Quantum Universe

Neil Turok

with thanks to
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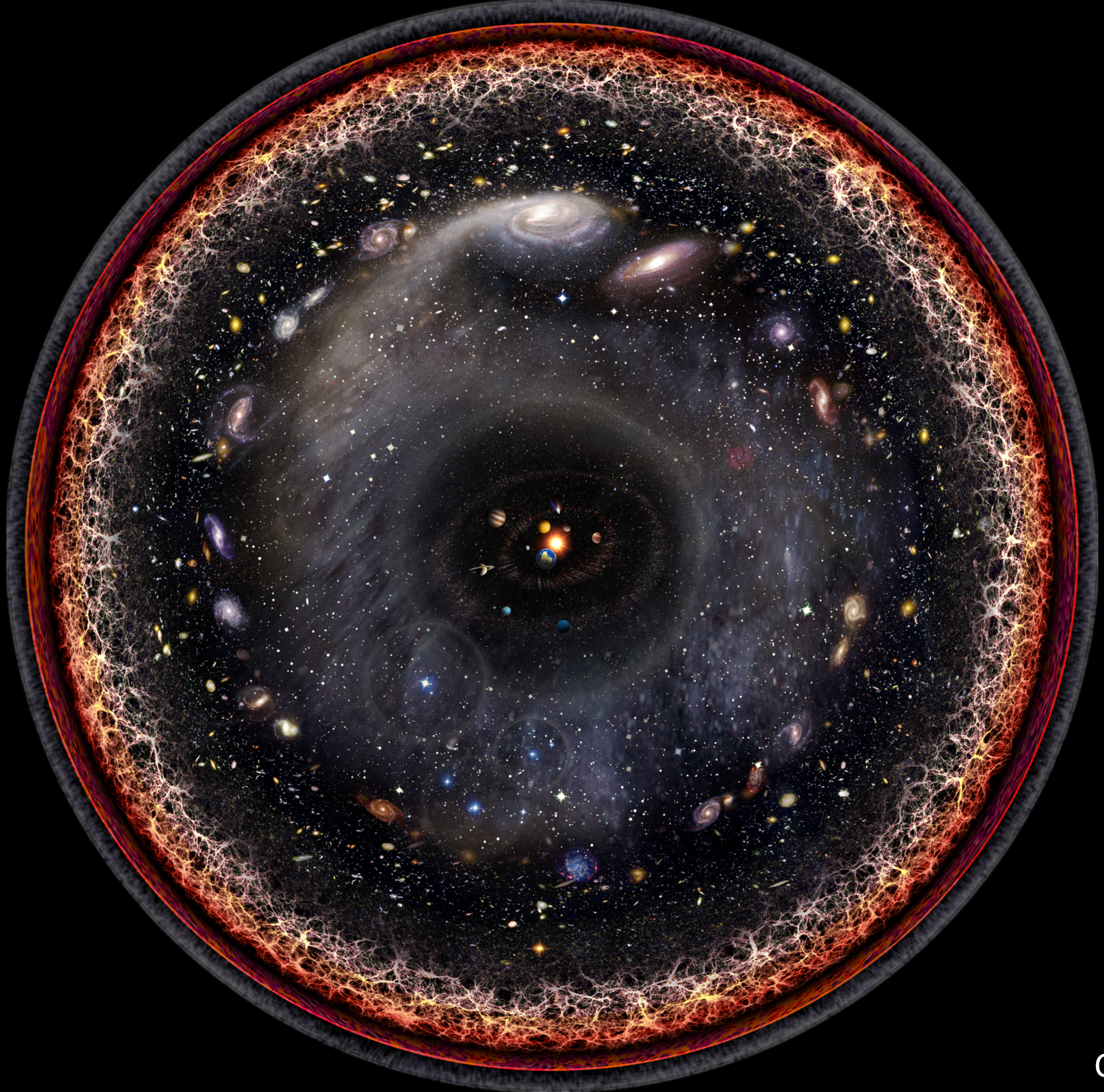


Credit: Pablo Carlos Budassi

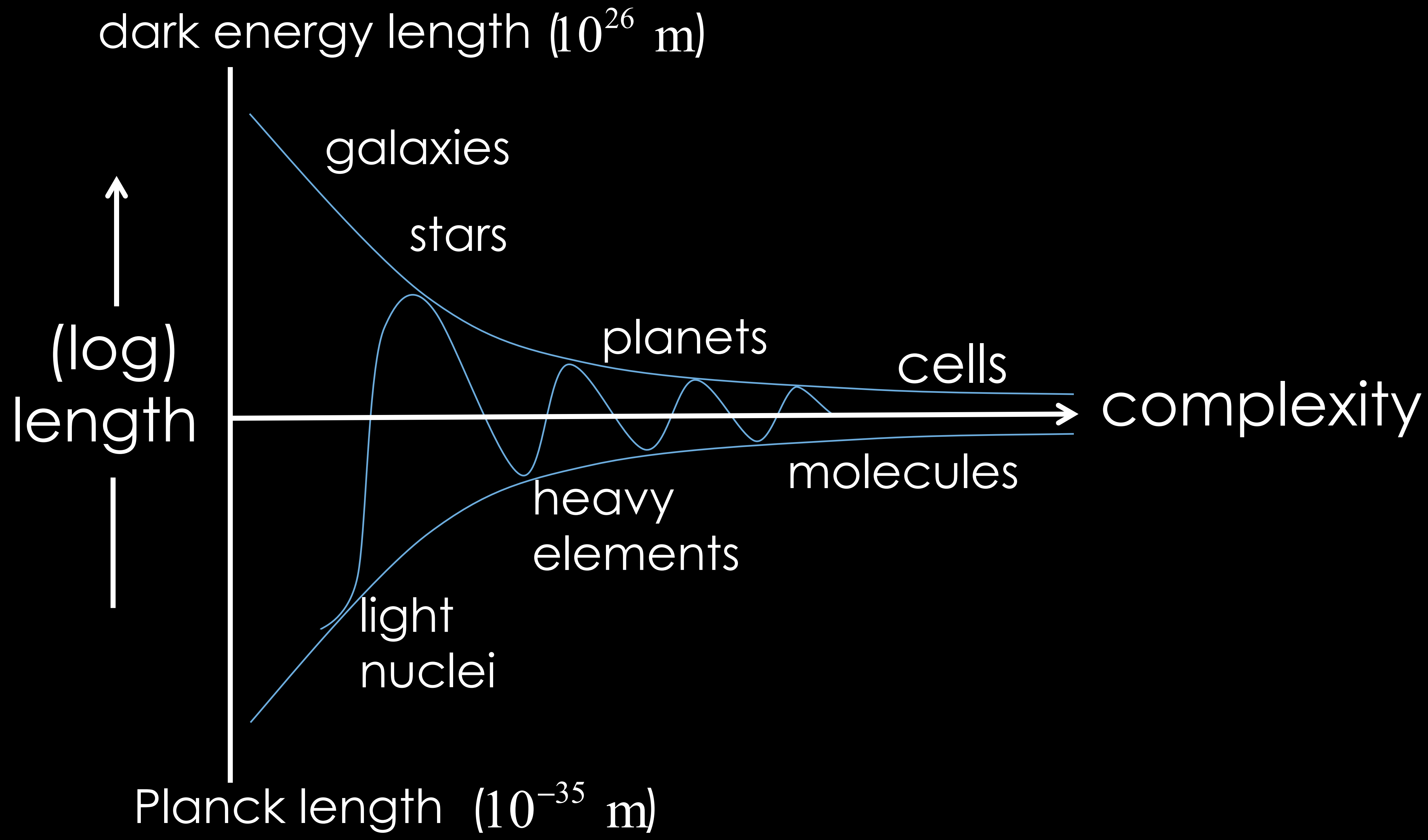
astonishing simplicity: just 5 numbers

			Measurement Error
today	Expansion rate:	67.8±0.9 km s^{-1} Mpc^{-1}	1%
	(Temperature)	2.728 ± 0.004 K	.1%
	(Age)	13.799 ±0.038 bn yrs	.3%
energy	Baryon-entropy ratio	6±.1x10^{-10}	1%
	Dark matter-baryon ratio	5.4± 0.1	2%
	Dark energy density	0.69±0.006 x critical	2%
geometry	Scalar amplitude	4.6±0.006 x 10^{-5}	1%
	Scalar spectral index n_s (scale invariant = 0)	-.033±0.004	12%

$+m_\nu$'s; but Ω_k, $1+w_{DE}$, $\frac{dn_s}{d\ln k}$, $\langle\delta^3\rangle, \langle\delta^4\rangle .., r = \frac{A_{gw}}{A_s}$ consistent with zero

Behind it all is surely an idea so simple, so beautiful, that when we grasp it - in a decade, a century or a millenium - we will all say to each other, how could it have been otherwise? How could we have been so stupid?

