Soft Hair on Black Holes

Malcolm Perry

University of Cambridge

January 11, 2017.

Stephen Hawking, Malcolm Perry, Andrew Strominger

"Soft Hair on Black Holes," arXiv 1601:00921
"Superrotation Charge and Supertranslation Hair on Black Holes," arXiv 1611:09175

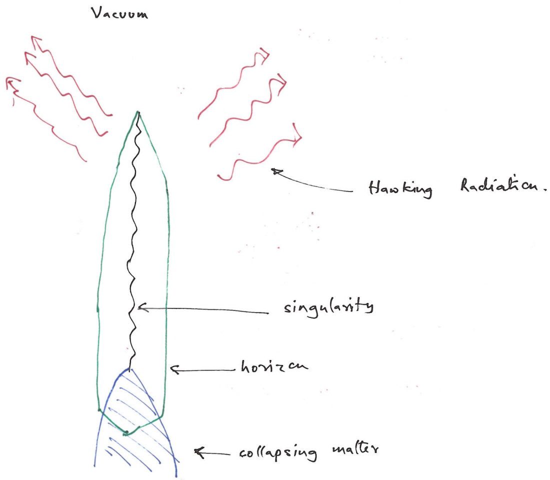
We are interested in finding a solution to the information paradox.

Minimal assumptions about black holes and quantum mechanics lead one to expect a breakdown of unitarity in gravitational physics.

No breakdown of quantum mechanics has ever been seen, and if virtual black holes exist, quantum mechanics would break down.

This would have been observed.

Loss of unitarity leads to a violation of conservative conservation laws such as conservation of energy and momentum.

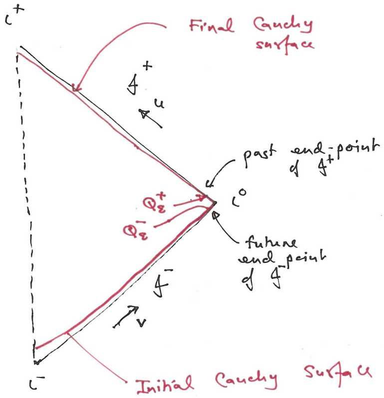


Original Hawking Picture.

The reason why the argument is compelling is the no-hair theorem.

Black holes are completely described by their mass, charge and angular momentum.

This means black holes are independent of the nature of their formation.


i^+
Final Cauchy surface
$\mathscr{I}^+$
u
past end-point of $\mathscr{I}^+$
i^0
Q_ε^+
Q_ε^-
future end-point of $\mathscr{I}^-$
$\mathscr{I}^-$
v
i^-
Initial Cauchy Surface

In electrodynamics, there are an infinity of charges Q^+_ϵ defined at the past end-point of future null infinity, $\mathcal{I}^+_-$.

$$Q^+_\epsilon = \int_{\mathcal{I}^+_-} d\Omega \, \epsilon * F.$$

where ϵ is any spherical harmonic and F is the electromagnetic 2-form field strength.

There are an infinity of charges Q^-_ϵ defined at the future end-point of past null infinity, $\mathcal{I}^-_+$.

$$Q^-_\epsilon = \int_{\mathcal{I}^-_+} d\Omega \, \epsilon * F.$$

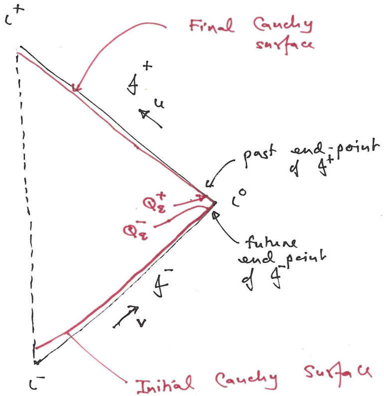
These charges are holographic in nature.

Although spacelike infinity is singular, Christodoulou and Klainerman showed that under certain conditions

$$Q_\epsilon^+ = Q_\epsilon^-$$

where the antipodal map between $\mathcal{I}^+_-$ and $\mathcal{I}^-_+$ is used in the evaluation of the integrals.

This gives an infinite number of conservation laws!

