Position-dependent Power Spectrum

~Attacking an old, but unsolved, problem with a new method~

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Motivation

• To gain a better insight into “mode coupling”
  • An interaction between short-wavelength modes and long-wavelength modes
  • Specifically, how do short wavelength modes respond to a long wavelength mode?
• We use the distribution of matter in the Universe as an example, but I would like to learn if a similar [or better!] technique has been used in other areas in physics
Two Approaches

• Global
  • “Bird’s view”: see both long- and short-wavelength modes, and compute coupling between the two directly

• Local
  • “Ant’s view”: Absorb a long-wavelength mode into a new background solution that a local observer sees, and compute short wavelength modes in the new background.
This presentation is based on

- **Chiang** et al. “Position-dependent power spectrum of the large-scale structure: a novel method to measure the squeezed-limit bispectrum”, JCAP 05, 048 (2014)

- **Chiang** et al. “Position-dependent correlation function from the SDSS-III BOSS DR10 CMASS Sample”, JCAP 09, 028 (2015)


Preparation I: Comoving Coordinates

• Space expands. Thus, a physical length scale increases over time

• Since the Universe is homogeneous and isotropic on large scales, the stretching of space is given by a time-dependent function, $a(t)$, which is called the “scale factor”

• Then, the physical length, $r(t)$, can be written as

  • $r(t) = a(t) \times$

  • $\times$ is independent of time, and called the “comoving coordinates”
Preparation II: Comoving Waveumbers

- Then, the physical length, \( r(t) \), can be written as
  - \( r(t) = a(t) \, x \)

- \( x \) is independent of time, and called the “comoving coordinates”

- When we do the Fourier analysis, **the wavenumber, \( k \), is defined with respect to \( x \)**. This “comoving wavenumber” is related to the physical wavenumber by \( k_{\text{physical}}(t) = k_{\text{comoving}}/a(t) \)
Preparation III: Power Spectrum

• Take these density fluctuations, and compute the density contrast:

\[ \delta(x) = \frac{[\rho(x) - \rho_{\text{mean}}]}{\rho_{\text{mean}}} \]

• Fourier-transform this, square the amplitudes, and take averages. The power spectrum is thus:

\[ P(k) = \langle |\delta_k|^2 \rangle \]
BOSS Collaboration,

arXiv:1203.6594

$\chi^2 = 81.5 / 59$

$z = 0.57$
A simple question within the context of cosmology

- How do the cosmic structures evolve in an overdense region?
Simple Statistics

- Divide the survey volume into many sub-volumes $V_L$, and compare locally-measured power spectra with the corresponding local over-densities.
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Simple Statistics

\[ \bar{\delta}(\mathbf{r}_L) = \frac{1}{V_L} \int_{V_L} d^3r \, \delta(\mathbf{r}) \]

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Divide the survey volume into many sub-volumes $V_L$, and compare locally-measured power spectra with the corresponding local over-densities $\bar{\delta}(r_L)$.
• A clear correlation between the local over-densities and the local power spectra
Integrated Bispectrum, $iB(k)$

- Correlating the local over-densities and power spectra, we obtain the “integrated bispectrum”:

$$i \hat{B}_L(k) = \frac{1}{N_{\text{cut}}^3} \sum_{i=1}^{N_{\text{cut}}} \hat{P}(k, \mathbf{r}_{L,i}) \hat{\delta}(\mathbf{r}_{L,i})$$

- This is a (particular configuration of) three-point function. The three-point function in Fourier space is called the “bispectrum”, and is defined as

$$\langle \delta(\mathbf{q}_1)\delta(\mathbf{q}_2)\delta(\mathbf{q}_3) \rangle = B(\mathbf{q}_1, \mathbf{q}_2, \mathbf{q}_3)(2\pi)^3 \delta_D(\mathbf{q}_1 + \mathbf{q}_2 + \mathbf{q}_3)$$
Shapes of the Bispectrum

(a) squeezed triangle 
\[ k_1 \approx k_2 >> k_3 \]

(b) elongated triangle 
\[ k_1 = k_2 + k_3 \]

(c) folded triangle 
\[ k_1 = 2k_2 = 2k_3 \]

(d) isosceles triangle 
\[ k_1 > k_2 = k_3 \]