John A. Wheeler, How Come the Quantum? Ann. N.Y.A.S., ***480****, 304-316 (1986).*


dark energy length (10^{26} m)
galaxies
stars
planets
cells
(log) length
complexity
molecules
heavy elements
light nuclei
Planck length (10^{-35} m)

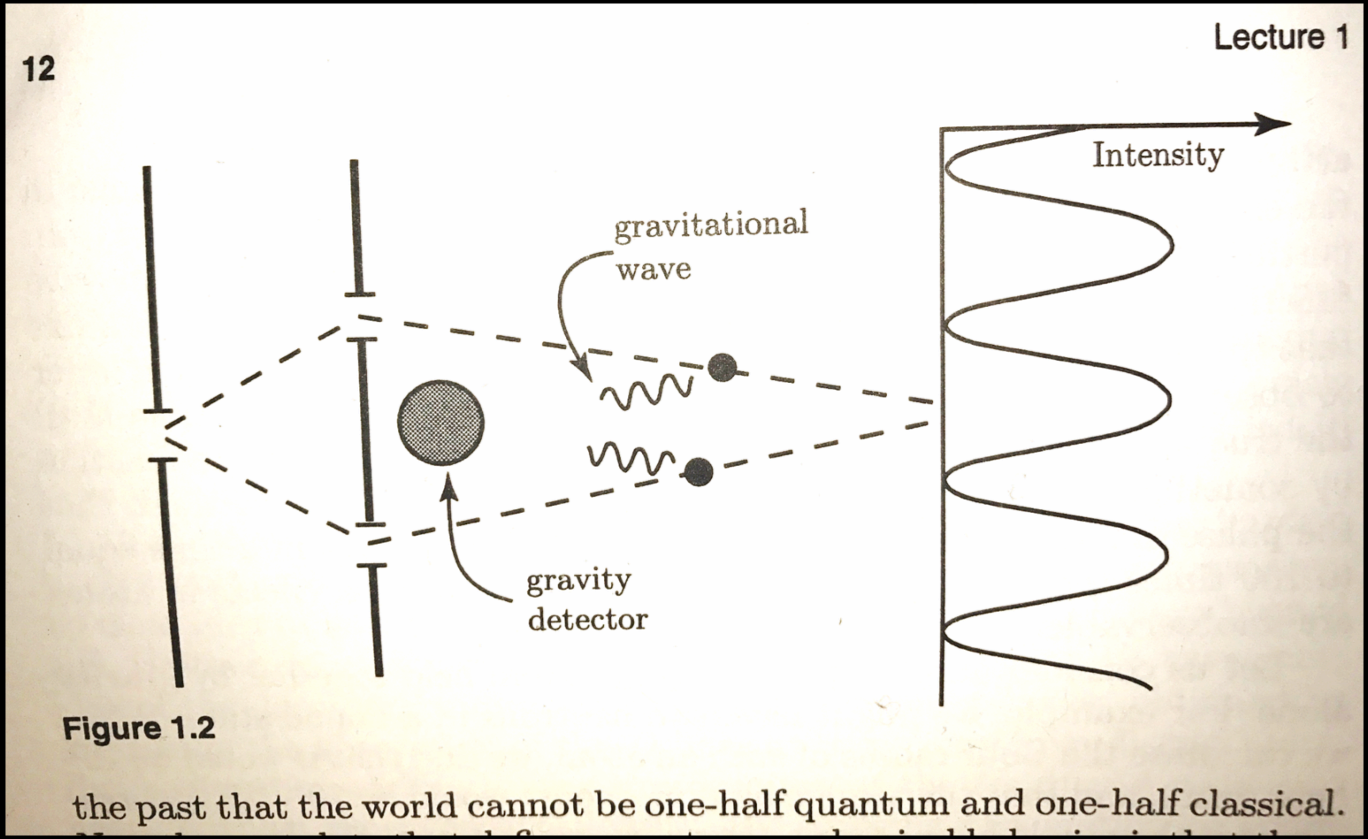
Nature has found a way to create a huge hierarchy of scales which appears simpler than any current theory.

This is a fascinating situation, demanding big new ideas.

One of the most conservative is quantum cosmology.

The universe must be quantum

1. because gravity must be quantum



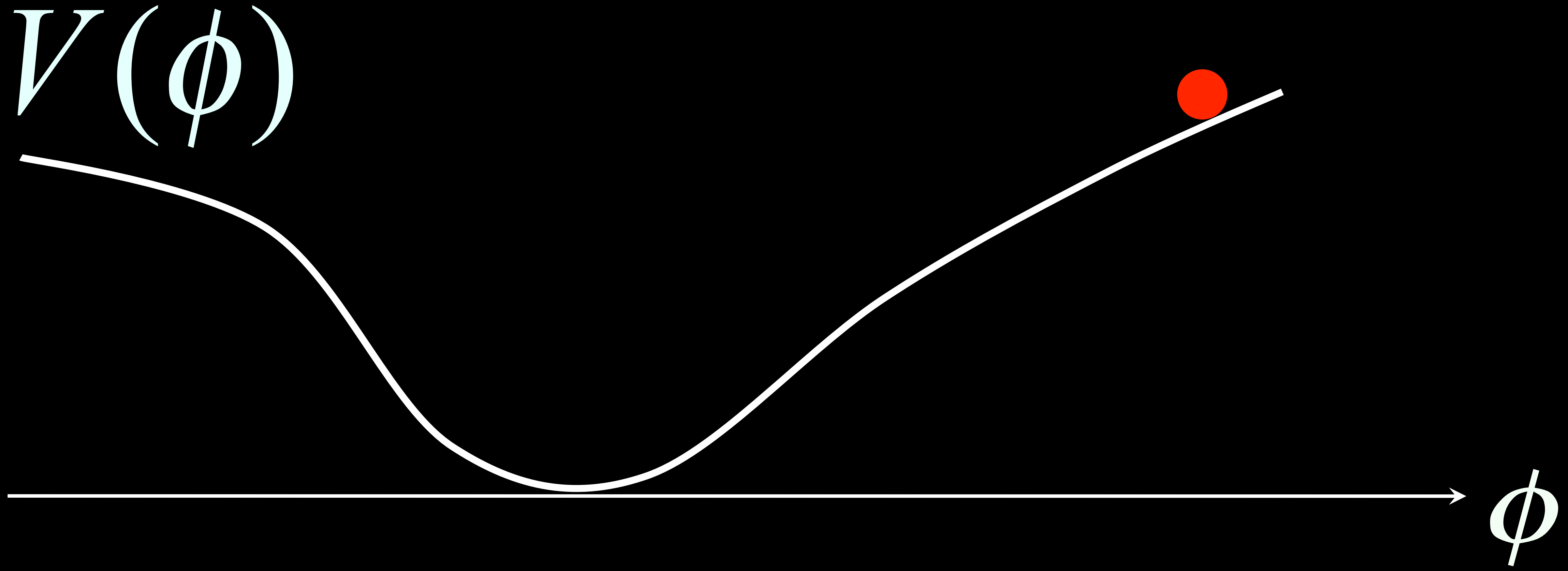
Feynman Lectures on Gravitation, Ed. B. Hatfield, Addison-Wesley (1995) p. 12

2. because classical cosmology is singular.
cf. hydrogen atom, UV catastrophe.

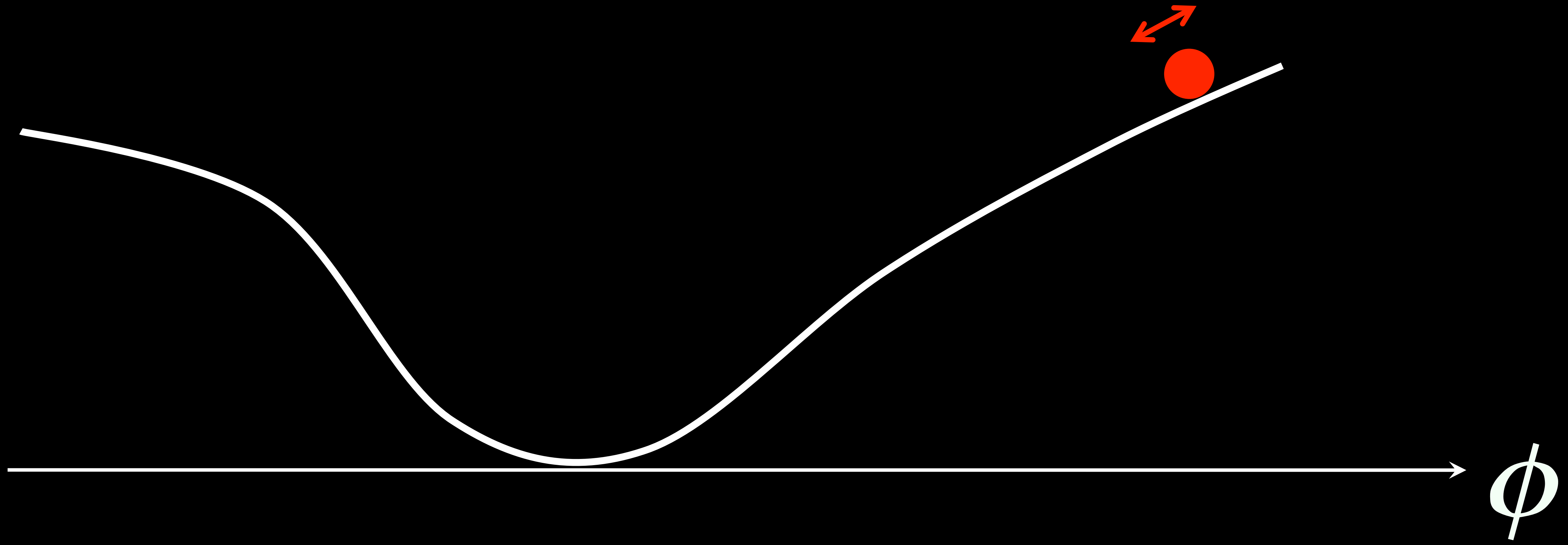
3. because the vacuum energy has an (apparently) divergent UV quantum contribution. It controls the causally accessible volume (UV $\leftrightarrow$ IR connection).

