The HVP contribution to the anomalous magnetic moment of the muon

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Edinburgh
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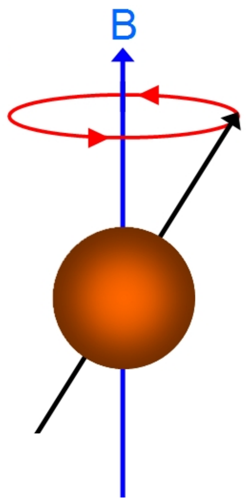
The muon is a heavier cousin of electron with same electric charge, e, and half integer spin.

Its magnetic moment is

$$\vec{\mu} = g\frac{e}{2m}\vec{S}$$



Difference of g from 2 by coupling to virtual particles gives anomalous magnetic moment

$$a_\mu = \frac{g-2}{2}$$

Experimental measurement at sub-ppm level allows test of the completeness of the Standard Model.

The largest corrections to g come from QED effects but weak and strong interaction (QCD) effects non-negligible.

For polarised muons circulating perpendicular to a B field, cyclotron frequency

$$\omega_c = \frac{eB}{\gamma m}$$

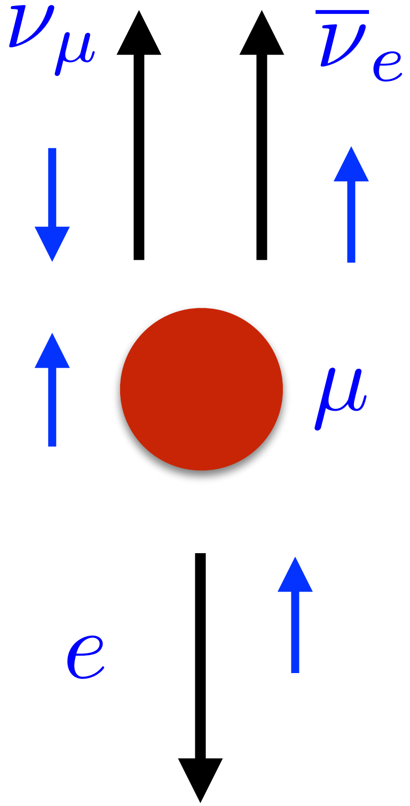
Difference of spin precession and cyclotron frequencies is proportional to anomaly at a ‘magic momentum’

$$\omega_s - \omega_c = a_\mu \frac{eB}{m}$$

$$\equiv \omega_a$$

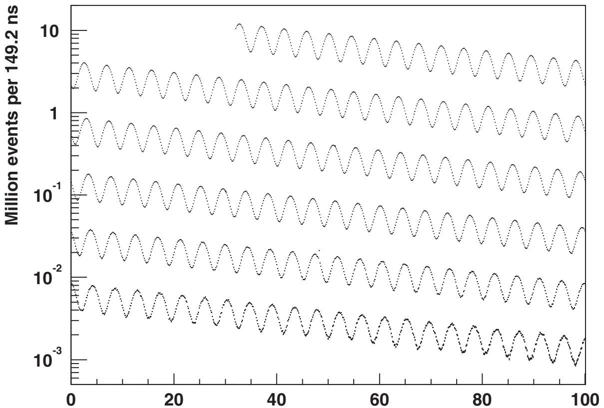
Muons decay to electrons via parity-violating weak interaction so electron emission direction tracks muon spin.

$$\mu \to e \nu_\mu \overline{\nu}_e$$



Number of electrons measured as a function of time then modulated with $\cos(\omega_a t)$

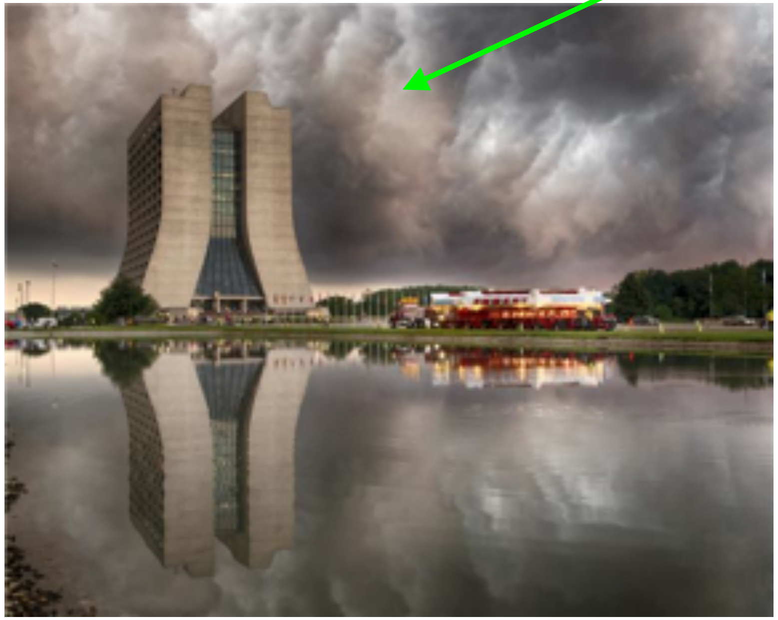
2001 BNL result (av. muon and antimuon):

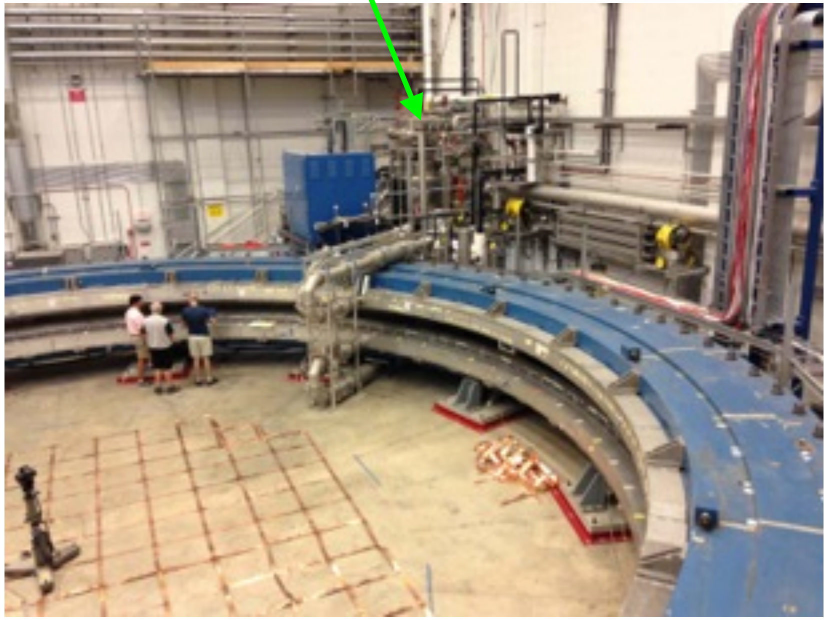


$$a_\mu^{expt} = 11659208.9(6.3) \times 10^{-10}$$

Discrepancy with Standard Model ~ 3σ
hint of new physics?

