A New Consistent Limit of 11D Supergravity

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based upon work in progress with C. Blair, J. Lahnsteiner and J. Rosseel

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Beyond Lorentzian Geometry II

rijksuniversiteit groningen

Motivation

- NR closed superstrings with $B \neq 0$ $\leftrightarrow$ NR $10D$ $\mathcal{N} = 1$ Supergravity

Klebanov, Maldacena (2000); Danielsson, Güijosa, Kruczenski (2000), Gomis, Ooguri (2001)

Lahnsteiner, Romano, Rosseel, Şimşek (2021)

$\Downarrow$

- NR Closed Supermembranes with $C \neq 0$ $\leftrightarrow$

Consistent Limit of 11D Supergravity?

García, Güijosa, Vergara (2002); Blair, Gallegos, Zinnato (2021)

see also talk by Ziqi

Outline

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Membrane Newton-Cartan geometry

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Generalizations

Defining a 'Particle' NR Limit

STEP 1: decomposing $E_\mu{}^{\hat{A}} = (E_\mu{}^0, E_\mu{}^a) =$ (clock, ruler) and introducing M_μ (for massive particles), perform an invertable field redefinition involving a dimensionless parameter ω:

$$E_\mu{}^0 = \omega\tau_\mu \,, \qquad E_\mu{}^a = e_\mu{}^a \,, \qquad M_\mu = \omega\tau_\mu - \omega^{-1} m_\mu$$

STEP 2: take the limit $\omega \to \infty$ and take care of possible divergences

Example: Particle coupled to general relativity

cancellation of infinities between kinetic term and Wess-Zumino term

Note : after taking the limit the field m_μ becomes a geometric field

The NC Gravity E.O.M.

$$\tau^\mu e^\nu{}_b\, r_{\mu\nu}{}^{0b}(G) = 0 : \qquad \mathbf{1}\ \text{ Poisson equation}$$

$$e^\nu{}_a r_{\mu\nu}{}^{ab}(J) = 0 : \qquad \mathbf{b}\,,\,(\mathbf{ab})$$

$$\partial_{[\mu}\,\tau_{\nu]} = 0 : \qquad \text{geometric constraint}$$

cp. to Van den Bleeken and Yunus (2016)

After gauge-fixing, assuming flat space, the above equations reduce to

$$\triangle\Phi = 0\,,$$

where the Newton potential Φ has been identified with the time component of the gauge field m_μ : $\Phi \sim m_0$

Intrinsic Torsion

$$\partial_{[\mu}\tau_{\nu]} = 0 \quad \rightarrow \quad \tau_\mu = \partial_\mu \rho \quad \text{with} \quad \tau_\mu \ \text{clock function}$$

$$\Delta T = \int_{\mathcal{C}} dx^\mu \tau_\mu = \int_{\mathcal{C}} d\rho \ \text{ is path-independent} \quad \rightarrow \quad \text{absolute time}$$

$$\text{Torsional NC gravity}: \quad \partial_\mu \tau_\nu - \Gamma_{\mu\nu}{}^\rho \tau_\rho = 0 \quad \rightarrow \quad \Gamma_{[\mu\nu]}{}^\rho \tau_\rho = \partial_{[\mu}\tau_{\nu]}$$

'Membrane NC' Geometry

A membrane couples to a 3-form gauge field $C_{\mu\nu\rho}$ with

$$C_{\mu\nu\rho} = -\omega^3 \epsilon_{ABC} \tau_\mu{}^A \tau_\nu{}^B \tau_\rho{}^C + c_{\mu\nu\rho}$$

defining a geometry with a co-dimension 3 foliation where $\tau_\mu \rightarrow \tau_\mu{}^A$ with $\hat{A} = (A, a) = (0, 1, 2, a)$

Note : after taking the limit the field $c_{\mu\nu\rho}$ becomes a geometric field

In the bosonic membrane sigma model there is a cancellation between the leading divergence of the kinetic term and the leading divergence of the Wess-Zumino term

The Variables of 'Membrane NC' Gravity

The redefinition leading to NC gravity

$$\{E_\mu{}^{\hat{A}}, M_\mu\} \quad \rightarrow \quad \{\tau_\mu, e_\mu{}^a, m_\mu\}$$

gets replaced by the following 'membrane' redefinition:

$$\{E_\mu{}^{\hat{A}}, C_{\mu\nu\rho}\} \quad \rightarrow \quad \{\tau_\mu{}^A, e_\mu{}^a, c_{\mu\nu\rho}\}$$

The Newton potential Φ can be identified with the worldvolume components c_{012} of the 3-form gauge field $c_{\mu\nu\rho}$

After gauge-fixing, this Newton potential should satisfy a Poisson equation in the transverse directions

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Expansion of the Bosonic Action

