}}, m_h^{\text{phys}}) \\ &\quad + C_{CL}^0 a^2 \\ &\quad + C_\chi^0 \left(m_\pi^2 - m_\pi^{2\text{phys}} \right) \\ &\quad + C_{P_h}^0 \Delta m_{P_h}^{-1} \end{aligned} \tag{1}$$

where $\Delta m_{P_h}^{-1} = m_{P_h}^{-1} - m_{P_h}^{\text{phys}}{}^{-1}$ with $P_h = D, D_s, \eta_c$

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Strange Quark Mass Correction

- Slight mistuning between unitary and physical strange quark mass.

ensemble	am_s^{unitary}	am_s^{physical}	mismatch
coarse	0.03620	0.03580(16)	-1.1%
medium	0.02661	0.02539(17)	-4.8%
fine	0.02144	0.02132(17)	-0.6%

- Parameterise mistuning in terms of dimensionless α :

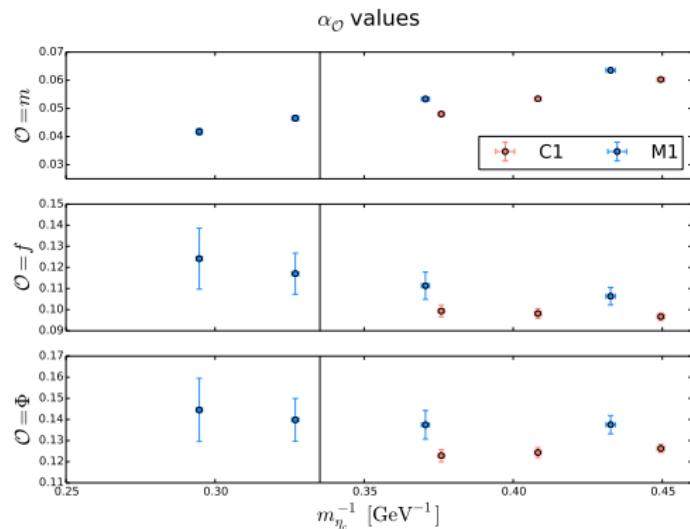
$$\mathcal{O}^{\text{phys}} = \mathcal{O}^{\text{uni}} \left(1 + \alpha_{\mathcal{O}} \frac{m_s^{\text{phys}} - m_s^{\text{uni}}}{m_s^{\text{phys}}} \right)$$

- Find $\alpha_{\mathcal{O}}$ from subset of ensembles and apply to others.

Strange Quark Mass Correction

$$\mathcal{O}^{\text{phys}} = \mathcal{O}^{\text{uni}} \left(1 + \alpha_{\mathcal{O}} \frac{m_s^{\text{phys}} - m_s^{\text{uni}}}{m_s^{\text{phys}}} \right)$$

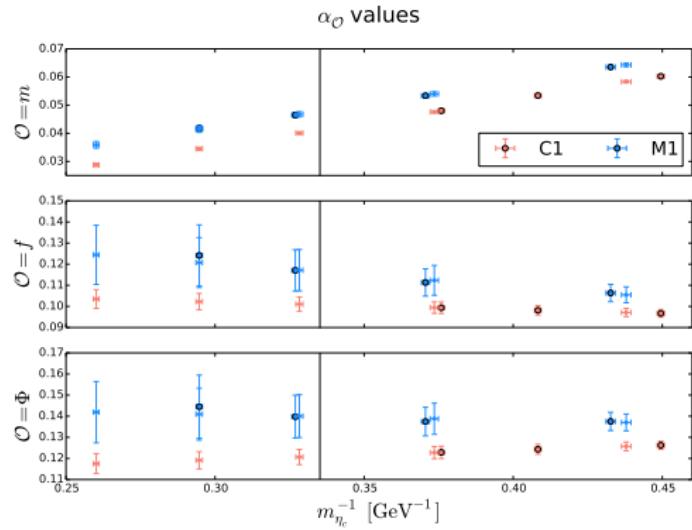
- Calculate 2 values of m_s on C1 and M1



Strange Quark Mass Correction

$$\mathcal{O}^{\text{phys}} = \mathcal{O}^{\text{uni}} \left(1 + \alpha_{\mathcal{O}} \frac{m_s^{\text{phys}} - m_s^{\text{uni}}}{m_s^{\text{phys}}} \right)$$

