Quirks, Novelties, and Curiosities of Fractons

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Beyond Lorentzian Geometry II
ICMS, University of Edinburgh
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Instituut-Lorentz
Universiteit Leiden
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Outline

1. Intro to fractons and some history.
2. Fractonic hydro.
3. Fractons on curved manifolds.
4. Fractonic RG.
5. Outlook.

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What is a fracton?

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A fracton is an excitation which cannot move in some spatial directions

- Lattice models
- Defects (e.g., disclinations and dislocations in solids)
- QFTs with peculiar dispersion relations

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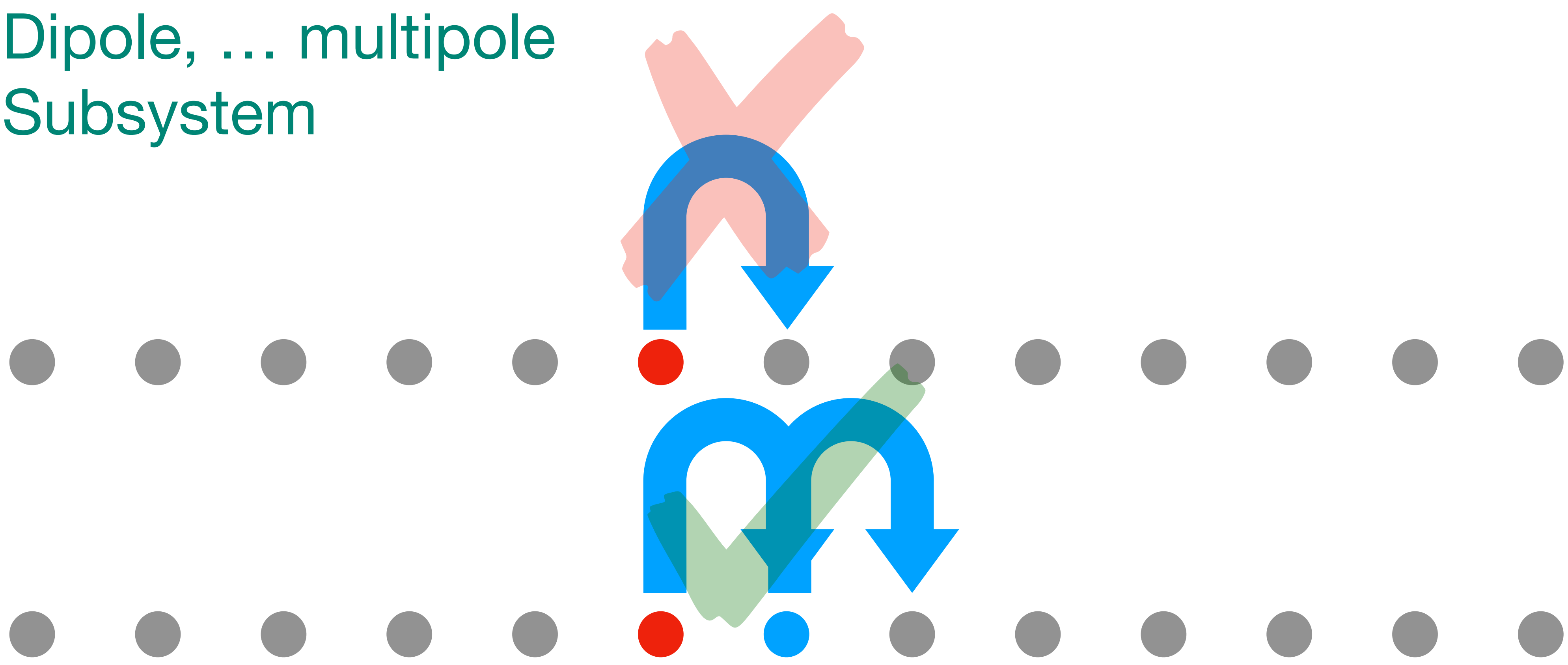
Mobility constraints imposed by spatially-dependent symmetries

- Dipole, … multipole
- Subsystem



A bit of history

Gapped fractonic lattice models

- Jan '11 Haah
  Haah code 1101.1962
- Mar '16 Vijay, Haah, Fu
  X-cube model 1603.04442
- Jun '18 Shirley, Slagle, Chen
  Foliated phases 1806.08625

Field theory

- Aug '17 Slagle, Kim
  X-cube EFT 1708.04619
- Jul '18 Pretko
  Fracton gauge principle 1807.11479
- Sep '19 Seiberg
  Vector global symmetry 1909.10544

Gapless fractonic models

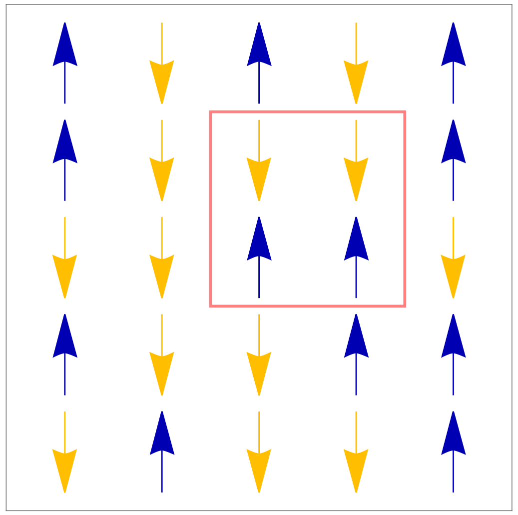
- Apr '16 Pretko
  Higher-rank spin liquid 1604.05329
- Jun '16 Pretko
  Tensor Gauge theory 1606.08857
- Nov '17 Pretko, Radzihovsky
  Fracton/Elasticity Duality 1711.11044

My involvement

- Aug '13 Griffin, G, Hořava, Yan
  Polynomial shifts 1308.5967
- Dec '18 Gromov
  Multipole algebra 1812.05104
- May '21 G, Hoyos, Peña-Benítez, Surówka
  Ideal hydro 2105.01084
- Dec '21 G, Lier, Surówka
  Fractonic RG and BKT 2207.14343