4. because the observed primordial fluctuations take the form of a free quantum field.

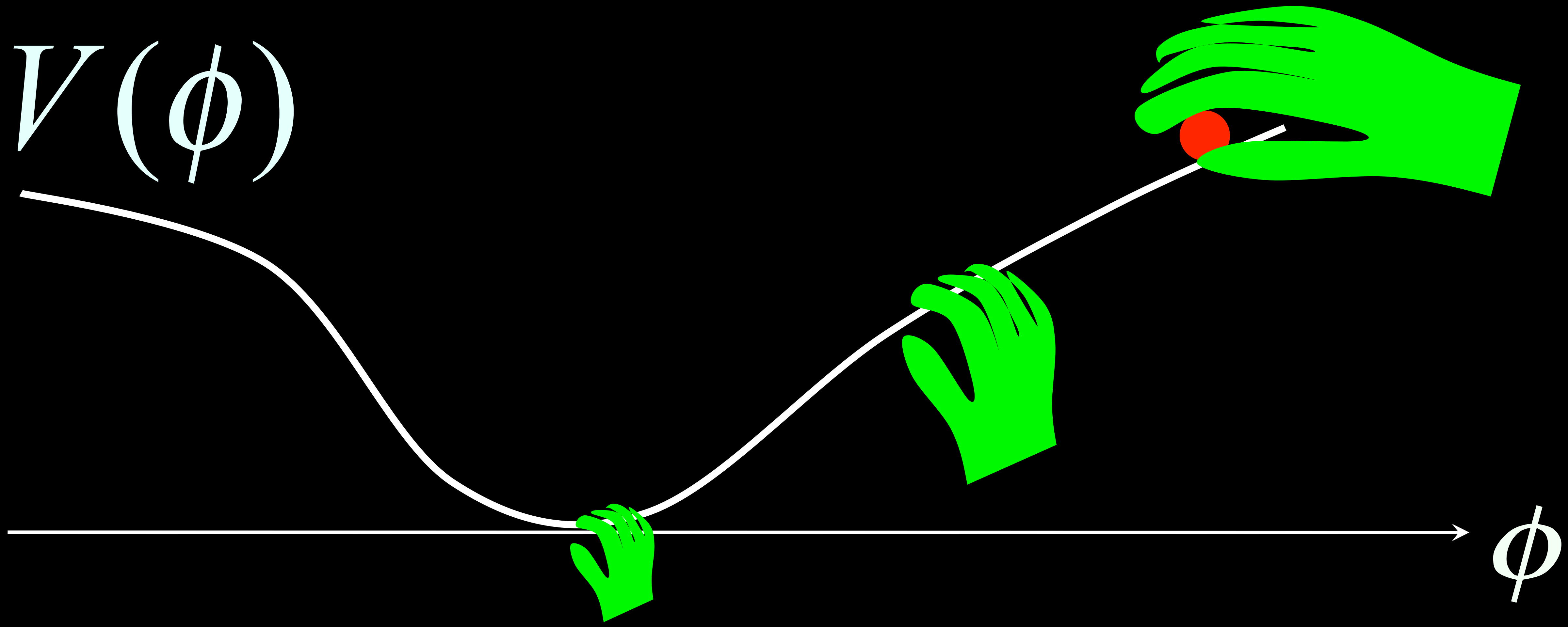
cosmic inflation



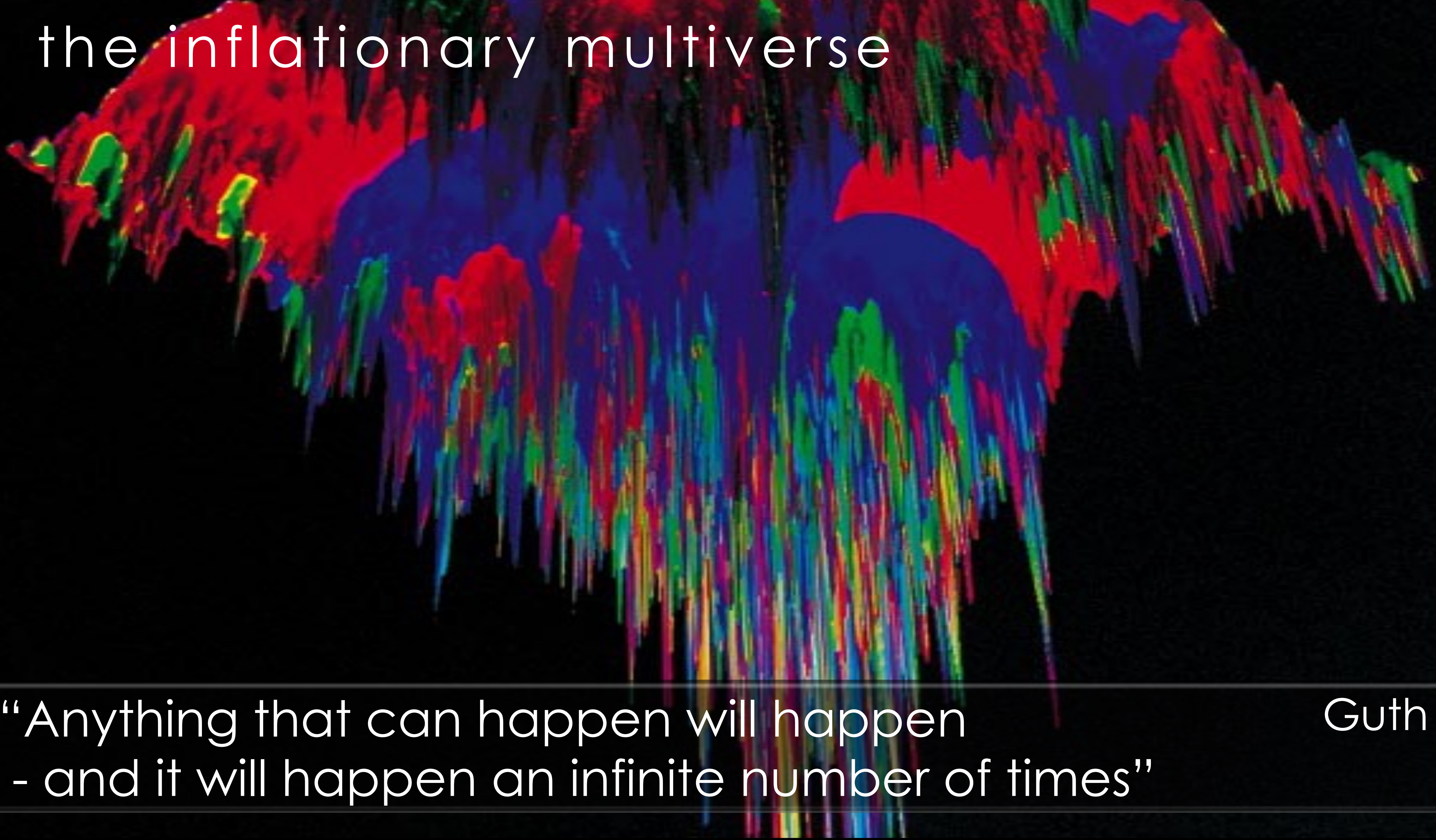
quantum fluctuations ⇒ large scale structure



