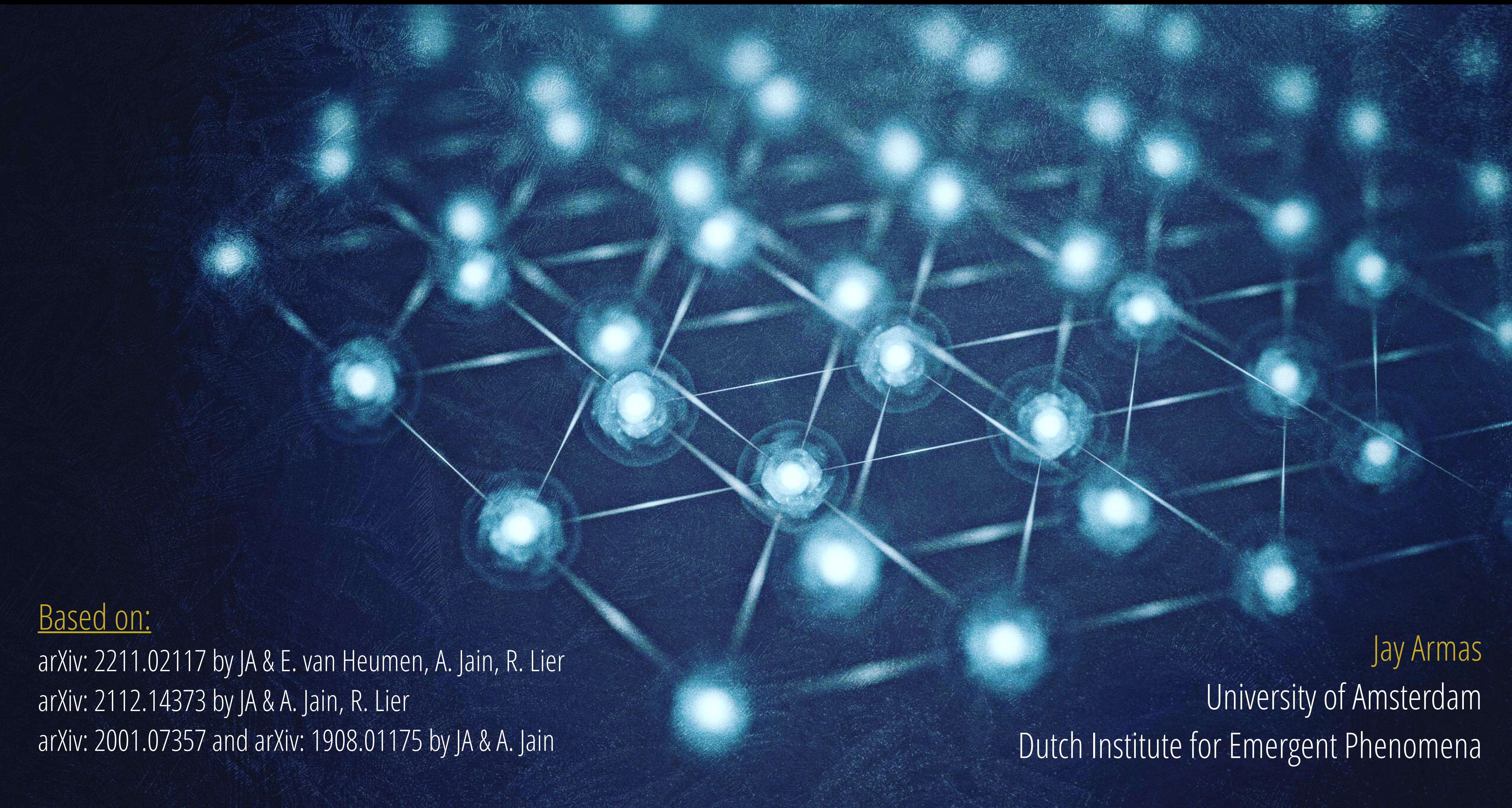
TOWARDS A HYDRODYNAMIC THEORY FOR CONDENSED MATTER SYSTEMS



Based on:

arXiv: 2211.02117 by JA & E. van Heumen, A. Jain, R. Lier

arXiv: 2112.14373 by JA & A. Jain, R. Lier

arXiv: 2001.07357 and arXiv: 1908.01175 by JA & A. Jain

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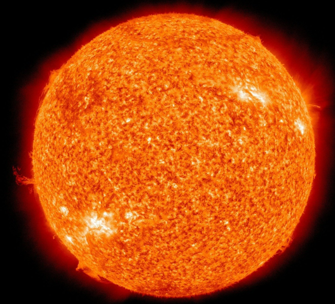
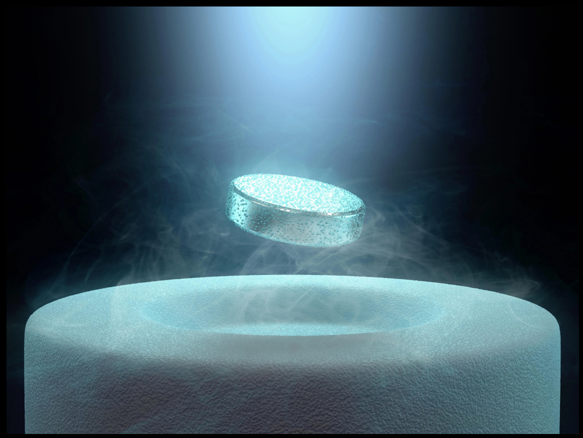
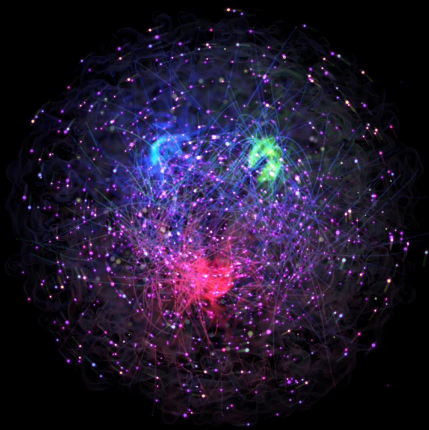
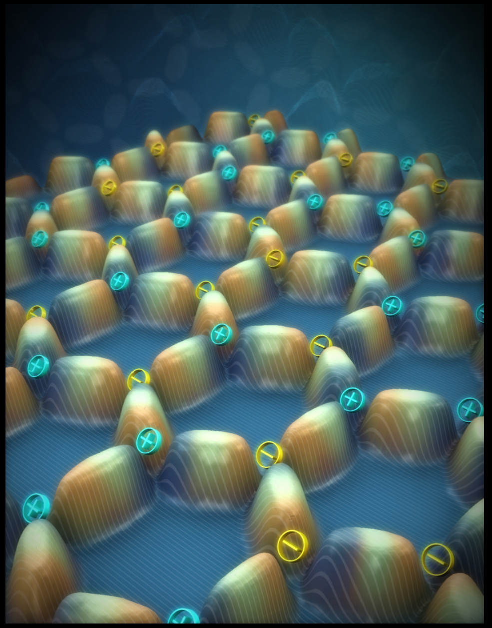
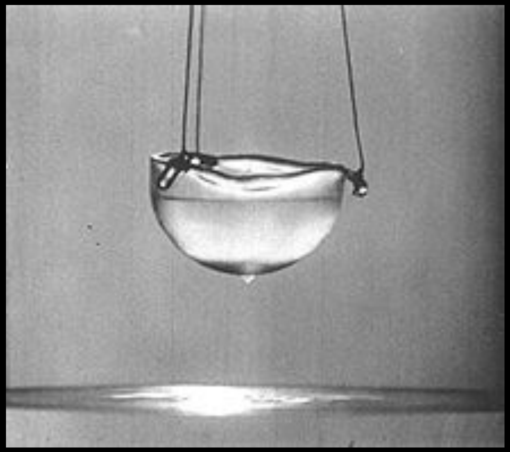
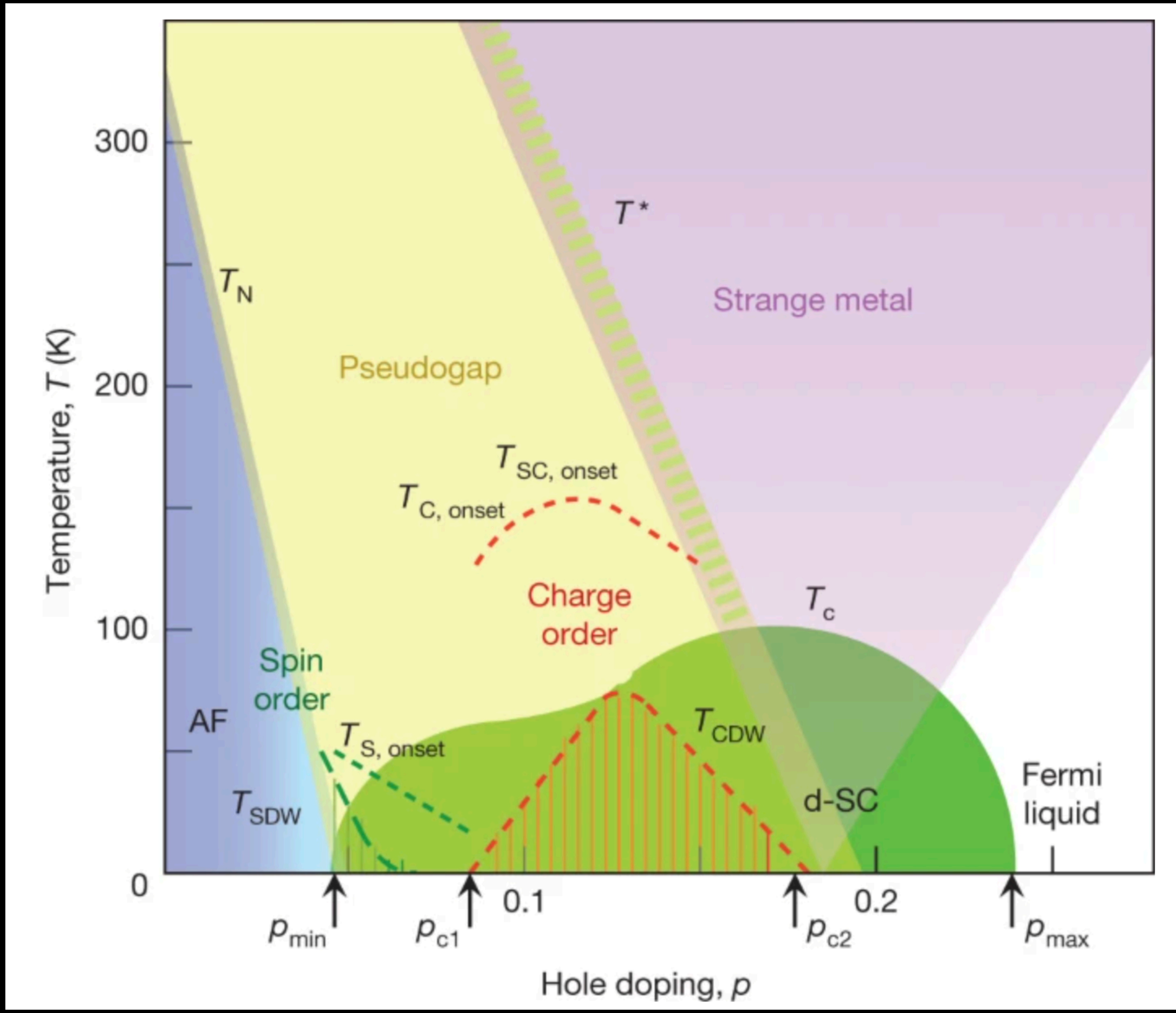