Subsystem Symmetries

Example from [Yan ('19)]:



$$H = -\sum_{\square} \prod_{i \in \square} S_i$$

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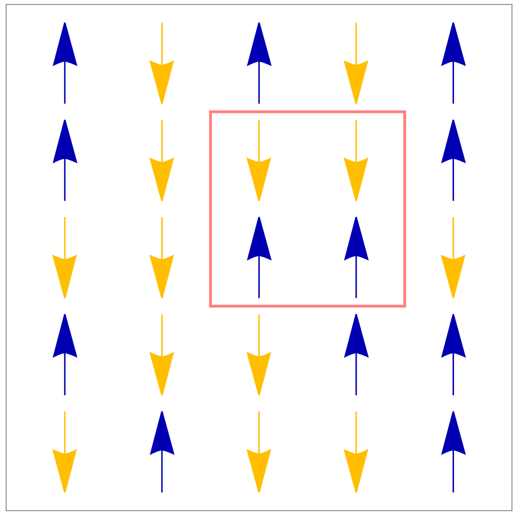

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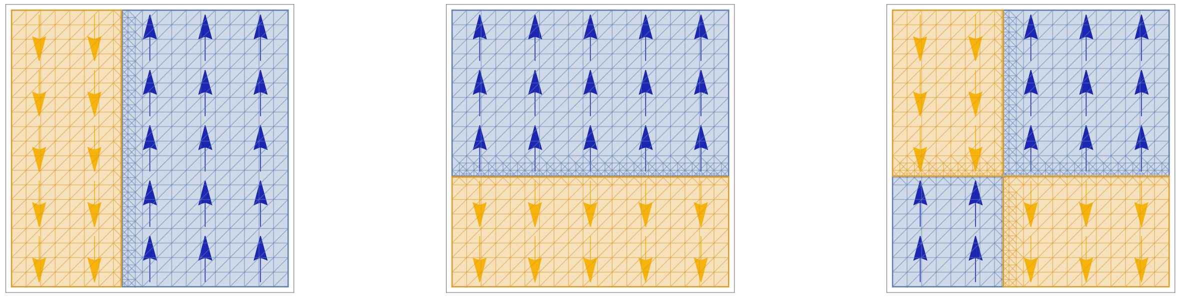
Flip spins on one side of a vertical or horizontal line or a combination:



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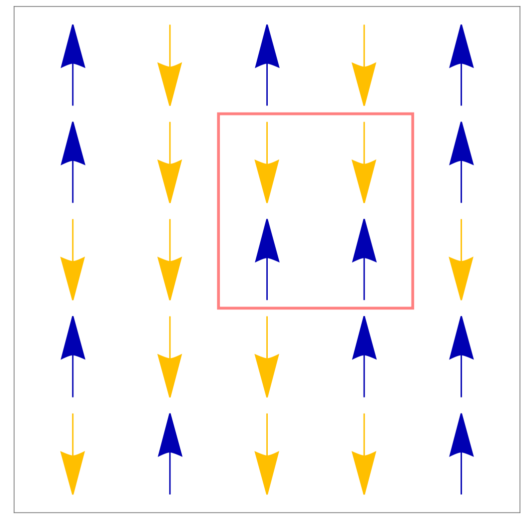

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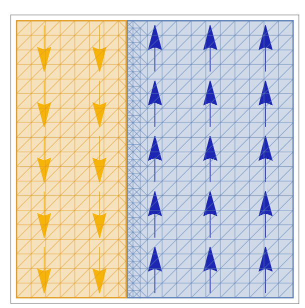
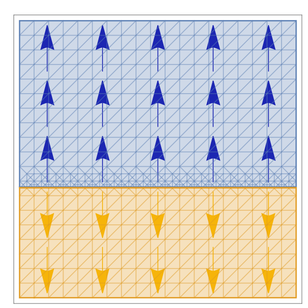
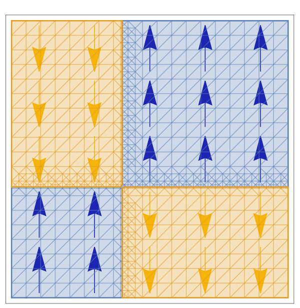
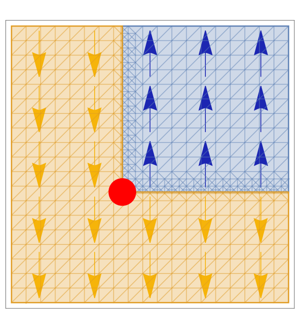


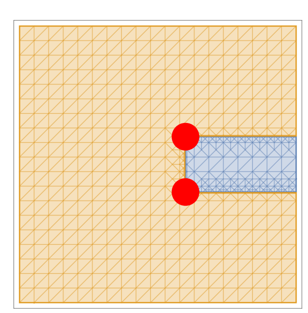




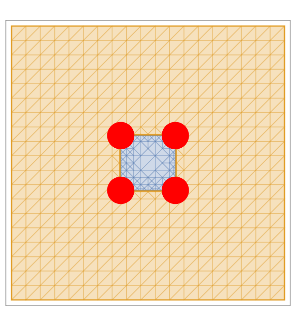
Elementary Fracton



2-Fracton bound state



4-Fracton bound state



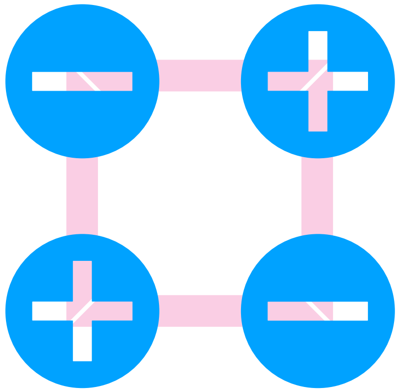
Subsystem Symmetries

Example from [Paramekanti, Balents, Fisher ('02)]: (2+1)d S^1-rotor bosons

Lattice site d.o.f.: phase $\phi_{\mathbf{r}}$ and momentum $\pi_{\mathbf{r}} = -i\partial_{\phi_{\mathbf{r}}}$

$$H = \sum_{\mathbf{r}} \left(\frac{U}{2}\pi_{\mathbf{r}}^2 - K\cos(\Delta_{xy}\phi_{\mathbf{r}}) \right)$$

$$\Delta_{xy}\phi_{\mathbf{r}} =$$



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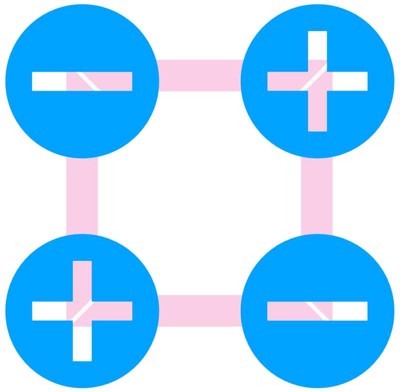
$\Delta_{xy}\phi_{\mathbf{r}} =$ 

