Exotic hadrons (mesons), resonances and Lattice QCD

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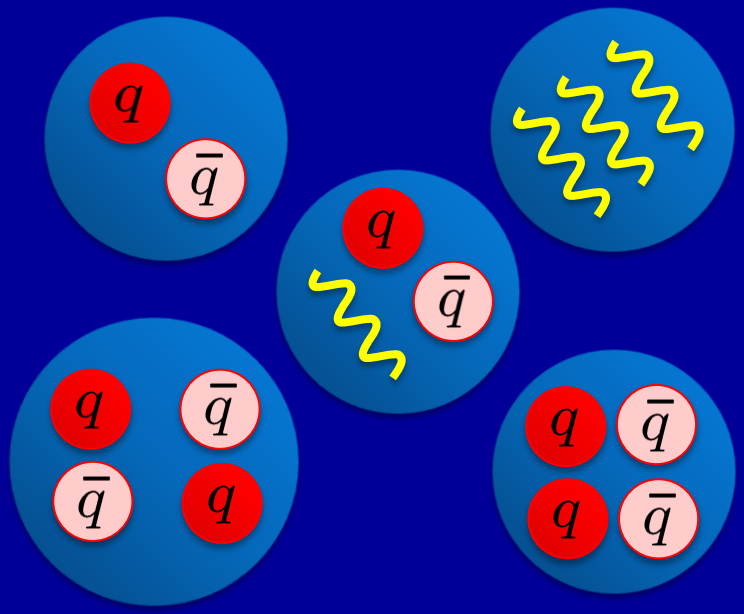
Workshop on Exotic Hadron Spectroscopy,
Edinburgh, 26 – 27 Sept 2016

UNIVERSITY OF CAMBRIDGE

Hadron Spectrum Collaboration

Meson spectroscopy



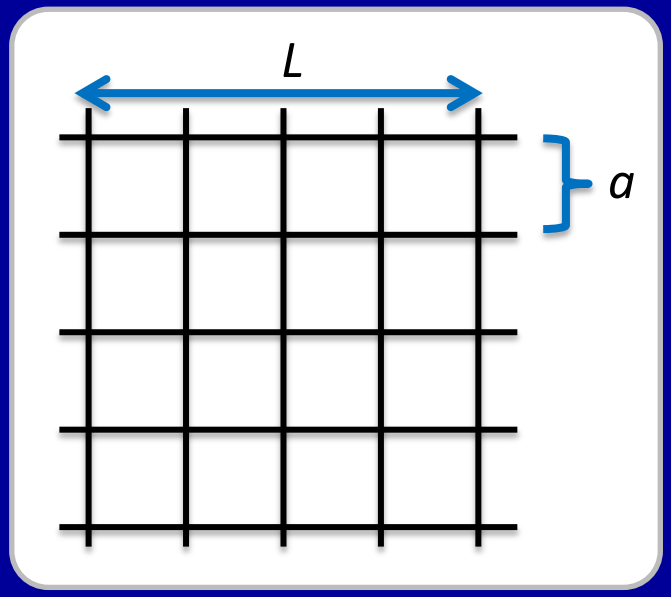
KLOE CLAS12

$X(3872)$, $Y(4260)$, $Z^+(4430)$, $Z_c^+(3900)$, Z_b^+, $X(5568)$, $D_s(2317)$, light scalars, $\pi_1(1600)$ [$J^{PC} = 1^{-+}$], ...

Exotic flavour or **exotic J^{PC}** (0^{--}, 0^{+-}, 1^{-+}, 2^{+-}, ...) quantum numbers – can't just be a $q\bar{q}$ pair

Lattice QCD – systematically-improvable first-principles calculations

Lattice QCD spectroscopy



- **Discretise** spacetime in a **finite volume**
- Compute correlation fns. numerically (Euclidean time, $t \rightarrow i\,t$)

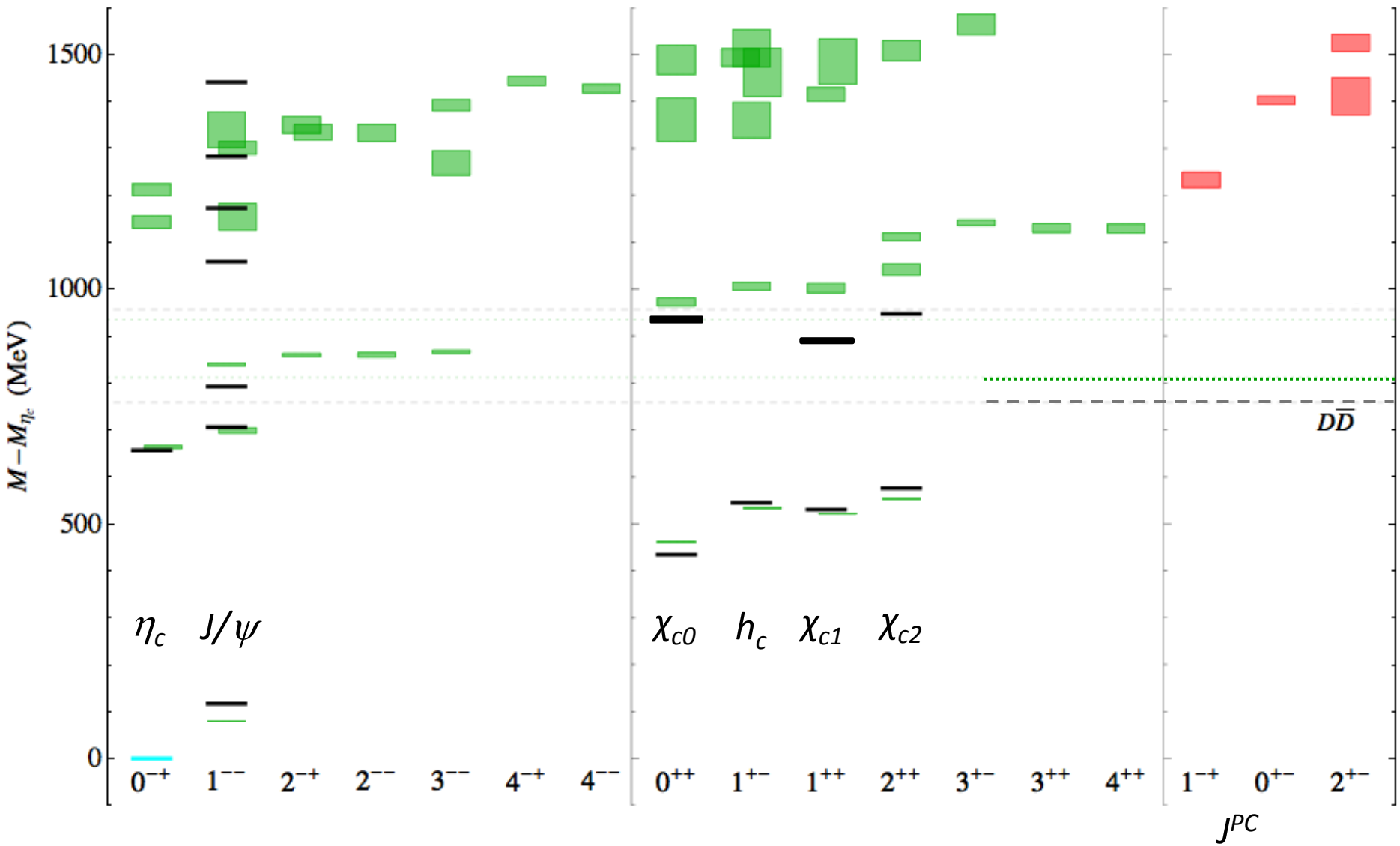
Note:

- Finite a and L
- Possibly unphysical m_π

Finite-volume energy eigenstates from:

$$C_{ij}(t) = \langle 0 | \mathcal{O}_i(t) \mathcal{O}_j^\dagger(0) | 0 \rangle$$

Excited charmonia



M_π = 391 MeV [HadSpec, JHEP 07 (2012) 126]

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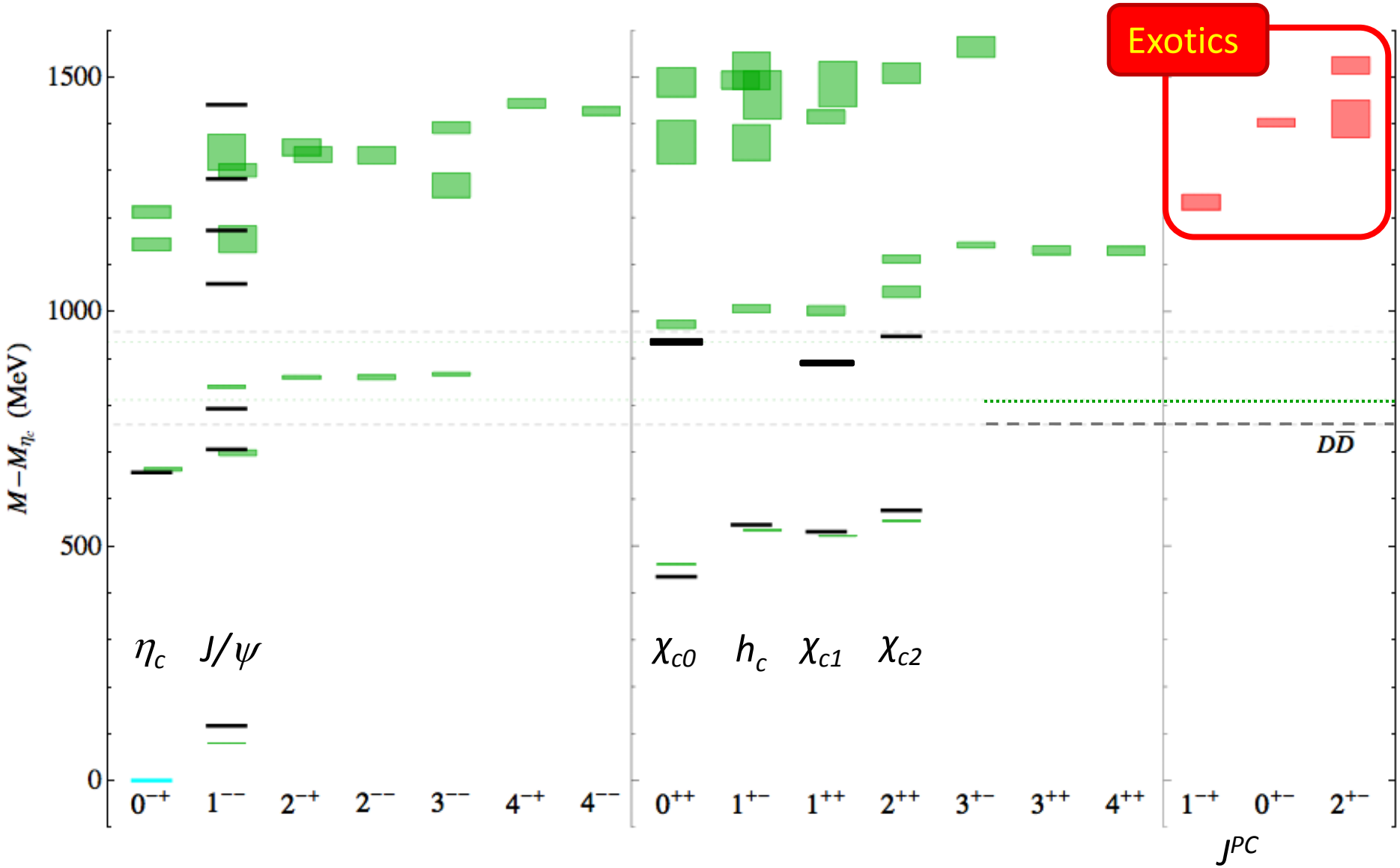

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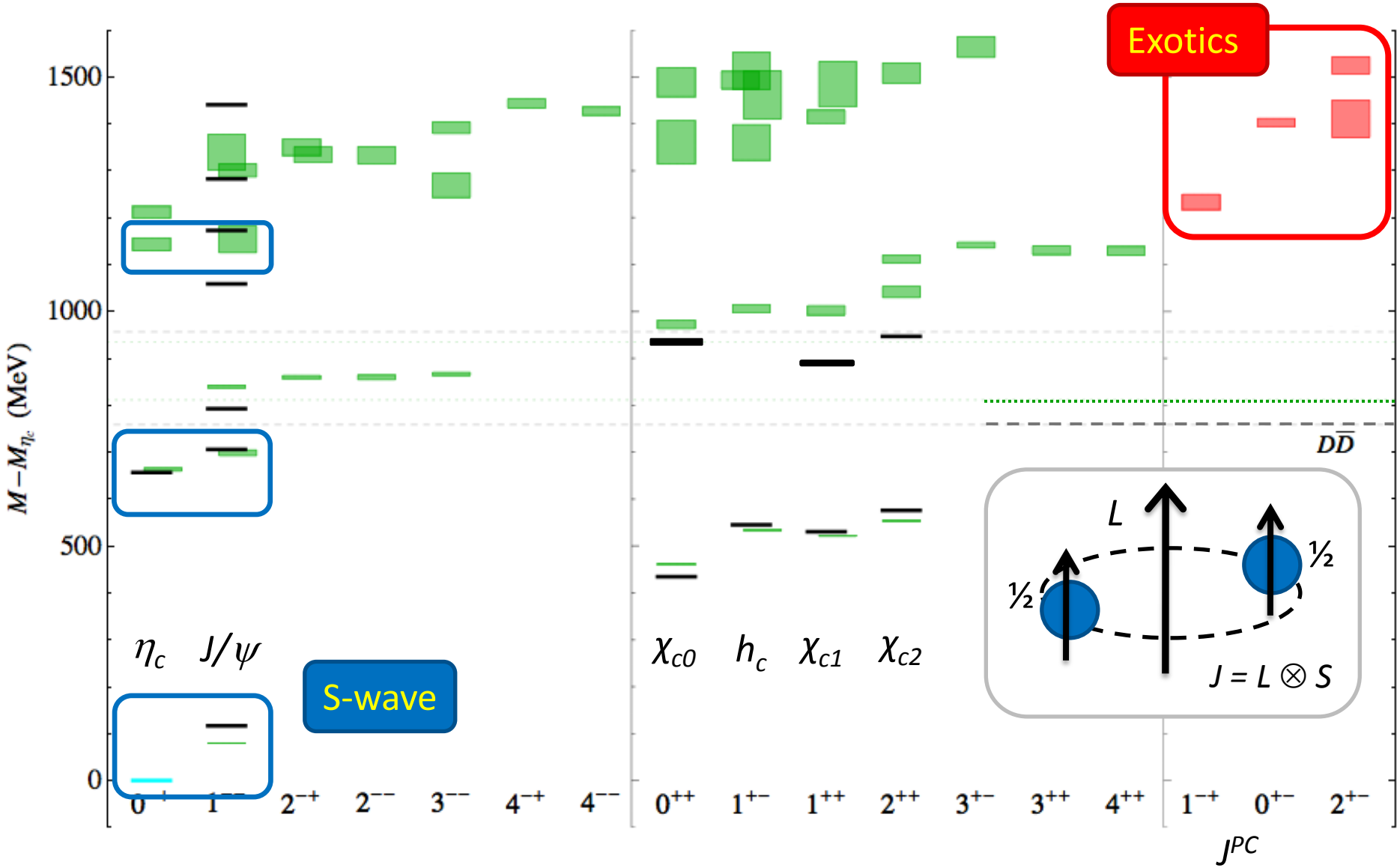

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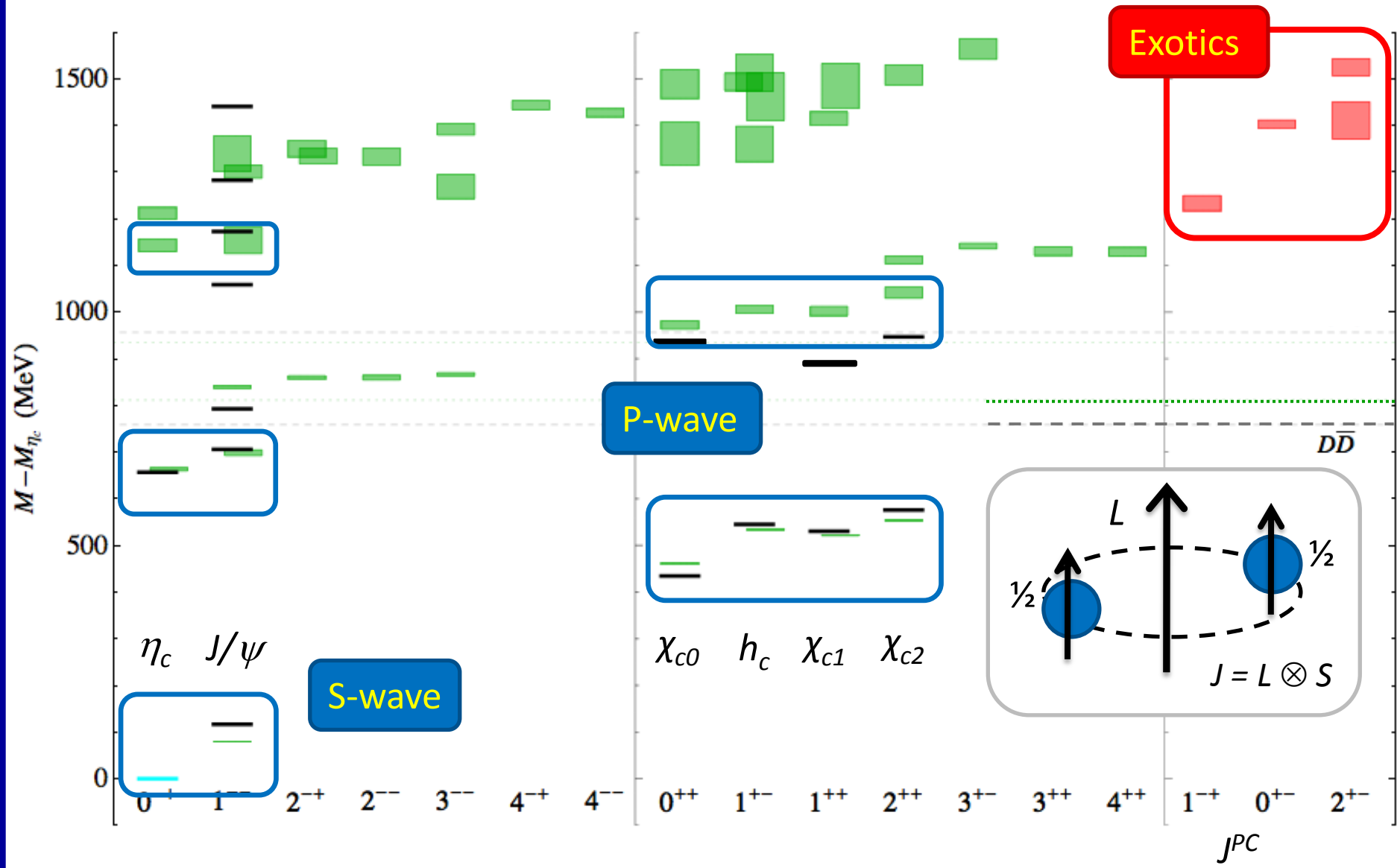