MOTIVATIONS

Nature Communications 518, (2015)



Schematic phase diagram for copper oxides

MOTIVATIONS

>> How do we consistently formulate such phases of matter?

[dissipative effects, gradient expansion, no ad-hoc assymptions on field content or transport, etc]

>> How do we characterise such phases?

[symmetries, how many transport coefficients, responses coefficients, etc]

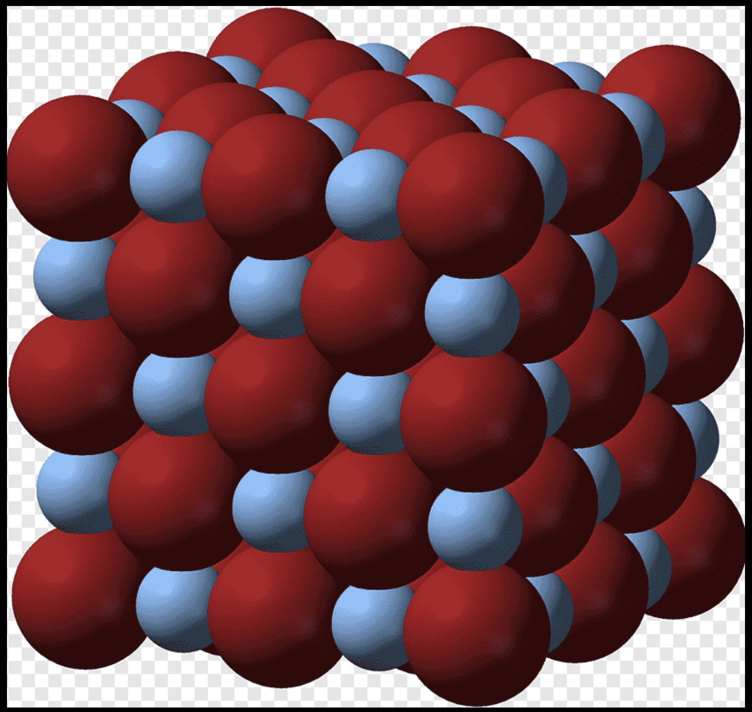
>> How do we describe all equilibrium states?

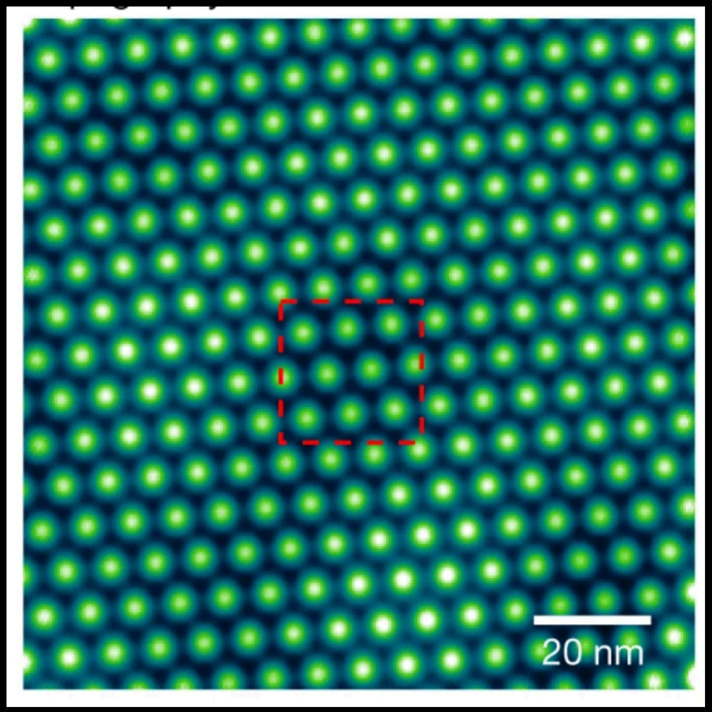
[effective action, partition functions, etc]

>> How do we incorporate in each phase various effects?

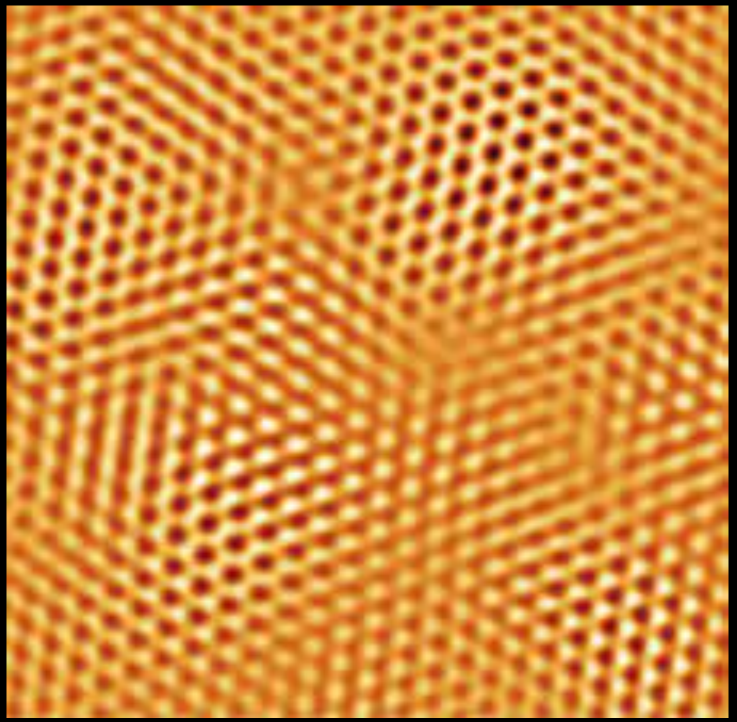
[phase relaxation, topological defects, plasticity, magnetic fields, dynamical EM fields, impurities, (additional symmetry principles), etc]