Subsystem shift symmetry: $\phi_{\mathbf{r}} \to \phi_{\mathbf{r}} + f(x) + g(y)$

log(ground state deg) $\sim N_x + N_y$

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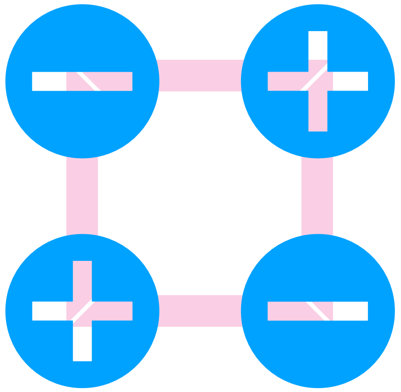

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In the continuum limit …

Subsystem Symmetries

Example from [Seiberg, Shao ('20)]: $\mathscr{L} = \frac{1}{2}(\partial_t\phi)^2 - \frac{1}{2}(\partial_x\partial_y\phi)^2$

Subsystem shift symmetry: $\phi \to \phi + f(x) + g(y)$
(see also [Karch, Raz ('20)] and [Casalbuonia, Gomis, Hidalgo ('21)])

Dispersion relation: $\omega^2 = k_x^2 k_y^2$

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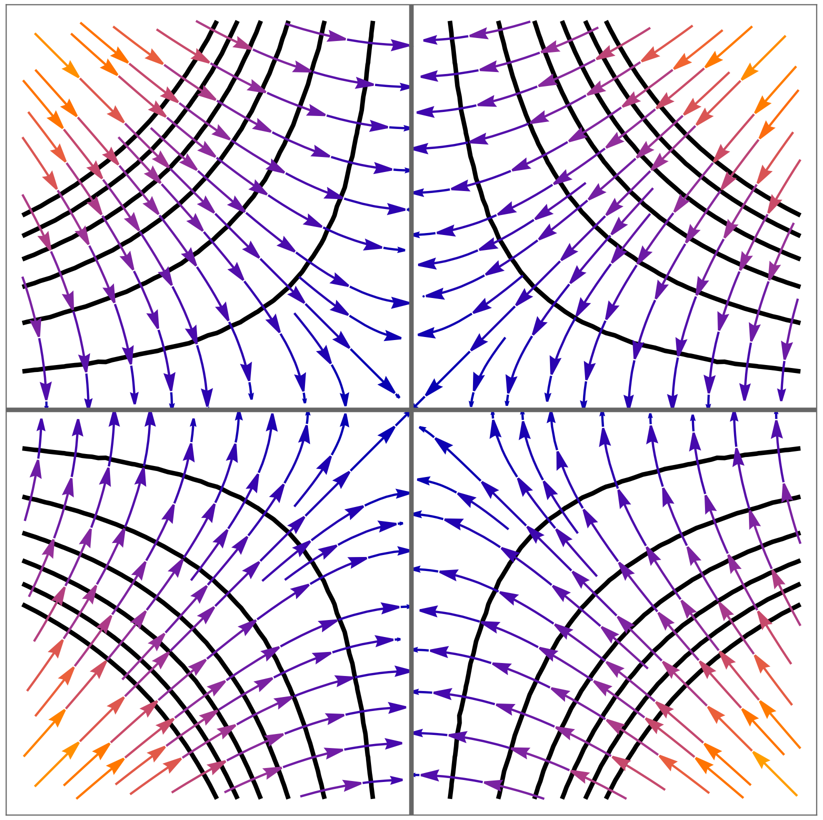

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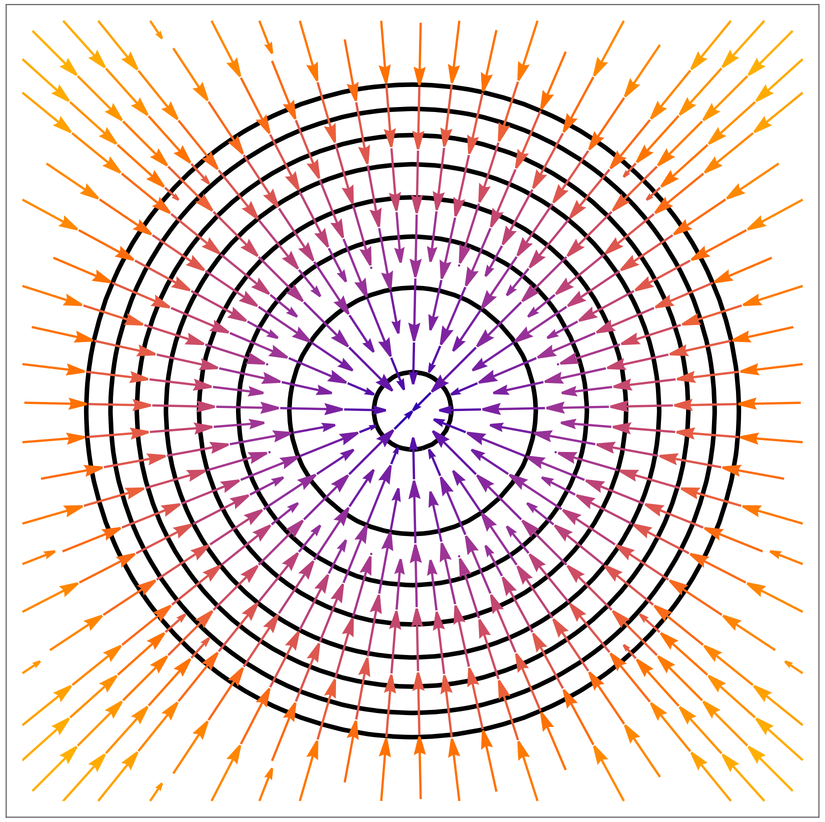

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Some References

[Submitted on 20 Mar 2020 (v1), last revised 22 Jul 2020 (this version, v3)]

Fracton hydrodynamics

Andrey Gromov, Andrew Lucas, Rahul M. Nandkishore

Dissipative, not translation-invariant

[Submitted on 3 May 2021]

Hydrodynamics of ideal fracton fluids

Kevin T. Grosvenor, Carlos Hoyos, Francisco Peña-Benitez, Piotr Surówka

MDMA, no dissipation (ideal)