fine tuning



the inflationary multiverse

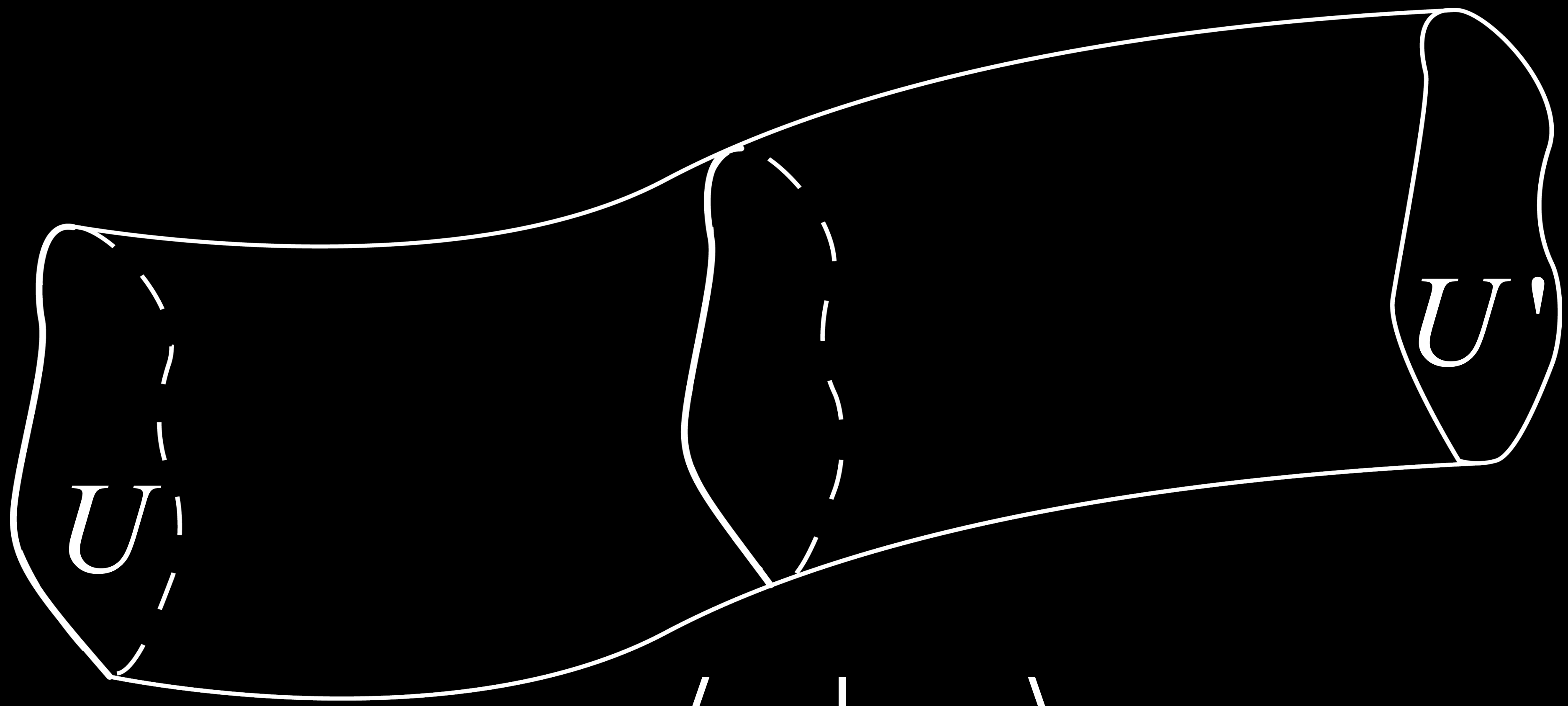


“Anything that can happen will happen
- and it will happen an infinite number of times”

Guth

rethinking quantum cosmology

s/ S. Gielen, 1612.0279, 1510.00699 (*Phys. Rev. Lett.* 117 (2016) 021301) quantum gravity



$$\langle U | U' \rangle$$

Perhaps the most impressive fact which emerges from a study of the quantum theory of gravity is that it is an extraordinarily economical theory. It gives one just exactly what is needed in order to analyze a particular physical situation, but not a bit more. Thus it will say nothing about time unless a clock to measure time is provided, and it will say nothing about geometry unless a device (either a material object, gravitational waves, or some other form of radiation) is introduced to tell when and where the geometry is to be measured.[50] In view of the strongly operational foundations of both the quantum theory and general relativity this is to be expected. When the two theories are united the result is an operational theory *par excellence*.[51]

B.S. DeWitt, *Phys. Rev.* 160, 1967 (p 1140)

Many beautiful theoretical works: basic conceptual problems remain unsolved

What equation determines the ‘the quantum state’?

What is the probability of a configuration?

This is a tremendous opportunity for theory: observations already provide vital clues.

Simplest case: conformal invariant matter $T^{\mu}_{\mu} = 0$

4d FRW $ds^2 \sim a(t)^2(-dt^2 + \gamma_{ij}dx^i dx^j);\quad R^{(3)} = 6\kappa$ $\begin{matrix} H^3, E^3, S^3 \\ \kappa < 0, = 0, > 0 \end{matrix}$

Friedmann $(\dot{a})^2 = 1 - \kappa a^2 \Rightarrow a \propto t$ as $a \to 0$

"perfect bounce" $ds^2 \sim t^2(-dt^2 + \gamma_{ij}dx^i dx^j)$ as $t \to 0$

unique analytic extension from negative to positive t (or a)

we call this a "perfect bounce" S. Gielen and N. Turok PRL, 117, 021301 (2016)

Consider Einstein gravity plus perfect radiation fluid and M conformally coupled scalar fields $\vec{\chi}$ (e.g. Higgs)

It is mathematically convenient to introduce another scalar ϕ via a Weyl transformation

$$S = \int \left[\tfrac{1}{2}\left((\partial\phi)^2 - (\partial\vec{\chi})^2 \right) + \tfrac{1}{12}\left(\phi^2 - \vec{\chi}^2 \right) R - \rho(n) - nU^{\mu}\partial_{\mu}\varphi \right]$$

Invariant under $\phi \to \Omega^{-1}\phi,\ \vec{\chi} \to \Omega^{-1}\vec{\chi},\ g_{\mu\nu} \to \Omega^2 g_{\mu\nu},\ \rho \to \Omega^{-4}\rho$ etc

ϕ may then be used to describe the expansion of the universe (in a Weyl gauge where the metric is fixed).

Pick Weyl gauge in which metric is static

$$ds^2 = -N(t)^2 dt^2 + \gamma_{ij} dx^i dx^j;\ \rho_r = r = const$$

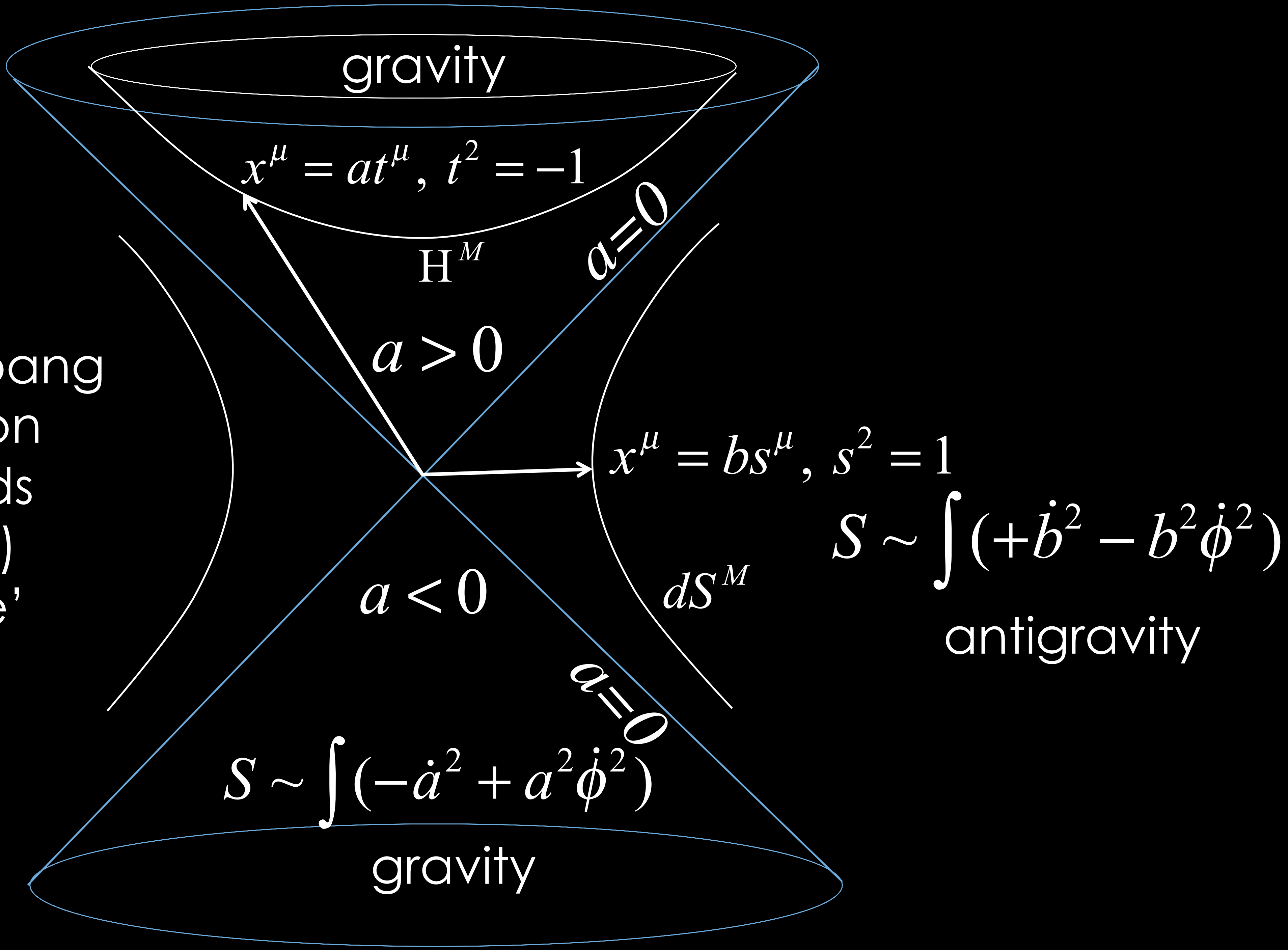
comoving volume

define $x^\alpha = \frac{1}{\sqrt{2r}}(\phi, \vec{\chi});\ \eta_{\alpha\beta} = \mathrm{diag}(-1,1,\ldots,1)$

