The Physics of Active Matter

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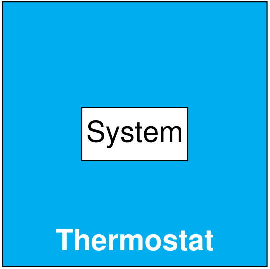
New Directions in Theoretical Physics II

Equilibrium Statistical Mechanics

- Large thermostat with chaotic dynamics
- Exchange energy with the system
- Drives the system towards thermal equilibrium



⟶ Boltzmann distribution $P_{\rm stat}(\mathcal{C}) \propto \exp[-\beta E(\mathcal{C})]$

⟶ Standard results of Thermodynamic hold

⟶ Time-reversal symmetry in steady-state

Non-equilib. phys. is like non-elephant biology

Some common definitions

- no steady state
- no Boltzmann weight
- no time-reversal symmetry

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Some examples

Glasses

Convection rolls

Biological systems

Non-equilib. phys. is like non-elephant biology

Some common definitions

- no steady state
- no Boltzmann weight
- no time-reversal symmetry

Some examples

Glasses

Convection rolls

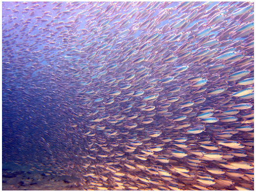
Biological systems

→ Identify interesting & coherent subclasses

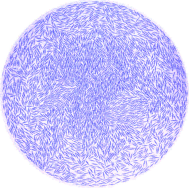
→ Say something smart & useful about them!

Active Matter

"Soft active systems are exciting examples of a new type of condensed matter where stored energy is continuously transformed into mechanical work at microscopic length scales." [Marchetti & Liverpool, PRL 97, 268101 (2006)]



Fish shoals



Vibrated rods

Birds flocks

- Rich phenomenology
- Simple models
- Experimental realisations

Active Matter

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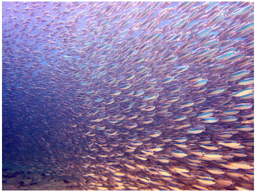

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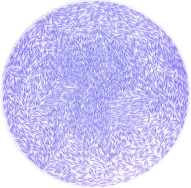


Vibrated rods

Birds flocks

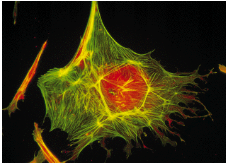
- Rich phenomenology
- Simple models → Generic description?
- Experimental realisations

Outline

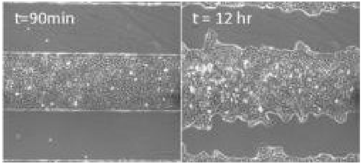
1. How thermodynamics fails: the pressure of active fluids
   [A Baskaran (Brandeis), M Cates (Cambridge), Y Fily (Brandeis), Y Kafri (Technion), M Kardar (MIT), A Solon (MIT)]

2. The emergence of collective motion
   [H Chate (CEA Saclay), A Solon (MIT)]

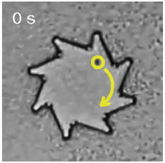
Active forces and mechanical pressure



Actin cortex

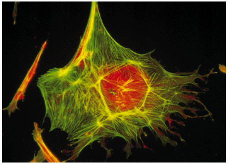


Wound healing



Rotating gear

Active forces and mechanical pressure



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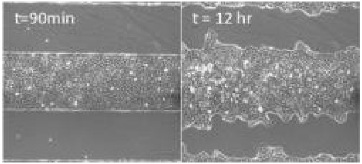

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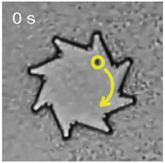
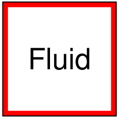


Rotating gear

- Much simpler: pressure of active fluid



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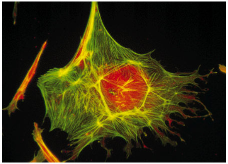

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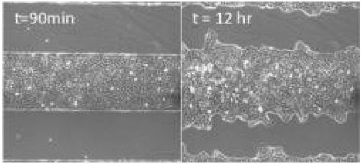

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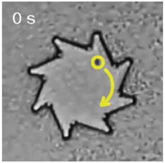

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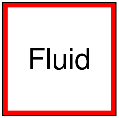