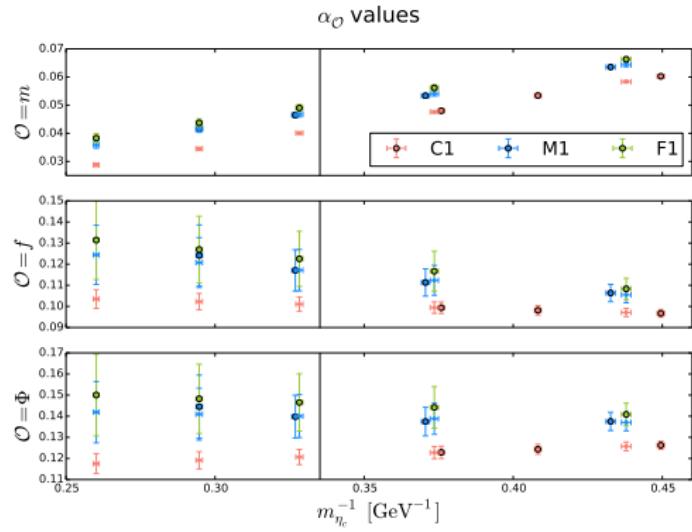
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- Extrapolate to F1 masses



Strange Quark Mass Correction

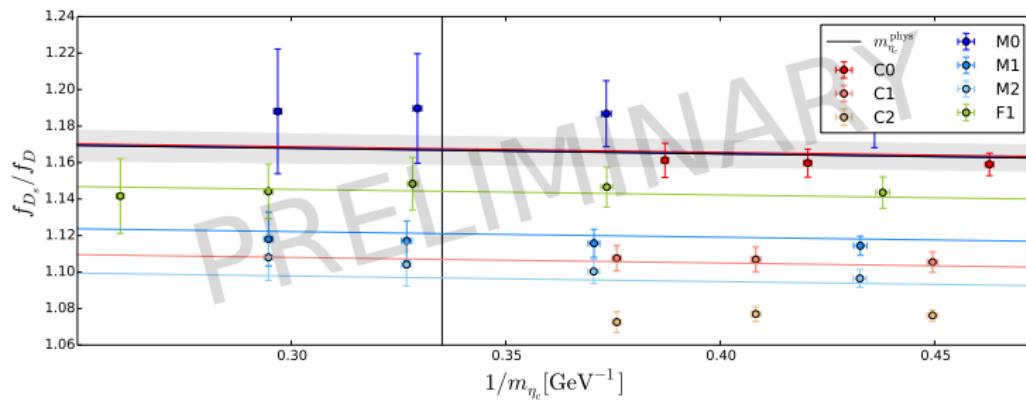
$$\mathcal{O}^{\text{phys}} = \mathcal{O}^{\text{uni}} \left(1 + \alpha_{\mathcal{O}} \frac{m_s^{\text{phys}} - m_s^{\text{uni}}}{m_s^{\text{phys}}} \right)$$

- Calculate 2 values of m_s on C1 and M1
- Extrapolate to F1 masses
- Extrapolate to F1 lattice spacing



f_{D_s}/f_D Global Fit

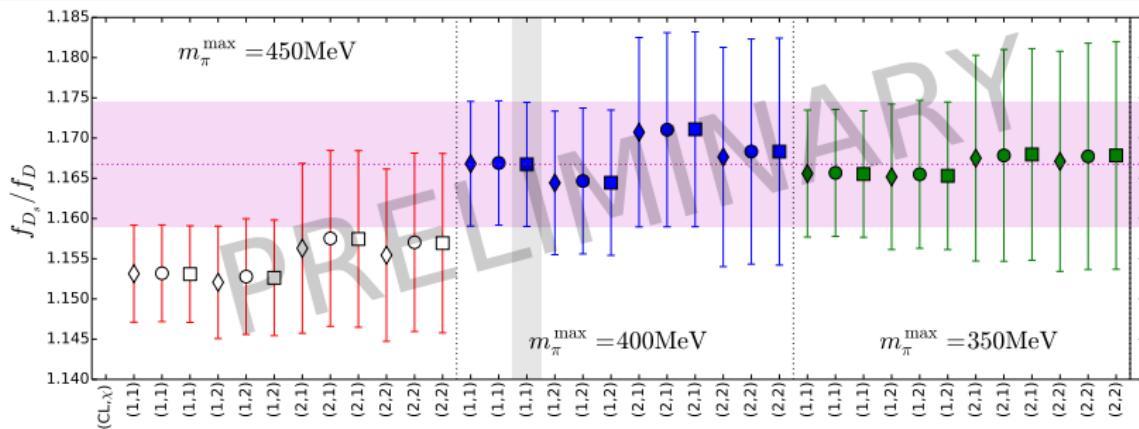
$$\mathcal{O}(a, m_I, m_h) = \mathcal{O}(0, m_I^{\text{phys}}, m_h^{\text{phys}}) + C_{CL}^0 a^2 + C_\chi^0 \Delta m_\pi^2 + C_{P_h}^0 \Delta m_{P_h}^{-1}$$