SPECIFIC MOTIVATIONS





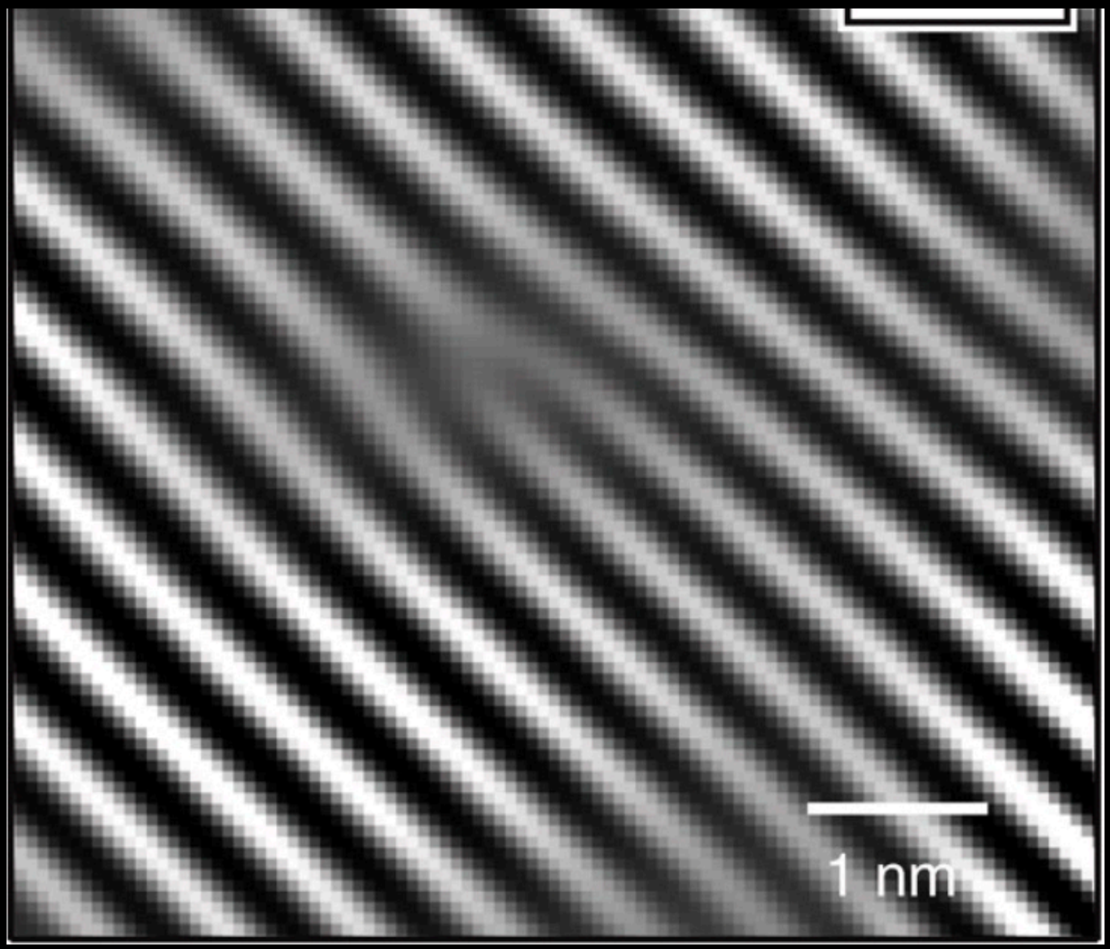
Nature 597 (2021)



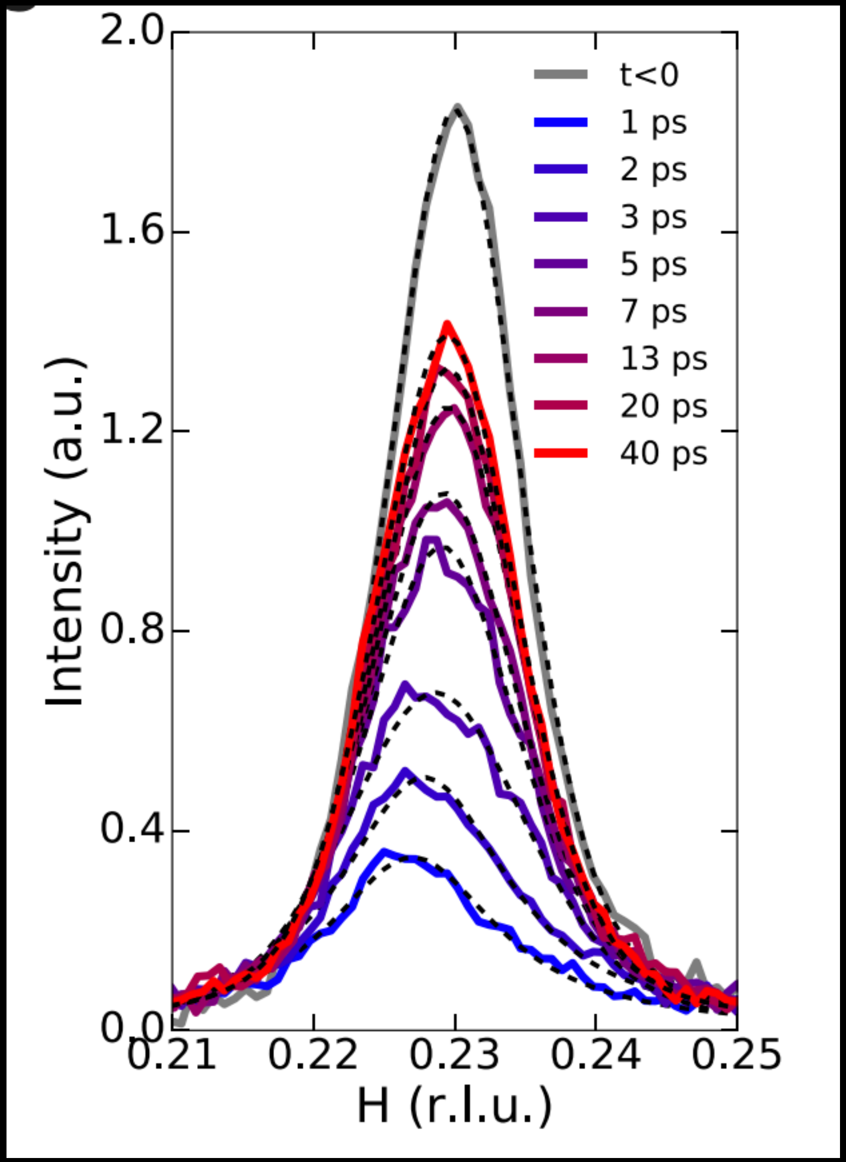
Nature Communications 6, (2015)

Crystallization happens at various scales.

SPECIFIC MOTIVATIONS



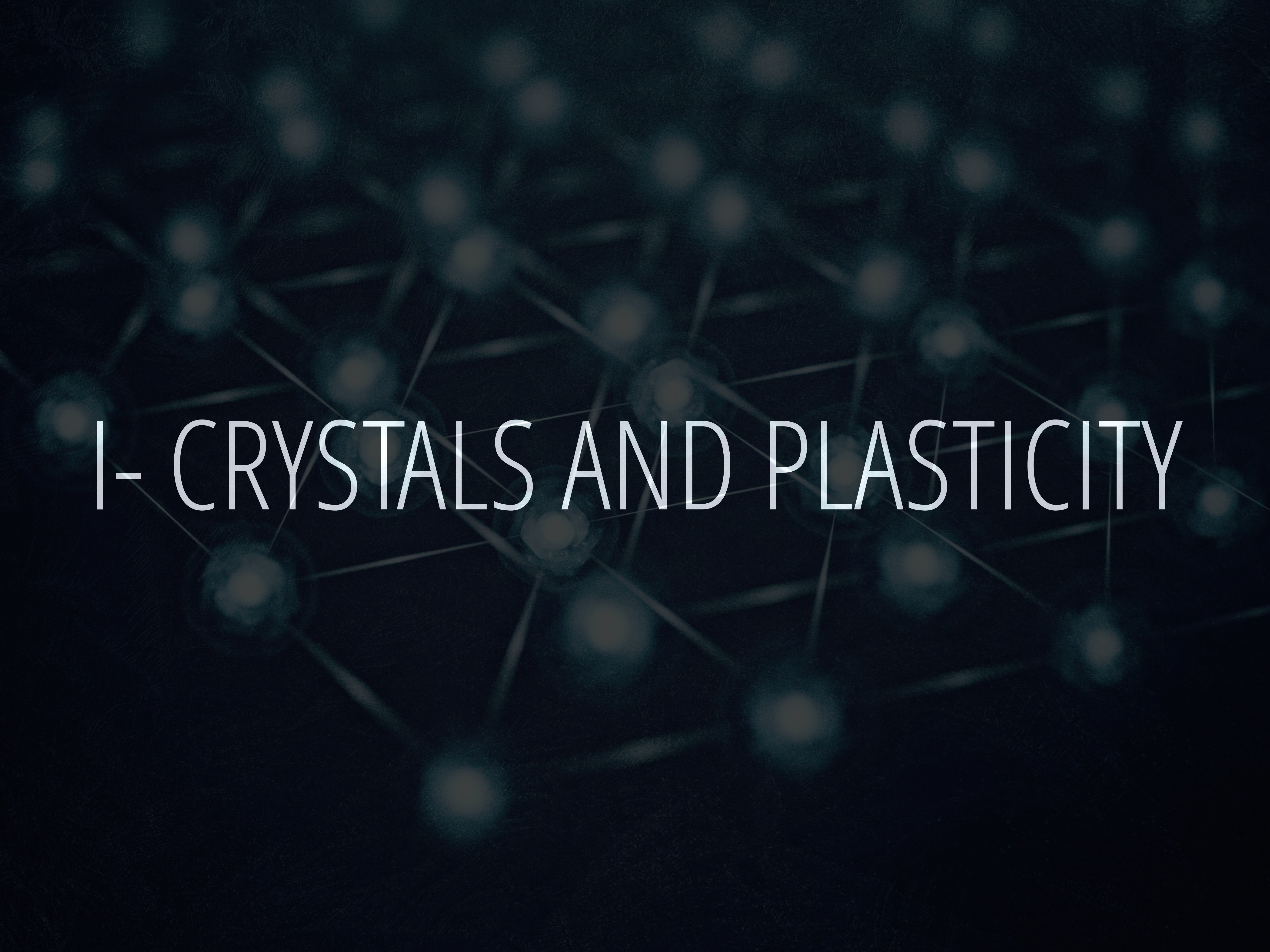
Science Vol 333 (2011)

