QFT in Galilean Superspace

Silvia Penati, University of Milano-Bicocca and INFN

February 6, 2023

“Beyond Lorentzian Geometry II”, ICMS Edinburgh

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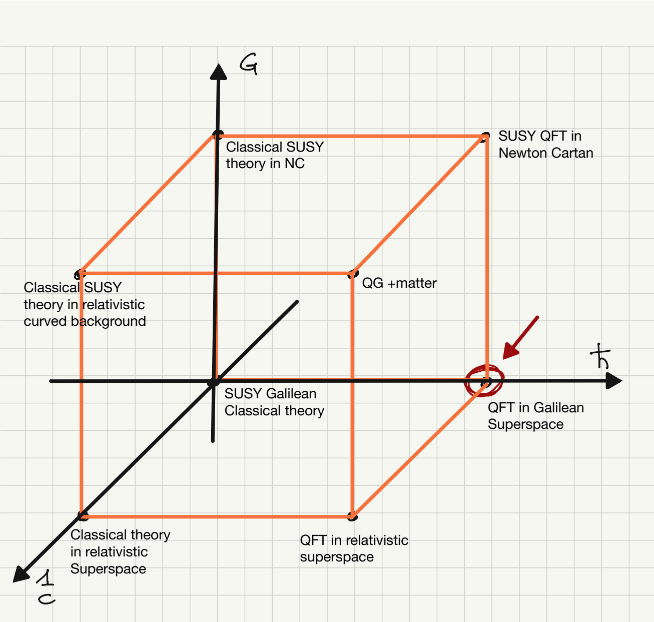
Two examples of non-relativistic susyQFTs

R. Auzzi, S. Baiguera, G. Nardelli, SP, JHEP 06 (2019) 048 [arXiv:1904.08404]

S. Baiguera, L. Cederle, SP, JHEP 09 (2022) 237 [arXiv:2207.06435]

SUSY Bronstein cube



Motivations

- In models describing CM systems SUSY has been observed to be an emergent symmetry, that is it appears in the effective theory describing the low-energy modes. On the other hand, at these scales the system is typically in a non-relativistic regime.

  Therefore, it is physically relevant to construct NR SUSY models

- Non-relativistic holography: Non-relativistic generalisation of the AdS/CFT is of interest for the holographic description of CM systems.

D.T. Son, PRD78 (2008)
K. Balasubramanian, J. McGreevy, PRL101 (2008)
W.D. Goldberger, JHEP03 (2009)
S. Kachru, X. Liu, M. Mulligan, PRD78 (2008)
S. Janiszewski, A. Karch, JHEP02 (2013)
M. H. Christensen, J. Hartong, N. A. Obers and B. Rollier, PRD89, JHEP01 (2014)
M. Taylor, CQG33 (2016)
A. Bagchi, R. Gopakumar, JHEP07 (2009)
A. Bagchi, R. Basu, D. Grumiller, M. Riegler, PRL114 (2015)

Which is the role of supersymmetry in NR holography?

- NR limits allow to improve our understanding of the relativistic theory:

  - NR corners of $\mathcal{N} = 4$ SYM to go beyond the planar limit

    T. Harmark, N. Wintergerst, PRL124 (2020)
    S. Baiguera, T. Harmark, N. Wintergerst, JHEP02 (2021)

  - NR corners of M-theory: Null M5-branes described by (4+1)d non-Lorentzian SUSY Lagrangians

    N. Lambert, A. Lipstein, P. Richmond, JHEP10 (2018)
    N. Lambert, A. Lipstein, R. Mouland, P. Richmond, JHEP01 (2020); JHEP03 (2021)

From a QFT point of view important open questions are:

- Which are the renormalization properties of NR SUSY theories?
- Does SUSY conspire with the NR space-time symmetry to mild UV divergences?
- Do non-renormalization theorems still work ?

