

The Next Theoretical Challenges for Gravitational Wave Observations

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Outline

- **New** astronomical **messengers**: gravitational waves
- **Detection** of **gravitational waves** by LIGO.
- Review of **theoretical work** that has **paved the way** to observe GW150914 & GW151226, and infer source's properties.
- The **science from GW experiments** stems on our **ability** to make **precise predictions**.
- GW observations in next several years: main **theoretical challenges** to take **full advantage** of **discovery potential**.

Gravitational waves: one of the greatest predictions of General Relativity

- In 1916 Einstein predicted **existence of gravitational waves**:

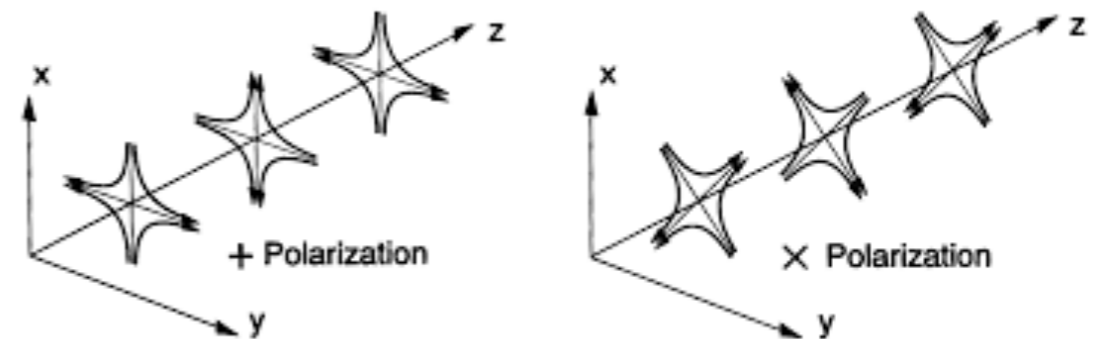
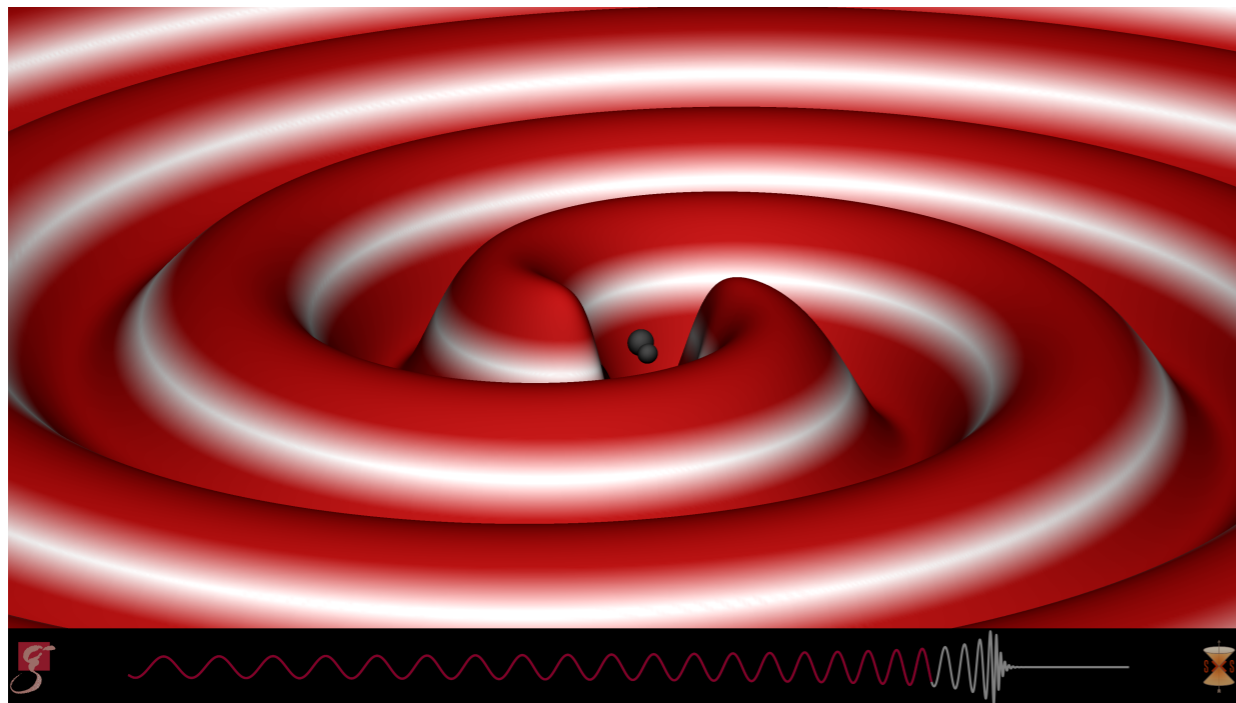
Linearized gravity (weak field): $g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$ $|h_{\mu\nu}| \ll 1$

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = \frac{8\pi G}{c^4}T_{\mu\nu} \longrightarrow \square \bar{h}_{\mu\nu} = -\frac{16\pi G}{c^4}T_{\mu\nu}$$

Distribution of **mass deforms spacetime** geometry in its neighborhood.

Deformations propagate away at finite speed **in form of waves** whose oscillations reflect temporal variation of matter distribution.

(visualization: Haas @ AEI)

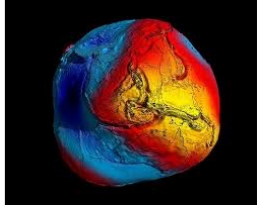


Two radiative degrees of freedom

Ripples in the curvature of spacetime

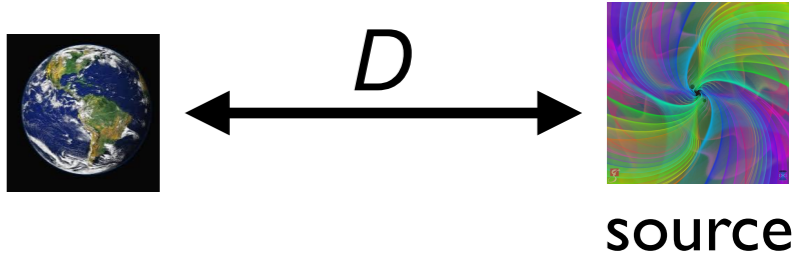
What makes gravitational waves unique astronomical messengers

- GW sources **dominated by gravity**



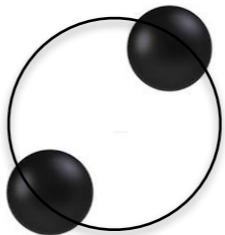
- Produced by variation in time of **quadrupole moment**: $h_{ij} \sim \frac{G}{c^4} \frac{\ddot{Q}_{ij}}{D}$

- Typical GW **strength**: $h \sim \epsilon \frac{G}{c^2} \frac{(E_{\text{kin}}/c^2)}{D}$



- Typical GW **luminosity**: $\mathcal{L}_{\text{GW}} \sim \epsilon^2 \frac{c^5}{G} \left(\frac{v}{c}\right)^{10}$ $\frac{c^5}{G} \sim 10^{59} \text{ erg/sec}$

Similar or larger to the one of **whole visible Universe!**



binary system

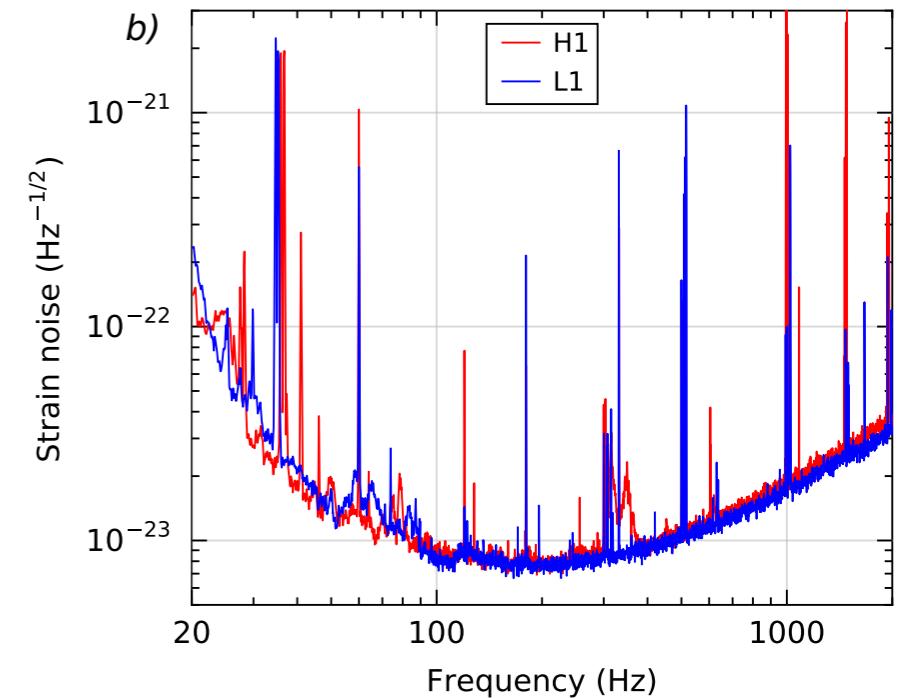
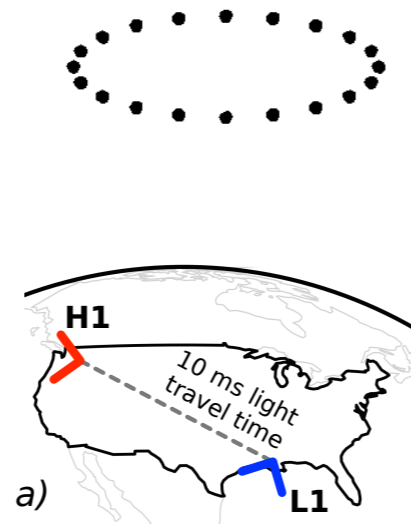
- Propagation unaffected by matter/energy: **pristine probes**

The two LIGO detectors

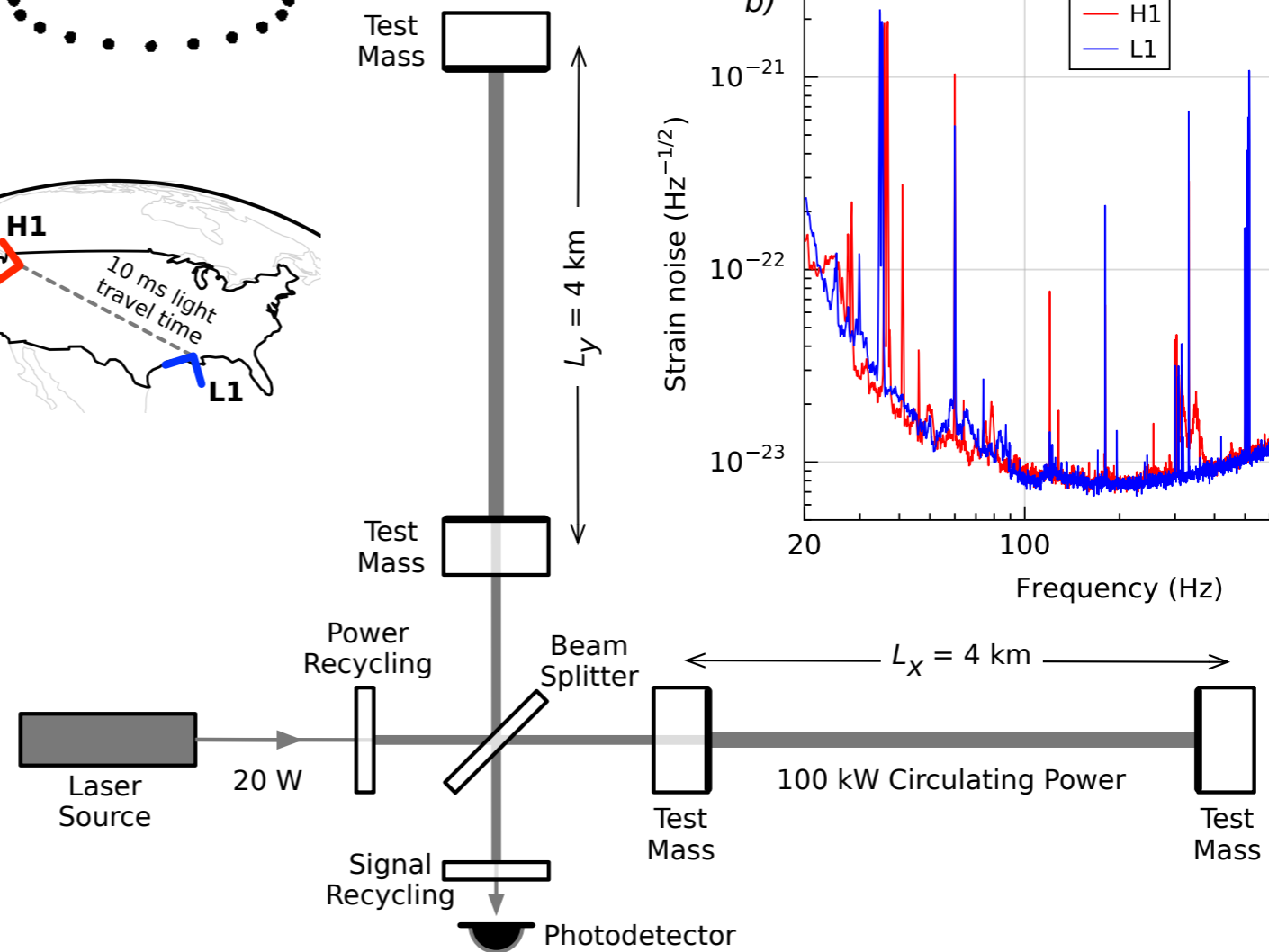
LIGO in Washington (H1)



(Abbott et al. PRL 116 (2016) 061102)



LIGO in Louisiana (L1)



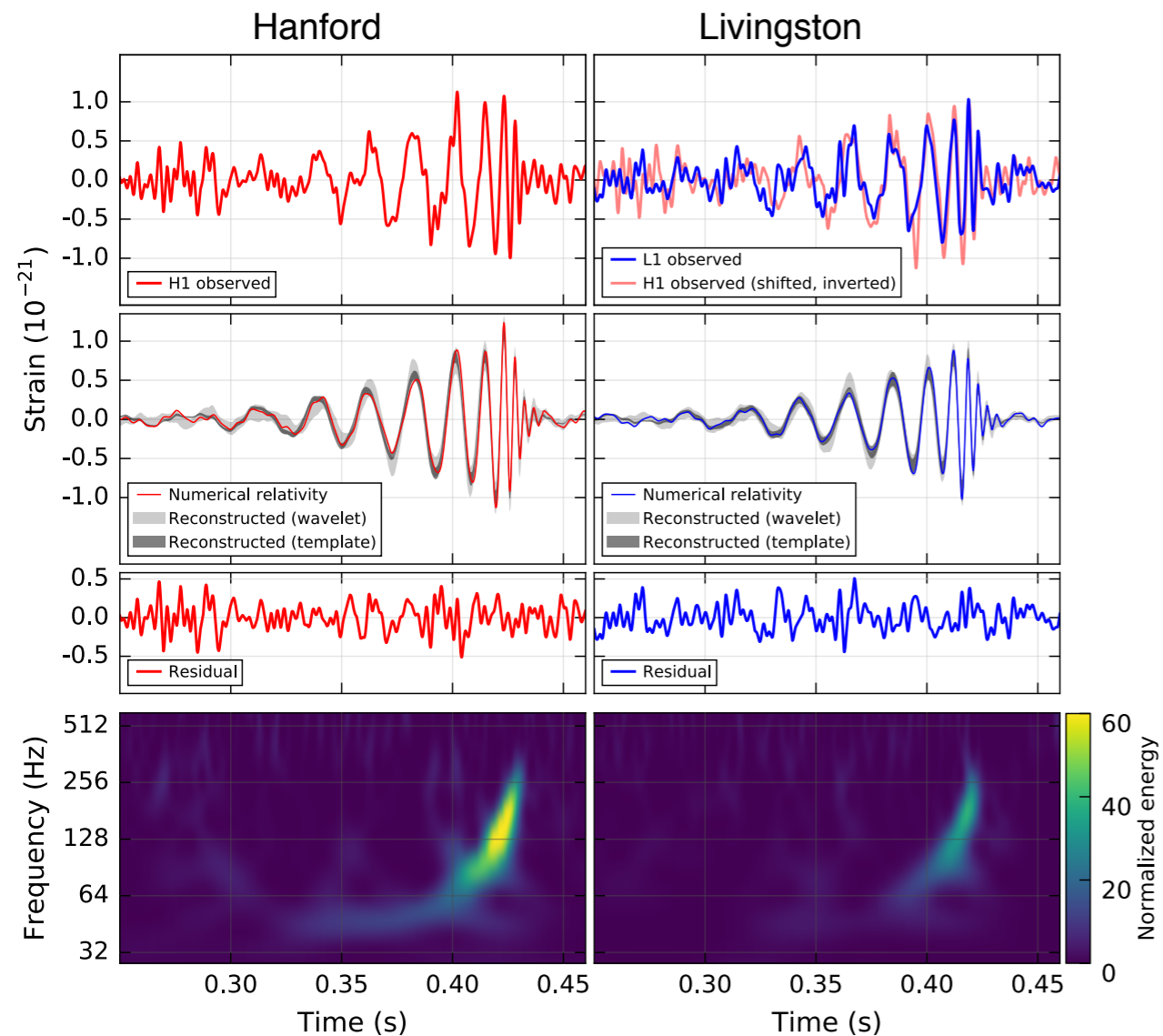
$$\Delta L = L h \sim 10^{-16} \text{ cm}$$

$$L = 4 \text{ km} \Rightarrow h \sim 10^{-21}$$

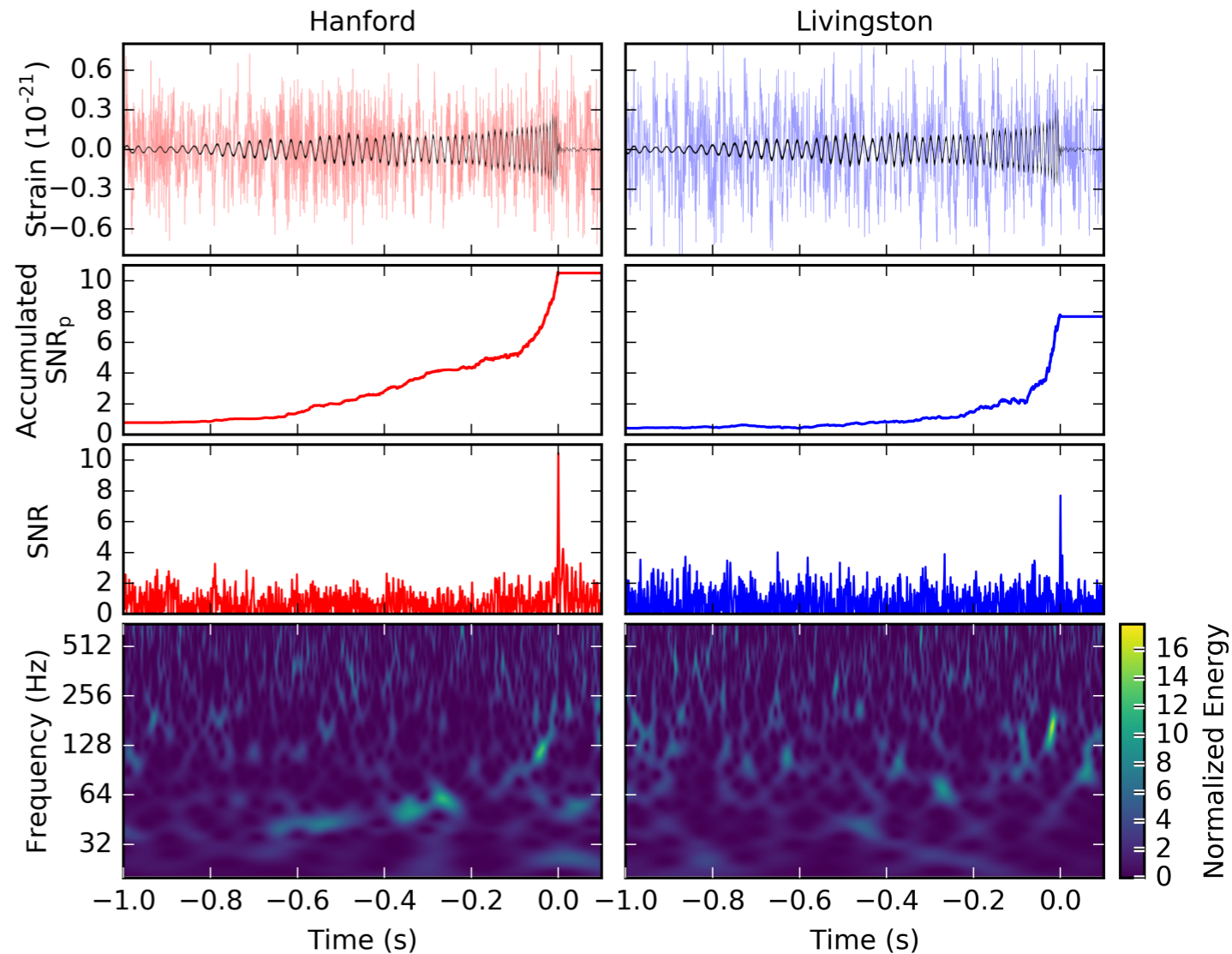
LIGOs measures **displacements of mirrors** at about a **ten-thousandth** of a **proton's diameter**.

LIGO detections during O1: GW150914 & GW151226

(Abbott et al. PRL 116 (2016) 061102)



(Abbott et al. PRL 116 (2016) 241103)



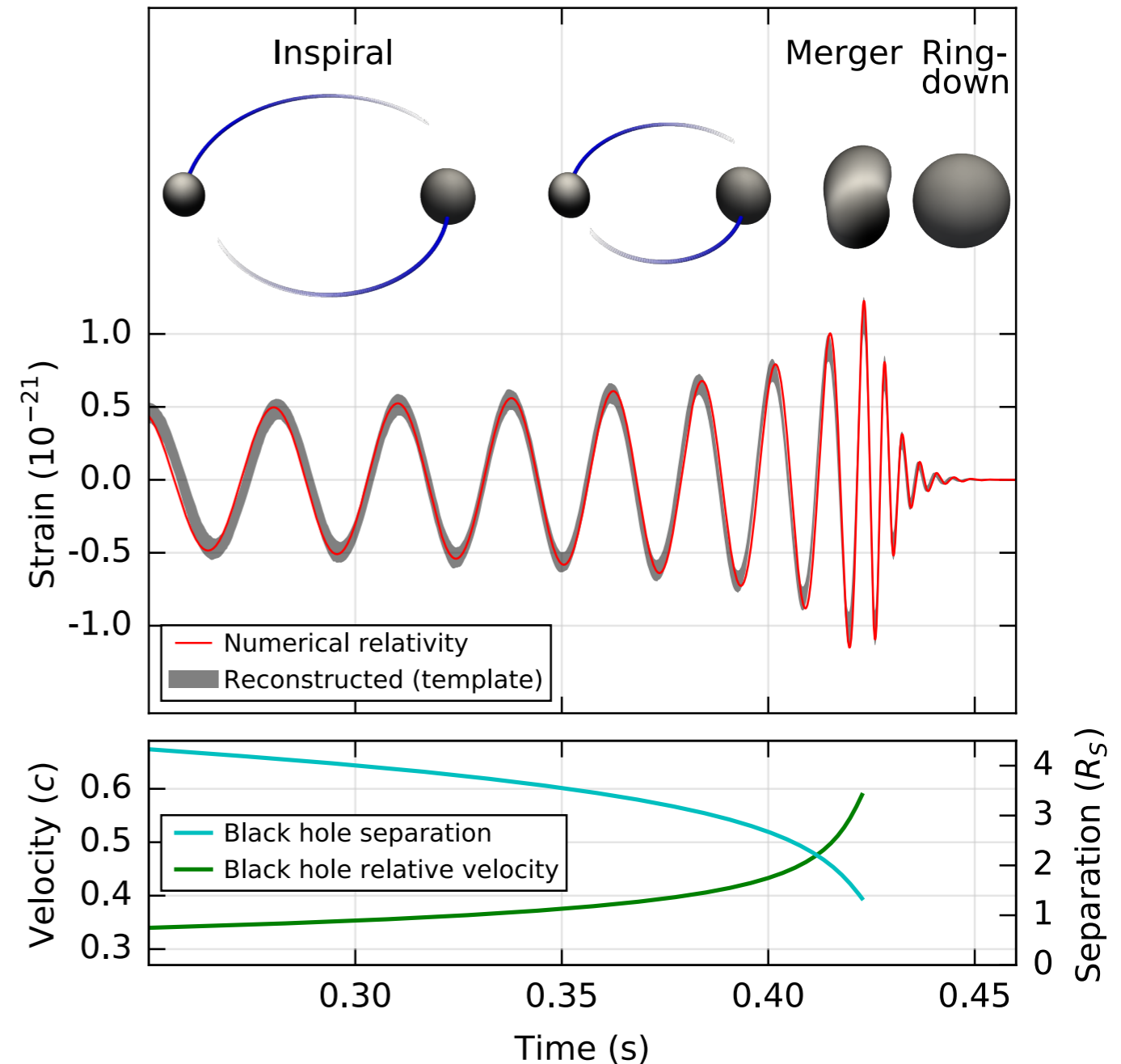
- **GW150914**: SNR=24 (very loud), 10 GW cycles, **0.2 sec.**

- **GW151226**: SNR=13 (quieter), 55 GW cycles, **1.5 sec.**

Characteristics of binary black-hole coalescence

- **Early inspiral:** low velocity & weak gravitational field.
- **Late inspiral/plunge:** high velocity & strong gravitational field.
- **Merger:** nonlinear & non perturbative effects; rapidly varying gravitational field
- **Ringdown:** excitation of quasi-normal modes/spacetime vibrations.