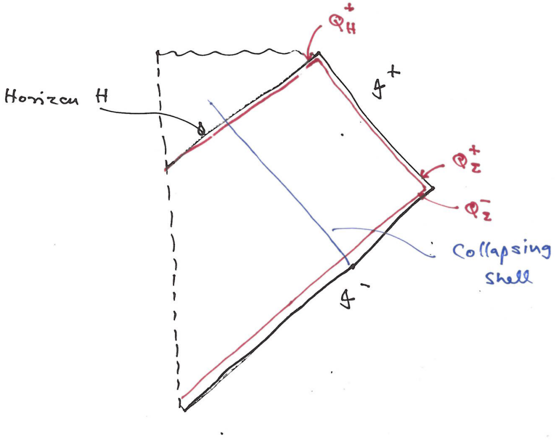


i^+
Final Cauchy surface
$\mathcal{I}^+$
u
past end-point of $\mathcal{I}^+$
i^0
Q_ε^+
Q_ε^-
future end-point of $\mathcal{I}^-$
$\mathcal{I}^-$
v
i^-
Initial Cauchy Surface

Each integral can be, by Gauss' theorem, extended into an integral over a Cauchy surface. In Minkowski space for example, assuming only massless degrees of freedom so that ι^+ can be neglected,

$$\Delta Q^+_\epsilon = \int_{\mathcal{I}^+} d\epsilon \wedge * F + \epsilon * j$$

The first term is interpreted as due to soft photons passing through $\mathcal{I}^+$. The second term is due to currents passing through $\mathcal{I}^+$.
Let $|0\rangle$ be a vacuum state. Then $|0'\rangle = Q^+_\epsilon |0\rangle$ is a different vacuum state, differing from the original by the addition of a soft photon. This is a large gauge transformation. The vacuum is thus infinitely degenerate.
Picking a particular vacuum state breaks this symmetry leaving the photon as the Nambu-Goldstone boson.



Eternal Black Hole
formed by collapsing shell

Define an horizon charge at the future end-point of the Horizon $\mathcal{H}^+$.

$$Q_\epsilon^{\mathcal{H}} = \int_{\mathcal{H}} \epsilon * F.$$

Extend this into an integral over the horizon just as we did for the integral over null infinity. The two terms represent a soft photon on the horizon and current flowing through the horizon.
The action of $Q_\epsilon^{\mathcal{H}}$ is again a large gauge transformation on the horizon.
Classically, the no-hair theorem gives us that $Q_\epsilon^{\mathcal{H}} = 0$ except for the case $\epsilon = 1$ or $l = 0$. Black holes can carry classical electric charge.

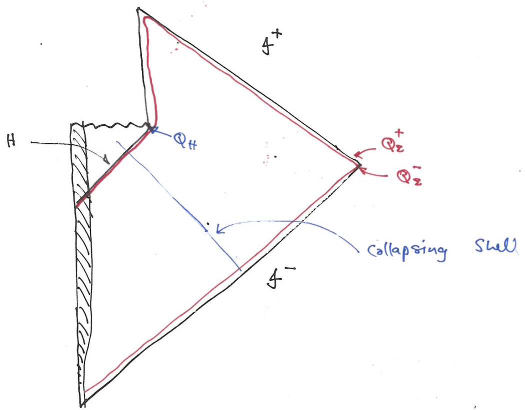
$$|BH'\rangle = Q_\epsilon^{\mathcal{H}}|BH\rangle \neq |BH\rangle.$$

Thus $Q_\epsilon^{\mathcal{H}}$ is an non-trivial operator even if its expectation value is zero.

Conservation law

$$Q_\epsilon^- = Q_\epsilon^+ + Q_\epsilon^{\mathcal{H}}.$$

This is in clear contradiction to classical ideas where there is only conservation of total electric charge.



Evaporating Black Hole

For evaporating black holes, an horizon forms and then disappears. Charges can be defined by integrals over a future Cauchy surface in the distant future on null infinity, Q_ϵ^+.
Charges can also be defined by the Cauchy surface consisting of part of the horizon, part of future null infinity, and a spacelike segment joining the two.

$$Q_\epsilon^- = Q_\epsilon^+$$

Throw a shockwave into a black hole at a constant advanced time $v = v_0$.

$$J_v = \frac{Y_{lm}(z, \bar{z})}{r^2}\delta(v - v_0), \quad l > 0.$$

Suppose initially no photons.
Then as $r \to \infty$, $F_{vr} = Y_{lm}\theta(v - v_0) + \ldots$ This gives soft photons on $\mathcal{I}^+$ with polarization vector $\partial_z Y_{lm}$ and on $\mathcal{H}$ similar soft photons.
So this way you can change the soft photon content of the horizon.

