Gravitational S-matrix & Carrollian holography

Laura DONNAY

Beyond Lorentzian Geometry II
Edinburgh, 6-8 Feb 2023

MA PER SEGUIR VIRTUTE E CANOSCENZA
SISSA

Intro and motivations

Quantum gravity in 4d asymptotically flat spacetimes ⟶ vanishing cosmological constant

$$\Lambda = 0$$

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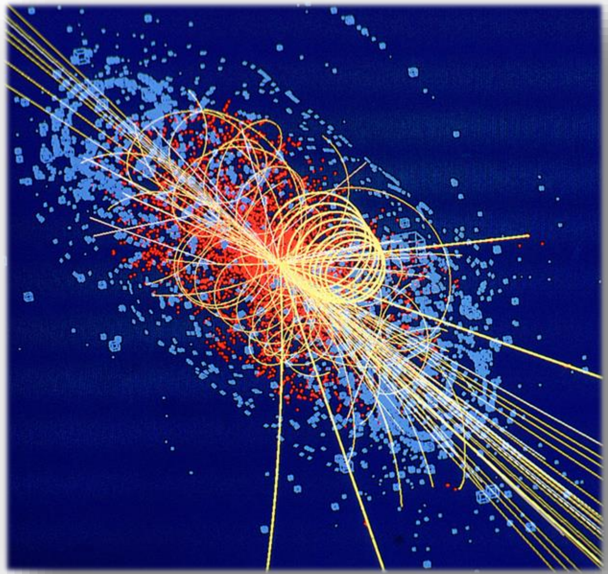
Quantum gravity in 4d asymptotically flat spacetimes ⟶ vanishing cosmological constant

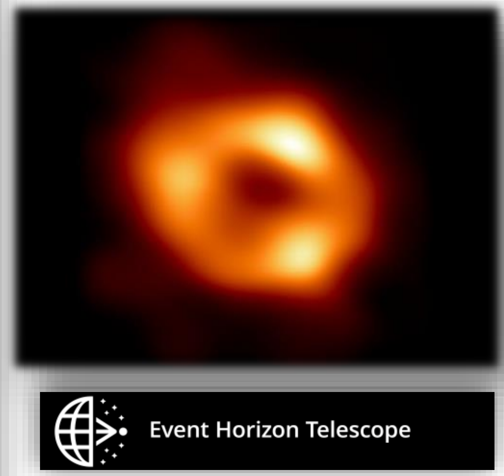
$$\Lambda = 0$$

These spacetimes are relevant



from collider physics ...

... to astrophysics
(< cosmological scales)

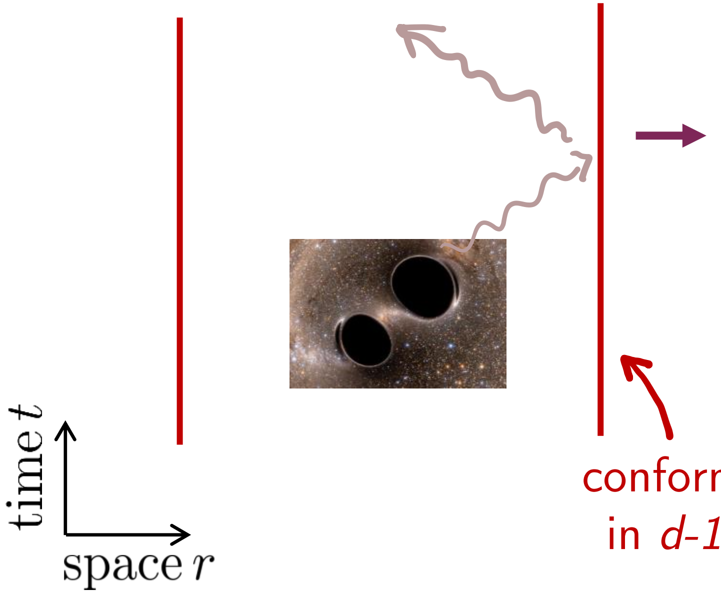


$$S_{BH} = \frac{Ac^3}{4G\hbar}$$

[Bekenstein][Hawking]

Holographic principle

Anti-de Sitter
in *d* dimensions



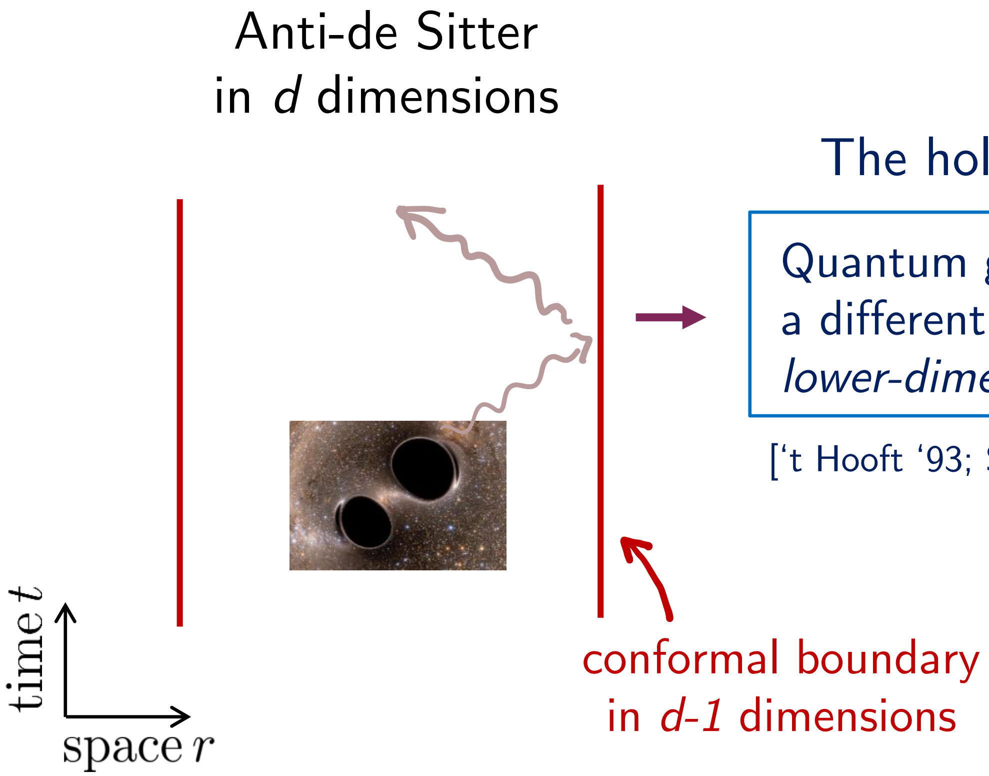
conformal boundary
in *d-1* dimensions

The holographic principle

> Quantum gravity is *encoded* in a different theory that lives in a *lower-dimensional* spacetime.

['t Hooft '93; Susskind '94; Maldacena '97]

Holographic principle



The holographic principle → **How general is it?**

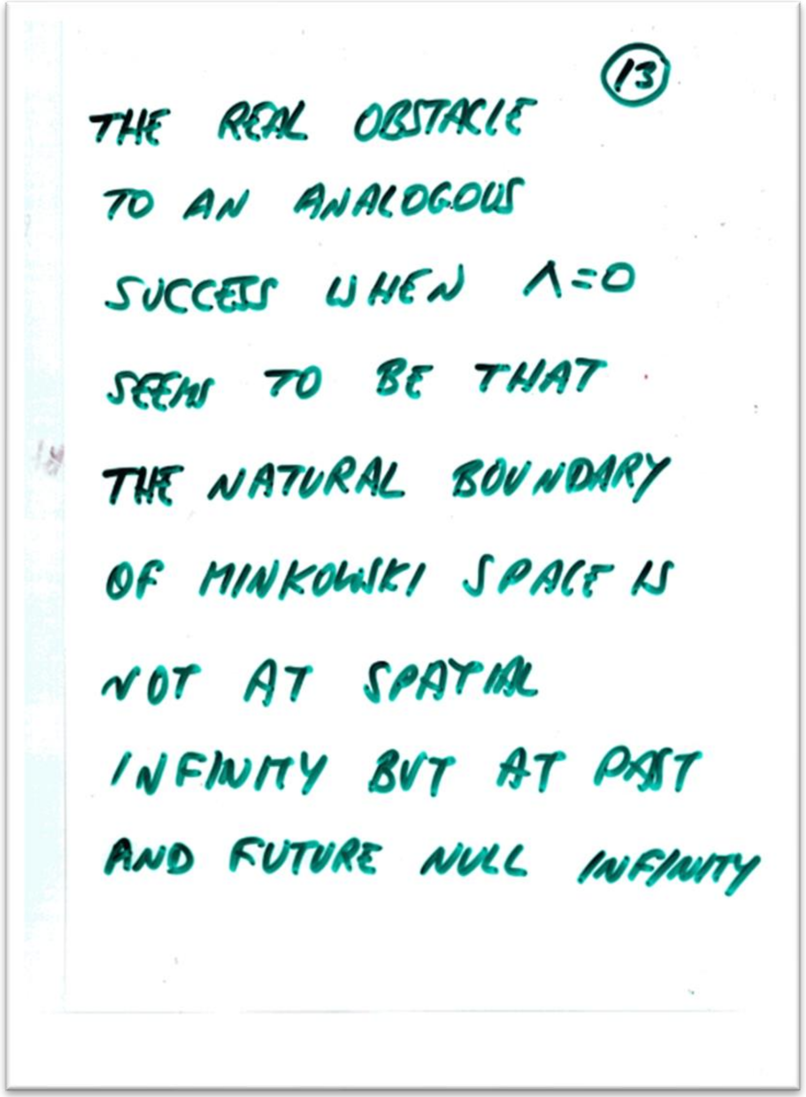
Quantum gravity is *encoded* in a different theory that lives in a *lower-dimensional* spacetime.

['t Hooft '93; Susskind '94; Maldacena '97]

Go beyond the canonical cases!

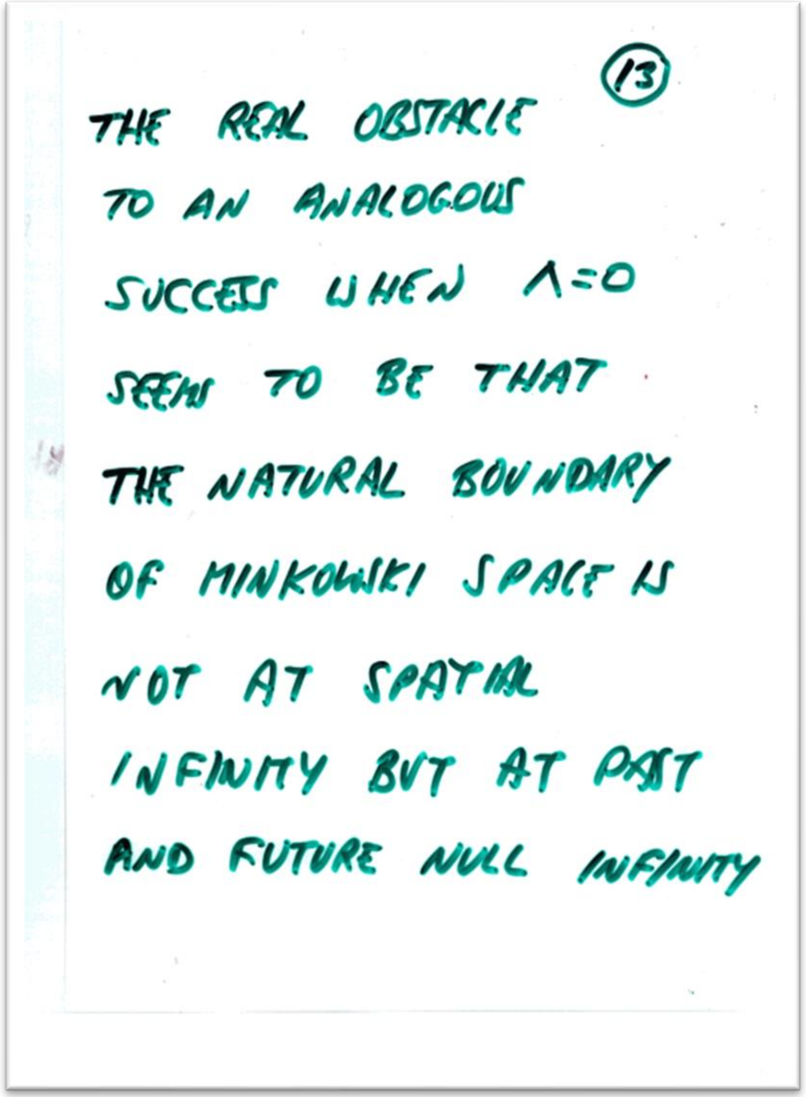
Anti-de Sitter	vs	**Flat**
$\Lambda < 0$		$\Lambda = 0$
CFT		**??**

Flat space holography: a structure X?



E. Witten's talk - *Strings* 1998

Flat space holography: a structure X?



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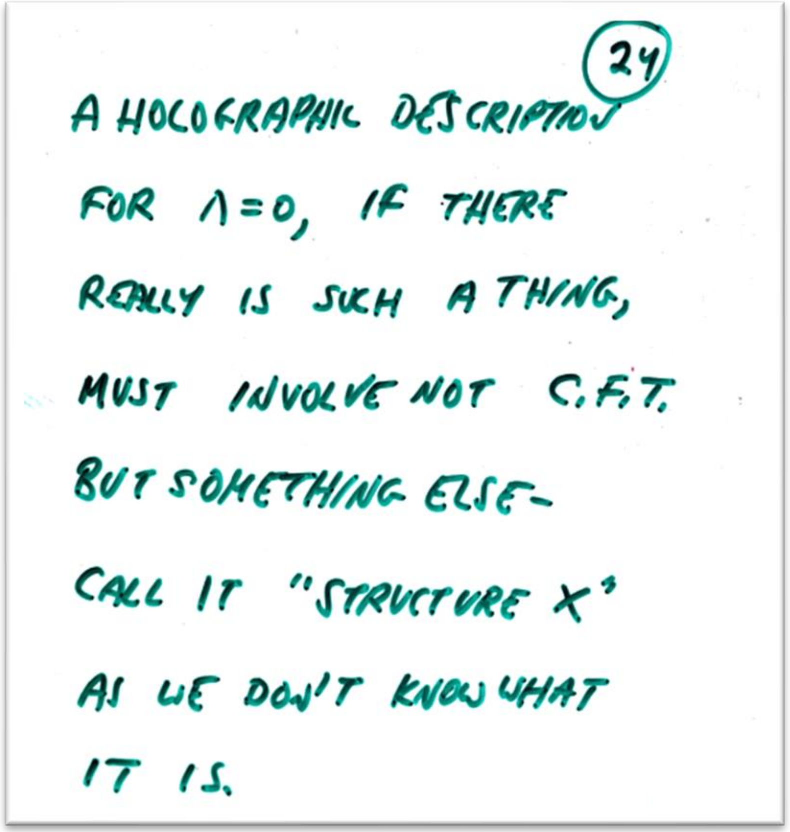
THE REAL OBSTACLE TO AN ANALOGOUS SUCCESS WHEN $\Lambda=0$ SEEMS TO BE THAT THE NATURAL BOUNDARY OF MINKOWSKI SPACE IS NOT AT SPATIAL INFINITY BUT AT PAST AND FUTURE NULL INFINITY



(24)

A HOLOGRAPHIC DESCRIPTION FOR $\Lambda=0$, IF THERE REALLY IS SUCH A THING, MUST INVOLVE NOT C.F.T. BUT SOMETHING ELSE – CALL IT "STRUCTURE X" AS WE DON'T KNOW WHAT IT IS.

E. Witten's talk - *Strings* 1998

Flat space holography

Holographic description of quantum gravity in 4d asymptotically flat spacetimes?