We focus on (2+1)D field theories with $\mathcal{N} = 2$ Super-Bargman symmetry

Plan of the talk

1) Construction of the non-relativistic (galilean) $\mathcal{N} = 2$ Superspace

2) NR Wess-Zumino Model: One-loop exactness

3) Supersymmetric Galilean Electrodynamics (SGED): A renormalizable non-linear sigma model

4) Conclusions and future directions

Super-Bargman algebra

Bargmann algebra $(H, \vec{P}, \vec{J}, \vec{G}, M)$ $\quad x' = Rx + vt + a, \quad t' = t + b$
$M = U(1)$ central extension

There are different ways to obtain the Super-Bargmann algebra in (d+1)D

- Completing the Bargmann algebra with a set of fermionic generators and impose constraints on the algebra
- Taking the Inönü-Wigner contraction of the (d+1)D super-Poincaré $\otimes\, U(1)$ algebra in the $c \to \infty$ limit
- By dimensionally reducing the ((d+1)+1)D relativistic SUSY algebra along a **null direction**

To construct a NR Superspace the most convenient approach is **null reduction**

Null reduction

- We start from the (3+1)D super-Poincarè algebra realized on the spacetime

$$(x^+, x^-, x^{i=1,2}) \qquad\qquad x^\pm = \frac{x^3 \pm x^0}{\sqrt{2}} \qquad \text{light} - \text{cone coords.}$$

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- Define $P_- \equiv M$ and select only the generators which commute with M $\Rightarrow P_+ \equiv H,\ P_{i=1,2},\ M_{12} \equiv J,\ M_{i-} \equiv G_i$ bosonic subalgebra

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- In the fermionic sector, write the (3+1)D anticommutator $\{\mathcal{Q}_\alpha, \bar{\mathcal{Q}}_{\dot\beta}\} = i\sigma^\mu_{\alpha\dot\beta}\partial_\mu$ in terms of the light-cone coordinates

$$\{\mathcal{Q}, \bar{\mathcal{Q}}\} = i \begin{pmatrix} \sqrt{2}\partial_+ & \partial_1 - i\partial_2 \\ \partial_1 + i\partial_2 & -\sqrt{2}\partial_- \end{pmatrix}$$

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- Identify $\partial_+ \to \partial_t\ ,\ \partial_- \to im \qquad \mathcal{Q}_\alpha \to Q_\alpha, \quad \bar{\mathcal{Q}}_{\dot\alpha} \to Q^\dagger_\alpha \equiv \bar{Q}_\alpha$

$\mathcal{N} = 2$ Super-Bargmann algebra

$$[P_j, G_k] = i\delta_{jk} M \,, \qquad [H, G_j] = iP_j \,,$$
$$[P_j, J] = -i\epsilon_{jk} P_k \,, \qquad [G_j, J] = -i\epsilon_{jk} G_k \,, \qquad\qquad j,k = 1,2$$

$$[Q_1, J] = \tfrac{1}{2} Q_1 \,, \qquad \{Q_1, Q_1^\dagger\} = \sqrt{2} H \,,$$
$$[Q_2, J] = -\tfrac{1}{2} Q_2 \,, \quad [Q_2, G_1 - iG_2] = -iQ_1 \,, \quad \{Q_2, Q_2^\dagger\} = \sqrt{2} M \,,$$
$$\{Q_1, Q_2^\dagger\} = -(P_1 - iP_2) \,, \quad \{Q_2, Q_1^\dagger\} = -(P_1 + iP_2)$$