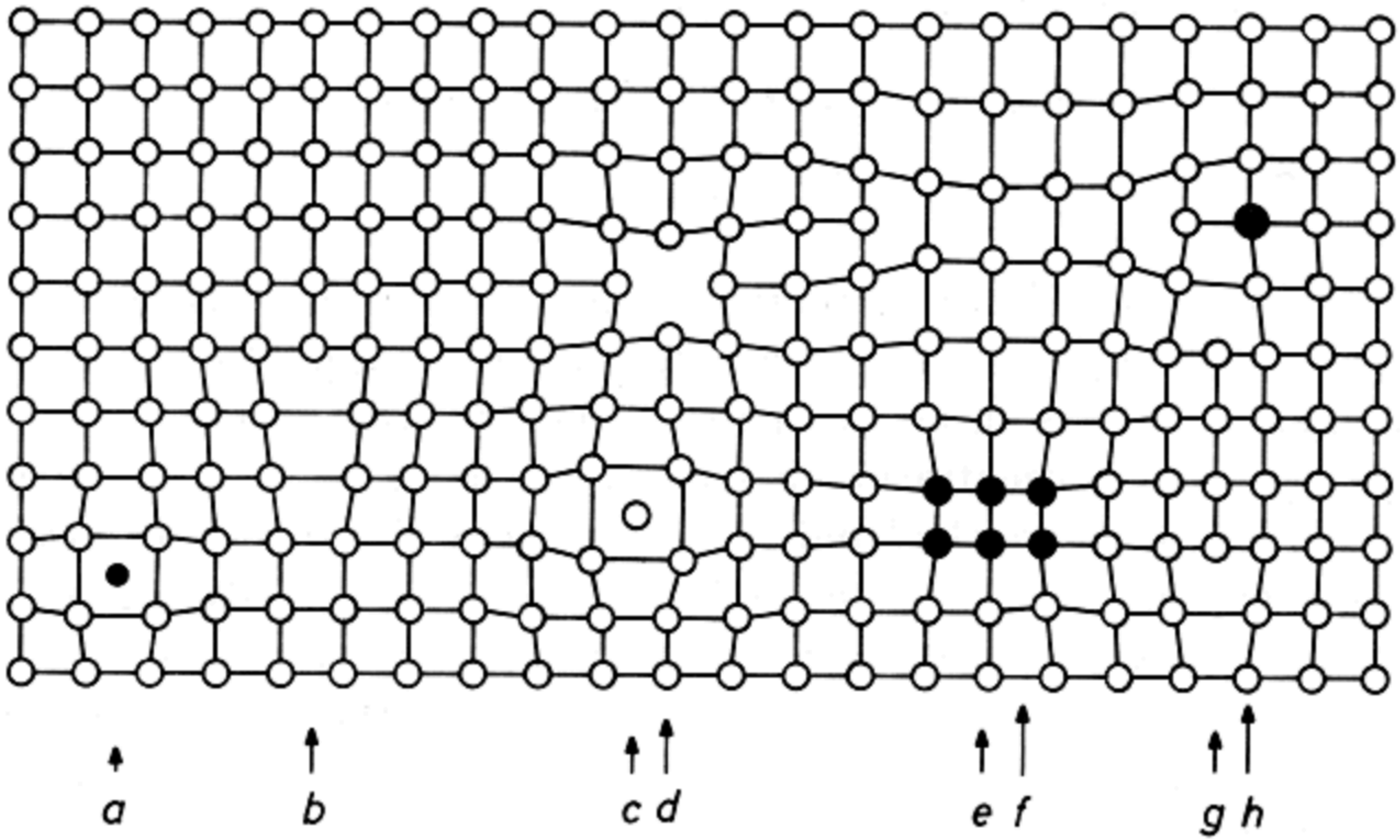


Science Advances Vol 5 (2019)

OUTLINE

(1) CRYSTALS AND PLASTICITY [crystal geometry, symmetries, near-equilibrium dynamics, topological defects]
(2) PINNING AND PLASTICITY [explicit symmetry breaking, competing effects, signatures]
(3) OUTLOOK

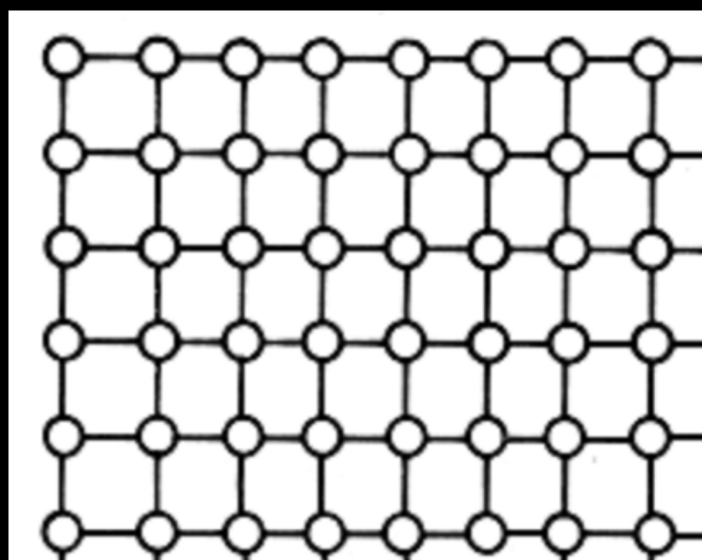

I- CRYSTALS AND PLASTICITY



a) Interstitial impurity atom, b) Edge dislocation, c) Self interstitial atom, d) Vacancy, e) Precipitate of impurity atoms, f) Vacancy type dislocation loop, g) Interstitial type dislocation loop, h) Substitutional impurity atom

Taken from lecture notes of H. Foll

Crystal space



ϕ^I

e_i^I

Background

Spacetime

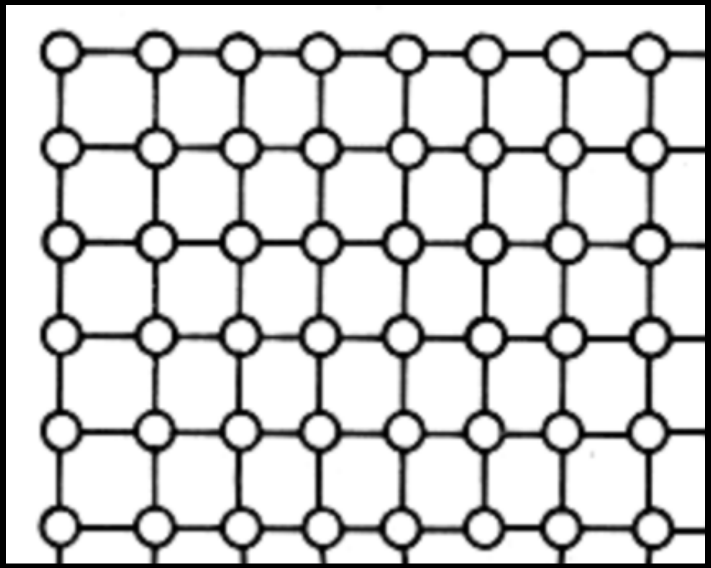
$g_{\mu\nu} \sim \eta_{\mu\nu}$

Space and time

$v^\mu \ , \ h^{\mu\nu}$

Homogeneous $\phi^I \to \phi^I + a^I$ $\Rightarrow$ $e_i^I = \partial_i \phi^I$

Crystal space



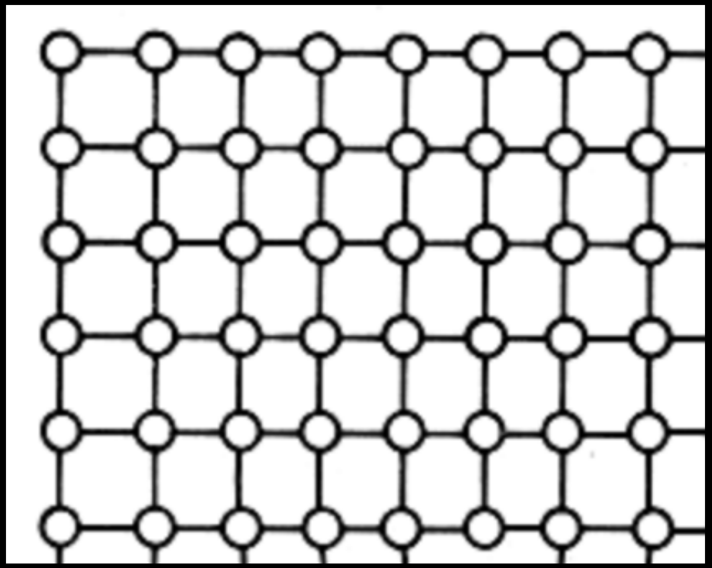
$$ds^2_{\text{crystal}} = h_{IJ} d\phi^I d\phi^J$$