$$f_{D_s}/f_D = 1.167(8)_{\text{stat}}$$

f_{D_s}/f_D Systematics

m_π and m_h , dependence

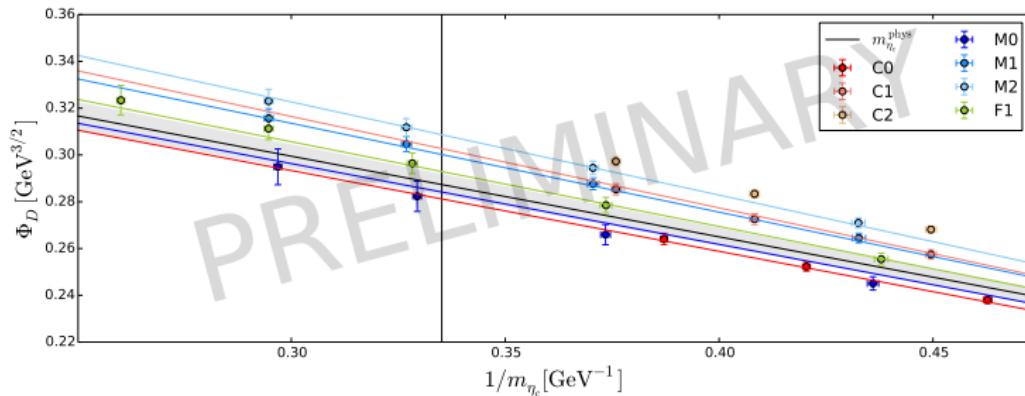


$$f_{D_s}/f_D = 1.167(8)_{\text{stat}} \left({}^{+4}_{-2} \right)_{\text{fit}}$$

m_c fixed with $1/m_{P_h}$ where $P_h = D(\diamondsuit), D_s(\circ), \eta_c^{\text{connected}}(\square)$

Φ_D

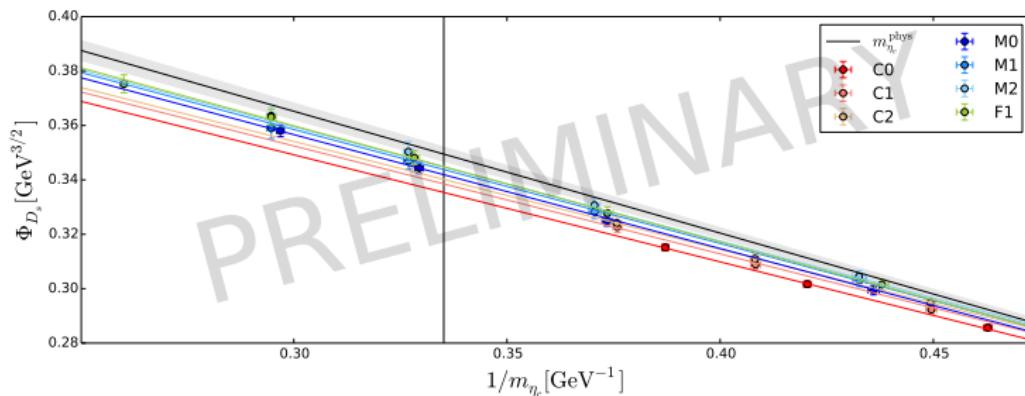
$$\begin{aligned} \mathcal{O}(a, m_I, m_h) &= \mathcal{O}(0, m_I^{\text{phys}}, m_h^{\text{phys}}) + C_{CL}^0 a^2 \\ &+ \left(C_\chi^0 + C_\chi^1 \Delta m_{P_h}^{-1} \right) \Delta m_\pi^2 + C_{P_h}^0 \Delta m_{P_h}^{-1} \end{aligned} \quad (2)$$