(e) equilateral triangle 
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- The expectation value of this quantity is an integral of the bispectrum that picks up the contributions mostly from the squeezed limit:

$$iB_L(k) = \langle \hat{P}(k, r_L) \delta(r_L) \rangle$$

$$= \frac{1}{V_L^2} \int \frac{d^2 k}{4\pi} \int \frac{d^3 q_1}{(2\pi)^3} \int \frac{d^3 q_3}{(2\pi)^3} B(k - q_1, -k + q_1 + q_3, -q_3) \times W_L(q_1) W_L(-q_1 - q_3) W_L(q_3)$$

"taking the squeezed limit and then angular averaging"
Power Spectrum Response

• The integrated bispectrum measures how the local power spectrum responds to its environment, i.e., a long-wavelength density fluctuation.
Response Function

- So, let us Taylor-expand the local power spectrum in terms of the long-wavelength density fluctuation:

\[ \hat{P}(k, r_L) = P(k)\big|_{\bar{\delta}=0} + \frac{dP(k)}{d\bar{\delta}}\big|_{\bar{\delta}=0} \bar{\delta} + \ldots \]

- The integrated bispectrum is then given as

\[ iB_L(k) = \sigma^2_L \frac{d \ln P(k)}{d\bar{\delta}}\big|_{\bar{\delta}=0} P(k) \]

response function
Response Function: N-body Results

- Almost a constant, but a weak scale dependence, and clear oscillating features. How do we understand this?
Non-linearity generates a bispectrum

- If the initial conditions were Gaussian, linear perturbations remain Gaussian.
- However, non-linear gravitational evolution makes density fluctuations at late times non-Gaussian, generating a non-vanishing bispectrum.

\[
\delta' + \nabla \cdot [(1 + \delta) \mathbf{v}] = 0 ,
\]
\[
\mathbf{v}' + (\mathbf{v} \cdot \nabla) \mathbf{v} = -\mathcal{H} \mathbf{v} - \nabla \phi ,
\]
\[
\nabla^2 \phi = 4\pi G a^2 \bar{\rho} \delta ,
\]

\[H = a'/a\]
1. Global, “Bird’s View”

overdensity

underdensity
Illustrative Example: SPT

• Second-order perturbation gives the lowest-order bispectrum as

\[ B_{\text{SPT}}(k_1, k_2, k_3) = 2[P_l(k_1) P_l(k_2) F_2(k_1, k_2) + 2 \text{ cyclic}] \]

“l” stands for “linear”

\[ F_2(k_1, k_2) = \frac{5}{7} + \frac{1}{2} \frac{k_1 \cdot k_2}{k_1 k_2} \left( \frac{k_1}{k_2} + \frac{k_2}{k_1} \right) + \frac{2}{7} \left( \frac{k_1 \cdot k_2}{k_1 k_2} \right)^2 \]

• Then

\[ iB_L(k) = \frac{1}{V_L^2} \int \frac{d^2 \hat{k}}{4\pi} \int \frac{d^3 q_1}{(2\pi)^3} \int \frac{d^3 q_3}{(2\pi)^3} B(k - q_1, -k + q_1 + q_3, -q_3) \times W_L(q_1) W_L(-q_1 - q_3) W_L(q_3) \]
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\]

- Then

\[
iB_{L,\text{SPT}}(k) \overset{k_L \to \infty}{\longrightarrow} \left[\frac{68}{21} - \frac{1}{3} \frac{d \ln k^3 P_l(k)}{d \ln k}\right] P_l(k) \sigma_L^2
\]
Illustrative Example: SPT

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- Then

\[ iB_{L, \text{SPT}}(k) \xrightarrow{kL \to \infty} \left[ \frac{68}{21} - \frac{1}{3} \frac{d \ln k^3 P_l(k)}{d \ln k} \right] P_l(k) \sigma_L^2 \]

Response, \( \frac{d \ln P(k)}{d \delta} \)
Illustrative Example: SPT

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- Then

\[ iB_{L,\text{SPT}}(k) \xrightarrow{kL\to\infty} \left[ \frac{68}{21} - \frac{1}{3} \frac{d \ln k^3 P_l(k)}{d \ln k} \right] P_l(k) \sigma^2_L \]

Oscillation in P(k) is enhanced
Lowest-order prediction

Less non-linear

More non-linear

$k$ [h Mpc$^{-1}$]