Induced metric $$h^{IJ} = e^{Ii} e^J_i$$

Reference metric $$ds^2_{\text{reference}} = h^{\text{ref}}{}_{IJ} d\phi^I d\phi^J$$

Strain tensor $$\kappa_{IJ} = \frac{1}{2}(h_{IJ} - h^{\text{ref}}{}_{IJ})$$

Physics should be invariant under redefinitions of lattice sites



$$\mathrm{Diff}(\phi) : d\phi^I \to \Lambda^I{}_J(\phi) d\phi^J, \ \partial_{[K}\Lambda^I{}_{J]} = 0,$$

$$h^{IJ} \to \Lambda^I{}_K \Lambda^J{}_L h^{KL}$$

$$h^{\mathrm{ref}}_{IJ} \to (\Lambda^{-1})^K{}_I (\Lambda^{-1})^L{}_J h^{\mathrm{ref}}_{KL}$$

Physical strain is invariant

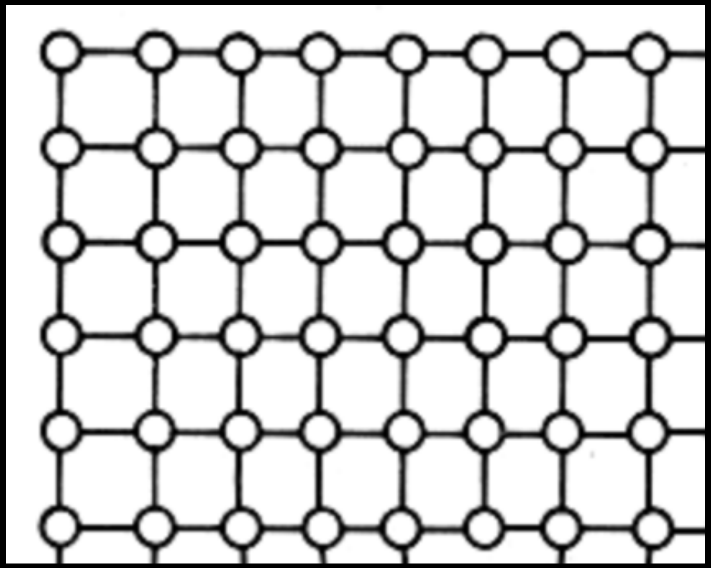
$$\kappa_{ij} = e^I_i e^J_j \kappa_{IJ}$$

Crystal velocity

$$u^i_\phi = -e^i_I \partial_t \phi^I$$

Local rest frame

$$(\partial_t + u^i_\phi \partial_i)\phi^I = 0$$



What is an elastic crystal?

$$\dot{h}^{\mathrm{ref}}_{IJ} \equiv \left(\partial_t + u^i_\phi \partial_i\right) h^{\mathrm{ref}}_{IJ} = 0$$

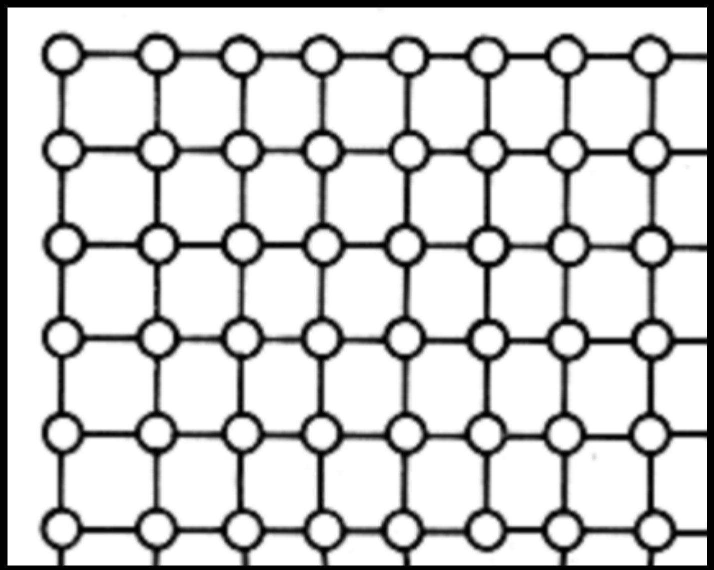
$$\mathrm{Diff}(\phi) \to SO(\phi)$$

$$h^{\mathrm{ref}}_{IJ} \to \delta_{IJ}$$

What is a plastic crystal?

h^{ref}_{IJ} evolves dynamically

(No assumptions on the microscopic origin of plastic deformations, ignored "bound-angle" plasticity)



What can you fix in a plastic crystal?

h^{ref}_{IJ} evolves dynamically

Distortion strain

$$\varepsilon_{IJ} = \frac{1}{2}(h_{IJ} - h^{\mathrm{ref}}_{IJ}(t=0)) = \frac{1}{2}(h_{IJ} - \delta_{IJ})$$

"Time derivative" of distortion strain

$$\dot{\varepsilon}_{IJ} \equiv (\partial_t + u^i_\phi \partial_i)\varepsilon_{IJ} = (\partial_t + u^i_\phi \partial_i) h_{IJ}/2 = e^i_I e^j_J \partial_{(i} u^\phi_{j)} \ .$$

Controlling plasticity

$$h^{\mathrm{ref}}_{IJ} = \delta_{IJ} + \ell \psi_{IJ}$$

$$\boxed{\ell \sim \mathcal{O}(\partial)}$$

CRYSTALS IN EQUILIBRIUM

Free energy:

$$F = -\int \mathrm{d}^d x \left(p(T, \mu, \vec{u}^2, h^{IJ}, \mathbb{h}_{IJ}) + T^{ij}_{\text{ext}} \kappa_{ij} \right)$$

Variations:

$$\delta S = \int \mathrm{d}t \mathrm{d}^d x \sqrt{\gamma} \bigg[\left(\pi^\mu v^\nu + \frac{1}{2} \tau^{\mu\nu} \right) \delta h_{\mu\nu} - \epsilon^\mu \delta n_\mu + j^\mu \delta A_\mu$$
$$+ K_I \delta\phi^I + \frac{1}{2} U^{IJ} \left(\delta\psi_{IJ} - e^\mu_K \partial_\mu \psi_{IJ} \delta\phi^K \right)$$

CRYSTALS IN EQUILIBRIUM

Thermodynamics:

$$\epsilon = -p + Ts + \mu n + u^i \pi_i.$$

$$\mathrm{d}p = s\mathrm{d}T + n\mathrm{d}\mu + \pi_i \mathrm{d}u^i + \frac{1}{2} r_{IJ} \mathrm{d}h^{IJ} + \frac{1}{2} \mathbb{r}^{IJ} \mathrm{d}\mathbb{h}_{IJ}$$

Equations of motion:

$$-\partial_i \left(r_{IJ} e^{Ji} \right) + \frac{\ell}{2} \mathbb{r}^{JK} e^i_I \partial_i \psi_{JK} + K^{\text{ext}}_I = 0$$

$$\ell \mathbb{r}^{IJ} + U^{IJ}_{\text{ext}} = 0$$

CRYSTALS IN EQUILIBRIUM

Small strains:

$$p = p_f + p_\ell \, \kappa^I{}_I - \frac{1}{2} C^{IJKL} \kappa_{IJ} \kappa_{KL}$$

Elasticity tensor:

$$C^{IJKL} = \left(B - \frac{2}{d} G \right) h^{IJ} h^{KL} + 2G \, h^{I(K} h^{L)J}$$

Expansion coefficeints:

$$\alpha_T = \frac{1}{B} \frac{\partial p_\ell}{\partial T}, \quad \alpha_m = \frac{1}{B} \frac{\partial p_\ell}{\partial \mu}, \quad \alpha_u = \frac{2}{B} \frac{\partial p_\ell}{\partial \vec{u}^2}$$

OUT OF EQUILIBRIUM

Equations of motion:

$$K_I + K_I^{\text{ext}} = 0$$
$$U^{IJ} + U^{IJ}_{\text{ext}} = 0$$

Conservation laws
(Ward identities)

$$\partial_t \epsilon + \partial_i \epsilon^i = -K_I \partial_t \phi^I - \frac{1}{2} U^{IJ} (\partial_t + u^k_\phi \partial_k) \psi_{IJ}$$
$$\partial_t \pi^i + \partial_j \tau^{ij} = K_I \partial^i \phi^I ,$$
$$\partial_t n + \partial_i j^i = 0 \quad ,$$

Second law
of thermodynamics:

$$\partial_t s^t + \partial_i s^i = \Delta \geq 0$$

OUT OF EQUILIBRIUM

Parametrising corrections:

$$\epsilon^i = (\epsilon + p)u^i + r_{IJ} e^{Ii} e^J_t + \mathcal{T}^{ij} u_j + \mathcal{E}^i \quad ,$$
$$\tau^{ij} = \rho\, u^i u^j + p\, \delta^{ij} - e^i_I e^j_J r^{IJ} + \mathcal{T}^{ij} \quad ,$$
$$j^i = n u^i + \mathcal{J}^i \quad ,$$
$$K_I = -\partial_i (r_{IJ} e^{Ji}) + \frac{\ell}{2} \mathbb{r}^{JK} e^i_I \partial_i \psi_{JK} + \mathcal{K}_I$$
$$U^{IJ} = \ell \mathbb{r}^{IJ} + \mathcal{U}^{IJ} \quad ,$$