The basic fields are $\{E_\mu{}^{\hat{A}}, C_{\mu\nu\rho}\}$ with relativistic action given by

$$S_{\rm rel} \sim \int d^{11}x \left(E\left[\mathcal{R} - \frac{1}{48}\mathcal{F}_{\mu\nu\rho\sigma}\mathcal{F}^{\mu\nu\rho\sigma}\right] + \frac{1}{144^2}\epsilon^{\mu_1\cdots\mu_{11}}\mathcal{F}_{\mu_1\cdots\mu_4}\mathcal{F}_{\mu_5\cdots\mu_8}C_{\mu_9\mu_{10}\mu_{11}}\right)$$

with $\mathcal{F}_{\mu\nu\rho\sigma} = 4\,\partial_{[\mu}C_{\nu\rho\sigma]}$. We decompose $\hat{A} = (A, a)$ and redefine

$$E_\mu{}^A = \omega\tau_\mu{}^A\,, \qquad E_\mu{}^a = \omega^{-1/2}e_\mu{}^a\,, \qquad C_{\mu\nu\rho} = -\omega^3\epsilon_{ABC}\tau_\mu{}^A\tau_\nu{}^B\tau_\rho{}^C + c_{\mu\nu\rho}$$

We find

$$S = \omega^3 \overset{(3)}{S} + \overset{(0)}{S} + \omega^{-3}\overset{(-3)}{S} + \cdots$$

Taming Divergences

(i) the metric and part of the 3-form contributions to $\overset{(3)}{S}$ precisely cancel !

(ii) There are remaining divergences coming from the 3-form kinetic term and the Chern-Simons term which precisely add up to a complete square:

$$\omega^3 \, f^{(+)}_{abcd} \, f^{abcd(+)}$$

where $f^{(+)}_{abcd}$ is the SO(8) self-dual part of the $c_{\mu\nu\rho}$ curvature

This divergence can be tamed by introducing an auxiliary field λ as follows:

$$\omega^3 X^2 \quad \text{is equivalent to} \quad -\frac{1}{\omega^3}\lambda^2 - 2\lambda X \quad \text{for any } X$$

After taking the limit this auxiliary field becomes a Lagrange multiplier

In our case we have $X^{(+)}_{abcd} = f^{(+)}_{abcd} \quad \rightarrow$ Lagrange multiplier $\lambda^{(+)}_{abcd}$

Conventional versus Geometric Tensors

conventional tensors are curvature components of $\{\tau_\mu^A\,, e_\mu{}^a\,, c_{\mu\nu\rho}\}$ that contain spin-connection fields. By setting these tensors to zero we can solve for some (but not all!) components of the spin-connection fields

geometric tensors are the remaining curvature components that do not contain any spin-connection. Setting them to zero leads to geometric constraints

The geometric tensors are given by the following projections of the $\tau_\mu{}^A$ curvature $T_{\mu\nu}{}^A$ and the $c_{\mu\nu\rho}$ curvature $f_{\mu\nu\rho\sigma}$:

$$T_{ab}{}^A\,, \quad T_a{}^{\{AB\}}\,, \quad T_a{}^A{}_A\,, \quad f^{(\pm)}_{abcd}\,, \quad f_{Abcd}$$

$\{AB\}$ means symmetric traceless in A and B

where we have defined longitudinal and transverse projections as follows:

$$V_A \equiv \tau^\mu{}_A V_\mu\,, \qquad V_a \equiv e^\mu{}_a V_\mu \qquad \text{for any vector } V_\mu$$

Divergences in the 11D Supergravity Action

The cancellation between divergences coming from the Einstein-Hilbert term and the kinetic term of the three-form continues to hold in the presence of fermions

The remaining divergences coming from the kinetic term of the three-form and the Chern-Simons term add up to a complete square as follows:

$$\omega^3 \hat{f}^{(+)}_{abcd} \hat{f}^{abcd(+)}$$

where $\hat{f}^{(+)}_{abcd}$ is the SO(8) self-dual part of the supercovariant $c_{\mu\nu\rho}$ curvature

This divergence can be tamed like in the bosonic case by introducing an auxiliary field but now with $X^{(+)}_{abcd} = \hat{f}^{(+)}_{abcd}$. We thus end up with an action with

$$\overset{(3)}{S} = 0$$

Divergences in the Supersymmetry Rule

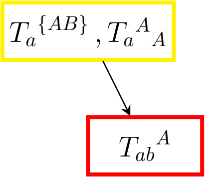
We find ω^3 divergent terms in the ‘Q-supersymmetry’ rule of the gravitino and Lagrange multiplier

Using $\overset{(3)}{S} = 0$, these divergences are tamed by emergent fermionic symmetries and by imposing geometric constraints:

- $\overset{(3)}{\delta_\epsilon} \overset{(0)}{S} = 0 \quad \rightarrow$ we find 2 ‘superconformal’ Stueckelberg symmetries
- imposing $\overset{(3)}{\delta_\epsilon} \overset{(-3)}{S} = 0$ we need to set all super-covariantized geometric tensors equal to zero