$z=3$

$z=2$

$z=1$

$z=0$

SPT

squeezed-limit

$F_{2}^{\text{eff}}$ and $P_{nl}$
1. Local, “Ant’s View”

overdensity

underdensity
Separate Universe Approach

• The meaning of the position-dependent power spectrum becomes more transparent within the context of the “separate universe approach”

• Each sub-volume with un over-density (or under-density) behaves as if it were a separate universe with different cosmological parameters

• In particular, if the global metric is a flat universe, then each sub-volume can be regarded as a different universe with non-zero curvature

Lemaitre (1933); Peebles (1980)
Mapping between two cosmologies

• The goal here is to compute the power spectrum in the presence of a long-wavelength perturbation $\delta$. We write this as $P(k, a|\delta)$

• We try to achieve this by computing the power spectrum in a modified cosmology with non-zero curvature. Let us put the tildes for quantities evaluated in a modified cosmology

$$\tilde{P}(\tilde{k}, \tilde{a}) \rightarrow P(k, a|\bar{\delta})$$
Separate Universe Approach: The Rules

• We evaluate the power spectrum in both cosmologies at the **same physical time** and **same physical spatial coordinates**

• Thus, the evolution of the scale factor is different:

\[
\tilde{a}(t) = a(t) \left[ 1 - \frac{1}{3} \delta(t) \right]
\]

*tilde: separate universe cosmology
Separate Universe Approach: The Rules

• We evaluate the power spectrum in both cosmologies at the same physical time and same physical spatial coordinates

• Thus, comoving coordinates are different too:

\[
\hat{x} = \frac{a(t)}{\hat{a}(t)} x = \left[ 1 + \frac{1}{3} \delta(t) \right] x
\]

*tilde: separate universe cosmology
Effect 1: Dilation

- Change in the comoving coordinates gives \( \frac{d \ln (k^3 P)}{d \ln k} \)

\[
\tilde{P}(k, t) \rightarrow \left[ 1 - \frac{1}{3} \bar{\delta}(t) \right]^3 P \left( k \left[ 1 - \frac{1}{3} \bar{\delta}(t) \right], t \right)
\]

\[
= \left[ 1 - \bar{\delta}(t) \right] P(k, t) \left[ 1 - \frac{1}{3} \frac{d \ln P(k, t)}{d \ln k} \bar{\delta}(t) \right]
\]

\[
= P(k, t) \left[ 1 - \frac{1}{3} \frac{d \ln k^3 P(k, t)}{d \ln k} \bar{\delta}(t) \right].
\]
Effect 2: Reference Density

• Change in the denominator of the definition of $\delta$:

$$\tilde{P}(\tilde{k}, t) \rightarrow [1 + \bar{\delta}(t)]^2 \tilde{P}(\tilde{k}, t) = [1 + 2\bar{\delta}(t)] \tilde{P}(\tilde{k}, t)$$

• Putting both together, we find a generic formula, valid to linear order in the long-wavelength $\delta$:

$$P(k, a|\bar{\delta}) = [1 + 2\bar{\delta}(t)] \tilde{P}(k, \bar{a}) \left[1 - \frac{1}{3} \frac{d \ln k^3 P(k, t)}{d \ln k} \bar{\delta}(t)\right]$$

$$= \tilde{P}\left(k, a \left[1 - \frac{1}{3} \bar{\delta}(a)\right]\right) \left[1 + \left(2 - \frac{1}{3} \frac{d \ln k^3 P(k, a)}{d \ln k}\right) \bar{\delta}(a)\right]$$
Example: Linear $P(k)$

- Let's use the formula to compute the response of the linear power spectrum, $P_l(k)$, to the long-wavelength $\delta$. Since $P_l \sim D^2$ [D: linear growth],

$$
\tilde{P}_l \left( k, a \left[ 1 - \frac{1}{3} \bar{\delta}(a) \right] \right) = \left( \frac{\tilde{D} \left( a \left[ 1 - \frac{1}{3} \bar{\delta}(a) \right] \right)}{D(a)} \right)^2 P_l(k, a)
$$

- Spherical collapse model gives

$$
\tilde{D} \left( a \left[ 1 - \frac{1}{3} \bar{\delta}(a) \right] \right) = D(a) \left[ 1 + \frac{13}{21} \bar{\delta}(a) \right]
$$
Response of $P_l(k)$