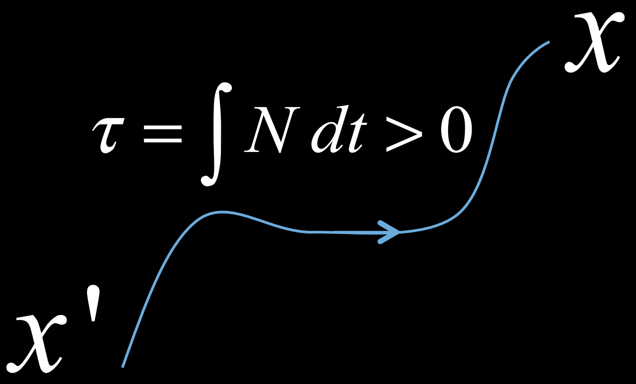
action $S = \frac{m}{2}\int dt \left[N^{-1}\dot{x}^\alpha \dot{x}_\alpha - N(\kappa x^\alpha x_\alpha + 1) \right]$ $m = 2V_0 r$ radiation density

- relativistic oscillator

Big crunch-Big bang coordinates on space of fields (zero modes) -'superspace'



Quantization: Hamiltonian $H = N\left(\frac{p^2}{2m} + \frac{m}{2}(1+\kappa x^2)\right)$



Causal (Feynman) propagator

gauge choice $0 \le t \le 1;\ N = \tau$

(*)

$$G_F(x,x') = \int_0^\infty dN \left\langle x \middle| e^{-\frac{i}{\hbar}N\hat{H}} \middle| x' \right\rangle = -i\left\langle x \middle| \hat{H}^{-1} \middle| x' \right\rangle \Rightarrow \hat{H}_x G_F(x,x') = -i\delta(x-x')$$

$$= \int_0^\infty dN \int Dx e^{\frac{i}{\hbar}S_{x'}^{x}}$$

(more formal but fixes bcs, pos. freq modes)

$$e.g.\ \kappa=0,\ \int_0^\infty d\tau \int Dx e^{i\frac{m}{2}\int_0^1 dt\left(\frac{\dot{x}^2}{\tau}-\tau\right)} = i\int_0^\infty d\tau \left(\frac{m}{2\pi i\tau}\right)^{\frac{M+1}{2}} e^{-i\frac{m}{2}\left(\frac{\sigma}{\tau}+\tau\right)}\ ;\ \sigma = -(x-x')^2$$

(usual Feynman propagator in Schwinger representation, in M+1 Minkowski)

Fourier transform on H^M, saddle point in τ determines behaviour at large $|a|$;

$$G_F(a,a') \sim e^{-i\omega a},\ a \to \infty$$

$$\sim e^{i\omega a'},\ a' \to -\infty$$

This defines the positive and negative frequency modes, from which G_F may be determined by solving (*) via the Wronskian method

Note: 1) in quantum cosmology, positive frequency $\leftrightarrow$ expanding!
2) imposing Neumann or Dirichlet bcs at a=0 (which most prior proposals do) *prohibits* an expanding universe
3) (for perturbation modes) no need for an independent specification of `Bunch-Davies vacuum'. Vacuum is consequence of path integral formulation.

$\kappa > 0$ no asymp states - non-normalizable in norm offered below
propagator ambiguous (multiple saddle points)

$\kappa < 0$

$G_F(x,x') \sim e^{i\frac{m}{2}\sqrt{-\kappa}x^2}$ as $x^2 \to \infty$
quantum tunneling to classically
allowed solutions in antigravity regime

Only $\kappa = 0$ seems quantum consistent†

path integrals are Gaussian, τ integral is not

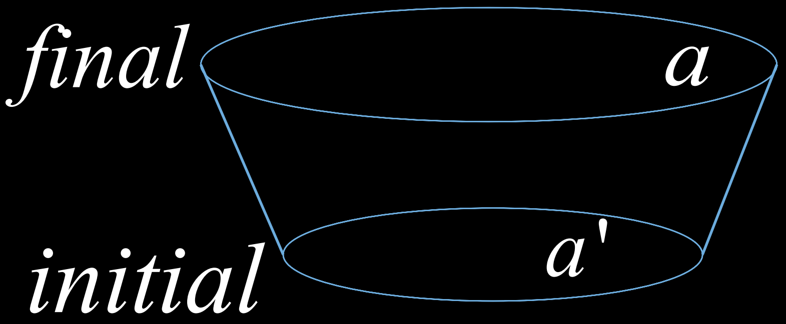
if m is large, FRW background is "heavy": quantum spreading
and backreaction are small except around $a = 0$

† when Λ included, among classically allowed universes with
asymptotic states, $\kappa = 0$ is the most probable

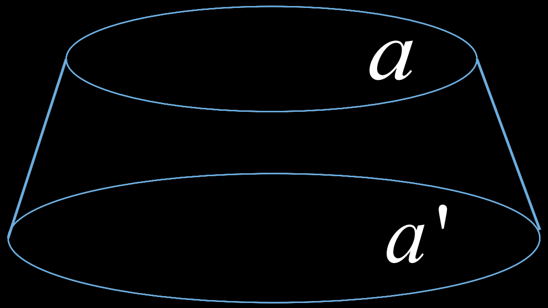
emergence of time (à la Wheeler)

w/ J. Feldbrugge

expanding



contracting

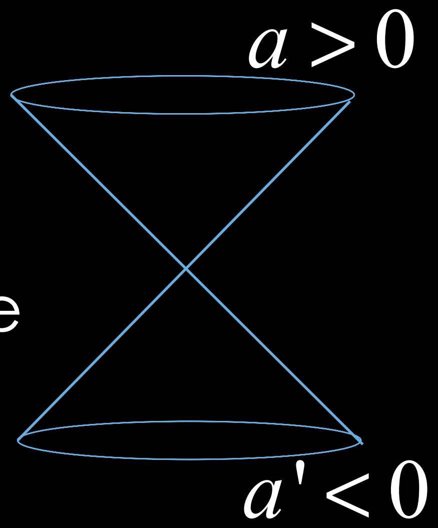


$$\langle t_P \rangle = \tfrac{1}{2}\,|\,a^2 - a'^2\,| + o(m^{-1})$$

$$\langle t_P^n \rangle_{irr} = o(m^{-1})$$ quantum corrections

Cosmological (Einstein-frame) proper time $t_P = \int_0^1 |a(t)|\,dt$

or,
in a
bounce



$$\langle t_P \rangle = \tfrac{1}{2}(a^2 + a'^2) + o(m^{-1})$$

$$\langle t_P^n \rangle = +o(m^{-1})$$ quantum fluctuations

Leading terms are
Just the classical results

time is determined by
boundary 3-geometries

Generic metrics: consider deviations around flat ($\kappa = 0$) FRW

Anisotropies: $$ds^2 \sim a(t)^2\left(-dt^2 + \sum_{i=1}^{3} e^{c\lambda_i} dx_i^2\right); \;\; \sum \lambda_i = 0$$

Line element on space of "moduli":

$$ds^2_{\mathrm{mod}} = -da^2 + a^2\left(dH_M^2 + dE_2^2\right)$$

scalars anisotropy moduli

(Note: standard model Higgs (M=1) sufficient to remove BKL chaos)

Ordering ambiguities resolved by invariance under coordinate transformations on superspace and redefinitions of lapse

$$N \to \Omega^{-2} N, \text{ which is equivalent to } \mathrm{g}^S_{\mu\nu} \to \Omega^2 g^S_{\mu\nu}, \ \mathrm{V}^S \to \Omega^{-2}\mathrm{V}^S$$

Halliwell
Moss

It follows that $\hat{H} = \hbar^2(-\Box^S + \xi R^S + V^S)$ with $\xi = \frac{D-2}{4(D-1)}$

The equation $\hat{H}_x G_F(x,x') = -i\delta(x-x')/\sqrt{-g^S}$ is then invariant

Under reasonable assumptions, near $a=0$ we have

$$\left(-\frac{d^2}{da^2}+\frac{c'}{a^2}\right)\Psi^{(\pm)}=m^2\Psi^{(\pm)},\ c'=-\vec{\zeta}^{\,2}-\frac{1}{4}+\frac{(M-1)}{2(M+2)},\ \vec{\zeta}=$$ conserved anisotropy and scalar momenta

This potential occurs in integrable models (Calogero-Sutherland) and, for $-\infty<a<\infty$ it is the simplest case of PT-invariant quantum mechanics.