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Early attempts:
[Susskind ‘99][Polchinski ‘99][Giddings ’99]
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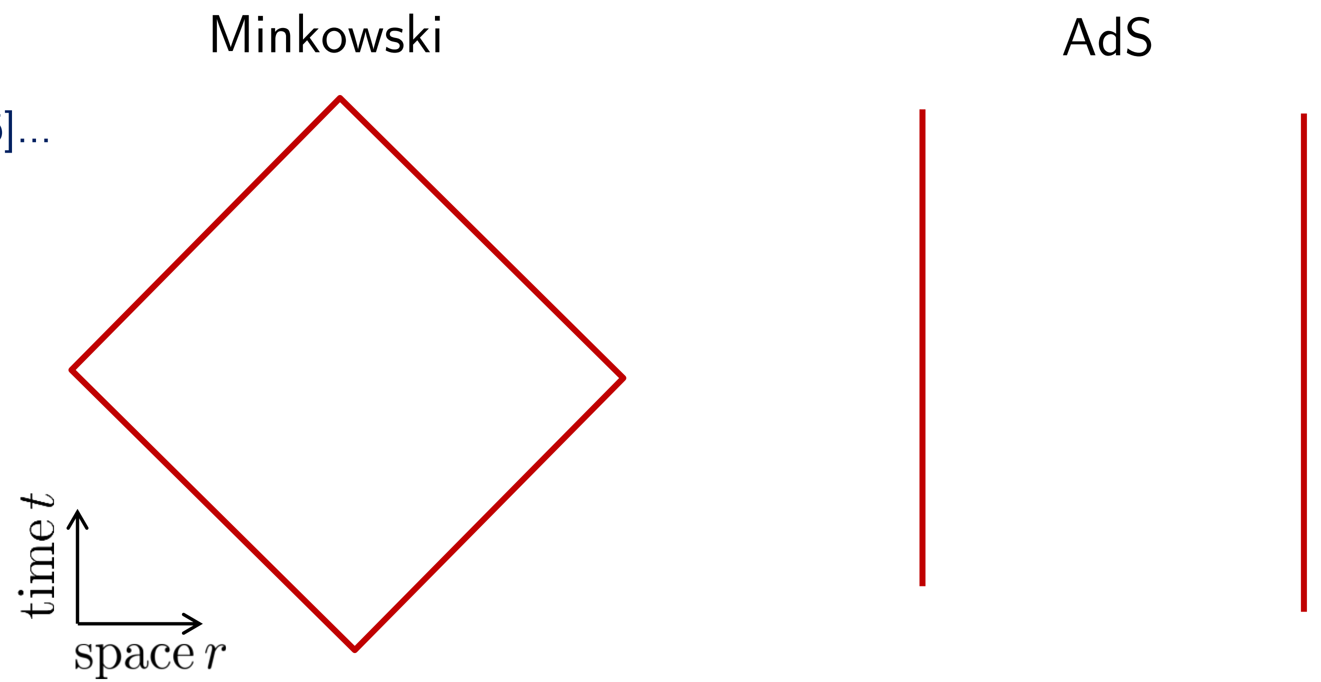
Main obstructions/difficulties:



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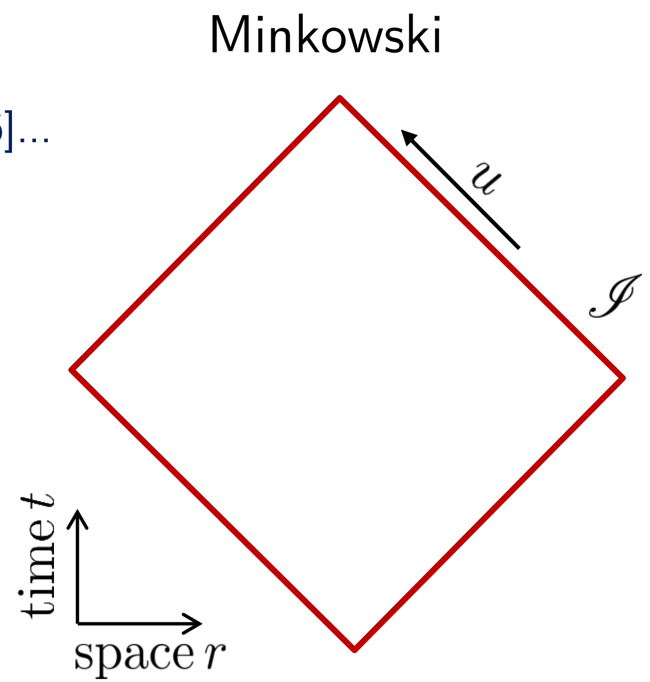
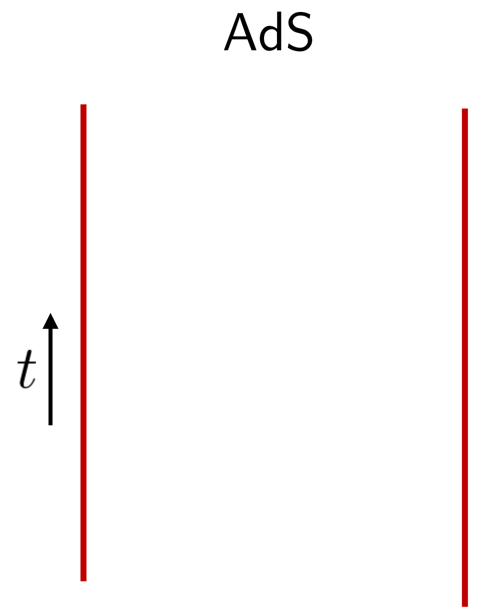
Main obstructions/difficulties:

① The boundary is a **null** hypersurface

$$u = t - r$$





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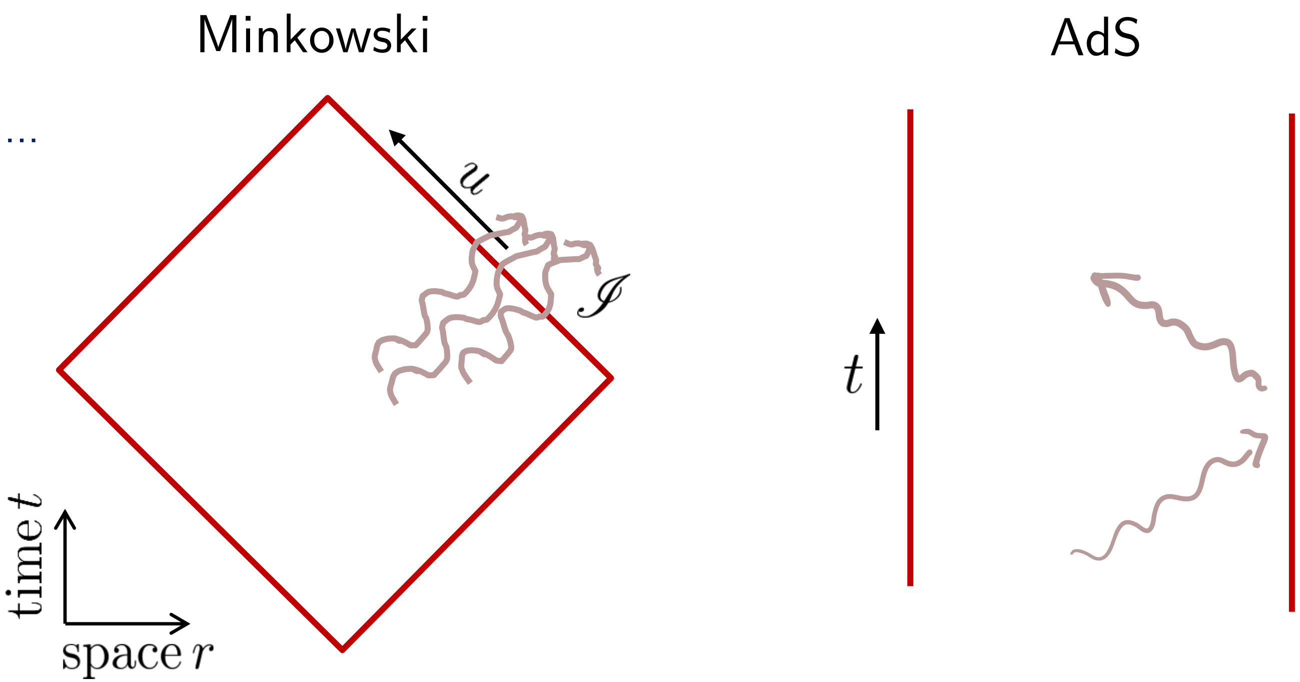
Main obstructions/difficulties:

1. The boundary is a **null** hypersurface

$$u = t - r$$

2. There are **fluxes** leaking out the boundary



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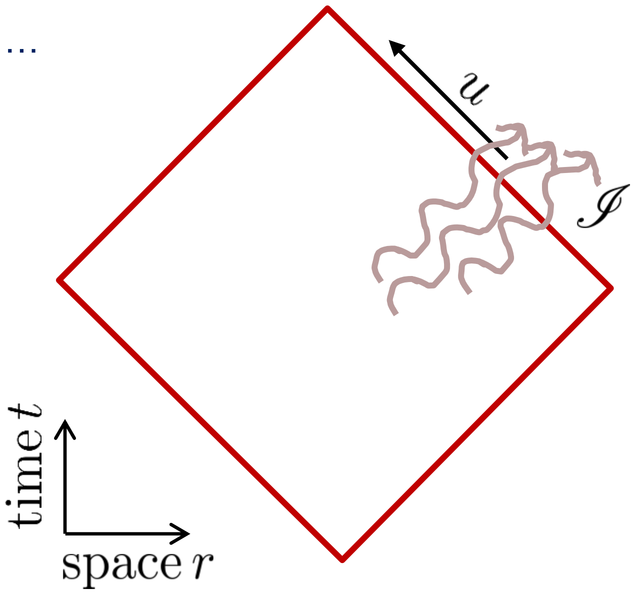
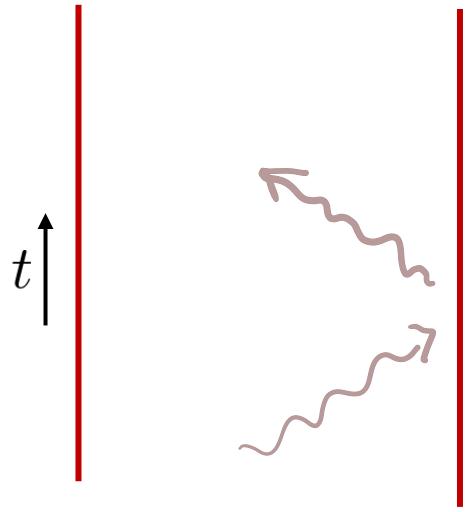


AdS



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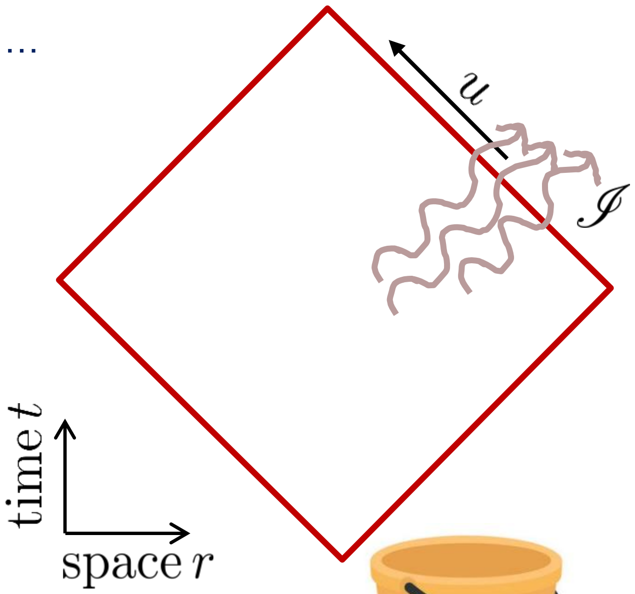

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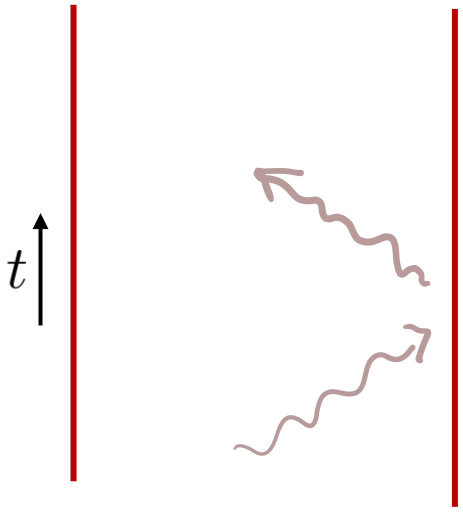

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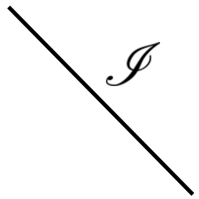

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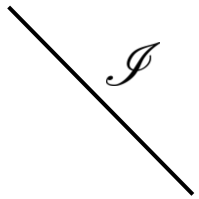

celestial sphere

Euclidean 2-sphere

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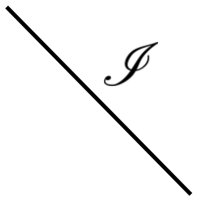

4d bulk/3d holography: 'Carroll holography'

Dual: 3d 'BMS field theory'

[Arcioni, Dappiaggi '03 '04] [Dappiaggi, Moretti, Pinamonti '06][Mann, Marolf '06] [Bagchi, Basu, Kakkar, Melhra '16] [Bagchi, Melhra, Nandi '20] [LD, Fiorucci, Herfray, Ruzziconi '22][Bagchi, Banerjee, Basu, Dutta '22][...]

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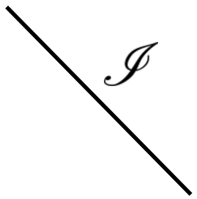

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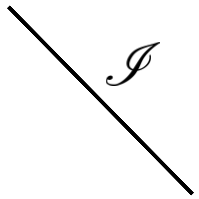

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Features: easier link to AdS/CFT ☺

treatment of fluxes ☹

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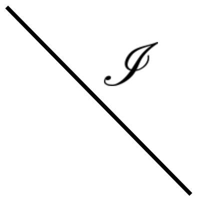

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Features: powerful CFT techniques at hand ☺
role of translations obscured ☹

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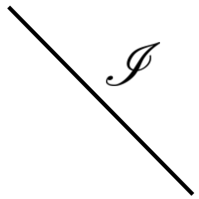

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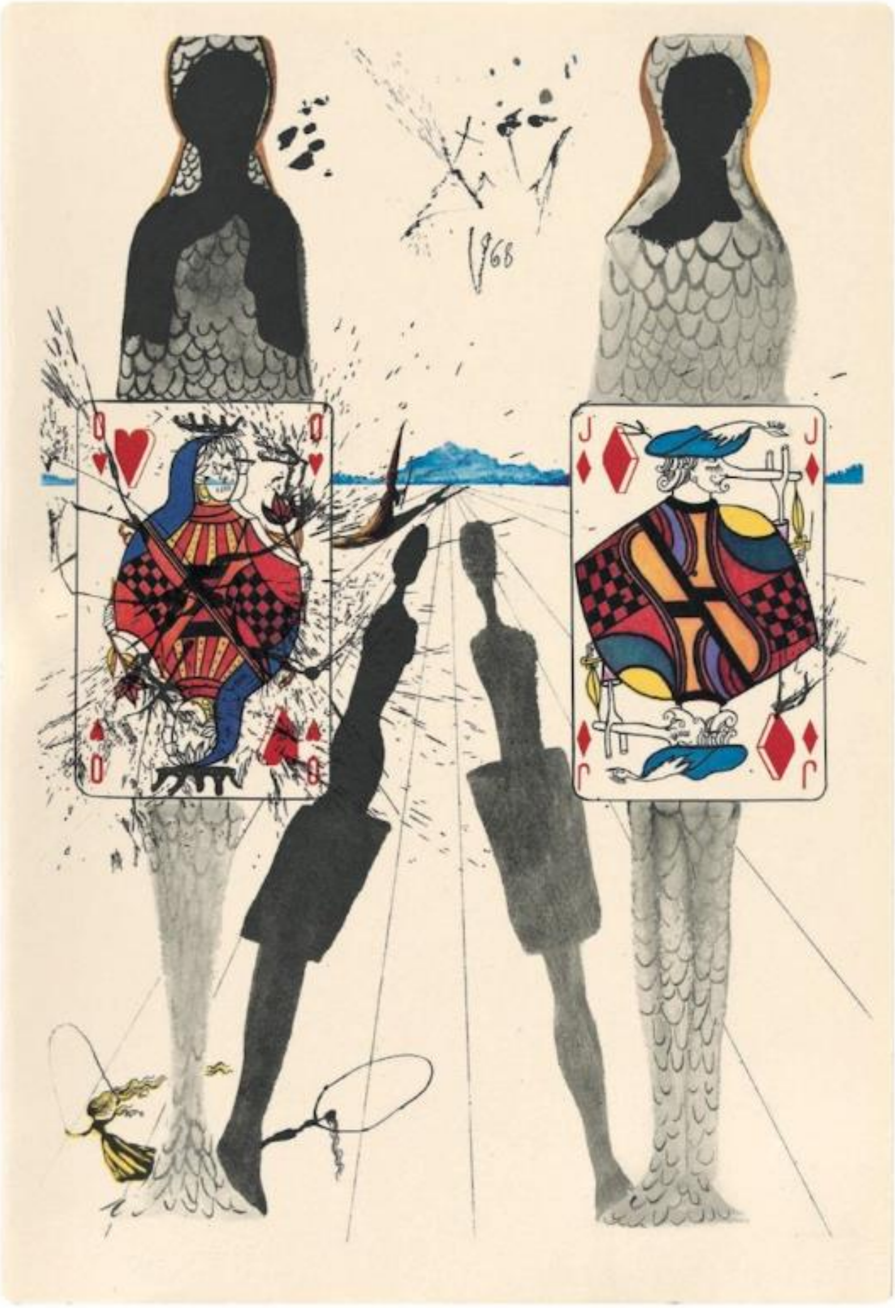
Salvador Dalí, illustrations for *Alice's Adventures in Wonderland*, 1969:



Outline

1. BMS & the S-matrix
2. Bases and boundary operators
3. Towards Carrollian holography
4. CCFT vs CCFT

based on 2202.04702 PRL (2022) & 2212.12553

w/ Adrien **FIORUCCI**, Yannick **HERFRAY** & Romain **RUZZICONI**

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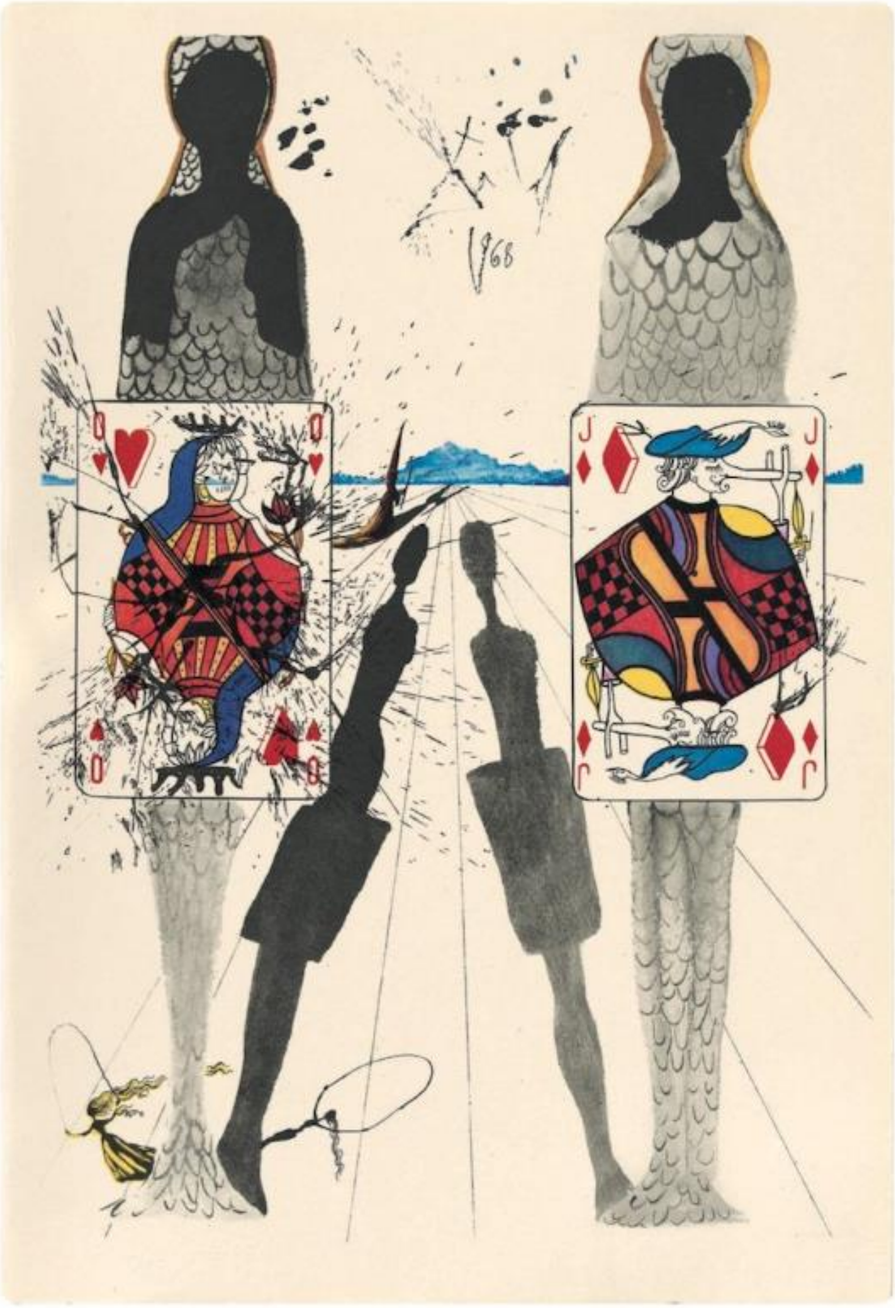


Outline

1. BMS & the S-matrix

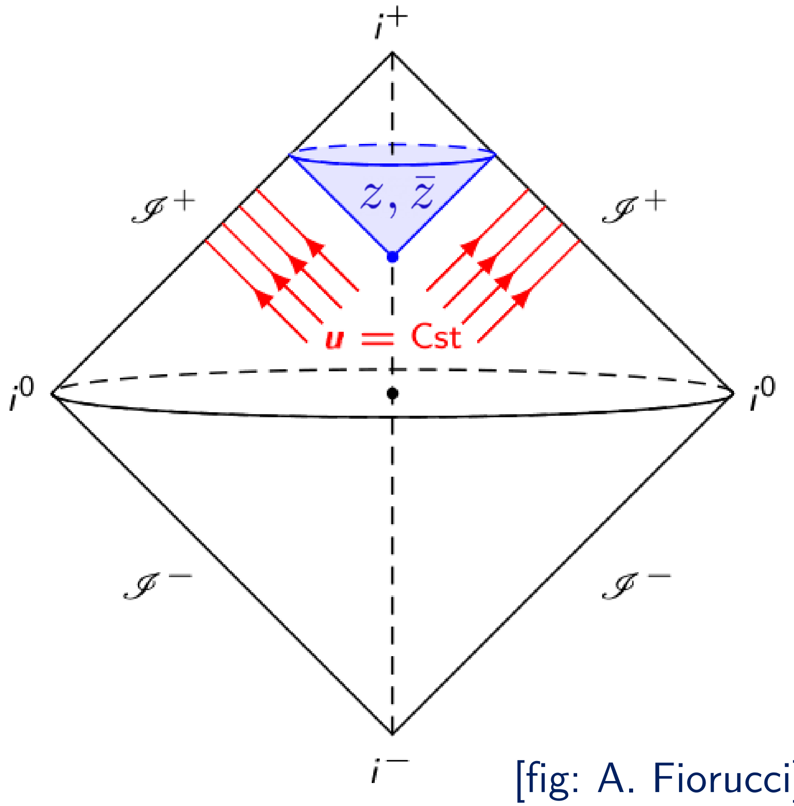
Gravitational solution space

[Bondi, van der Burg, Metzner '62] [Sachs '62]
[Barnich, Troessaert '10]

Asymptotically flat spacetimes in Bondi gauge:

$$r \to \infty \qquad\qquad (u, r, x^A)\,, \quad x^A = (z, \bar{z})$$

$$\begin{aligned} ds^2 = &-\mathrm{d}u^2 - 2\mathrm{d}u\mathrm{d}r + 2r^2\gamma_{z\bar{z}}\,\mathrm{d}z\mathrm{d}\bar{z} \\ &+\frac{2M}{r}\mathrm{d}u^2 + rC_{zz}\mathrm{d}z^2 + D^zC_{zz}\mathrm{d}u\mathrm{d}z \\ &+\frac{1}{r}\left(\frac{4}{3}(N_z + u\partial_z m_B) - \frac{1}{4}\partial_z(C_{zz}C^{zz})\right)\mathrm{d}u\mathrm{d}z + c.c. + \cdots \end{aligned}$$



[fig: A. Fiorucci]

BONDI

METZNER

SACHS

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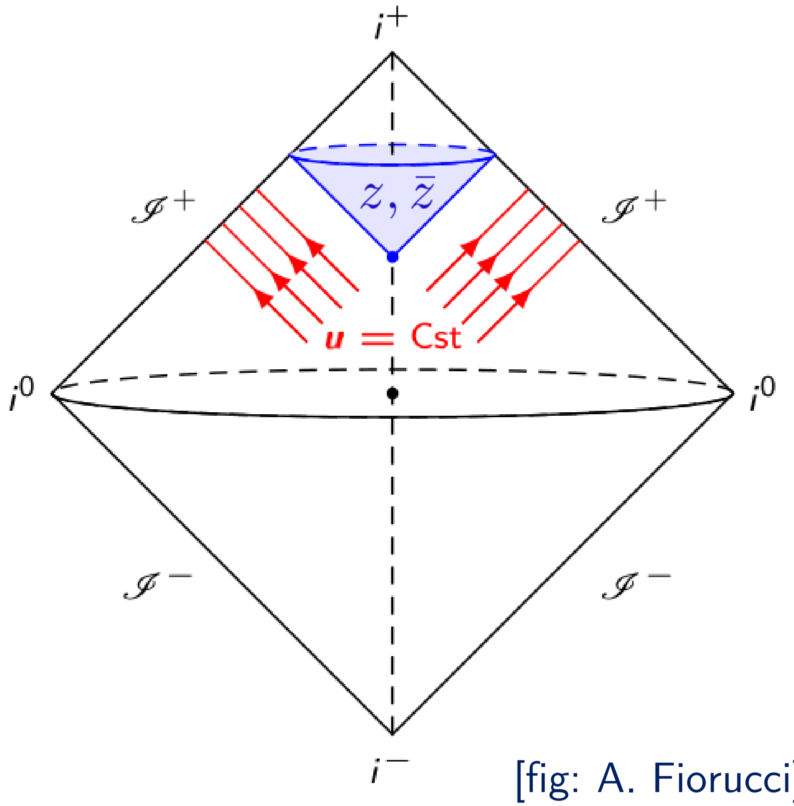

[fig: A. Fiorucci]

The Bondi **mass** and **angular momentum** aspects satisfy

$$\partial_u M = -\frac{1}{8}N_{AB}N^{AB} + \frac{1}{4}\partial_A\partial_B N^{AB},$$

$$\begin{aligned} \partial_u N_A &= \partial_A M + \frac{1}{16}\partial_A(N_{BC}C^{BC}) - \frac{1}{4}N^{BC}\partial_A C_{BC} \\ &\quad - \frac{1}{4}\partial_B(C^{BC}N_{AC} - N^{BC}C_{AC}) - \frac{1}{4}\partial_B\partial^B\partial^C C_{AC} + \frac{1}{4}\partial_B\partial_A\partial_C C^{BC} \end{aligned}$$

$N_{AB} \equiv \partial_u C_{AB}$

Bondi news: encodes **gravitational waves!**

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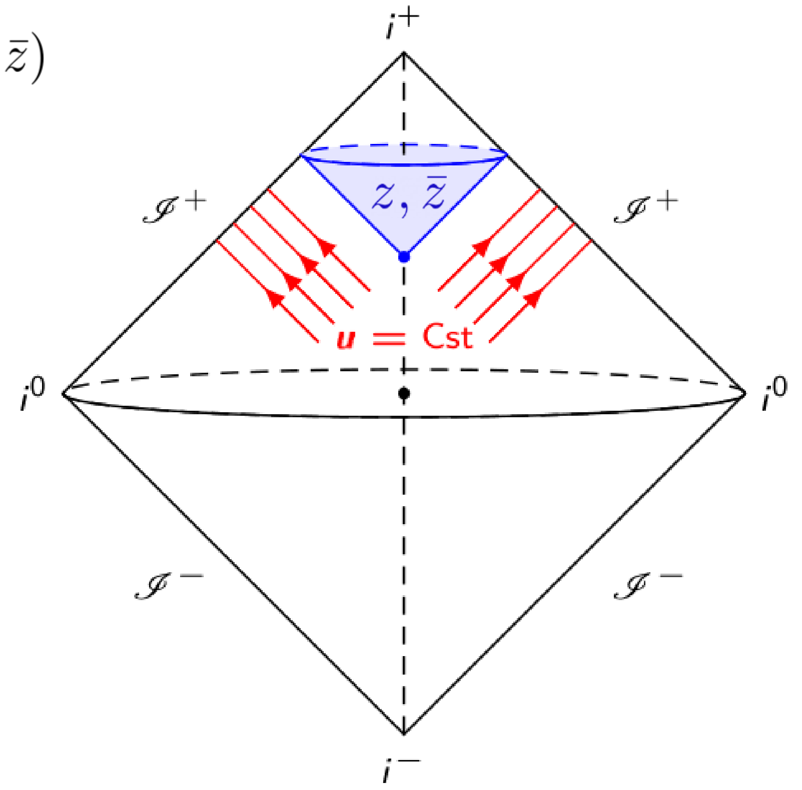
$$



BMS symmetries:

Poincaré = 4 translations ⋉ 6 Lorentz
↓ ↓ ↓
BMS = supertranslations$_\infty$ ⋉ superrotations$_\infty$

$\xi(T) = T(z,\bar{z})\partial_u$

SUPERTRANSLATIONS

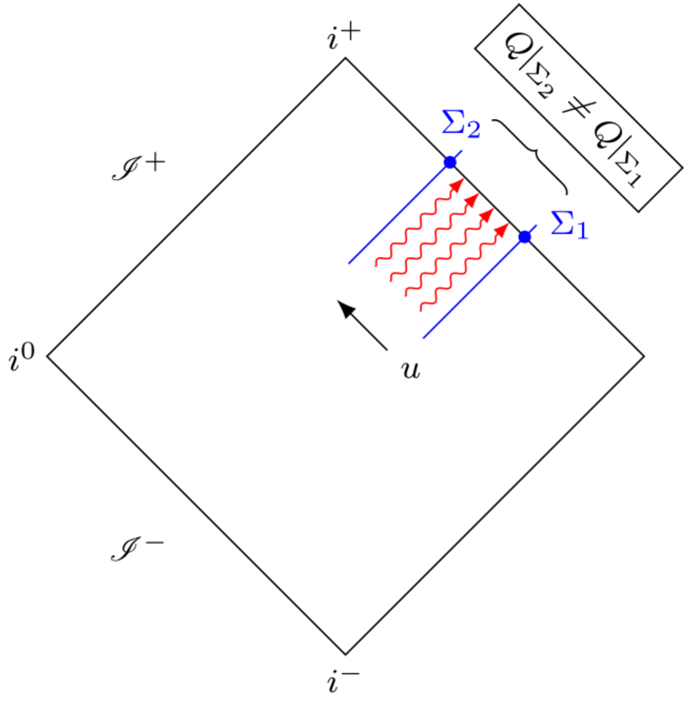
$\xi(Y) = Y^z(z)\partial_z + \frac{u}{2}D_zY^z\partial_u + c.c.$

SUPERROTATIONS

BMS charges and fluxes

- At each cut $\{u = \text{constant}\}$ of $\mathscr{I}^+$, one can construct 'surface charges' associated to BMS symmetries.

Outgoing radiation ⟶ BMS charges are *not* conserved.



$$\int_{-\infty}^{+\infty} du\, \partial_u Q_\xi = F_\xi \neq 0$$

outgoing flux

BMS charges and fluxes

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Outgoing radiation $\longrightarrow$ BMS charges are *not* conserved.

A ‘good prescription’ for BMS charges has emerged in recent years:

[Barnich, Troessaert ’11][He, Lysov, Mitra, Strominger ’14][Kapec, Lysov, Pasterski, Strominger ’14][Compère, Fiorucci, Ruzziconi ’19 ’20][Campiglia, Peraza ’20]
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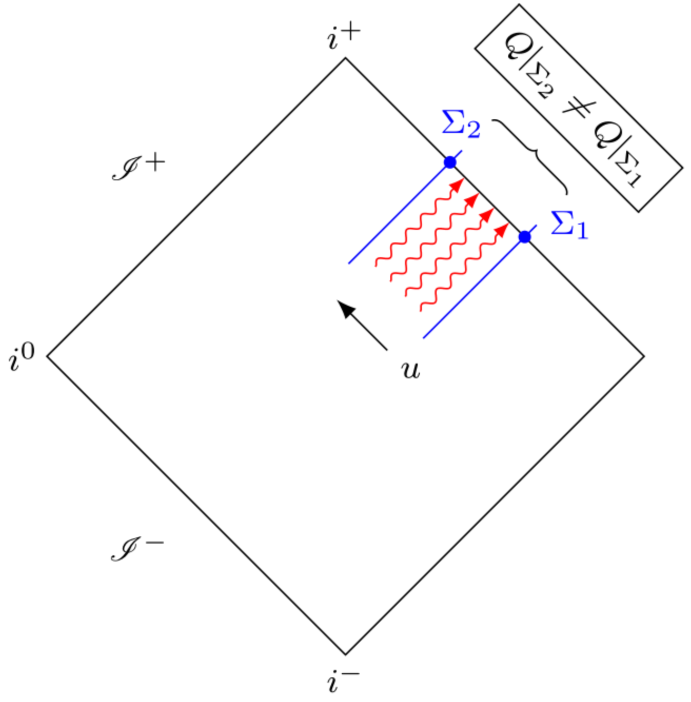

$$Q_\xi = \frac{1}{8\pi G}\int_{\mathcal{S}} d^2z\,[2\mathcal{T}\widetilde{M} + \mathcal{Y}\tilde{\bar{N}} + \bar{\mathcal{Y}}\tilde{N}]\,,$$

$$\widetilde{M} = M + \frac{1}{8}(C_{zz}N^{zz} + C_{\bar z\bar z}N^{\bar z\bar z})$$

$$\tilde{N} = N_{\bar z} - u\bar\partial M + \frac{1}{4}C_{\bar z\bar z}\bar\partial C^{\bar z\bar z} + \frac{3}{16}\bar\partial(C_{zz}C^{zz}) + \frac{u}{4}\bar\partial\Big[\Big(\partial^2 - \frac{1}{2}N_{zz}\Big)C^z_{\bar z} - \Big(\bar\partial^2 - \frac{1}{2}N_{\bar z\bar z}\Big)C^{\bar z}_z\Big]$$