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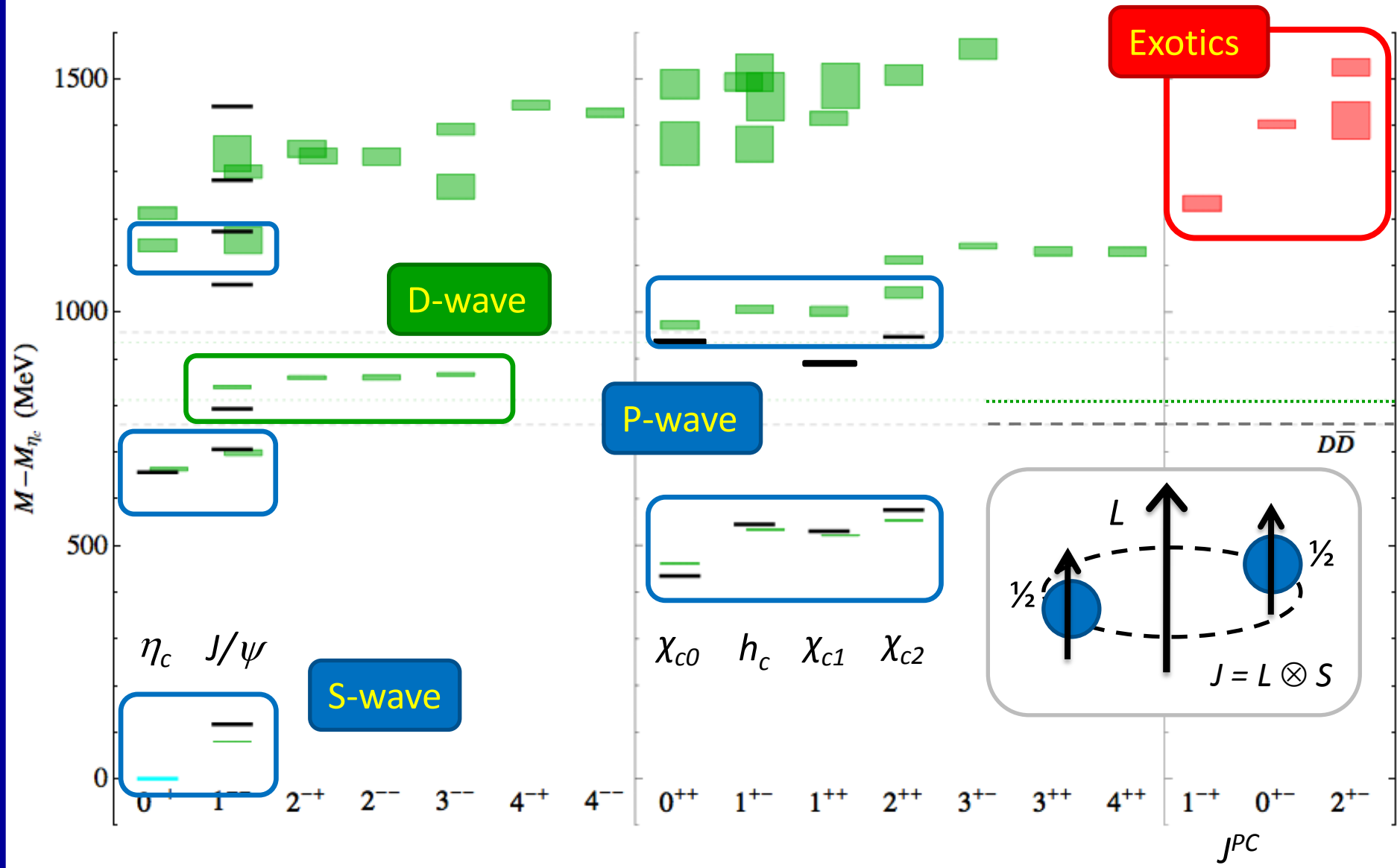

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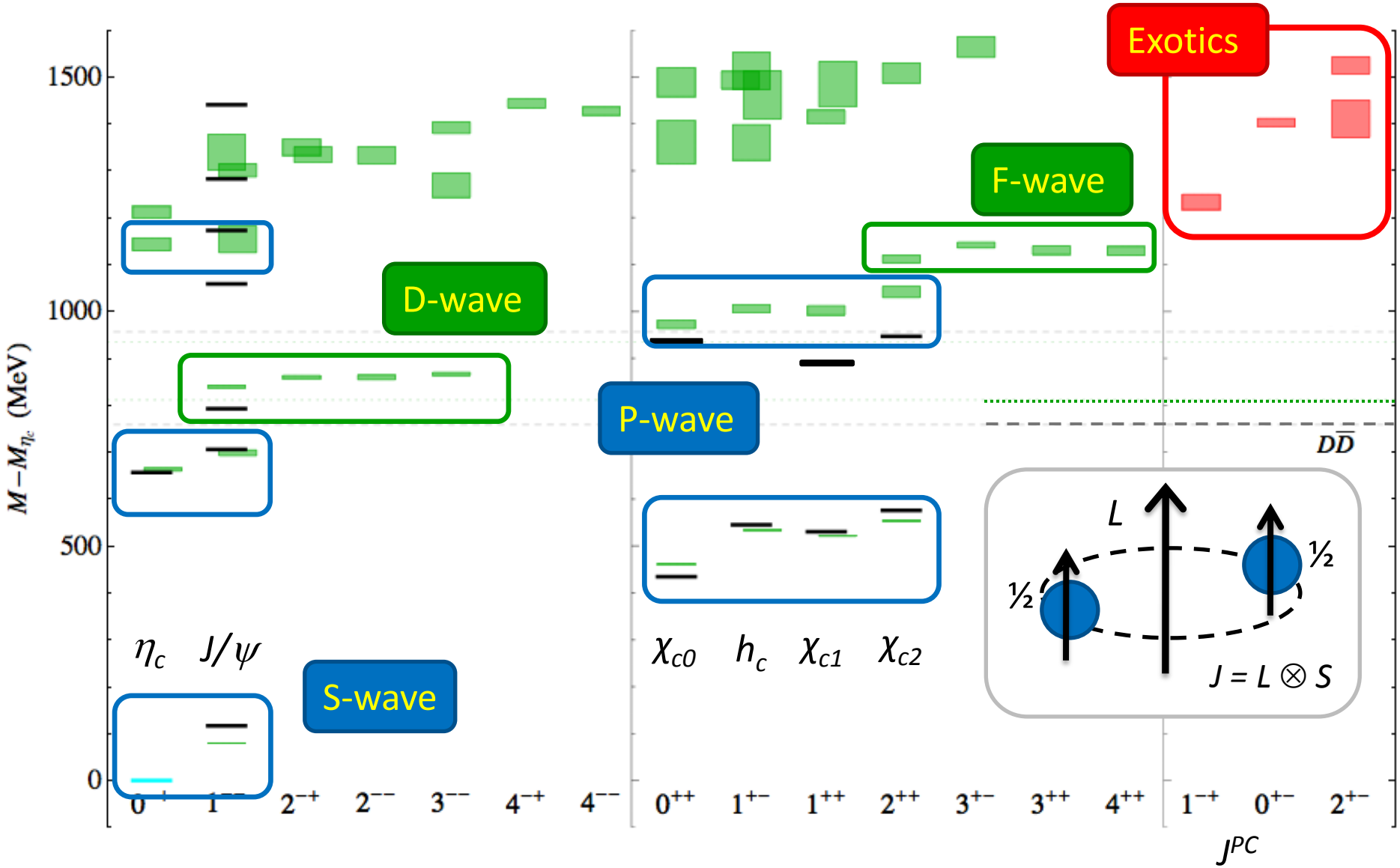

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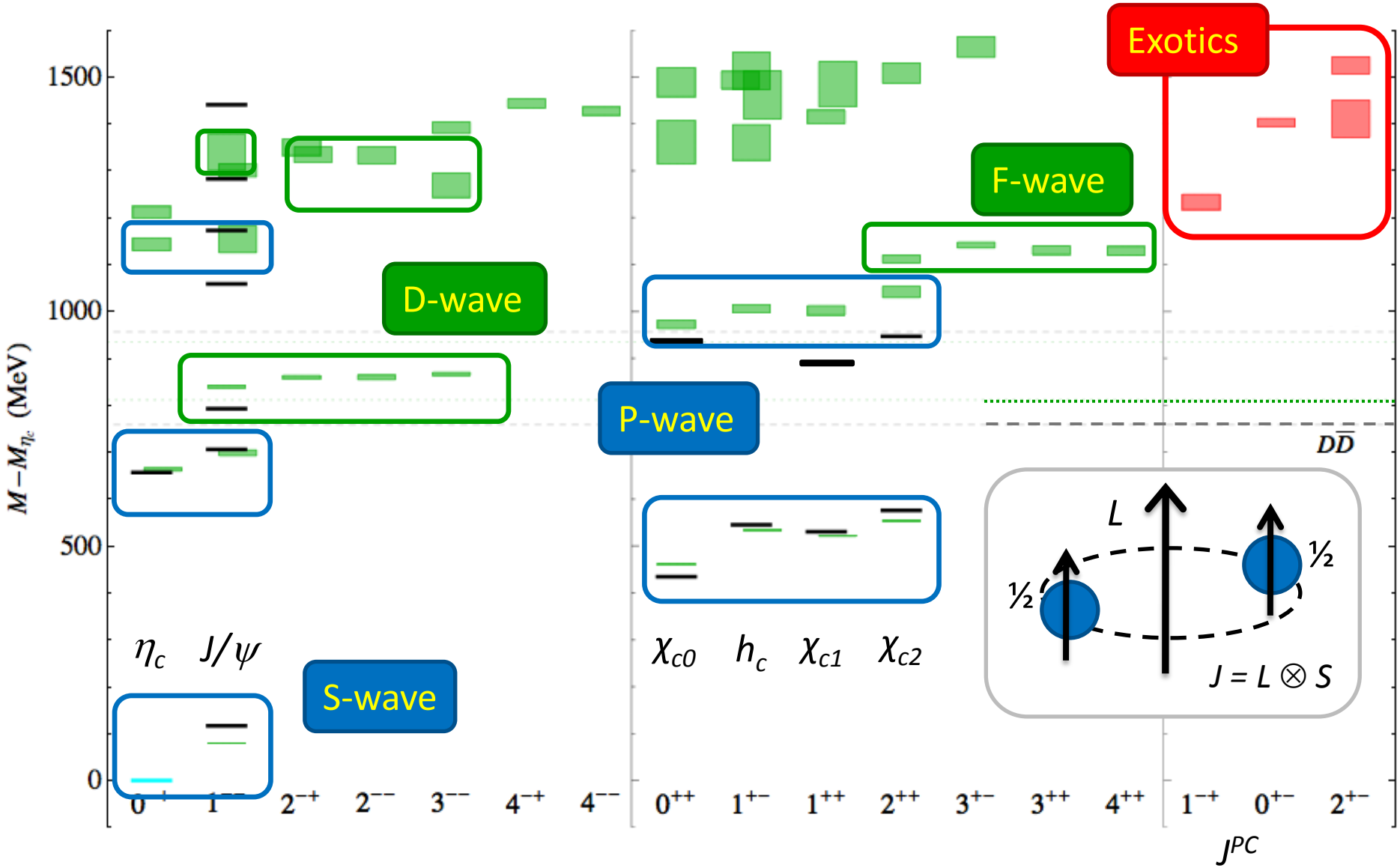

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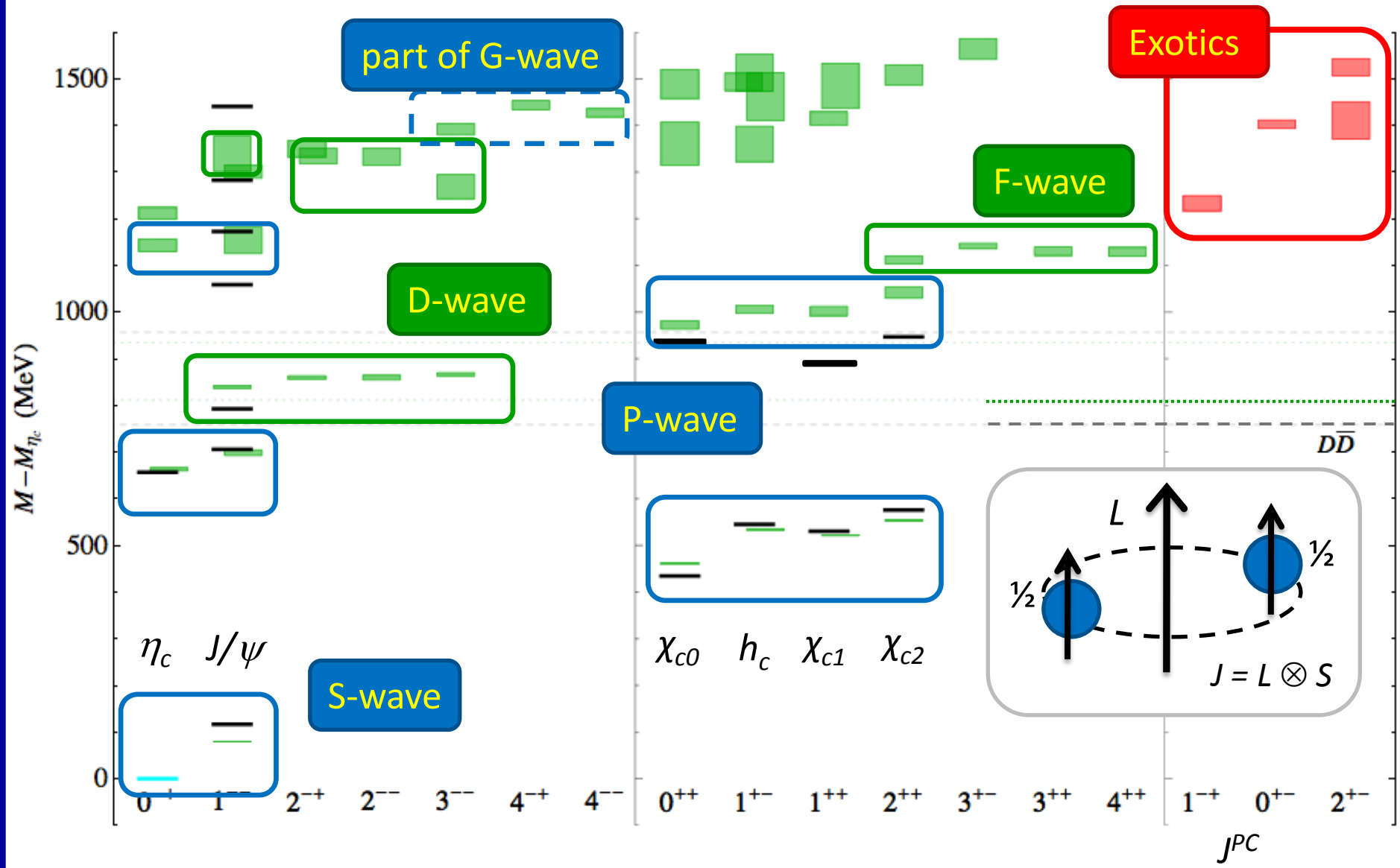

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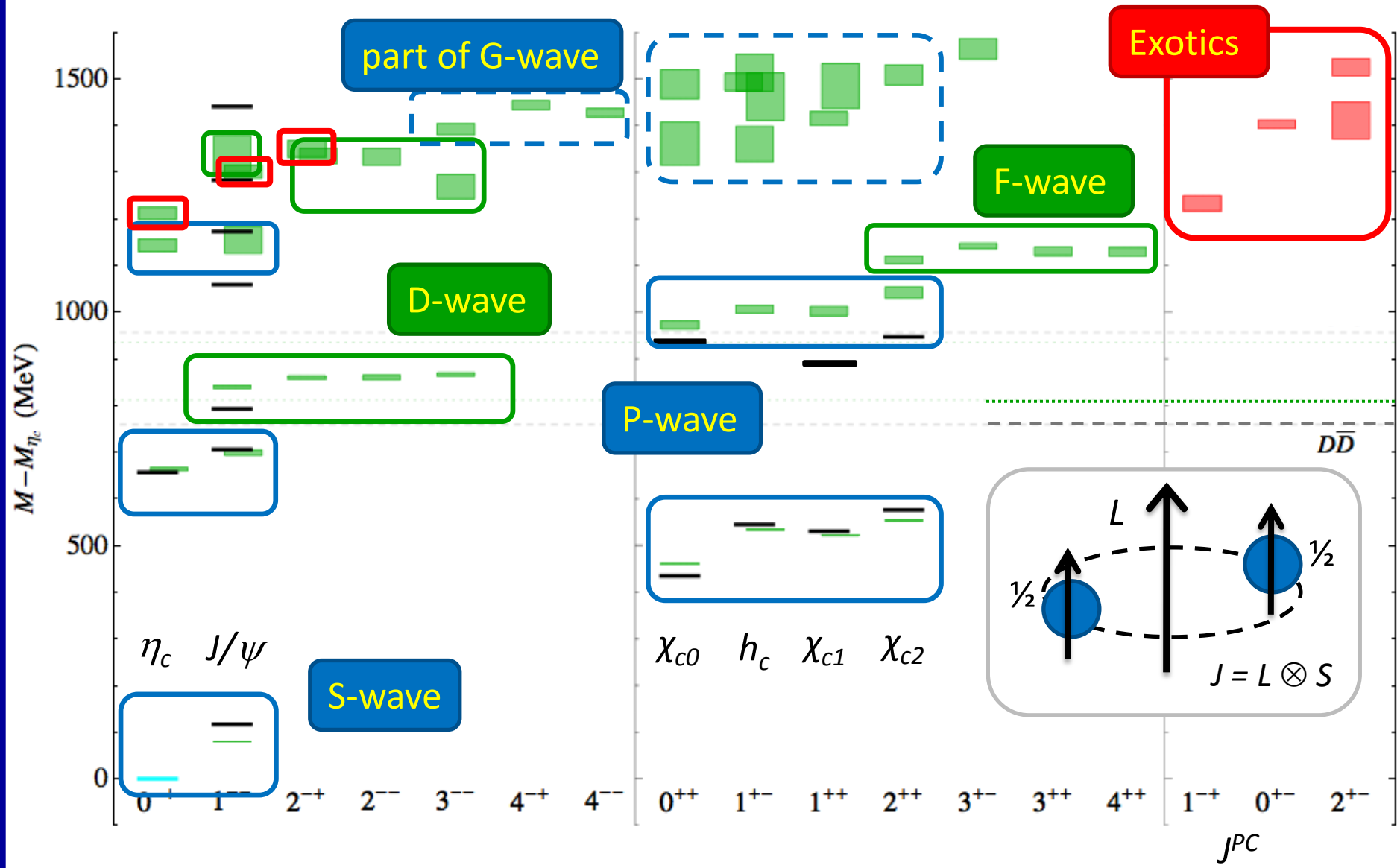