Then we obtain:

$$\frac{d \ln P_l(k, a)}{d \delta(a)} = \frac{68}{21} - \frac{1}{3} \frac{d \ln k^3 P_l(k, a)}{d \ln k}$$

Remember the response computed from the leading-order SPT bispectrum:

$$i B_{L,SPT}(k) \xrightarrow{kL \to \infty} \left[ \frac{68}{21} - \frac{1}{3} \frac{d \ln k^3 P_l(k)}{d \ln k} \right] P_l(k) \sigma_L^2$$

So, the leading-order SPT bispectrum gives the response of the linear $P(k)$. Neat!!
Response of $P_{3\text{rd-order}}(k)$

- So, let’s do the same using third-order perturbation theory!

\[ P(k, a) = P_i(k, a) + P_{22}(k, a) + 2P_{13}(k, a) \]

\[ P_{22}(k, a) = 2 \int \frac{d^3q}{(2\pi)^3} P_i(q, a)P_i(|k - q|, a) [F_2(q, k - q)]^2 \]

\[ 2P_{13}(k, a) = \frac{2\pi k^2}{252} P_i(k, a) \int_0^{\infty} \frac{dq}{(2\pi)^3} P_i(q, a) \]

\[ \times \left[ 100 \frac{q^2}{k^2} - 158 + 12 \frac{k^2}{q^2} - 42 \frac{q^4}{k^4} + \frac{3}{k^5q^3} (q^2 - k^2)^3(2k^2 + 7q^2) \ln \left( \frac{k + q}{|k - q|} \right) \right] \]

- Then we obtain:

\[ \frac{d \ln P(k, a)}{d\delta(a)} = \frac{68}{21} - \frac{1}{3} \frac{d \ln k^3P(k, a)}{d \ln k} + \frac{26}{21} \frac{P_{22}(k, a) + 2P_{13}(k, a)}{P(k, a)} \]
3rd-order does a decent job

Less non-linear

More non-linear
This is a powerful formula

\[ P(k, a|\bar{\delta}) = \tilde{P} \left( k, a \left[ 1 - \frac{1}{3} \bar{\delta}(a) \right] \right) \left[ 1 + \left( 2 - \frac{1}{3} \frac{d \ln k^3 P(k, a)}{d \ln k} \right) \bar{\delta}(a) \right] \]

• The separate universe description is powerful, as it provides physically intuitive, transparent, and straightforward way to compute the effect of a long-wavelength perturbation on the small-scale structure growth

• The small-scale structure can be arbitrarily non-linear!
Do the data show this?
SDSS-III/BOSS DR11

• OK, now, let’s look at the real data (BOSS DR10) to see if we can detect the expected influence of environments on the small-scale structure growth

• Bottom line: we have detected the integrated bispectrum at 7.4σ. Not bad for the first detection!
L = 220 Mpc/h
L = 120 Mpc/h
Results: $\chi^2$/DOF = 46.4/38

- Because of complex geometry of DR10 footprint, we use the local correlation function, instead of the power spectrum.

- Integrated three-point function, $i\zeta(r)$, is just Fourier transform of $iB(k)$:

$$i\zeta_L(r) = \int \frac{d^3k}{(2\pi)^3} iB_L(k)e^{i\mathbf{r} \cdot \mathbf{k}}$$
Results: $\chi^2/\text{DOF} = 46.4/38$

- $L = 120 \text{ Mpc/h}$
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$$i \zeta_L(r) = \int \frac{d^3 k}{(2\pi)^3} \ i B_L(k) e^{i r \cdot k}$$

7.4σ measurement of the squeezed-limit bispectrum!!
Nice, but what is this good for?

- Primordial non-Gaussianity from the early Universe
  - The constraint from BOSS is work in progress, but we find that the integrated bispectrum is a nearly optimal estimator for the squeezed-limit bispectrum from inflation
  - We no longer need to measure the full bispectrum, if we are just interested in the squeezed limit
Nice, but what is this good for?

- We can also learn about galaxy bias
  - Local bias model:
    - \( \delta_g(x) = b_1 \delta_m(x) + \frac{b_2}{2} [\delta_m(x)]^2 + \ldots \)
  - The bispectrum can give us \( b_2 \) at the leading order, unlike for the power spectrum that has \( b_2 \) at the next-to-leading order
Result on $b_2$

- We use the leading-order SPT bispectrum with the local bias model to interpret our measurements.
  