Solution:

$$\begin{pmatrix} \mathcal{E}^I \\ \mathcal{J}^I \\ \mathcal{K}^I \end{pmatrix} = - \begin{pmatrix} \sigma_\epsilon & \gamma_{\epsilon n} & \gamma_{\epsilon\phi} \\ \gamma'_{\epsilon n} & \sigma_n & \gamma_{n\phi} \\ \gamma'_{\epsilon\phi} & \gamma'_{n\phi} & \sigma_\phi \end{pmatrix} \begin{pmatrix} e^I_i \frac{1}{T} \partial^i T \\ e^I_i T \partial^i \frac{\mu}{T} \\ (\partial_t + u^i \partial_i) \phi^I \end{pmatrix}$$

$$\begin{pmatrix} \mathcal{T}^{IJ} \\ \mathcal{U}^{IJ} \end{pmatrix} = -h^{IJ} \begin{pmatrix} \zeta_\tau & \zeta_{\tau \mathbb{h}} \\ \zeta'_{\tau \mathbb{h}} & \zeta_{\mathbb{h}} \end{pmatrix} \begin{pmatrix} \partial_k u^k \\ \frac{1}{2} h^{KL} (\partial_t + \bar{u}^i \partial_i) \psi_{KL} \end{pmatrix}$$

$$- \begin{pmatrix} \eta_\tau & \eta_{\tau \mathbb{h}} \\ \eta'_{\tau \mathbb{h}} & \eta_{\mathbb{h}} \end{pmatrix} \begin{pmatrix} 2 e_i^{\langle I} e_j^{J\rangle} \partial^i u^j \\ h^{K\langle I} h^{J\rangle L} (\partial_t + \bar{u}^i \partial_i) \psi_{KL} \end{pmatrix}$$

OUT OF EQUILIBRIUM

Constraints Onsager:

$$\gamma'_{\epsilon n} = \gamma_{\epsilon n}, \qquad \gamma'_{\epsilon\phi} = -\gamma_{\epsilon\phi}, \qquad \gamma'_{n\phi} = -\gamma_{n\phi},$$
$$\eta'_{\tau\mathbb{h}} = \eta_{\tau\mathbb{h}}, \qquad \zeta'_{\tau\mathbb{h}} = \zeta_{\tau\mathbb{h}}.$$

Constraints entropy:

$$\eta_\tau \geq 0, \qquad \zeta_\tau \geq 0, \qquad \sigma_\epsilon \geq 0, \qquad \sigma_\phi \geq 0,$$
$$\sigma_\epsilon \sigma_n \geq \gamma_{\epsilon n}^2, \qquad \eta_\tau \eta_{\mathbb{h}} \geq \eta_{\tau\mathbb{h}}^2, \qquad \zeta_\tau \zeta_{\mathbb{h}} \geq \zeta_{\tau\mathbb{h}}^2.$$

Galiliean symmetry: $\gamma_{\epsilon n}, \gamma'_{\epsilon n}, \gamma_{n\phi}, \gamma'_{n\phi},$ vanish

Relativistic symmetry: $\gamma_{\epsilon n}, \gamma'_{\epsilon n}, \gamma_{\epsilon\phi}, \gamma'_{\epsilon\phi}, \sigma_\epsilon$ vanish

LINEARIZATION

$$u^i, \qquad \mu = \mu_0 + \delta\mu,$$
$$\phi^I = x^I - \delta^I_i \delta\phi^i, \qquad \psi_{IJ}.$$

Josephson equation:

$$u^i_\phi = u^i + D^{\parallel}_\phi \partial^i \kappa^k{}_k + 2D^{\perp}_\phi \left(\partial_j \kappa^{ij} - \partial^i \kappa^k{}_k\right) - \left(\gamma_n + \frac{B\alpha_m}{\sigma_\phi}\right)\partial^i \mu \ ,$$

Diffusion coefficients:

$$D^{\parallel}_\phi = \frac{B + 2\frac{d-1}{d}G}{\sigma_\phi}, \quad D^{\perp}_\phi = \frac{G}{\sigma_\phi}, \quad \gamma_n = \frac{\gamma_{n\phi}}{\sigma_\phi}$$

LINEARIZATION

$$u^i, \qquad \mu = \mu_0 + \delta\mu,$$
$$\phi^I = x^I - \delta^I_i \delta\phi^i, \qquad \psi_{IJ}.$$

Strain evolution:

$$\dot{\kappa}_{ij} = \frac{1}{d}\lambda_B \delta_{ij}\partial_k u^k + \lambda_G \partial_{\langle i} u_{j\rangle} - \left(\gamma_n + \frac{B\alpha_m}{\sigma_\phi}\right)\partial_i\partial_j\mu$$
$$+ D^{\parallel}_{\phi}\partial_i\partial_j\kappa^k{}_k + 2D^{\perp}_{\phi}\left(\partial_{(i}\partial^k\kappa_{j)k} - \partial_i\partial_j\kappa^k{}_k\right)$$
$$- \frac{1}{d}\delta_{ij}\Omega_B\left(\kappa^k{}_k - \alpha_m\delta\mu\right) - \Omega_G\kappa_{\langle ij\rangle} \; .$$

Relaxation coefficients:

$$\lambda_G = 1 + \frac{\ell\eta_{\tau\mathbb{h}}}{\eta_{\mathbb{h}}}, \qquad \lambda_B = 1 + \frac{\ell\zeta_{\tau\mathbb{h}}}{\zeta_{\mathbb{h}}},$$
$$\Omega_G = \frac{\ell^2 G}{\eta_{\mathbb{h}}}, \qquad \Omega_B = \frac{\ell^2 B}{\zeta_{\mathbb{h}}}.$$

NOTE ON INTERSTIALS/VACANCIES

Interstials density and flux:

$$n_\Delta = n - m_0 v,$$
$$j^i_\Delta = j^i - m_0 v\, u^i_\phi,$$

Local volume element:

$$v = \sqrt{\det\left(e^I_i e^J_j \mathbb{h}_{IJ}\right)},$$

Interstials conservation:

$$\partial_t n_\Delta + \partial_i j^i_\Delta = -m_0(\partial_t + u^k_\phi \partial_k) \det \mathbb{h}$$

NOTE ON INTERSTIALS/VACANCIES

Corrections to interstial density and flux

$$n_\Delta = \delta n + \kappa^k{}_k,$$
$$j^i_\Delta = -\sigma_\Delta \partial^i \mu - nD^{\parallel}_\Delta \partial^i \kappa^k{}_k - 2D^{\perp}_\Delta \left(\partial_j \kappa^{ij} - \partial^i \kappa^k{}_k\right)$$

Coefficients:

$$\sigma_\Delta = \sigma - n\gamma_n - \left(1 - \frac{\gamma_{n\phi}}{n}\right) \frac{nB\alpha_m}{\sigma_\phi},$$
$$D^{\parallel}_\Delta = \left(1 - \frac{\gamma_{n\phi}}{n}\right) D^{\parallel}_\phi, \quad D^{\perp}_\Delta = \left(1 - \frac{\gamma_{n\phi}}{n}\right) D^{\perp}_\phi$$

$$\sigma = \sigma_n + \frac{\gamma^2_{n\phi}}{\sigma_\phi},$$

NOTE ON INTERSTIALS/VACANCIES

Glide constraint:

$$\partial_t n_\Delta + \partial_i j^i_\Delta = -m_0(\partial_t + u^k_\phi \partial_k) \det \mathbb{h}$$

$$\Omega_B = 0$$

MODE SPECTRUM

Solid regime: $\ell \ll k \ll L_T^{-1}$

Transverse sound mode: $$\omega = \pm v_\perp k - \frac{i}{2}\left(\Gamma_\perp k^2 + \Omega_\perp\right) + \ldots,$$

Coefficients: $$v_\perp^2 = \frac{G}{\rho}, \qquad \Gamma_\perp = \frac{\eta}{\rho} + \frac{G}{\sigma_\phi}, \qquad \Omega_\perp = \Omega_G.$$

Longitudinal sound and diffusion: $$\omega = \pm v_\parallel k - \frac{i}{2}\left(\Gamma_\parallel k^2 + \Omega_\parallel\right) + \ldots,$$
$$\omega = -iD_\parallel k^2 - i\Omega_D + \ldots.$$

MODE SPECTRUM

Liquid regime: $k \ll \ell \ll L_T^{-1}$

Transverse shear mode: $\omega = -i\frac{\eta_l}{\rho}k^2 + \dots,$

Longitudinal sound: $\omega = \pm\sqrt{\frac{n^2}{\rho\chi_l}}k - \frac{i}{2}\left(\frac{\zeta_l + 2\frac{d-1}{d}\eta_l}{\rho} + \frac{\sigma}{\chi_l}\right)k^2$