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$$\widetilde{M} = -\frac{1}{2}(\Psi_2^0 + \bar{\Psi}_2^0)$$

$$\widetilde{N} = -\Psi_1^0 + u\bar{\partial}\Psi_2^0$$

[Barnich, Troessaert '11][He, Lysov, Mitra, Strominger '14][Kapec, Lysov, Pasterski, Strominger '14][Compère, Fiorucci, Ruzziconi '19 '20][Campiglia, Peraza '20]
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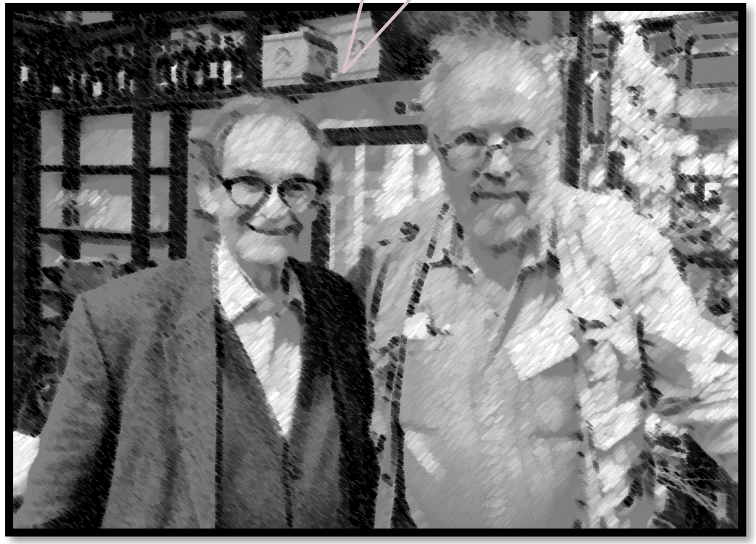
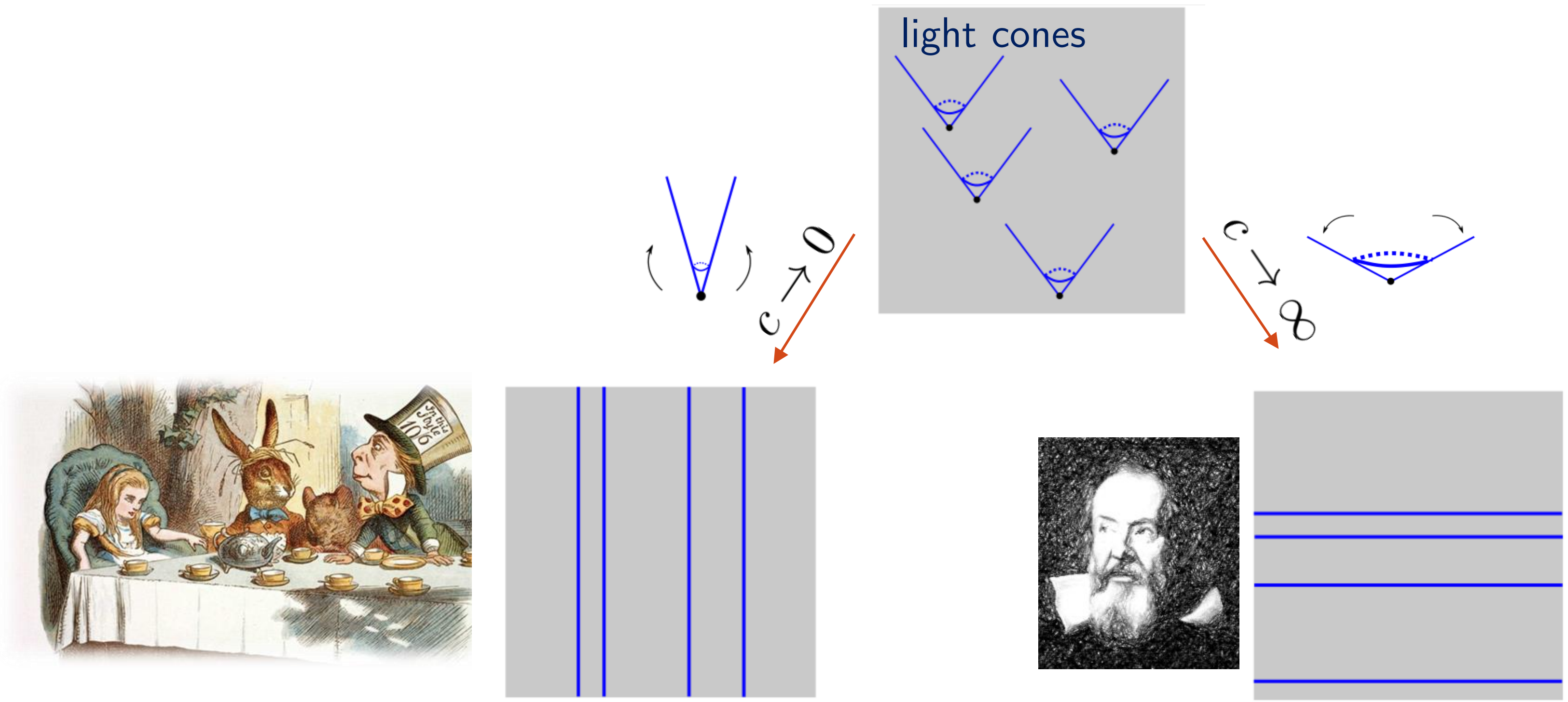
$$Q_\xi = \frac{1}{8\pi G}\int_{\mathcal{S}} d^2z\,[2\mathcal{T}\widetilde{M} + \mathcal{Y}\bar{\widetilde{N}} + \bar{\mathcal{Y}}\widetilde{N}]\,,$$

$$\widetilde{M} = M + \frac{1}{8}(C_{zz}N^{zz} + C_{\bar{z}\bar{z}}N^{\bar{z}\bar{z}})$$

$$\widetilde{N} = N_{\bar{z}} - u\bar{\partial}\mathcal{M} + \frac{1}{4}C_{\bar{z}\bar{z}}\bar{\partial}C^{\bar{z}\bar{z}} + \frac{3}{16}\bar{\partial}(C_{zz}C^{zz}) + \frac{u}{4}\bar{\partial}\Big[\Big(\partial^2 - \frac{1}{2}N_{zz}\Big)C^z_{\bar{z}} - \Big(\bar{\partial}^2 - \frac{1}{2}N_{\bar{z}\bar{z}}\Big)C^{\bar{z}}_z\Big]$$



BMS = conformal Carrollian symmetries



[Levy-Leblond '65] Carrollian spacetime
(space is absolute)

Galilean spacetime
(time is absolute)

[fig. Yannick Herfray]

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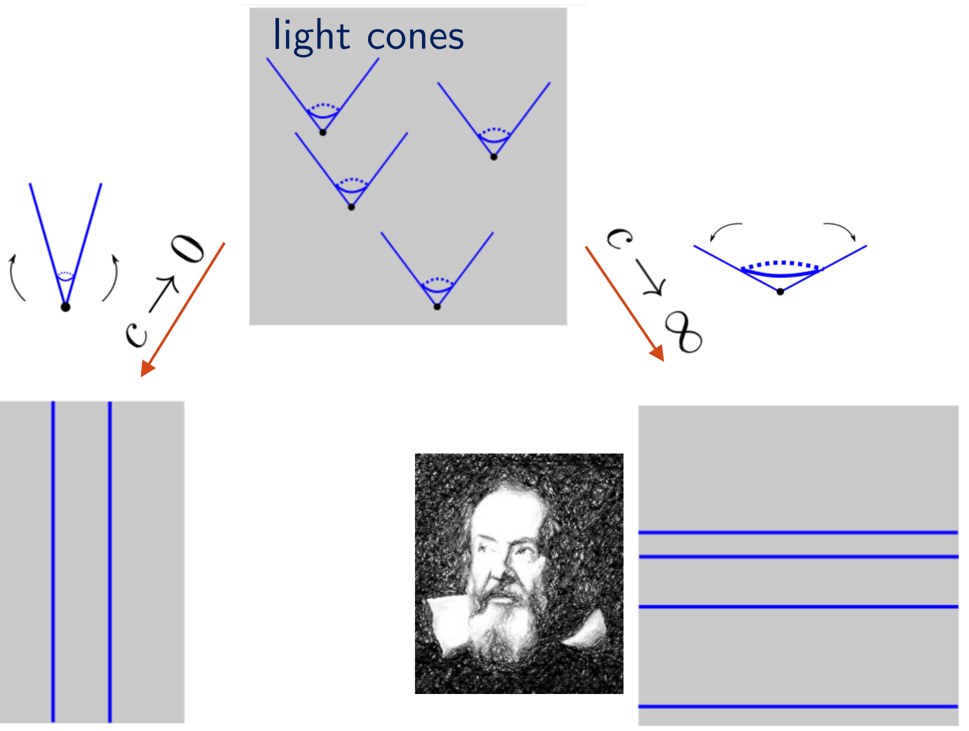
- BMS symmetries = **conformal** symmetries of a **Carrollian** structure at null infinity
  [Geroch][Penrose][Duval, Gibbons, Horvathy] [Hartong][Ciambelli, Leigh, Marteau, Petropoulos][Bekaert, Morand][Herfray]...

$$x^a = (u, z, \bar{z})$$

q_{ab} : a degenerate metric $\longrightarrow$ $q_{ab}dx^a dx^b = 0 \times du^2 + 2\gamma_{z\bar{z}}dzd\bar{z}$

a vector field satisfying $q_{ab}n^b = 0 \rightarrow n = \partial_u$



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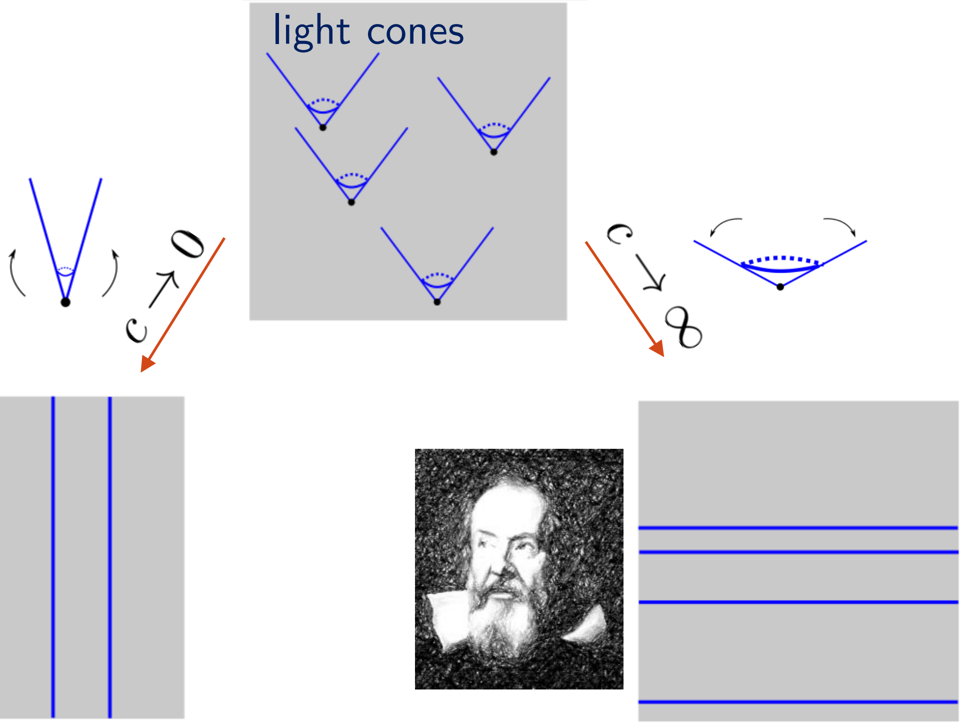

Conformal Carrollian symmetries:

$\mathcal{L}_{\bar{\xi}} q_{ab} = 2\alpha q_{ab}$ $\qquad$ $\mathcal{L}_{\bar{\xi}} n^a = -\alpha n^a$

$\alpha := \frac{1}{2}(D\mathcal{Y} + \bar{D}\bar{\mathcal{Y}})$

$$\bar{\xi} = \left[\mathcal{T} + \frac{u}{2}(D\mathcal{Y} + \bar{D}\bar{\mathcal{Y}})\right]\partial_u + \mathcal{Y}\partial + \bar{\mathcal{Y}}\bar{\partial}$$

[Levy-Leblond '65] Carrollian spacetime
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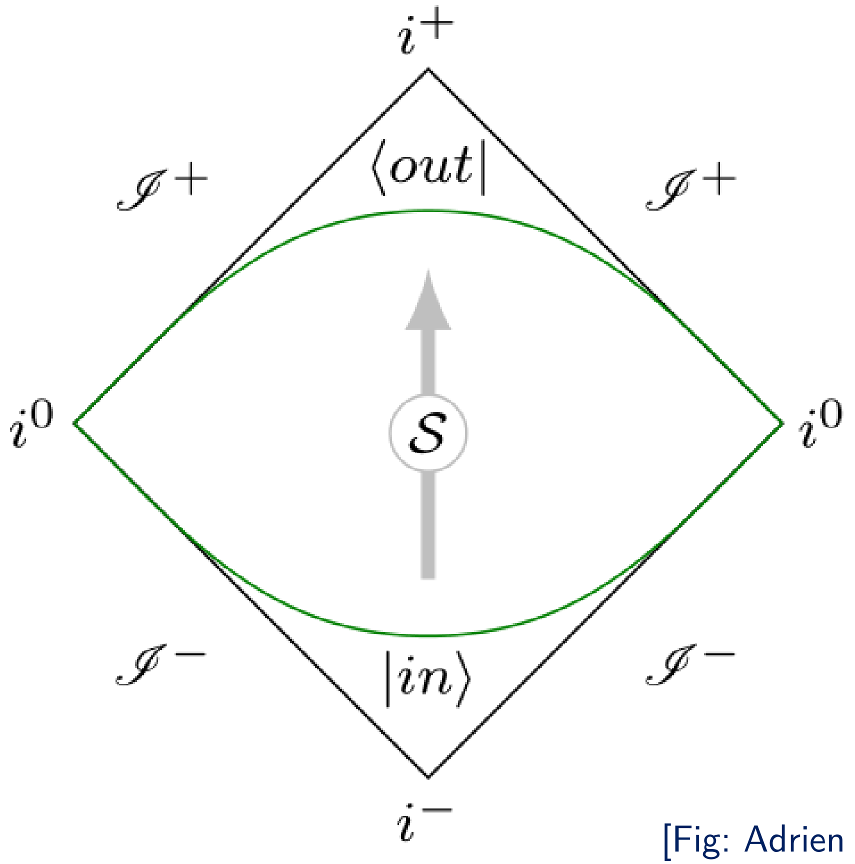
BMS and the scattering problem

Seminal observation: BMS symmetries constrain the gravitational scattering problem! [Strominger ’14]

$\langle out|S|in\rangle$



[Fig: Adrien Fiorucci]

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→ 2 key ingredients

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(1) Noether charges for BMS symmetries

[Barnich, Troessaert '10]

$$Q_{\mathcal{T}} = \frac{1}{4\pi G} \int d^2 z \sqrt{\gamma}\, \mathcal{T} M$$

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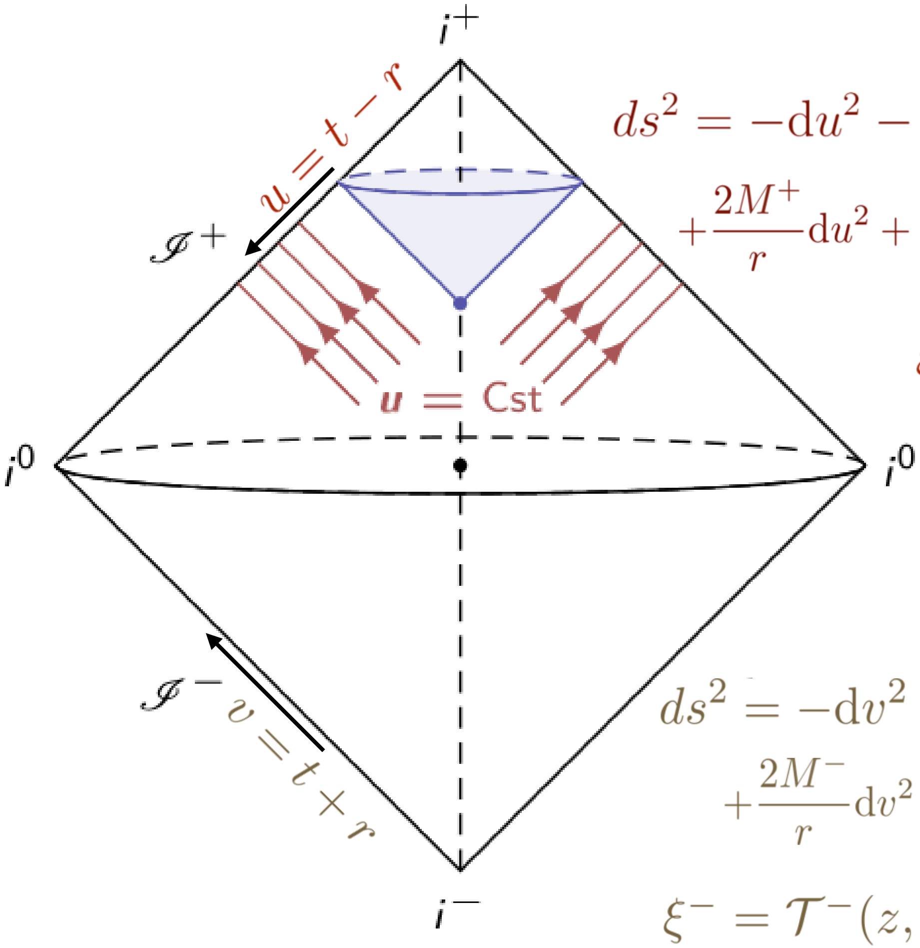
$$Q_{\mathcal{T}} = \frac{1}{4\pi G}\int d^2z\sqrt{\gamma}\,\mathcal{T} M$$

2. Relating the *past* and the *future*



$$ds^2 = -\mathrm{d}u^2 - 2\mathrm{d}u\mathrm{d}r + 2r^2\gamma_{z\bar{z}}\,\mathrm{d}z\mathrm{d}\bar{z} + \frac{2M^+}{r}\mathrm{d}u^2 + rC^+_{zz}\mathrm{d}z^2 + D^zC^+_{zz}\mathrm{d}u\mathrm{d}z + \cdots$$