[Submitted on 27 May 2021]

Breakdown of hydrodynamics below four dimensions in a fracton fluid

Paolo Glorioso, Jinkang Guo, Joaquin F. Rodriguez-Nieva, Andrew Lucas

MDMA plus dissipation

[Gromov, Lucas, Nandkishore ('20)]

MDA Hydro

Monopole: $Q = \int d^d x \, \rho$

Dipole: $Q^i = \int d^d x \, \rho x^i$

$[Q^i, Q] = 0$

$\partial_t \rho + \partial_i \partial_j J^{ij} = 0$

[Gromov, Lucas, Nandkishore ('20)]

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Why is this non-trivial? Suppose $\partial_t \rho + \partial_i J^i = 0$ and expand J^i in derivatives of ρ. The term $J_i = -D\partial_i \rho$ gives the usual Fick's law $\partial_t \rho = D\nabla^2 \rho$. This still ensures $\dot{Q} = \dot{Q}^i = 0$ assuming no boundary terms.

Fluctuation-dissipation (or Einstein relation) relates the diffusion constant to charge mobility.

Expect a uniform electric field to excite a charge current.

But, force on a dipole $\mathbf{d}$ is $\mathbf{F} = (\mathbf{d} \cdot \nabla)\mathbf{E}$, which is 0 for a uniform electric field.

Key point: a solitary charge is immobile by itself.

[Gromov, Lucas, Nandkishore ('20)]

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Key point: a solitary charge is immobile by itself.

Generalized Fick's law: $J_{ij} = -D\partial_i \partial_j \rho \quad \Longrightarrow \quad \partial_t \rho = D\nabla^4 \rho$ (subdiffusive)

[G, Hoyos, Peña-Benítez, Surówka ('21)]

Monopole-Dipole-Momentum Algebra (MDMA)

Monopole: Q

Dipole: $Q^i,\ i = 1,\ldots,d$

Momentum: $P_i,\ i = 1,\ldots,d$

$$\{Q, Q\} = \{Q, P_i\} = \{Q, Q^i\} = 0$$

$$\{P_i, Q^j\} = Q\delta_i^j$$

(Heisenberg algebra)

[Peña-Benítez '21]

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$$\{P_i, Q^j\} = Q\delta_i^j$$

(Heisenberg algebra)
[Peña-Benítez '21]

Transformation law: $\delta_{(\alpha,\beta,\gamma)}\Phi = \{\Phi, \alpha Q + \beta_i Q^i + \gamma^i P_i\}$

Scalar example: $\delta_\alpha \phi(t, \vec{x}) = \alpha$ $\delta_\beta \phi(t, \vec{x}) = \beta_i x^i$ $\delta_\gamma \phi(t, \vec{x}) = \gamma^i \partial_i \phi(t, \vec{x})$

[Nambu '60; Goldstone '61]

Galileons [Nicolis et al. '08]
Polynomial shifts [Griffin et al. '14]

[G, Hoyos, Peña-Benítez, Surówka ('21)]

MDMA Hydro

$\{P_i, Q^j\} = Q\delta_i^j$ ➡ $\delta_\beta p_i = \beta_i \rho$ ➡ $V_i := \frac{p_i}{\rho}$ ➡ $\delta_\beta V_i = \beta_i$ ↩ $\delta_\beta \partial_i V_j = 0$ ⬅ $H = \int d^d x\, h(\rho, \partial_i V_j, \partial_i \rho \ldots)$

Linear perturbations around equilibrium state $\rho = \rho_0 + \delta\rho$ $\quad p_i = 0 + \delta p_i$

General t-rev.-inv. quadratic Hamiltonian: $h = \frac{\mu_0}{2\rho_0}\rho^2 + \frac{1}{2\rho_0}\mu_1^{ij}\partial_i\rho\partial_j\rho + \frac{\rho_0}{2}v_1^{ijkl}\partial_i V_j \partial_k V_l$

Linearized hydro equations:

$$\partial_t \delta\rho - v_1^{ijkl}\partial_i\partial_j\partial_k \delta p_l = 0$$

$$\partial_t \delta p_i + (\mu_0 - \mu_1^{jk}\partial_j\partial_k)\partial_i \delta\rho = 0$$

➡

$$\omega^2 = \mu(q) v_1^{ijkl} q_i q_j q_k q_l$$

$$\mu(q) = \mu_0 + \mu_1^{ij} q_i q_j$$

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Fractons on General 3D Manifolds [Shirley, Slagle, Wang, Chen ('18)]

Chiral Topological Elasticity and Fracton Order [Gromov ('19)]

Symmetric Tensor Gauge Theories on Curved Spaces [Slagle, Prem, Pretko ('19)]

Fractons, Symmetric Gauge Fields and Geometry [Peña-Benítez ('21)]

Fractons on Curved Space [Jain, Jensen ('22)]

Fractons, Dipole Symmetries and Curved Spacetime
[Bidussi, Hartong, Have, Musaeus, Prohazka ('22)]

Couple to Gauge Fields

Coupling [Pretko ('16)] : $-\int dt\, d^d x\, (\rho A_t + J^{ij} A_{ij})$

Gauge transformation: $A_t \rightarrow A_t + \partial_t \alpha$ and $A_{ij} \rightarrow A_{ij} + \partial_i \partial_j \alpha$

Field strength: $E_{ij} = -\partial_t A_{ij} - \partial_i \partial_j A_t$ and $B_{ijk} = 2\partial_{[i} A_{j]k}$