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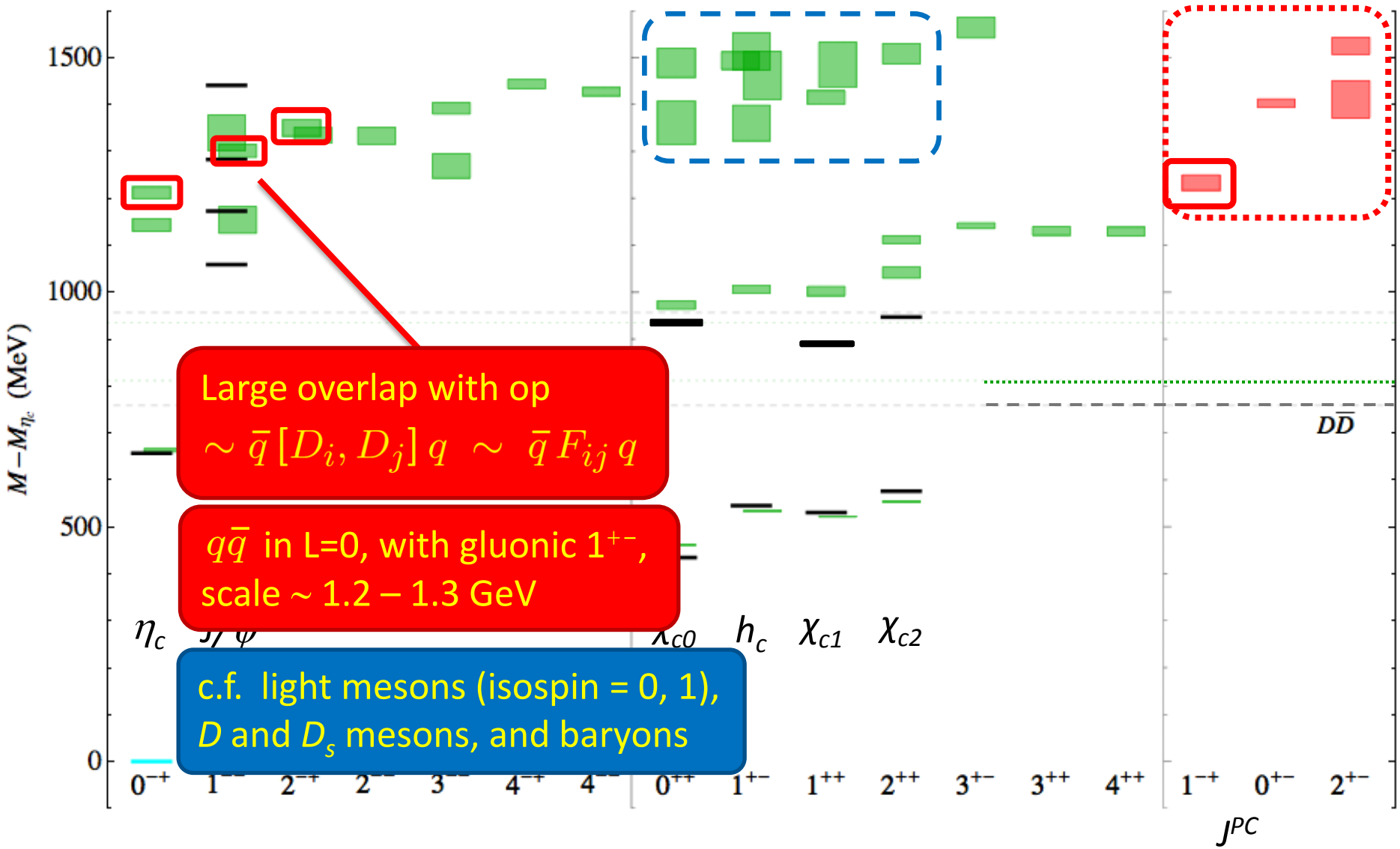

Large overlap with op
$\sim \bar{q}\,[D_i, D_j]\,q \;\sim\; \bar{q}\,F_{ij}\,q$

$q\bar{q}$ in L=0, with gluonic 1^{+-},
scale ~ 1.2 – 1.3 GeV

c.f. light mesons (isospin = 0, 1),
D and D_s mesons, and baryons

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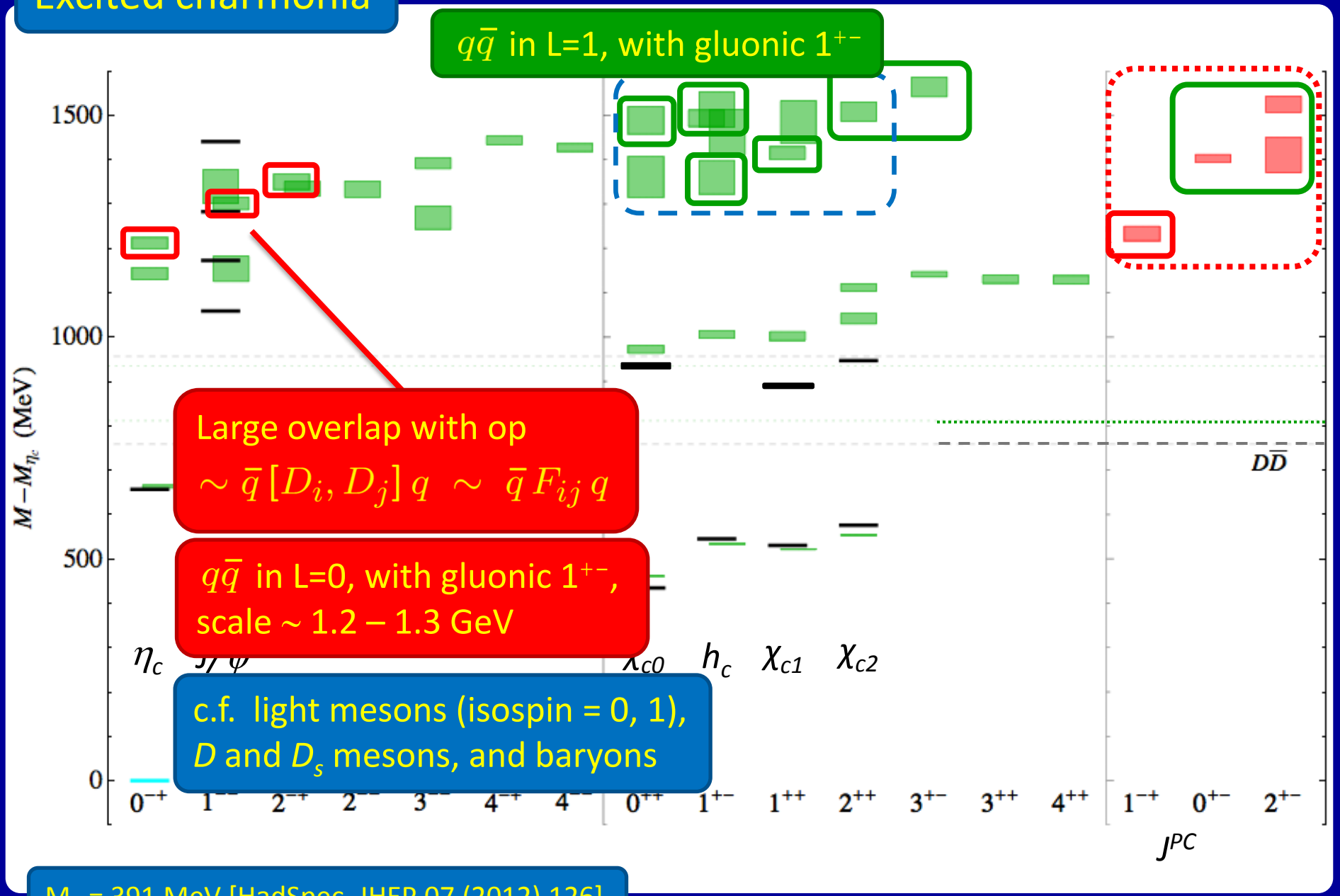

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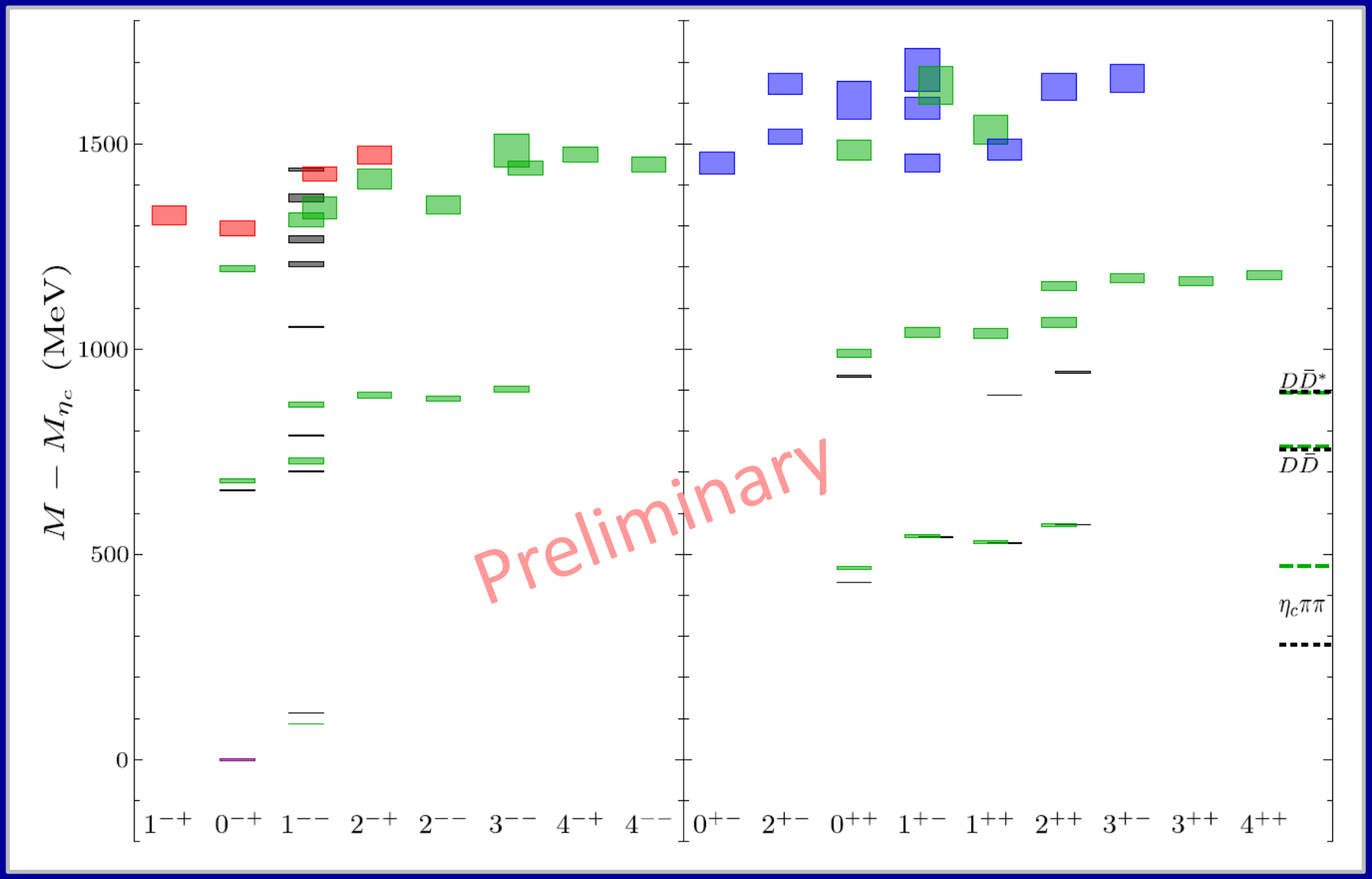
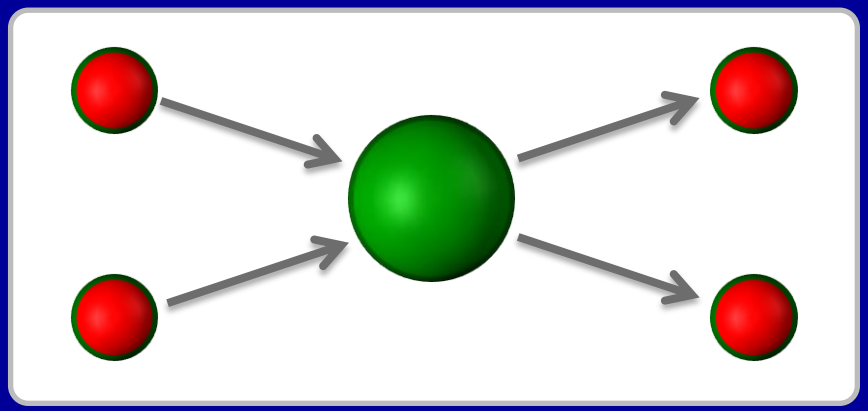
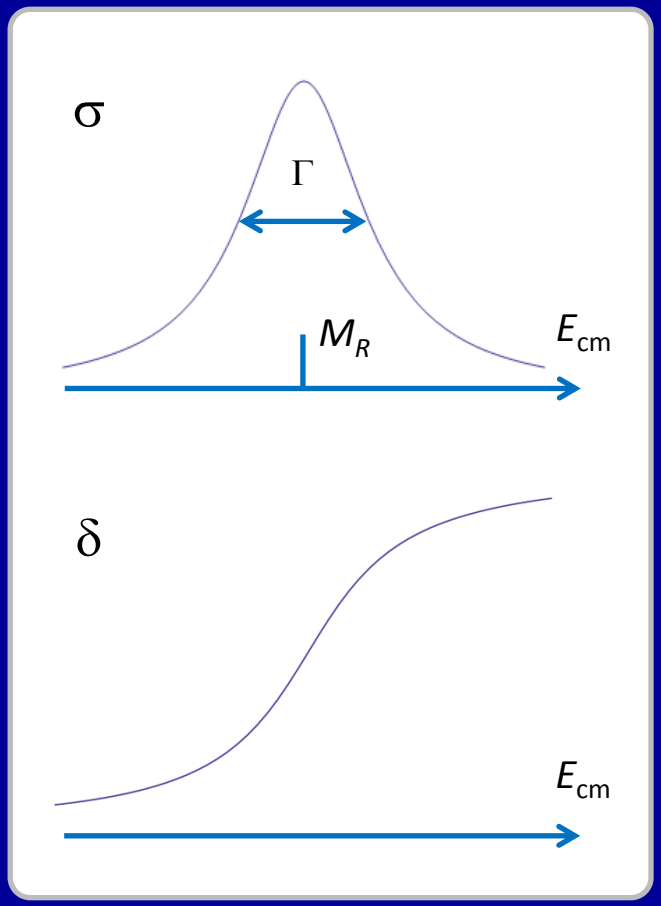


NEW: M_π = 236 MeV [HadSpec, to appear soon]

Scattering and resonances

Most hadrons appear as resonances in scattering of lighter hadrons





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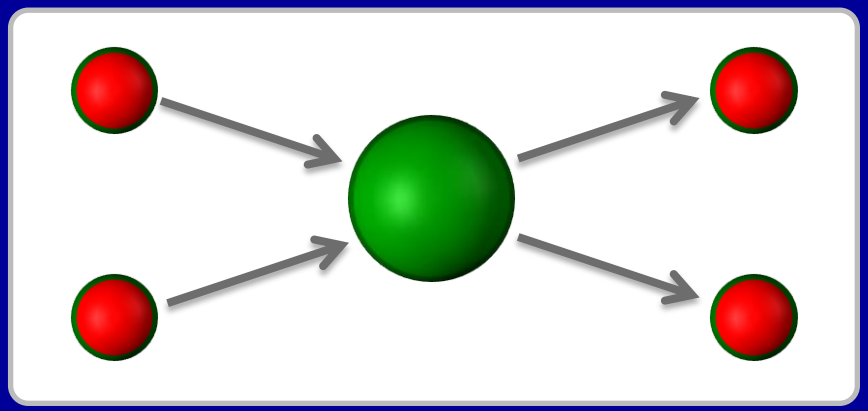
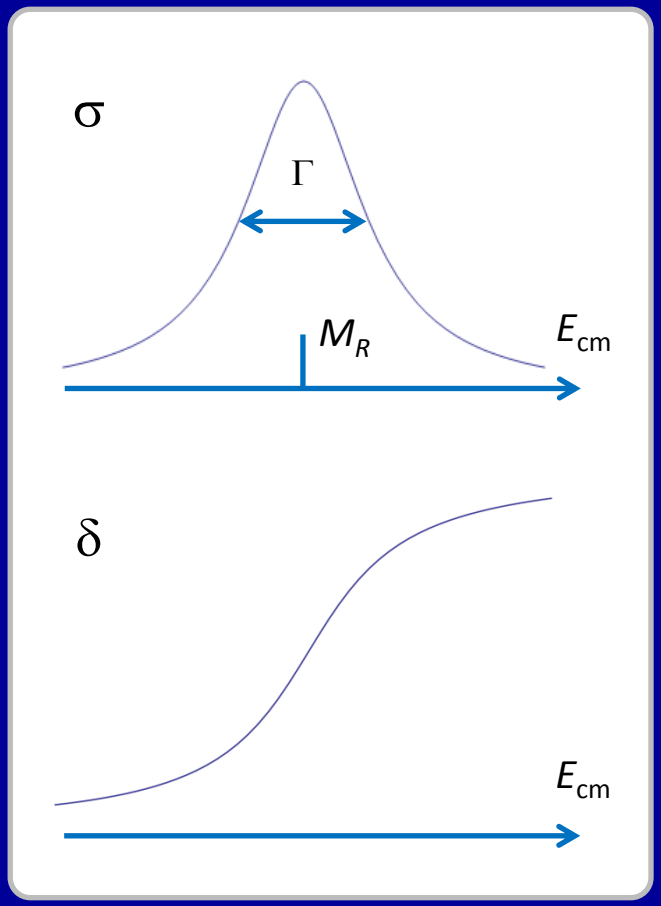
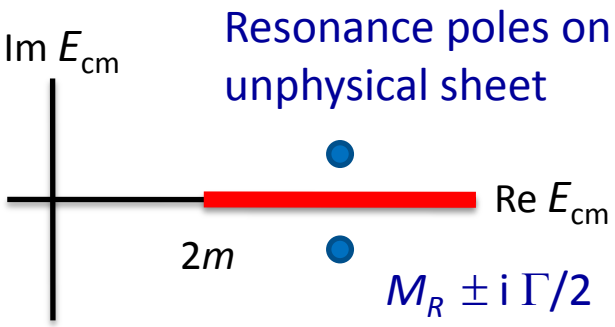