(Abbott et al. PRL 116 (2016) 061102)



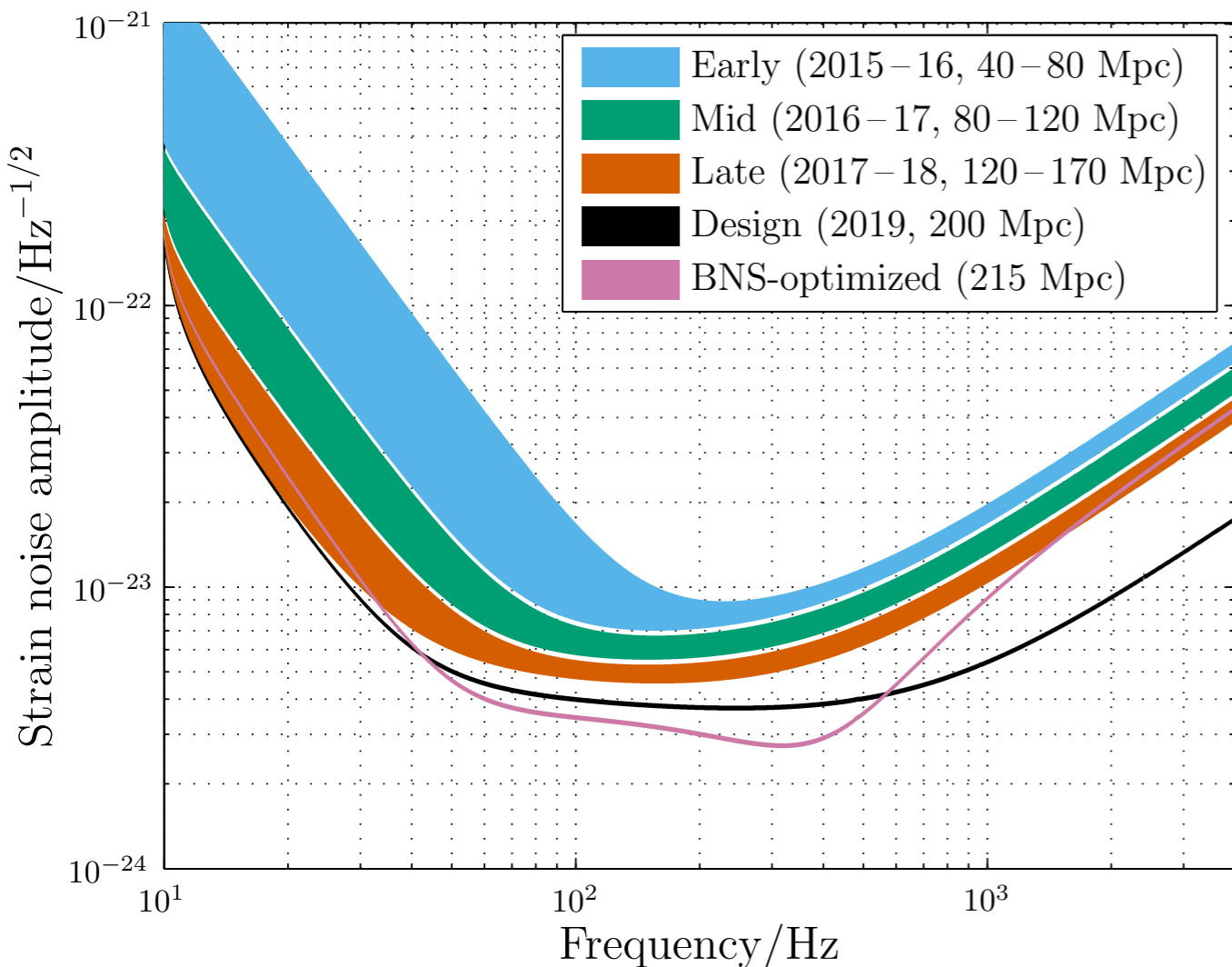
Phase/amplitude evolution **encodes unique information** about the source

Black holes of radius of 90 km at separation of 350 km are making 75 orbits per second before merging!

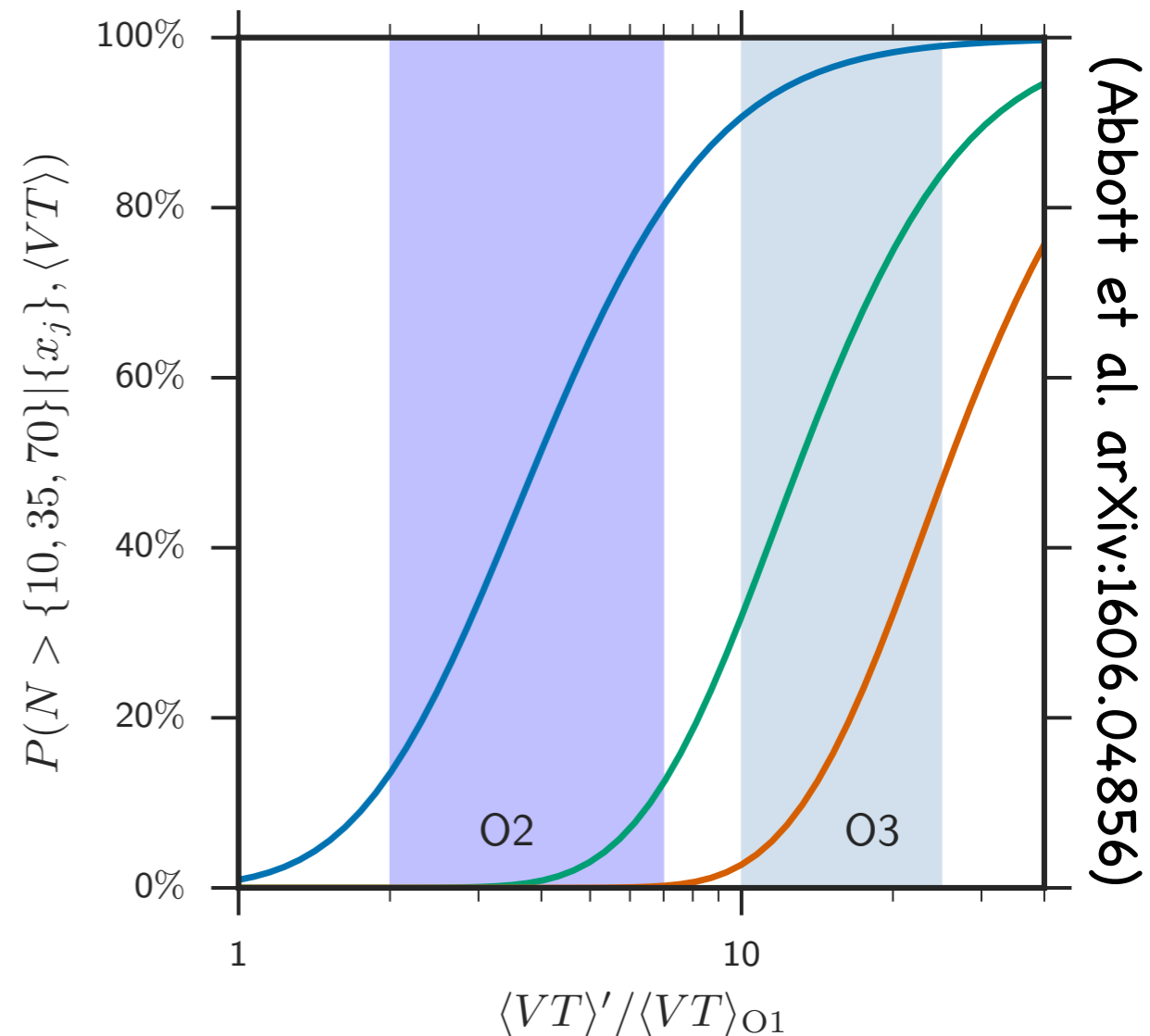
Advanced detectors' roadmap and rates

(Aasi et al. arXiv:1304.0670)

Advanced LIGO



• Rates: $9 - 240 /(\text{Gpc})^3 / \text{yr}$

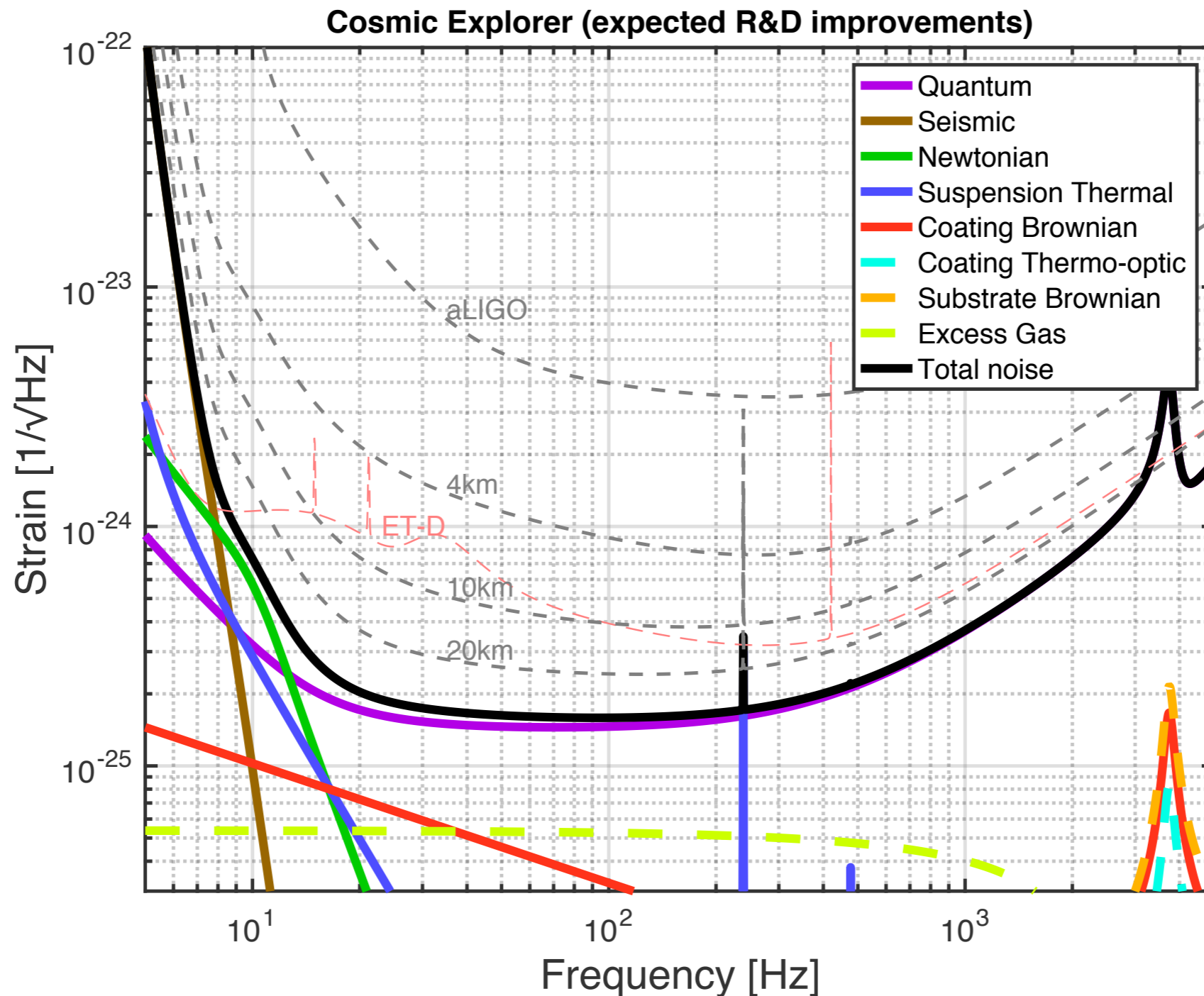


(Abbott et al. arXiv:1606.04856)

Detection rates @ **design sensitivity**:

- **Binary neutron stars: 0.2 - 200 per year**
- **Binary black holes: tens to hundreds per year!**

Looking more ahead: Einstein Telescope/Cosmic Explorer (2028?)



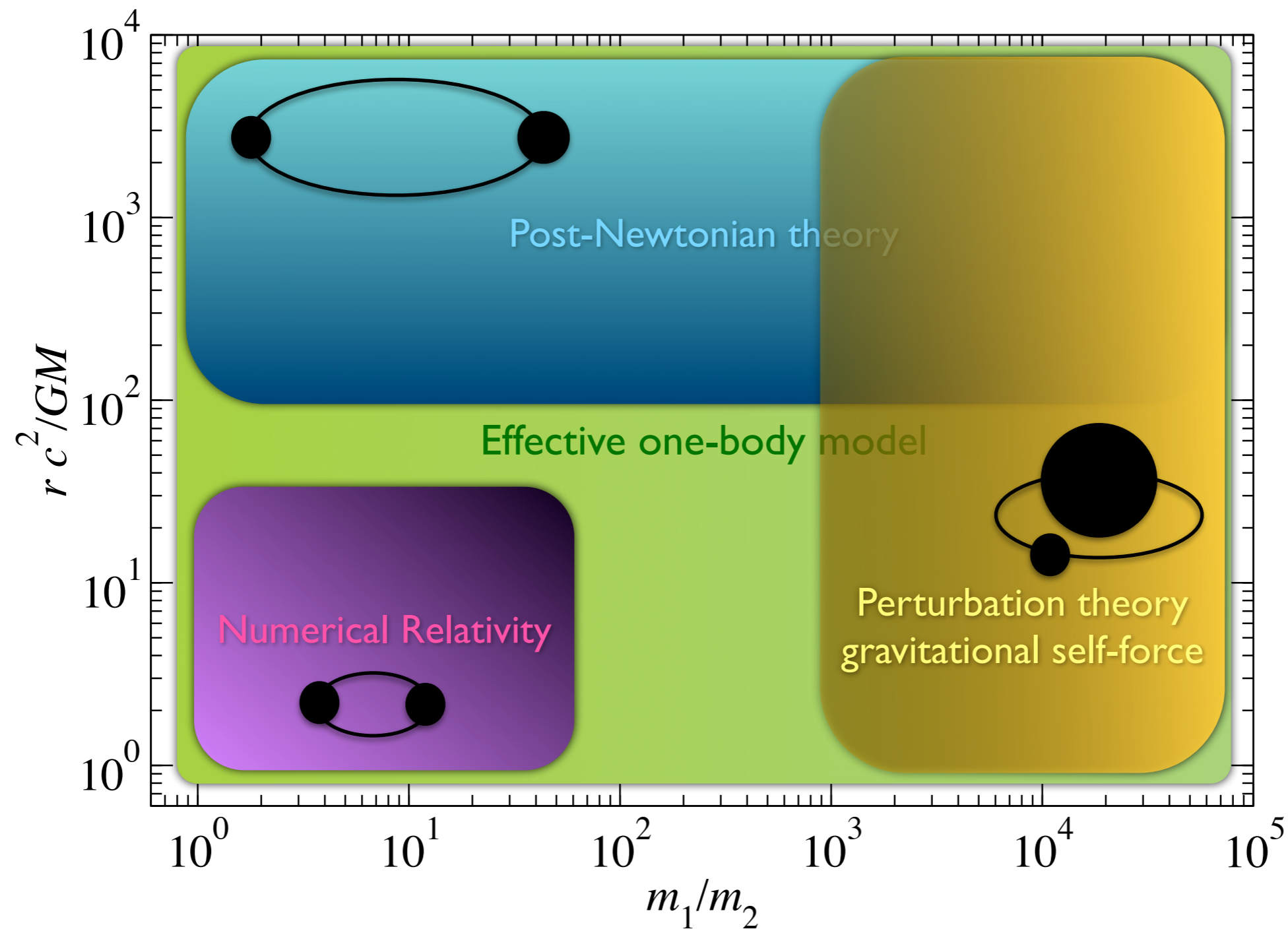
(Abbott et al. arXiv:1607.08697)

- Observing **binary black-hole coalescences with high SNR** (> 20) even **at high redshift** ($z > 10$) or $\text{SNR} > 100$ and $z < 2$!

Bright future of GW observations comes with challenges

- In the next 5 years with detectors on the ground:
 1. **Signal-to-noise** ratios of **$O(100)$** ?
 2. **$O(100)$** or more **detections**?
 3. Binaries with **large mass ratios** and **generic spins**?
 4. Detection of compact **binaries with matter**? Eccentricity?
- Challenges:
 1. **Do current waveform models contain all the physics**?
 2. Do we **control systematics** in all parameter space?
 3. Are there **more efficient and accurate ways** to tackle the two-body problem **analytically**?
 4. Will we constrain **binary formation scenarios**?
 5. Will we **rule out modified theories** of GR?
 6. Will we identify compact-objects as **Kerr black holes**?
 7. Will we **measure equation-of-state** of **neutron stars**?
 8. Will we extract **cosmological parameters**?

Waveform modeling to detect and infer source's properties



(AB & Sathyaprakash 14)

- Two parameters determine the **range of validity** of each method:

$$\frac{GM}{r c^2} \sim \frac{v^2}{c^2} \quad \& \quad \frac{m_2}{m_1}$$

Post-Newtonian approximation

- **First developed in 1917** (Droste & Lorentz 1917, and Einstein, Infeld & Hoffmann 1938)

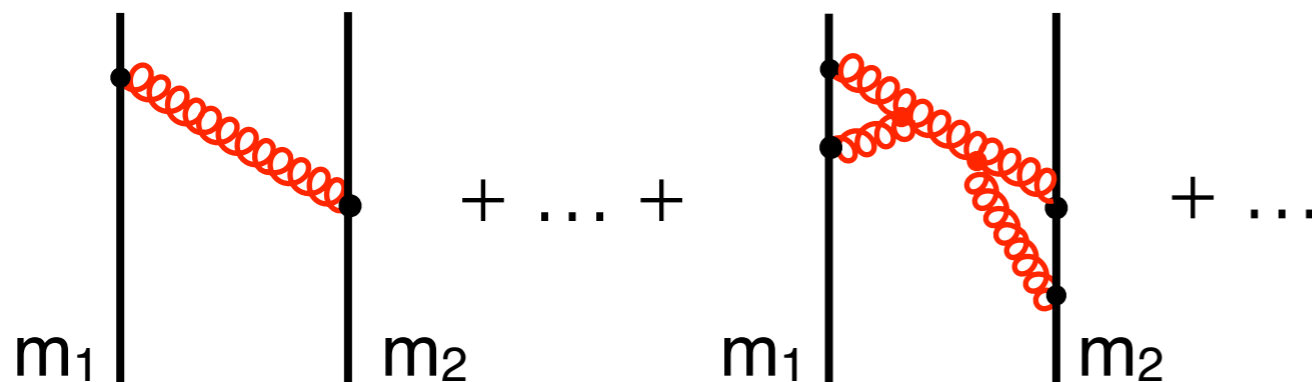
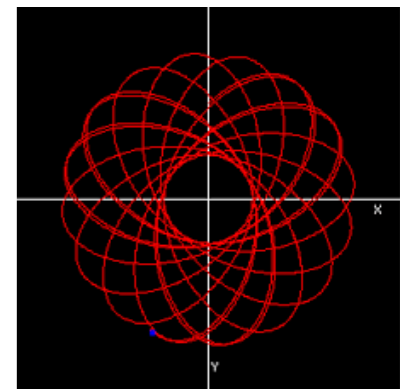
(Blanchet, Damour, Iyer, Faye, AB, Bohe', Marsat; Jaranowski, Schaefer, Steinhoff; Will, Wiseman; Goldberger, Porto, Rothstein, Levi, Foffa, Sturani; Flanagan, Hinderer, Vines ...)

$$\hat{H}_N(\mathbf{r}, \mathbf{p}) = \frac{\mathbf{p}^2}{2} - \frac{1}{r}$$

Small parameter is v/c

$$\hat{H}_{1PN}(\mathbf{r}, \mathbf{p}) = \frac{1}{8}(3\nu - 1)(\mathbf{p}^2)^2 - \frac{1}{2} \left\{ (3 + \nu)\mathbf{p}^2 + \nu(\mathbf{n} \cdot \mathbf{p})^2 \right\} \frac{1}{r} + \frac{1}{2r^2}$$

$$\hat{H}_{2PN}(\mathbf{r}, \mathbf{p}) = \frac{1}{16}(1 - 5\nu + 5\nu^2)(\mathbf{p}^2)^3 + \frac{1}{8} \left\{ (5 - 20\nu - 3\nu^2)(\mathbf{p}^2)^2 + \dots \right.$$



- Compact object is **point-like object endowed** with time-dependent **multipole moments**.

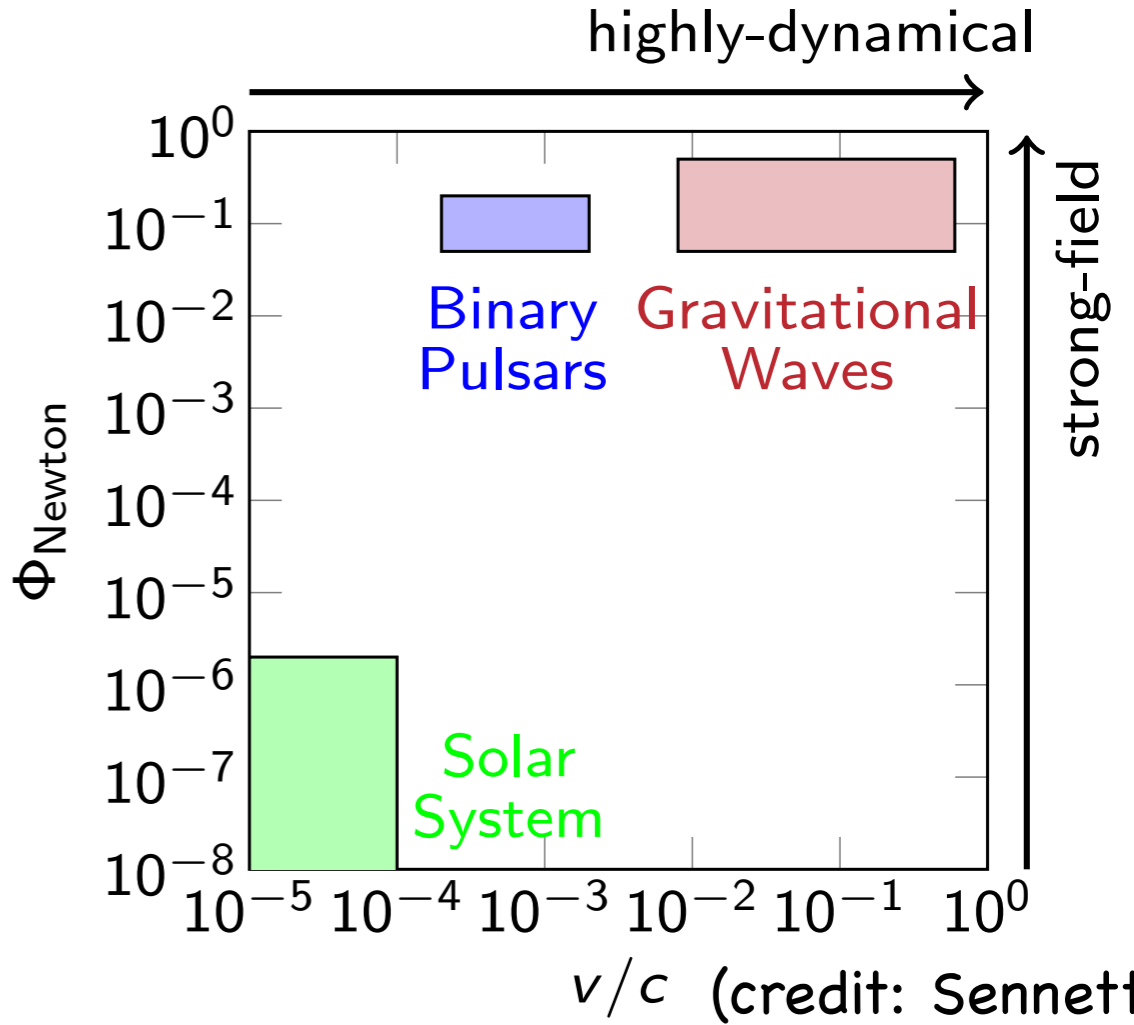
$$F_{PN} = -\frac{G m_1 m_2}{r^2} \left(1 + a_{1PN} \frac{v^2}{c^2} + a_{2PN} \frac{v^4}{c^4} + a_{2.5PN} \frac{v^5}{c^5} + a_{3PN} \frac{v^6}{c^6} + a_{3.5PN} \frac{v^7}{c^7} + \dots \right)$$