- Mechanics $P_M = \frac{F_{\rm wall}}{S}$
- Hydrodynamics $P_H = -\frac{{\rm Tr}\, \sigma}{d}$
- Statistical Mechanics $P_S = -\left.\frac{\partial \mathcal{F}}{\partial V}\right|_N$

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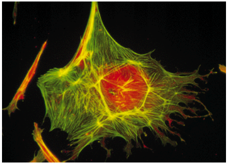

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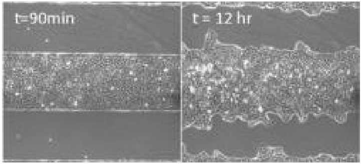

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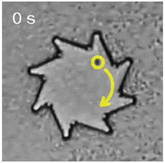


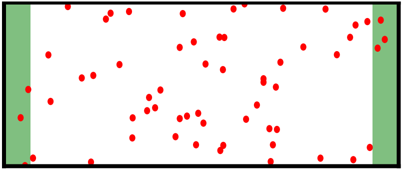
Rotating gear

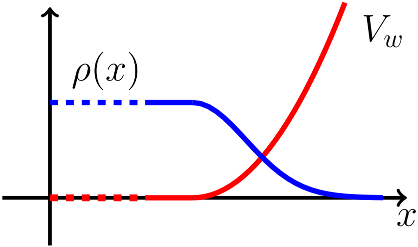
- Much simpler: pressure of active fluid

Fluid

- Mechanics $P_M = \frac{F_{\text{wall}}}{S}$
- Hydrodynamics $P_H = -\frac{\text{Tr}\,\sigma}{d}$
- Statistical Mechanics $P_S = -\left.\frac{\partial \mathcal{F}}{\partial V}\right|_N$

How to measure the mechanical pressure





- Pressure: $P = \langle \int_0^\infty \rho(x) V_w'(x) \rangle$

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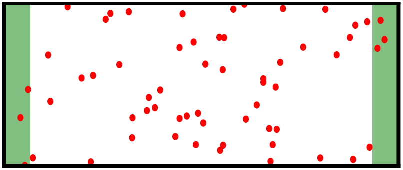

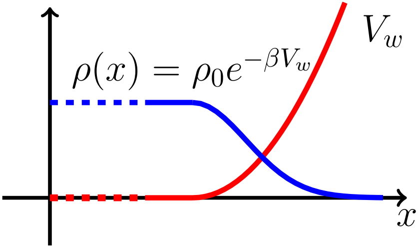

- Pressure: $P = \langle \int_0^\infty \rho(x) V_w'(x) \rangle$
- Perfect gas: $\rho(x) = e^{-V_w(x)/kT} \longrightarrow P = \rho_0 kT$

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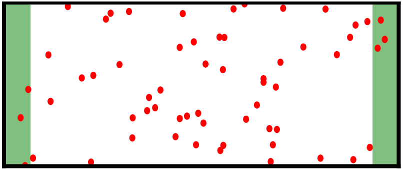
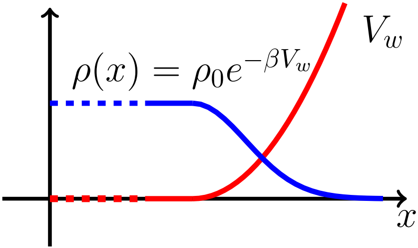
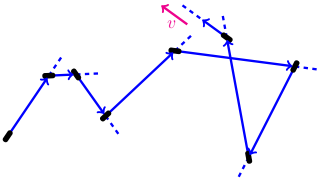


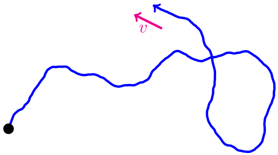


- Pressure: $P = \langle \int_0^\infty \rho(x) V_w'(x) \rangle$
- Perfect gas: $\rho(x) = e^{-V_w(x)/kT} \longrightarrow P = \rho_0 kT$
- P Independent of $V_w \longrightarrow$ Equation of state $P(\rho)$

Pressure of an active perfect gas

- Two types of active particles: tumble at rate α, rot. diff. D_r





Run and Tumble Particles • Active Brownian Particles •

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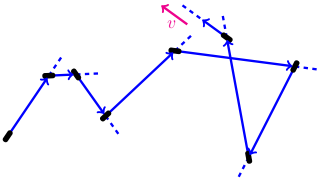