Coefficients:

$$\chi_l = \chi + B\alpha_m^2,$$

$$\eta_l = \eta + \frac{G}{\Omega_G},$$

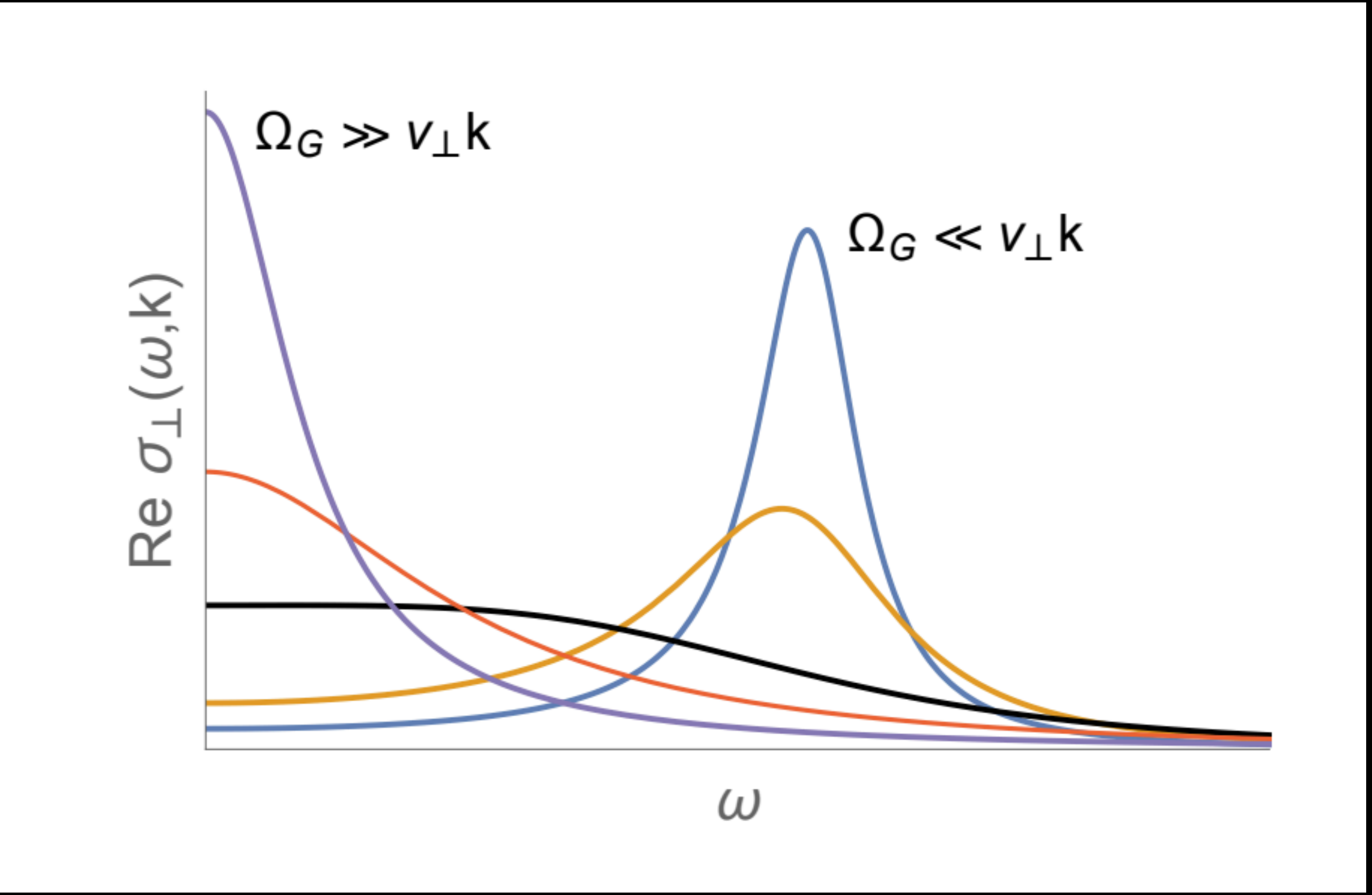
$$\zeta_l = \zeta + \left(1 + \frac{n\alpha_m}{\chi + B\alpha_m^2}\right)^2 \frac{B}{\Omega_B}.$$

+ gapped modes

CORRELATION FUNCTIONS:

Current-current:

$$G^R_{j^y j^y}(\omega,k) = \frac{n^2}{\rho} - i\omega\sigma - \frac{Gk^2\left(n/\rho - i\omega\gamma_n\right)^2}{i\omega(i\omega-\Omega_G)+k^2\left(v_\perp^2 - i\omega D_\phi^\perp\right)}\,.$$

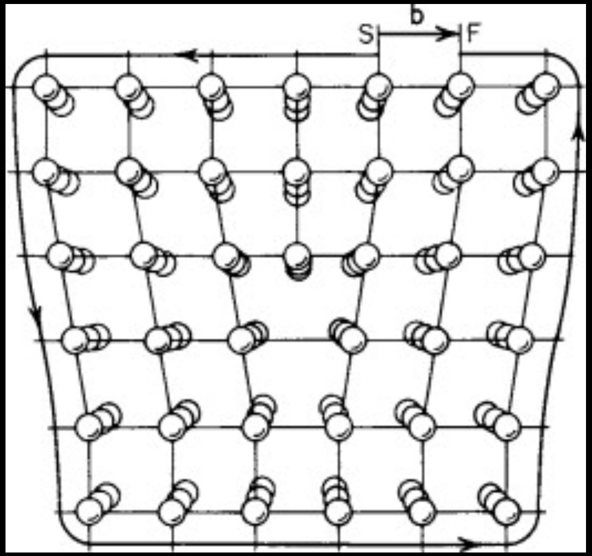


CORRELATION FUNCTIONS:

Strain-strain:

$$G^R_{\kappa^{ij}\kappa^{kl}}(\omega) = -\delta^{ij}\delta^{kl}\frac{1}{4B}\frac{\Omega_B}{\left(1+\frac{B\alpha_m^2}{\chi}\right)\Omega_B - i\omega} - 2\delta^{i\langle k}\delta^{l\rangle j}\frac{1}{4G}\frac{\Omega_G}{\Omega_G - i\omega},$$

TOPOLOGICAL DEFECTS:



Crystal frame field:

$$\tilde{e}_i^I = \partial_i \tilde{\varphi}^I$$

Defect density:

$$\epsilon^{ij}\partial_i \tilde{e}_j^I = \ell n_{\mathrm{dloc}}^I \neq 0$$

Burgers vector:

$$\begin{aligned} B^I &= \int_U \mathrm{d}^2x\, \ell n_{\mathrm{dloc}}^I \\ &= \oint_{\partial U} \mathrm{d}x^i \partial_i \tilde{\varphi}^I \ . \end{aligned}$$

Conservation law:

$$\partial_t n_{\mathrm{dloc}}^I + \partial_i j_{\mathrm{dloc}}^I = 0$$

$$\ell j_{\mathrm{dloc}}^{Ii} = -\epsilon^{ij}\partial_t \tilde{e}_j^I + \epsilon^{ij}\partial_j \tilde{e}_t^I$$

TOPOLOGICAL DEFECTS:

Split into smooth and singular parts:

$$\partial_i \tilde{\varphi}^I = \partial_i \phi^I + \ell V_i^I$$
$$\partial_t \tilde{\varphi}^I = \partial_t \phi^I + \ell V_t^I$$

Define strain:

$$\tilde{\kappa}_{IJ} = \frac{1}{2}\left(\tilde{h}_{IJ} - \delta_{IJ}\right)$$

Identify the formulations:

$$\kappa_{ij} = e_i^I e_j^J \kappa_{IJ} = \tilde{e}_i^I \tilde{e}_j^J \tilde{\kappa}_{IJ}$$

$$e_i^I e_j^J \psi_{IJ} = 2\delta_{IJ} e^I_{(i} V^J_{j)} + \ell^2 \delta_{IJ} V_i^I V_j^J$$

II- PINNING AND PLASTICITY

PINNING

Explicitly break translation symmetry: $\Phi^I(x)$

Transformations: $\Phi^I(x) \to \Phi^I(x) + a^I$

$$\phi^I(x) \to \phi^I(x) + a^I$$

Pinning term: $p \sim -\frac{1}{2}\ell'^2 m^2 h_{IJ}\left(\phi^I - \Phi^I\right)\left(\phi^J - \Phi^J\right)$

$$\ell' \sim \mathcal{O}(\partial)$$

PINNING

Conservation laws:

$$\partial_t \epsilon + \partial_i \epsilon^i = -K_I \partial_t \phi^I - \frac{1}{2} U^{IJ} (\partial_t + \bar{u}^k \partial_k) \psi_{IJ} - \ell' L_I \partial_t \Phi^I,$$

$$\partial_t \pi^i + \partial_j \tau^{ij} = K_I \partial^i \phi^I + \ell' L_I \partial^i \Phi^I,$$

$$\partial_t n + \partial_i j^i = 0 \quad .$$

Ansatz:

$$K_I = -\partial_i \left(r_{IJ} e^{Ji} \right) + \frac{\ell}{2} \Gamma^{JK} e^i_I \partial_i \psi_{JK} - \ell'^2 m^2 h_{IJ} \left(\phi^I - \Phi^I \right) + \mathcal{K}_I,$$

$$L_I = \ell' m^2 h_{IJ} \left(\phi^I - \Phi^I \right) + \mathcal{L}_I \quad .$$

Solution:

$$\begin{pmatrix} \mathcal{E}^I \\ \mathcal{J}^I \\ \mathcal{K}^I \\ \mathcal{L}^I \end{pmatrix} = - \begin{pmatrix} \sigma_\epsilon & \gamma_{\epsilon n} & \gamma_{\epsilon \phi} & \gamma_{\epsilon \Phi} \\ \gamma'_{\epsilon n} & \sigma_n & \gamma_{n\phi} & \gamma_{n\Phi} \\ \gamma'_{\epsilon\phi} & \gamma'_{n\phi} & \sigma_\phi & \sigma_\times \\ \gamma'_{\epsilon\Phi} & \gamma'_{n\Phi} & \sigma'_\times & \sigma_\Phi \end{pmatrix} \begin{pmatrix} e^I_i \frac{1}{T} \partial^i T \\ e^I_i T \partial^i \frac{\mu}{T} \\ (\partial_t + u^i \partial_i) \phi^I \\ \ell' (\partial_t + u^i \partial_i) \Phi^I \end{pmatrix} .$$