Non-relativistic Superspace

$(3+1)$ $\mathcal{N}=1$ relativistic superspace $\Longrightarrow$ $(2+1)$ $\underline{\mathcal{N}=2 \text{ NR superspace}}$

$$(x^+, x^-, x^i, \theta^\alpha, \bar\theta^{\dot\alpha}) \quad \Longrightarrow \quad (t, x^i, \theta^1, \theta^2, \bar\theta^1, \bar\theta^2)$$

$$[t] = -2, \ [x^i] = -1, \ [\theta^1] = -1, \ [\theta^2] = 0$$

- ⋆ Reduction of a generic superfield

$$\Phi(x^M, \theta^\alpha, \bar\theta^{\dot\alpha}) = e^{imx^-}\,\tilde\Phi(t, x^i, \theta^1, \theta^2, \bar\theta^1, \bar\theta^2) \qquad m \to M - \text{eigenvalue}$$

- ⋆ Covariant derivatives

$$\begin{cases} \mathcal{D}_\alpha = \frac{\partial}{\partial\theta^\alpha} - \frac{i}{2}\bar\theta^{\dot\beta}\partial_{\alpha\dot\beta} \\ \bar{\mathcal{D}}_{\dot\alpha} = \frac{\partial}{\partial\bar\theta^{\dot\alpha}} - \frac{i}{2}\theta^{\beta}\partial_{\beta\dot\alpha} \end{cases} \Longrightarrow \begin{cases} D_1 = \frac{\partial}{\partial\theta^1} - \frac{i}{2}\bar\theta^2(\partial_1 - i\partial_2) - \frac{i}{\sqrt{2}}\bar\theta^1\partial_t \\ \bar D_1 = \frac{\partial}{\partial\bar\theta^1} - \frac{i}{2}\theta^2(\partial_1 + i\partial_2) - \frac{i}{\sqrt{2}}\theta^1\partial_t \\ D_2 = \frac{\partial}{\partial\theta^2} - \frac{i}{2}\bar\theta^1(\partial_1 + i\partial_2) - \frac{1}{\sqrt{2}}\bar\theta^2 M \\ \bar D_2 = \frac{\partial}{\partial\bar\theta^2} - \frac{i}{2}\theta^1(\partial_1 - i\partial_2) - \frac{1}{\sqrt{2}}\theta^2 M \end{cases}$$

$$\{\mathcal{D}_\alpha, \bar{\mathcal{D}}_{\dot\alpha}\} = -i\partial_{\alpha\dot\alpha} \quad \Longrightarrow \quad \{D_\alpha, \bar D_\beta\} = -i\partial_{\alpha\beta} \qquad [D_1] = [\bar D_1] = 1$$

$$[D_2] = [\bar D_2] = 0$$

⋆ (Anti)chiral superfields $\bar{D}_\alpha \Sigma = 0, \qquad D_\alpha \bar{\Sigma} = 0$

$$\Sigma(x_L, \theta^\alpha) = \varphi(x_L) + \theta^\alpha \psi_\alpha(x_L) - \theta^2 F(x_L)$$

$$\bar{\Sigma}(x_R, \bar{\theta}^\beta) = \bar{\varphi}(x_R) + \bar{\theta}_\gamma \bar{\psi}^\gamma(x_R) - \bar{\theta}^2 \bar{F}(x_R) \qquad x^{\alpha\beta}_{L,R} = x^{\alpha\beta} \mp i\, \theta^\alpha \bar{\theta}^\beta$$

⋆ Berezin and spacetime integrations

$$\int d^4x d^4\theta\, \Psi = \int d^4x\, \mathcal{D}^2 \bar{\mathcal{D}}^2 \Psi \Big|_{\theta=\bar{\theta}=0} \qquad (\Psi = e^{imx^-}\, \tilde{\Psi})$$

$$\longrightarrow \underbrace{\int d^3x D^2 \bar{D}^2 \tilde{\Psi}\Big|_{\theta=\bar{\theta}=0}} \times \frac{1}{2\pi} \int_0^{2\pi} dx^-\, e^{imx^-}$$

$$\downarrow \qquad\qquad \Downarrow$$

$$\equiv \int d^3x\, d^4\theta\, \tilde{\Psi} \qquad \text{Non-vanishing result only if } \mathrm{M}(\Psi) = 0$$