NOW: experimental ring moved to Fermilab





E989 (FNAL) will reduce exptl uncty to 1.6, improving systematics, starting 2017.

Theory calculation also needs improvement.

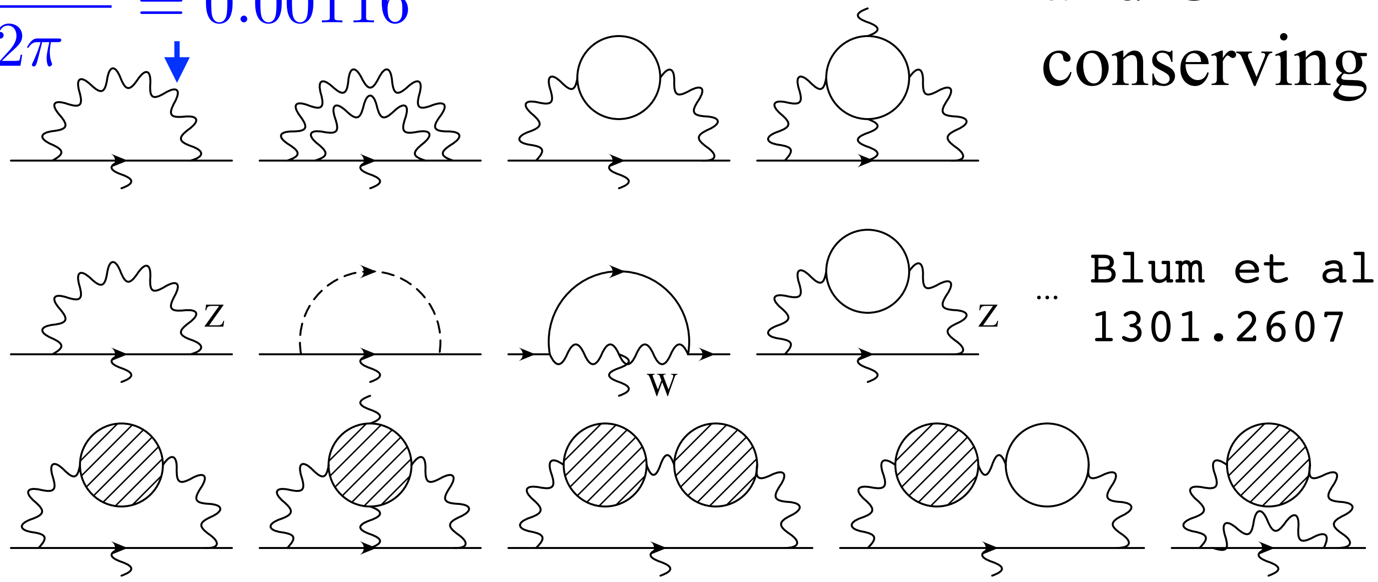
Standard Model theory expectations

Contributions from QED, EW and QCD interactions.

QED dominates.

QCD contribs start at α_{QED}^2

$\frac{\alpha_{QED}}{2\pi} = 0.00116$



flavour
and CP
conserving

`Blum et al, 1301.2607`

LO Hadronic vacuum polarisation (HVP) dominates uncertainty in SM result

$$a_\mu^{QED} = 11658471.885(4) \times 10^{-10}$$

$$a_\mu^{EW} = 15.4(2) \times 10^{-10}$$

$$a_\mu^{E821} = 11659208.9(6.3) \times 10^{-10}$$

Hadronic (and other) contributions = EXPT - QED - EW

$$a_\mu^{expt} - a_\mu^{QED} - a_\mu^{EW} = 721.7(6.3) \times 10^{-10}$$

$$= a_\mu^{HVP} + a_\mu^{HOHVP} + a_\mu^{HLBL} + a_\mu^{new\, physics}$$

Focus on lowest order hadronic vacuum polarisation, so assume:

$$a_\mu^{HLbL} = 10.5(2.6) \times 10^{-10}$$

$$a_\mu^{HOHVP} = -8.85(9) \times 10^{-10}$$

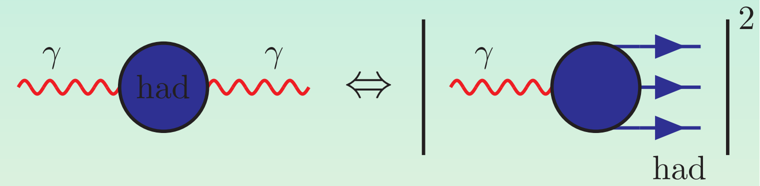
NLO+NNLO

Kurz et al, 1403.6400

$$a_\mu^{HVP, no\, new\, physics} = 719.8(6.8) \times 10^{-10}$$

Best method to date for HVP uses exptl e+e- cross-section

$$a_\mu^{HVP} = \frac{1}{4\pi^3}\int_{m_\pi^2}^{\infty} ds\,\sigma^0_{had}(s)K(s)$$