PINNING | linearization

Josephson equation:

$$u^i_\phi = \lambda u^i + D^{\parallel}_\phi \partial^i \kappa^k{}_k + 2D^{\perp}_\phi \left(\partial_j \kappa^{ij} - \partial^i \kappa^k{}_k\right) - \left(\gamma_n + \frac{B\alpha_m}{\sigma_\phi}\right) \partial^i \mu - \Omega_\phi \delta\phi^i \ ,$$

$$\Omega_\phi = \frac{\ell'^2 m^2}{\sigma_\phi}, \qquad \lambda = 1 + \frac{\ell' \sigma_\times}{\sigma_\phi}$$

Strain evolution:

$$\dot{\kappa}_{ij} = \frac{1}{d}\lambda'_B \delta_{ij}\partial_k u^k + \lambda'_G \partial_{\langle i} u_{j\rangle} - \left(\gamma_n + \frac{B\alpha_m}{\sigma_\phi}\right)\partial_i\partial_j\mu + D^{\parallel}_\phi \partial_i\partial_j \kappa^k{}_k + 2D^{\perp}_\phi \left(\partial_{(i}\partial^k \kappa_{j)k} - \partial_i\partial_j\kappa^k{}_k\right) - \frac{1}{d}\delta_{ij}\Omega_B\left(\kappa^k{}_k - \alpha_m \delta\mu\right) - \Omega_G \kappa_{\langle ij\rangle} - \Omega_\phi \varepsilon_{ij},$$

$$\lambda'_B = 1 + \frac{\ell \eta_{\tau\mathbb{h}}}{\eta_{\mathbb{h}}} + \frac{\ell'\sigma_\times}{\sigma_\phi}, \qquad \lambda'_G = 1 + \frac{\ell\zeta_{\tau\mathbb{h}}}{\zeta_{\mathbb{h}}} + \frac{\ell'\sigma_\times}{\sigma_\phi}$$

PINNING | linearization

Damping attenuation:

$$\Omega_\phi = D_\phi^\perp \omega_0^2 / v_\perp^2 = D_\phi^{||} \omega_0^2 / v_{||\phi}^2$$

Pinning frequency:

$$\omega_0^2 = \frac{\lambda^2 \ell'^2 m^2}{\rho}$$

PINNING | modes

Solid regime: $\ell \ll k \ll L_T^{-1}$

Transverse sound mode:

$$\omega = \pm\sqrt{\omega_0^2 + v_\perp^2 k^2} - \frac{i}{2}\Big(\Gamma_\perp k^2 + \Omega_\perp(k) + \Gamma\Big)$$

Coefficients (Pinning dominates):

$$v_\perp^2 = \frac{\lambda^2 G}{\rho},$$

$$\Omega_\perp(k) = \frac{v_\perp^2 k^2}{\omega_0^2 + v_\perp^2 k^2}\Omega_G + \Omega_\phi$$

Longitudinal sound and diffusion:

$$\omega = \pm\sqrt{\omega_0^2 + v_\parallel^2 k^2} - \frac{i}{2}\left(\Gamma_\parallel(k)k^2 + \Omega_\parallel(k) + \Gamma\right)$$

$$\omega = -iD_\parallel k^2 - i\Omega_D,$$

PINNING | modes

Liquid regime: $k \ll \ell \ll L_T^{-1}$

Transverse sound mode:

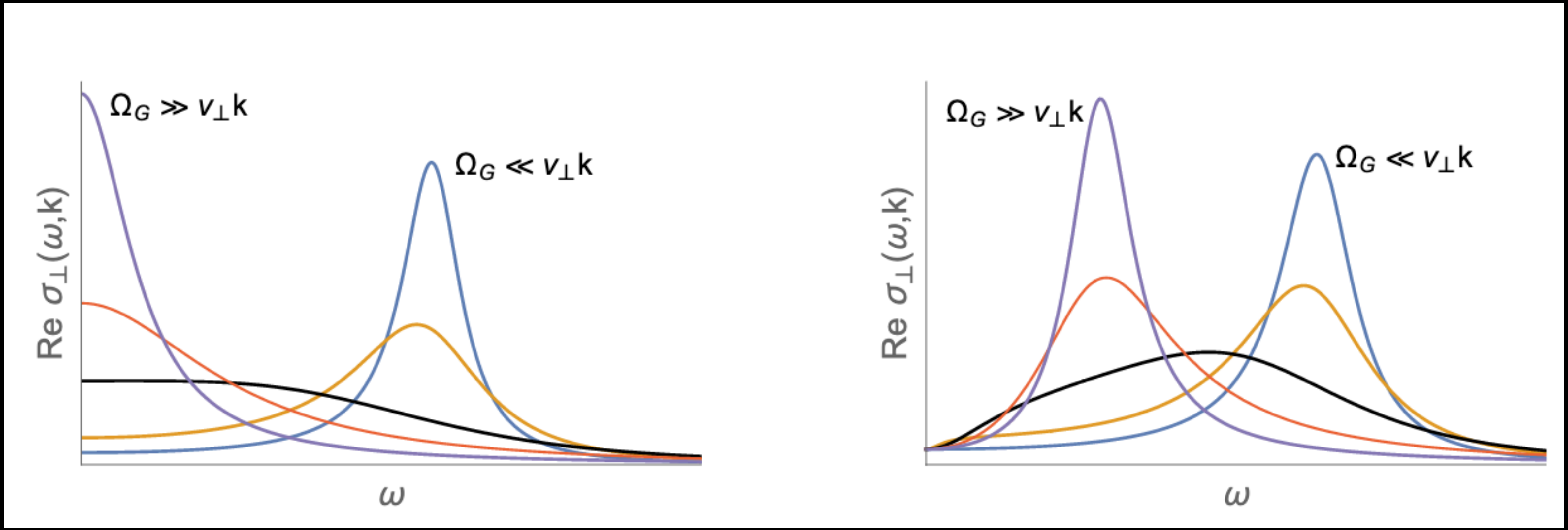
$$\omega = \pm\omega_0 - \frac{i}{2}\left(\frac{\eta_f}{\rho}k^2 + \Gamma + \Omega_\phi\right) + \dots,$$

Longitudinal sound and diffusion:

$$\omega = \pm\sqrt{\omega_0^2 + v_f^2 k^2} - \frac{i}{2}\left(\frac{\zeta_f + 2\frac{d-1}{d}\eta_f}{\rho}k^2 + \frac{\sigma}{\chi_f}k^2 - \frac{\omega_0^2\left(\sigma - 2n\gamma_n + \frac{n^2}{\sigma_\phi}\right)}{\chi_f(\omega_0^2 + v_f^2 k^2)}k^2 + \Gamma + \Omega_\phi\right) + \dots,$$

$$\omega = -\frac{\omega_0^2\left(\sigma - 2n\gamma_n + \frac{n^2}{\sigma_\phi}\right)}{\chi_f(\omega_0^2 + v_f^2 k^2)}k^2 + \dots,$$

CORRELATION FUNCTIONS:



Plasticity, no pinning

Pinning and plasticity

III- OUTLOOK

$$\delta S = \int \mathrm{d}^{d+1}x\sqrt{-g}\Big[\frac{1}{2}T^{\mu\nu}\delta g_{\mu\nu} + J^\mu \delta A_\mu + K_I\delta\phi^I + \frac{1}{2}U^{IJ}\left(\delta\psi_{IJ} - e^\mu_K\partial_\mu\psi_{IJ}\delta\phi^K\right) + \ell' L_I \delta\Phi^I\Big].$$

Action variation

$$\nabla_\mu T^{\mu\nu} = K_I e^{I\nu} - \frac{1}{2}U^{IJ}u^\mu_\phi u^\nu_\phi \partial_\nu \psi_{IJ} + F^{\nu\rho}J_\rho + \ell' L_I \nabla^\nu \Phi^I \quad ,$$

$$\nabla_\mu J^\mu = 0 \quad .$$

Ward identities

OUTLOOK

(1) What other regimes are there?
(2) Can this be directly related to experimental setups?
(3) Unidirectional (smectic) crystals?
(4) Is there a symmetry understanding of topological defects?
(5) How to simultaneously account for disclinations?
(6) Combine it with dynamical EM fields?
(7) Other phases of matter?