Post-Newtonian/post-Minkowskian formalism/effective field theory

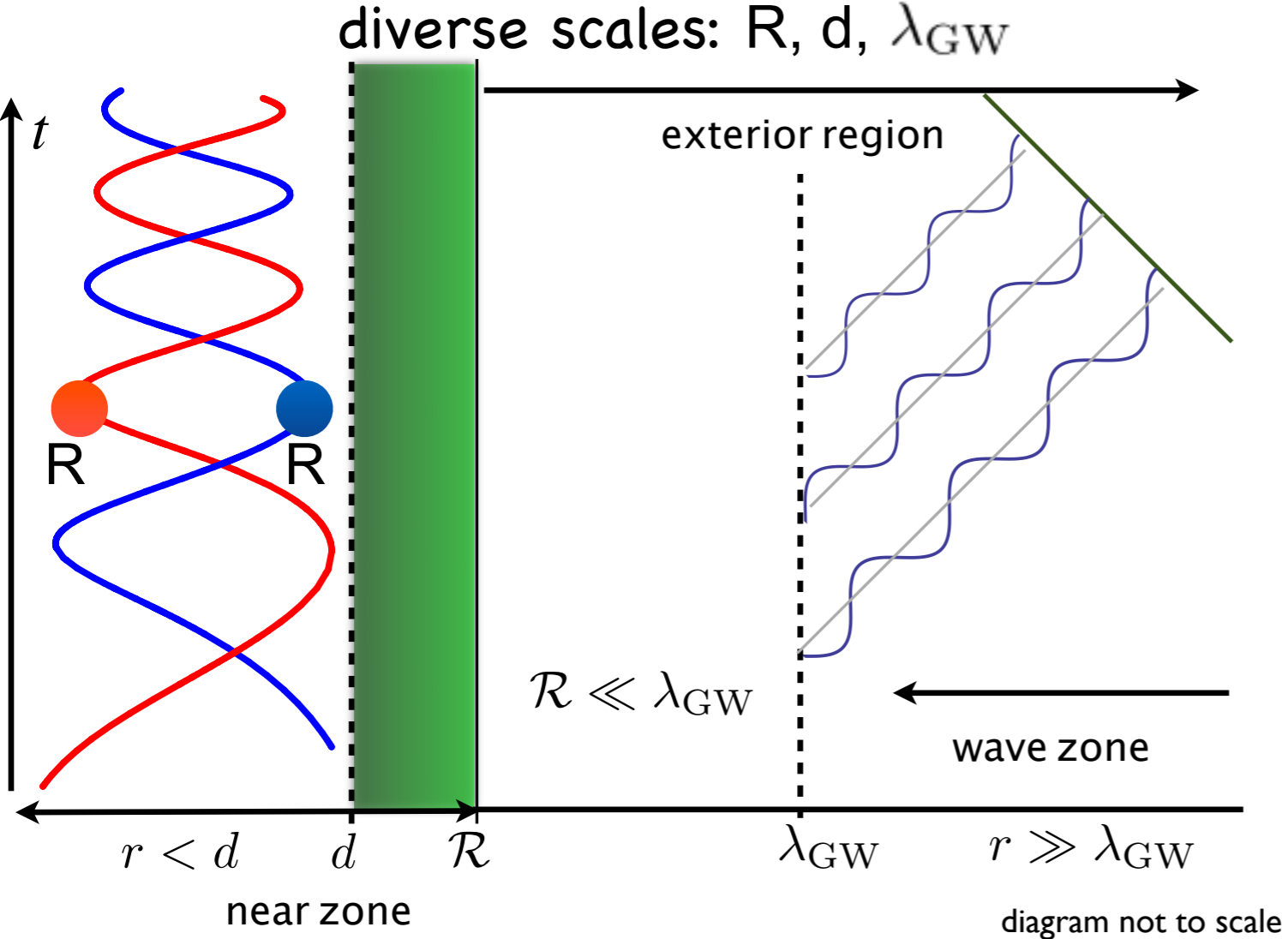
Solar system: $\frac{v}{c} \sim 10^{-5} - 10^{-4}$

Binary pulsars: $\frac{v}{c} \sim 10^{-3}$

LIGO: $\frac{v}{c} \geq 0.1$



(AB & Sathyaprakash 14)



- **Generation** and **radiation-reaction** problems
- **Equations of motion** of compact objects
- Physical **observables are waveforms** at null infinity

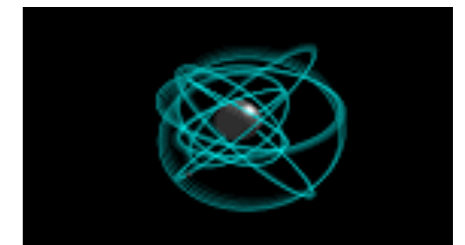
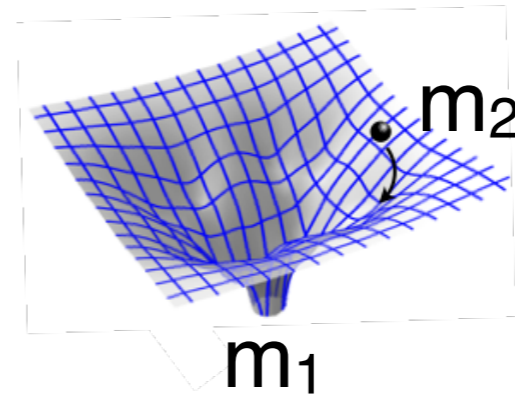
Perturbation theory and gravitational self force (GSF)

- **First works in 50-70s** (Regge & Wheeler 56, Zerilli 70, Teukolsky 72)

Small parameter is m_2/m_1

Equation of gravitational perturbations in black-hole spacetime:

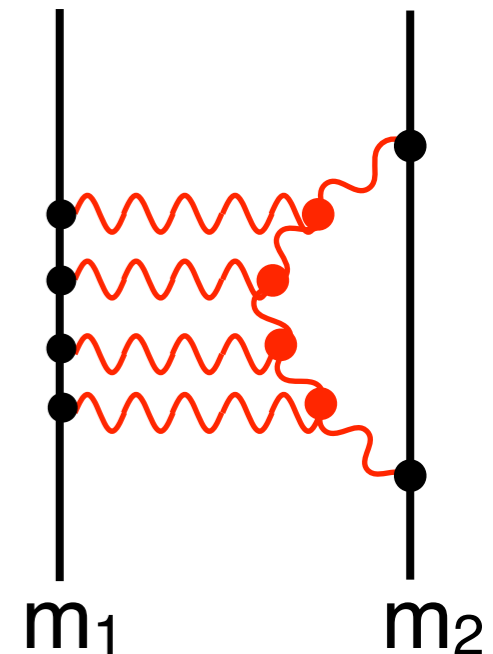
$$\frac{\partial^2 \Psi}{\partial t^2} - \frac{\partial^2 \Psi}{\partial r_{\star}^2} + V_{lm} \Psi = \mathcal{S}_{lm}$$



Green functions in Schwarzschild/Kerr spacetimes.

(Fujita, Poisson, Sasaki, Shibata, Khanna, Hughes, Bernuzzi, Harms, ...)

- **GSF**: Accurate modeling of **relativistic dynamics of large mass-ratio** inspirals **requires** to include **back-reaction effects** due to interaction of small object with its own gravitational perturbation field.



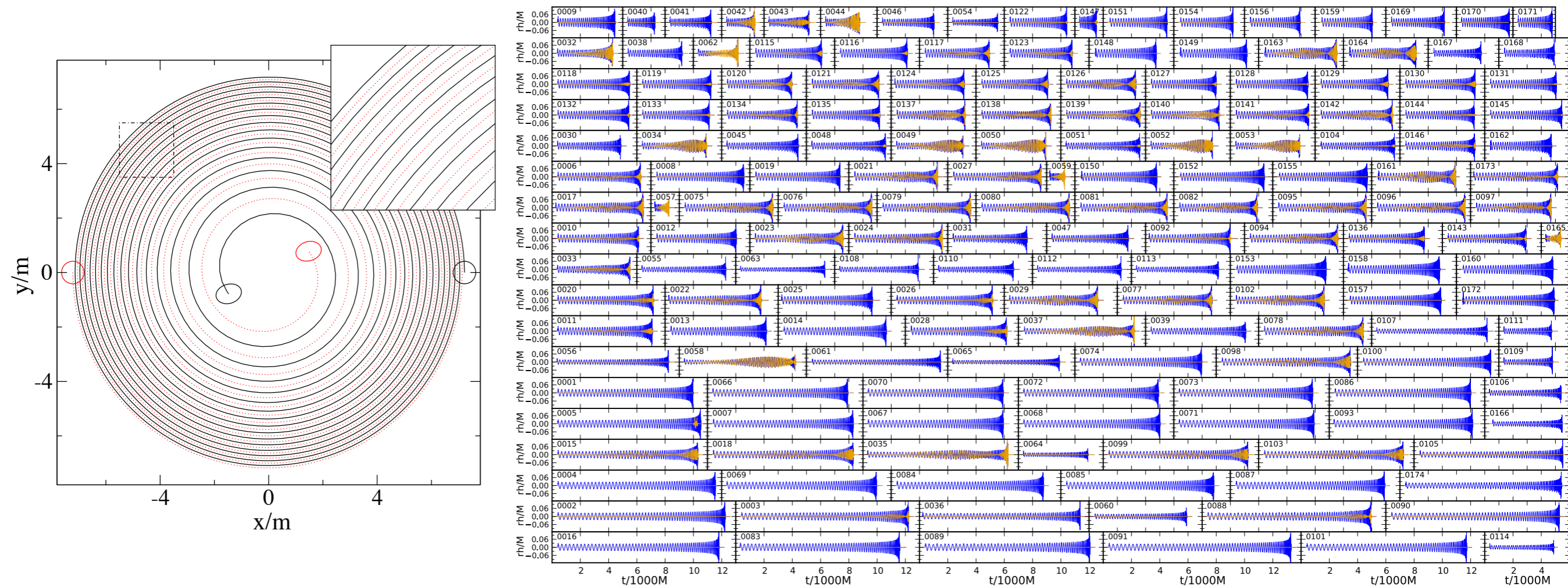
(Deitweiler, Whiting, Mino, Poisson, Quinn, Sasaki, Tanaka, Barack, Ori, Pound, van de Meent...)

Numerical Relativity

- **Breakthrough** in 2005 (Pretorius 05, Campanelli et al. 06, Baker et al. 06)

Kidder, Pfeiffer, Scheel, Lindblom, Szilagyi; Bruegmann, Hannam, Husa, Tichy; Laguna, Shoemaker; ...

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = \frac{8\pi G}{c^4}T_{\mu\nu}$$

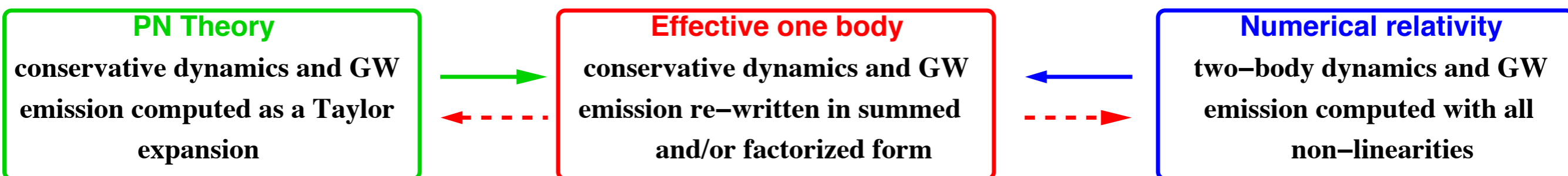


- **Simulating eXtreme Spacetime (SXS) collaboration** (Mroue et al. 13)
- **Numerical-Relativity & Analytical Relativity** collaboration (Hinder et al. 13)

The effective-one-body (EOB) approach

- **EOB** approach **introduced before NR** breakthrough

AB, Pan, Taracchini, Bohe', Shao, Barausse, Hinderer, Steinhoff; Damour, Nagar, Bernuzzi, Bini, Balmelli; Iyer, Sathyaprakash; Jaranowski, Schaefer;



- **EOB** model uses best information available in PN theory, but **resums PN terms** in suitable way to describe accurately dynamics and radiation during inspiral and plunge.
- **EOB** assumes **comparable-mass** description is **smooth deformation of test-particle limit**. It employs non-perturbative ingredients and **models analytically merger-ringdown** signal.

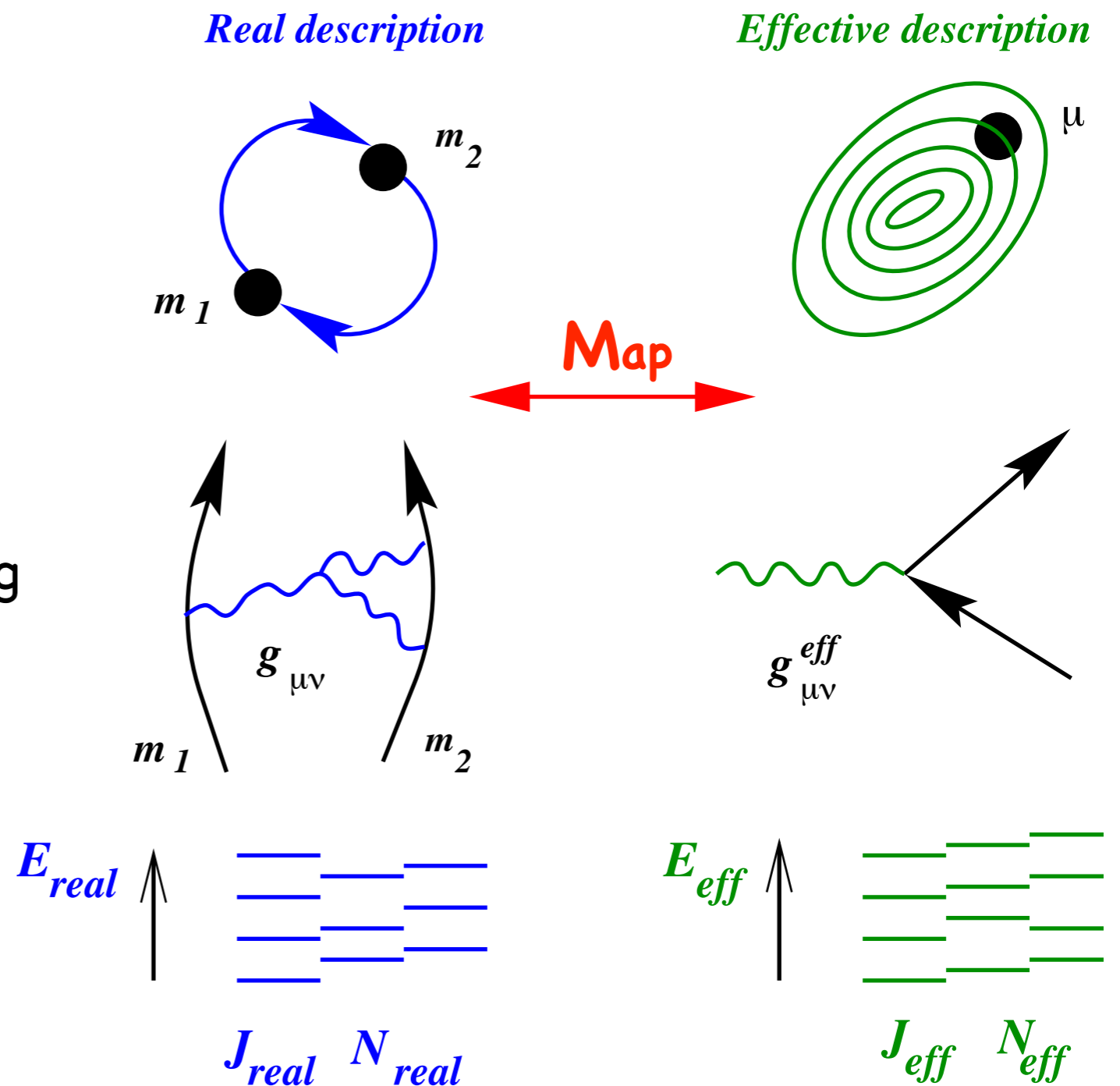
The effective-one-body approach in a nutshell

$$\nu = \frac{\mu}{M} \quad 0 \leq \nu \leq 1/4$$

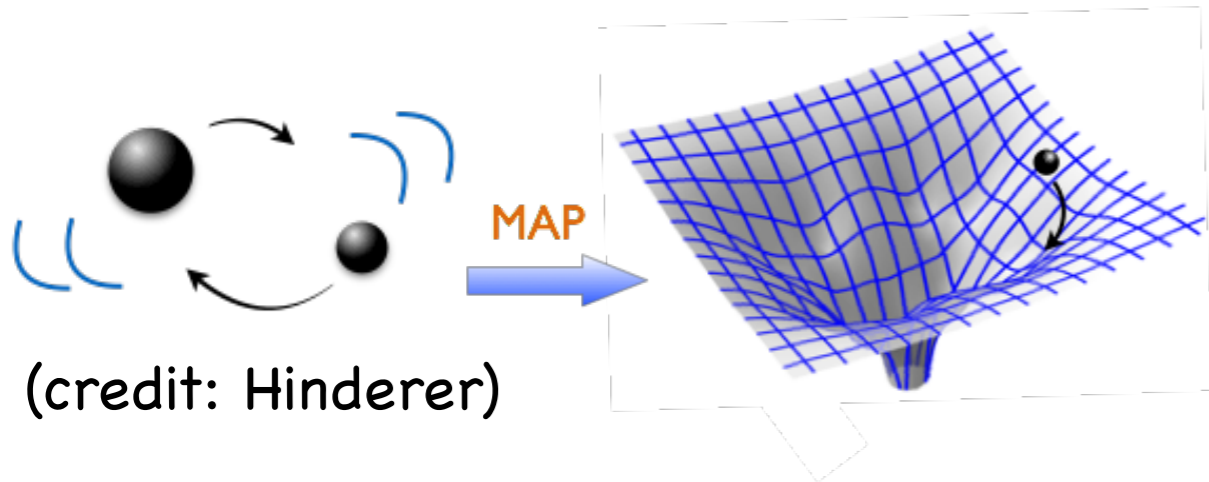
$$\mu = \frac{m_1 m_2}{M} \quad M = m_1 + m_2$$

- **Two-body dynamics** is mapped into dynamics of **one-effective body** moving in **deformed black-hole spacetime**, deformation being the mass ratio.

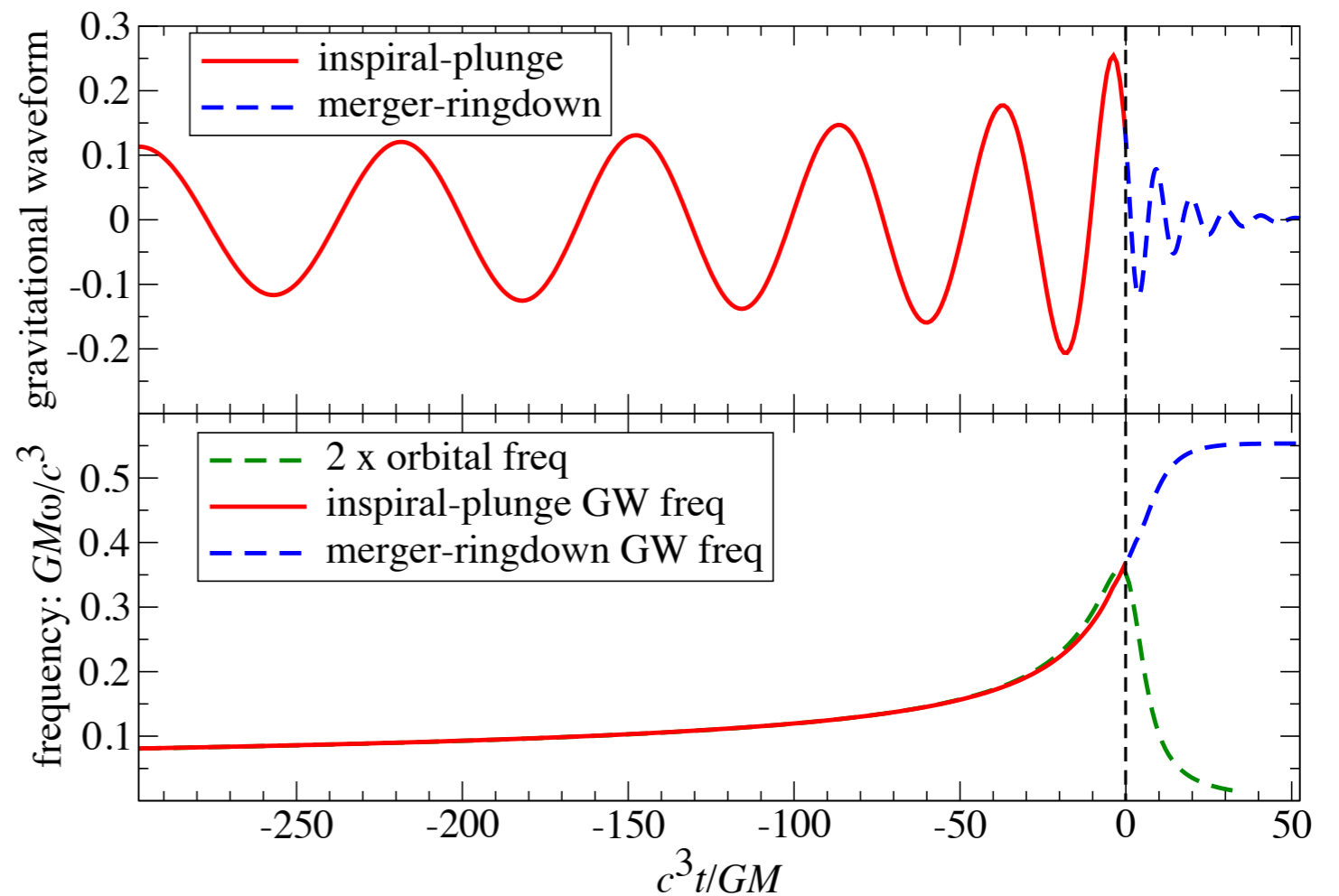
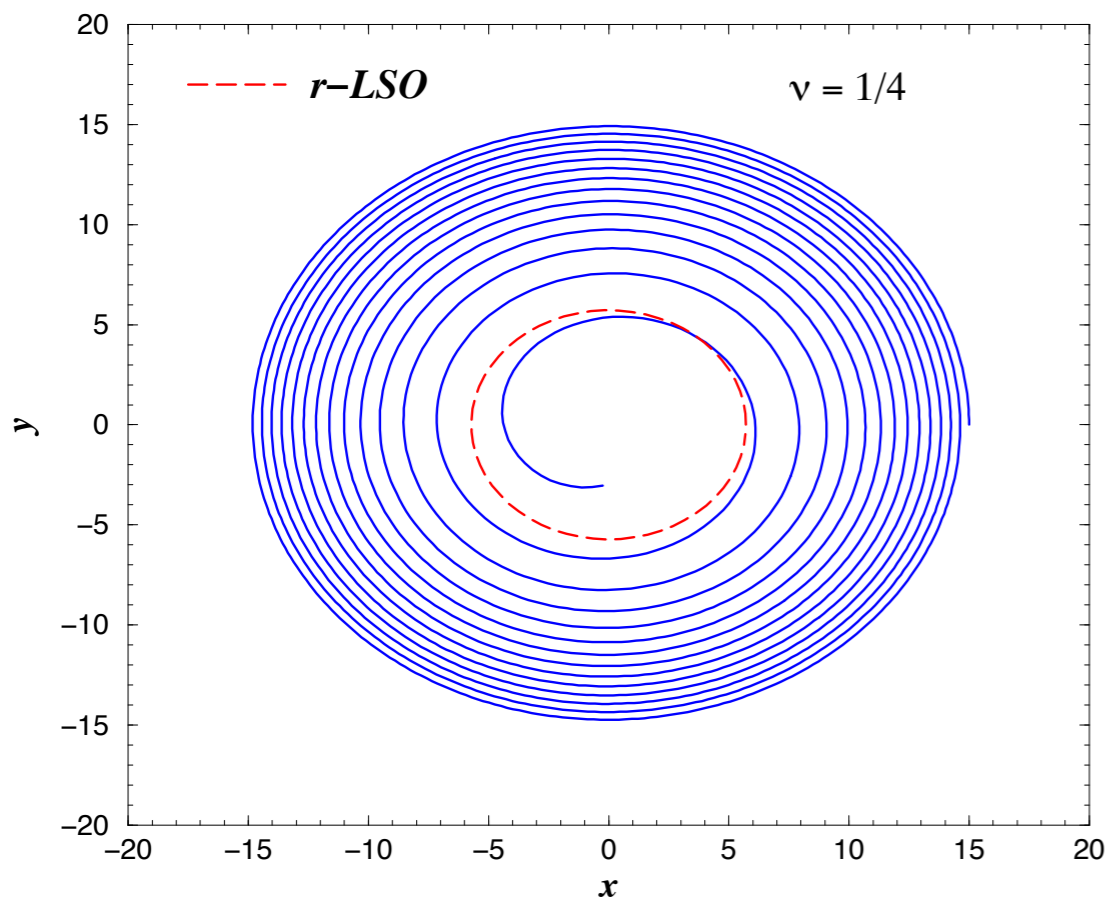
- Some key **ideas** of EOB model were **inspired by quantum field theory** when describing energy of comparable-mass charged bodies.