$$\Phi_D = 0.2857(44)_{\text{stat}} \text{GeV}^{3/2}$$

Φ_{D_s}

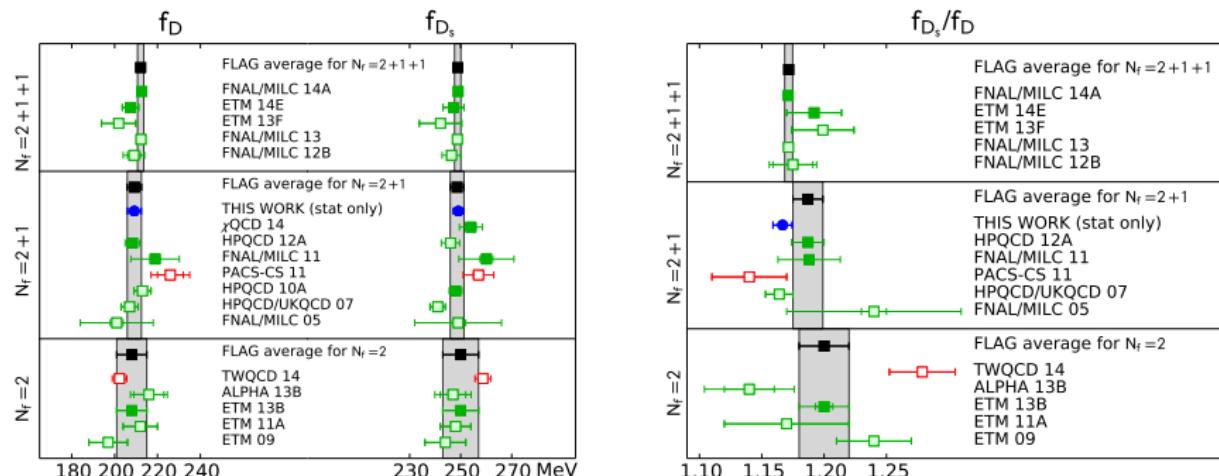
$$\begin{aligned} \mathcal{O}(a, m_I, m_h) = & \mathcal{O}(0, m_I^{\text{phys}}, m_h^{\text{phys}}) + \left(C_{CL}^0 + C_{CL}^1 \Delta m_{P_h}^{-1} \right) a^2 \\ & + C_\chi^0 \Delta m_\pi^2 + C_{P_h}^0 \Delta m_{P_h}^{-1} \end{aligned} \quad (3)$$



$$\Phi_{D_s} = 0.3495(26)_{\text{stat}} \text{GeV}^{3/2}$$

comparison with FLAG (decay constants)

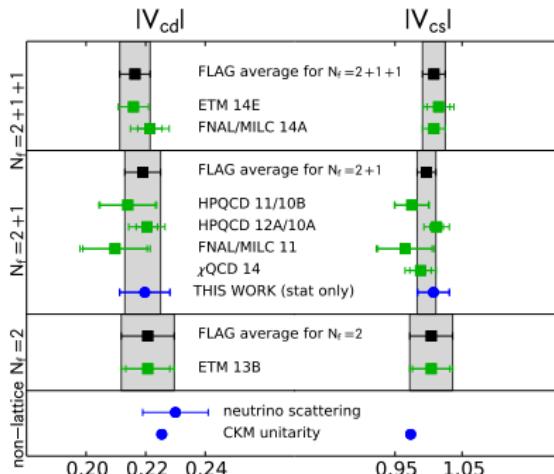
adapted from FLAG 2016 (arXiv:1607.00299)



PRELIMINARY and STATISTICAL ERROR ONLY

V_{cd} and V_{cs} comparison with literature

adapted from FLAG 2016
 (arXiv:1607.00299)
 using data from arXiv:1509.02220



PRELIM, STAT ERROR ONLY

Fit systematics:

- m_π^{cuts}
- ways to set m_c
- mass-dependent chiral and CL coeffs.