Non-relativistic Wess-Zumino model in (2+1)D

Relativistic $\mathcal{N} = 1$ WZ model in (3+1)D

$$S = \int d^4x\, d^4\theta\, \bar{\Sigma}\Sigma + \int d^4x\, d^2\theta \left(\frac{\mu}{2}\Sigma^2 + \frac{\lambda}{3!}\Sigma^3 \right) + \text{h.c.} \qquad \bar{\mathcal{D}}\Sigma = \mathcal{D}\bar{\Sigma} = 0$$

- **The WZ model is renormalizable**
- **Non-renormalization theorem**

  → Perturbative **M.T. Grisaru, W. Siegel, M. Rocek, NPB 159 (1979) 429**

  → Non-perturbative (Holomorphicity, SUSY, $U(1) \times U(1)_R$ symmetry)

  N. Seiberg, PLB 318 (1993) 469

$$\mathcal{L} \to \mathcal{L}_{\text{ren}} = \int d^4\theta\, Z_\Sigma(\bar{\Sigma}\Sigma) + \int d^2\theta\, Z_\lambda Z_\Sigma^{3/2}(\frac{\lambda}{3!}\Sigma^3)$$

The absence of chiral divergences implies

$$Z_\lambda Z_\Sigma^{3/2} = 1 \implies Z_\lambda = Z_\Sigma^{-3/2}$$

Non-relativistic Wess-Zumino model in (2+1)D

Particle number conservation requires at least two superfields ($\mu = 0$)

$$S = \int d^3x d^4\theta \left(\bar{\Phi}_1\Phi_1 + \bar{\Phi}_2\Phi_2\right) + g \int d^3x d^2\theta \, \Phi_1^2\Phi_2 + \text{h.c.} \qquad \bar{D}\Phi_a = D\bar{\Phi}_a = 0$$

$$M(\Phi_1) = m, \; M(\Phi_2) = -2m$$

Manifestly invariant under NR $\mathcal{N} = 2$ SUSY

$$\Phi_1 = \varphi_1 + \theta^1\xi_1 + \theta^2 \, 2^{\frac{1}{4}}\sqrt{m}\,\chi_1 - \frac{1}{2}\theta^\alpha\theta_\alpha F_1$$
$$\Phi_2 = \varphi_2 + \theta^1\xi_2 + \theta^2 \, i2^{\frac{1}{4}}\sqrt{2m}\,\chi_2 - \frac{1}{2}\theta^\alpha\theta_\alpha F_2$$

(ξ_1, F_1) (ξ_2, F_2) $\longrightarrow$ auxiliary (non-dynamical) fields [dim $= 2$]

Action in components

$$S = \int d^3x \left[\bar{\varphi}_1 \left(2im\partial_t + \nabla^2\right) \varphi_1 + \bar{\varphi}_2 \left(4im\partial_t + \nabla^2\right) \varphi_2 \right.$$
$$\left. + \bar{\chi}_1 \left(2im\partial_t + \nabla^2\right) \chi_1 + \bar{\chi}_2 \left(4im\partial_t + \nabla^2\right) \chi_2 \right] + S_{\text{int}}$$

$$S_{\text{int}} = \int d^3x \Big[- 4|g|^2|\varphi_1\varphi_2|^2 - |g|^2|\varphi_1|^4$$
$$- ig \left(\sqrt{2}\varphi_1\chi_1(\partial_1 - i\partial_2)\bar{\chi}_2 - 2\bar{\varphi}_2\chi_1(\partial_1 - i\partial_2)\chi_1 + 2\sqrt{2}\varphi_1((\partial_1 - i\partial_2)\chi_1)\bar{\chi}_2\right) + \text{h.c.}$$
$$+ 2|g|^2 \left(-|\varphi_1|^2\bar{\chi}_1\chi_1 - 4|\varphi_1|^2\bar{\chi}_2\chi_2 + 2|\varphi_2|^2\bar{\chi}_1\chi_1 + 2\sqrt{2}\varphi_1\varphi_2\bar{\chi}_1\bar{\chi}_2 + 2\sqrt{2}\bar{\varphi}_1\bar{\varphi}_2\chi_2\chi_1\right) \Big]$$