EOB inspiral-merger-ringdown analytic waveform



$$E_{\text{real}}^2 = m_1^2 + m_2^2 + 2m_1m_2 \left(\frac{E_{\text{eff}}}{\mu} \right)$$



(AB & Damour PRD62 (2000) 064015)

EOB Hamiltonian: resummed PN conservative dynamics

- Real Hamiltonian

$$H_{\text{real}}^{\text{PN}} = H_{\text{Newt}} + H_{1\text{PN}} + H_{2\text{PN}} + \dots$$



- Effective Hamiltonian

$$H_{\text{eff}}^{\nu} = \mu \sqrt{A_{\nu}(r) \left[1 + \frac{\mathbf{p}^2}{\mu^2} + \left(\frac{1}{B_{\nu}(r)} - 1 \right) \frac{p_r^2}{\mu^2} \right]}$$

$$ds_{\text{eff}}^2 = -A_{\nu}(r) dt^2 + B_{\nu}(r) dr^2 + r^2 d\Omega^2$$

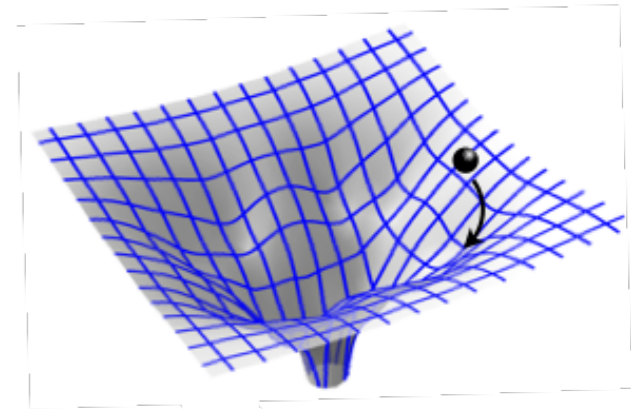
- EOB Hamiltonian:

$$H_{\text{real}}^{\text{EOB}} = M \sqrt{1 + 2\nu \left(\frac{H_{\text{eff}}^{\nu}}{\mu} - 1 \right)}$$

- Dynamics condensed in $A_{\nu}(r)$ and $B_{\nu}(r)$

- $A_{\nu}(r)$, which encodes the energetics of circular orbits, is quite simple:

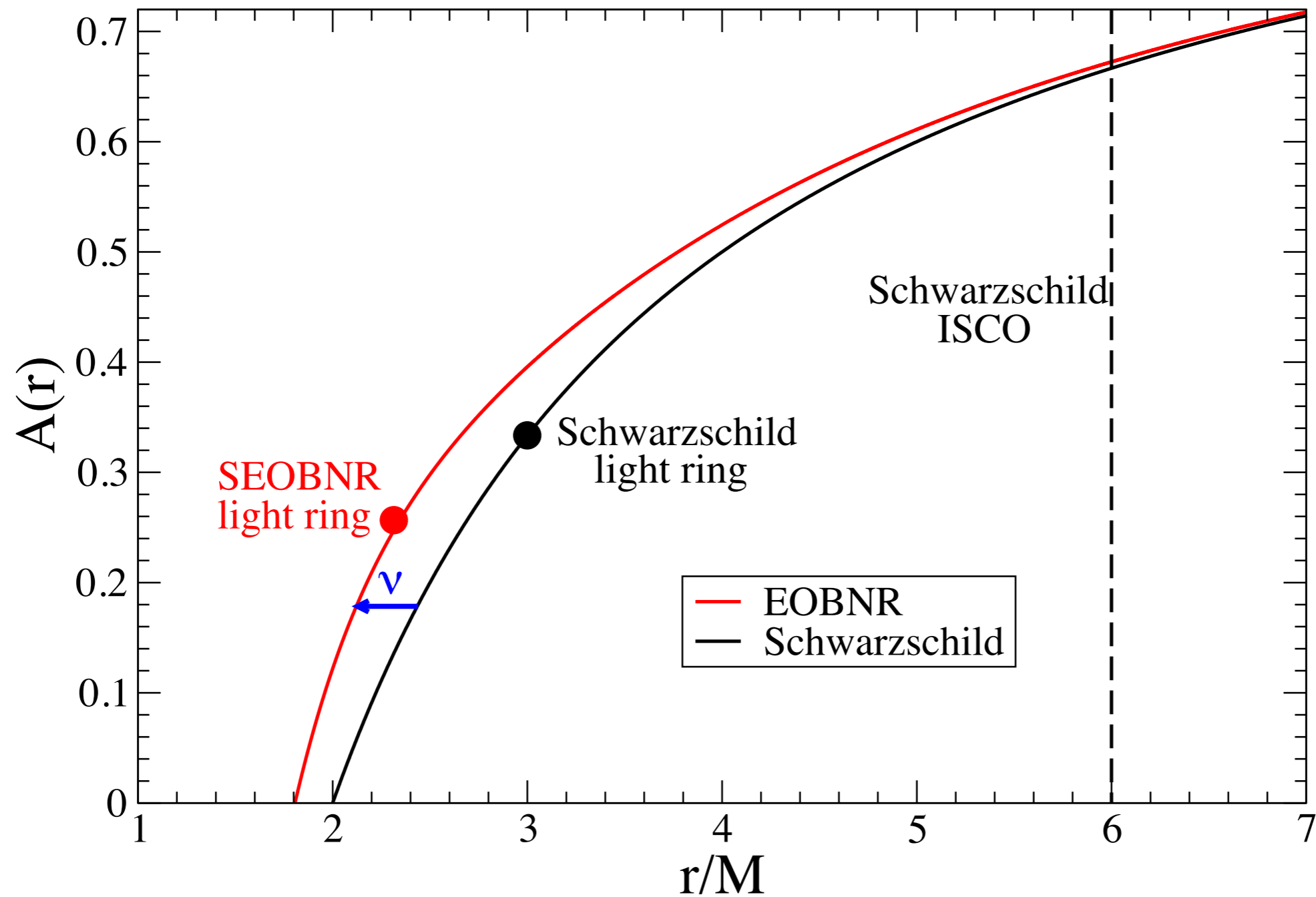
$$A_{\nu}(r) = 1 - \frac{2M}{r} + \frac{2M^3\nu}{r^3} + \left(\frac{94}{3} - \frac{41}{32}\pi^2 \right) \frac{M^4\nu}{r^4} + \frac{a_5(\nu) + a_5^{\log}(\nu) \log(r)}{r^5} + \frac{a_6(\nu)}{r^6} + \dots$$



(credit: Hinderer)

Strong-field effects in binary black holes

Finite mass-ratio effects make gravitational interaction less attractive

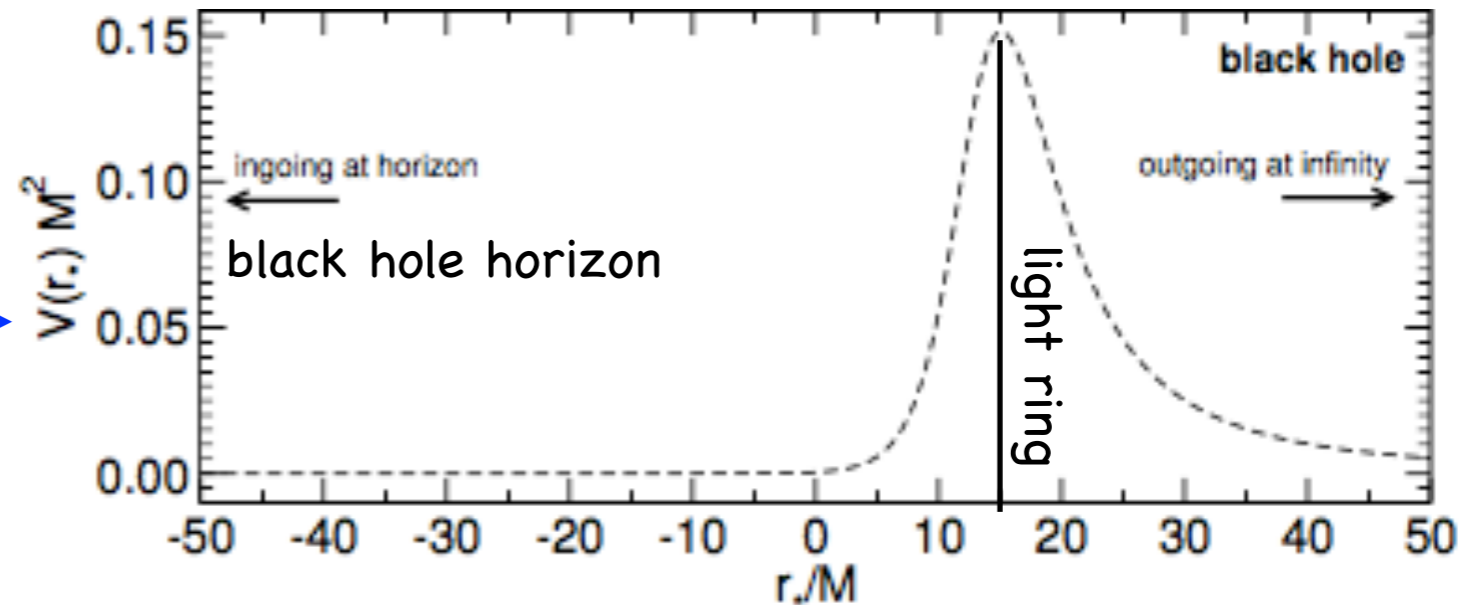


On the simplicity of merger signal

equation of gravitational perturbations
in black-hole spacetime

(Regge & Wheeler 56, Zerilli 70,
Teukolsky 72)

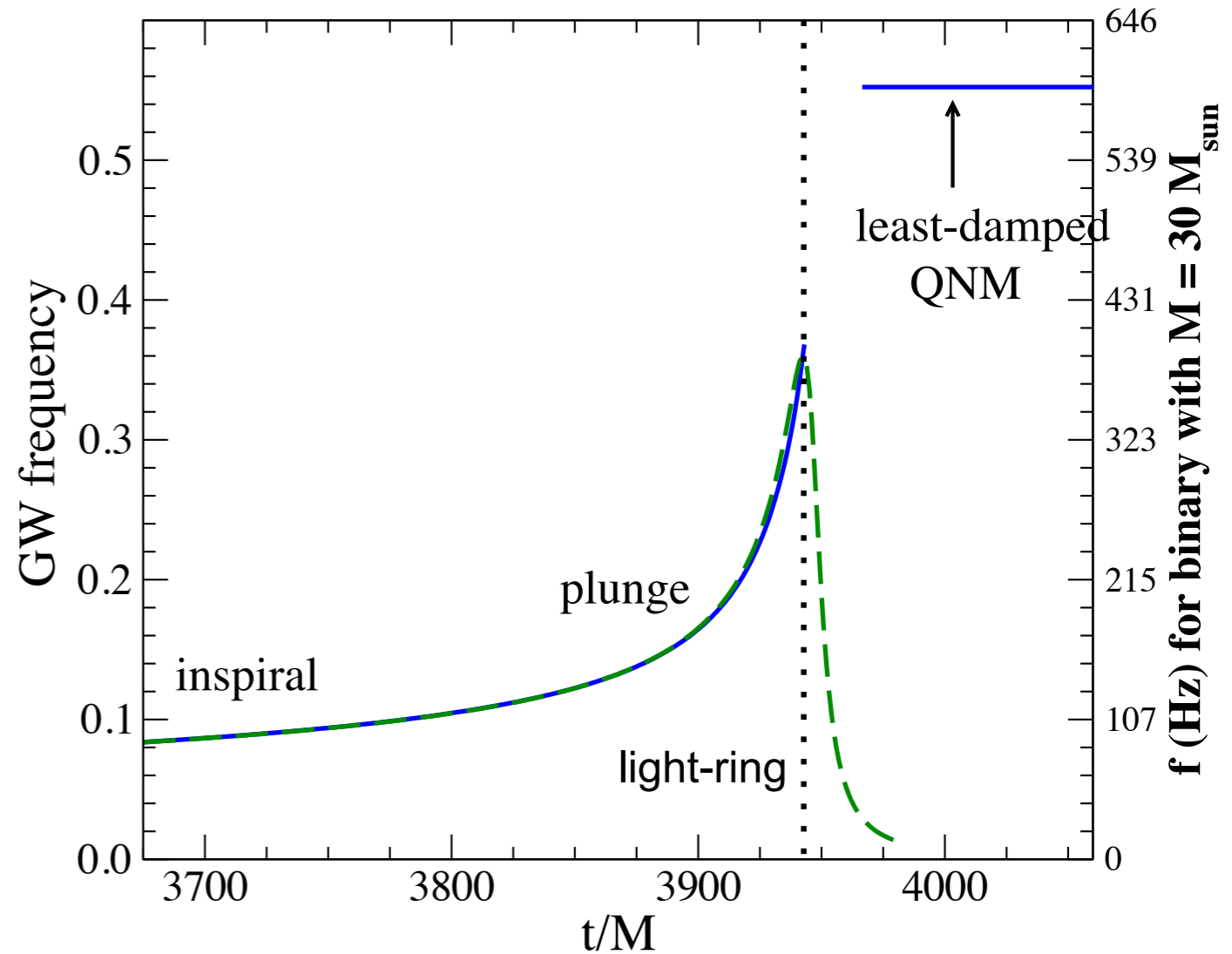
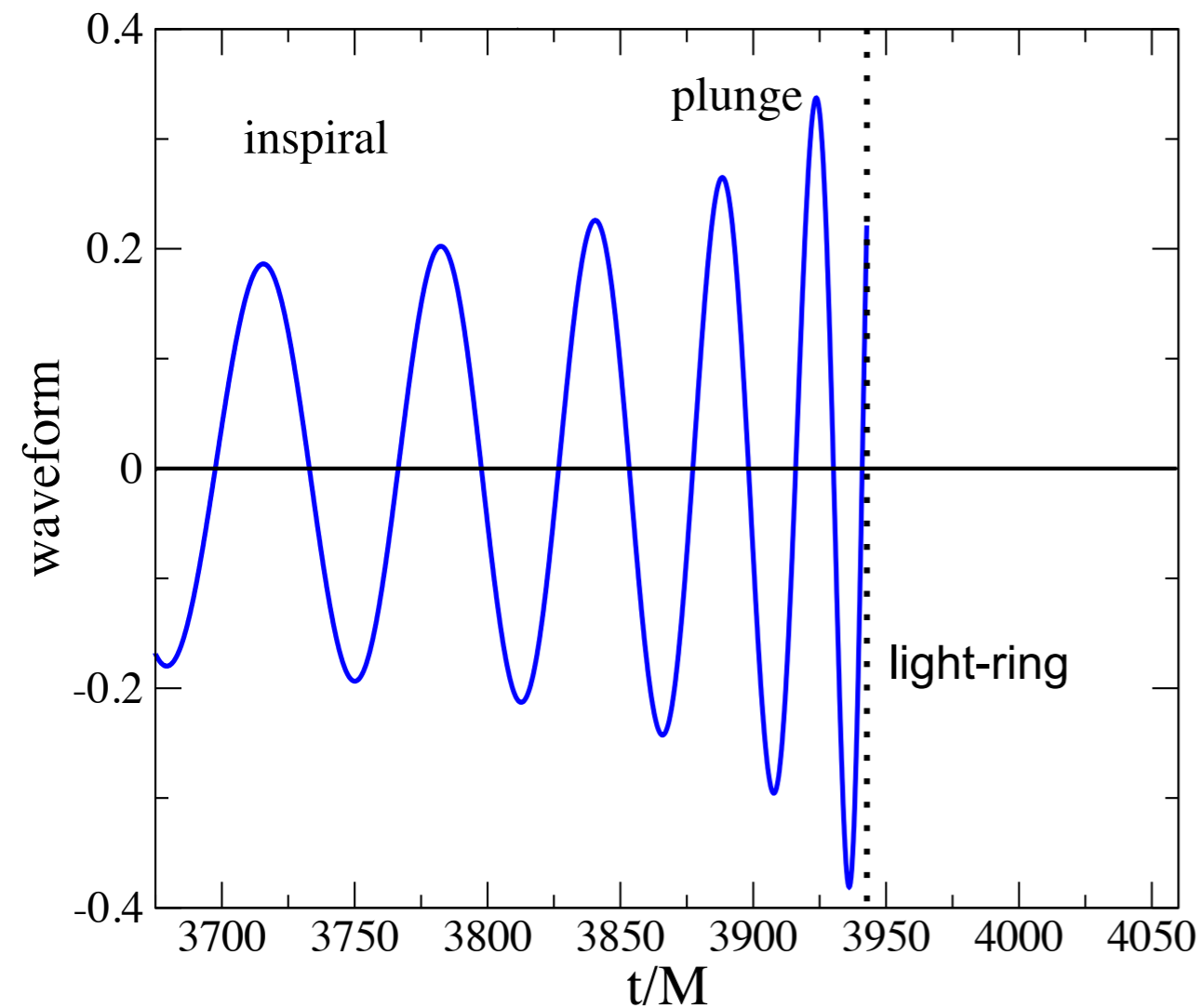
$$\frac{\partial^2 \Psi}{\partial t^2} - \frac{\partial^2 \Psi}{\partial r_{\star}^2} + V_{lm} \Psi = S_{lm}$$



- **Peak** of black-hole potential **close to "light ring"**.
- Once particle is inside potential, **direct gravitational radiation** from its motion is **strongly filtered** by potential barrier (**high-pass filter**).
- Only **black-hole spacetime vibrations** (quasi-normal modes) **leaks out** black-hole potential.

EOB inspiral-plunge waveform & frequency

Evolve **two-body dynamics up to light ring** (or photon orbit) and then ...

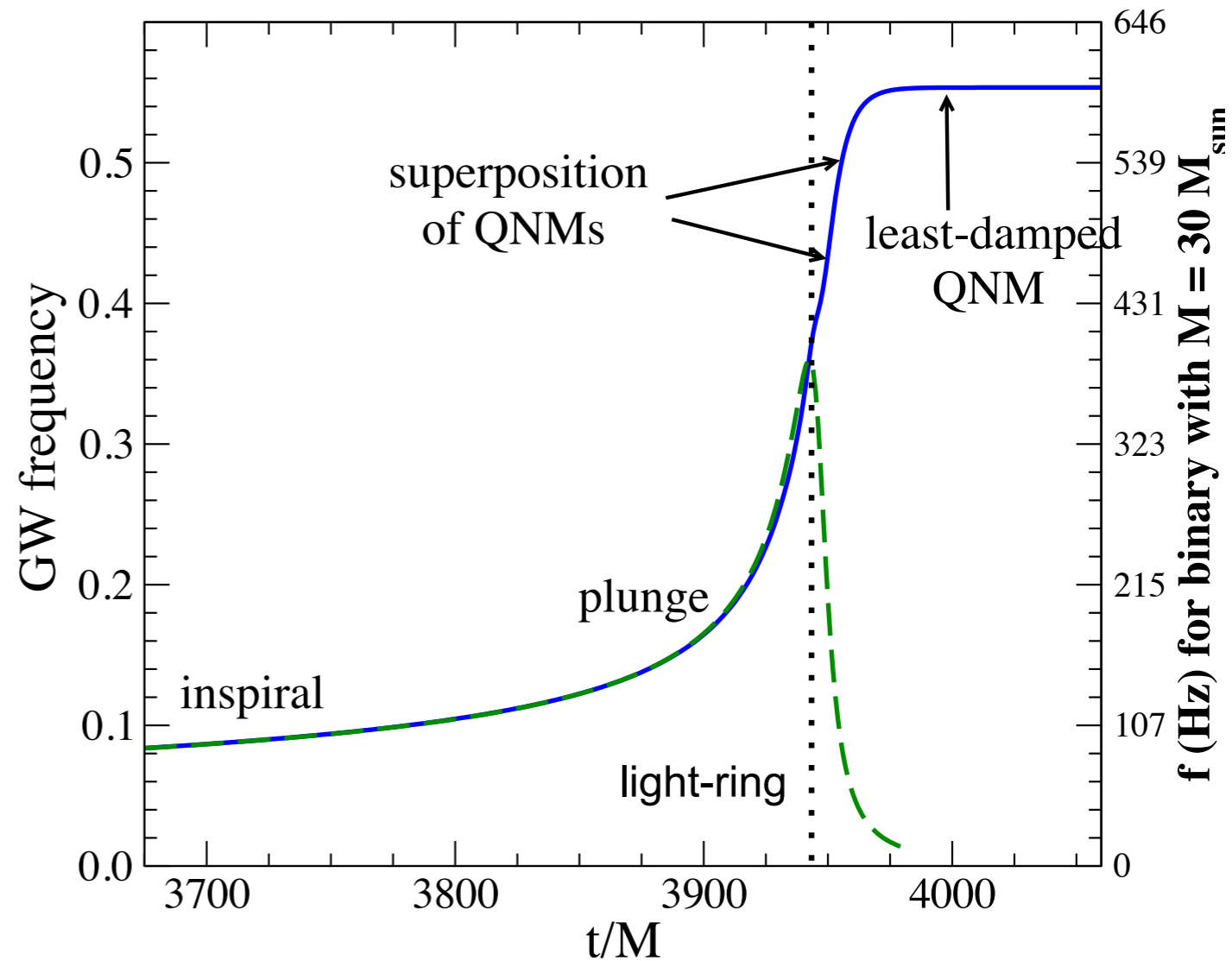
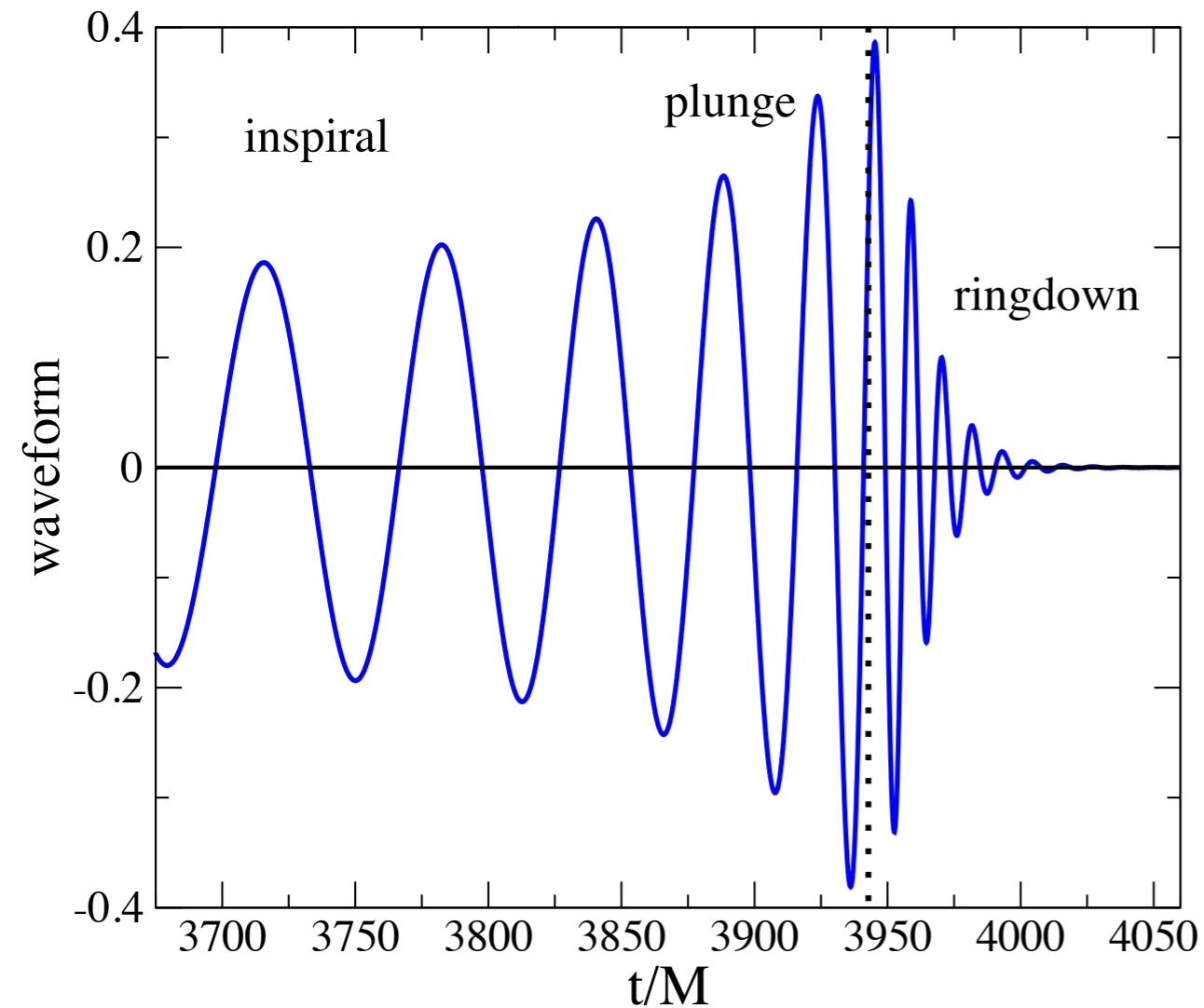


Quasi-normal modes excited at **light-ring crossing**

(Goebel 1972, Davis et al. 1972, Ferrari et al. 1984, Damour et al. 07, Barausse et al. 11, Price et al. 15)

EOB inspiral-merger-ringdown waveform & frequency

... attach **superposition of quasi-normal modes** of **remnant** black hole.



- Very **short/simple transition** plunge-merger-ringdown: "easy" to model!