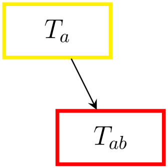
Five Bosonic Membrane Geometries

J. Figueroa-O'Farrill, Iisakki, Rosseel, ter Veldhuis, van Helden + E.B., *work in progress*



1. The intrinsic torsion is unconstrained
2. $\mathbf{T_{ab}{}^A = 0}$: the foliation is integrable
3. $\mathbf{T_a{}^{\{AB\}} = T_{ab}{}^A = 0}$: the vectors $e^\mu{}_a$ are homothetic Killing vectors
4. $\mathbf{T_a{}^A{}_A = T_{ab}{}^A = 0}$: the worldvolume is absolute
5. $\mathbf{T_{\mu\nu}{}^A = 0}$: the foliation is integrable, the vectors $e^\mu{}_a$ are homothetic Killing vectors and the worldvolume is absolute

Particle and Domain Wall Geometries



three particle geometries (Galilei gravity: $T_{ab} = 0$)

cp. to Christensen, Hartong, Obers, Rollier (2014)

$$\boxed{T^{\{AB\}}\,, T^{A}{}_{A}}$$

four domain wall geometries

cp. to Figueroa-O'Farrill (2020)

Carroll gravity after $a \leftrightarrow A$: $T^{(ab)} = 0$ Gomis, Rollier, Rosseel, ter Veldhuis (2017)

cp. to Barducci, Casalbuoni and Gomis (2018); Izquierdo, Romano + E.B. (2020)

Supergeometry

We introduce two S-supersymmetry gauge fields

Like in the bosonic case we divide the Q- and S-covariant curvatures of the gravitino into conventional and geometric tensors

Under susy we find

$$\text{bosonic G.C.} \rightarrow \text{fermionic G.C.} \rightarrow \partial\,(\text{bosonic G.C.}) + \text{constraints on } r(\omega)$$

The Poisson equation is a singlet constraint on $r(\omega)$ (boost)

work in progress!

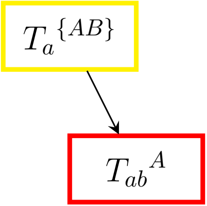
Three Supergeometries

In the bosonic case we find an an-isotropic local dilatation symmetry D

The field equations (including the Poisson equation) form a reducible but indecomposable representation

In the supersymmetric case we find consistency with the emergent fermionic symmetries: $\{Q\,,S\}\sim D$

This leads to three possible modified geometries:



Alternative Approach: Gauge-fixing

Gauge-fixing has been done for non-relativistic 3D particle supergravity with bosonic field content

$$\{\tau_\mu\,, e_\mu{}^a\,, m_\mu\}$$

Rosseel, Zojer + E.B. (2015)

After gauge-fixing,

$$\tau_\mu = \delta_\mu{}^0\,, \quad e_\mu{}^a = (0, \delta_i{}^a)\,, \quad m_\mu = (\Phi = m_0, 0)$$

the (bosonic part of) the theory reduces to a 3D particle Newtonian supergravity multiplet in flat space:

$$\boxed{\{\Phi = m_0, \Psi\}}$$

Although there are differences (particle versus membrane, emergent symmetries) we expect after gauge fixing to find a similar 11D Membrane Newtonian supergravity multiplet (to be confirmed!)

$$\boxed{\{\Phi = C_{012}, \Psi, \lambda^{ijkl(+)}\}}$$

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Other Supergravity Theories

Our results pave the way for constructing

- NR IIA string supergravity by double dimensional reduction
- NR IIB string supergravity: branched SL(2,Z) duality

  see Ziqi's talk; Grosvenor, Lahnsteiner, Yan, Zorba + E.B. (2022)
- NR Heterotic Supergravity: non-Lorentzian Chern-Simons term?

Much more p-brane supergravity theories are suggested such as a NR 11D five-brane supergravity !

One can now classify the supersymmetric solutions including the NR D-brane solutions $\rightarrow$ holographic applications ?

Relation to Null Reduction

Going back to 10D there is an underlying sigma model formulation with a NR string T-duality which is the $R \to \infty$ limit of the relativistic T-duality

$$R \leftrightarrow 1/R \qquad \text{giving}$$

$$\text{NR string theory} \quad \longleftrightarrow \quad \text{DLCQ of string theory}$$

What we did is in some sense the M-theory uplift (or the Type IIA string analogue) of this NR string T-duality giving information about the null-reduction of M-theory

What about α'-corrections ?

Take-Home Message

This talk provides one more example of the

many interesting applications/connnections of

non-Lorentzian (Super-)Gravity Theories in Diverse Dimensions!

Oling and Yan (2022); Figueroa-O'Farrill, Gomis + E.B. (2022); Rosseel + E.B. (2022); Hartong, Obers, Oling (2022)