- [We also use information from BOSS’s 2-point correlation function on $f\sigma_8$ and BOSS’s weak lensing data on $\sigma_8$]

- We find: $b_2 = 0.41 \pm 0.41$
Simulating Ant’s Views
This is a powerful formula

\[ P(k, a|\bar{\delta}) = \tilde{P} \left( k, a \left[ 1 - \frac{1}{3} \bar{\delta}(a) \right] \right) \left[ 1 + \left( 2 - \frac{1}{3} \frac{d \ln k^3 P(k, a)}{d \ln k} \right) \bar{\delta}(a) \right] \]

- How can we compute \( \tilde{P}(k, a) \) in practice?
  - **Small N-body simulations with a modified cosmology ("Separate Universe Simulation")**
  - Perturbation theory
Separate Universe Simulation

- How do we compute the response function beyond perturbation theory?

- Do we have to run many big-volume simulations and divide them into sub-volumes? No.

- Fully non-linear computation of the response function is possible with separate universe simulations

- E.g., we run two small-volume simulations with separate-universe cosmologies of over- and under-dense regions with the same initial random number seeds, and compute the derivative $d\ln P/d\delta$ by, e.g.,

$$
\frac{d \ln P(k)}{d\delta} = \frac{\ln P(k| + \delta) - \ln P(k| - \delta)}{2\delta}
$$
Separate Universe Cosmology

\[ \rho(t) [1 + \delta_\rho(t)] = \tilde{\rho}(t) \]

\[ \frac{\Omega_m h^2}{a^3(t)} [1 + \delta_\rho(t)] = \frac{\tilde{\Omega}_m \tilde{h}^2}{\tilde{a}^3(t)} \]

\[ \frac{\tilde{K}}{H_0^2} = \frac{5}{3} \frac{\Omega_m}{a(t_i)} \delta_\rho(t_i) \]

\[ \delta_H = \left( 1 - \frac{\tilde{K}}{H_0^2} \right)^{1/2} - 1 \]

\[ \tilde{H}_0 = H_0[1 + \delta_H] \]

\[ \tilde{\Omega}_m = \Omega_m[1 + \delta_H]^{-2} \]

\[ \tilde{\Omega}_\Lambda = \Omega_\Lambda[1 + \delta_H]^{-2} \]
\[ \tilde{D}(t) = D(t) \left[ 1 + \frac{13}{21} \delta_L(t) + \frac{71}{189} \delta_L^2(t) + \frac{29609}{130977} \delta_L^3(t) ight. \\
+ \left. \frac{691858}{5108103} \delta_L^4(t) + \mathcal{O}(\delta_L^5(t)) \right]. \]
\[ R_1 = d \ln P / d \delta \]

- The symbols are the data points with error bars. You cannot see the error bars!
\[ R_2 = d^2 \ln P / d\delta^2 \]

- More derivatives can be computed by using simulations run with more values of \( \delta \)
$$R_3 = \frac{d^3 \ln P}{d \delta^3}$$

- But, what do $d^n \ln P / d \delta^n$ mean physically??
More derivatives: Squeezed limits of N-point functions

- $R_1$: 3-point function
- $R_2$: 4-point function
- $R_3$: 5-point function
- $R_N$: N–2-point function

Why do we want to know this? I don’t know, but it is cool and they have not been measured before!
One more cool thing

• We can use the separate universe simulations to test **validity of SPT to all orders in perturbations**

• The fundamental prediction of SPT: the non-linear power spectrum at a given time is given by the linear power spectra at the same time

• In other words, the only time dependence arises from the linear growth factors, $D(t)$
One more cool thing

- We can use the separate universe simulations to test **validity of SPT to all orders in perturbations**

\[
\begin{align*}
\delta' + \nabla \cdot [(1 + \delta)v] &= 0, \\
v' + (v \cdot \nabla)v &= -\mathcal{H}v - \nabla \phi, \\
\nabla^2 \phi &= 4\pi G \alpha^2 \bar{\rho} \delta,
\end{align*}
\]

**SPT at all orders: Exact solution of the pressureless fluid equations**

We can test validity of SPT as a description of collisions particles
Example: $P_{3\text{rd-order}}(k)$