$$e^+e^- \to \gamma^* \to hadrons$$

"bare" cross-section
but inc. final-state radiation

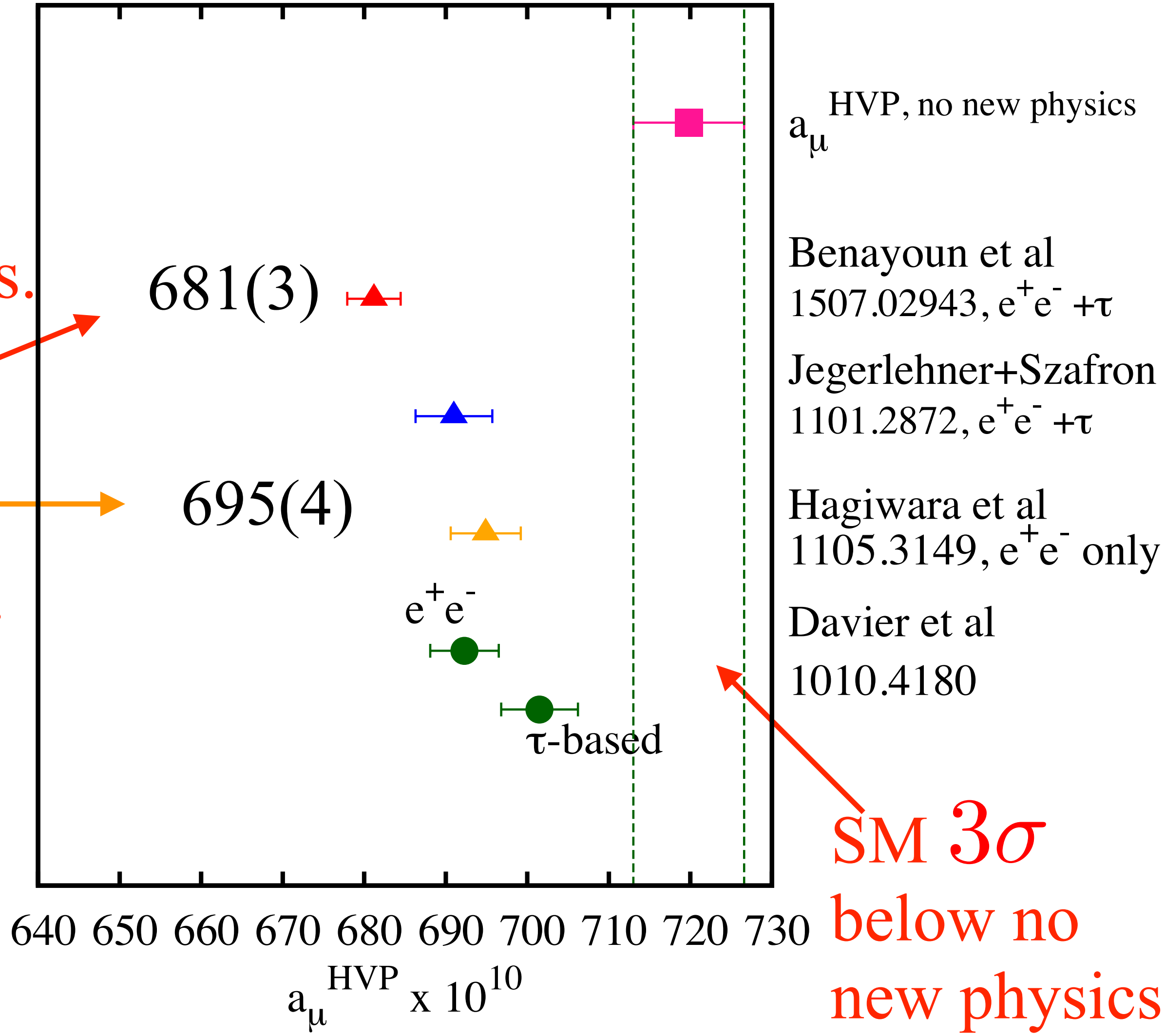
some "tension" between results.
Difference is
use of BaBar radiative
return data.
BES III data appearing now …

For Hagiwara et al:

$$a_\mu^{HVP} = 694.9(4.3)\times 10^{-10}$$

SM discrepancy:

$$24.9(8.0)\times 10^{-10}$$



SM 3σ
below no
new physics

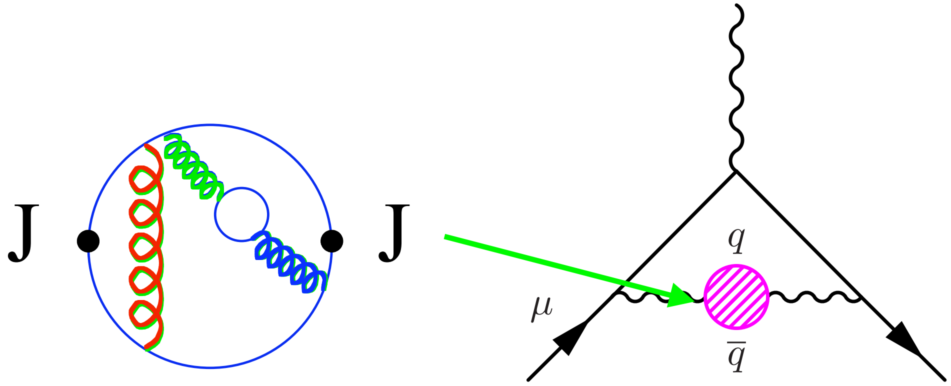
Lattice QCD calculation of the LO HVP contribution

Key quantity to be calculated is vacuum polarisation function $\hat{\Pi}(q^2)$ from vector-vector correlator

$$a_\mu^{HVP,i} = \frac{\alpha}{\pi}\int_0^\infty dq^2 f(q^2)(4\pi\alpha e_i^2)\hat{\Pi}_i(q^2)$$

← small q^2 dominates integrand

Time-moments of correlator give small q^2 expansion



Blum, hep-lat/0212018

$$G_n \equiv \sum_{t,\vec{x}} t^n Z_V^2 \langle J^j(\vec{x},t) J^j(0)\rangle$$

calculate on ensembles of gluon fields generated by importance sampling of QCD path integral and average

$$\hat{\Pi}(q^2) = \sum_{k=1}^{\infty}(-1)^{k+1}\frac{G_{2k+2}}{(2k+2)!}q^k$$

HPQCD, 1403.1778

Darwin@Cambridge

We use gluon field configurations on discrete space-time generated by MILC collaboration, inc. effect of sea quarks.

Multiple sets with different lattice spacing, quark masses.

Most realistic snapshots of QCD to date.

We solve Dirac equation for valence quark propagators and combine for correlators.

numerically costly, data intensive

Darwin allows us to calculate quark propagators rapidly and store them for flexible re-use.