Znojil
Dorey et al

For $c'\geq-\frac{1}{4}$ the QM Hamiltonian has a real, positive spectrum.

So, at least for M>1 and for an open set of anisotropy+scalar momenta, continuation across a=0 seems completely safe.

The surprising inverse square potential:

classically,

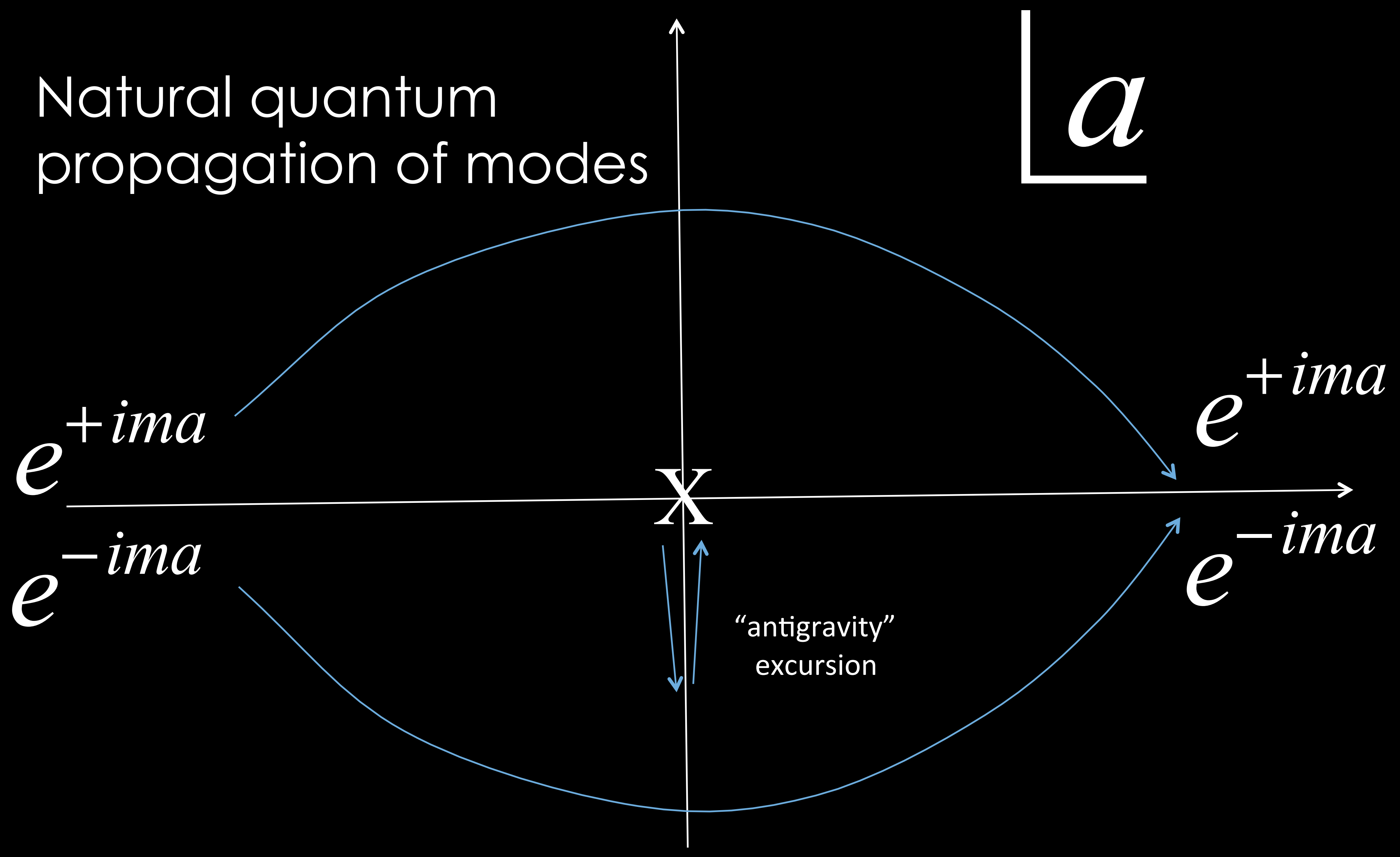
$$m\frac{\dot{a}^2}{\tau^2}+\frac{C}{ma^2}=m \;\Rightarrow\; a^2=\frac{C}{m^2}+\tau^2(t-t_0)^2$$

no time delay: invisible!

quantum mechanically, there isn't even a phase shift (consequence of scale invariance)

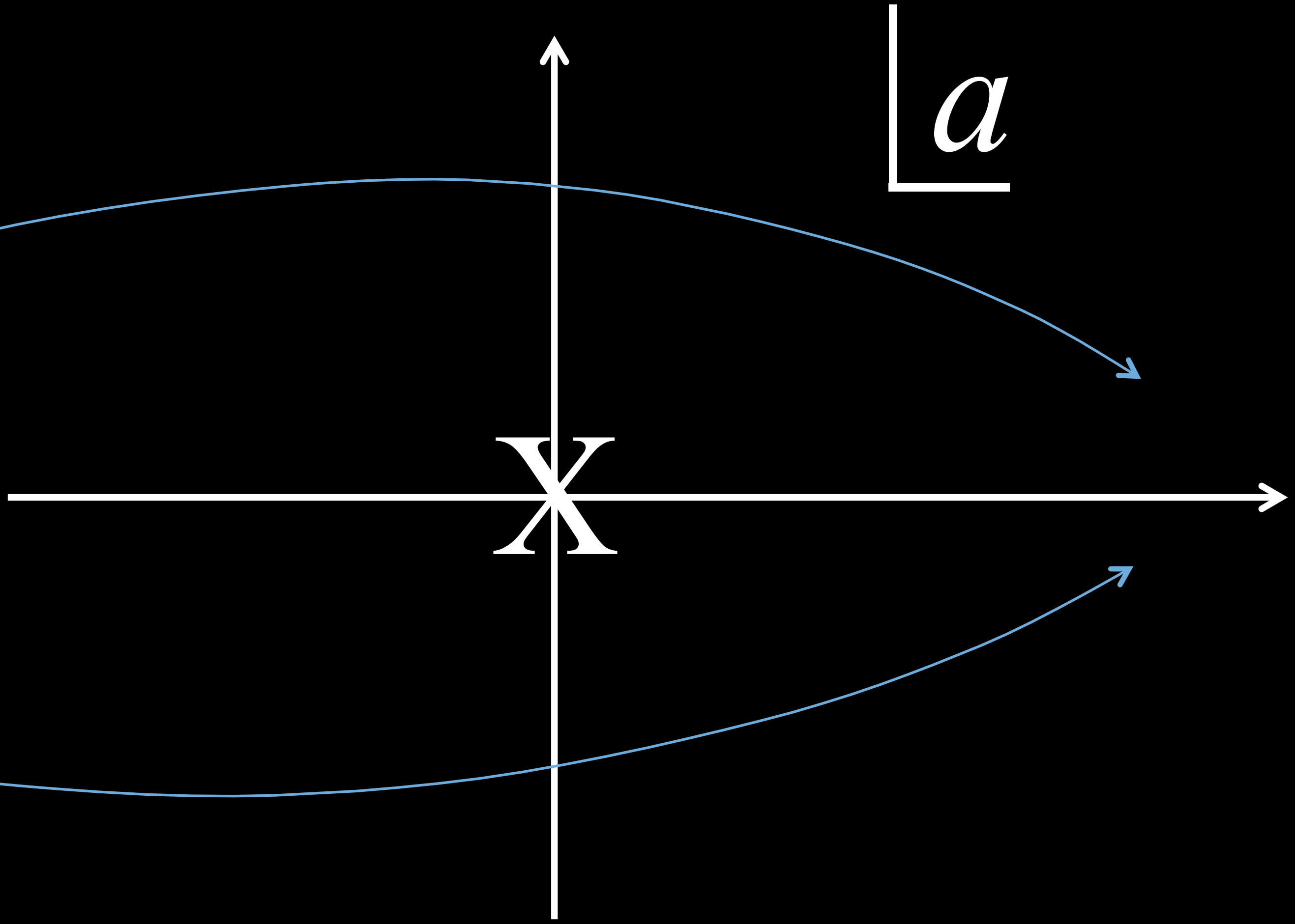
Proposals which fix the wavefunction at a=0 violate the correspondence principle and have other undesirable features (superposition of expanding and contracting).