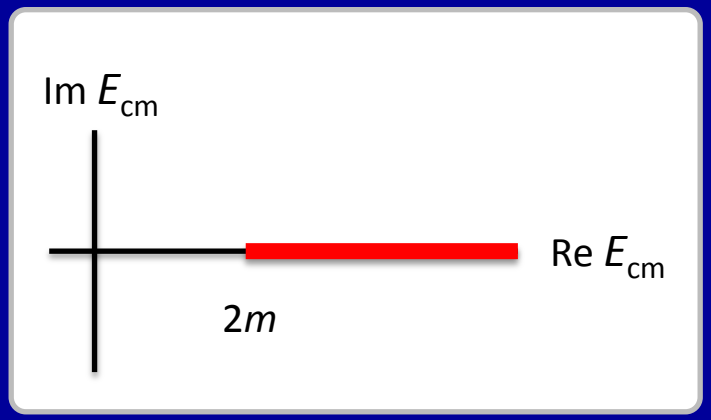


Singularity structure
of scattering matrix



Scattering in Lattice QCD

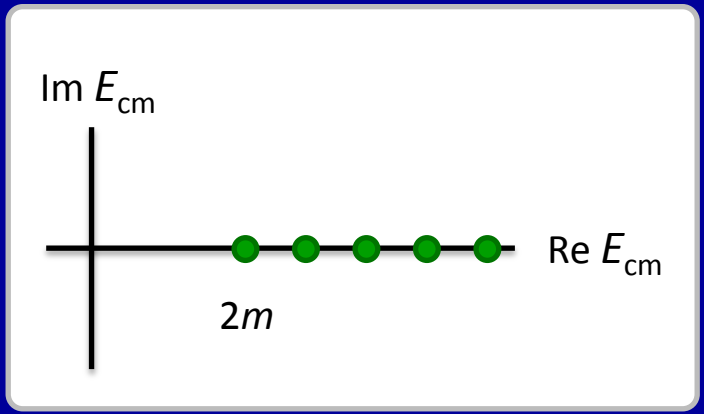
Infinite volume – continuous spectrum above threshold

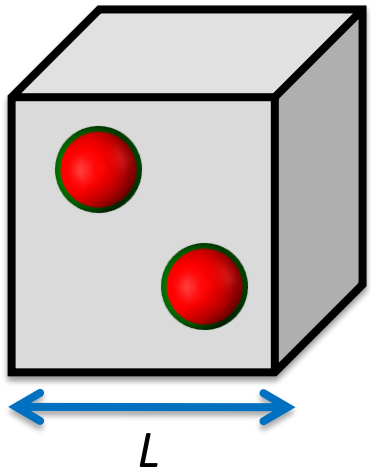


Scattering in Lattice QCD

Infinite volume – continuous spectrum above threshold

Finite volume – discrete spectrum





[periodic b.c.s]

Non-interacting: $\vec{k}_{A,B} = \frac{2\pi}{L}(n_x, n_y, n_z)$

Interacting: $\vec{k}_{A,B} \neq \frac{2\pi}{L}(n_x, n_y, n_z)$

c.f. 1-dim: $k = \frac{2\pi}{L}n + \frac{2}{L}\delta(k)$

scattering phase shift

Scattering in Lattice QCD

Lüscher method (and extensions): relate **finite-volume energy levels** $\{E_{cm}\}$ to **infinite-volume scattering *t*-matrix**

Scattering in Lattice QCD

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Elastic scattering: from E_{cm} get $t(E_{cm})$ or equivalently $\delta(E_{cm})$
[Complication: reduced symmetry of lattice volume → partial wave mixing]

Scattering in Lattice QCD

Lüscher method (and extensions): relate **finite-volume energy levels** $\{E_{cm}\}$ to **infinite-volume scattering *t*-matrix**

Elastic scattering: from E_{cm} get $t(E_{cm})$ or equivalently $\delta(E_{cm})$
[Complication: reduced symmetry of lattice volume → partial wave mixing]

Coupled-channel scattering:

E.g. $$\mathbf{t}(E_{\rm cm}) = \begin{pmatrix} t_{\pi\pi\to\pi\pi}(E_{\rm cm}) & t_{\pi\pi\to K\bar{K}}(E_{\rm cm}) \\ t_{K\bar{K}\to\pi\pi}(E_{\rm cm}) & t_{K\bar{K}\to K\bar{K}}(E_{\rm cm}) \end{pmatrix}$$

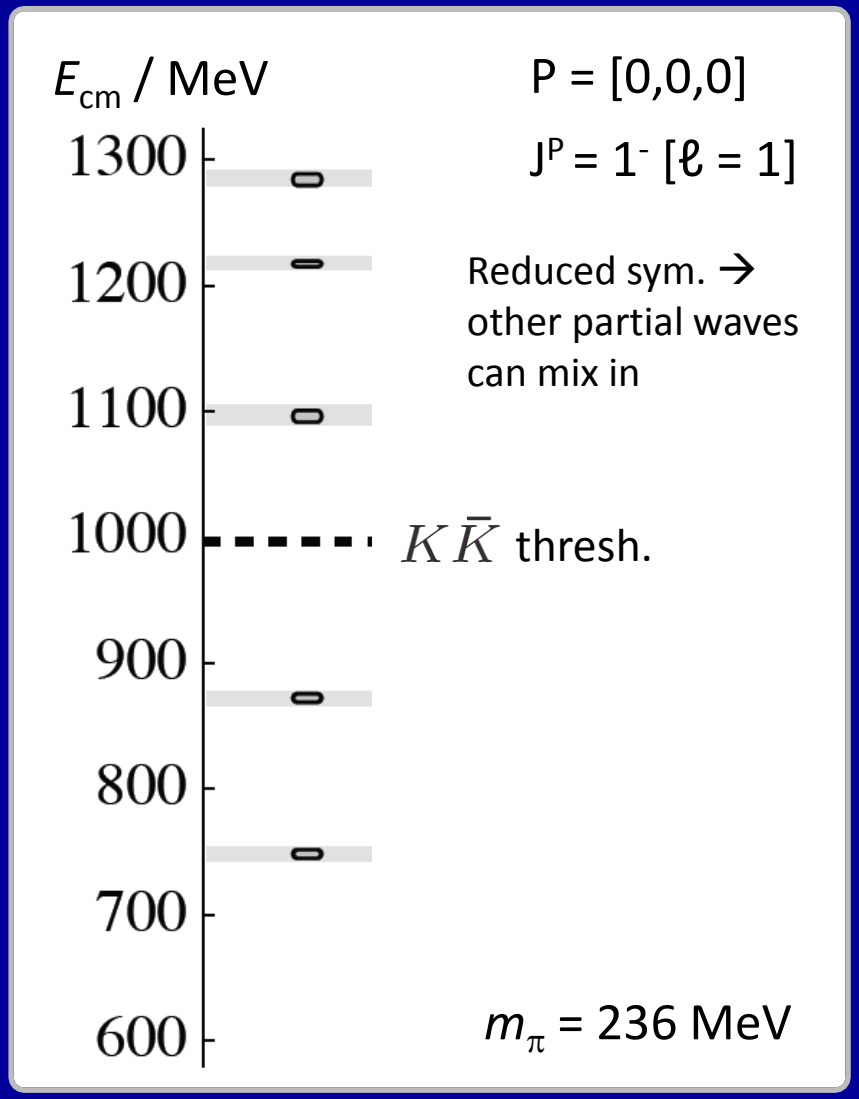
→ Determinant equation for $\mathbf{t}(E_{cm})$ at each E_{cm}
→ **Under-constrained problem** (e.g. 2 channels: 3 unknowns but 1 equ.)
→ Parameterize E_{cm} dependence of *t*-matrix and fit $\{E_{lat}\}$ to $\{E_{param}\}$

Try different parameterizations, e.g. various *K*-matrix forms
(for elastic scattering also Breit Wigner, effective range expansion).

Larger set of E_{cm} by e.g. overall non-zero mom., twisted b.c.s, different vols.

The ρ resonance in ππ scattering

($J^{PC} = 1^{--}$, I = 1)



Experimentally

$$\mathrm{BR}(\rho \to \pi\pi) \sim 100\%$$

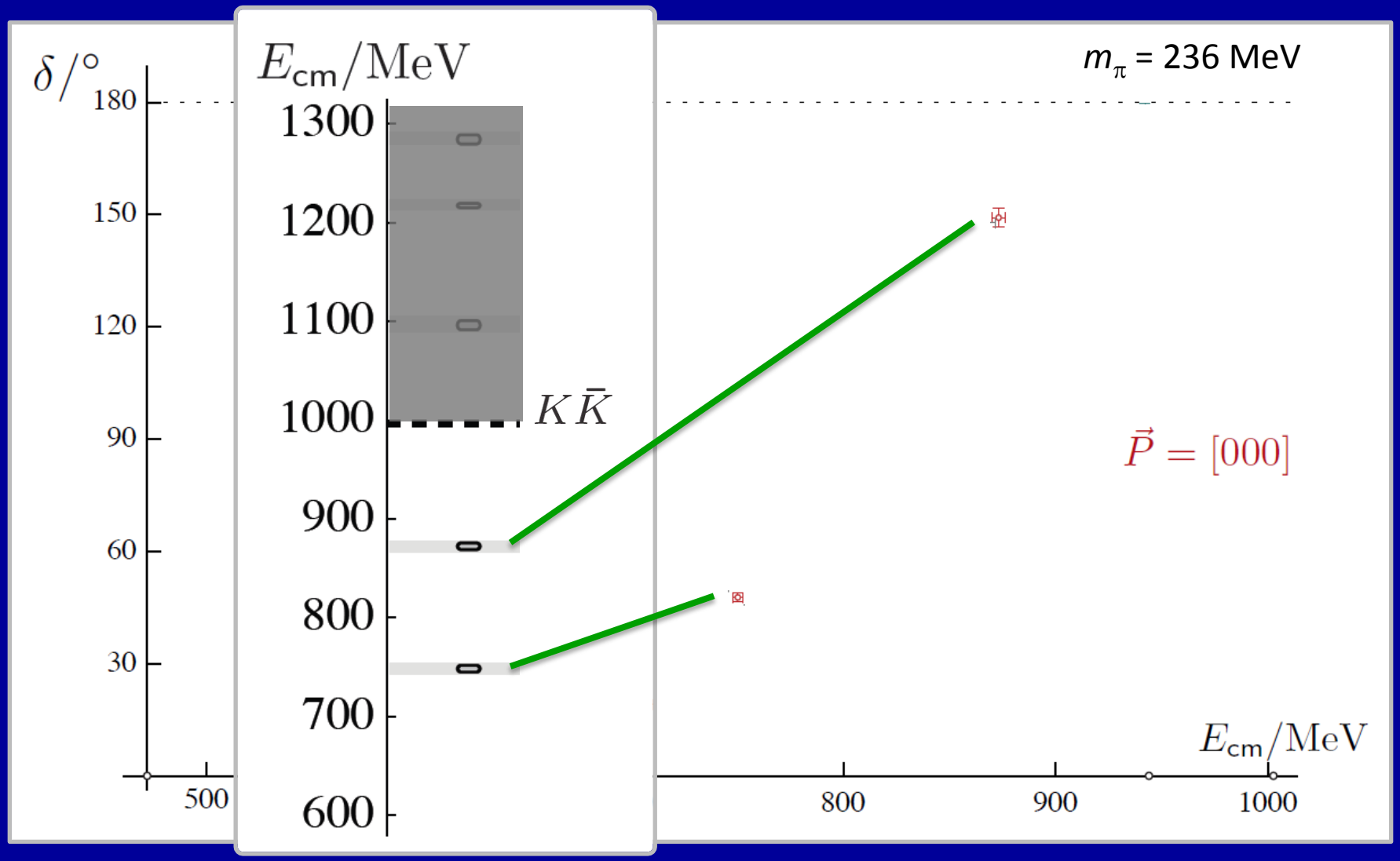
Finite volume spectrum from:

$$C_{ij}(t) = < 0|\mathcal{O}_i(t)\mathcal{O}_j^\dagger(0)|0 >$$

Use many different operators

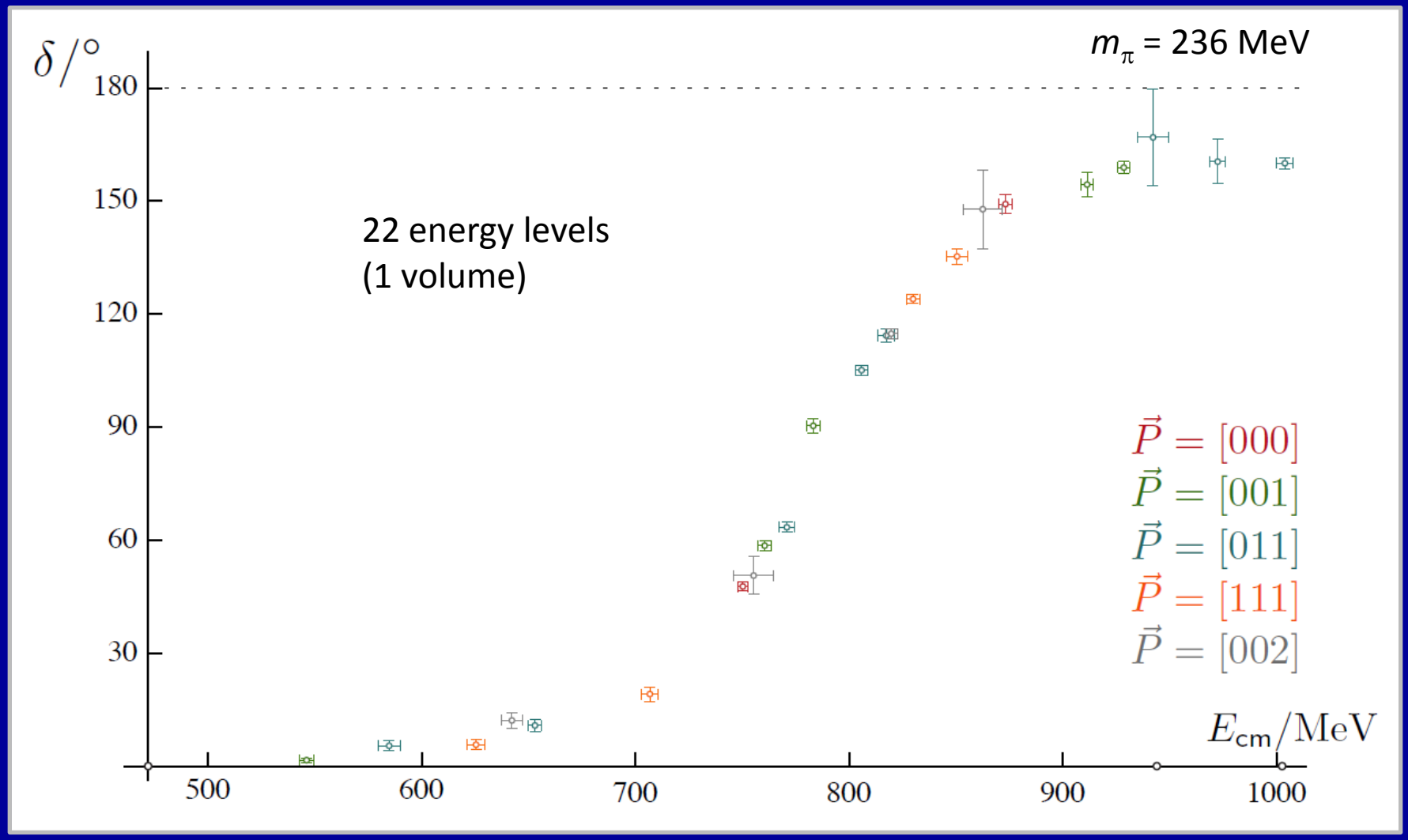
Wilson *et al* (HadSpec) [PR D92, 094502 (2015)] and Dudek, Edwards, CT (HadSpec) [PR D87, 034505 (2013)]

The ρ resonance: **elastic** ππ scattering



(HadSpec) [PR D87, 034505 (2013); PR D92, 094502 (2015)]