Renormalization in Superspace

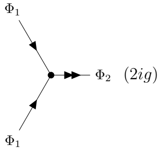
- $\langle \Phi_a \bar{\Phi}_a \rangle$ propagators

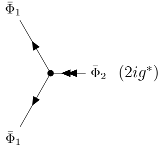
$$D_a(\omega, \vec{p}, \theta_1, \bar{\theta}_1, \theta_2, \bar{\theta}_2) = \frac{i}{2m_a\omega - \vec{p}^{\,2} + i\varepsilon}\, \delta^{(4)}(\theta_1 - \theta_2) \qquad a = 1, 2$$

In configuration space

$$D_a(\vec{x}, t, \theta_1, \bar{\theta}_1, \theta_2, \bar{\theta}_2) = -\frac{i\,\Theta(t)}{4\pi\, t} e^{i\frac{m_a \vec{x}^2}{2t}}\, \delta^{(4)}(\theta_1 - \theta_2)$$

- Supervertices



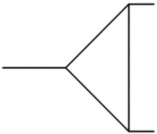


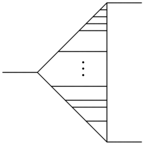
Number of incoming arrows = Number of outgoing arrows

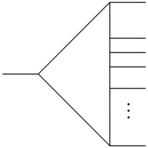
Selection rules

Loop diagrams are formally the same as in the relativistic 2-field WZ model, but....

- Selection rule 1 - Particle number conservation at each vertex

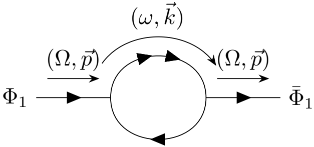


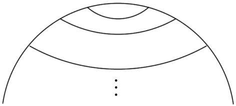




- Selection rule 2 - Arrows inside a Feynman diagram cannot form a closed loop.

O. Bergman, PRD 46 (1992) 5474





Results

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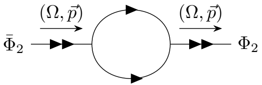

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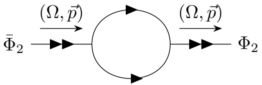

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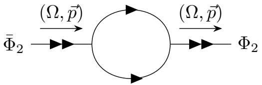

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- **Non-relativistic non-renormalization theorem**

  No vertex corrections allowed. Seiberg's argument can be easily imported and a non-perturbative non-renormalization theorem holds Similar results in Lifshitz theories I.Arav, Y.Oz, A.Raviv-Moshe, JHEP11 ('19)

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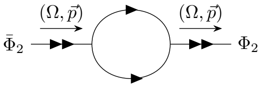

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- **Exact beta-function**

  No UV divergent vertex corrections implies $Z_g = Z_2^{-1/2}$

$$\boxed{\beta_g = \frac{g^3}{4\pi m}} \implies g^2(\mu) = \frac{2\pi m \bar{g}^2}{2\pi m - \bar{g}^2 \log \mu/\Lambda}$$