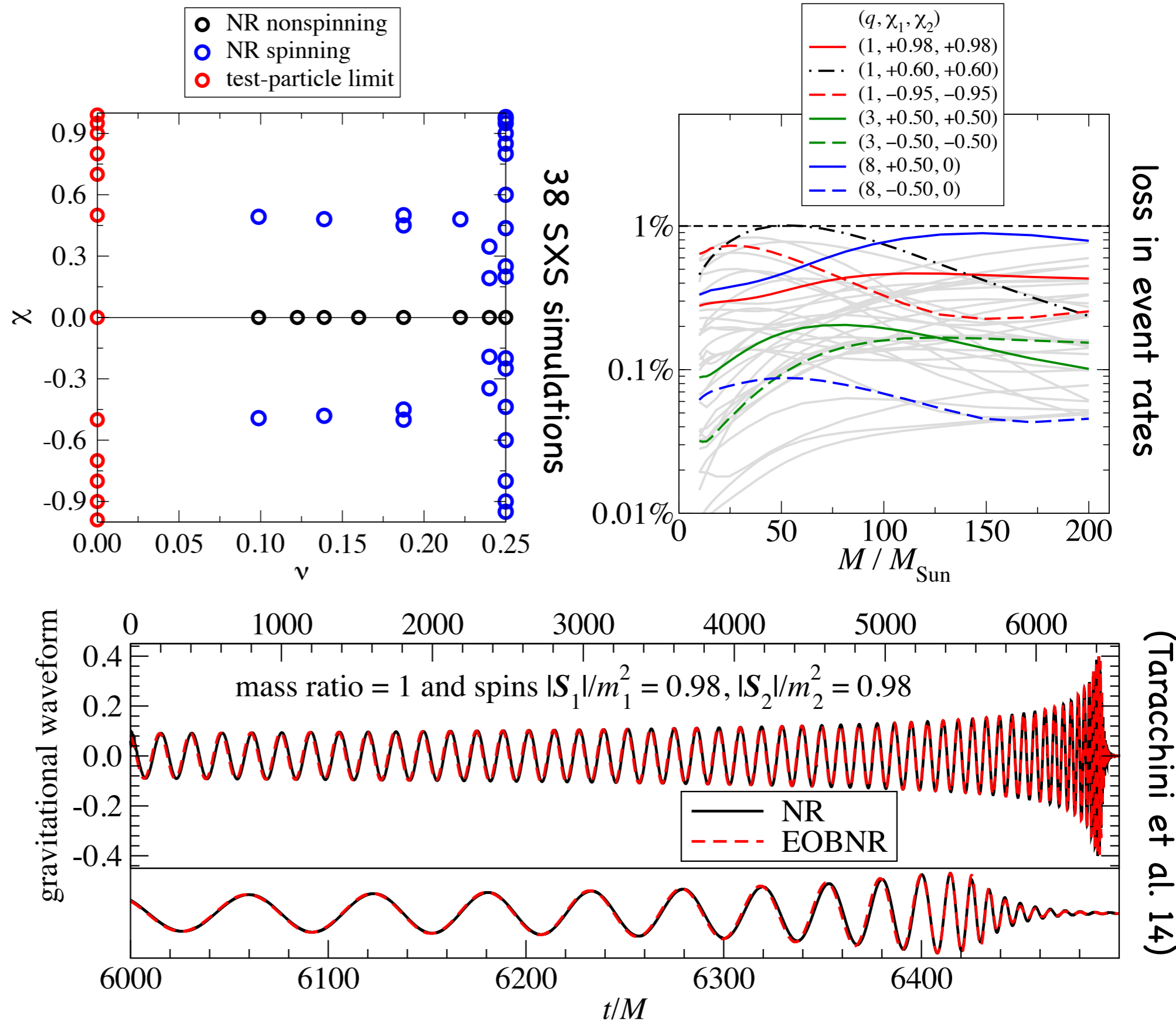
- Energy **quickly released** during merger: 2%-12% M

EOBNR waveforms used in LIGO O1 modeled-search

- EOB waveforms with **nonprecessing spins** calibrated to NR waveforms

(Taracchini, AB, Pan, Hinderer, SXS 14)

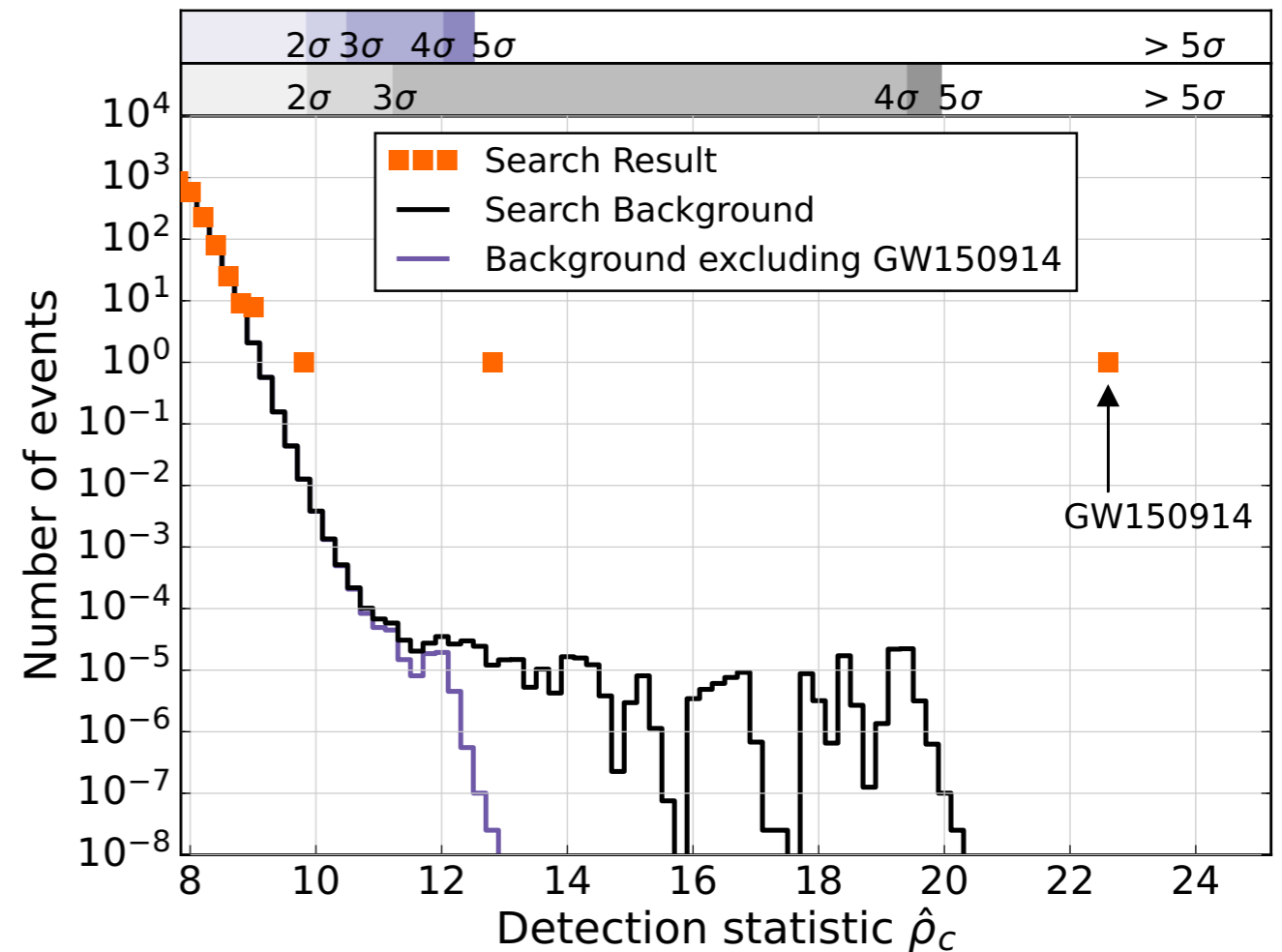
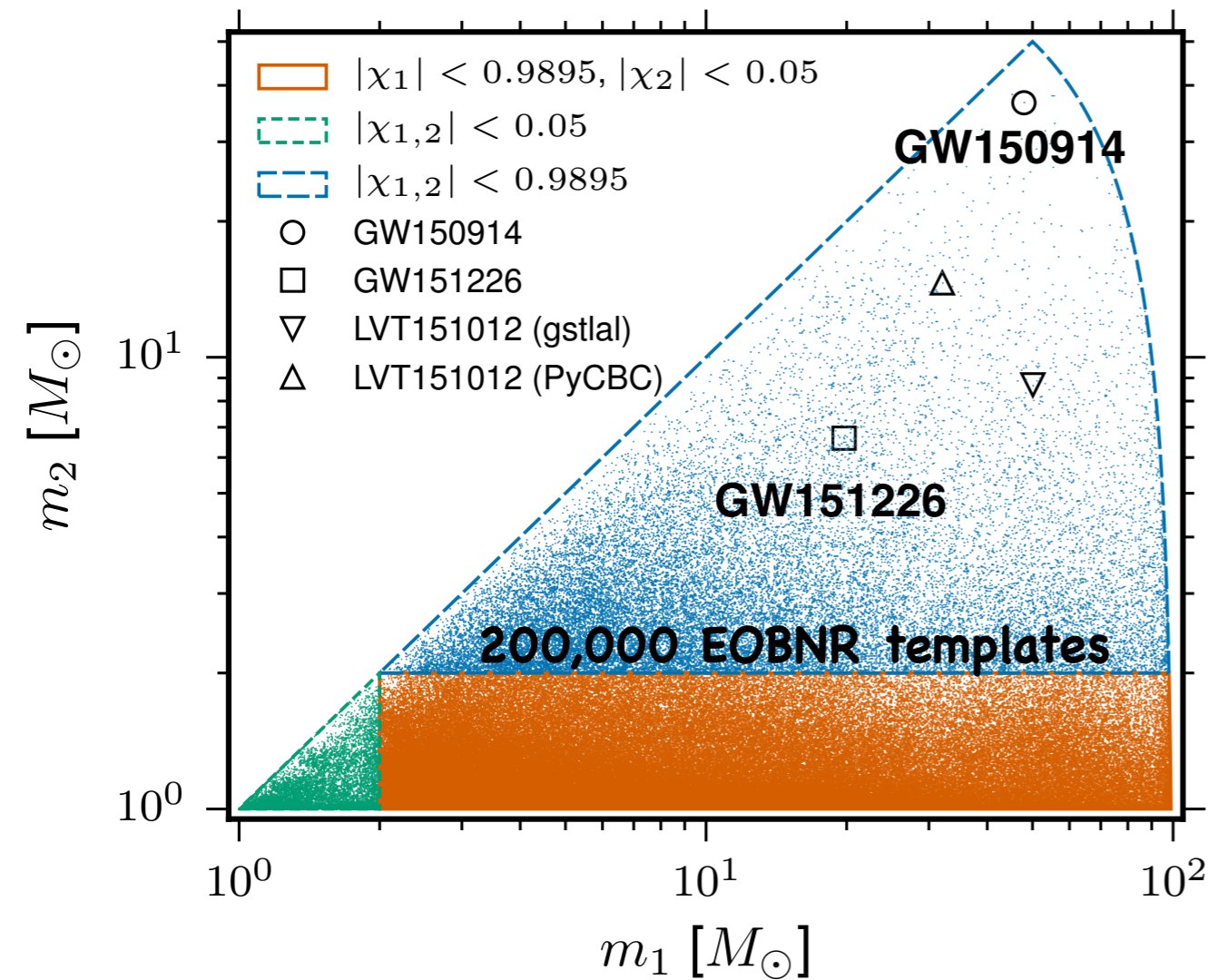
(Barausse & AB 10, 11, Barausse et al. 09, Damour & Nagar 09, Pan et al. 08, Damour et al. 08)



Detection confidence with modeled search in O1

- **Matched filtering** employed

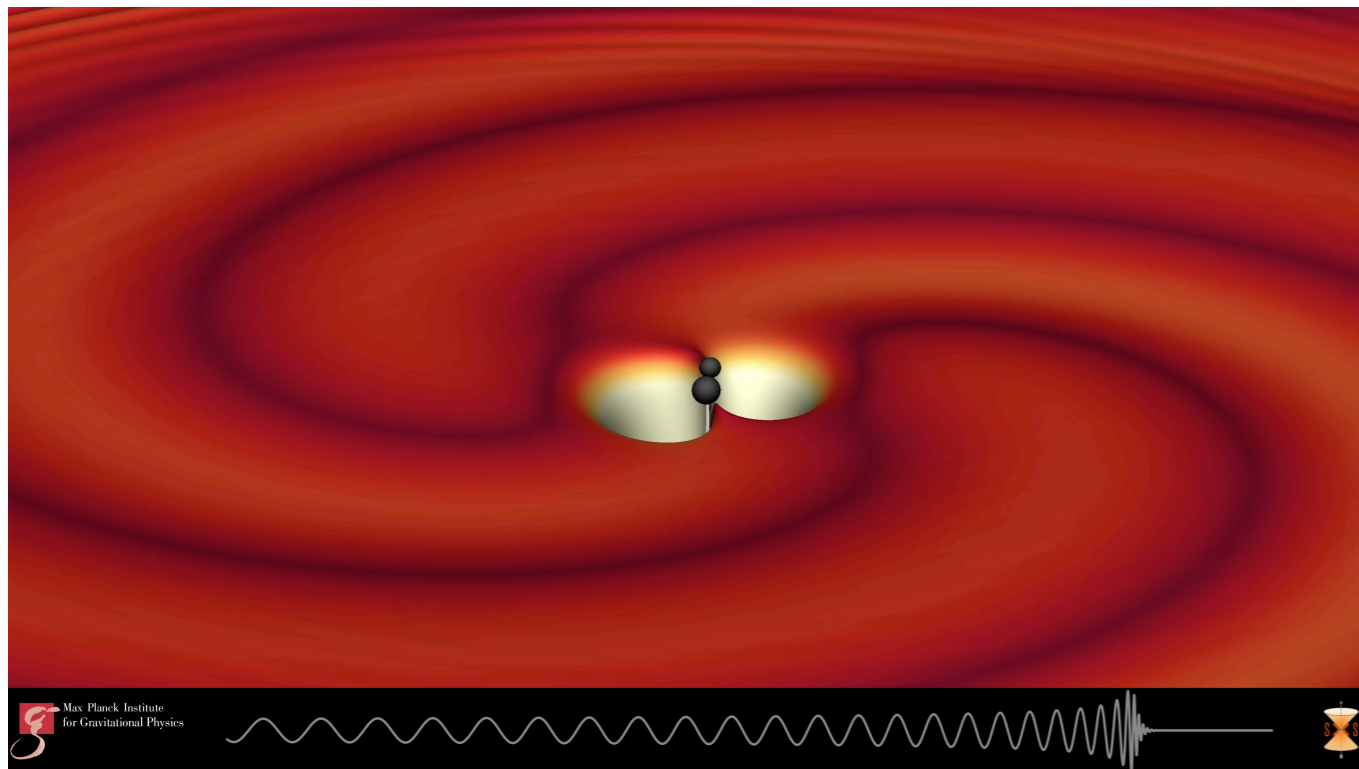
(Abbott et al. arXiv:1606.04856)



- **Confidence (FAR) $> 5.3\sigma$ ($< 1/200,000$ year)** that GW150914 & GW151226 were real gravitational-wave signals.
- **Minimal-assumption search** reached high detection confidence (**$> 4.6\sigma$**) **only for GW150914.**

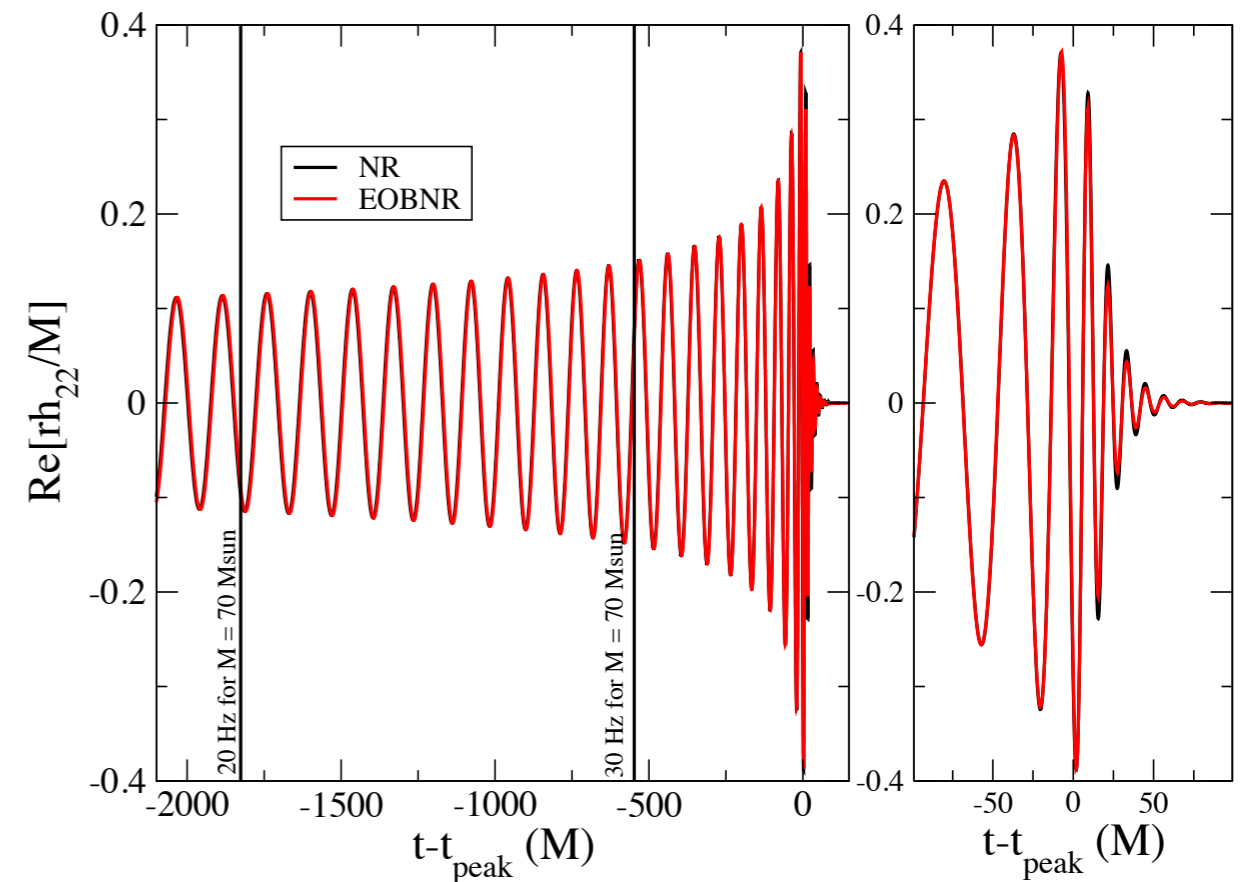
Numerical-relativity simulation of a binary black-hole merger with parameters close to GW150914

(visualization's credit: Haas @ AEI)



(Ossokine, AB & SXS project)

(Ossokine, AB & SXS project)



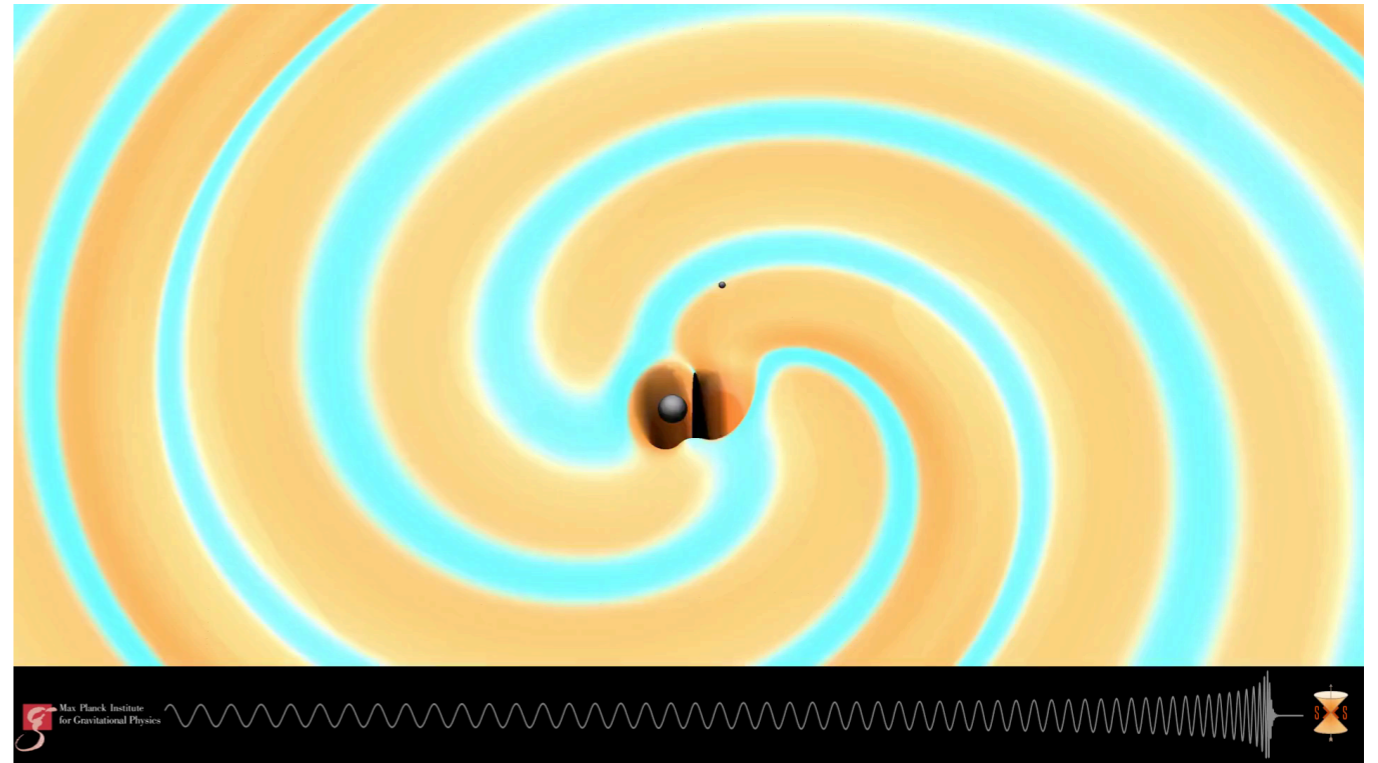
- **Waveform models** very closely **match** the **exact solution** from Einstein equations around GW150914 & GW151226.
- **Systematics** due to modeling **are smaller than statistical** errors.