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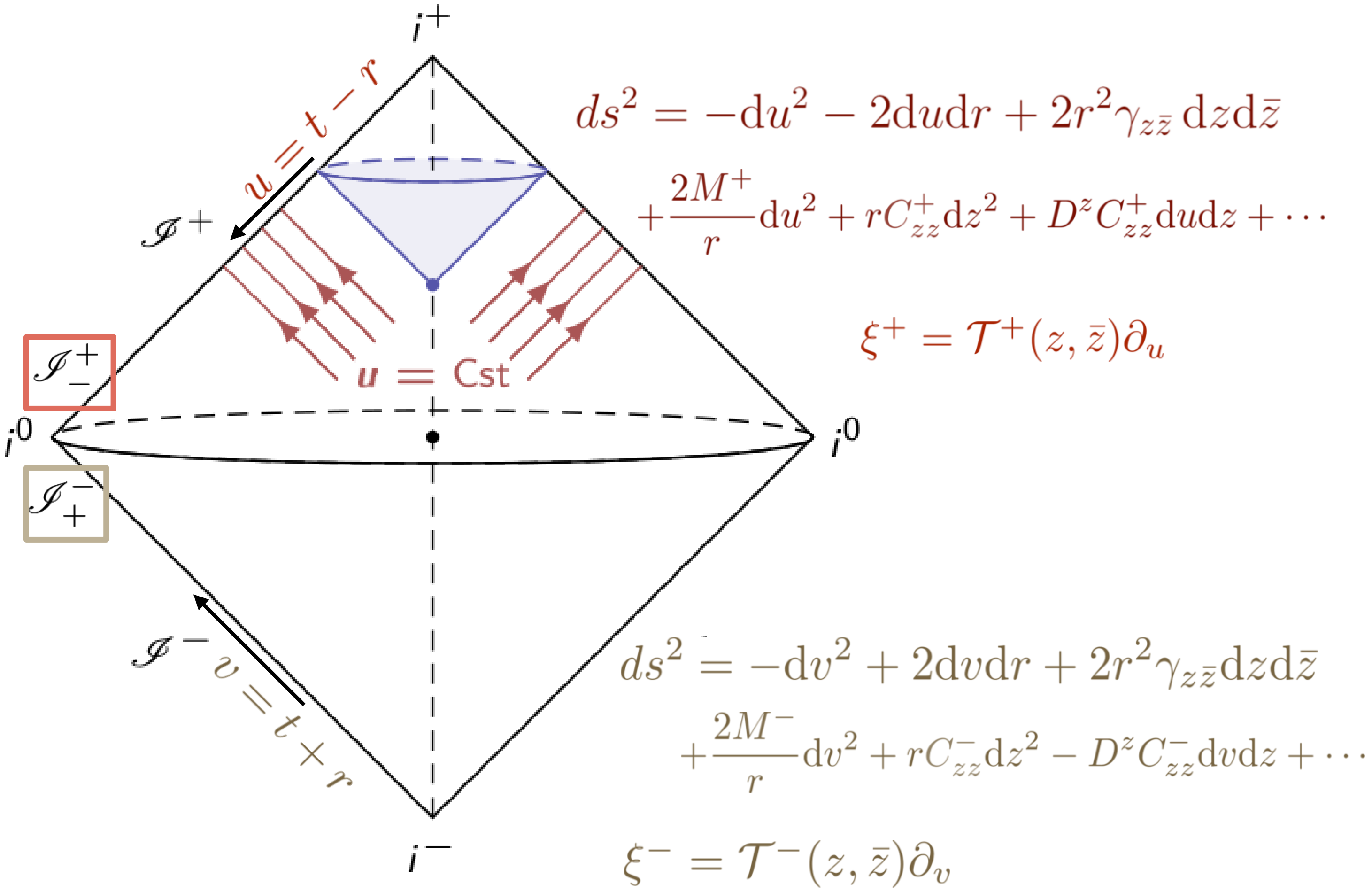

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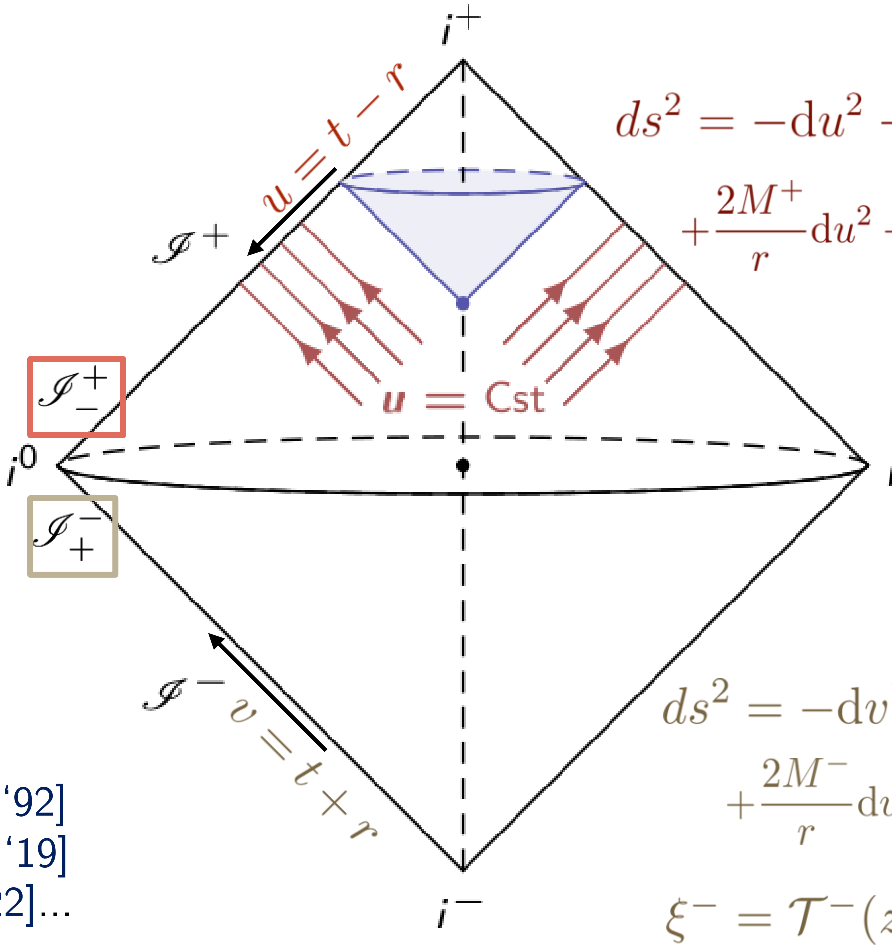
Antipodal matching conditions

$$M^-(v,z,\bar{z})|_{\mathscr{I}^-_+} = M^+(u,z,\bar{z})|_{\mathscr{I}^+_-}$$

$$\mathcal{T}^-(z,\bar{z})|_{\mathscr{I}^-_+} = \mathcal{T}^+(z,\bar{z})|_{\mathscr{I}^+_-}$$

[Strominger '14]; see also [Herberthson, Ludvigsen '92] [Troessaert '18][Henneaux, Troessaert '18][Prabhu '19] [Kroon, Mohamed '21][Capone, Nguyen, Parisini '22]...



$$ds^2 = -du^2 - 2dudr + 2r^2\gamma_{z\bar{z}}\,dzd\bar{z} + \frac{2M^+}{r}du^2 + rC^+_{zz}dz^2 + D^zC^+_{zz}dudz + \cdots$$

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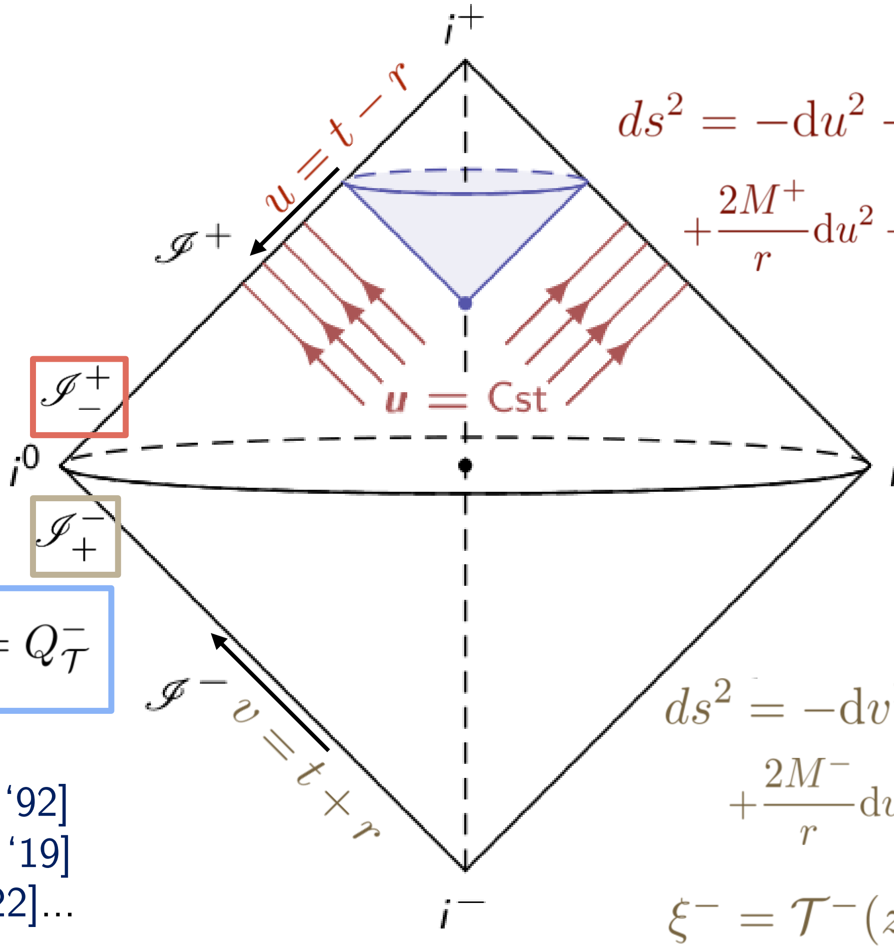

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BMS and the scattering problem

Prime example:

The **leading soft graviton theorem** [Weinberg '65]

$\mathcal{A}_n = \langle out|S|in\rangle$

+soft particle (energy $\omega \to 0$)

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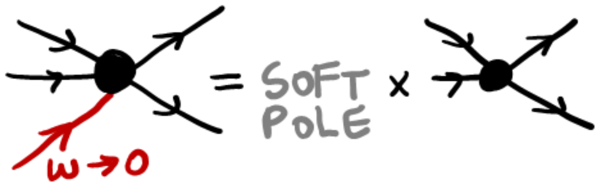
$$n \text{ hard particles } (p_k) + \text{external graviton } (q)$$

$$\lim_{\omega \to 0} \mathcal{A}_{n+1}(q) = S^{(0)} \mathcal{A}_n + \mathcal{O}(q^0)$$

$$S^{(0)} = \sum_{k=1}^{n} \frac{p_k^\mu p_k^\nu \varepsilon_{\mu\nu}(q)}{p_k \cdot q}$$

$\mathcal{A}_n = \langle out | S | in \rangle$

+soft particle (energy $\omega \to 0$)



BMS and the scattering problem

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$\mathcal{A}_n = \langle out|S|in\rangle$

+soft particle (energy $\omega \to 0$)

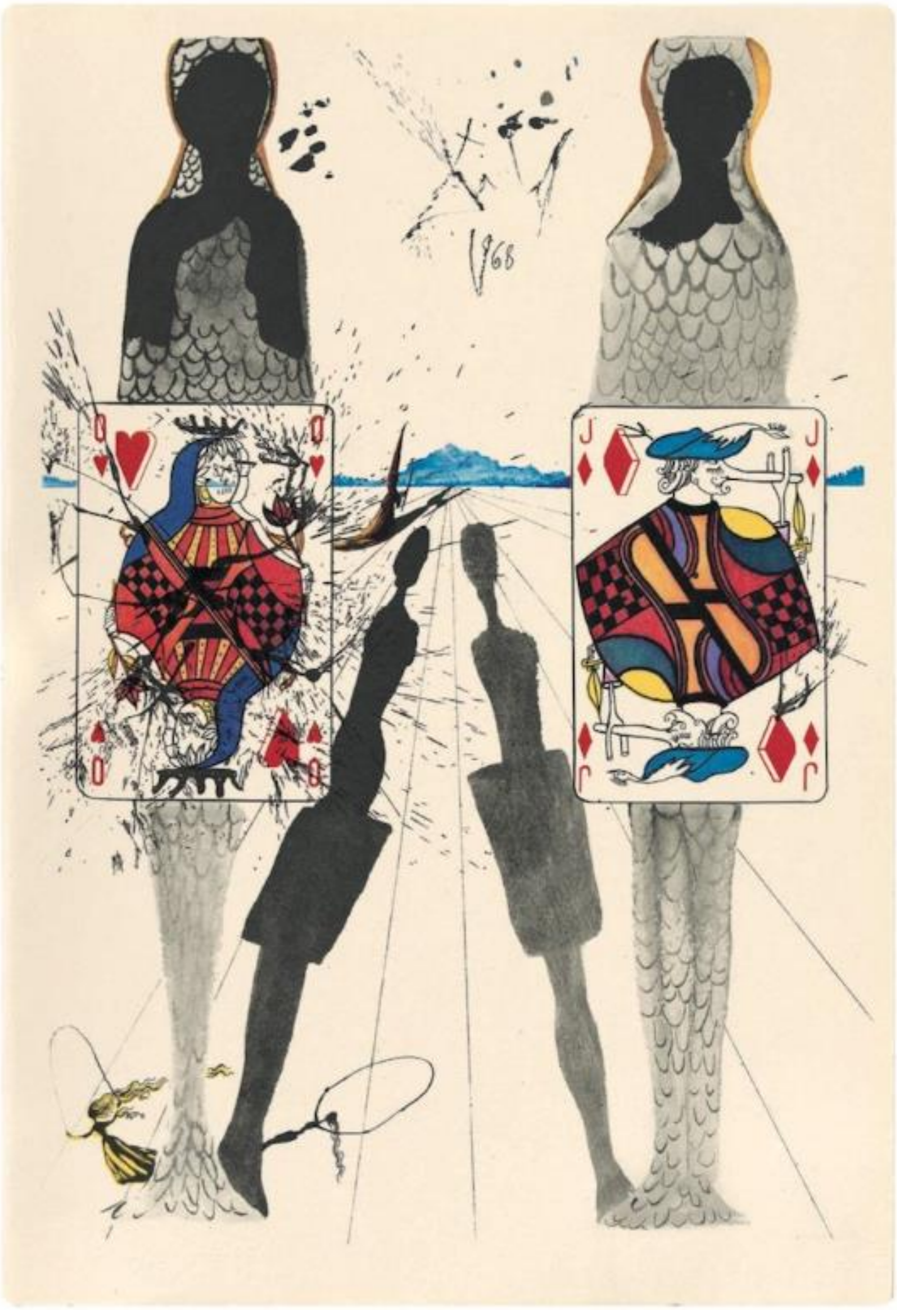
is nothing but the **Ward identity** associated to **supertranslation** symmetry [He, Lysov, Mitra, Strominger '15]

$$\langle out|Q^+_{\mathcal{T}}\mathcal{S} - \mathcal{S}Q^-_{\mathcal{T}}|in\rangle = 0$$

supertranslation charge

$$Q_{\mathcal{T}} = \frac{1}{4\pi G}\int d^2z\sqrt{\gamma}\,\mathcal{T}M$$

Salvador Dalí, illustrations for *Alice's Adventures in Wonderland*, 1969:



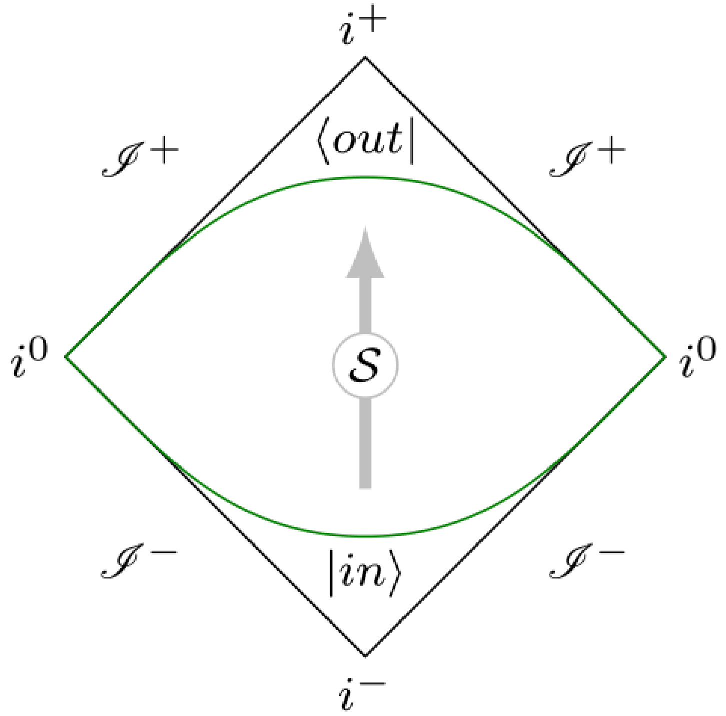
Outline

1. BMS & the S-matrix
2. Bases and boundary operators

3 bases for the scattering problem

Consider the scattering of N massless spin-s in flat spacetimes

Momentum of a massless particle $p^\mu = \omega q^\mu(z,\bar z)$
heading to a point $(z,\bar z)$ on the celestial sphere



Momentum basis $\mathcal{A}_N = \langle \text{out}|\mathcal{S}|\text{in}\rangle_{\text{momentum}}$

i.e. the usual formulation of the scattering amplitudes

$$|\omega, z, \bar z, \pm s\rangle = a_\pm^{(s)\text{in}}(\omega, z, \bar z)^\dagger\,|0\rangle$$

$$\langle \omega, z, \bar z, \pm s| = \langle 0| a_\pm^{(s)\text{out}}(\omega, z, \bar z)$$

$$|\text{in}\rangle = |\omega_1, z_1, \bar z_1, \pm s_1\rangle \otimes \cdots \otimes |\omega_n, z_n, \bar z_n, \pm s_n\rangle$$

$$\langle \text{out}| = \langle \omega_{n+1}, z_{n+1}, \bar z_{n+1}, \pm s_{n+1}| \otimes \cdots \otimes \langle \omega_N, z_N, \bar z_N, \pm s_N|$$