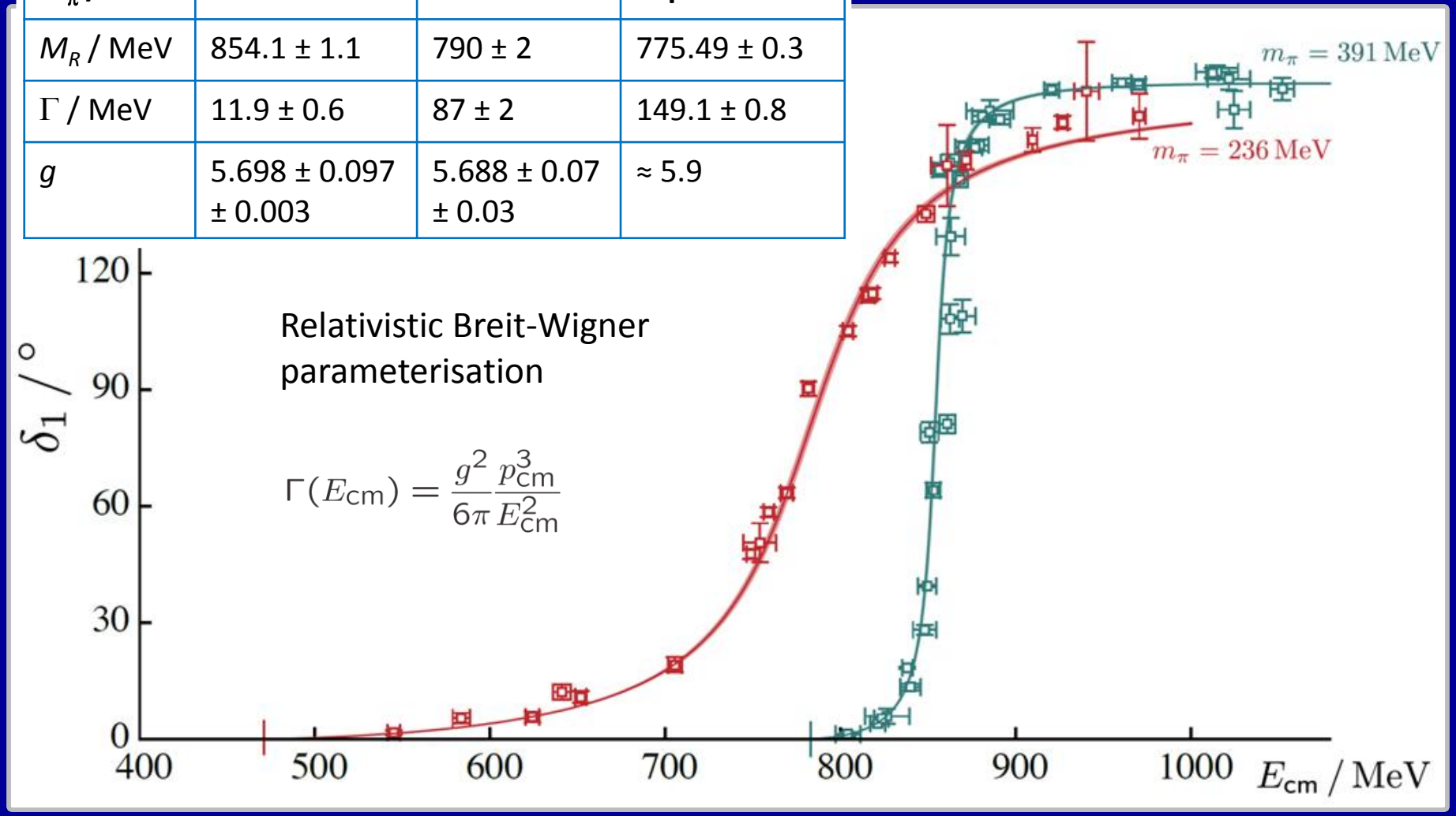
The ρ resonance: **elastic** ππ scattering



(HadSpec) [PR D87, 034505 (2013); PR D92, 094502 (2015)]

The ρ resonance: **elastic** ππ scattering

m_π / MeV	391	236	Experimental
M_R / MeV	854.1 ± 1.1	790 ± 2	775.49 ± 0.3
Γ / MeV	11.9 ± 0.6	87 ± 2	149.1 ± 0.8
g	5.698 ± 0.097 ± 0.003	5.688 ± 0.07 ± 0.03	≈ 5.9

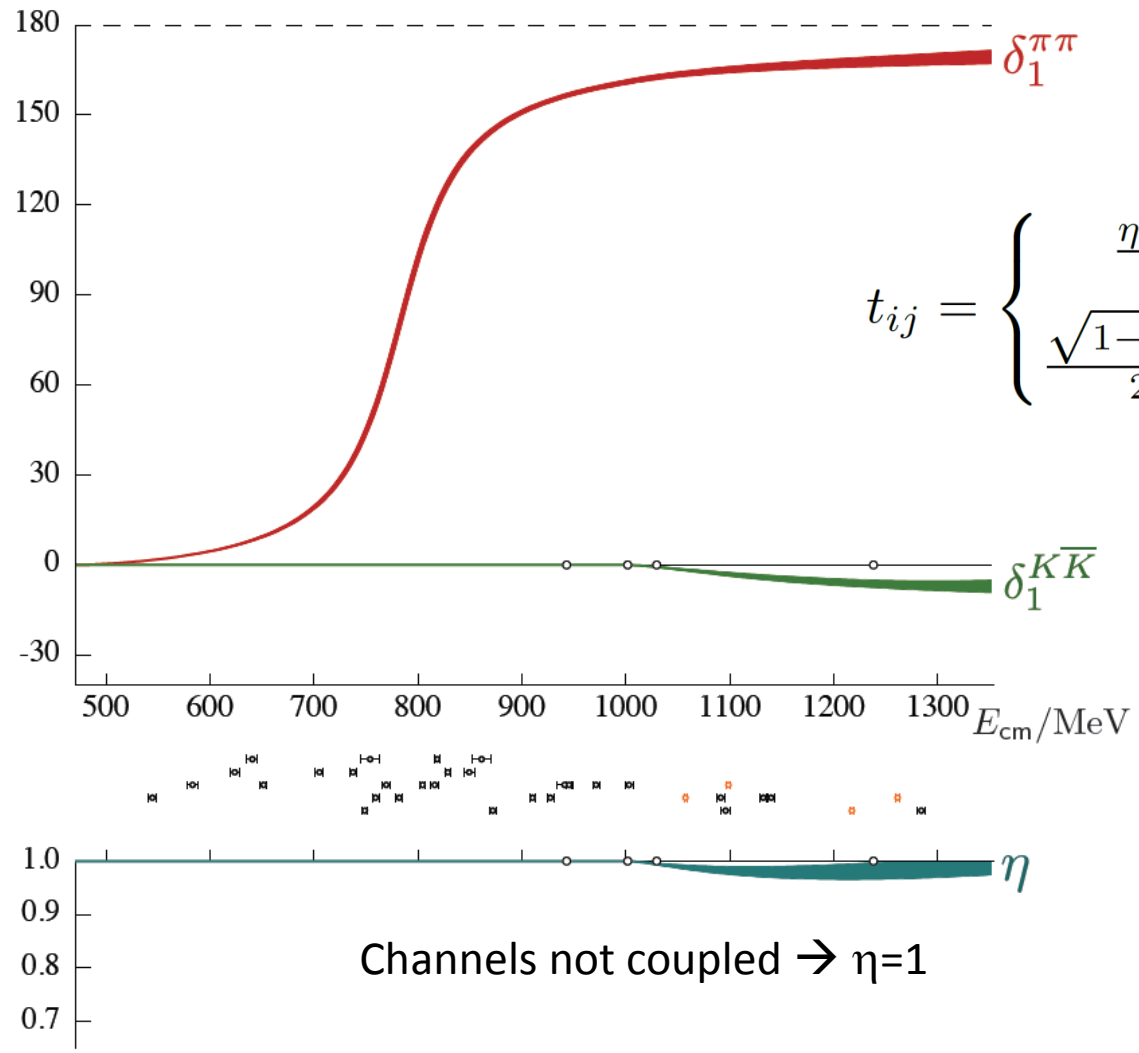


Relativistic Breit-Wigner parameterisation

$$\Gamma(E_{\rm cm}) = \frac{g^2}{6\pi}\frac{p_{\rm cm}^3}{E_{\rm cm}^2}$$

(HadSpec) [PR D87, 034505 (2013); PR D92, 094502 (2015)]

The ρ resonance: **coupled-channel** $\pi\pi, K\bar{K}$



m_π = 236 MeV

$$t_{ij} = \begin{cases} \dfrac{\eta\, e^{2i\delta_i} - 1}{2i\,\rho_i} & (i = j) \\ \dfrac{\sqrt{1-\eta^2}\, e^{i(\delta_i+\delta_j)}}{2\sqrt{\rho_i\,\rho_j}} & (i \neq j) \end{cases}$$

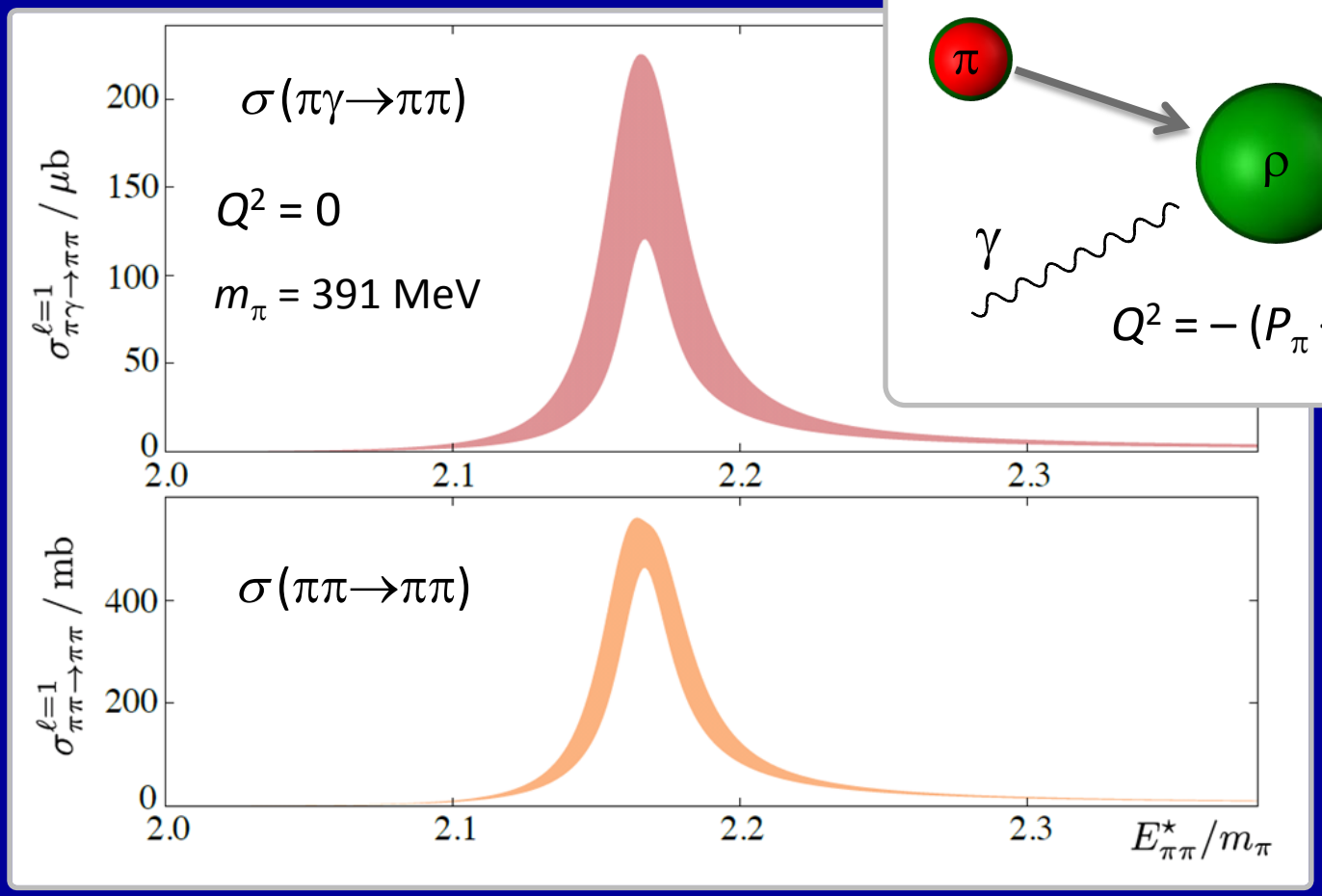
34 energy levels
(1 volume)

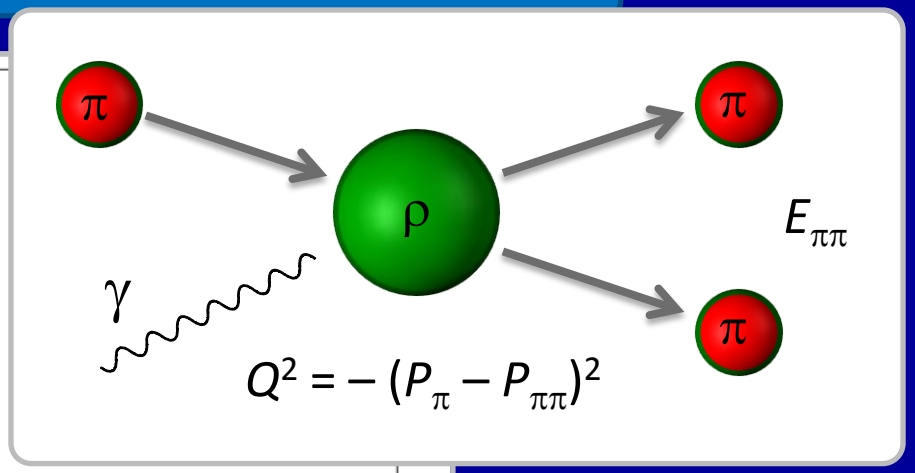
Channels not coupled → η=1

(HadSpec) [PR D92, 094502 (2015)]

Resonant $\pi^+ \gamma \to \rho \to \pi^+ \pi^0$ amplitude

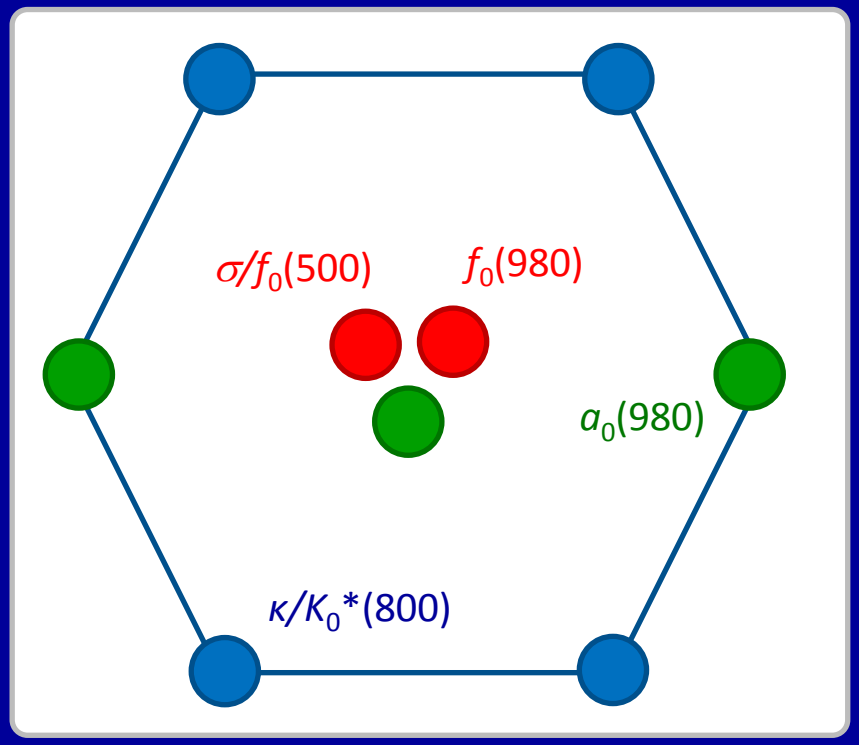
Need: $C_{ij}(t_f, t, t_i) = < 0|O_i(t_f)\ \bar{\psi}(t)\gamma^\mu\psi(t)\ O_j(t_i)|0 >$





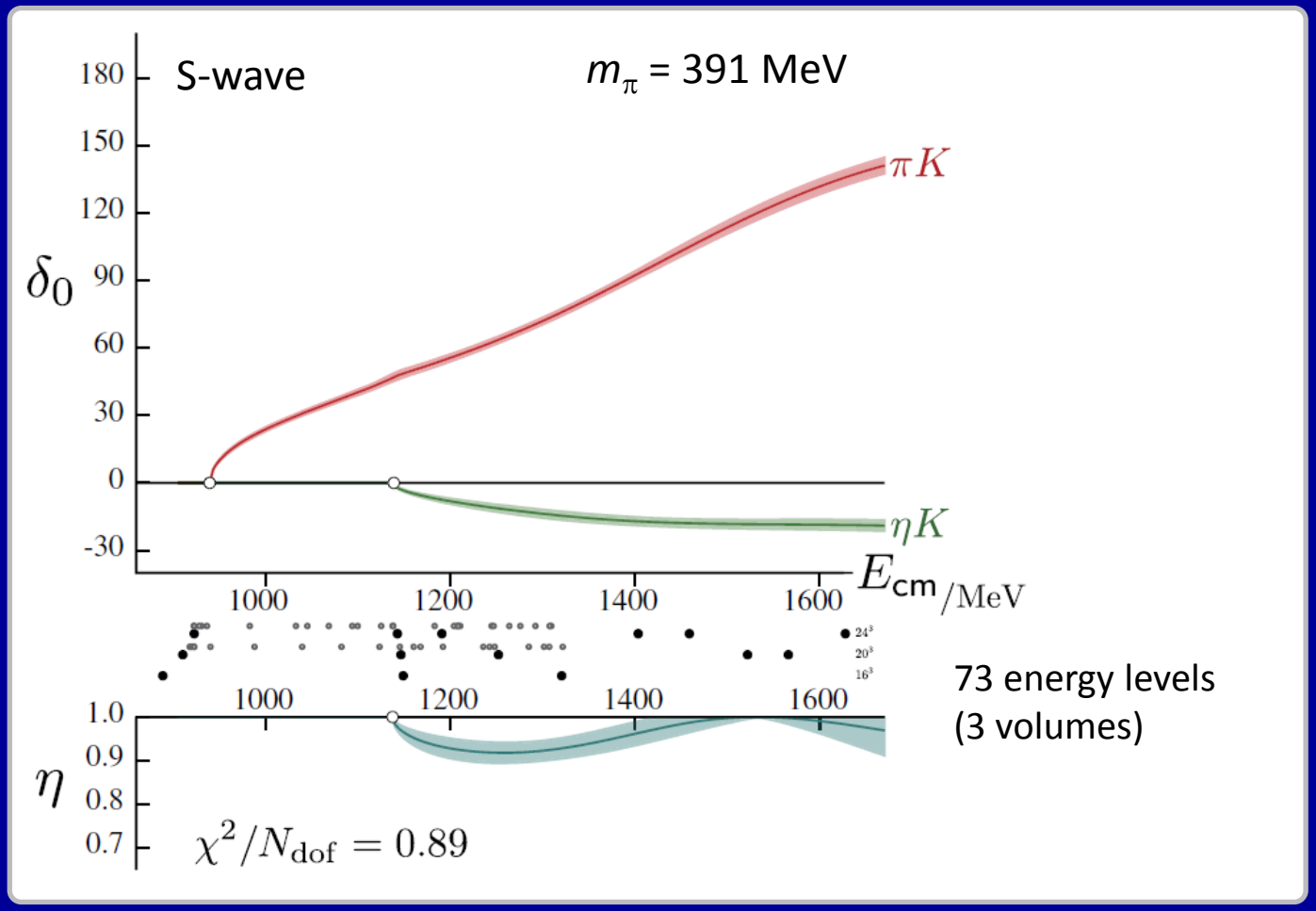
Briceño *et al* (HadSpec) [PRL 115, 242001 (2015); PRD 93, 114508 (2016)]