Galilean Electrodynamics (GED) in d=2+1

M. L. Bellac, J. M. Levy-Leblond, Nuovo Cim. B14 (1973)
A. Bagchi, R. Basu, A. Mehra, JHEP11 (2014)

Null reduction of 4d scalar QED: $A_\mu(x) = A_\mu(t, x_{i=1,2})$ coupled to

$$\phi(x) = e^{imx^-}\phi(t, x_{i=1,2})$$

$$A_\mu \to (A_- \equiv \varphi, A_t, A_{i=1,2})$$

$$S_{GED} = \int dt d^2x \left[\frac{1}{2}(\partial_t\varphi)^2 + E^i\partial_i\varphi - \frac{1}{4}f_{ij}f^{ij} + \frac{i}{2}\bar{\phi}\nabla_t\phi - \phi\nabla_t\bar{\phi} - \frac{1}{2\mathcal{M}}\nabla_i\bar{\phi}\nabla^i\phi\right]$$

where $E_i = \partial_t A_i - \partial_i A_t \quad f_{ij} = \partial_i A_j - \partial_j A_i, \qquad \mathcal{M} \equiv m - e\varphi$

$$\nabla_t\phi = (\partial_t - ieA_t)\phi \qquad \nabla_i\phi = (\partial_i - ieA_i)\phi$$

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At classical level:

- The real scalar field φ is invariant under gauge and galilean transformations. Moreover, $[\varphi] = 0$

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At classical level:

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- The theory exhibits Schroedinger symmetry
- There are no propagating gauge dof

At quantum level: S. Chapman, L. Di Pietro, K.T. Grosvenor, Z. Yan, JHEP10 (2020)

- Non-renormalization of the coupling e

At quantum level: S. Chapman, L. Di Pietro, K.T. Grosvenor, Z. Yan, JHEP10 (2020)

- Non-renormalization of the coupling e
- The theory is not renormalizable: infinitely many marginal couplings get turned on along the RG flow

$$\Delta S_{\text{GED}} = \int dt\, d^2x \left(\mathcal{J}[\mathcal{M}]\, \partial^i \mathcal{M} \partial_i \mathcal{M}\, \bar{\phi}\phi - \frac{1}{4}\lambda \mathcal{V}[\mathcal{M}]\, (\bar{\phi}\phi)^2 - \mathcal{E}[\mathcal{M}]\, (\partial^i \partial_i \mathcal{M} - e^2 \bar{\phi}\phi)\, \bar{\phi}\phi \right)$$

$$\mathcal{M} = m - e\varphi$$

Renormalizable theory is $S_{GED} + \Delta S_{\text{GED}}$

At quantum level: S. Chapman, L. Di Pietro, K.T. Grosvenor, Z. Yan, JHEP10 (2020)

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Renormalizable theory is $S_{GED} + \Delta S_{\rm GED}$

- Conformal manifold of fixed points where the theory exhibits Schroedinger symmetry. This is peculiar of the theory in d=2+1

SuperGalilean Electrodynamics (SGED)

$\mathcal{N} = 2$ SUSY generalization of the Galilean scalar electrodynamics in d=2+1 by null reduction in superspace

$$S_{\rm nSGED} = \frac{1}{2} \int d^3x d^2\theta \; W^\alpha W_\alpha + \int d^3x d^4\theta \; \bar{\Phi} e^{gV} \Phi$$

$$W_\alpha = i\bar{D}^2 D_\alpha V \qquad \bar{D}_\alpha \Phi = D_\alpha \bar{\Phi} = 0$$

At classical level:

- $U(1)_M$ assignment: $M(V) = 0 \quad M(\Phi) = m$

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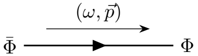
- $U(1)_M$ assignment: $M(V)=0 \quad M(\Phi)=m$
- Super-Schroedinger $SU(2,2|1)$ and $U(1)$ supergauge invariance
- Gauge field components

$$A_t = \frac{1}{2}[\bar{D}_1, D_1]V\Big| \qquad A_1 + iA_2 = \frac{1}{\sqrt{2}}[\bar{D}_1, D_2]V\Big| \qquad \varphi = D_2\bar{D}_2 V\Big|$$

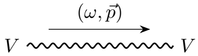
The superfield $D_2\bar{D}_2 V \equiv D\bar{D}V$ is supergauge invariant and dimensionless