Test on STRANGE quark contribution

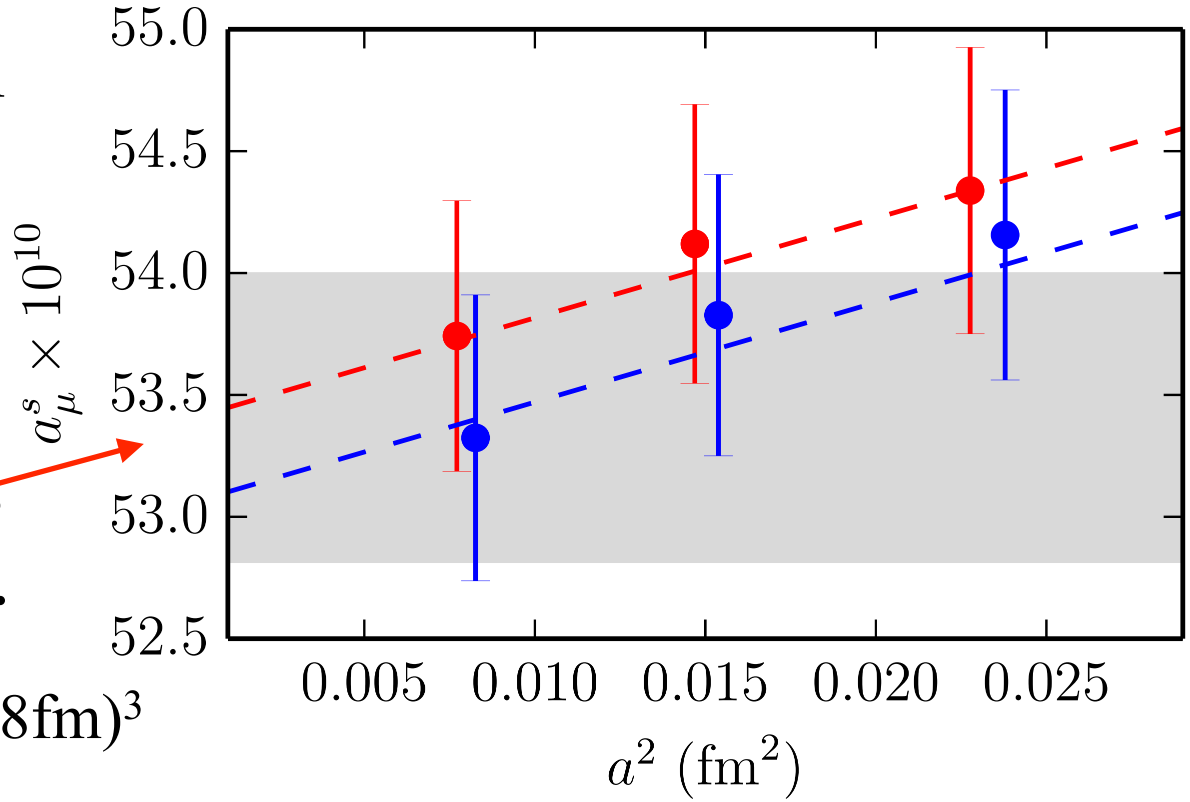
`HPQCD 1403.1778, 1408.5768`

HISQ valence quarks on MILC 2+1+1 HISQ configs. Local J_v - nonpert. Z_v.

multiple a (fixed by w_0), m_l (inc. phys.), volumes.

Tune s from η_s

up to $(5.8\text{fm})^3$



	a_μ^s
Uncertainty in lattice spacing (w_0, r_1):	1.0%
Uncertainty in Z_V:	0.4%
Monte Carlo statistics:	0.1%
$a^2 \to 0$ extrapolation:	0.1%
QED corrections:	0.1%
Quark mass tuning:	0.0%
Finite lattice volume:	$< 0.1\%$
Padé approximants:	$< 0.1\%$
Total:	1.1%