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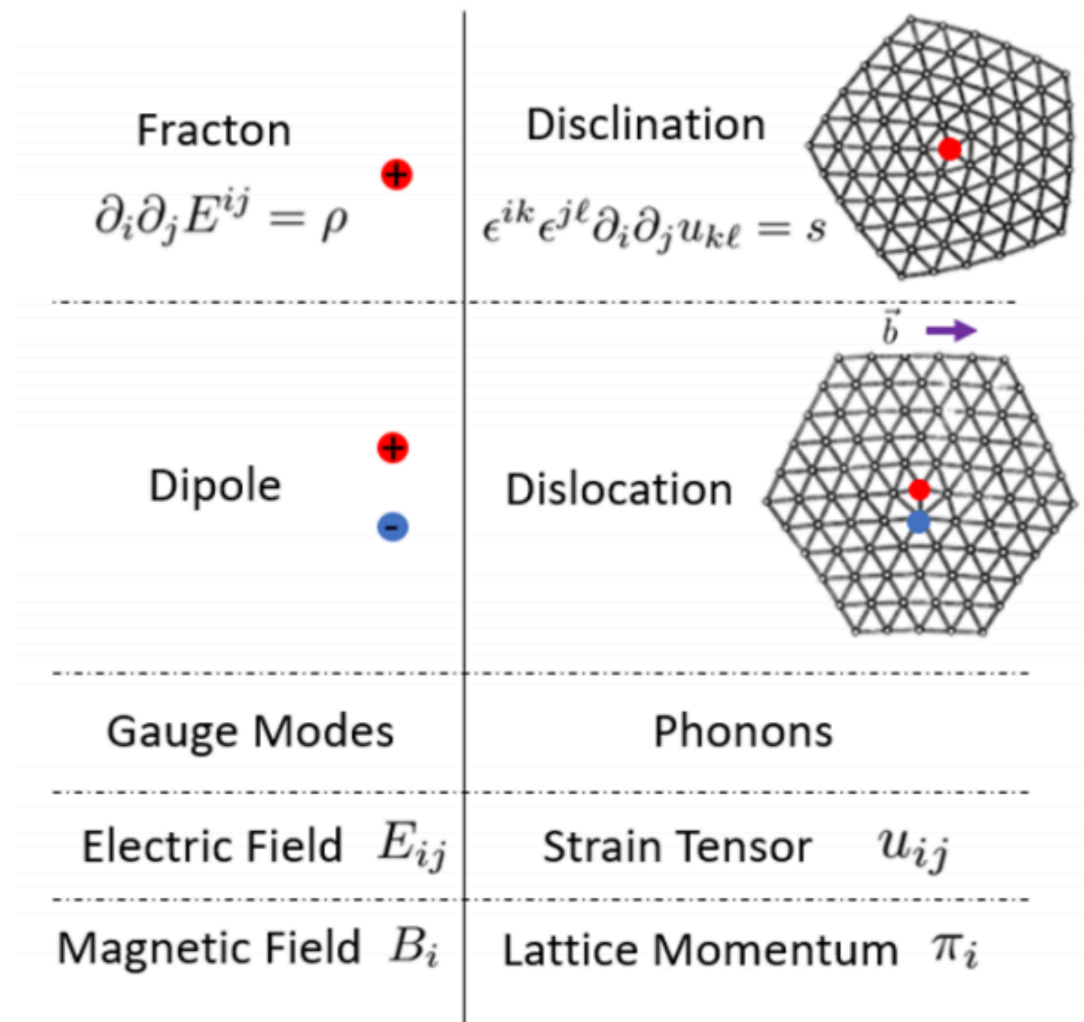
2d elasticity duality [Pretko, Radzihovsky ('17)]:



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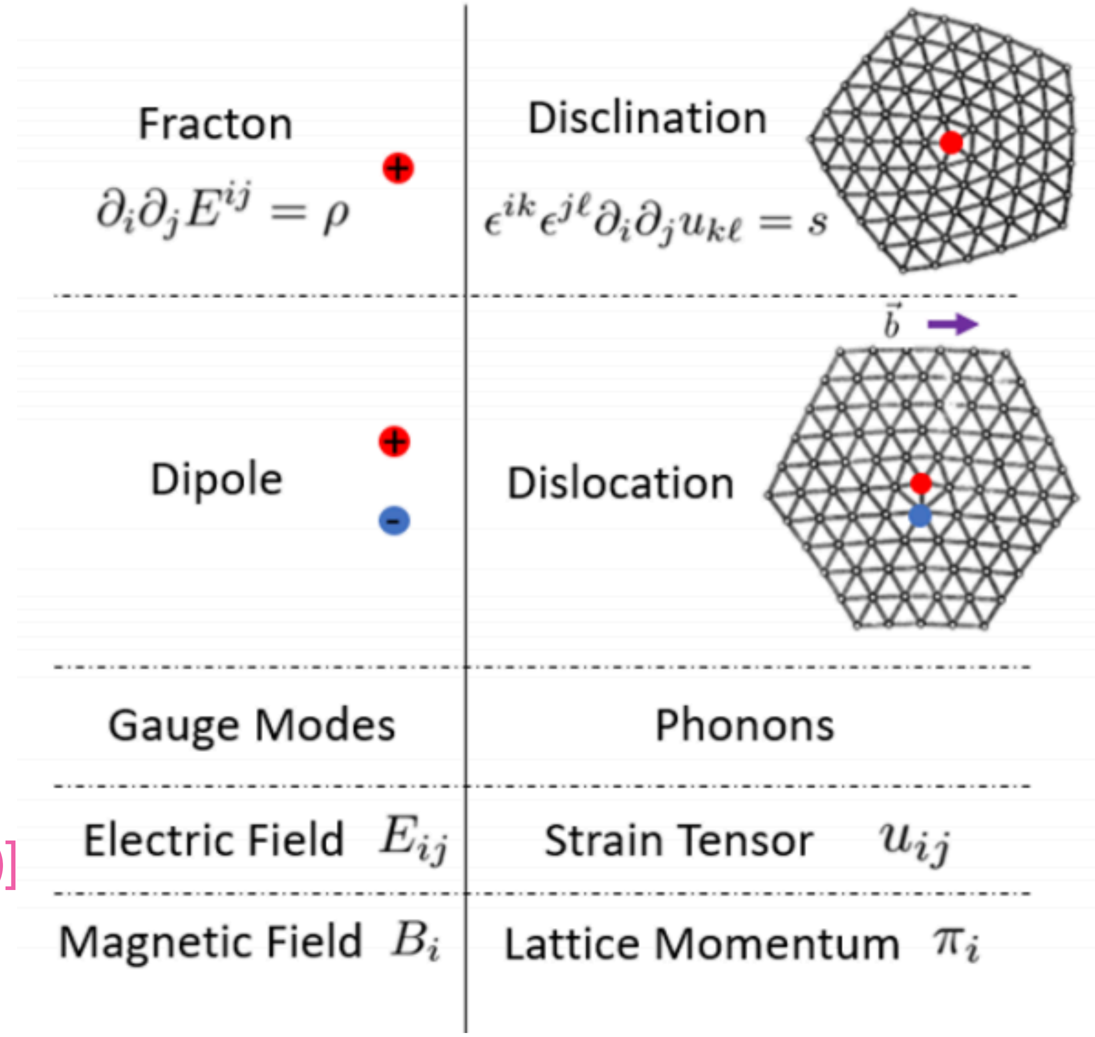