3 bases for the scattering problem

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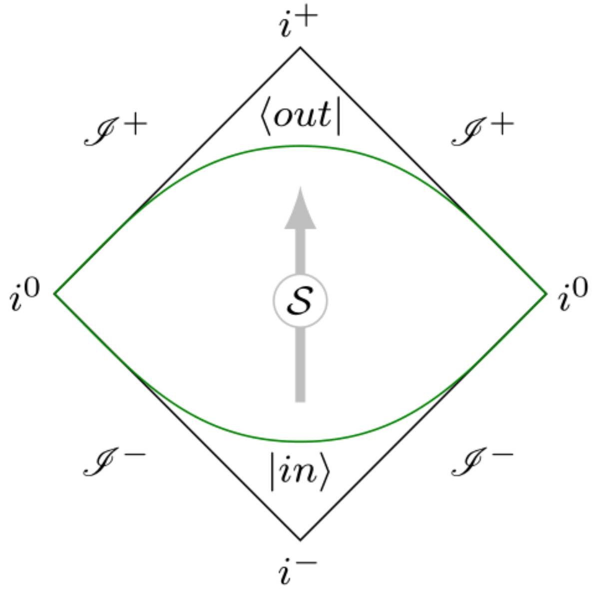
Celestial basis $\mathcal{M}_N = \langle \text{out} | \mathcal{S} | \text{in} \rangle_{\text{boost}}$

used in celestial holography, obtained via Mellin transforms

$$|\Delta, z, \bar{z}, \pm s\rangle = a^{(s)}_{\Delta,\pm}(z, \bar{z})^\dagger |0\rangle = \int_0^{+\infty} d\omega \, \omega^{\Delta - 1} |\omega, z, \bar{z}, \pm s\rangle$$



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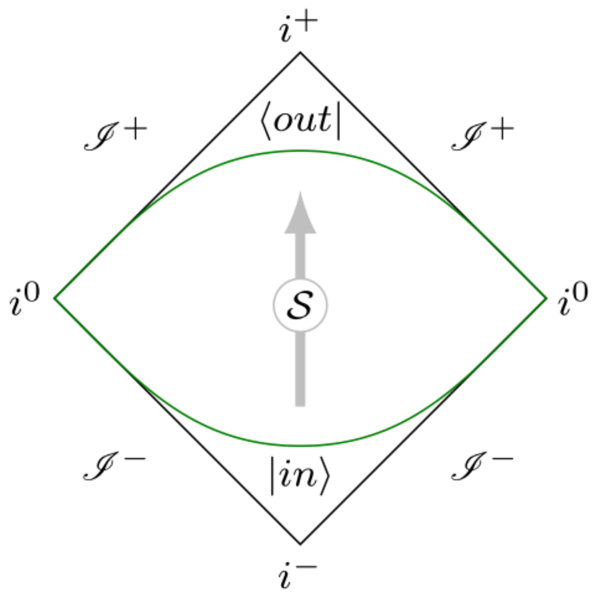

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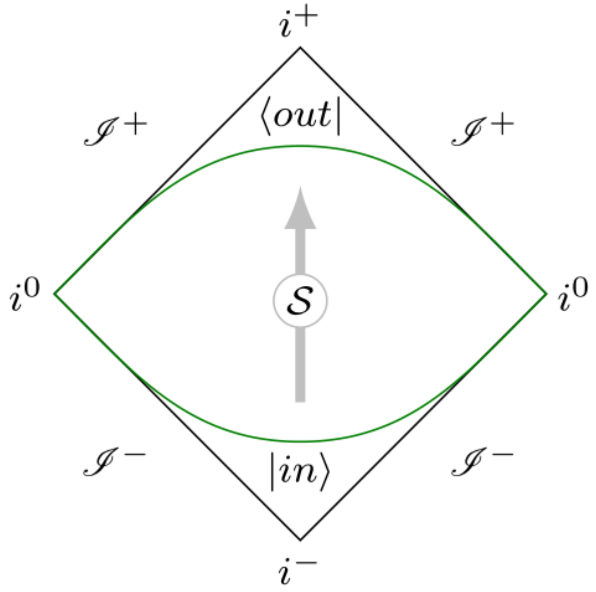

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loads of these celestial amplitudes have been explicitly computed recently

Advantage: this basis makes the conformal transformation more manifest
(but obscures the translation transformations)

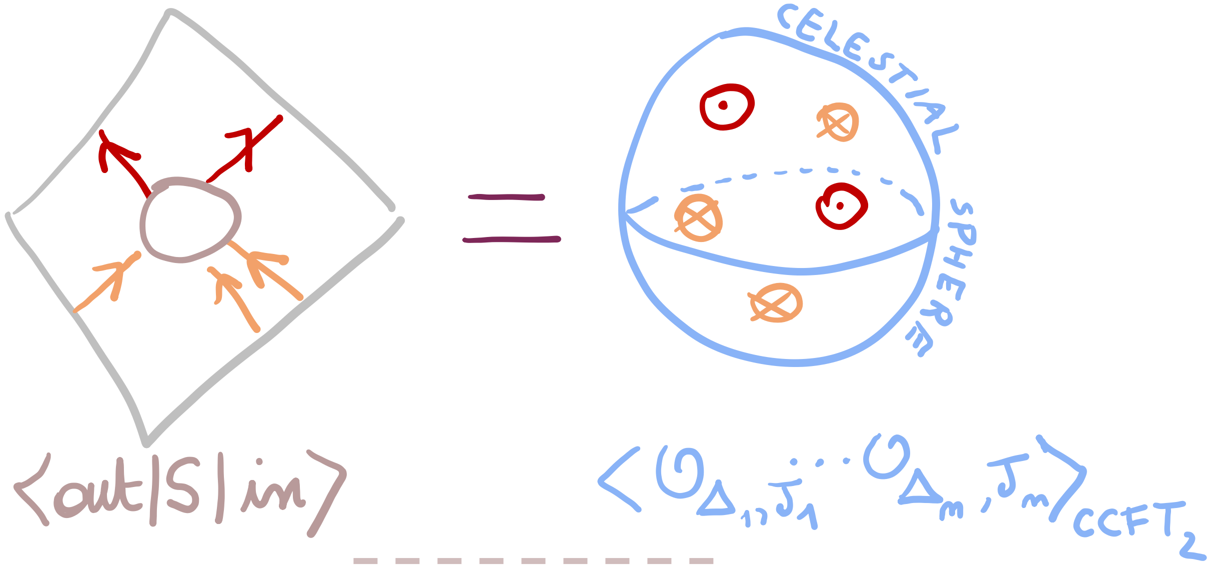
[de Boer, Solodukhin '03][Pasterski, Shao, Strominger '17]

Celestial holography in 1 slide

momentum of a massless particle

$p^\mu = \omega q^\mu(z,\bar{z})$

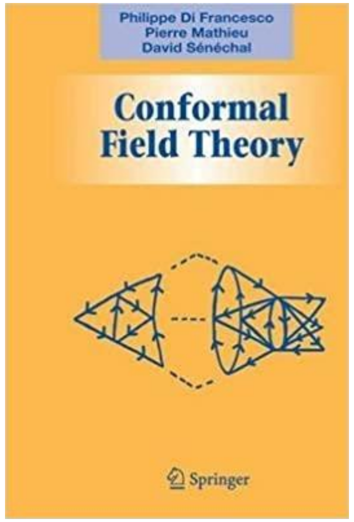
$(z,\bar{z})$: a point on $\mathcal{CS}^2$



$$\int_0^\infty d\omega\, \omega^{\Delta-1}$$

$\Delta = h + \bar{h}$: conformal dimension

J : 2d spin



?

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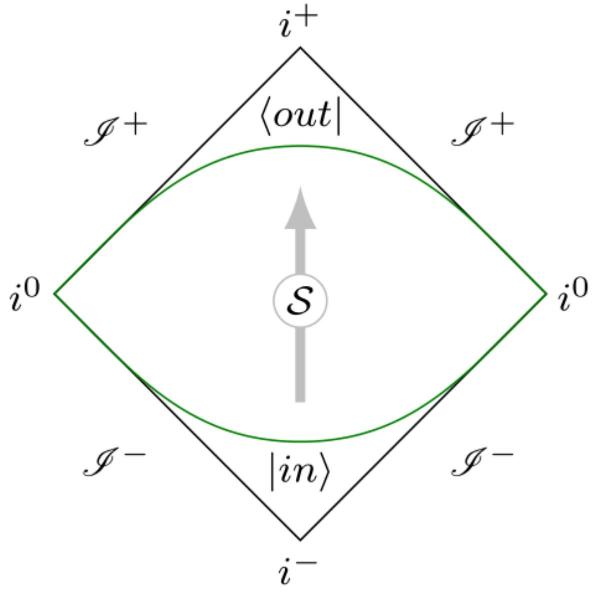
Position space basis

Fourier transforms from momentum basis

$$\mathcal{C}_N = \int_0^{+\infty} d\omega_1\, e^{-i\omega_1 u_1} \dots \int_0^{+\infty} d\omega_N\, e^{i\omega_N v_N}\, \mathcal{A}_N$$

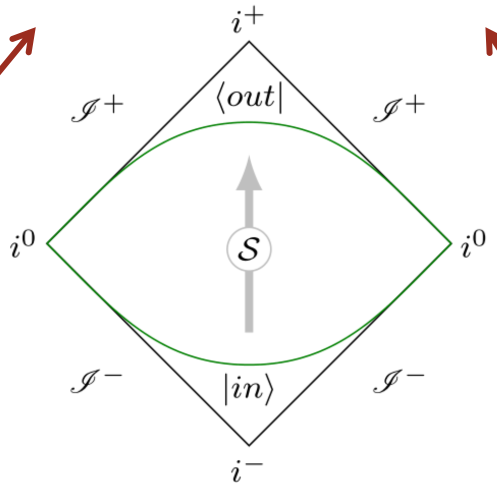


Momentum basis

$$\mathcal{A}_N = \langle \text{out} | \mathcal{S} | \text{in} \rangle_{\text{momentum}}$$

Mellin

$$\int_0^{+\infty} d\omega \, \omega^{\Delta - 1}$$



Fourier

$$\int_0^{+\infty} d\omega \, e^{-i\omega u}$$

Celestial basis

$$\mathcal{M}_N = \langle \text{out} | \mathcal{S} | \text{in} \rangle_{\text{boost}}$$

Mellin + Fourier

$$\int_{-\infty}^{+\infty} du \, u^{-\Delta}$$

Position space basis

$$\mathcal{C}_N = \langle \text{out} | \mathcal{S} | \text{in} \rangle_{\text{position}}$$

Ex: Two-point amplitudes

Momentum basis

$$\mathcal{A}_2(\omega_1, \omega_2) = \omega_1^{-1}\delta(\omega_1 - \omega_2)\delta^{(2)}(z_1 - z_2)\delta_{\alpha_1,\alpha_2}$$

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Celestial basis

$$\mathcal{M}_2(\Delta_1,\Delta_2) = \delta(\nu_1+\nu_2)\delta^{(2)}(z_1-z_2)\delta_{\alpha_1,\alpha_2}$$

$$\Delta_i = 1 + i\nu_i$$ [de Boer, Solodukhin '03]

Normalizable wavepackets lie on the principal series

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Normalizable wavepackets lie on the principal series

$$\int_0^{+\infty} d\omega\, e^{-i\omega u}$$

Position space basis

$$\mathcal{C}_2(u_1,u_2) = \int_0^{+\infty} \frac{d\omega_1}{\omega_1} e^{-i\omega_1(u_1-u_2)}\delta^{(2)}(z_1-z_2)\delta_{\alpha_1,\alpha_2}$$

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divergent integral → can be regulated as

$$= \lim_{\beta\to 0^+}\left[\frac{1}{\beta} - \left(\gamma + \ln|u_1-u_2| + \frac{i\pi}{2}\text{sign}(u_1-u_2)\right)\right]\delta^{(2)}(z_1-z_2)\delta_{\alpha_1,\alpha_2}$$

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$$\int_{-\infty}^{+\infty} du\, u^{-\Delta} \qquad \int_0^{+\infty} d\omega\, e^{-i\omega u}$$

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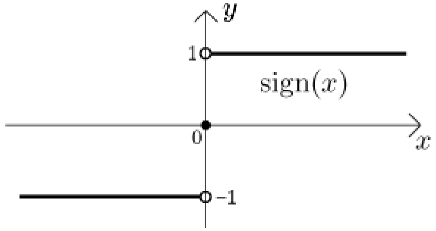
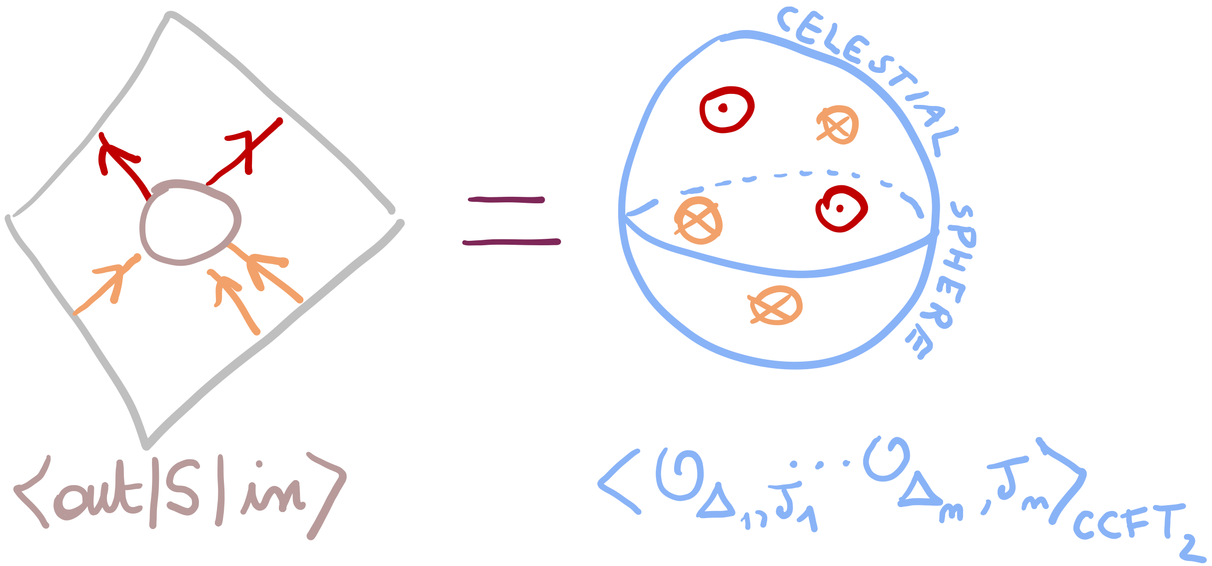


Towards Carrollian holography...

The S-matrix has an intrinsic holographic flavor.

In celestial holography, scattering elements -written in a boost eigenstate basis- are interpreted as correlation functions of a 'celestial CFT'.



Towards Carrollian holography...

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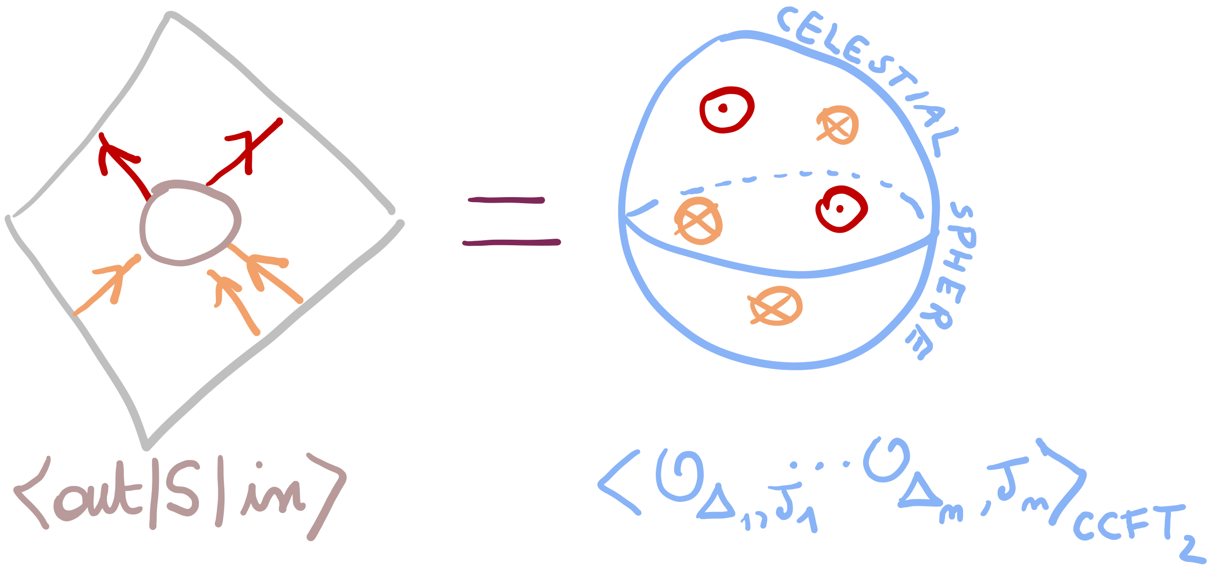

Can we interpret S-matrix elements as correlation functions of a '**conformal Carrollian field theory**'?

$$\langle \sigma^{\text{out}}_{(k_1,\bar{k}_1)}(x_1)\dots\sigma^{\text{out}}_{(k_n,\bar{k}_n)}(x_n)\sigma^{\text{in}}_{(k_{n+1},\bar{k}_{n+1})}(x_{n+1})\dots\sigma^{\text{in}}_{(k_N,\bar{k}_N)}(x_N)\rangle \equiv \prod_{k=1}^{n}\int_0^{+\infty} d\omega_k\, e^{-i\omega_k u_k}\prod_{\ell=n+1}^{N}\int_0^{+\infty} d\omega_\ell\, e^{i\omega_\ell v_\ell}\mathcal{A}_N(p_1;\dots;p_N),$$

From bulk to boundary operators (and back)

From **bulk** to **boundary** (large r expansion):

$$\Phi(X) = \int \frac{d^3p}{(2\pi)^3 2p^0} \left[a(p)e^{ip\cdot X} + a(p)^\dagger e^{-ip\cdot X}\right]$$

$p^\mu = \omega q^\mu(\vec{w})$

momentum of a massless particle heading towards the celestial sphere

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scalar: $\Phi \sim \frac{1}{r} \int_0^{+\infty} d\omega \left[\, a(\omega, z, \bar{z})e^{-i\omega u} - a(\omega, z, \bar{z})^\dagger e^{+i\omega u}\right]$

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spin s: $$\Phi^{(s)}_{z\ldots z}(X) \sim r^{s-1}\int_0^{+\infty} d\omega \left[a_+^{(s)}(\omega,z,\bar{z})e^{-i\omega u} - a_-^{(s)}(\omega,z,\bar{z})^\dagger e^{+i\omega u}\right]$$

(photon) $A_z \sim A_z^{(0)}(u,z,\bar{z})$

(graviton) $h_{zz} \sim rC_{zz}(u,z,\bar{z})$

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$$|||$$

$$\bar{\Phi}_{z\dots z}(u, z, \bar{z})$$

This is the boundary operator: it encodes the asymptotic behavior at null infinity. Later we will identify it with a 'Carrollian primary'.