(see also Abbott et al. arXiv:1611.07531)

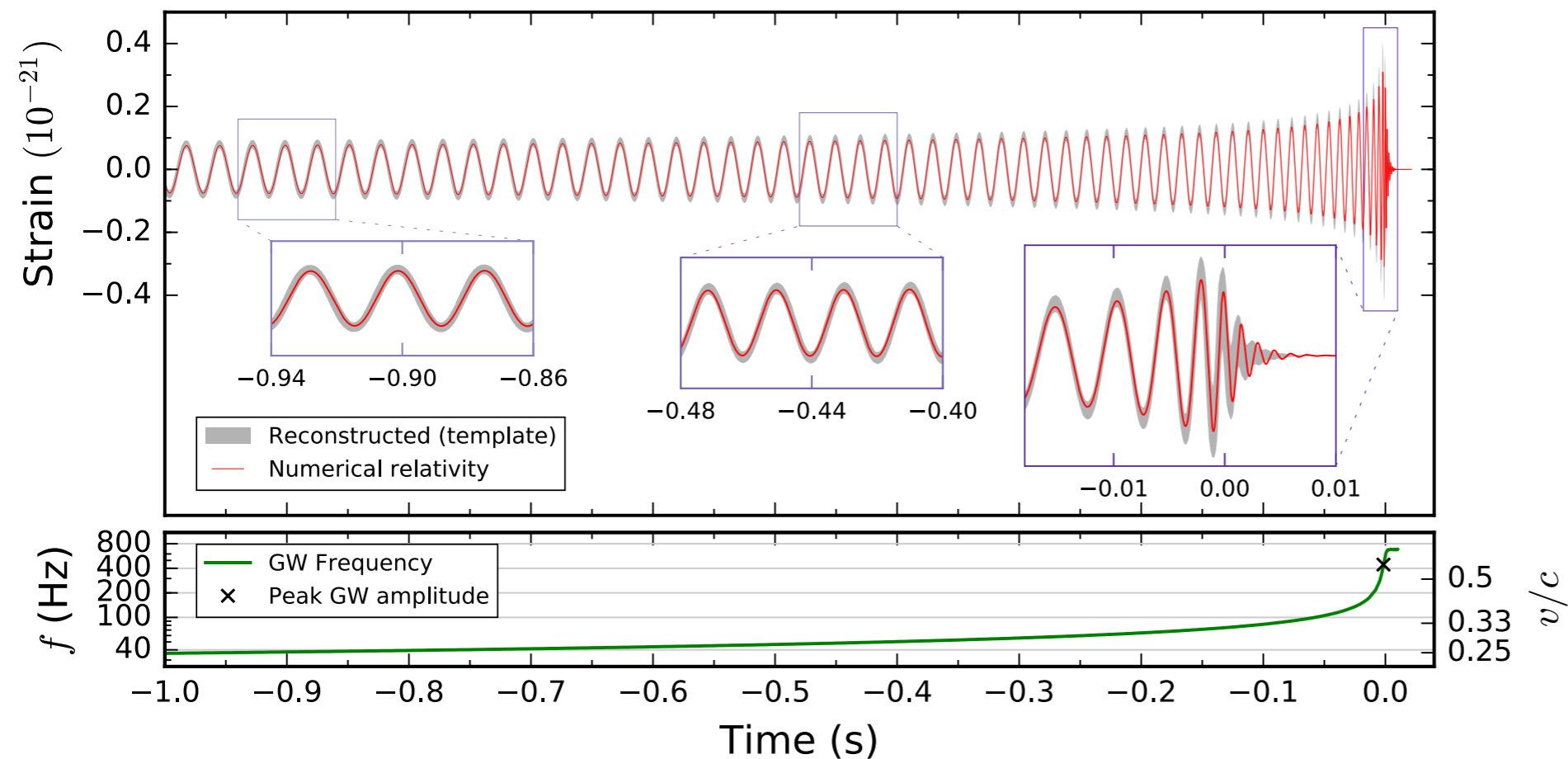
Numerical-relativity simulation of a binary black-hole merger with parameters close to GW151226

(visualization's credit: Dietrich, Haas @AEI)

(Ossokine, AB & SXS project)



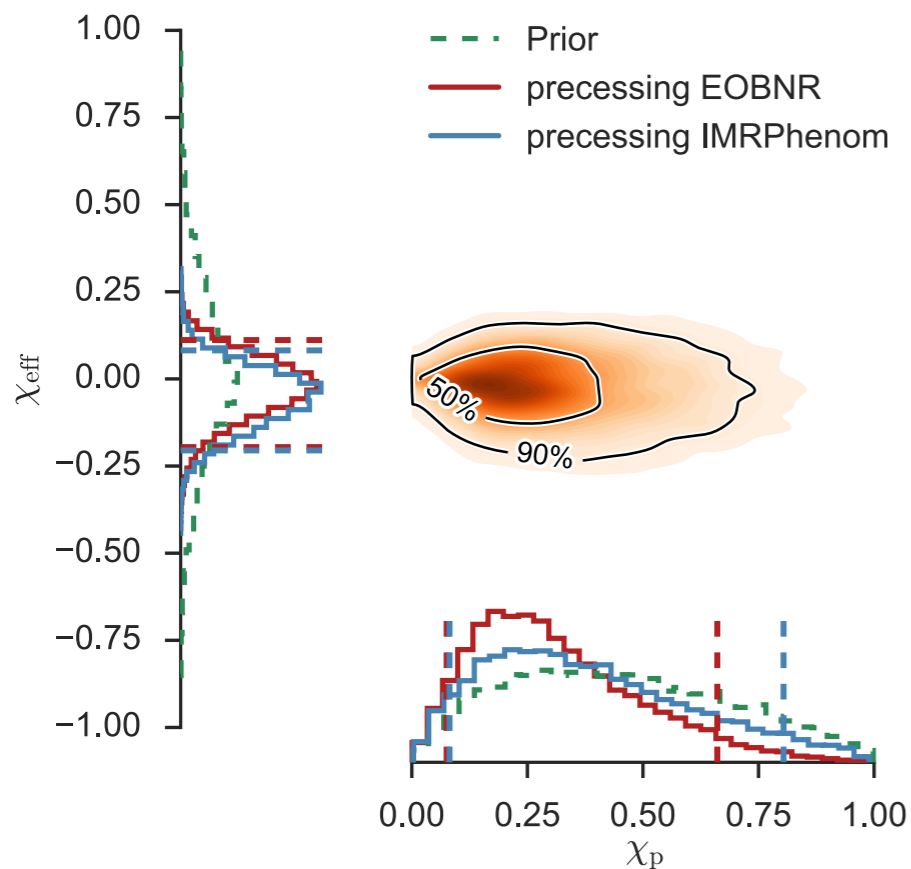
(Abbott et al. PRL 116 (2016) 241103)



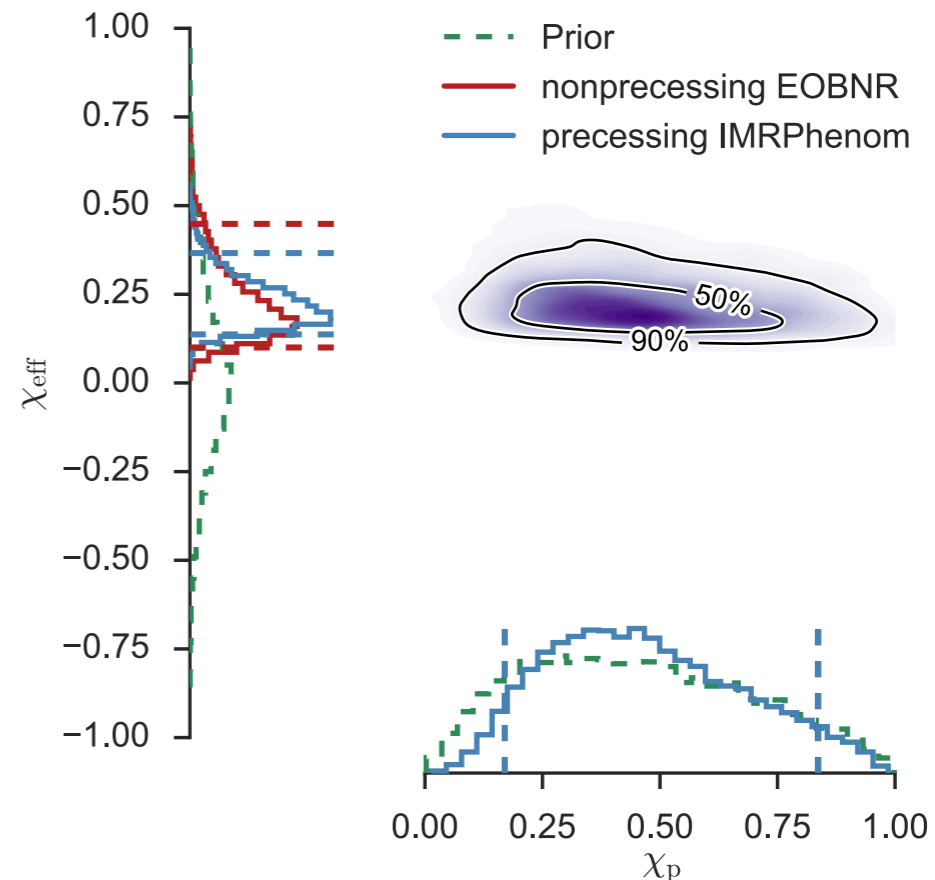
Unveiling binary black-holes properties: spins

(Abbott et al. arXiv:1606.01210)

GW150914

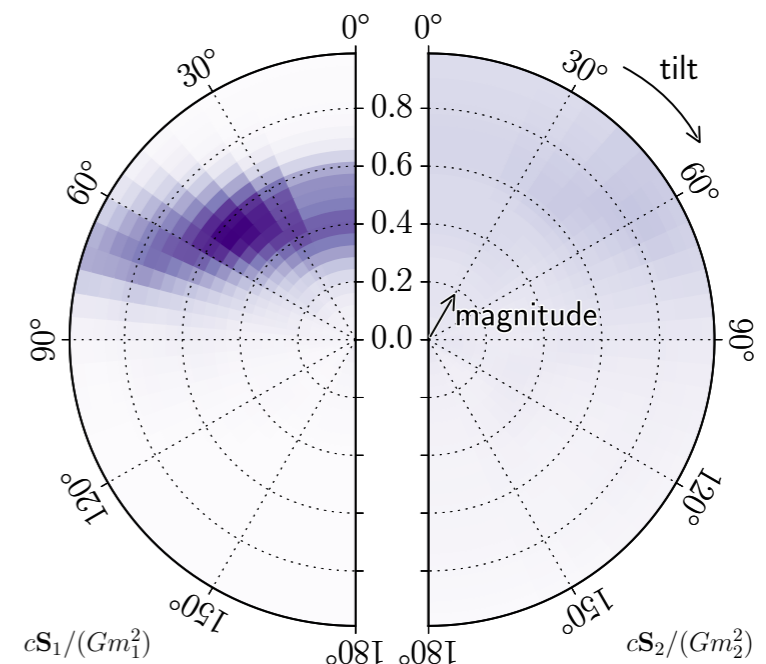


GW151226



$$\chi_{\text{eff}} = \left(\frac{\mathbf{S}_1}{m_1} + \frac{\mathbf{S}_2}{m_2} \right) \cdot \left(\frac{\hat{\mathbf{L}}}{M} \right)$$

- BHs' spins not maximal, and for GW151226 one BH's spin larger than 0.2 at 99% confidence.
- Spins < 0.7. No information about precession.



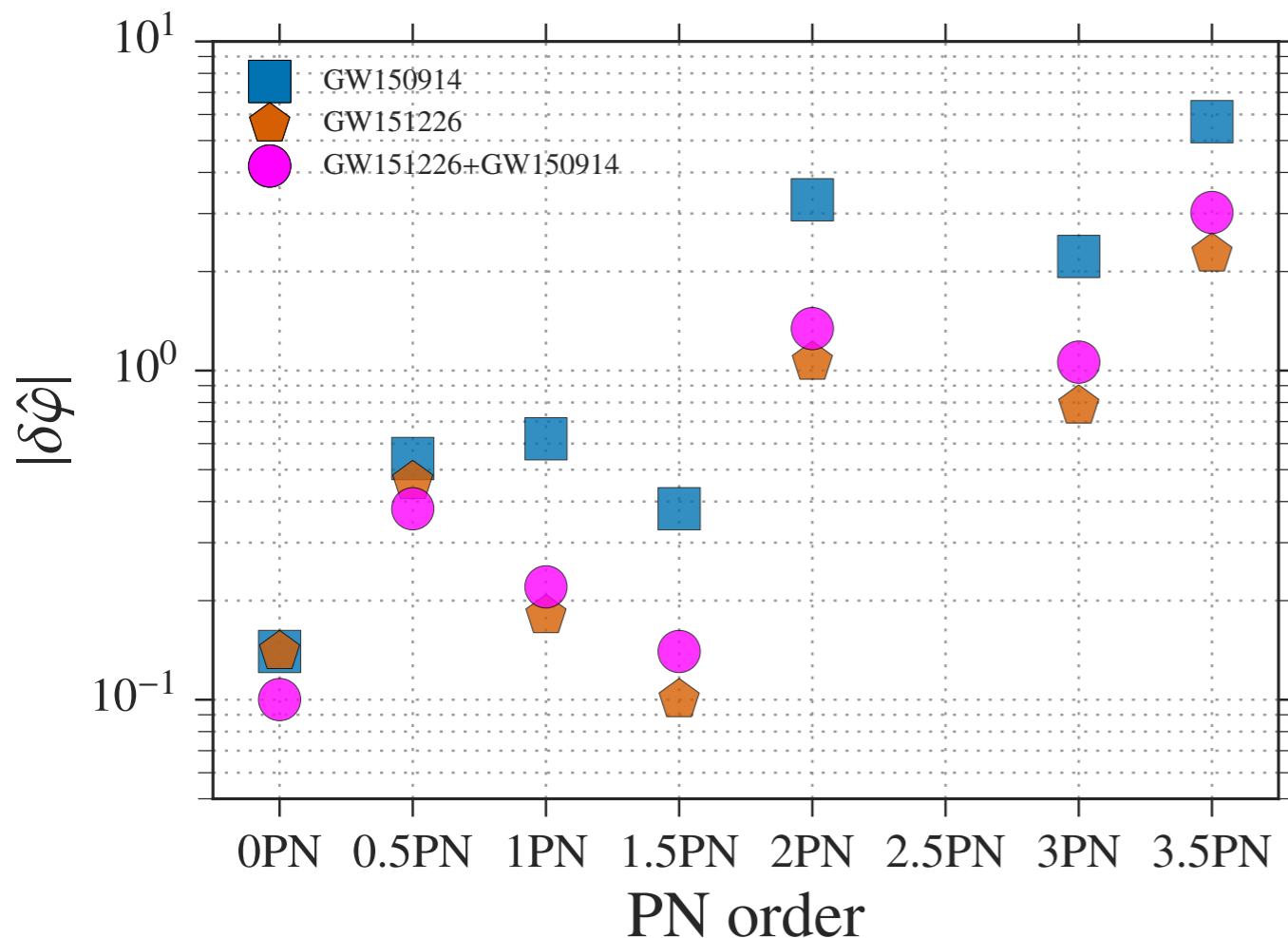
(Abbott et al. PRL 116 (2016) 241103)

Tests of GR with first LIGO's black holes: inspiral

- GW150914/GW122615's **rapidly varying orbital periods** allow us to **bound higher-order PN coefficients** in gravitational phase.

$$\tilde{h}(f) = \mathcal{A}(f)e^{i\varphi(f)} \quad \varphi(f) = \varphi_{\text{ref}} + 2\pi f t_{\text{ref}} + \varphi_{\text{Newt}}(Mf)^{-5/3} + \varphi_{0.5\text{PN}}(Mf)^{-4/3} + \varphi_{1\text{PN}}(Mf)^{-1} + \varphi_{1.5\text{PN}}(Mf)^{-2/3} + \dots$$

(Abbott et al. arXiv:1606.04856)

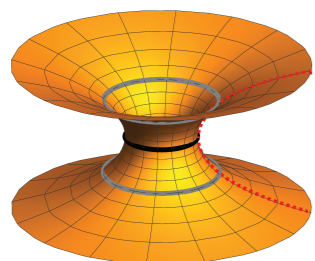


(Arun et al. 06 , Mishra et al. 10, Yunes & Pretorius 09, Li et al. 12)

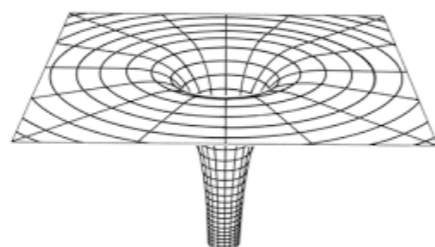
- PN parameters** describe: **tails** of radiation due to backscattering, **spin-orbit** and **spin-spin** couplings.
- First **GR test** in the genuinely dynamical, **strong-field regime**.

Can we probe BH horizon from GW ringdown?

- What determines **ringdown signal**? **Light ring or horizon?**



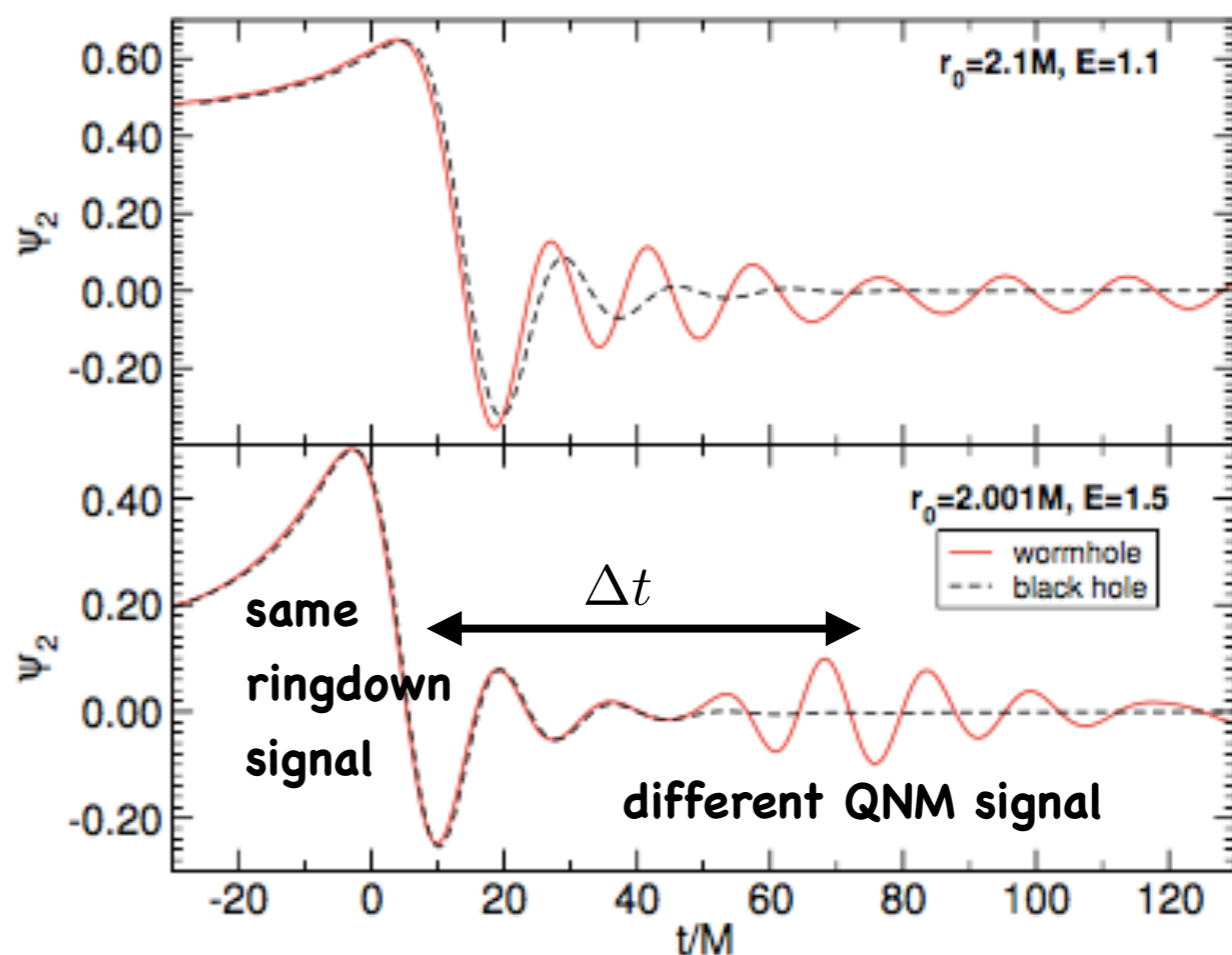
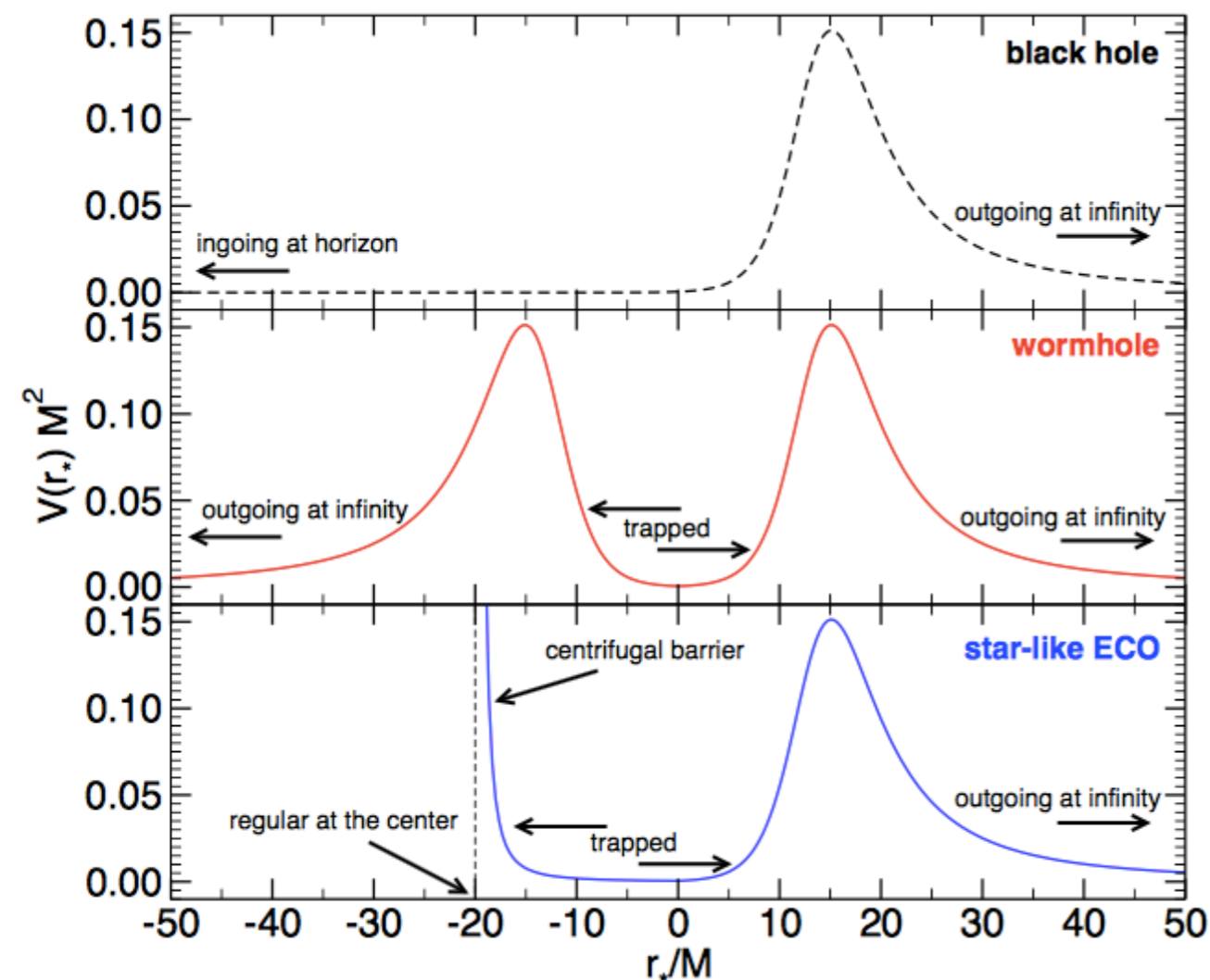
horizonless object



black hole

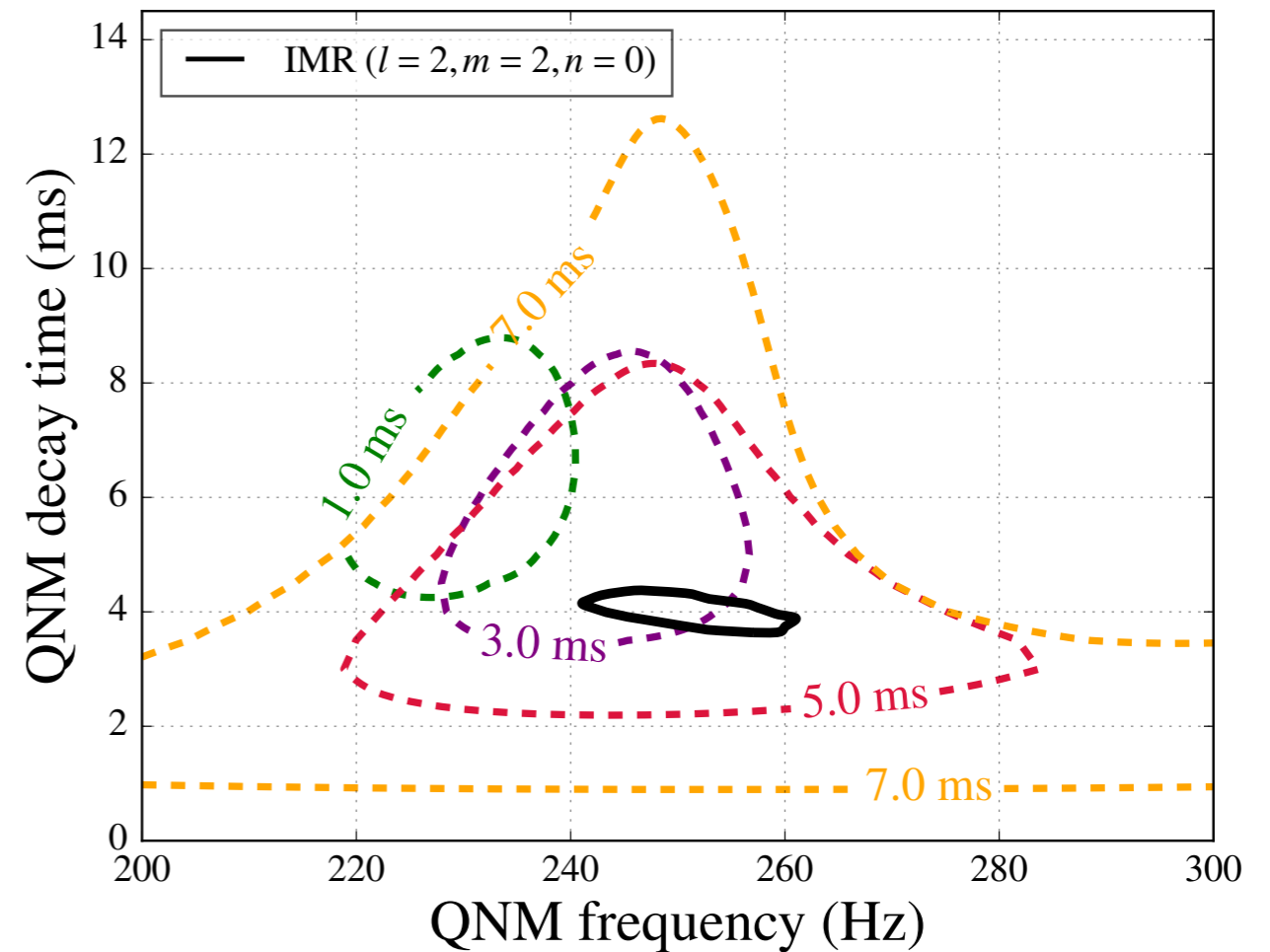
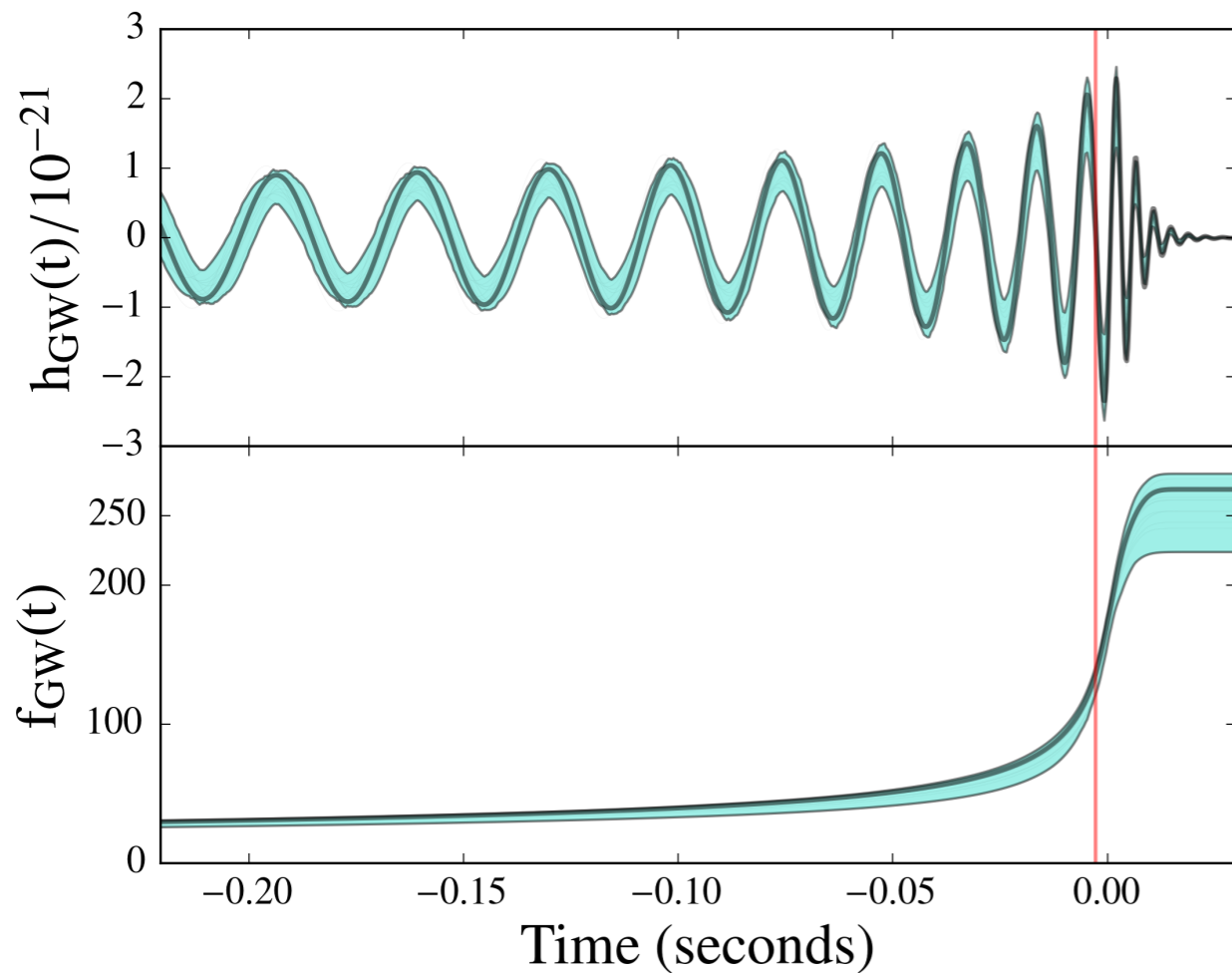
(Cardoso, Franzin & Pani 16)

- **QNM spectrum differs in BH and horizonless object.**
- **Ringdown and QNM signals can be different in horizonless objects. GW echoes!**



radial infall

Could we prove that GW150914's remnant is a BH?