Generalizations:

Cosserat elasticity [Gromov, Surówka ('19)]
Superfluid vortices [Nguyen, Gromov, Moroz ('20)]
Quasicrystals [Surówka ('21)]
Moiré lattices [Gaa, Palle, Fernandes, Schmalian ('21)]

The Problem of Curvature

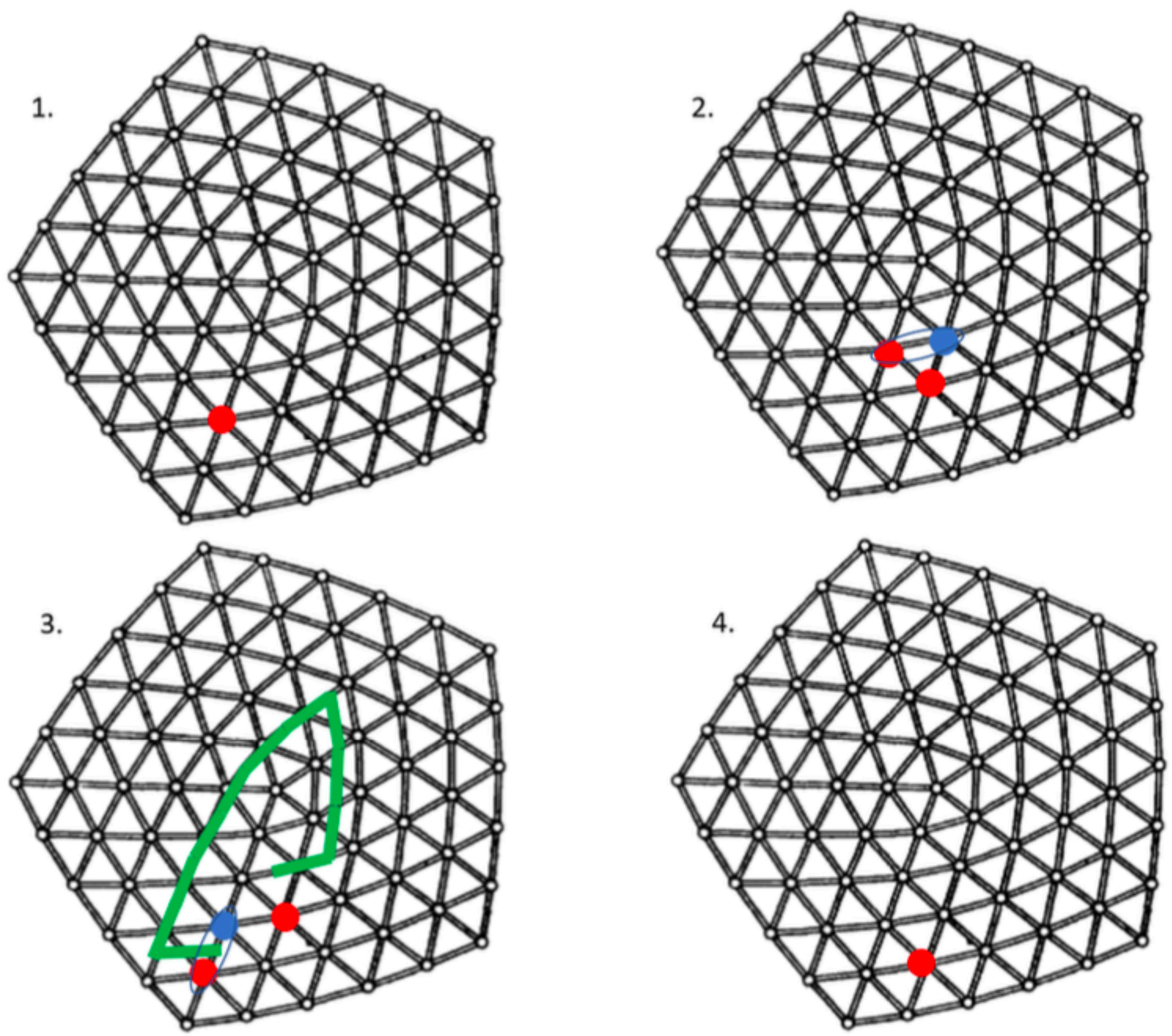

Figure 2: On a lattice, the mobility restriction on fractons is violated by a (two-dimensional) virtual dipole propagating around a disclination defect, *i.e.* a quantized unit of curvature (and torsion), which rotates the dipole and results in net motion of the fracton.

[Slagle, Prem, Pretko ('19)]

The Problem of Curvature

[Bidussi, Hartong, Have, Musaeus, Prohazka ('22)]

Curved space:

Dim.	Theory	Spatial geometry
$d=2$	magnetic theory with $h_1+h_2>0$	flat
	electric theory (traceful and traceless)	any
	CS-like theory	constant sectional curvature
$d\geq 3$	magnetic theory with $h_2\neq -(d-1)h_1$	flat
	magnetic theory with $h_2=-(d-1)h_1$	constant sectional curvature
	electric theory (traceful and traceless)	any

Curved spacetime:

Theory	Curved torsion-free Aristotelian background
Magnetic theory (7.38) with $h_2=-(d-1)h_1$ $(d\geq 3)$	obeying $h^{\mu\kappa}h^{\nu\lambda}R_{\mu\nu\rho}{}^{\sigma}=\dfrac{R}{d(d-1)}\left(P^{\kappa}_{\rho}h^{\sigma\lambda}-h^{\sigma\kappa}P^{\lambda}_{\rho}\right)$
Magnetic theory (7.38) with $h_2\neq -(d-1)h_1$ $(d\geq 2)$	flat
Traceful electric theory for $d\geq 2$ (7.41)	obeying $v^{\nu}R_{\nu\lambda\kappa}{}^{\alpha}=0$

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Fractonic Berezinskii-Kosterlitz-Thouless transition from a renormalization group perspective

Kevin T. Grosvenor,[1,*] Ruben Lier,[2,3,†] and Piotr Surówka[4,5,6,‡]