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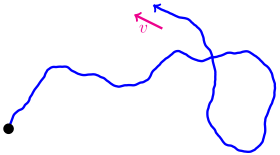


Active Brownian Particles •

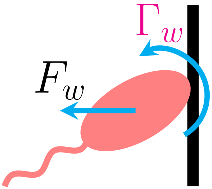
- Particles exchange momentum with a substrate

- This talk: neglect what happens to the substrate

- For bulk swimmers ⟶ Osmotic pressure

Pressure of an active perfect gas

- Master equation + V_W + Torque



Pressure of an active perfect gas

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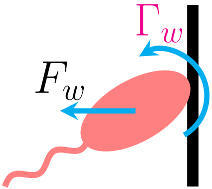

$$P = \rho_0 kT_{\text{eff}} - \frac{v}{D_r + \alpha} \int_0^\infty dx \int_0^{2\pi} d\theta \, \Gamma_w(x, \theta) \sin(\theta) \rho(x)$$

- $kT_{\text{eff}} = \frac{v^2}{2(D_r+\alpha)} + D_t$

Pressure of an active perfect gas

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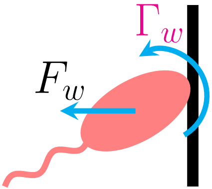

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- $kT_{\text{eff}} = \frac{v^2}{2(D_r+\alpha)} + D_t$

- Torque $\longrightarrow$ P depends on the type of walls!

- Pressure not a property of the sole fluid $\longrightarrow$ No equation of state

- Self-propelled ellipsoids of principal axes a and b
- Anisotropy: $\kappa = (a^2 - b^2)/8$
- Harmonic wall $V_w(x) = \lambda \frac{(x-x_w)^2}{2}$

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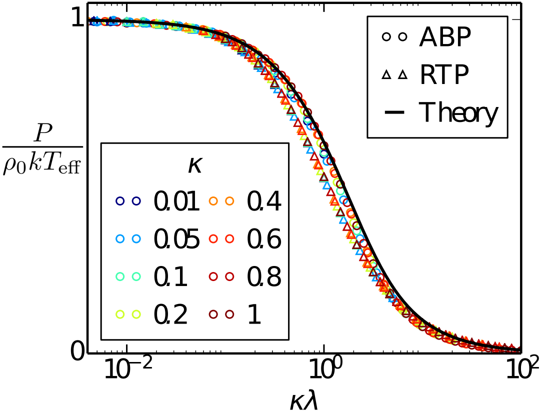
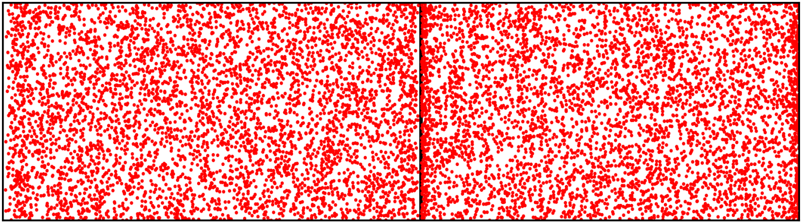
$$P_{ABP} \simeq \frac{\rho_0 v^2}{2\lambda\kappa}\left[1 - e^{-\frac{\lambda\kappa}{D_r}}\right]$$



A ‘simple’ test of the equation of state

- Place an asymmetric wall in the middle of a cavity
- Equilibrium: wall always static.

Spherical particles •



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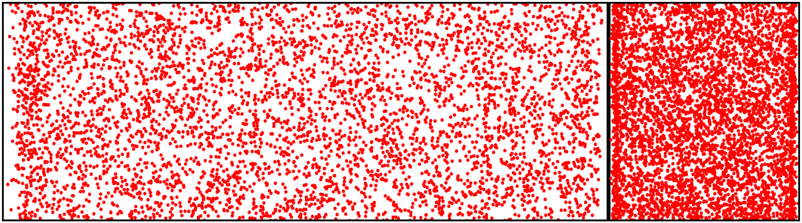
Ellipses •



- No equation of state ⟶ Spontaneous compression

Outline

1. Mechanical pressure of active fluids

   [A Solon, Y Fily, A Baskaran, M Cates, Y Kafri, M Kardar, JT, Nat Phys 11, 673 (2015)]