Feynman rules in Superspace

Propagators:



$$= \frac{i}{2m\omega - \vec{p}^{\,2} + i\epsilon}\, \delta^{(4)}(\theta' - \theta)$$



$$= -\frac{i}{-\vec{p}^{\,2} + i\epsilon}\, \delta^{(4)}(\theta' - \theta)$$

Vertices: $\bar{\Phi} V^n \Phi$

$\Rightarrow$ Background field method, $V \to V_0 + V$

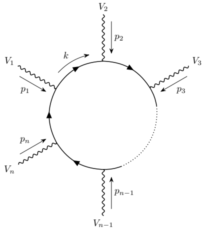
$$\nabla_\alpha = e^{-V} \boldsymbol{\nabla}_\alpha e^{V}\,, \qquad \boldsymbol{\nabla}_\alpha = e^{-V_0/2} D_\alpha e^{V_0/2}\,, \qquad \bar{\nabla}_\beta = \bar{\boldsymbol{\nabla}}_\beta = e^{V_0/2} \bar{D}_\beta e^{-V_0/2}$$

$$\tilde{\Phi} = e^{V_0/2}\,\Phi \qquad \bar{\tilde{\Phi}} = \bar{\Phi}\, e^{V_0/2}$$

Selection rules & non-renormalization theorems

1. Any 1PI Feynman diagram with negative superficial degree of divergence in the ω variable, vanishes identically.

2. All loop corrections to the effective action with purely vector external lines vanish, $\Gamma^{(n)}(V) = 0$.



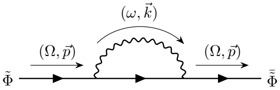
Therefore, V does not renormalize and for gauge invariance g does not renormalize

Non-renormalizability of nSGED

At one-loop the action S_{nSGED} acquires infinite UV divergent contributions

$$\Gamma^{(1L)} \to -\frac{ig^2}{16\pi m\varepsilon} \int d^3x d^4\theta \; \bar{\Phi} e^{gV} \Phi \; \frac{1}{1 - \frac{g}{\sqrt{2}m} \bar{D}DV} \qquad\qquad d = 2 - 2\varepsilon$$

Technical explanation:



$$\frac{1}{\Box_c} = \frac{1}{\Box} + \frac{1}{\Box}\sqrt{2}g(\bar{D}DV)\omega\frac{1}{\Box} + \frac{1}{\Box}\sqrt{2}g(\bar{D}DV)\omega\frac{1}{\Box}\sqrt{2}g(\bar{D}DV)\omega\frac{1}{\Box} + \dots$$

Infinitively marginal couplings turn on. The model is not renormalizable! :-(

A renormalizable SGED

Consider the more general non-linear sigma model

$$S_{SGED} = \frac{1}{2}\int d^3x d^2\theta \, W^\alpha W_\alpha + \int d^3x d^4\theta \, \bar{\Phi} e^{gV} \Phi \, \mathcal{F}(\bar{D}DV)$$

$$\mathcal{F}(\bar{D}DV) = \sum_n \frac{1}{n!} \mathcal{F}^{(n)} (\bar{D}DV)^n$$

We have infinite new $n$pt vertices with couplings $\mathcal{F}^{(n)}$

Non-trivial renormalization

$$\delta\Phi = \frac{g}{16\pi m}\left(g - 2\sqrt{2} m \mathcal{F}^{(1)}\right)\frac{1}{\varepsilon} + \cdots \qquad \delta\mathcal{F}^{(n)} = \frac{g^{n+2}\, n!}{16\pi m (\sqrt{2} m)^n}\frac{1}{\varepsilon} + \cdots$$