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[2] *Max Planck Institute for the Physics of Complex Systems, 01187 Dresden, Germany*
[3] *Würzburg-Dresden Cluster of Excellence ct.qmat, Germany*
[4] *Department of Theoretical Physics, Wrocław University of Science and Technology, 50-370 Wrocław, Poland*
[5] *Institute for Theoretical Physics, University of Amsterdam, 1090 GL Amsterdam, The Netherlands*
[6] *Dutch Institute for Emergent Phenomena (DIEP), University of Amsterdam, 1090 GL Amsterdam, The Netherlands*

Proliferation of defects is a mechanism that allows for topological phase transitions. Such a phase transition is found in two dimensions for the XY-model, which lies in the Berezinskii-Kosterlitz-Thouless (BKT) universality class. The transition point can be found using renormalization group analysis. We apply renormalization group arguments to determine the nature of BKT transitions for the three-dimensional plaquette-dimer model, which is a model that exhibits fractonic mobility constraints. We show that an important part of this analysis demands a modified dimensional analysis that changes the interpretation of scaling dimensions upon coarse-graining. Using this modified dimensional analysis we compute the beta functions of the model and predict a finite critical value above which the fractonic phase melts, proliferating dipoles. Importantly, the transition point and its value are found unequivocally within the formalism of renormalization group.

Ruben Lier

U.v. Amsterdam

Piotr Surówka

Wrocław U.

Fractonic RG and application to BKT

The fractonic dimer plaquette model [You-Moessner ('22)]

$$L = \frac{\kappa_{xy}}{2}(\partial_x\partial_y\theta)^2 + \frac{\kappa_z}{2}(\partial_z\theta)^2 + 2\sum_I \alpha_I \cos(2\pi f_I)$$

$f_0 = \theta$ vortex monopole

$f_x = a_x\partial_x\theta$

$f_y = a_y\partial_y\theta$

vortex dipoles

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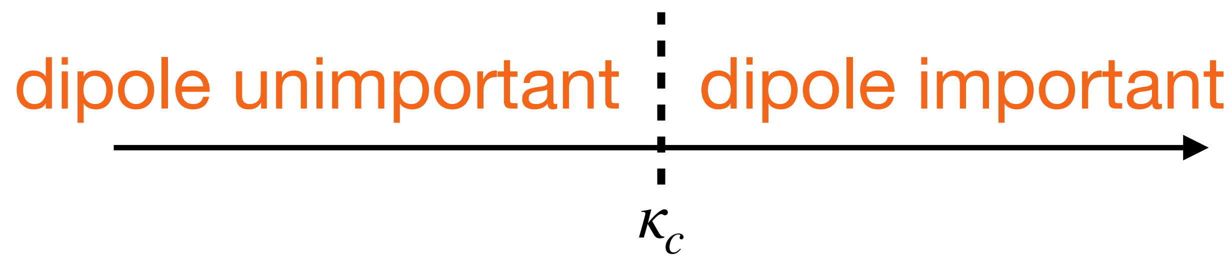
monopole always decays quickly



Kevin T. Grosvenor

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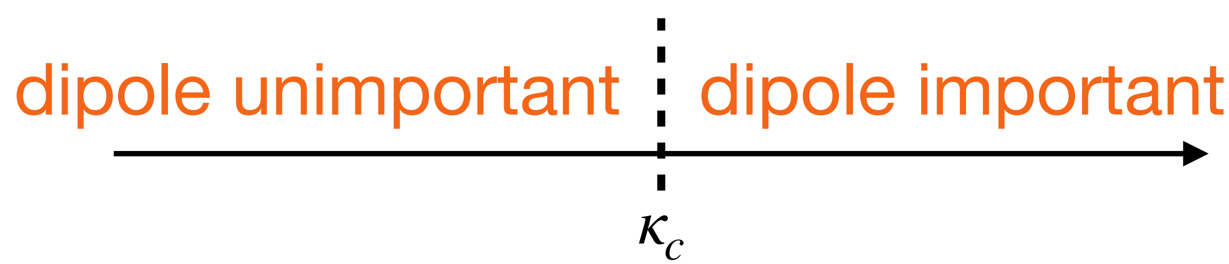

Upsets the usual RG pattern (lower derivatives usually more relevant)

This transition is "beyond the renormalization group paradigm as the low-energy behavior at criticality is manipulated by local fluctuation at short wave-lengths."

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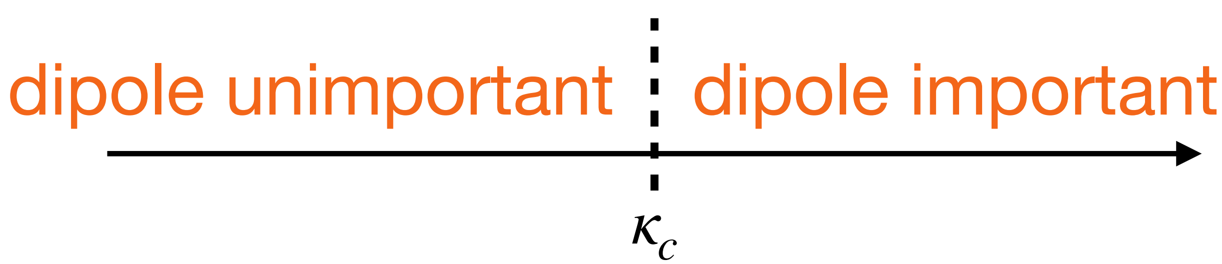
monopole always decays quickly



Upsets the usual RG pattern (lower derivatives usually more relevant)

This transition is "beyond the renormalization group paradigm as the low-energy behavior at criticality is manipulated by local fluctuation at short wave-lengths."

We worked out how to explain all of this using RG.