$$a_\mu^{HVP,s} = 53.41(59) \times 10^{-10}$$

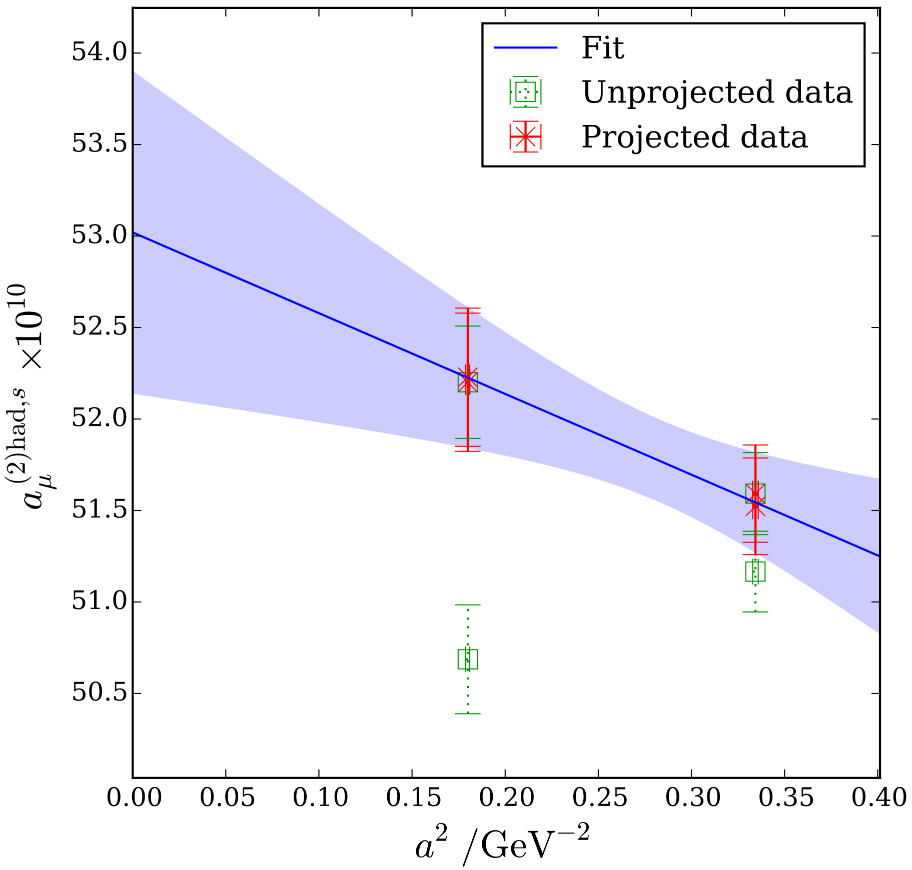
Also

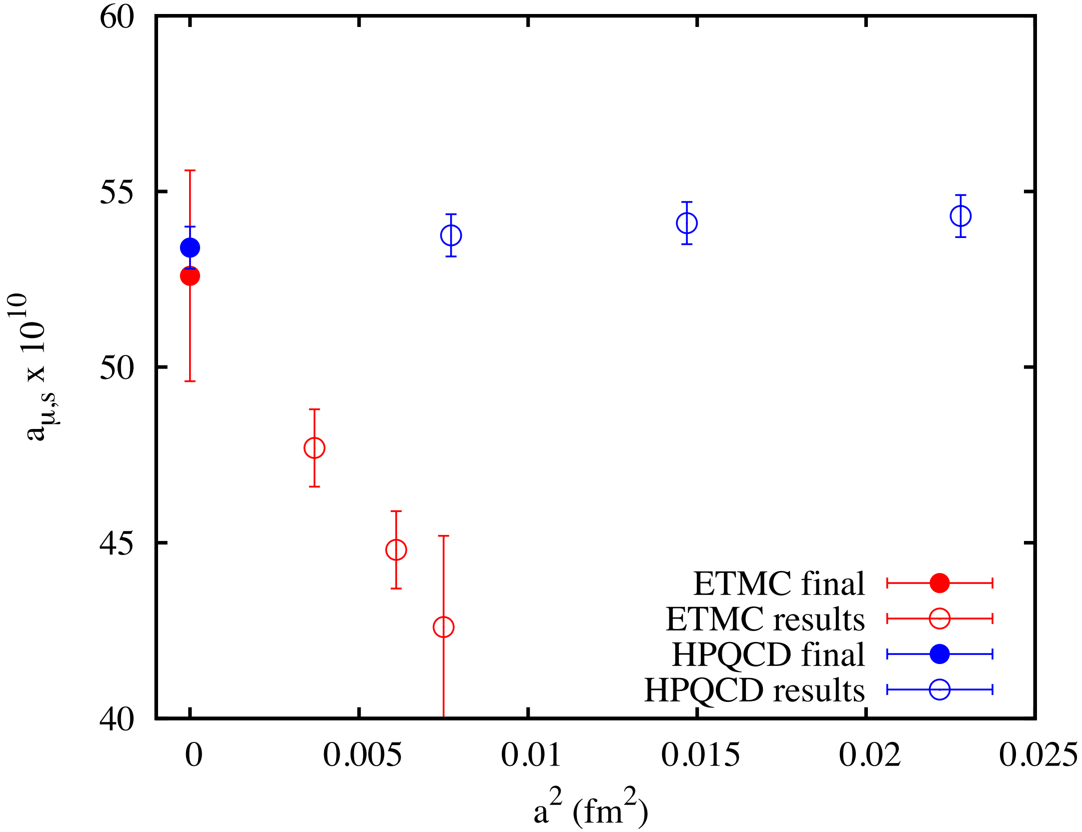
$$a_\mu^{HVP,c} = 14.4(4) \times 10^{-10}$$

$$a_\mu^{HVP,b} = 0.27(4) \times 10^{-10}$$

NRQCD

New results from other formalisms provide good check





RBC/UKQCD domain wall

$$a_\mu^{HVP,s} = 53.1(9) \times 10^{-10}$$

1607.01767

ETMC twisted mass

$$a_\mu^{HVP,s} = 53(3) \times 10^{-10}$$

From $R_{e^+e^-}$ we estimate $a_\mu^{HVP,s} <\approx 55 \times 10^{-10}$

UP/DOWN contribution $m_u = m_d$

Much noisier and sensitive to u/d mass. Use

$$G(t) = \begin{cases} G_{\mathrm{data}}(t) & \text{for } t \leq t^* \\ G_{\mathrm{fit}}(t) & \text{for } t > t^* \end{cases}$$