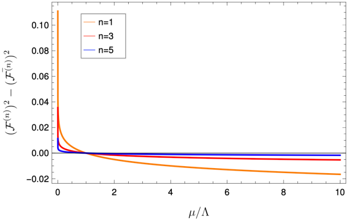
RG flows & conformal manifold

- One-loop beta functions for the couplings

$$\beta^{(n)}_{\mathcal{F}} = \frac{d\mathcal{F}^{(n)}}{d\log\mu} = -g^{n+2}\frac{n!\,n}{16\pi m(\sqrt{2}m)^n\mathcal{F}^{(n)}} \qquad \beta_g = 0$$

Solutions

$$(\mathcal{F}^{(n)})^2(\mu) - (\bar{\mathcal{F}}^{(n)})^2 = - g^{n+2}\frac{n!\,n}{16\pi m(\sqrt{2}m)^n}\log\left(\frac{\mu}{\Lambda}\right)$$



- Anomalous dimensions

$$\gamma_\Phi = \frac{1}{2}\frac{d\log(1+\delta_\Phi)}{d\log\mu} = \frac{g}{8\sqrt{2}\pi}\mathcal{F}^{(1)}$$

• IR interacting fixed point at $g = 0$

$$\gamma_\Phi = 0 \qquad\qquad \beta_{\mathcal{F}}^{(n)} = 0$$

At the fixed point the gauge-matter minimal coupling disappears, but the model contains an infinite number of gauge-matter couplings driven by $\bar{D}DV$

$$S_{SGED} = \frac{1}{2}\int d^3x d^2\theta\, W^\alpha W_\alpha + \int d^3x d^4\theta\, \bar{\Phi}\Phi\, \mathcal{F}(\bar{D}DV)$$

Matrix of anomalous dimensions, $I, J \in \{g, \mathcal{F}^{(1)}, \mathcal{F}^{(2)}, \cdots, \mathcal{F}^{(n)}, \cdots\}$

$$\partial_I \beta^J \to \frac{1}{16\pi m}\left(0, \frac{1}{\sqrt{2}m}\frac{g^3}{(\mathcal{F}^{(1)})^2}, \dots, \frac{n!n}{(\sqrt{2}m)^n}\frac{g^{n+2}}{(\mathcal{F}^{(n)})^2}, \dots\right)$$

At $g = 0$, infinite number of exactly marginal couplings $\Rightarrow$ infinite dimensional superconformal manifold

Conclusions

We have studied quantum properties of NR $\mathcal{N} = 2$ SUSY models in d=(2+1). Working in NR Superspace we have found

1. NR Wess-Zumino model is one-loop exact. Scale invariance is broken by one-loop effects.
   Our results are consistent with the ones for nonSUSY NR $\lambda\phi^4$ theory
   (O. Bergman, PRD46 (1992))

2. SGED: The model which is consistent at quantum level is a non-linear sigma model. This is peculiar of d=(2+1). The topology of the conformal manifold is peculiar of one-loop approximation. At higher loops we expect a richer spectrum of fixed points and further constraints on $\mathcal{F}$ for the existence of a conformal manifold.

Future directions

1. What is the meaning of the coupling $\mathcal{F}(\bar{D}DV)$?

2. Generalization to non-abelian theories
   A. Bagchi, R. Basu, M. Islam, K.S. Kolekar, A. Mehra, JHEP04 (2022)

3. Coupling to supergravity. Models coupled to Newton-Cartan supergravity as null reduction of relativistic SUSY models coupled to Poincaré supergravity
   E.Bergshoeff, A.Chatzistavrakidis, J.Lahnsteiner, L.Romano, J.Rosseel, JHEP07 ('20)

4. NR localization

5. Coupling to Chern-Simons terms and theories with more SUSY.
   Example: NR ABJM **Y. Nakayama, M. Sakaguchi, K. Yoshida, JHEP04 (2009)**
   Y. Nakayama Lett. Math. Phys. 89 (2009)
   K.-M. Lee, S. Lee, S. Lee JHEP09 (2009)
   Y. Nakayama, S-J. Rey, JHEP08 (2009)

6. Defects in NR QFTs, Integrability in NR systems, etc....