   - P is not a state function except in exceptional cases
   - Thermodynamics is strongly altered

2. The emergence of collective motion

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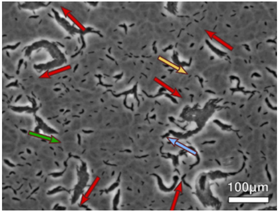
2. The emergence of collective motion

Collective motion

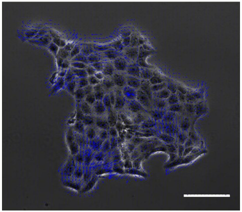
Birds

Fishes

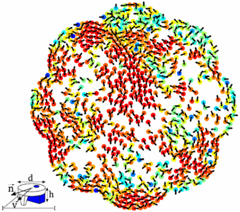
Sheeps



Myxobacteria



Cell migration



Vibrated disks

Minimal models: Self-propulsion + Alignment

The Vicsek model

[Vicsek et al. PRL 95]

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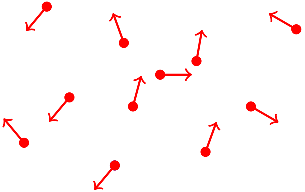

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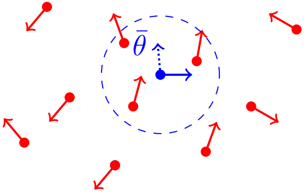

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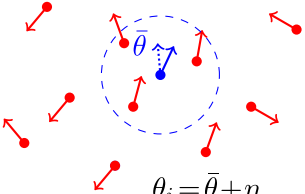

$$\theta_i = \bar{\theta} + \eta$$

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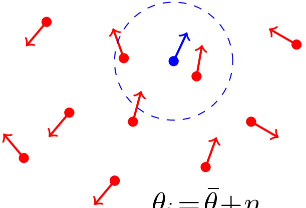

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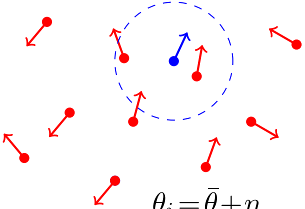

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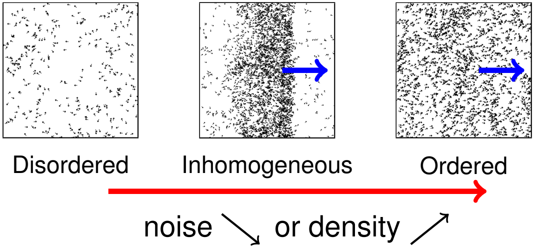
Transition to collective motion

[Grégoire, Chaté, PRL 2004]



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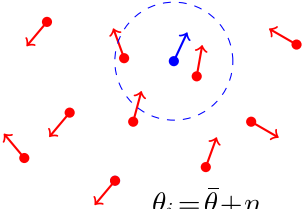

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- Order of the transition ?
- Large finite-size effects
- Complicated hydrodynamic equations

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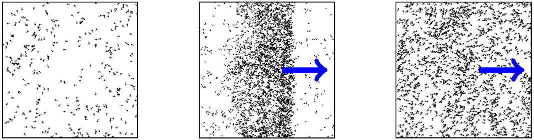

Disordered Inhomogeneous Ordered

noise ↘ or density ↗

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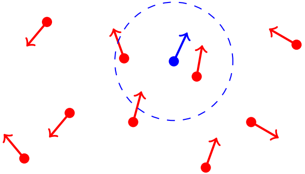

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Active XY model

⟶ Active Ising model

Transition to collective motion

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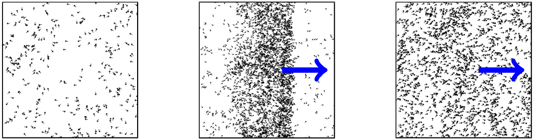

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The Active Ising model

[Solon, Tailleur PRL 2013]

- Ising model: simplest model for ferromagnetic transition
- Spin ± 1 on lattice that stochastically align with their neighbours

$$W(\oplus \to \ominus) = \exp(-\frac{1}{kT}\frac{m}{\rho})$$

Ferromagnetic alignement

The Active Ising model

[Solon, Tailleur PRL 2013]