From bulk to boundary operators (and back)

From **bulk** to **boundary** (large r expansion):

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$$\equiv$$

$$\bar{\Phi}_{z\dots z}(u, z, \bar{z})$$

This is the boundary operator: it encodes the asymptotic behavior at null infinity. Later we will identify it with a 'Carrollian primary'.

Using the usual commutation relations $[a^{(s)}_{\alpha}(\vec{p}), a^{(s)}_{\alpha'}(\vec{p}')^\dagger] = (2\pi)^3 2p^0 \delta^{(3)}(\vec{p} - \vec{p}')\delta_{\alpha,\alpha'}$, one gets

$$[\bar{\Phi}_{z\dots z}(u, z, \bar{z}), \bar{\Phi}_{\bar{z}\dots\bar{z}}(u', z', \bar{z}')] = \text{sign}(u - u')\delta^{(2)}(z - z')$$

Ex: gravitational shear obeys the canonical relations $[C_{zz}(u, z, \bar{z}), C_{\bar{z}\bar{z}}(u', z', \bar{z}')] = \text{sign}(u - u')\delta^{(2)}(z - z')$

From bulk to boundary operators (and back)

From **bulk** to **boundary** (large r expansion): $\Phi^{(s)}_{z\dots z}(X) \sim r^{s-1} \; \bar{\Phi}_{z\dots z}(u, z, \bar{z})$

From **boundary** to **bulk**:

$$\Phi^{(s)}_{I}(X) = \int_0^{+\infty} d\omega d^2 z \left[\epsilon^{*\alpha}_I \, a^{(s)}_\alpha(\omega, z, \bar{z}) e^{ip\cdot X} + h.c.\right]$$

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$$a^{(s)}_+(\omega, z, \bar{z}) = \int_{-\infty}^{+\infty} d\tilde{u}\, e^{i\omega\tilde{u}} \bar{\Phi}_{z\dots z}(\tilde{u}, z, \bar{z})$$

$$\Phi^{(s)}_I(X) = \int d^2 z\, \epsilon^{*+}_I\, \partial_{\tilde{u}} \bar{\Phi}_{z\dots z}(\tilde{u} = -q \cdot X, z, \bar{z}) + h.c.$$

Kirchhoff-d'Adhémar formula

Allows to reconstruct the bulk field from its boundary value at $\mathscr{I}^+$

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  destroys (creates) outgoing spin-s particles with positive (negative) helicity

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$$\delta_\xi \bar{\Phi}^{(s)}(u,z,\bar{z}) = \left[\left(\mathcal{T} + \tfrac{u}{2}(\partial\mathcal{Y} + \bar{\partial}\bar{\mathcal{Y}})\right)\partial_u + \mathcal{Y}\partial + \bar{\mathcal{Y}}\bar{\partial} + k\,\partial\mathcal{Y} + \bar{k}\,\bar{\partial}\bar{\mathcal{Y}}\right]\bar{\Phi}^{(s)}(u,z,\bar{z})$$

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  Ex: gravitational shear $C_{zz}(u,z,\bar{z})$ is a (quasi-)Carrollian primary of weights $\left(\frac{3}{2}, -\frac{1}{2}\right)$.

  $J = +2$

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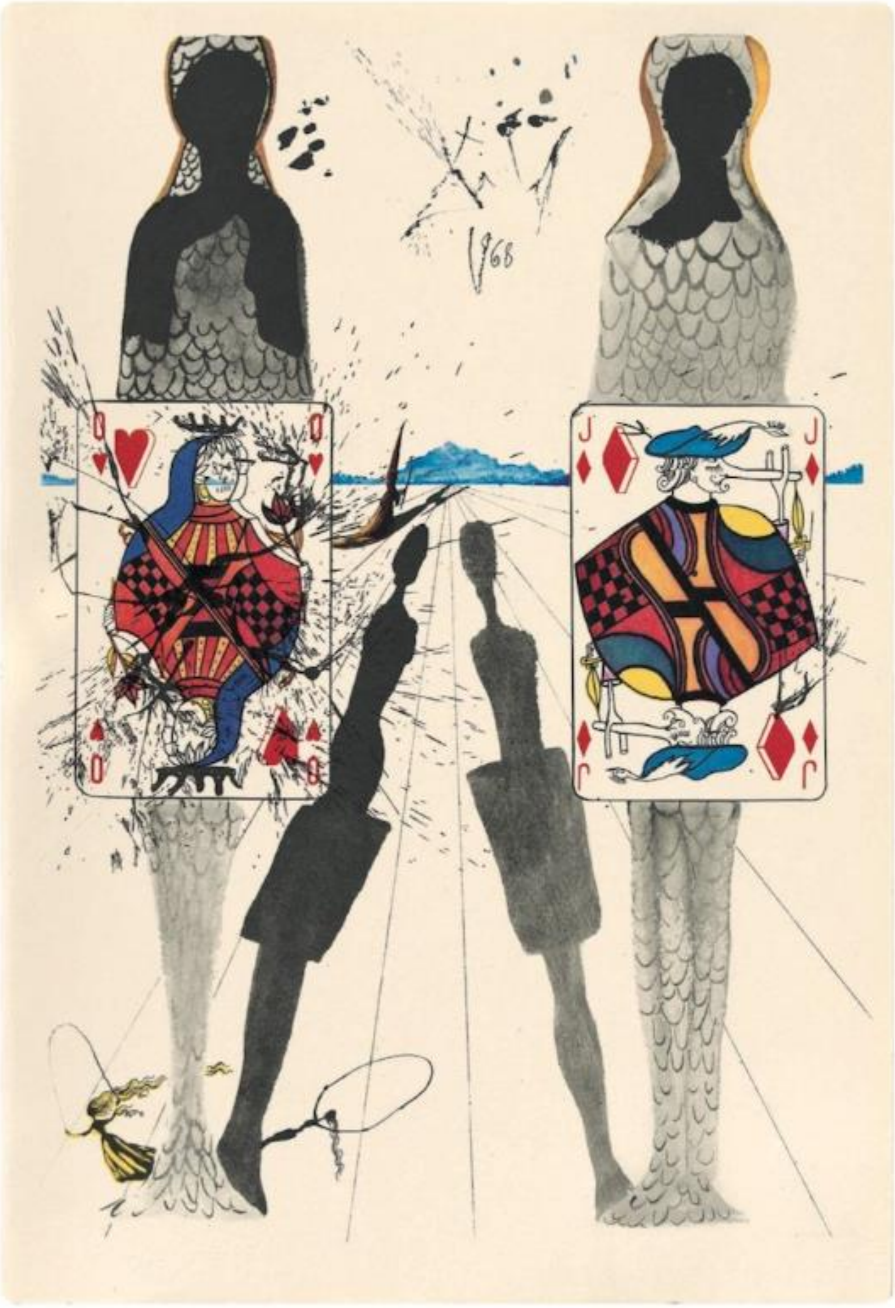
$$k_{12}^{\pm} \equiv \sum_{i=1,2} (k_i \pm \bar{k}_i)$$

Ex: two-point function

$$\mathcal{C}_2(u,v) = \left[\frac{1}{\beta} - \left(\gamma + \ln|u-v| + \frac{i\pi}{2} \text{sign}(u-v) \right) \right] \delta^{(2)}(z_1 - z_2) \delta_{k_{12}^+,0} \delta_{k_{12}^-,0}$$

Salvador Dalí, illustrations for *Alice's Adventures in Wonderland*, 1969:



Outline

1. BMS & the S-matrix
2. Bases and boundary operators
3. Towards Carrollian holography

Sourced Ward identities

Set up: consider a theory in n dimensions with action

$$S[\Psi|\sigma] = \int d^n x \, L[\Psi|\sigma]$$

Ψ^i : dynamical fields

σ^m : sources (fields without e.o.m)

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Idea: promote the Noetherian symmetries $\delta_K \Psi^i = K^i[\Psi]$ of the theory without sources $(\sigma = 0)$ to generalized symmetries of the sourced theory [Troessaert '16][Barnich, Fiorucci, Ruzziconi, *to appear*] :

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In the presence of external sources, Noether currents j^a_K are no longer conserved:

$$\partial_a j^a_K = F_K[\Psi|\sigma] \neq 0$$

flux term

Sourced Ward identities

Ward identities associated to generalized symmetries:

$$\delta_K \Psi^i = K^i[\Psi|\sigma] \qquad \delta_K \sigma^m = K^m[\sigma]$$

$$\partial_a \langle j_K^a(x) X_N^\Psi \rangle = \sum_{k=1}^{N} \delta^{(n)}(x - x_k)\, \delta_{K^{i_k}} \langle X_N^\Psi \rangle + \langle F_K(x) X_N^\Psi \rangle$$

$$\partial_a \langle j_K^a(x) X_N^\sigma \rangle = \langle F_K(x) X_N^\sigma \rangle$$

'sourced Ward identities'

$$X_N^\Psi \equiv \Psi^{i_1}(x_1) \dots \Psi^{i_N}(x_N) \qquad \delta_{K^{i_k}} X_N^\Psi \equiv \Psi^{i_1}(x_1) \dots K^{i_k}[\Psi(x_k)] \dots \Psi^{i_N}(x_N)$$

$$X_N^\sigma \equiv \sigma^{m_1}(x_1) \dots \sigma^{m_N}(x_N)$$

[LD, Fiorucci, Herfray, Ruzziconi '22]

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$$\partial_a \langle j^a_K(x) X^\sigma_N \rangle = \langle F_K(x) X^\sigma_N \rangle$$

→ let's apply this to a 'conformal Carrollian theory'

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$$j^a_{\bar{\xi}} = \mathcal{C}^a{}_b \bar{\xi}^b$$

$$x^a = (u, z, \bar{z})$$

$$\mathcal{C}^a{}_b = \begin{bmatrix} \mathcal{M} & \mathcal{N}_{\mathcal{B}} \\ \mathcal{B}^A & \mathcal{A}^A{}_B \end{bmatrix}$$

Carrollian stress tensor

: encodes Carrollian momenta

[Ciambelli, Marteau, Petkou, Petropoulos, Siampos '18]²

[Ciambelli, Marteau '18][LD, Marteau '19]

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Carrollian stress tensor

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[Ciambelli, Marteau, Petkou, Petropoulos, Siampos '18]²
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- Global conformal Carrollian symmetries (Carrollian rotation, translations, boosts, dilatation, ~~special CT~~) impose the following constraints

$z\partial_z - \bar z\partial_{\bar z}$ $\quad \partial_a \quad$ $z\partial_u, \bar z\partial_u$ $\quad x^a\partial_a$

$$\begin{aligned} \partial_u \mathcal{M} &= F_u\,, & \mathcal{B}^A &= 0\,, \\ \partial_u \mathcal{N}_z - \tfrac{1}{2}\partial\mathcal{M} + \bar\partial\mathcal{A}^{\bar z}{}_z &= F_z\,, & 2\mathcal{A}^z{}_z + \mathcal{M} &= 0\,, \\ \partial_u \mathcal{N}_{\bar z} - \tfrac{1}{2}\bar\partial\mathcal{M} + \partial\mathcal{A}^z{}_{\bar z} &= F_{\bar z}\,, & 2\mathcal{A}^{\bar z}{}_{\bar z} + \mathcal{M} &= 0 \end{aligned}$$

[LD, Fiorucci, Herfray, Ruzziconi '22]

Sourced conformal Carrollian Ward identities

$X \equiv \Psi^{i_1}(x_1)\dots\Psi^{i_N}(x_N)$

The sourced Ward identities $\partial_a \langle j^a_K(x) X\rangle = \sum_{k=1}^{N} \delta^{(n)}(x-x_k)\,\delta_{K^{i_k}}\langle X\rangle + \langle F_K(x) X\rangle$

of a conformal Carrollian field theory imply

$j^a_{\bar\xi} = \mathcal{C}^a{}_b \bar\xi^b \qquad \mathcal{C}^a{}_b = \begin{bmatrix} \mathcal{M} & \mathcal{N}_B \\ \mathcal{B}^A & \mathcal{A}^A{}_B \end{bmatrix}$

$$\partial_u \langle \mathcal{M}\, X\rangle + \sum_i \delta^{(3)}(x-x_i)\partial_{u_i}\langle X\rangle = \langle F_u\, X\rangle$$

$$\partial_u \langle \mathcal{N}_z\, X\rangle - \frac{1}{2}\partial\langle \mathcal{M}\, X\rangle + \bar\partial\langle \mathcal{A}^{\bar z}{}_z\, X\rangle + \sum_i \left[\delta^{(3)}(x-x_i)\partial_i\langle X\rangle - \partial\left(\delta^{(3)}(x-x_i)\, k_i\,\langle X\rangle\right)\right] = \langle F_z\, X\rangle$$

$$\partial_u \langle \mathcal{N}_{\bar z}\, X\rangle - \frac{1}{2}\bar\partial\langle \mathcal{M}\, X\rangle + \partial\langle \mathcal{A}^{z}{}_{\bar z}\, X\rangle + \sum_i \left[\delta^{(3)}(x-x_i)\bar\partial_i\langle X\rangle - \bar\partial\left(\delta^{(3)}(x-x_i)\, \bar k_i\,\langle X\rangle\right)\right] = \langle F_{\bar z}\, X\rangle$$

$$\langle \mathcal{B}^A\, X\rangle = 0$$

$$\langle (\mathcal{A}^z{}_z + \frac{1}{2}\mathcal{M})X\rangle + \sum_i \delta^{(3)}(x-x_i)\, k_i\,\langle X\rangle = 0\,,$$

$$\langle (\mathcal{A}^{\bar z}{}_{\bar z} + \frac{1}{2}\mathcal{M})X\rangle + \sum_i \delta^{(3)}(x-x_i)\, \bar k_i\,\langle X\rangle = 0$$

[LD, Fiorucci, Herfray, Ruzziconi '22]

Duality Carrollian momenta/gravitational data

We propose

$$\langle \mathcal{M} \rangle = \frac{\widetilde{M}}{4\pi G} \, ,$$
$$\langle \mathcal{N}_A \rangle = \frac{1}{8\pi G} \left(\widetilde{N}_A + u \partial_A \widetilde{M} \right) \, ,$$
$$\langle \mathcal{C}^A{}_B \rangle + \frac{1}{2} \delta^A{}_B \langle \mathcal{M} \rangle = 0 \, .$$

[LD, Fiorucci, Herfray, Ruzziconi '22]

recall e.g.

$$\partial_u \langle \mathcal{M} \, X \rangle + \sum_i \delta^{(3)}(x - x_i) \partial_{u_i} \langle X \rangle = \langle F_u \, X \rangle$$

Duality Carrollian momenta/gravitational data

We propose

$$\langle \mathcal{M} \rangle = \frac{\widetilde{M}}{4\pi G}\,,$$
$$\langle \mathcal{N}_A \rangle = \frac{1}{8\pi G}\left(\widetilde{N}_A + u\partial_A \widetilde{M}\right)\,,$$
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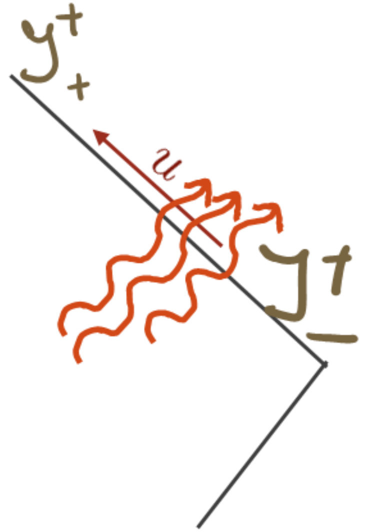
The external sources at the boundary are identified with the asymptotic shear

$$\sigma_{AB} = C_{AB}$$

Fluxes:

$$F_u = \frac{1}{16\pi G}\left[\partial_z^2 \partial_u \sigma_{\bar{z}\bar{z}} + \frac{1}{2}\sigma_{\bar{z}\bar{z}}\partial_u^2 \sigma_{zz} + \text{c.c.}\right],$$
$$F_z = \frac{1}{16\pi G}\left[-u\partial_z^3 \partial_u \sigma_{\bar{z}\bar{z}} + \sigma_{zz}\partial_z \partial_u \sigma_{\bar{z}\bar{z}} - \frac{u}{2}(\partial_z \sigma_{zz}\partial_u^2 \sigma_{\bar{z}\bar{z}} + \sigma_{zz}\partial_z \partial_u^2 \sigma_{\bar{z}\bar{z}})\right]$$

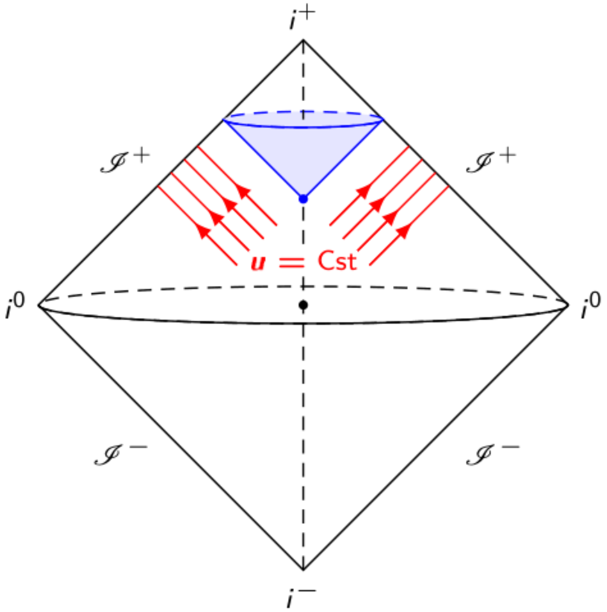


Consistently, these expressions plugged into the sourced Ward id. of the conformal Carrollian theory reproduce the time evolution $\partial_u \widetilde{M} = \ldots$ and $\partial_u \widetilde{N}_A = \ldots$ (no correlator insertion)

Constraints for a holographic conformal Carrollian theory

Gluing the future and the past

So far we have looked at future null infinity. Analogous results hold for past null infinity.