Natural quantum propagation of modes



General prescription

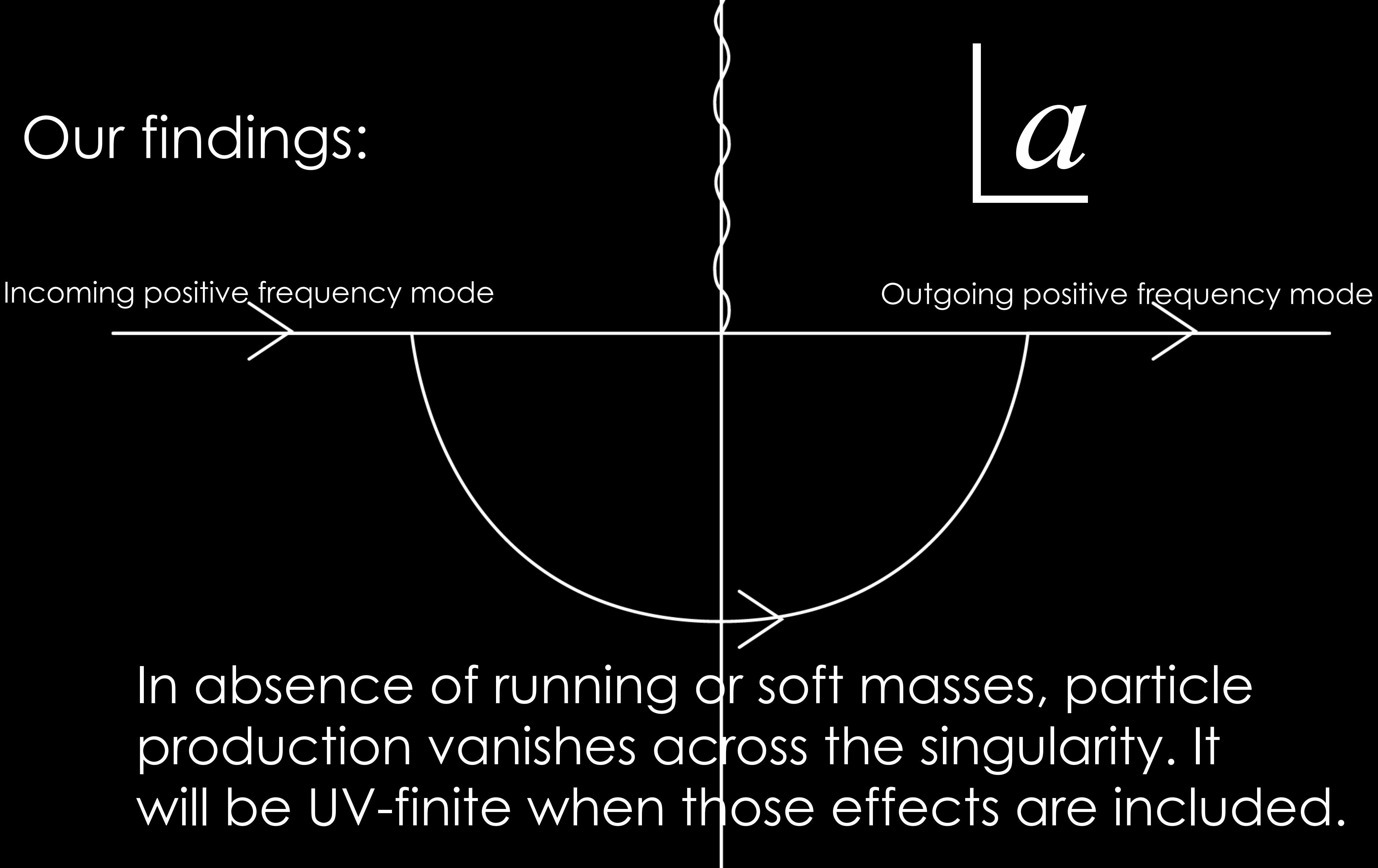
$$\Psi = \underset{\text{analytic lhp}}{\overset{\text{Positive frequency}}{\Psi^{(+)}}} + \underset{\text{analytic uhp}}{\overset{\text{Negative frequency}}{\Psi^{(-)}}}$$



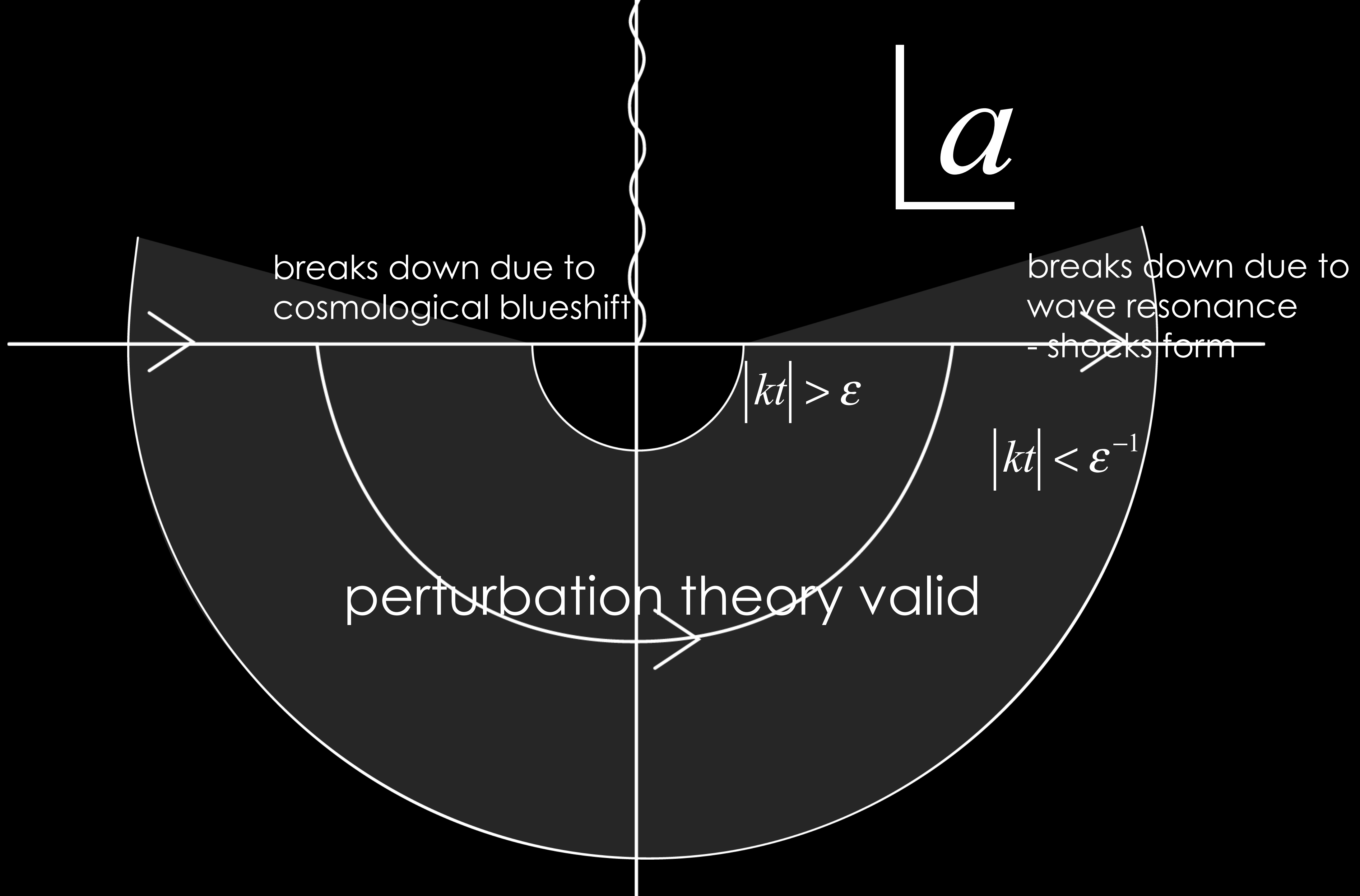
We have solved for the scalar and tensor perturbations at linear and nonlinear order, concluding that for a perfect radiation fluid there is precisely zero particle production across a bounce. We also found some surprises…

S. Gielen and N. Turok
PRL, 117, 021301 (2016)

Our findings:



In absence of running or soft masses, particle production vanishes across the singularity. It will be UV-finite when those effects are included.

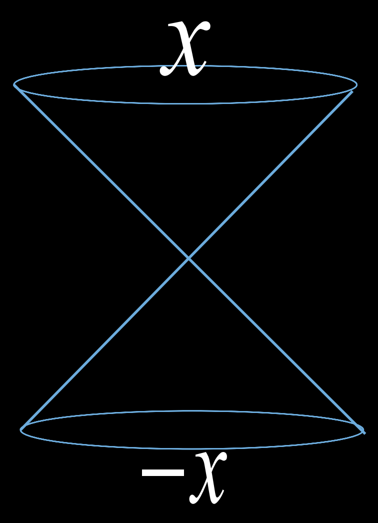


a
breaks down due to
cosmological blueshift
breaks down due to
wave resonance
- shocks form
$|kt| > \varepsilon$
$|kt| < \varepsilon^{-1}$
perturbation theory valid

A quantum measure for the universe

The fundamental object in quantum geometrodynamics is the propagator. Let us try to use it to define probabilities.