Light scalar mesons



κ in πK, ηK

$J^P = 0^+$, Isospin = ½, Strangeness = 1



73 energy levels
(3 volumes)

Wilson, Dudek, Edwards, CT (HadSpec) [PRL 113, 182001 2014); PR D91, 054008 (2015)]

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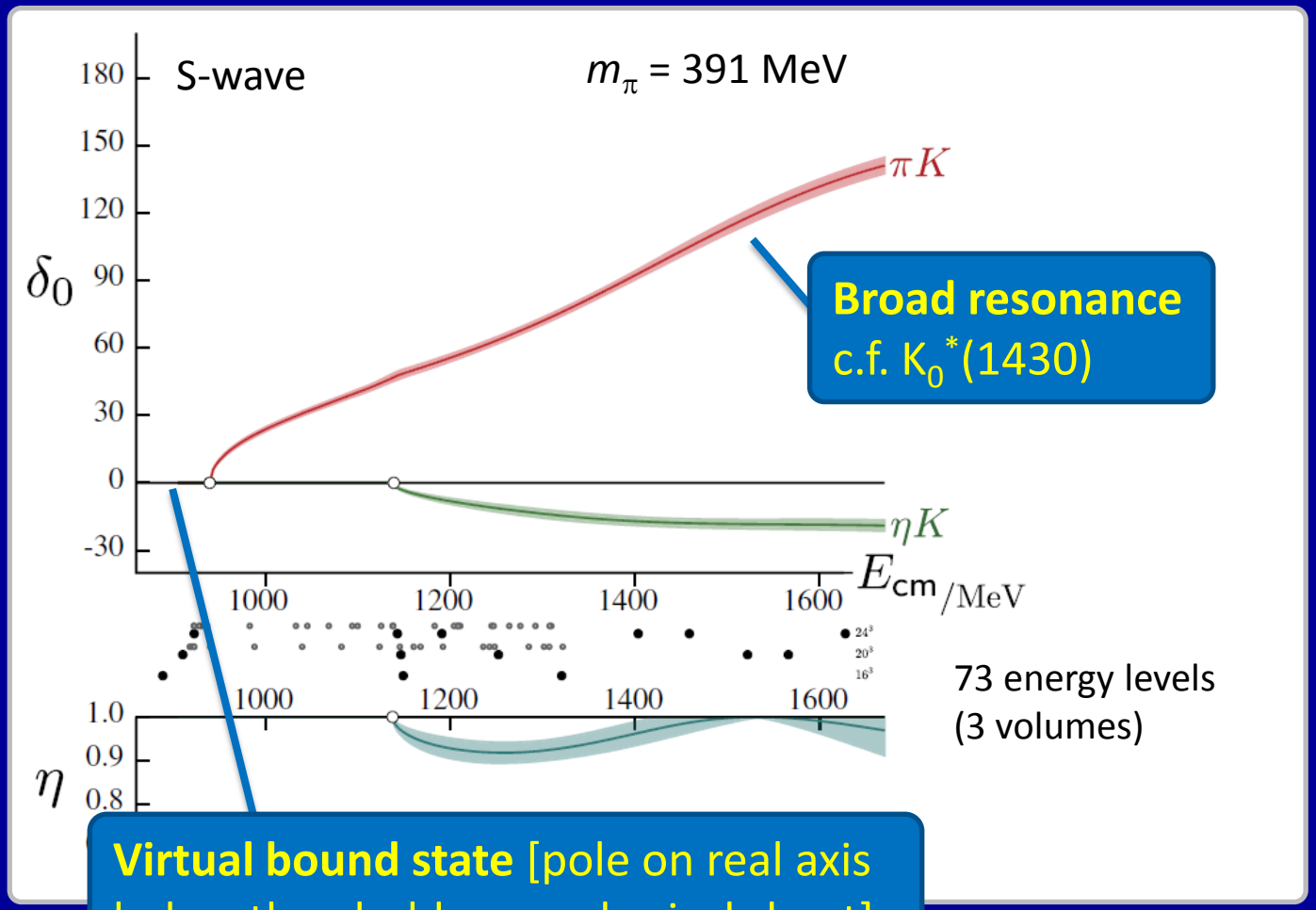

Broad resonance
c.f. $K_0^*(1430)$

73 energy levels
(3 volumes)

Virtual bound state [pole on real axis below threshold on unphysical sheet]

c.f. κ in unitarised χpt [Nebreda & Pelaez, PR D81, 054035 (2010)]

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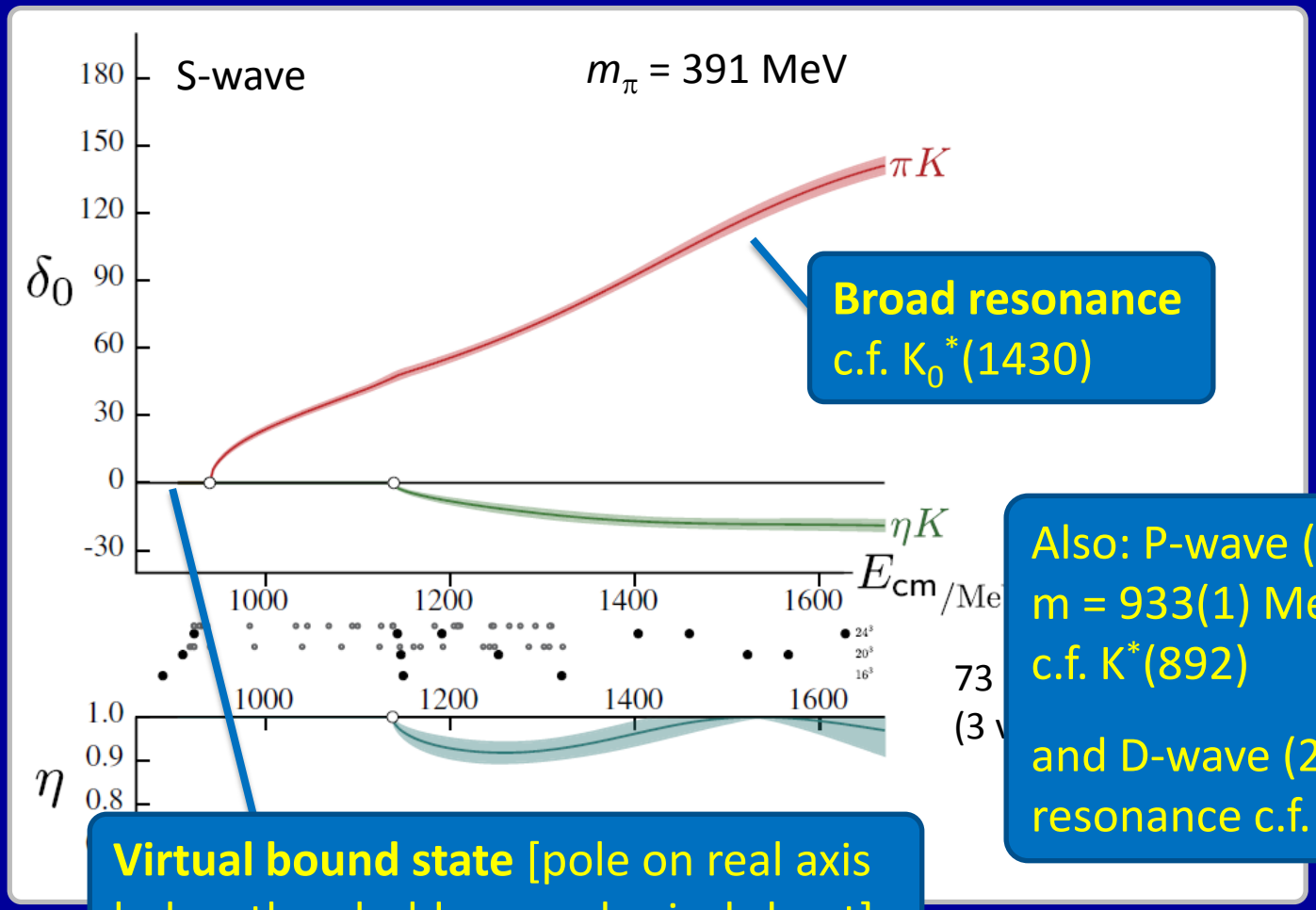

Broad resonance
c.f. $K_0^*(1430)$

Also: P-wave (1^-) bound state,
m = 933(1) MeV, g = 5.93(26)
c.f. $K^*(892)$

and D-wave (2^+) narrow resonance c.f. $K_2^*(1430)$

Virtual bound state [pole on real axis below threshold on unphysical sheet]

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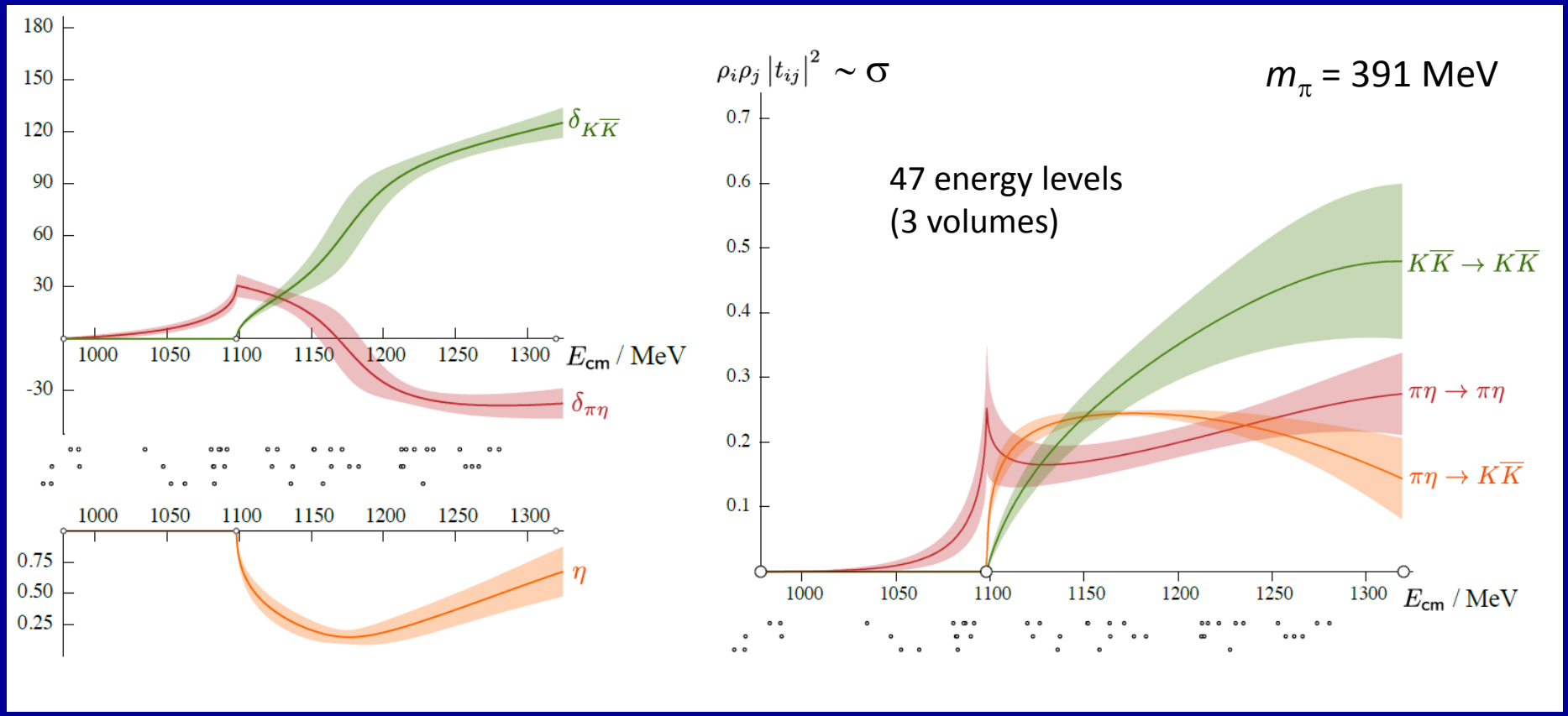
a_0 resonance in $\pi\eta$, $K\bar{K}$

$J^P = 0^+$, $I = 1$



Strongly coupled to both $\pi\eta$ and $K\bar{K}$

Dudek, Edwards, Wilson (HadSpec)
[PR D93, 094506 (2016)]

a_0 resonance in $\pi\eta$, $K\bar{K}$

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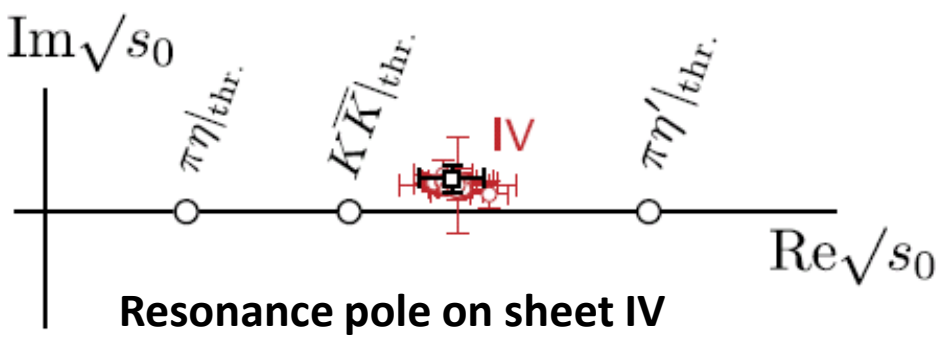


Resonance pole on sheet IV
very close to sheet II.

$$\sqrt{s_0} = \left((1177 \pm 27) + \frac{i}{2}(49 \pm 33)\right) \text{ MeV}$$

$$|c_{K\bar{K}}/c_{\pi\eta}| = 1.30(37) \qquad t_{ij} \sim \frac{c_i c_j}{s_0 - s}$$

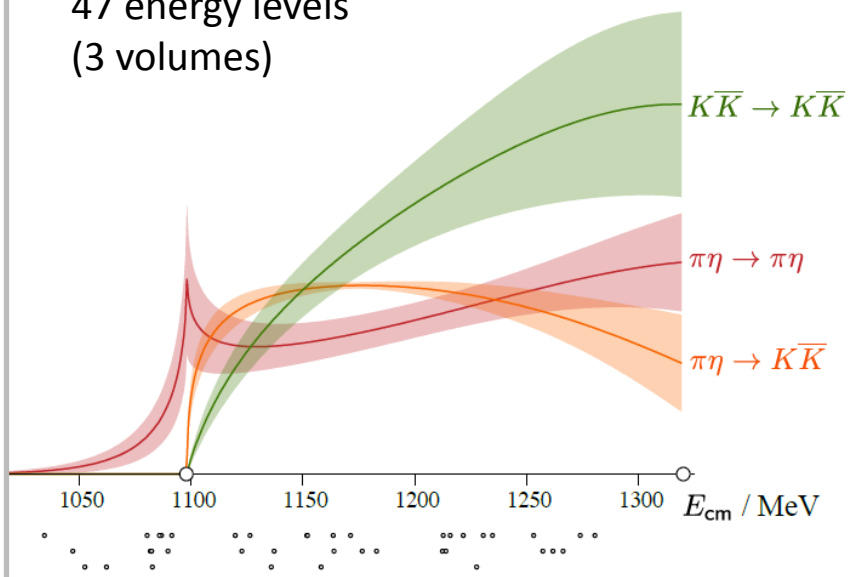
C.f. analysis of exp. data,
Baru *et al* [EPJ A23, 523 (2005)]

Sheet	$\mathrm{Im}k_{\pi\eta}$	$\mathrm{Im}k_{K\bar{K}}$
I	+	+
II	−	+
III	−	−
IV	+	−

$\sim \sigma$

m_π = 391 MeV

47 energy levels
(3 volumes)



Dudek, Edwards, Wilson (HadSpec)
[PR D93, 094506 (2016)]

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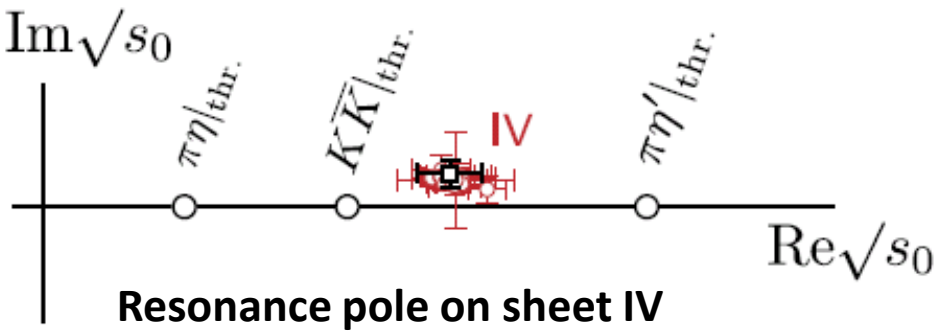

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Sheet	$\mathrm{Im}k_{\pi\eta}$	$\mathrm{Im}k_{K\bar{K}}$
I	+	+
II	−	+
III	−	−
IV	+	−