(Abbott et al. PRL 116 (2016) 221101)

- We measured frequency & decay time of **damped sinusoid** in the data after GW150914's peak.
- **Multiple QNMs** need to be measured to extract mass and spin of remnant, test **no-hair theorem** and **second-law black-hole mechanics** (Israel 69, Carter 71; Hawking 71, Bardeen 73).

How to constrain or rule out modified theories of GR?

- Is it possible to **parameterize GR** and **modified theories of GR** in terms of relevant physical parameters during the **non-perturbative merger-ringdown** stage?
- Need **NR merger simulations** in **modified theories of GR**: scalar-tensor theories, Einstein-Aether theory, dynamical Chern-Simons, Einstein-dilaton Gauss-Bonnet theory, massive gravity theories, etc.
- Need NR merger simulations of binaries composed of **exotic objects**, such as boson stars, gravastar, etc.
- Can we **disprove** the **presence** of BH “**horizon**” in binary mergers?
- Can we **probe quantum gravity** with binary **black hole mergers**?
Will **new** physical **scales be observable**?

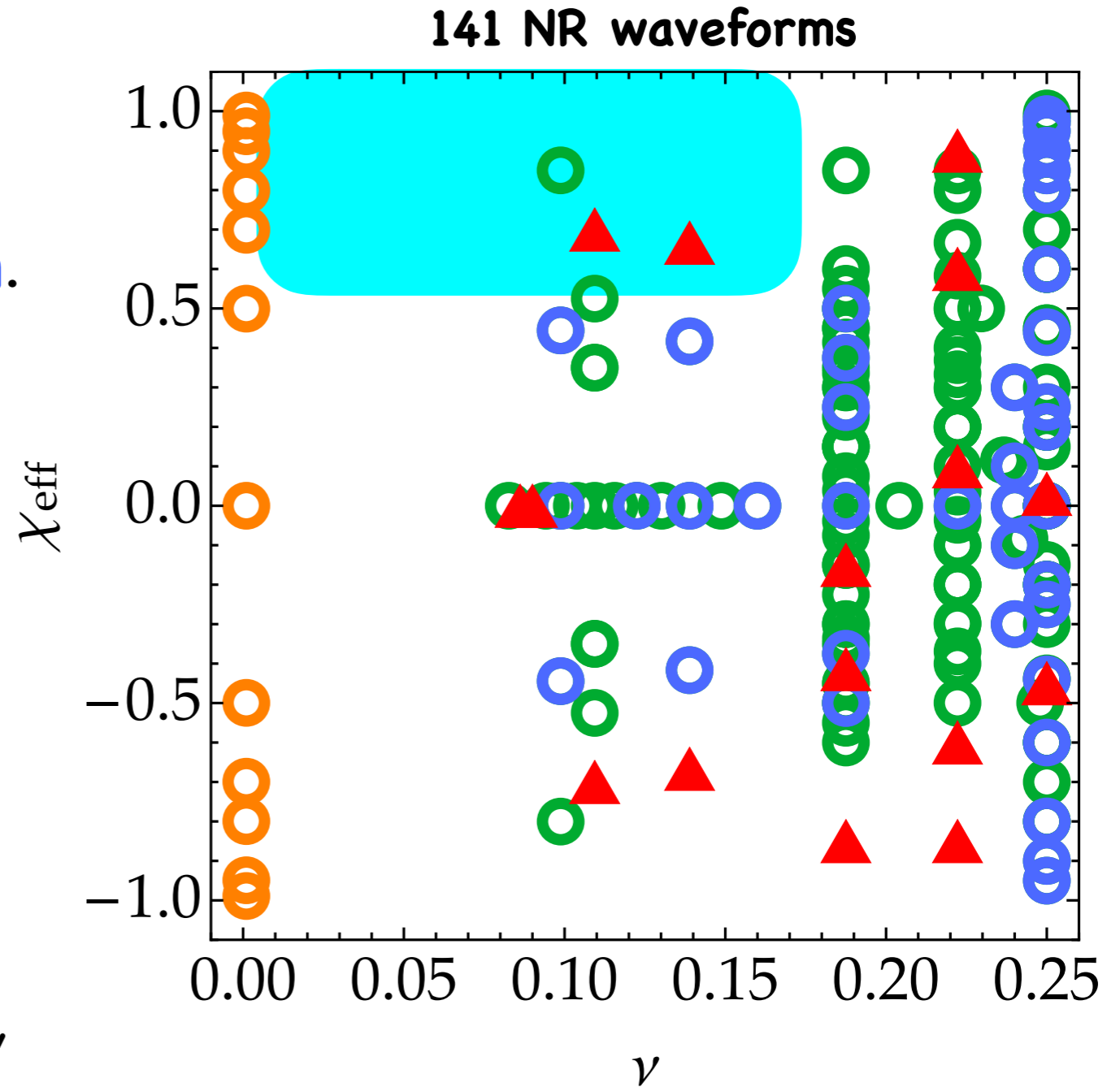
Extending waveform model in all binary parameter space

- **Best EOBNR** waveform model is **calibrated to 141 NR** waveforms and **used** in ongoing **O2 LIGO** run.

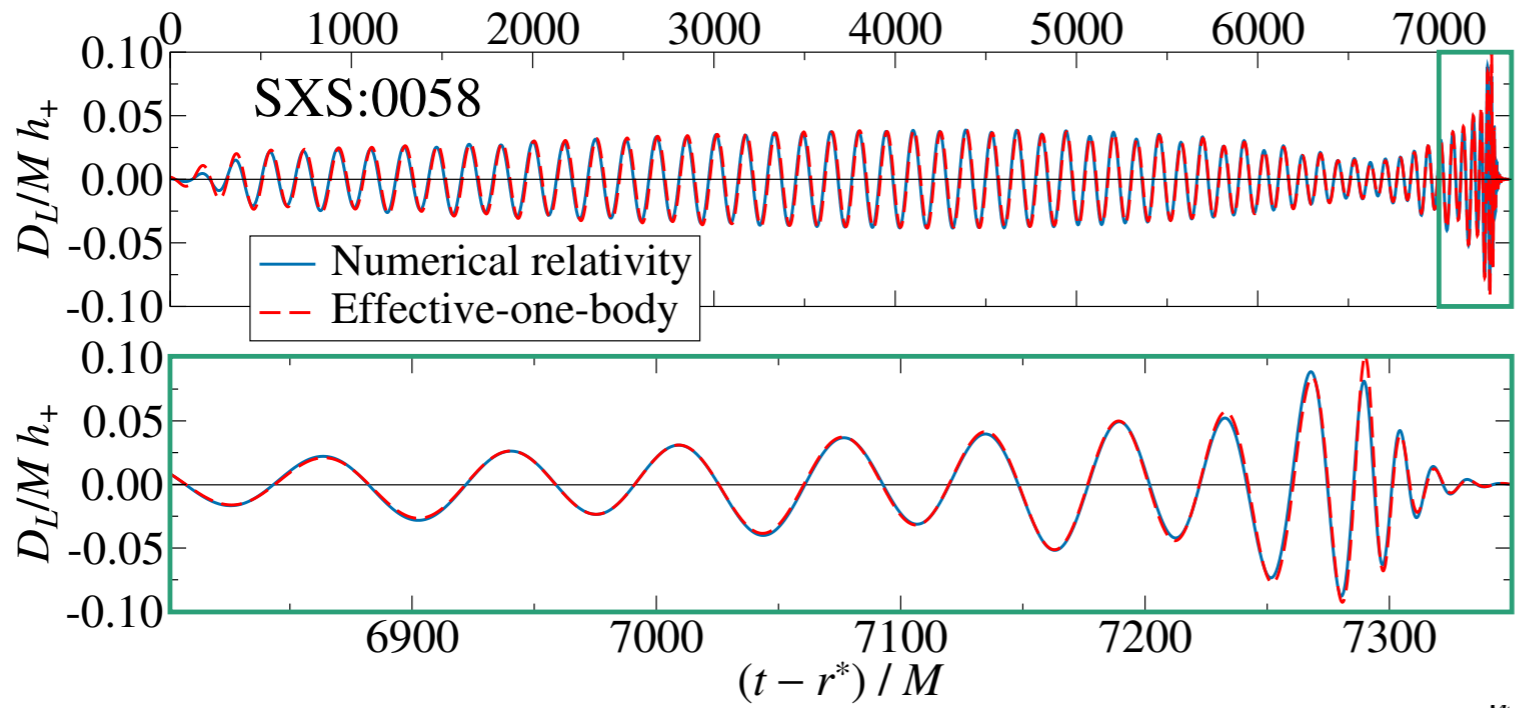


- Perform **EOB internal consistency checks** to control **systematics** due to limited length of NR simulations, interpolation and extrapolation.

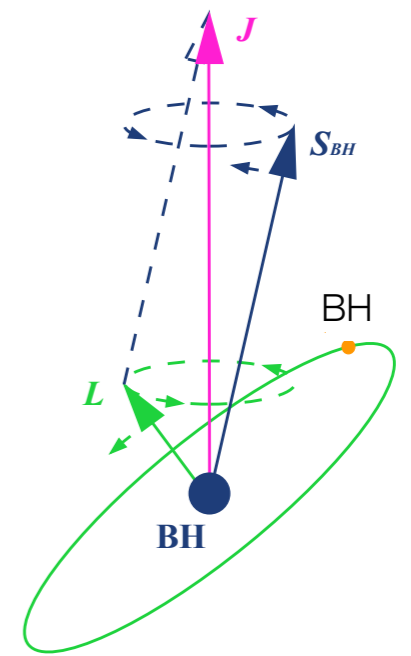
- Difficult to run **NR simulations** for **large mass ratios** (> 4) and **large spins** (> 0.8), with **large number** of GW **cycles** (> 50). 



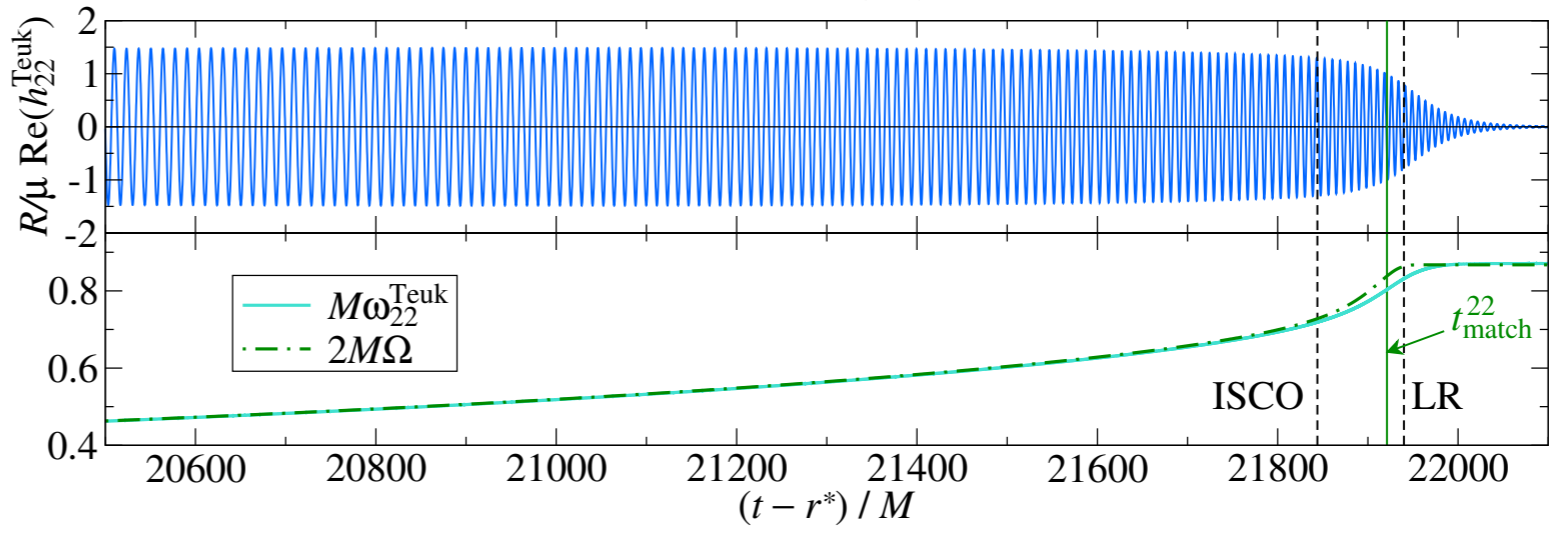
More challenges: spin-precession, extremal BHs, eccentricity ...



(Babak et al. 16)



$\omega/M = 0.99$, (2,2) mode



(Taracchini et al. 13)

(Hinder et al. in prep)

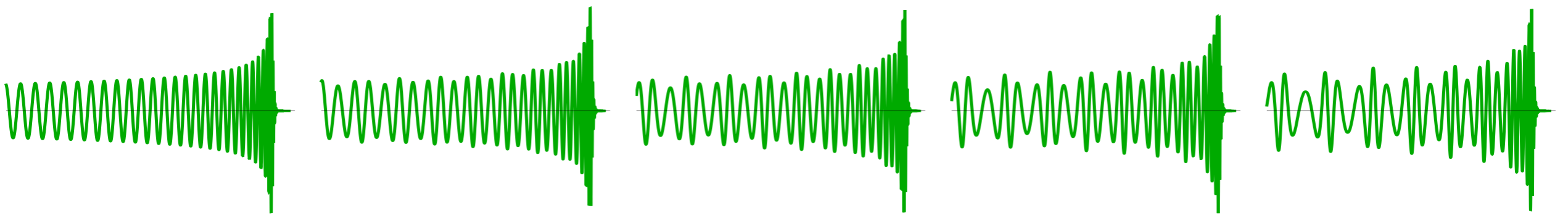
$e_0 = 0.00$

$e_0 = 0.05$

$e_0 = 0.10$

$e_0 = 0.15$

$e_0 = 0.20$



Insights & new results on resumming two-body problem

- In test-body limit, spinning EOB Hamiltonian includes **linear and quadratic terms in spin of test body at all PN orders.**

(Barausse et al. 10, Barausse & AB 11, 12; Vines et al. 15)

$$(S + S^2) \left(1 + \frac{1}{c^2} + \dots \right)$$

- Is EOB **mapping unique** at all orders?

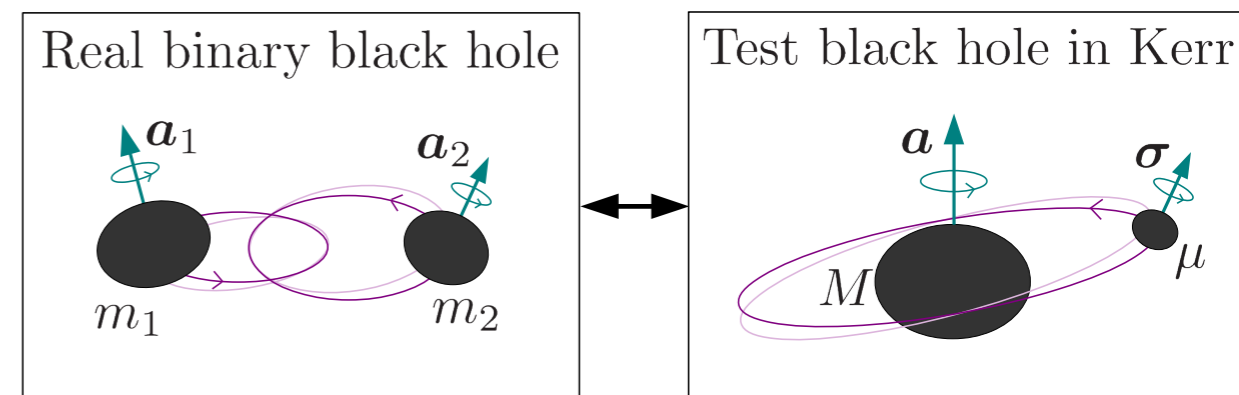
$$H_{\text{real}}^{\text{EOB}} = M \sqrt{1 + 2\nu \left(\frac{H_{\text{eff}}^{\nu}}{\mu} - 1 \right)}$$

At 1PM: mapping unique & 2-body relativistic motion equivalent to 1-body motion in Schwarzschild (Damour 16)

$$G \left(1 + \frac{1}{c^2} + \dots \right)$$

- Can **2-body dynamics** be fully obtained from gravitational **scattering**?

- Can **two-body problem be solved** using **modern amplitude techniques** (on shell scattering amplitudes)?



exact mapping at the leading PN orders

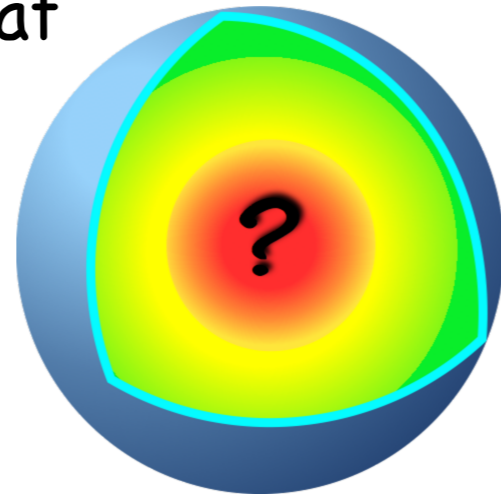
- Results at **leading PN order** but **all orders in spin.**

(Vines & Steinhoff 16; Vines & Harte 16)

$$\frac{1}{c^2} (S + S^2 + \dots)$$

Probing NS's equation of state with LIGO and Virgo

- **NSs** are **unique laboratories** to study baryonic matter at supra-nuclear density



(credit: Hinderer)

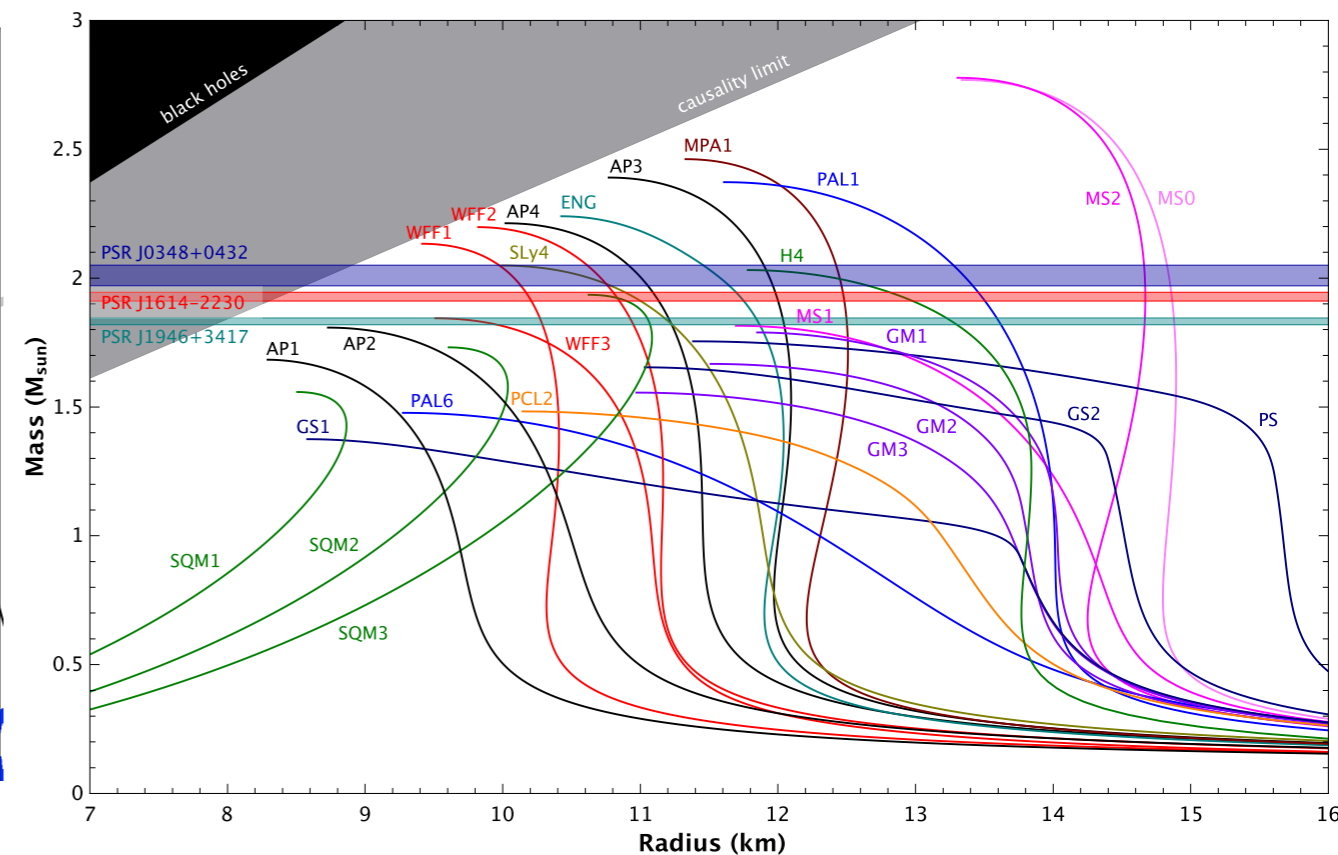
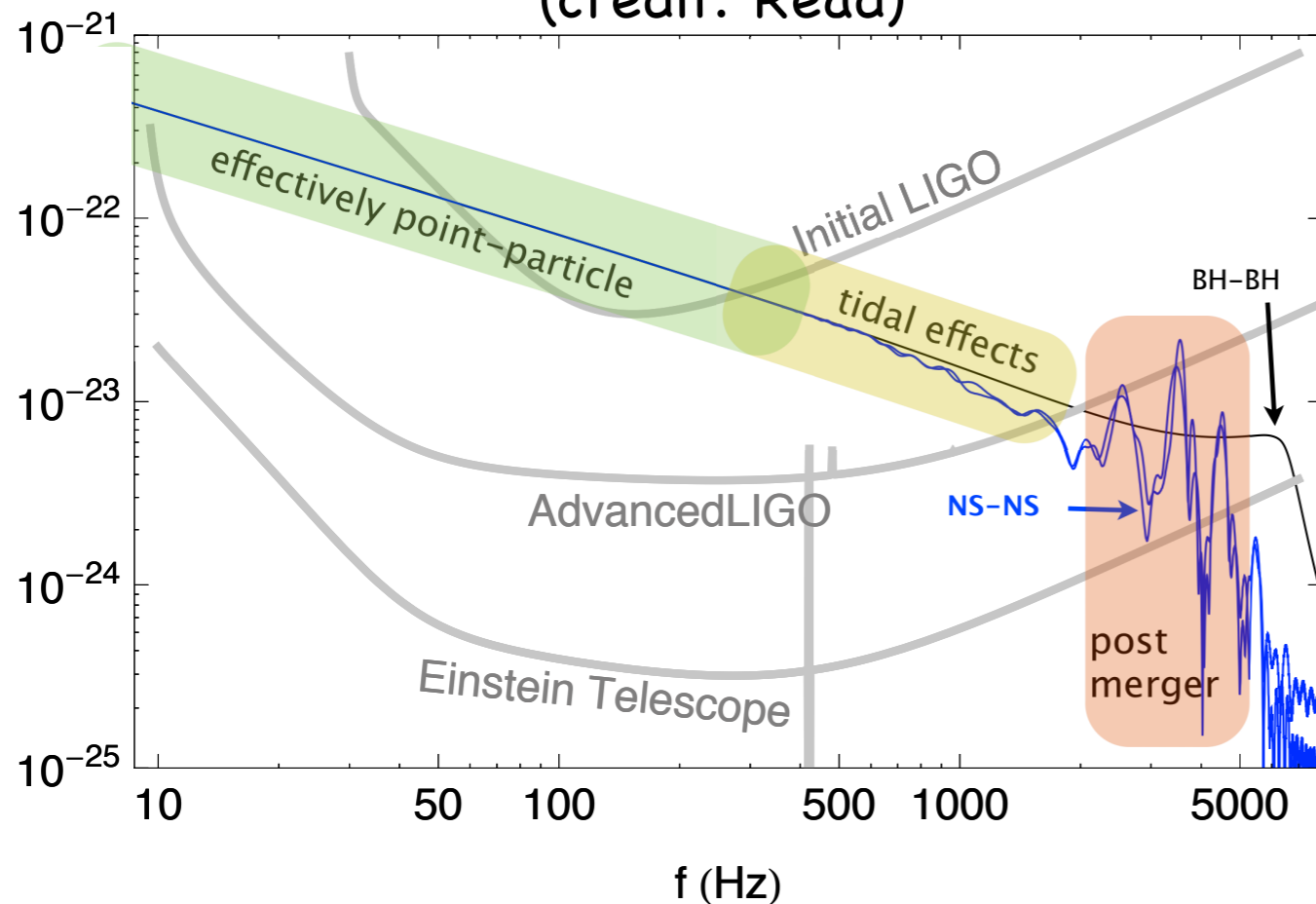
(Baade & Zwicky 1934, Gamow 1937, Landau 1938, Oppenheimer & Volkoff 1939, Cameron 1959, Wheeler 1966)