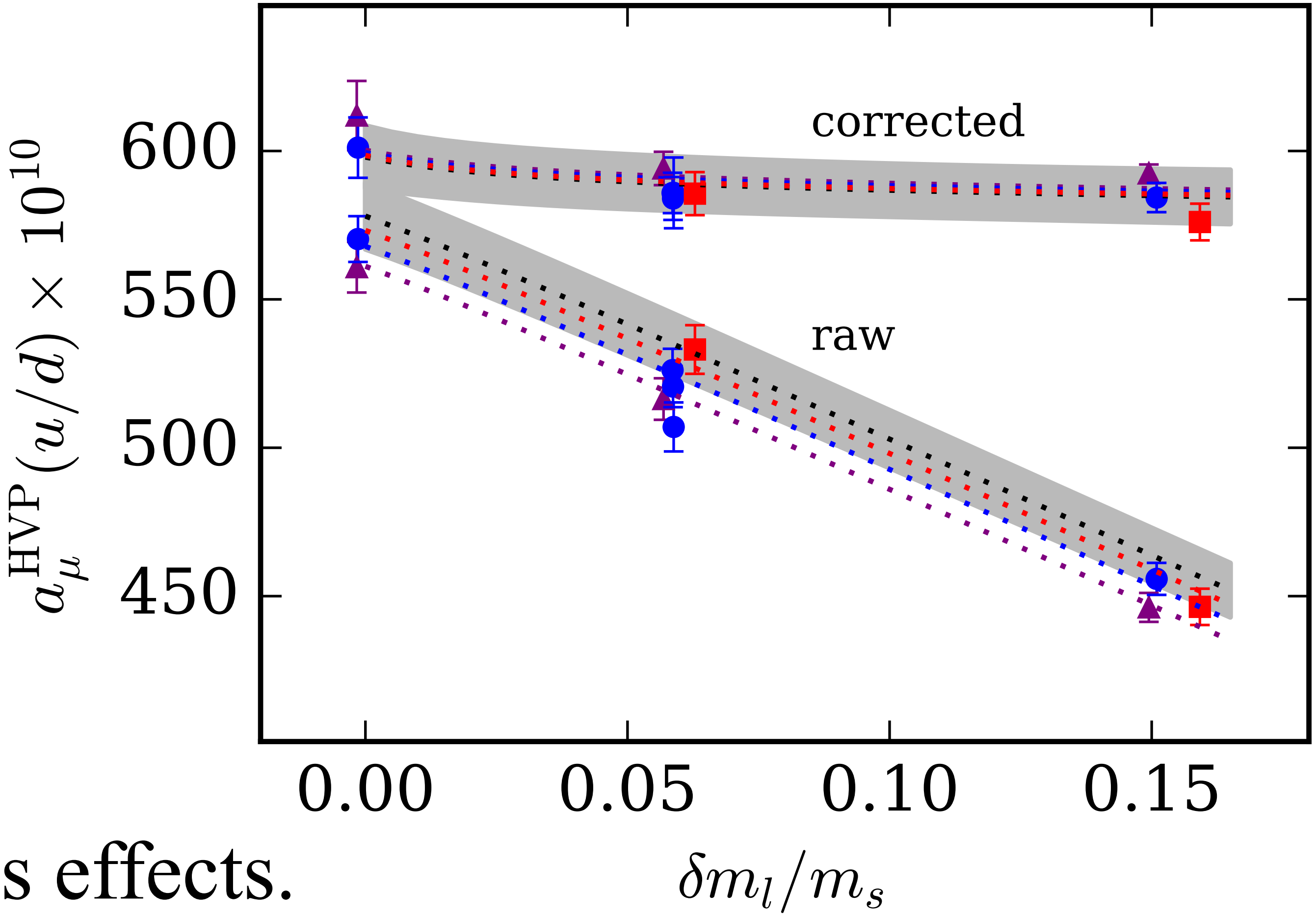
← from Monte Carlo

← from multi-exponential fit

$t^* = 1.5\mathrm{fm} = 6/m_\rho$ so 70% of result from G_{data}

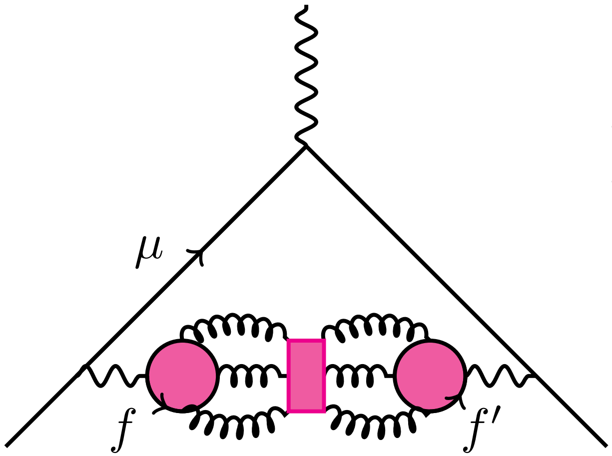
Must correct for finite vol. effects in $\pi\pi$ contribn using scalar QED (7%)

Rescale Π_j by $(m_\rho^{latt}/m_\rho^{expt})^{2j}$ to reduce u/d mass effects.