$\sim \sigma$

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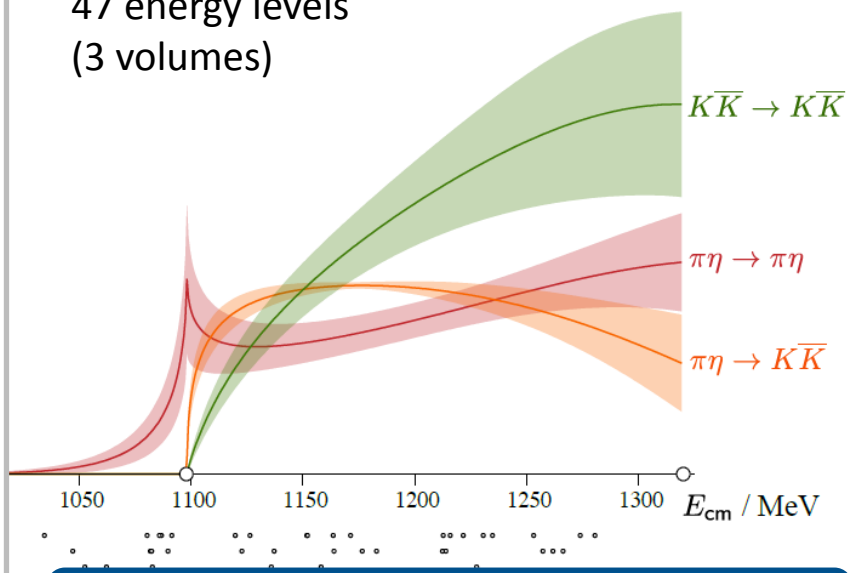
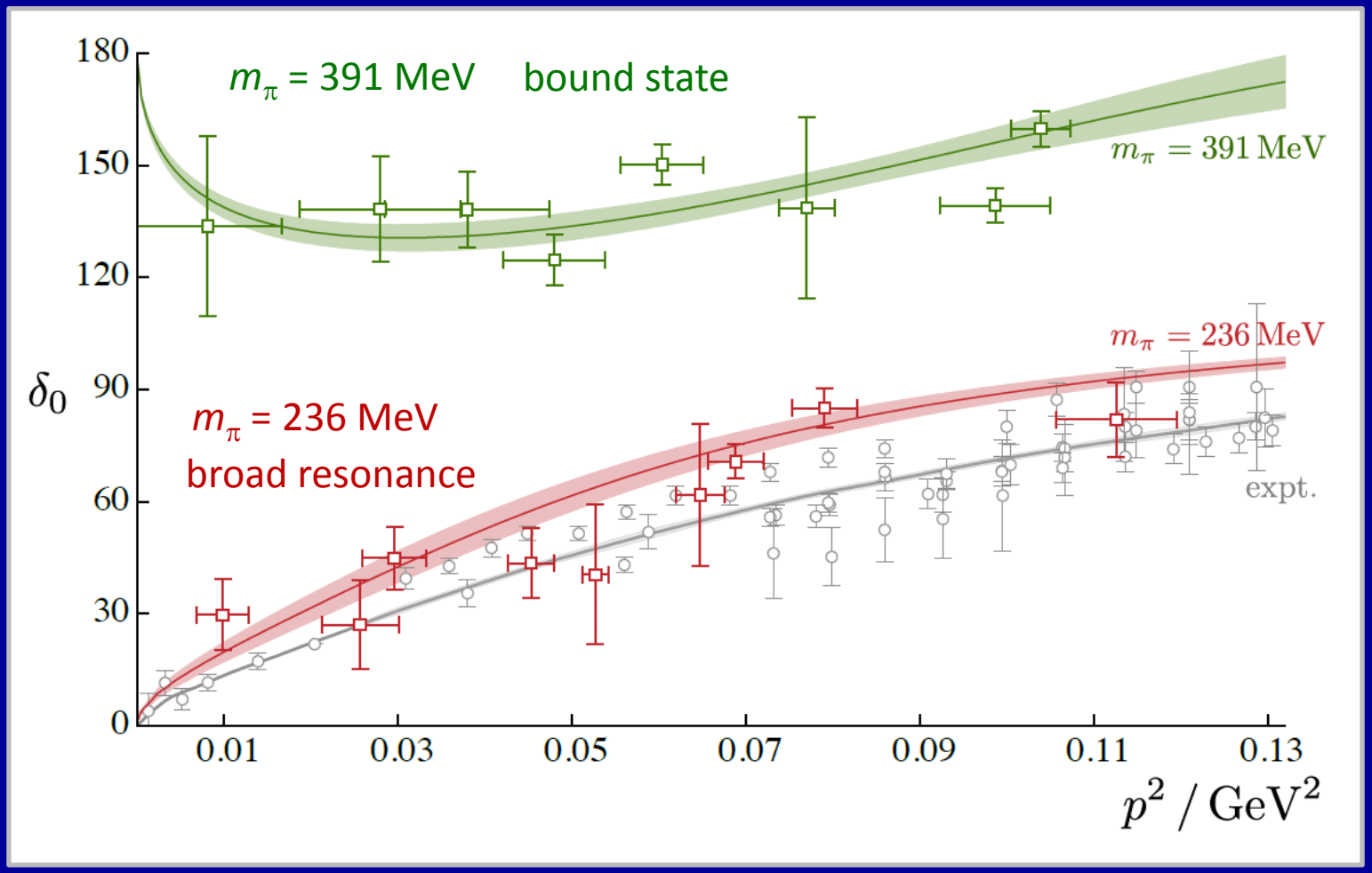


Also: including $\pi\eta'$ in S-wave, and a D-wave (2^+) resonance c.f. a_2

Dudek, Edwards, Wilson (HadSpec)
[PR D93, 094506 (2016)]

$f_0(500)/\sigma$ in $\pi\pi$ scattering

$J^P = 0^+, I = 0$



Briceño, Dudek, Edwards, Wilson (HadSpec) [arXiv:1607.05900]

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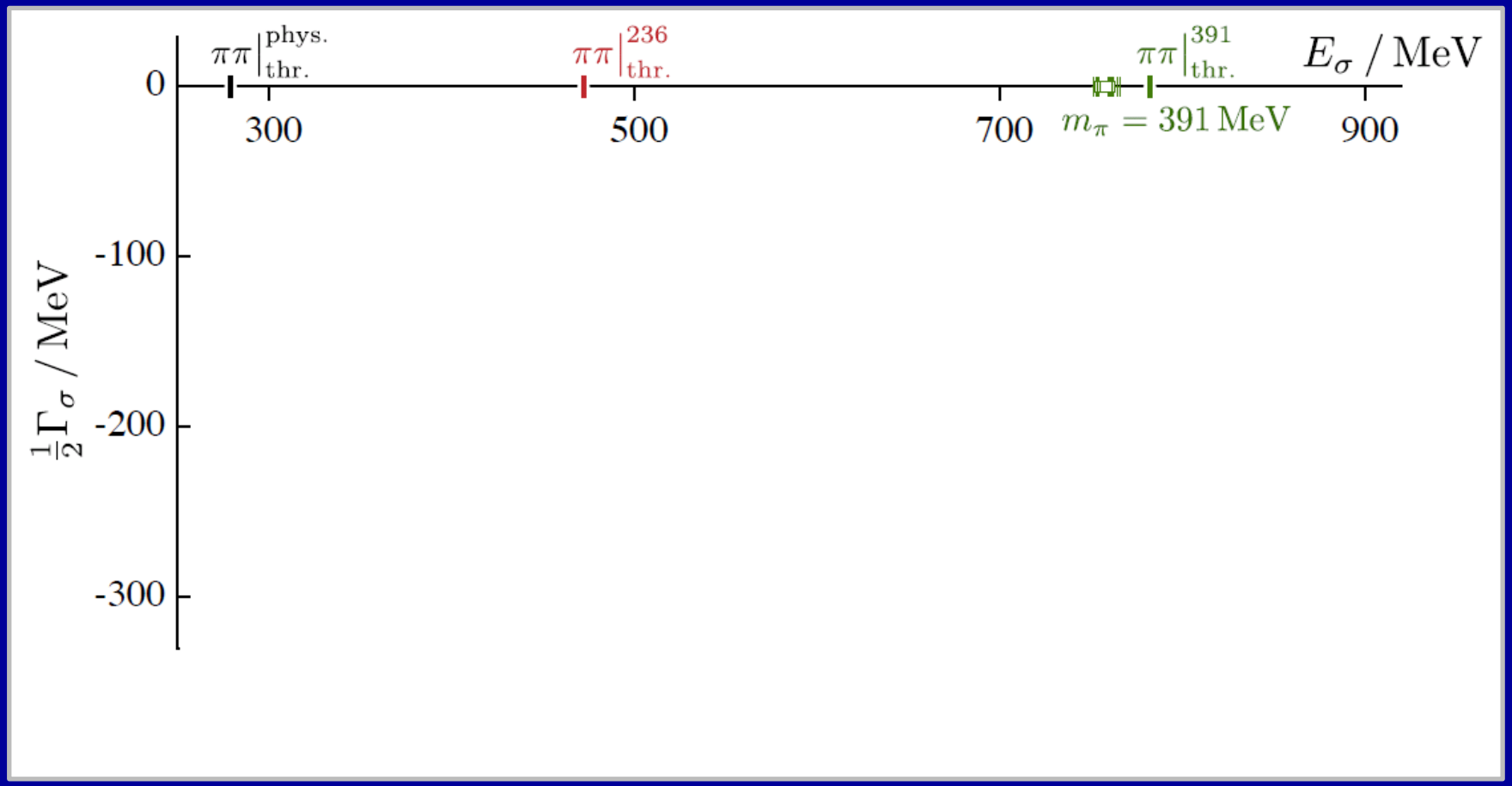

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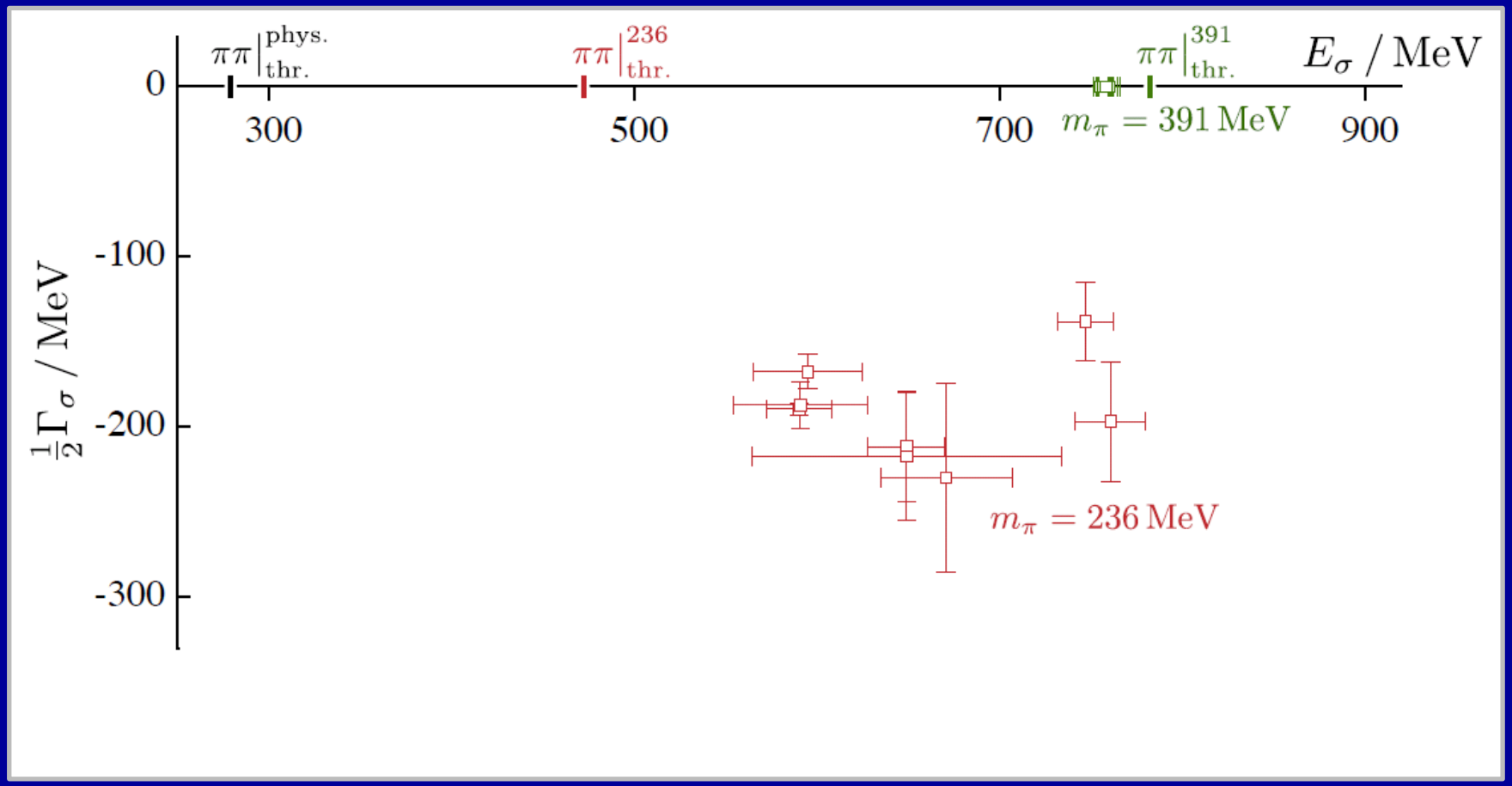

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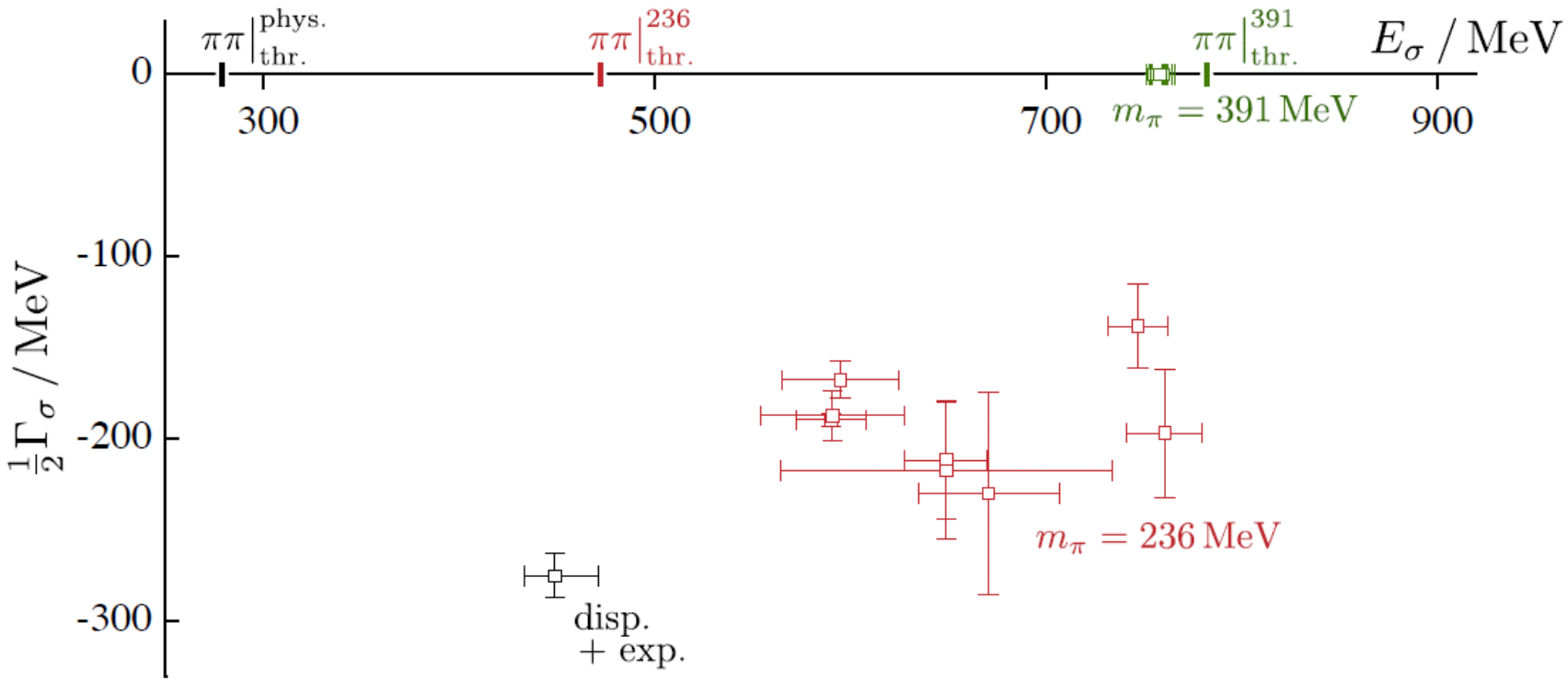
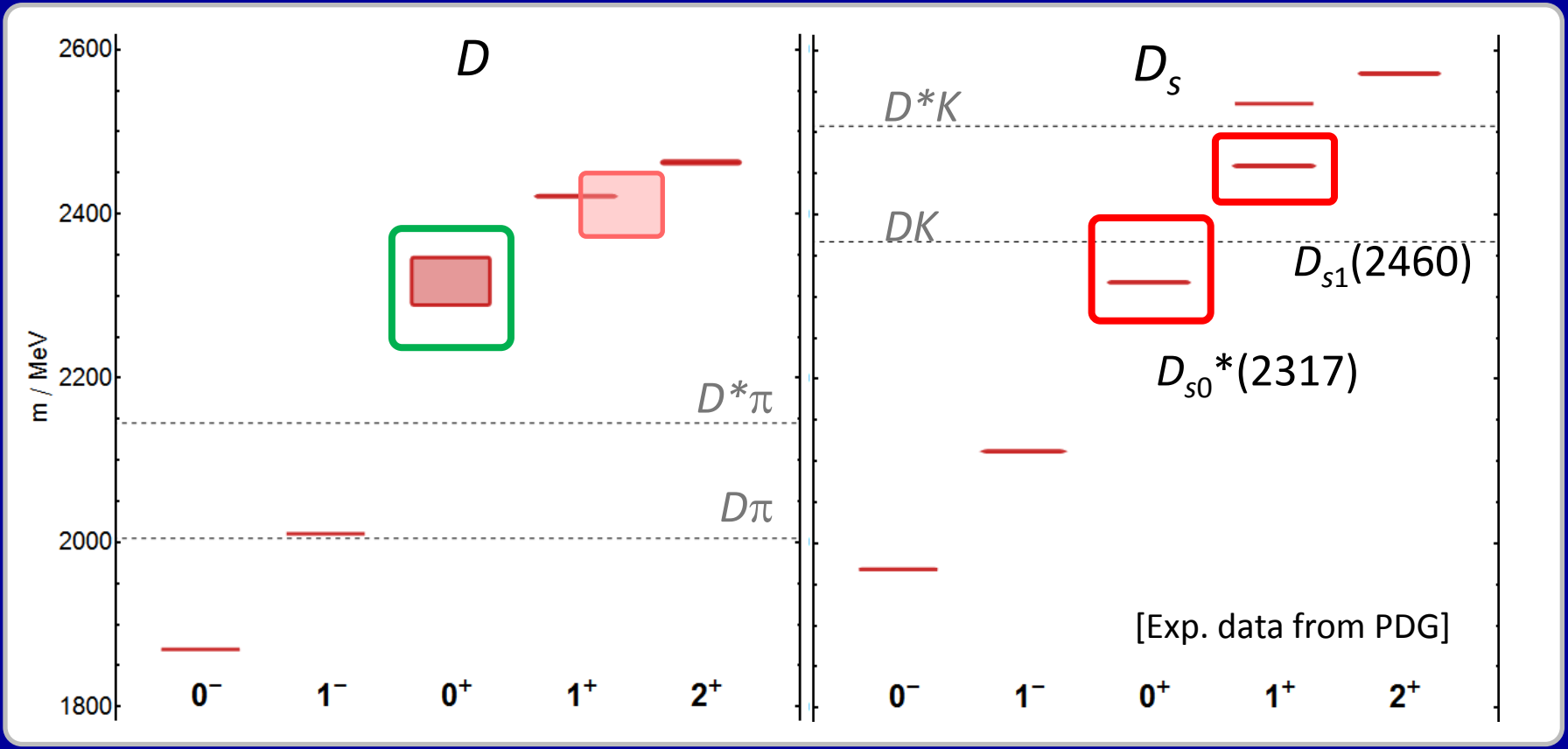


analysis of exp. data, Pelaez [arXiv:1510.00653]