- **NS's** characteristics:

- mass: 1-3 M_{sun}
- radius: 9-15 km
- core density $> 10^{14} \text{g/cm}^3$

(credit: Read)

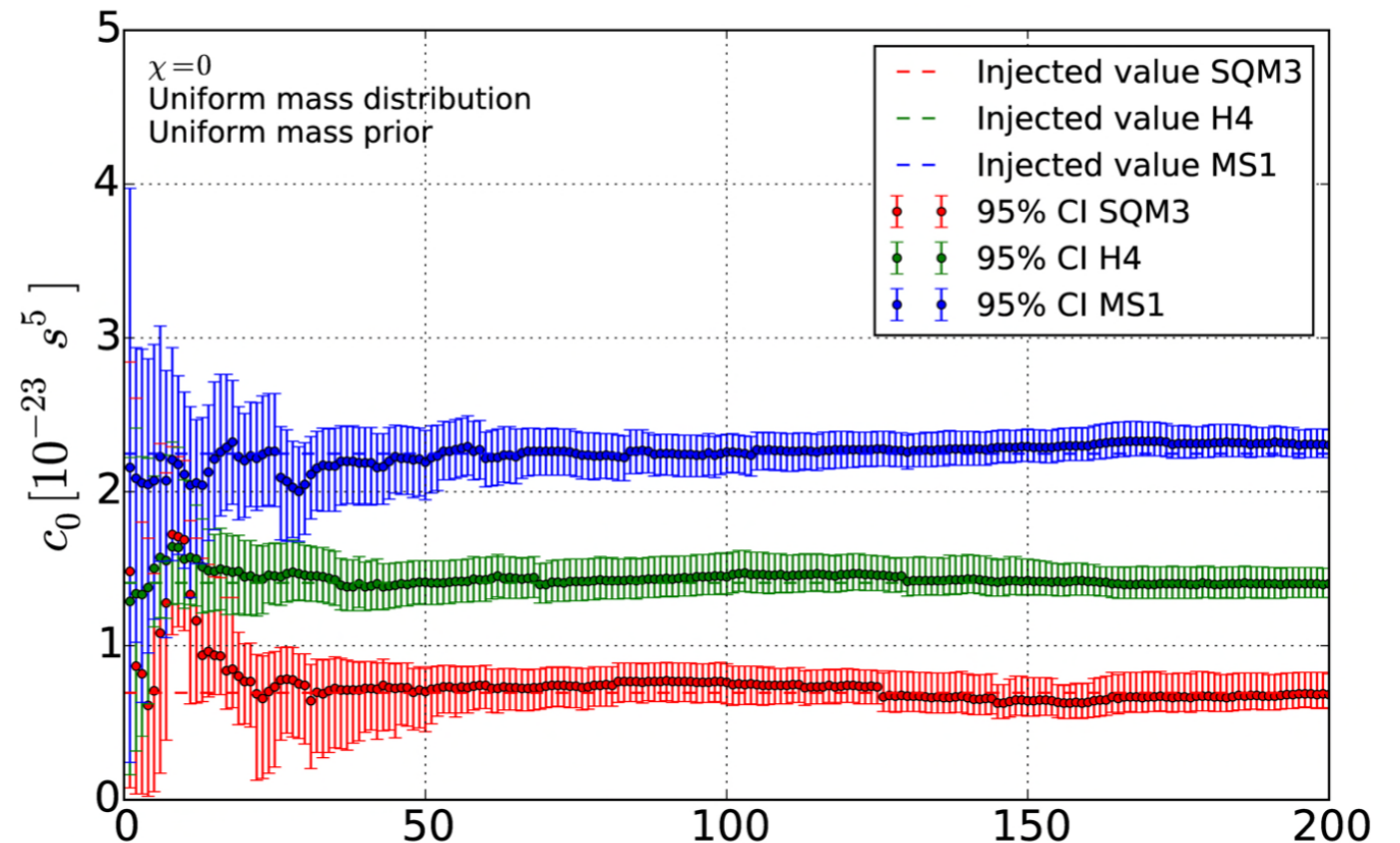
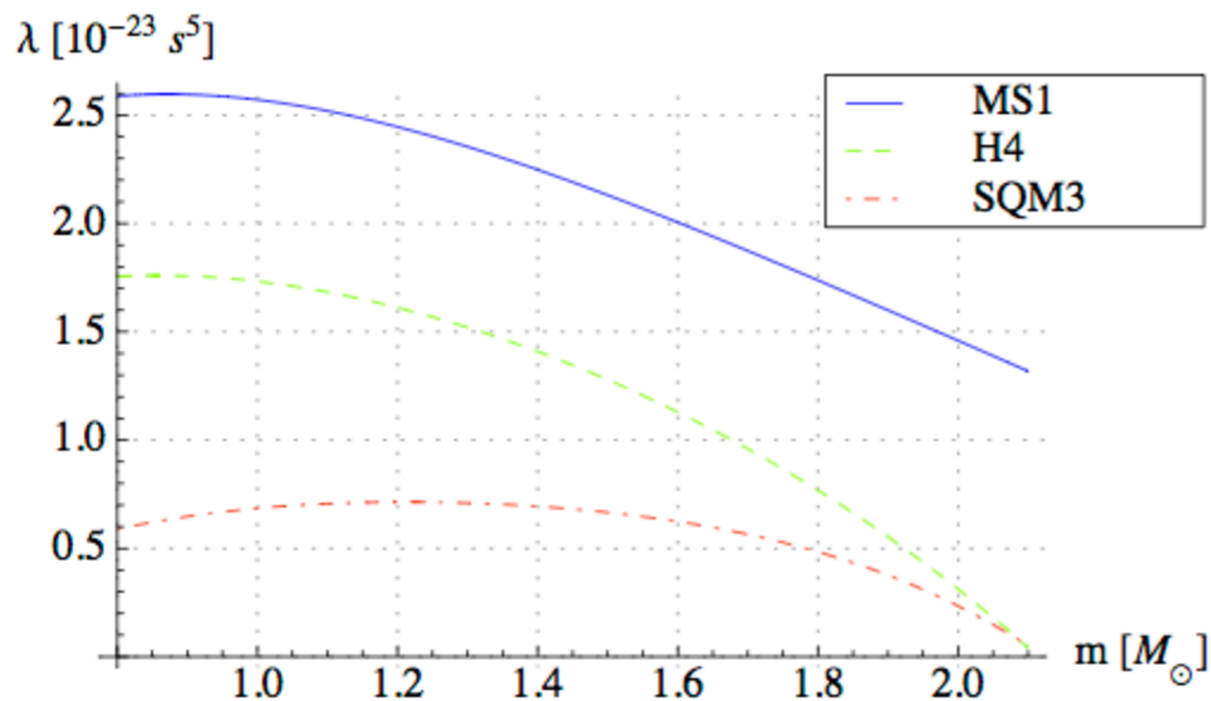
(Antoniadis et al. 16)



With several tens of events, possible to distinguish “some” EOS

- The influence of NS’s **internal structure** on the waveform is **characterized** by a single (constant) parameter, the **tidal deformability** λ
- λ **measures** star’s **quadrupole deformation** in response to the companion **perturbing tidal field**: $Q_{ij} = -\lambda \mathcal{E}_{ij}$

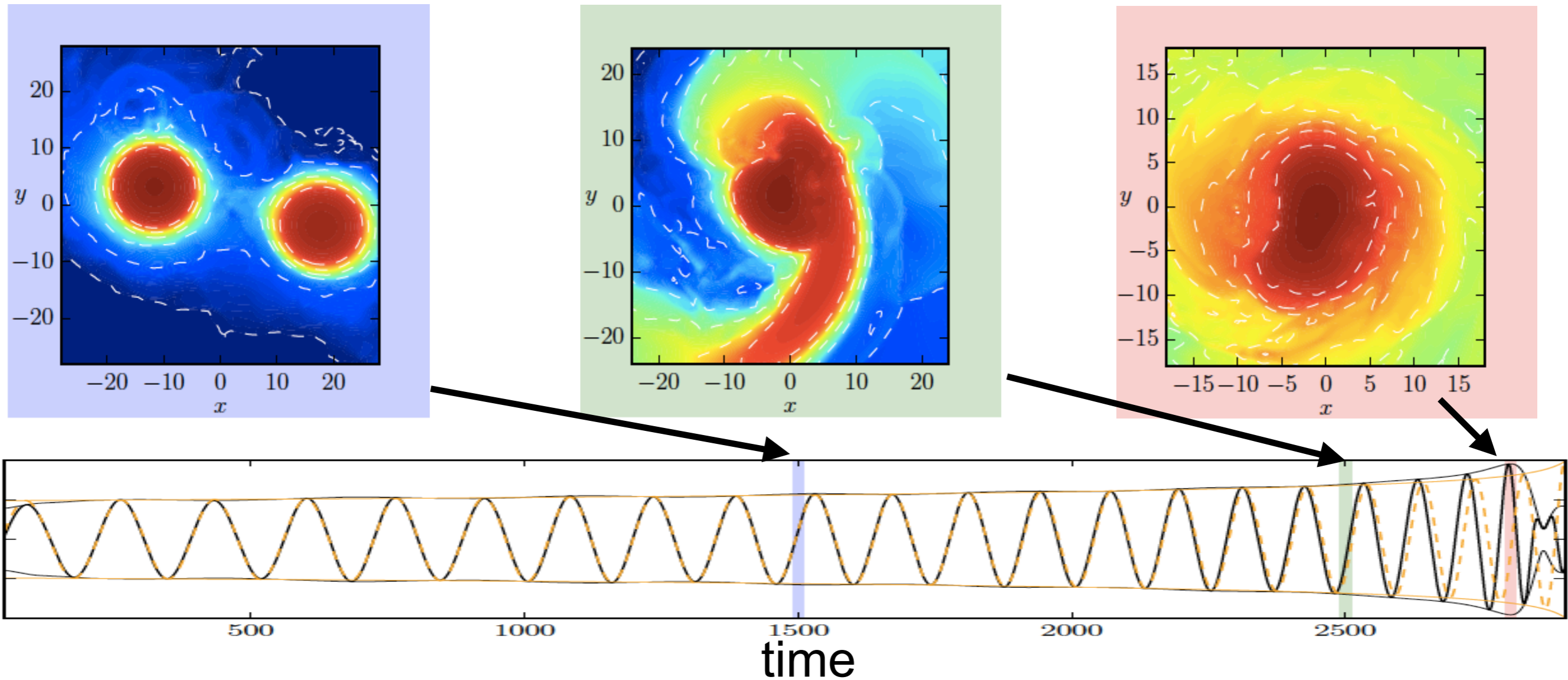
(Kochanek 92, Bildsten & Cutler 92, Lai et al. 93, Lai 94)



(Agathos et al. 15, Del Pozzo et al. 13 (see also Lackey et al. 14; Lynch et al. 14; Yagi et al. 15))

Waveform modeling for NS-NS binaries up to merger

mass ratio = 1.5, EOS = MS1b



(Dietrich, Hinderer et al. in prep)

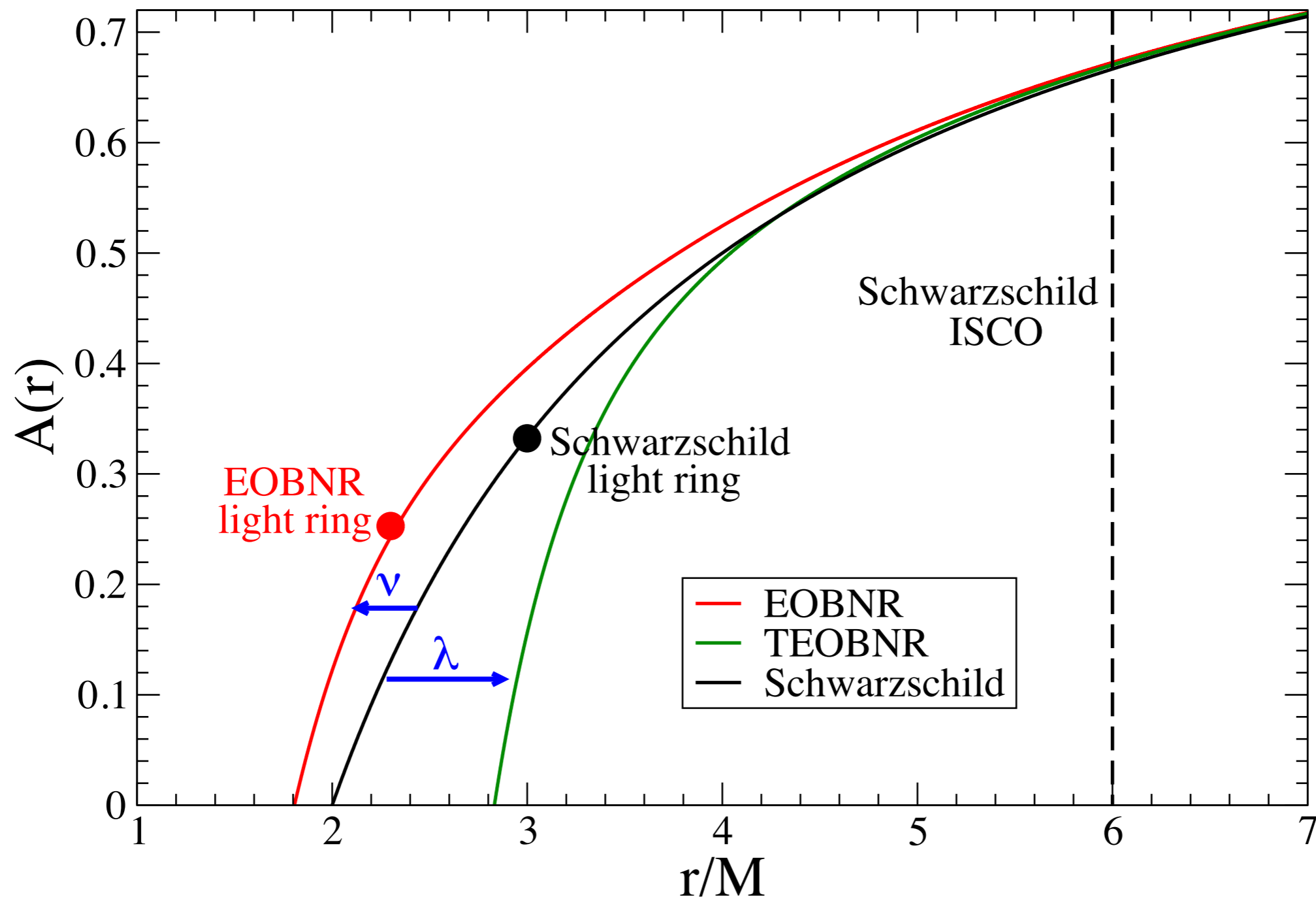
tidal EOB waveform model versus NR

$$A(r) = A_{\nu}(r) + A_{\text{tides}}(r)$$

(Damour & Nagar 12; Bernuzzi et al. 15; Hinderer et al. 16, Steinhoff et al. 16)

Strong-field effects in presence of matter

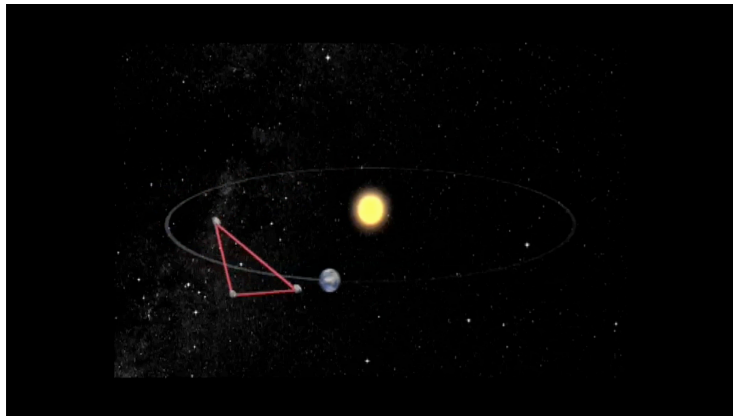
$$A(r) = A_\nu(r) + A_{\text{tides}}(r)$$



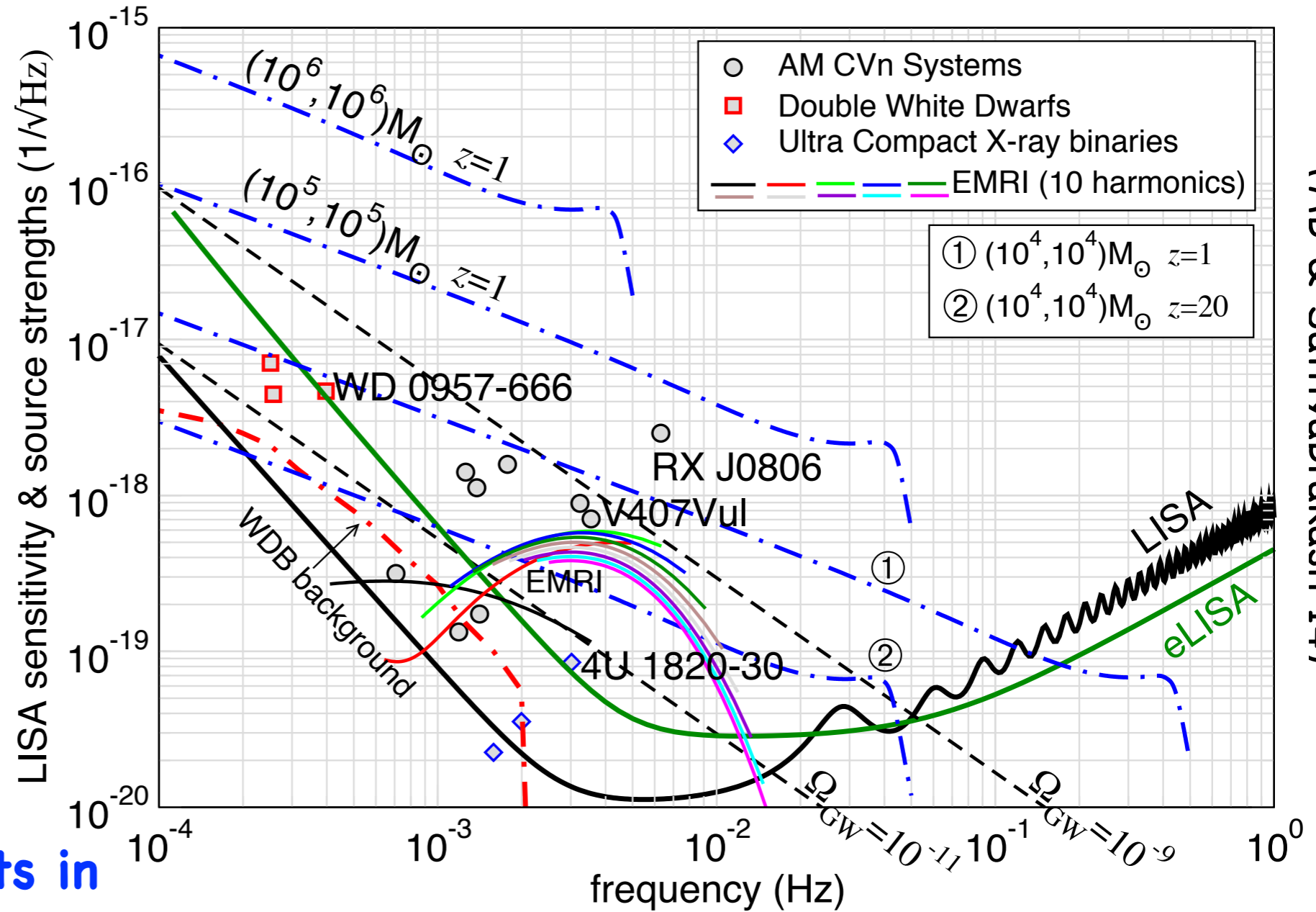
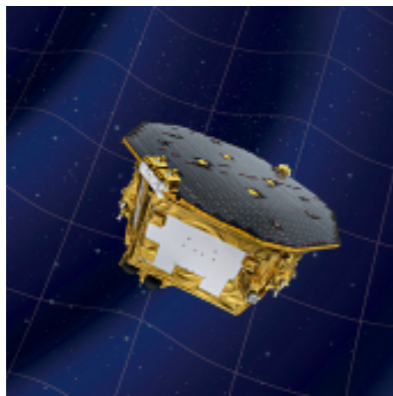
(Bernuzzi et al. PRL 114 (2015) 161103,
Hinderer et al. PRL 116 (2016) 181101)

Tides make **gravitational** interaction **more attractive**

The future of GW astronomy lies also in space: LISA (2034)



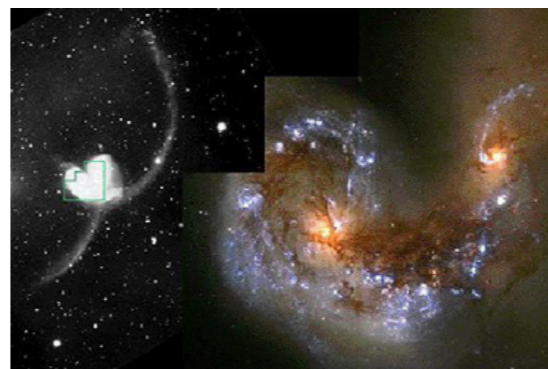
Credit: AEI/Milde Marketing



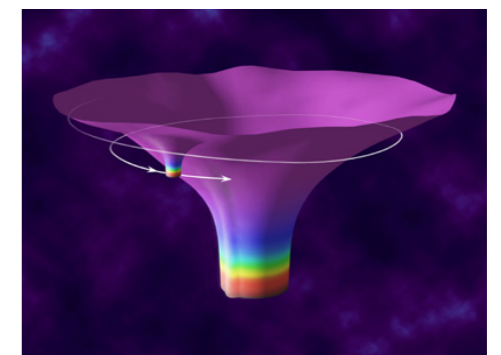
(AB & Sathyaprakash 14)

- **LISA Pathfinder results in 2016: extremely successful technology mission. LISA works!**

(Armano et al. PRL 116 (2016) 231101)



SBBH



EMRI

Motivations for improving theoretical techniques in gravity

- We can now **probe** the **most extreme astrophysical objects** in the universe, and learn **how they formed**.
- We can now **learn about gravity** in the genuinely **highly dynamical, strong field** regime.
- We can now **unveil properties of neutron stars** inaccessible in other ways.
- We can now provide the **most convincing evidence** that **black holes in our Universe** are the objects **predicted** by GR.
- **Unique science** done so far and even more exciting science in next years and decades **if able to make precise analytical predictions**.
- **“Simplicity” of two-body problem**, even more **to be discovered**.