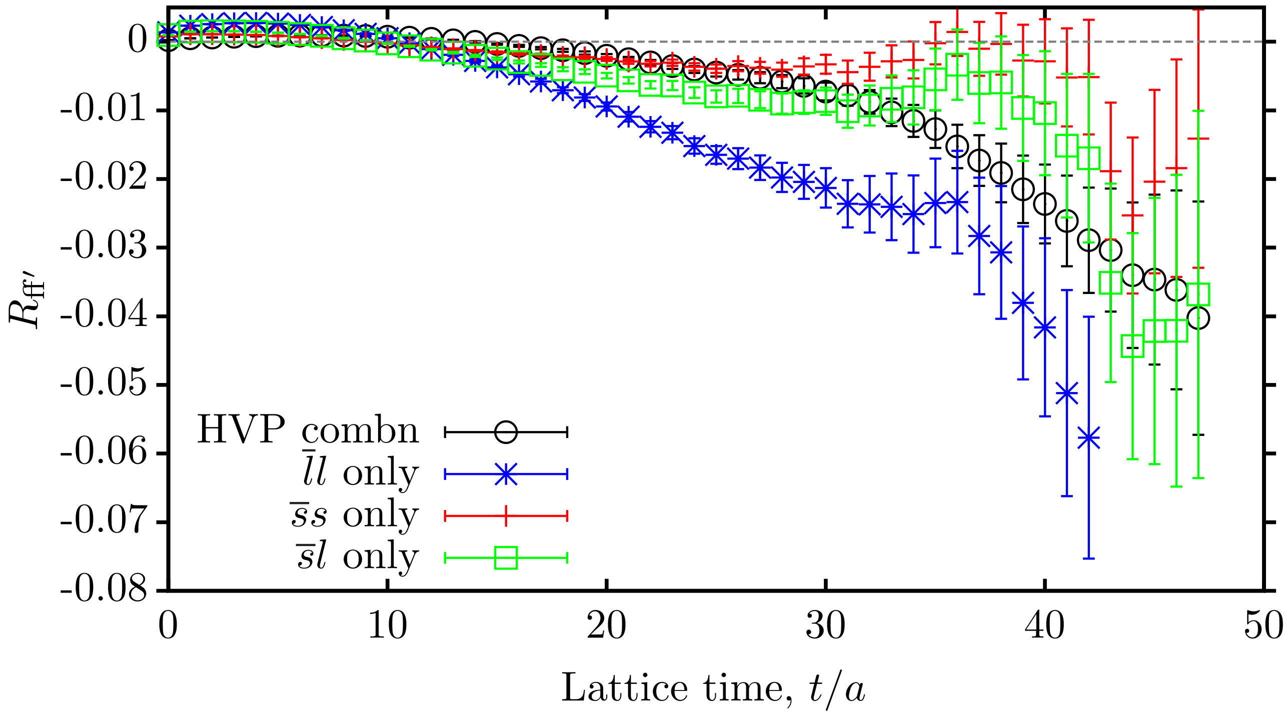


Quark-line disconnected contribution

`HPQCD/Hadspec`
`1512.03270,`
`see also RBC/UKQCD`
`1512.09054`



Hard to calculate but small.
Suppressed by masses since $\sum_{u,d,s} Q_f = 0$

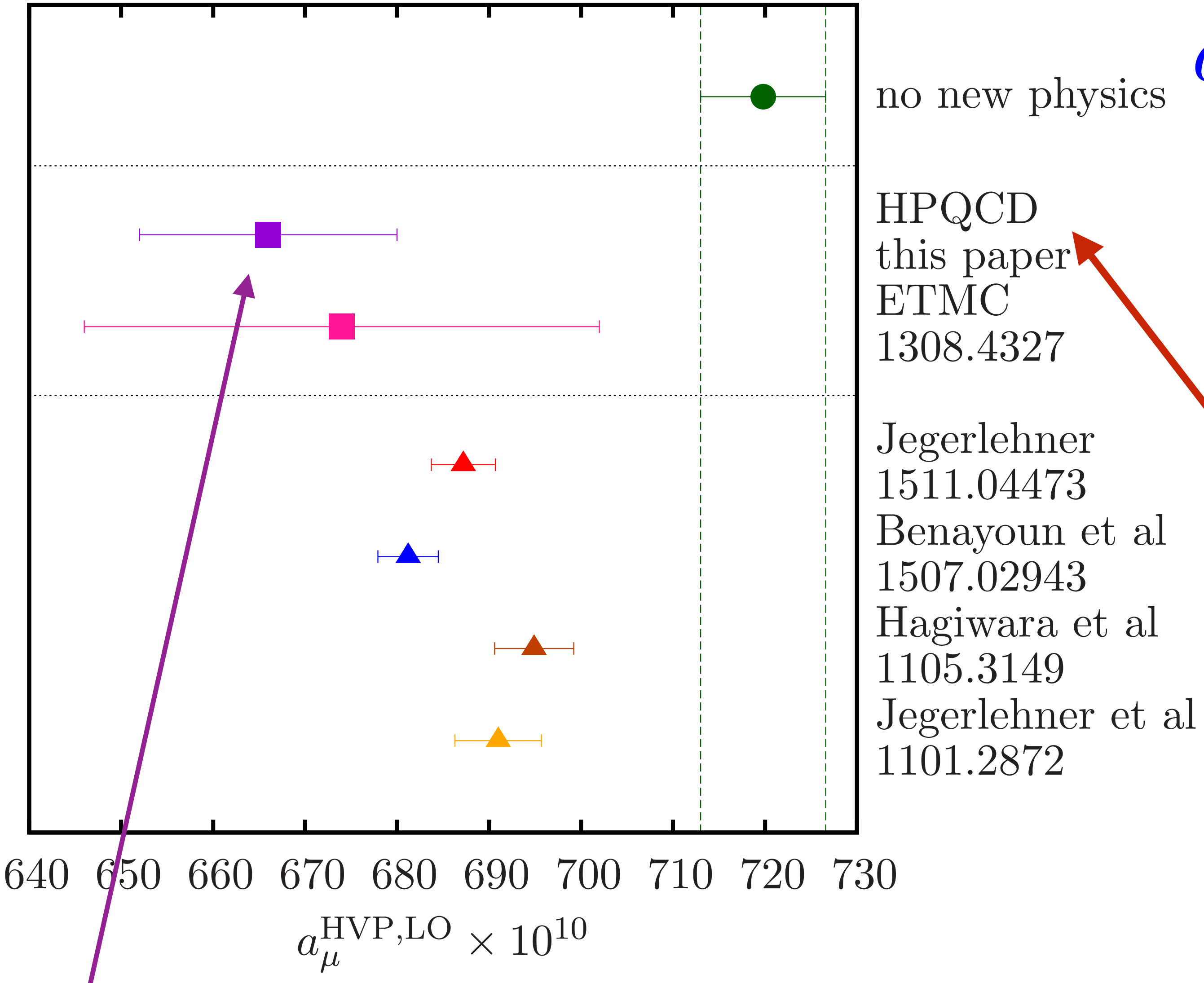


Ratio of disc. to conn. correlators small and further suppressed (by factor 5 by quark charge combns)
Estimate (after fitting):

$$a_\mu^{HVP,disc} = 0(9) \times 10^{-10}$$

`see also RBC/UKQCD 1512.09054:`
`-9.6(4.0) x` 10^{-10}

Combining numbers for a total



$$a_\mu^{\mathrm{HVP,LO}} \times 10^{-10}$$

$598(11)\ u/d$

$53.4(6)\ s$

$14.4(4)\ c$

$0.27(4)\ b$

Total $666(6)(12)$

add syst from
disc. diags
(1.5%) in quad

3.5σ discrepancy with no new physics

Conclusion

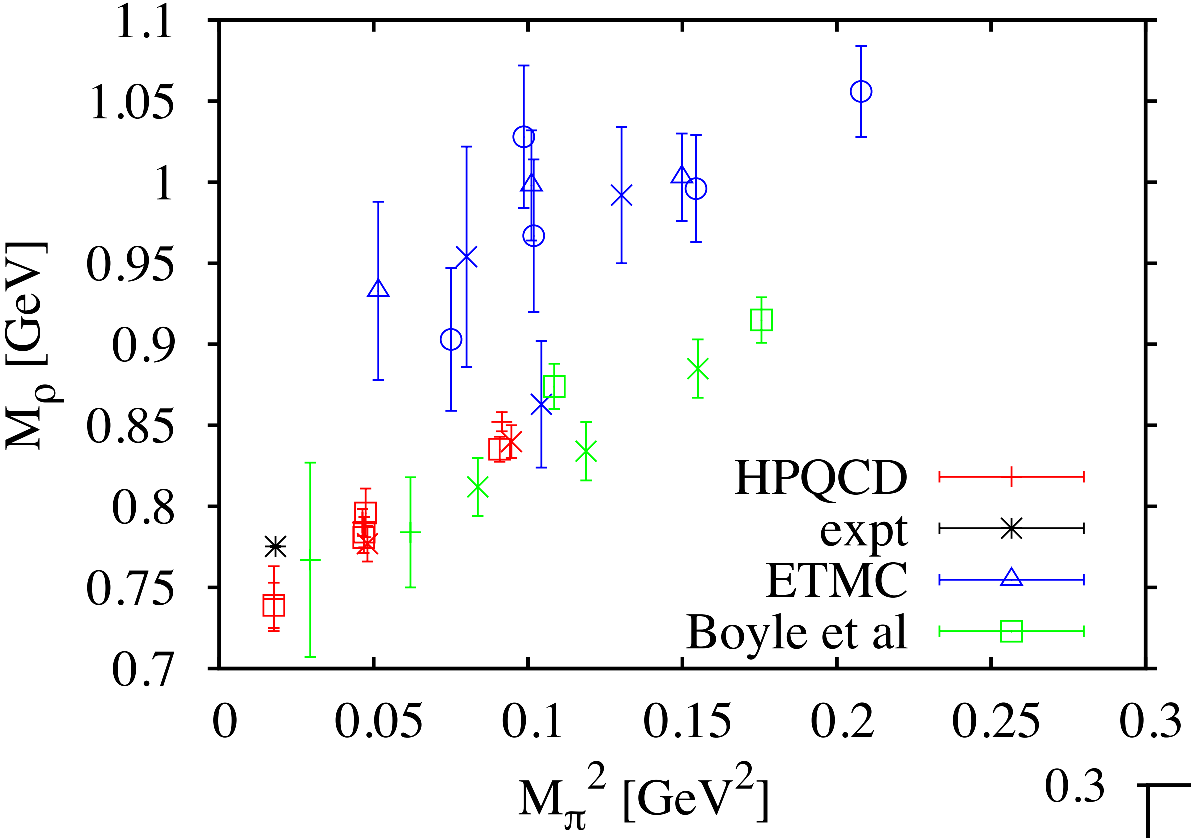
- Lattice QCD calculations now on ‘2nd generation’ gluon configs with charm in the sea and $m_{u,d}$ at physical value (so no extrapoln).