Not finalised yet

- Take all correlation into account
- FV
- Isospin
- sea-charm effects
- renormalisation

Limitations and how to reach further

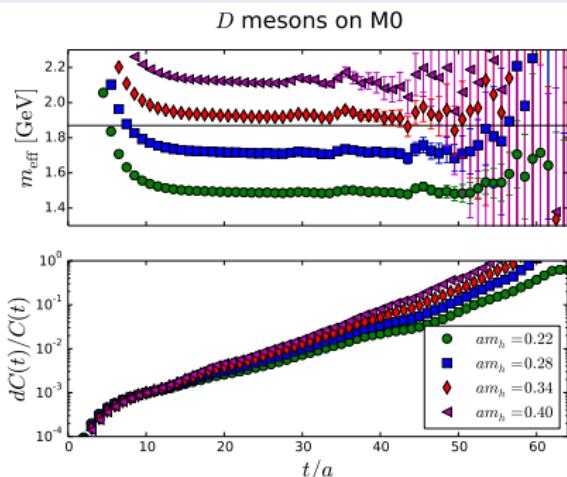
Problems

- $am_h^{\max} < am_c^{\text{phys}}$ on Coarse
- Poor Signal-to-noise for $m_\pi = 139\text{MeV}$ and $m_h \gtrsim m_c^{\text{phys}}$

Solutions?

- Stout Smeared charm
- Gaussian Smearing for source + sink

Noise growth



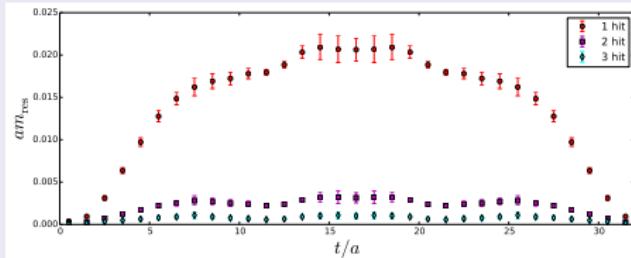
“Smeared Runs”: Tests

Smearing

Found sweetspot for

- $M_5 = 1.0$
- 3 hits of stout smearing
- Standard Stout parameter $\rho = 0.1$

hit comparison



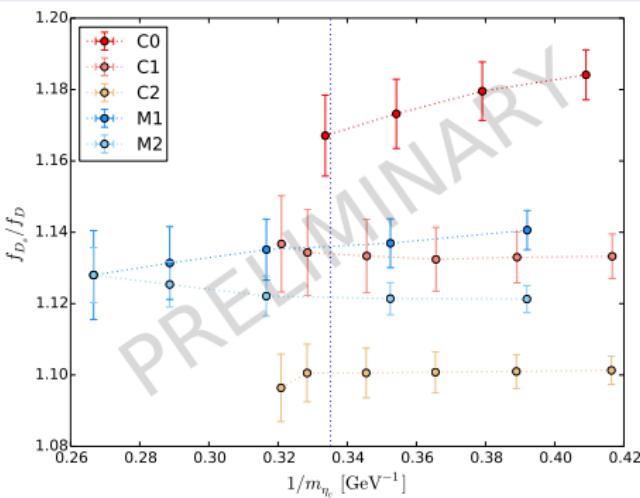
Conclusion

- ⇒ Gaussian smearing of light and strange
- ⇒ Stout smearing of heavy quarks
- ⇒ $am_h \lesssim 0.7$

“Smeared Runs”

WORK IN PROGRESS:

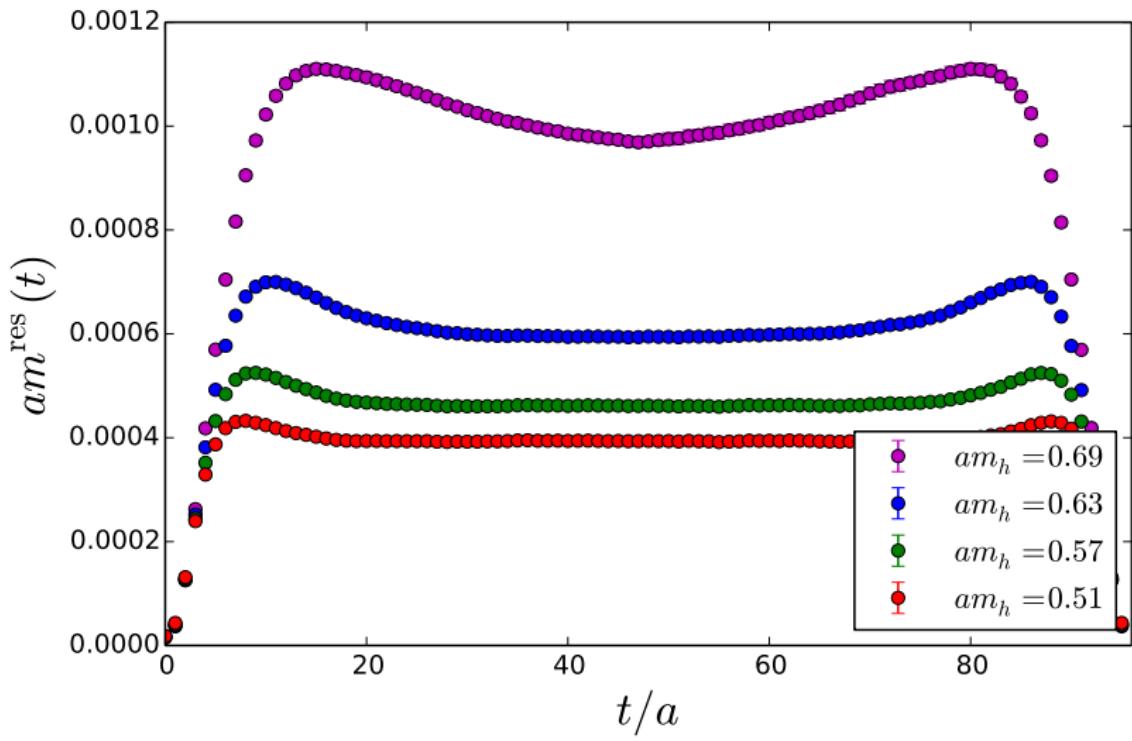
First Data (Limited statistic)



am_h reach

- Sweetspot: $M_5 = 1.0$
 3 hits of stout smearing
 $(\rho = 0.1)$
 $\Rightarrow am_h \lesssim 0.7$
- **Can reach charm** on all ensembles
- **Physical program:**
 f_D , f_{D_s} , semi-leptonics,
 bag parameters, ξ , HVP
- Ratio Method to reach b .

"Smeared Runs" - residual mass



Summary

What we have done:

- Established heavy Moebius DWFs: $M_5 = 1.6$, $am_h \lesssim 0.4$
- D and D_s decay constants
- at **Physical Pion Masses** ($2 + 1f$ simulation) in an automatically $\mathcal{O}(a)$ -improved setting.
- Continuum Limit with 3 lattice spacings.
- Global fit with control of systematics

Current work:

- Finalising systematic error budget
- Produce smeared data on other ensembles
- semi-leptonics, ratio method, combined analysis of all data

ADDITIONAL SLIDES

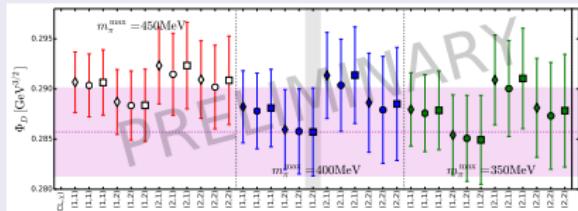
Definitions

- Decay constants: $\langle 0 | A_{cq}^\mu | D_q(p) \rangle = f_{D_q} p_{D_q}^\mu$
 \Rightarrow Extraction of $|V_{cd}|$ and $|V_{cs}|$
- Bag Parameters: $B_P = \frac{\langle P^0 | O_{VV+AA} | \bar{P}^0 \rangle}{8/3 f_P^2 m_P^2}$ where
 $O_{VV+AA} = (\bar{h}\gamma_\mu q)(\bar{h}\gamma_\mu q) + (\bar{h}\gamma_5\gamma_\mu q)(\bar{h}\gamma_5\gamma_\mu q)$
- ξ :
$$\xi = \frac{f_{hs}\sqrt{B_{hs}}}{f_{hl}\sqrt{B_{hl}}}$$

 \Rightarrow Extraction of $|V_{td}/V_{ts}|$

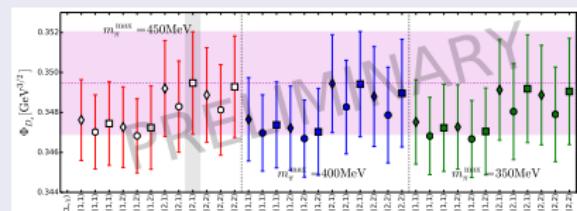
f_{D_s} and f_D Systematics

Φ_D



Can resolve C_χ^1

Φ_{D_s}



Can resolve C_{CL}^1

To Do:

- Renormalisation
- FV
- Isospin
- Fully correlated fits
- m_{η_c} -disconnected
- sea-charm