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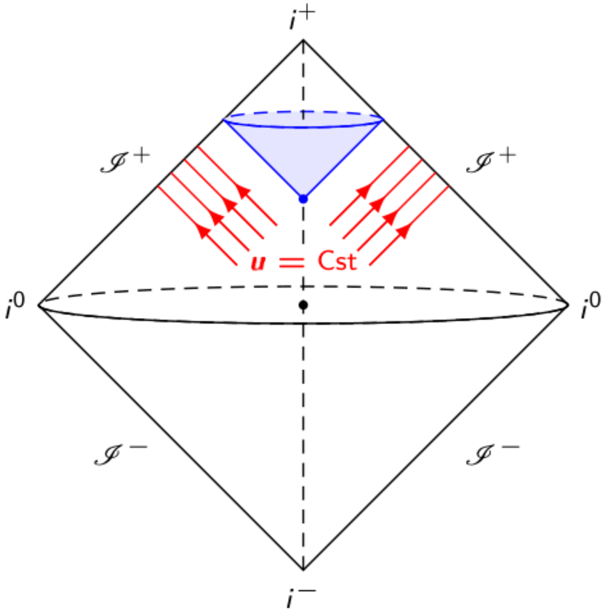
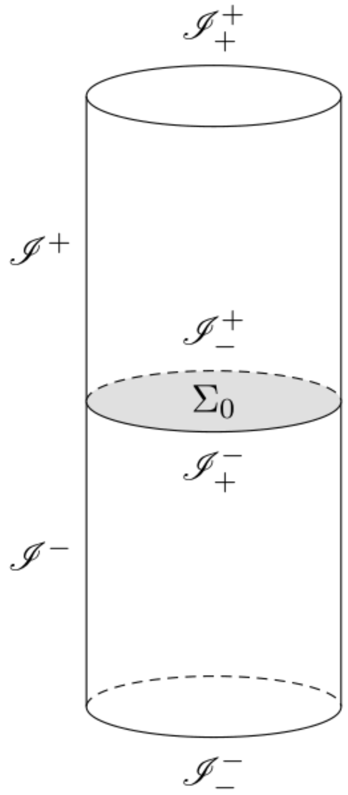


- We want to treat the conformal boundary as a **whole** by gluing the two pieces around spatial infinity.

$$\hat{\mathscr{I}} \equiv \mathscr{I}^- \sqcup \mathscr{I}^+$$



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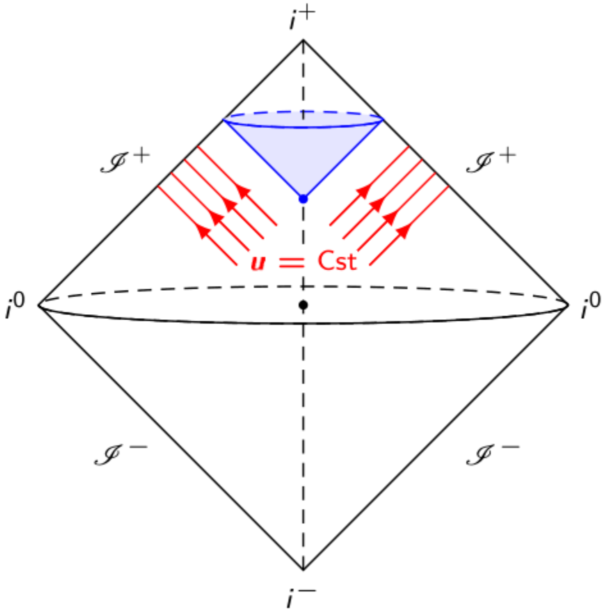

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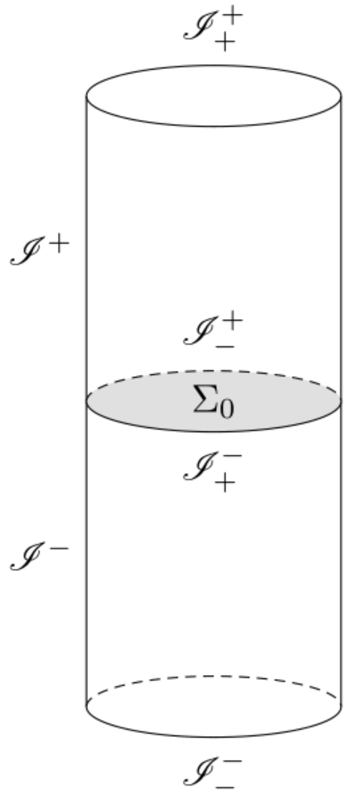
Separating surface
= locus where the Carrollian vector n^a vanishes

- We get only one smooth automorphism of $\hat{\mathscr{I}}$.
  Consistent with antipodal matching of [Strominger '13].



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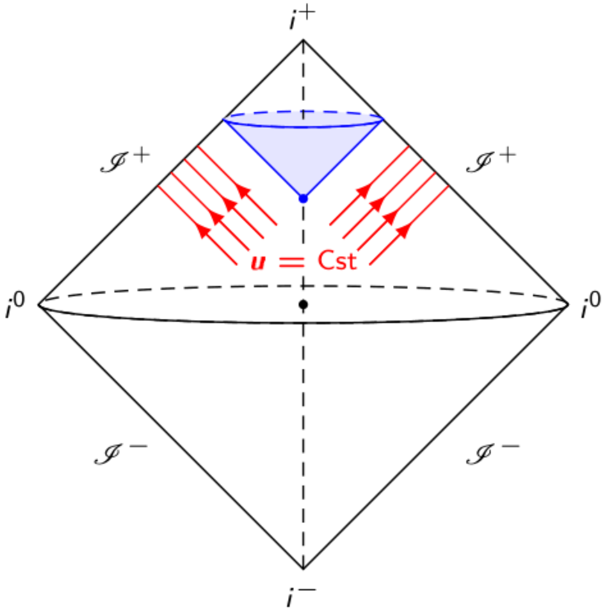

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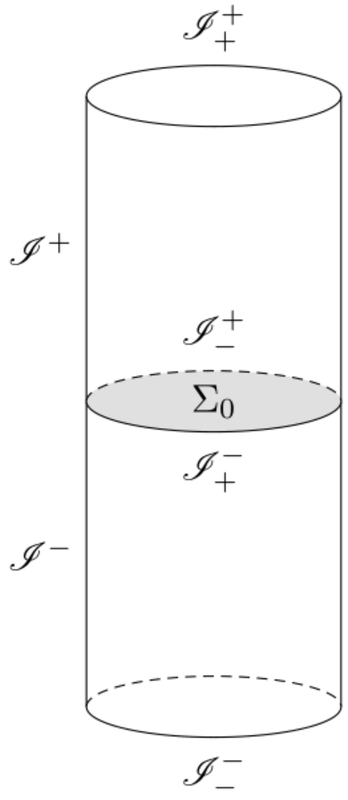

Ward id. for massless scattering

Assuming that the Noether current vanishes at $\mathscr{I}^-_-$ and $\mathscr{I}^+_+$: $\langle \int_{\hat{\mathscr{I}}} F_{\bar{\xi}}(x) X^\sigma_N \rangle = 0 \longrightarrow \boxed{\delta_{\bar{\xi}} \langle X^\sigma_N \rangle = 0}$

Invariance of the correlators under conformal Carroll symmetries

Conformal Carroll invariant low-point correlators

$$\boxed{\delta_{\bar{\xi}} \langle X_N \rangle = 0}$$

$$\langle X_2 \rangle = \langle \Phi_{(k_1,\bar{k}_1)}(u_1, z_1, \bar{z}_1), \Phi_{(k_2,\bar{k}_2)}(u_2, z_2, \bar{z}_2) \rangle$$

[Bagchi, Mandal ‘09][Bagchi, Gary, Zodinmawia ‘17][Chen, Liu, Zheng ‘21][Bagchi, Banerjee, Basu, Dutta ‘22]

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like a 2d CFT, but not interesting in this context

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$$k_{12}^{\pm} \equiv \sum_{i=1,2} (k_i \pm \bar{k}_i)$$

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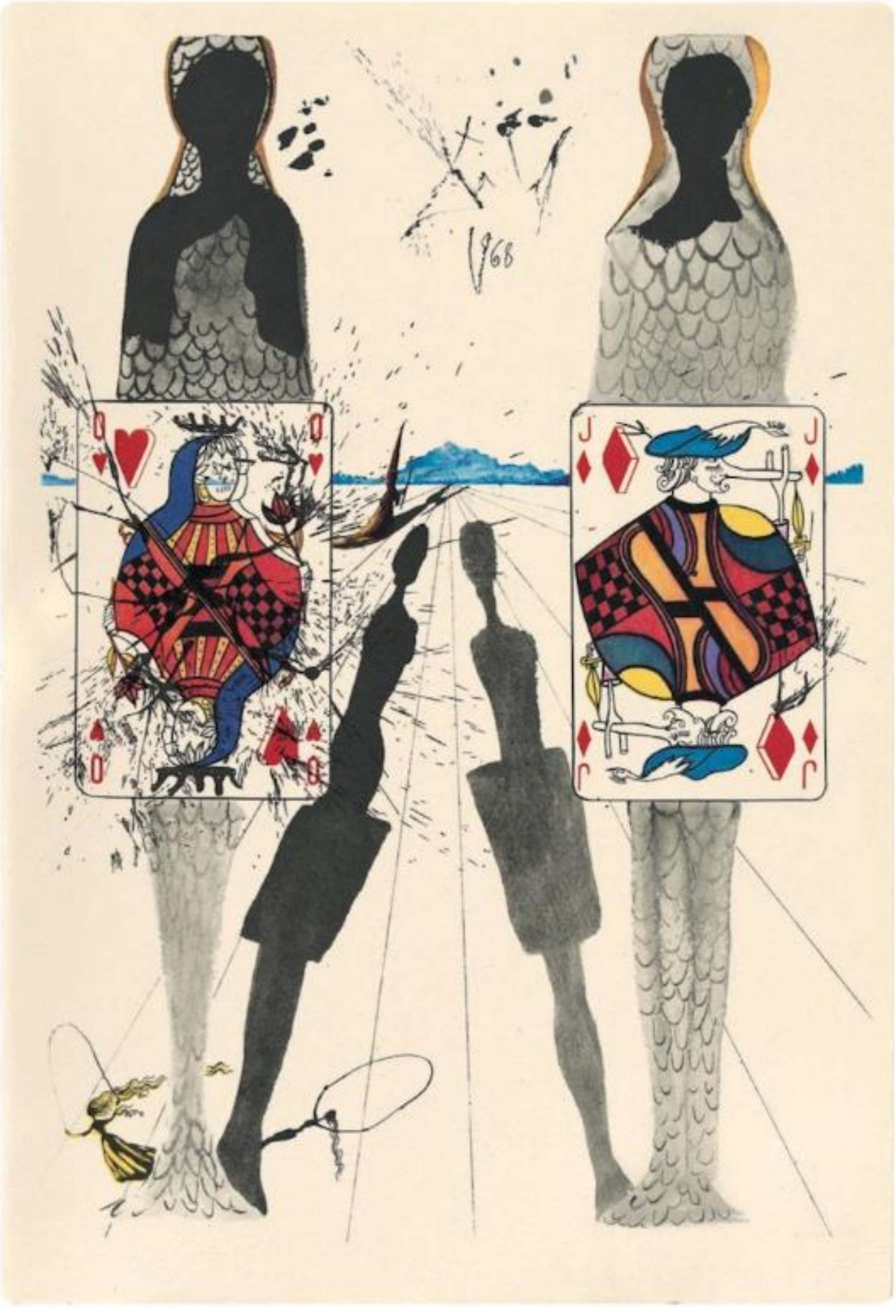
$$\langle X_2 \rangle^g_{(n)} = c_3 \frac{d^n}{du_1^n} \mathrm{sign}(u_1 - u_2)\, \delta^{(2)}(z_{12})\, \delta_{k_{12}^-,0}\, \delta_{k_{12}^+,2+n}$$

$n \in \mathbb{N}$

Salvador Dalí, illustrations for *Alice's Adventures in Wonderland*, 1969:



Outline

1. BMS & the S-matrix
2. Bases and boundary operators
3. Towards Carrollian holography
4. CCFT vs CCFT

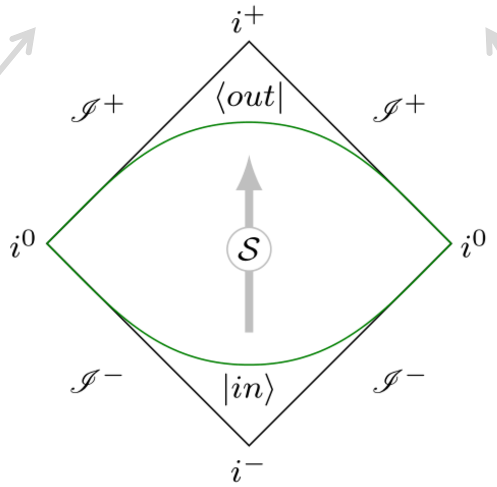
From Carrollian to celestial

Momentum basis

$$\mathcal{A}_N = \langle \text{out}|\mathcal{S}|\text{in}\rangle_{\text{momentum}}$$

$$\int_0^{+\infty} d\omega\, \omega^{\Delta-1}$$

Mellin



Fourier

$$\int_0^{+\infty} d\omega\, e^{-i\omega u}$$

Celestial basis

$$\mathcal{M}_N = \langle \text{out}|\mathcal{S}|\text{in}\rangle_{\text{boost}}$$

Position space basis

$$\mathcal{C}_N = \langle \text{out}|\mathcal{S}|\text{in}\rangle_{\text{position}}$$

Mellin + Fourier

$$\int_{-\infty}^{+\infty} du\, u^{-\Delta}$$

see also ‘extrapolate dictionary’ [Pasterski, Puhm, Trevisani ’21]

Relationship with celestial Ward identities

- The map between conformal Carrollian and celestial operators is [LD, Fiorucci, Herfray, Ruzziconi '22]

$$\mathcal{O}^{\text{out}}_{(\Delta_i,J_i)}(z_i,\bar{z}_i) = \lim_{\epsilon\to 0^+}\int_{-\infty}^{+\infty}\frac{du_i}{(u_i+i\epsilon)^{\Delta_i}}\,\sigma^{\text{out}}_{(k_i,\bar{k}_i)}(u_i,z_i,\bar{z}_i),$$

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$$k=\frac{1}{2}(1\pm J)\,,\qquad \bar{k}=\frac{1}{2}(1\mp J)$$

- Conformal Carrollian Ward identities can reproduce the ones for celestial CFT:

$$\Big\langle P(z,\bar{z})\prod_{i=1}^{N}\mathcal{O}_{\Delta_i,J_i}(z_i,\bar{z}_i)\Big\rangle + \sum_{q=1}^{N}\frac{1}{z-z_q}\Big\langle \ldots\mathcal{O}_{\Delta_q+1,J_q}(z_q,\bar{z}_q)\ldots\Big\rangle = 0$$

$$\Big\langle T(z)\prod_{i=1}^{N}\mathcal{O}_{\Delta_i,J_i}(z_i,\bar{z}_i)\Big\rangle + \sum_{q=1}^{N}\Big[\frac{\partial_q}{z-z_q}+\frac{h_q}{(z-z_q)^2}\Big]\Big\langle\prod_{i=1}^{N}\mathcal{O}_{\Delta_i,J_i}(z_i,\bar{z}_i)\Big\rangle = 0$$

leading & subleading soft graviton theorem

[He, Lysov, Mitra, Strominger '15][Kapec, Mitra, Raclariu, Strominger '17]
[LD, Puhm, Strominger '18][Fan, Fotopoulos, Taylor '19]

Summary and outlook

Conformal Carrollian field theory living at null infinity $\longleftrightarrow$ quantum gravity in flat spacetime

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What is a Conformal Carrollian FT?
$\longrightarrow$ Beyond kinematics? Top-down constructions?

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link with AdS/CFT, dS/CFT
building representations
log corrections
bootstrapping CCFT
higher dimensions
massive particles
relationship to string theory
adding black holes
...

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amplitudes

gravitational waves observation

conformal field theory

twistor theory

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Thank you.