- Allows calculation of the LO HVP contribution to the anomalous magnetic moment of the muon with 2% uncertainty.

- Uncertainty dominated by systematics from using m_u=m_d, from not including QED and from uncertainties in the quark-line disconnected piece. All these are being tackled now by multiple collaborations.

- sub-1% uncertainties on lattice QCD results for HVP contribution to a_μ are within sight, in time for new expt.

Spares



Analysis of ρ parameters

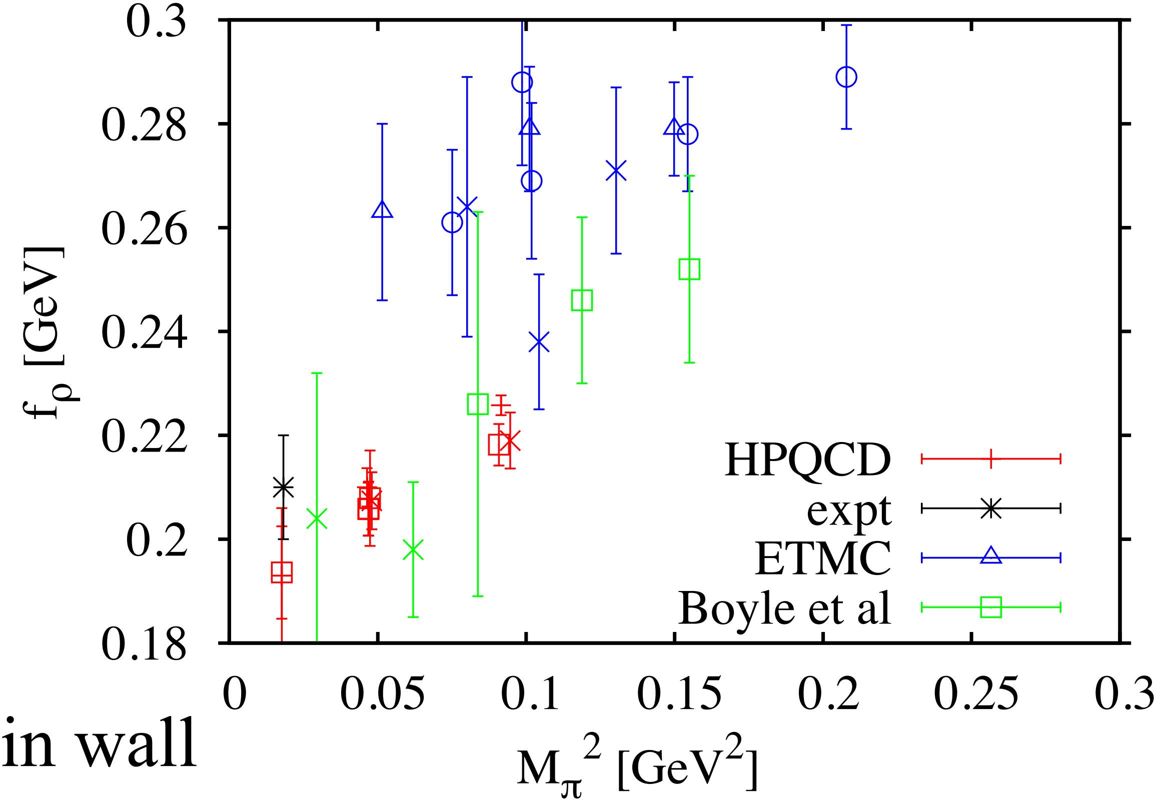
Direct comparison with ETMC (1308.4327) and Boyle et al (1107.1497) possible

ETMC a 0.06-0.08fm
L 2.5- 2.9fm

HPQCD a 0.09-0.15fm
L 2.5-5.8fm

Boyle et al a 0.09-0.14fm
L 2.7-4.6fm

domain wall



Error budget for HVP,LO calculation

TABLE III: Error budget for the connected contributions to the muon anomaly a_μ from vacuum polarization of u/d quarks.

	$a_\mu^{\mathrm{HVP,LO}}(u/d)$
QED corrections:	$1.0\,\%$
Isospin breaking corrections:	$1.0\,\%$
Staggered pions, finite volume:	$0.7\,\%$
Noise reduction (t^*):	$0.5\,\%$
Valence m_ℓ extrapolation:	$0.4\,\%$
Monte Carlo statistics:	$0.4\,\%$
Padé approximants:	$0.4\,\%$
$a^2 \to 0$ extrapolation:	$0.3\,\%$
Z_V uncertainty:	$0.4\,\%$
Correlator fits:	$0.2\,\%$
Tuning sea-quark masses:	$0.2\,\%$
Lattice spacing uncertainty:	$< 0.05\,\%$
Total:	$1.9\,\%$