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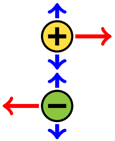
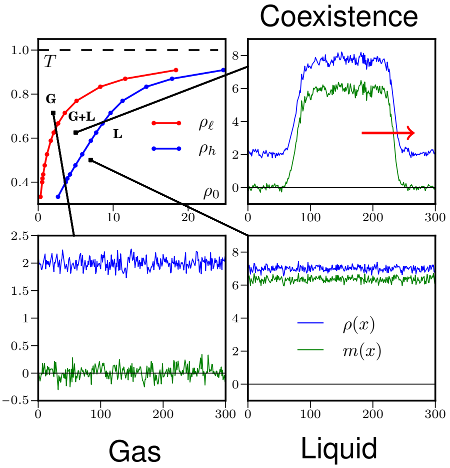


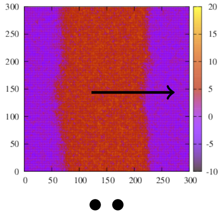
Ferromagnetic alignement

Self-propulsion along x
Diffusion along y

Simpler model, same phenomenology ?

Liquid-gas transition in the Active Ising model





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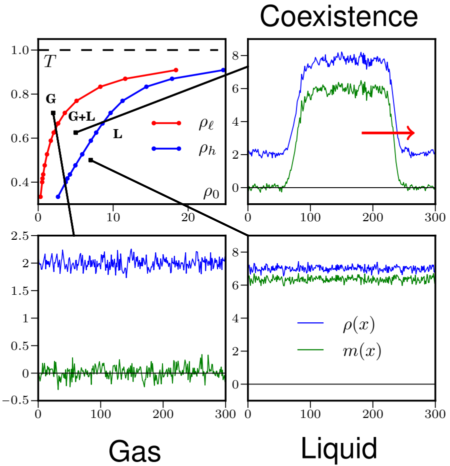
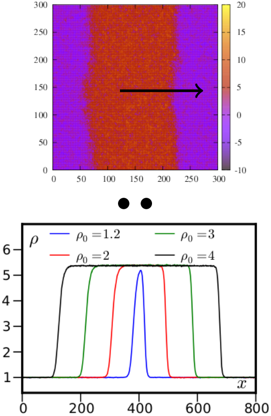
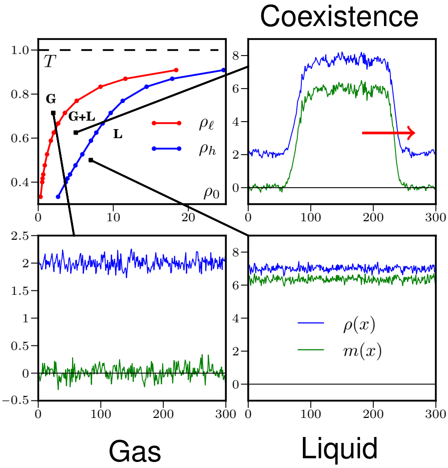
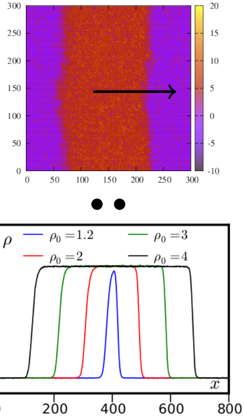




As in equilibirum: nucleation, hysteresis, lever rule

Liquid-gas transition in the Active Ising model

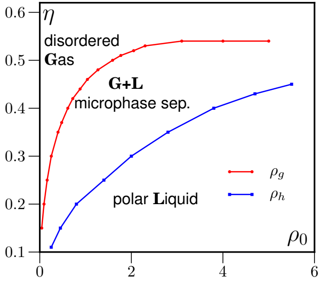




As in equilibirum: nucleation, hysteresis, lever rule

Everything can be understood analytically

Back to the Vicsek model

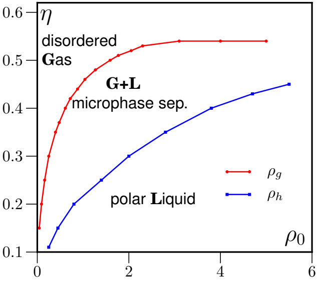


Vicsek model:

Same phase diagram

[Solon, Chaté, Tailleur PRL (2015)]