- SPT to 3rd order

$$P(k, a) = P_l(k, a) + P_{22}(k, a) + 2P_{13}(k, a)$$

$$P_{22}(k, a) = 2 \int \frac{d^3q}{(2\pi)^3} P_l(q, a) P_l(|k - q|, a) [F_2(q, k - q)]^2$$

$$2P_{13}(k, a) = \frac{2\pi k^2}{252} P_l(k, a) \int_0^\infty \frac{dq}{(2\pi)^3} P_l(q, a)$$

$$\times \left[ 100 \frac{q^2}{k^2} - 158 + 12 \frac{k^2}{q^2} - 42 \frac{q^4}{k^4} + \frac{3}{k^5 q^3} (q^2 - k^2)^3 (2k^2 + 7q^2) \ln \left( \frac{k + q}{|k - q|} \right) \right]$$

- The only time-dependence is in $P_l(k, a) \sim D^2(a)$

- Is this correct?
Rescaled simulations vs Separate universe simulations

• To test this, we run two sets of simulations.

• **First**: we rescale the initial amplitude of the power spectrum, so that we have a given value of the linear power spectrum amplitude at some later time, $t_{out}$

• **Second**: full separate universe simulation, which changes all the cosmological parameters consistently, given a value of $\delta$

• We choose $\delta$ so that it yields the same amplitude of the linear power spectrum as the first one at $t_{out}$
Results: 3-point function

- To isolate the effect of the growth rate, we have removed the dilation and reference-density effects from the response functions.
Results: 4-point function

- To isolate the effect of the growth rate, we have removed the dilation and reference-density effects from the response functions.
Results: 5-point function

- To isolate the effect of the growth rate, we have removed the dilation and reference-density effects from the response functions.
Break down of SPT at all orders

- At $z=0$, SPT computed to all orders breaks down at $k \sim 0.5 \text{ Mpc/h}$ with 10% error, in the squeezed limit 3-point function
- Break down occurs at lower $k$ for the squeezed limits of the 4- and 5-point functions
- Break down occurs at higher $k$ at $z=2$
- I find this information quite useful: it quantifies accuracy of the perfect-fluid approximation of density fields
Summary

• **New observable**: the position-dependent power spectrum and the integrated bispectrum

• Straightforward interpretation in terms of the separate universe

• Easy to measure; easy to model!

• Useful for primordial non-Gaussianity and non-linear bias

• Lots of applications: e.g., QSO density correlated with Lyman-alpha power spectrum

• All of the results and much more are summarised in Chi-Ting Chiang’s PhD thesis: [arXiv:1508.03256](https://arxiv.org/abs/1508.03256)
More on $b_2$

- Using slightly more advanced models, we find:

<table>
<thead>
<tr>
<th></th>
<th>baseline</th>
<th>eff kernel</th>
<th>tidal bias</th>
<th>both*</th>
</tr>
</thead>
<tbody>
<tr>
<td>$b_2$</td>
<td>0.41 ± 0.41</td>
<td>0.51 ± 0.41</td>
<td>0.48 ± 0.41</td>
<td>0.60 ± 0.41</td>
</tr>
</tbody>
</table>

*The last value is in agreement with $b_2$ found by the Barcelona group (Gil-Marín et al. 2014) that used the full bispectrum analysis and the same model*
<table>
<thead>
<tr>
<th></th>
<th>$b_1$</th>
<th>$b_2$</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>r-space</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>baseline</td>
<td>1.971 ± 0.076</td>
<td>0.58 ± 0.31</td>
</tr>
<tr>
<td>eff kernel</td>
<td>1.973 ± 0.076</td>
<td>0.62 ± 0.31</td>
</tr>
<tr>
<td>tidal bias</td>
<td>1.971 ± 0.076</td>
<td>0.64 ± 0.31</td>
</tr>
<tr>
<td>both</td>
<td>1.973 ± 0.076</td>
<td>0.68 ± 0.31</td>
</tr>
<tr>
<td><strong>z-space</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>baseline</td>
<td>1.931 ± 0.077</td>
<td>0.54 ± 0.35</td>
</tr>
<tr>
<td>eff kernel</td>
<td>1.933 ± 0.077</td>
<td>0.65 ± 0.35</td>
</tr>
<tr>
<td>tidal bias</td>
<td>1.932 ± 0.077</td>
<td>0.60 ± 0.35</td>
</tr>
<tr>
<td>both</td>
<td>1.933 ± 0.077</td>
<td>0.71 ± 0.35</td>
</tr>
</tbody>
</table>