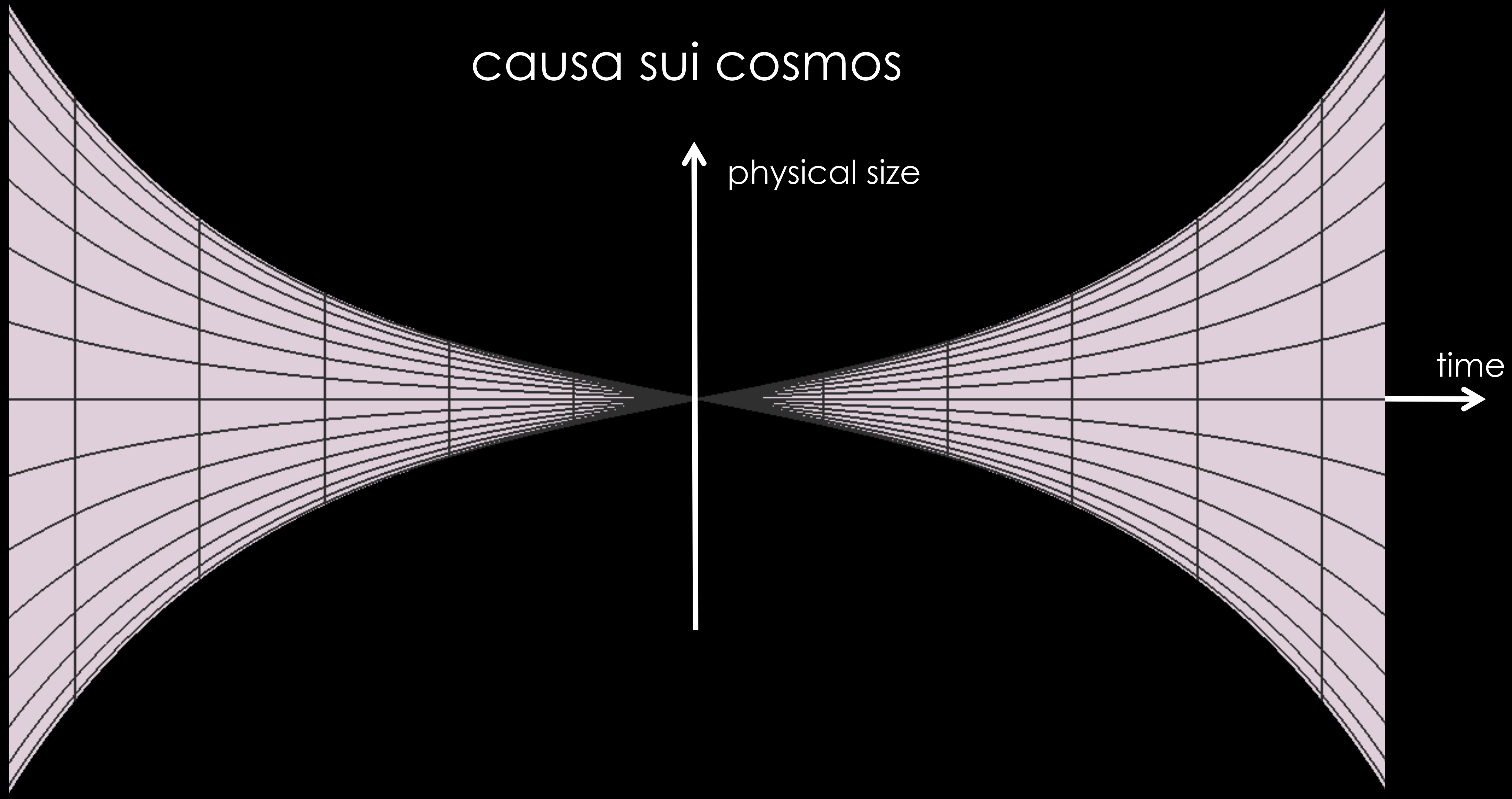
The scale factor a is a natural dynamical variable.
But the theory is invariant under $PT : a \to -a$.
Consider the amplitude to go from one copy to the other.



$$\int d^{M+1}x\sqrt{-g^S}\ \left|G_F(x,-x)\right|^2$$

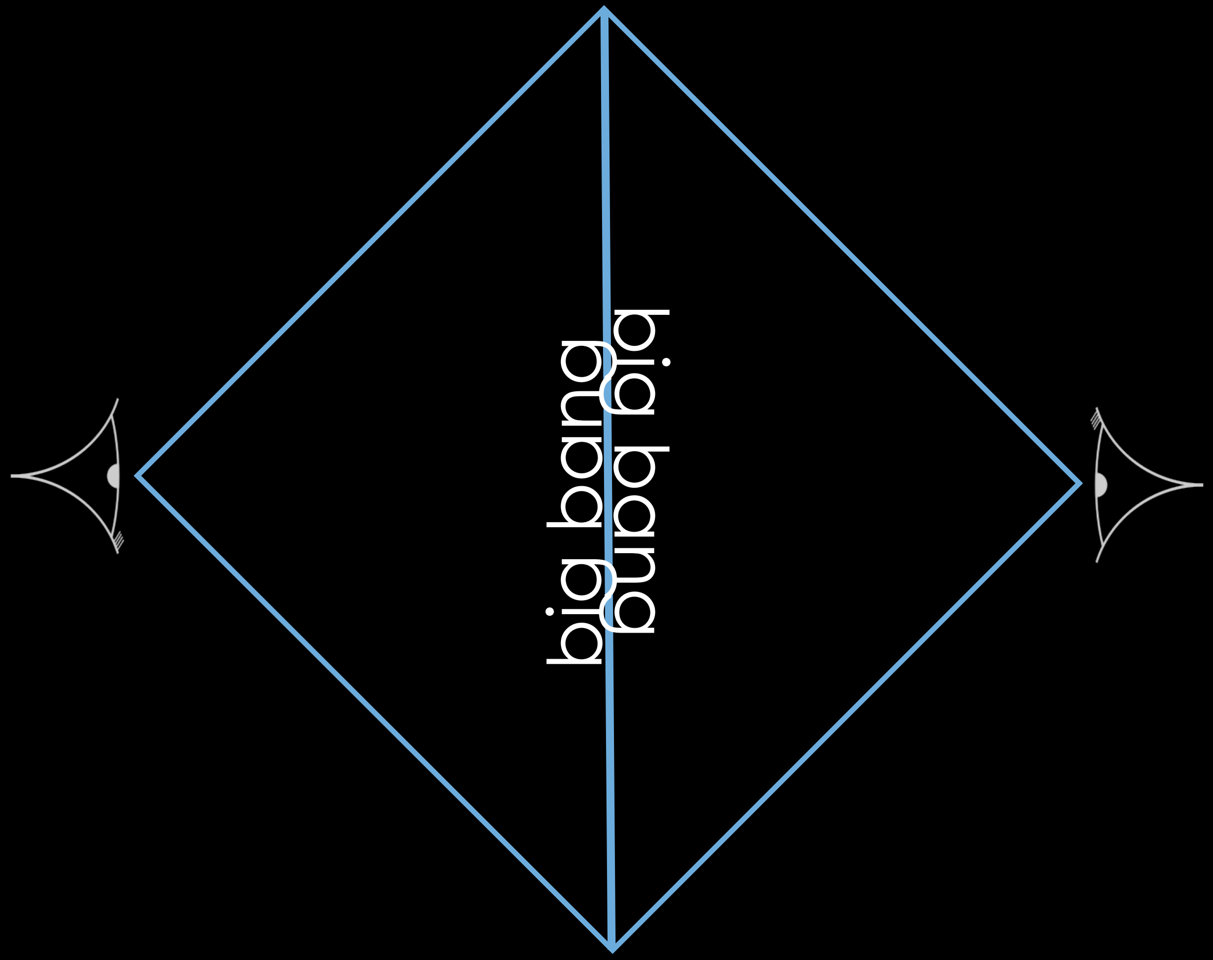
Normalizable if Λ is positive but only invariant if number of zero modes M=3. In addition to two anisotropy dofs, there must be a scalar (Higgs!)

causa sui cosmos



“By causa sui I understand that whose essence involves existence, or that whose nature cannot be conceived unless existing.” *Spinoza, Ethics, 1677*

causal structure



Among universes with these asymptotic states, fixing Λ and maximising the probability as a function of κ, one finds $\kappa = 0$ is most probable

This scenario explains

Why the universe is expanding
Why it apparently `began' in a singularity
Why dark energy is essential

with a modest addition (one extra scalar) it can also explain the quantum generation of scale-invariant curvature perturbations

$\Rightarrow$ a more minimal and predictive cosmology