Back to the Vicsek model



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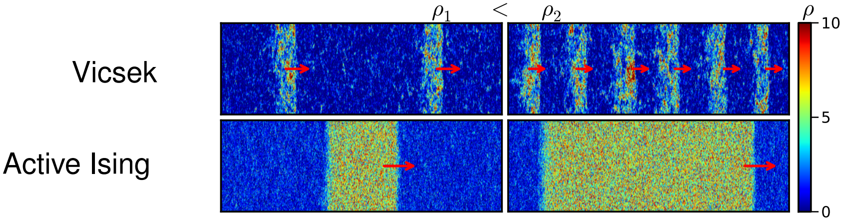

phase vs micro-phase separation

Hydrodynamic equations

$$\left.\begin{aligned}\dot{\rho} &= D\Delta\rho - v\partial_x m \\ \dot{m} &= D\Delta m - v\partial_x \rho + \beta_t(\rho) m - \alpha\frac{m^3}{\rho^2}\end{aligned}\right\} \quad \text{Scalar } m$$

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Hydrodynamic equations

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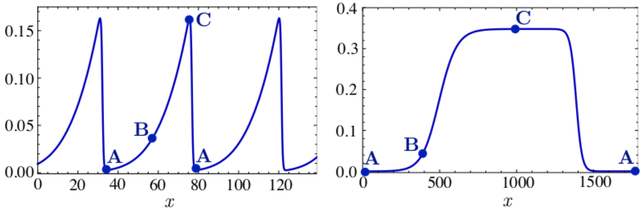
- Hydrodynamic equations have generically the same types of solutions [Caussin et al., PRL 2014]



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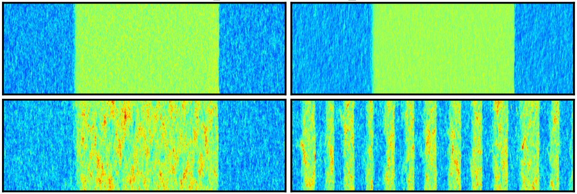
Vectorial $\mathbf{m}$

$t_1 < t_2$



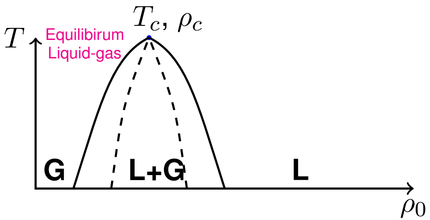
Scalar m

Vectorial $\mathbf{m}$

- Fluctuations select the good solution !

Transition to collective motion

- Liquid-gas transition
- G and L have different symmetries
  $\longrightarrow$ $\rho_c = \infty$;



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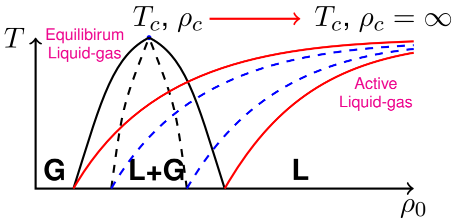

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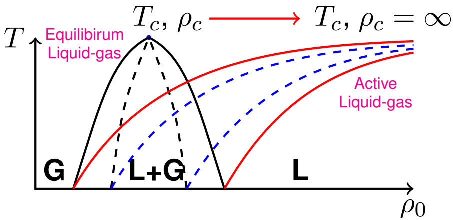

- Two universality classes:

Active Ising class	Active XY class
Discrete symmetry Phase separation.	Continuous symmetry Microphase separation.

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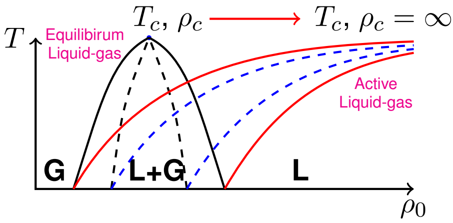

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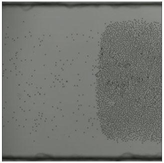
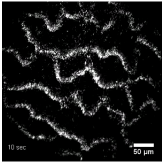
Active Ising class	Active XY class
Discrete symmetry	Continuous symmetry
Phase separation.	Microphase separation.

- Experimentally:





Conclusion

- Active matter offers a variety of interesting phenomenologies
- Needs a new thermodynamics
- New routes to collective behaviour
- Acknowledgments & References

Pressure

[A Solon, Y Fily, A Baskaran, M Cates, Y Kafri, M Kardar, JT, Nat Phys 11, 673-678 (2015)]

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Thank You!