Not all modes can be excited. There is a maximum value of l. The size of a particle is given by its Compton wavelength $\frac{1}{m}$. But if m is increased, the size will shrink. Eventually at m_{Planck}, the size is l_{Planck} but this is described by a black hole. Beyond this, one gets bigger black holes.

The smallest area on the horizon to be excited has area $\sim l^2_{Planck}$. Thus the number of soft photon degrees of freedom is $\sim A$, the black hole area.

Supertranslations are gravitational analogs of the electromagnetic gauge transformations. They are coordinate transformations that preserve the structure of null infinity - asymptotic symmetries. They form an infinite dimensional Abelian group contained in the BMS group.

At null infinity, the diffeomorphism

$$f\partial_u - \frac{1}{2}D^2 f\partial_r + \frac{1}{r}D^A f\partial_A$$

is a supertranslation, where f is any spherical harmonic.

The charge and the past endpoint of future null infinity is given by

$$Q_f^+ = \frac{1}{4\pi}\int_{\mathcal{I}^+_-} f m_B$$

where m_B is the Bondi mass aspect.

Asymptotically

$$ds^2 = -(1 - \frac{2m_b}{r})du^2 - 2dudr + r^2\gamma_{AB}dz^A dz^B + \dots$$

defines the Bondi mass aspect.

There are similar flux formulae and an extension to the horizon.

$$\Delta Q_f^+ = \frac{1}{4\pi}\int_{\mathcal{I}^+} dud^2z\sqrt{\gamma}\, f(4\pi r^2 T_{uu} - \frac{1}{4}N_{AB}N^{AB} - \frac{1}{4}D_A D_B N^{AB}).$$

T_{uu} is the energy density of matter passing through null infinity. N_{AB} is the Bondi News function which describes the gravitational radiation going off to null infinity. The second term on the right hand side describes the energy carried away by gravitational radiation.

The last term describes soft gravitons.

For gravity, there is also something a bit different: superrotations. At null infinity these are given by a diffeomorphism

$$\frac{1}{2}u\psi\partial_u - (\frac{1}{2}r\psi - \frac{1}{4}uD^2\psi)\partial_r + (Y^A - \frac{u}{2r}D^A\psi)\partial_A$$

where Y^A is any conformal Killing vector on the celestial sphere, that is the sphere at fixed retarded time on null infinty and $\psi = D_A Y^A$.

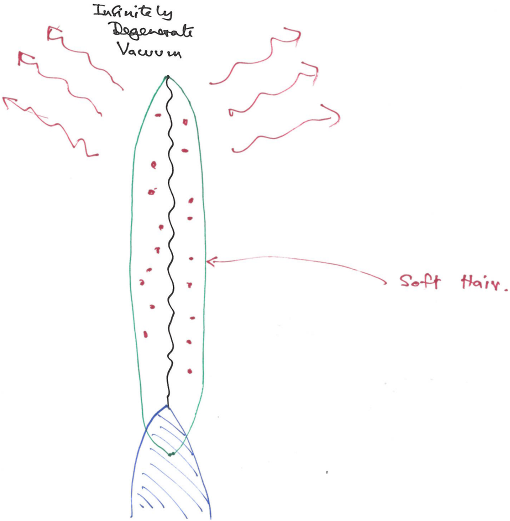
There is a charge defined in a similar way to supertranslations.

$$Q_Y = \frac{1}{8\pi}\int Y^A N_A,$$

where N_A is the angular momentum aspect. Again, there are similar flux formulae and again these can be extended to the horizon.

The BMS group is the semi-direct product of supertranslations with superrotations. The Poincaré group is a subgroup of BMS.

The generalised BMS group is BMS together with locally defined conformal Killing vectors. (Controversial).



New Picture

Issues

- Super-rotations
- Is this too much hair, the right kind of hair?
- Is this enough hair?
    - Species problem
    - Entropy Calculation
- How does all the information get recorded
    - Global Symmetries
- How does all the information get retrieved in the Hawking radiation
- Black Hole Complementarity
    - No cloning theorem
    - The edge of spacetime
- The singularity at the end of the horizon

Conclusions

Black Holes have an infinite amount of soft hair.

The vacuum is infinitely degenerate

A step towards resolving the information paradox.

Work Continues