Briceño, Dudek, Edwards, Wilson (HadSpec) [arXiv:1607.05900]

Charm-light (D) and charm-strange (D_s) mesons

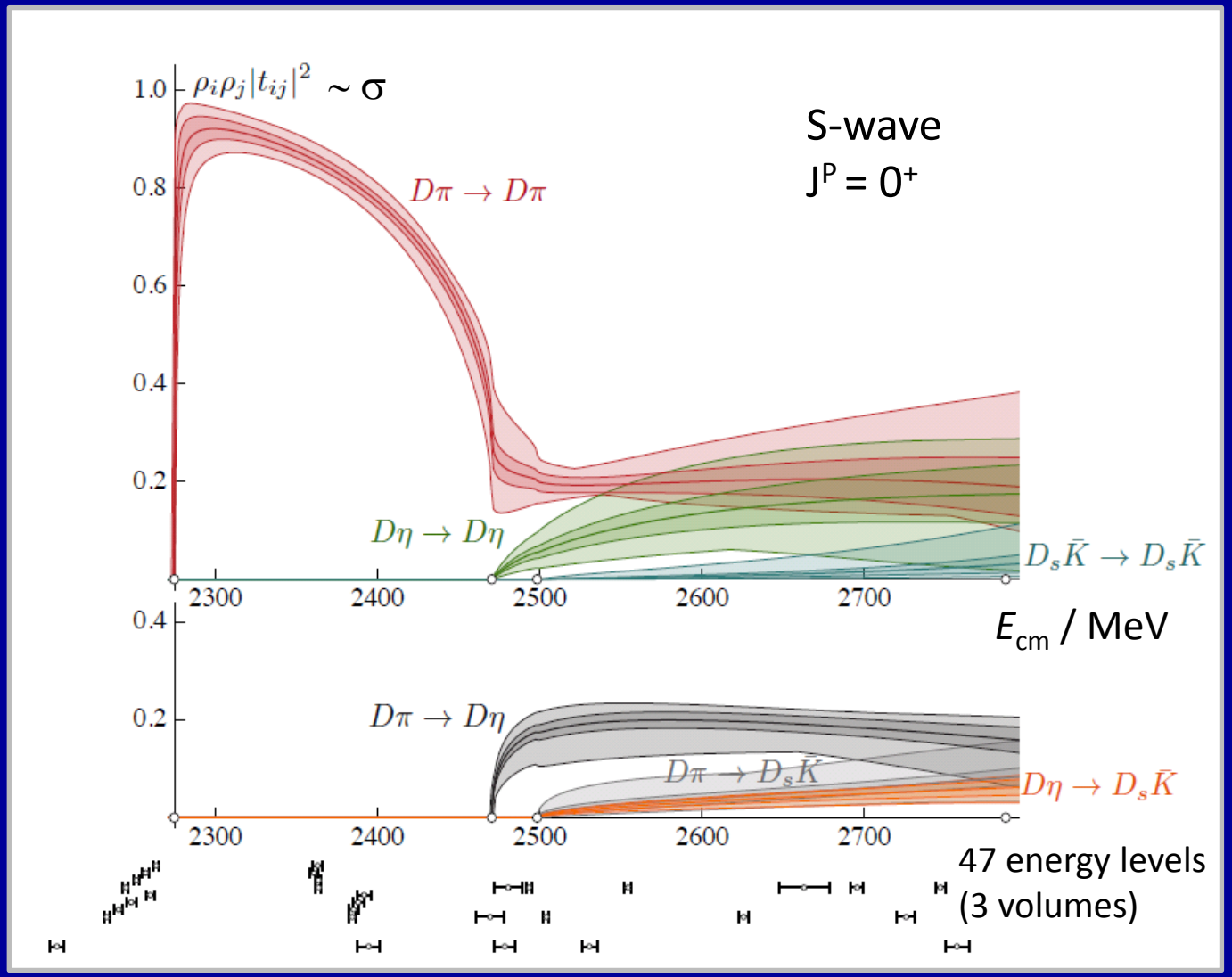


Some earlier LQCD studies:

- Mohler *et al* [PR D87, 034501 (2012)] – 0^+ $D\,\pi$ and 1^+ $D^*\,\pi$ resonances
- Mohler *et al* [PRL 111, 222001 (2013)] – 0^+ $D_s(2317)$ below $D\,K$ threshold
- Lang *et al* [PRD 90, 034510 (2014)] – 0^+ $D_s(2317)$ and 1^+ $D_{s1}(2460)$, $D_{s1}(2536)$

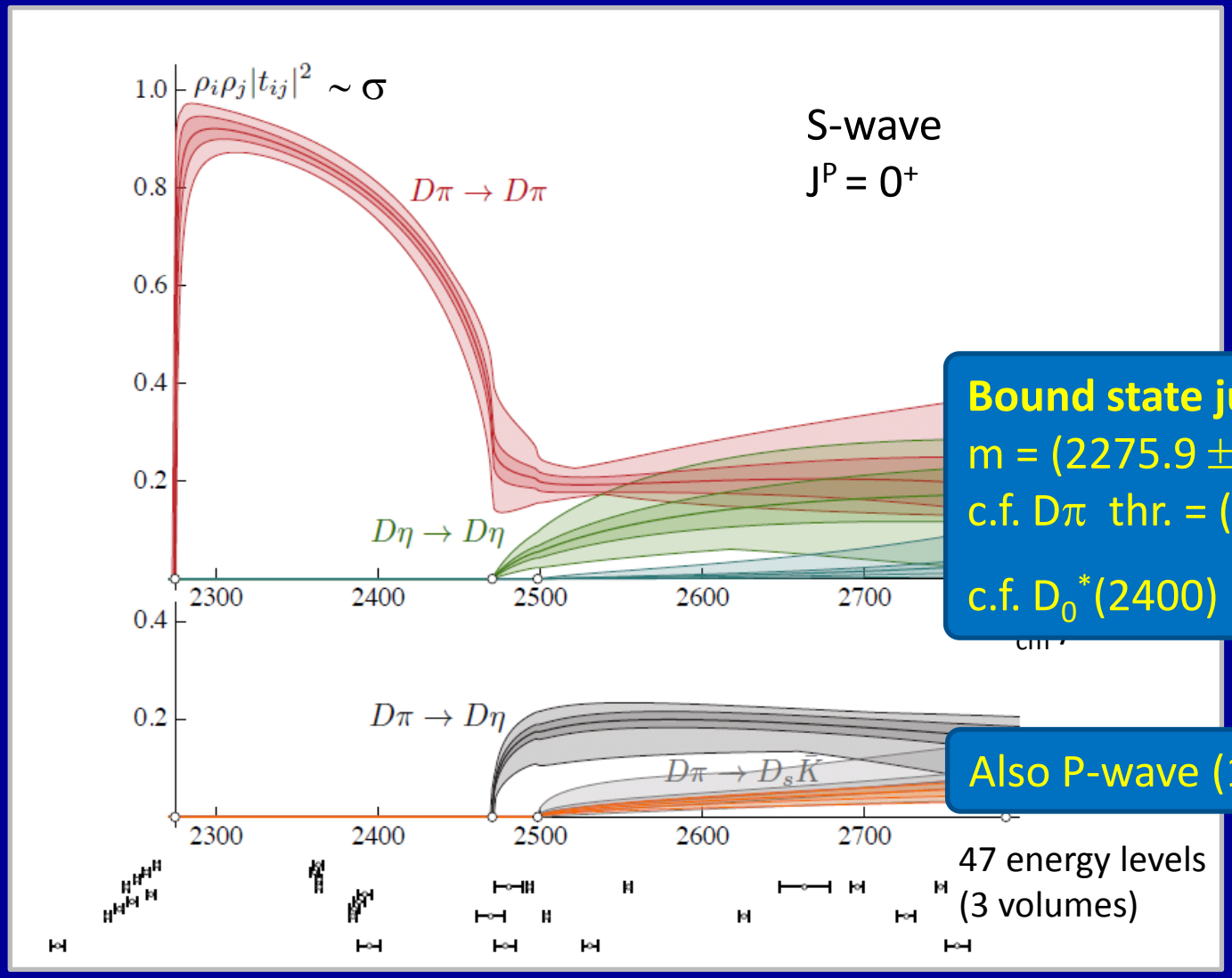
$D\pi$, $D\eta$, $D_s\bar{K}$ (I=½)

m_π = 391 MeV



Moir, Peardon, Ryan, CT, Wilson (HadSpec) [arXiv:1607.07093]

$D\pi$, $D\eta$, $D_s\bar{K}$ (I=½)



m_π = 391 MeV

Bound state just below thresh.
m = (2275.9 ± 0.9) MeV
c.f. $D\pi$ thr. = (2276.4 ± 0.9) MeV

c.f. $D_0^*(2400)$

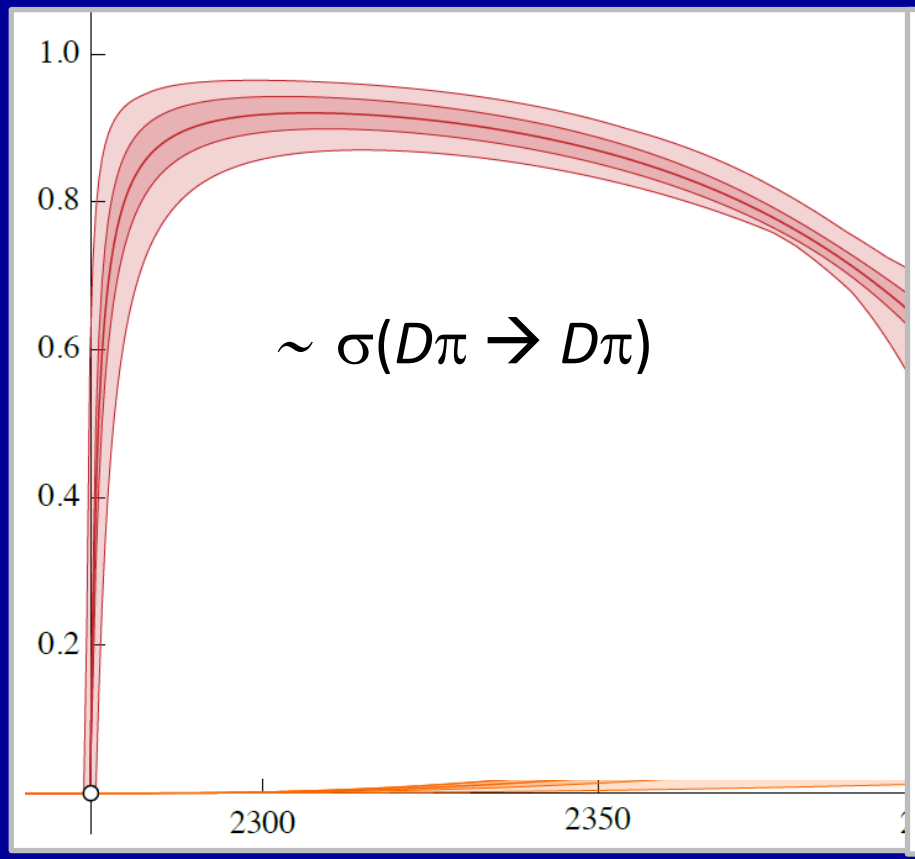
Also P-wave (1^-) and D-wave (2^+)

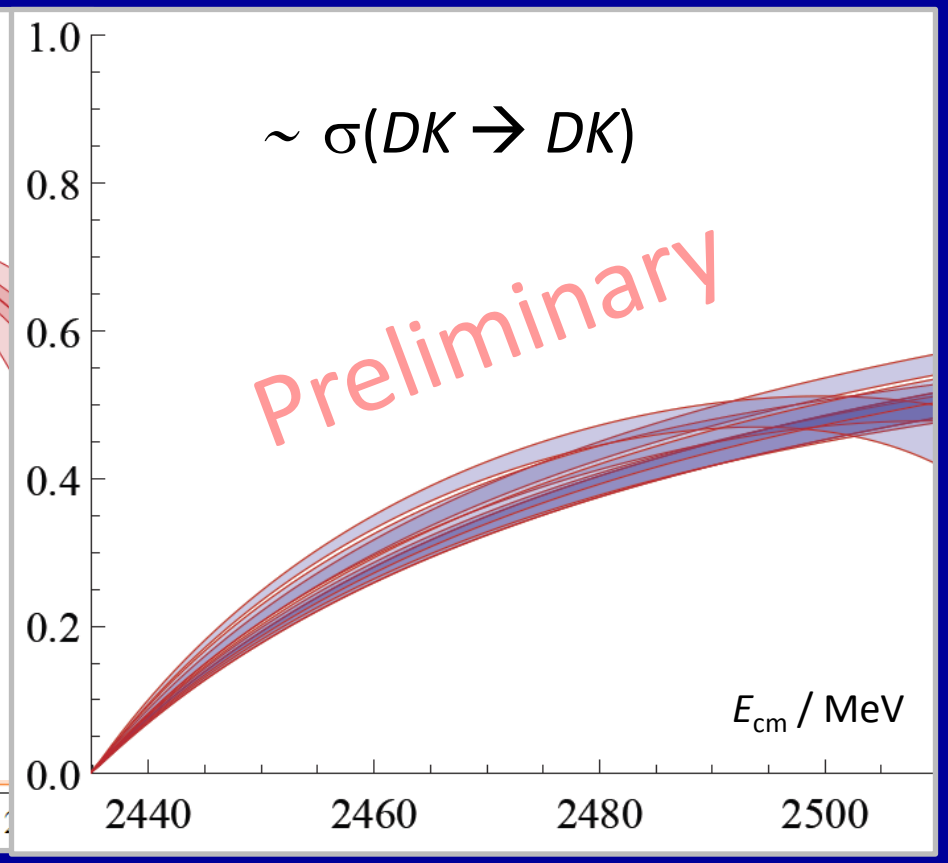
Moir, Peardon, Ryan, CT, Wilson (HadSpec) [arXiv:1607.07093]

Dπ (I=½) c.f. DK (I=0)

0^+ in Dπ at (2275.9 ± 0.9) MeV
c.f. Dπ threshold (2276.4 ± 0.9) MeV

0^+ in DK at ≈ 2380 MeV
c.f. DK threshold ≈ 2430 MeV





m_π = 391 MeV

(HadSpec) [arXiv:1607.07093]

Summary

- Excited spectra of charmonia and light mesons including **exotic** J^{PC} – supermultiplets of **hybrid mesons**
- Need to compute properties of **resonances**, near-threshold states, etc – **significant progress** in LQCD in recent years
- Coupled-channel scattering for the first time.
- Some examples of recent work:
  - ρ resonance, light scalars (σ, $a_0(980)$, κ)
  - Charm-light mesons
- Use m_π dependence as a tool
- Ongoing work on formalism (e.g. 3-hadron scattering)
- Connections with analysis of experimental data

Hadron Spectrum Collaboration

Jefferson Lab, USA:
Jozef Dudek, Robert Edwards, David Richards,
Raul Briceño

Trinity College Dublin:
Mike Peardon, Sinéad Ryan, *Cian O'Hara*, *David Tims*

University of Cambridge:
CT, **Graham Moir**, **David Wilson** (→ Dublin),
Gavin Cheung, Antoni Woss

Tata Institute:
Nilmani Mathur

The ρ resonance: other elastic ππ calcs.

Some other recent lattice QCD calculations:

- Bali *et al* (RQCD) [PR D93, 054509 (2016)] (N_f = 2)
- Bulava *et al* [NP B910, 842 (2016)]
- Guo *et al* [PR D94, 034501 (2016)] (N_f = 2)

Some other recent work on charmonium(-like) mesons:

- Ozaki, Sasaki [PR D87, 014506 (2013)] – no sign of *Y*(4140) in J/ψ φ
- Prelovsek & Leskovec [PRL 111, 192001 (2013)] – 1^{++} I=0 near $D\bar{D}^*$ – *X*(3872)?
- Prelovsek *et al* [PL B727, 172; PR D91, 014504 (2015)] – no sign of Z^+(3900) in 1^{+-}
- Chen *et al* (CLQCD) [PR D89, 094506 (2014)] – 1^{++} I=1 $D\bar{D}^*$ weakly repulsive
- Padmanath *et al* [PR D92, 034501 (2015)] – 1^{++} I=0 [X(3872)?]; no I=1 or *Y*(4140)
- Lang *et al* [JHEP 1509, 089 (2015)] – I=0 $D\bar{D}$: 1^{--} ψ(3770) and 0^{++}
- Chen *et al* (CLQCD) [PR D92, 054507 (2015)] – 1^{+-} I=1 $D^*\bar{D}^*$ weakly repulsive?
- Chen *et al* (CLQCD) [PR D93, 114501 (2016)] – 0^{--}, 1^{+-} I=1$D^*\bar{D}_1$ some attraction?
- Ikeda *et al* (HAL QCD) [arXiv:1602.03465] – π J/ψ, ρ η_c, $D\bar{D}^*$ using HAL QCD method – suggest Z^+(3900) is a threshold cusp
- Albaladejo *et al* [1606.03008] – different scenarios for PR D91, 014504 (2015)

Bottom mesons:

- Lang *et al* [PL B750, 17 (2015)] – *BK* (0^+) and *B*K* (1^+) I=0 bound states
- Lang *et al* [arXiv:1607.03185] – $B_s\pi$, *BK* (I=1) $J^P = 0^+$ no sign of *X*(5568)

Heavy-flavour tetraquarks:

- Bicudo *et al* [PR D92, 014507 (2015); PR D93, 034501 (2016); 1602.07621] – compute potential between two *B* mesons (or *B* and $\bar{B}$) in static approximation
- Francis *et al* [1607.05214] – $ud\bar{b}\bar{b}$ I=0 and $ls\bar{b}\bar{b}$ I=1/2 1^+ tetraquarks
- Peters *et al* [1609.00181] – $ud\bar{b}\bar{b}$ I=0 1^+ tetraquarks