Unconventional Dimensional Analysis

Unconventional Dimensional Analysis

Usual case:

$$S = \frac{1}{2}\int \underbrace{d^4x}_{-4}\left(\underbrace{\partial_\mu}_{+1}\ \underbrace{\phi}_{+1}\ \underbrace{\partial^\mu}_{+1}\ \underbrace{\phi}_{+1} - \underbrace{m^2}_{+2}\underbrace{\phi^2}_{+2} - \underbrace{\lambda_3}_{+1}\underbrace{\phi^3}_{+3} - \underbrace{\lambda_4}_{0}\underbrace{\phi^4}_{+4} + \cdots\right)$$

Kevin T. Grosvenor

Unconventional Dimensional Analysis

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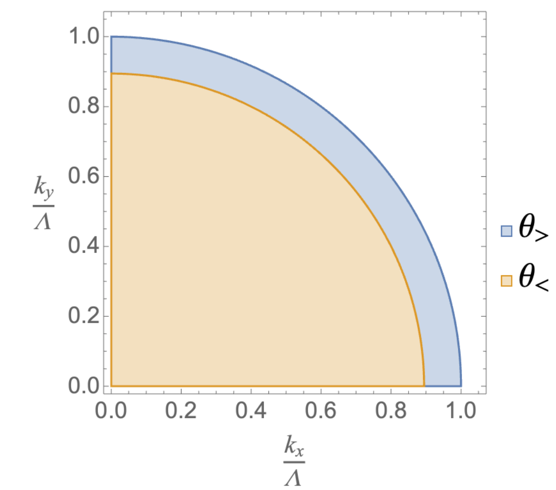

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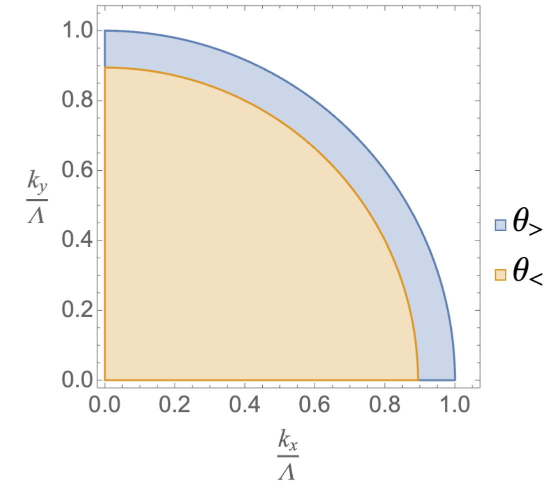

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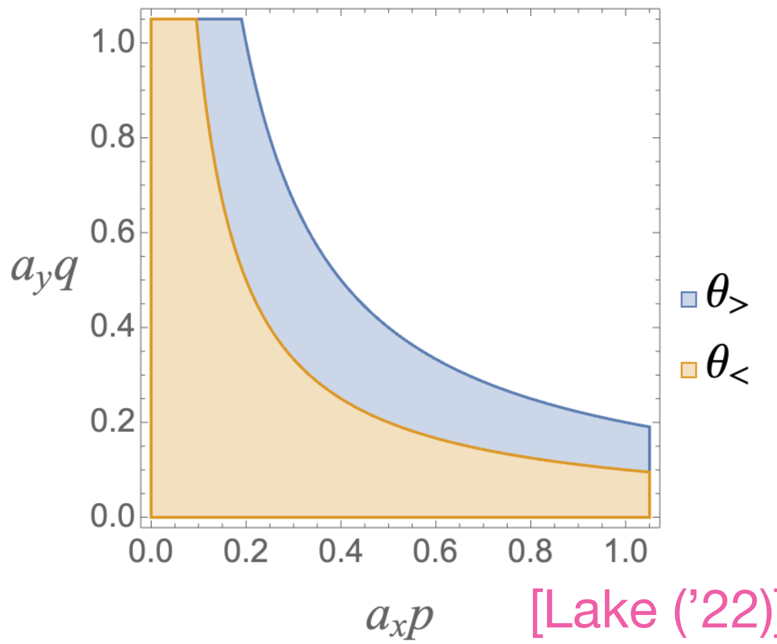
Our case: $(\partial_x\partial_y\theta)^2 + (\partial_z\theta)^2$

Measure dimensions w.r.t. dilatation from axes.

$$[k_x] = \frac{k_y^2}{k_x^2 + k_y^2} \qquad [k_y] = \frac{k_x^2}{k_x^2 + k_y^2} \qquad [k_z] = 1$$



[Lake ('22)]

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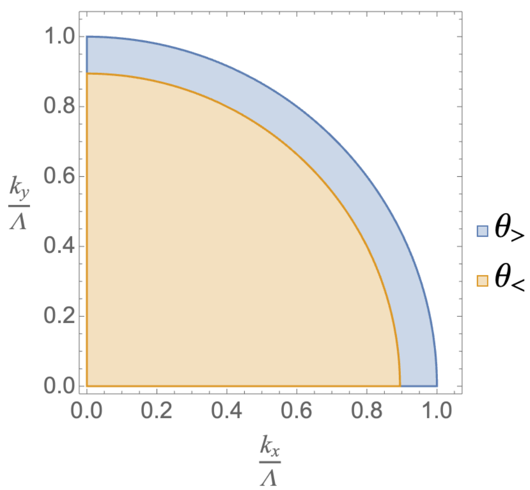

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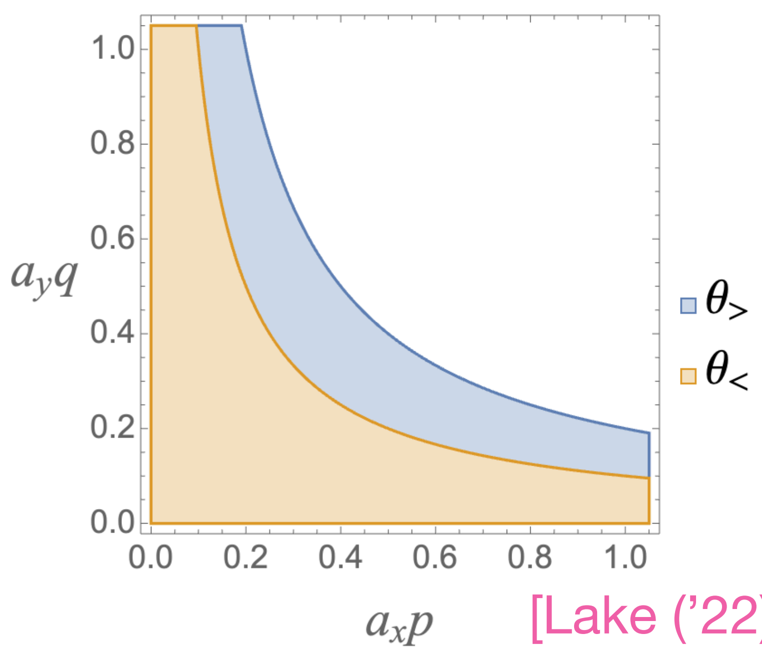
$(\partial_x\theta)^2$ not obviously more relevant than $(\partial_x\partial_y\theta)^2$



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RG of Vortex Monopoles

The fractonic dimer plaquette model

$$L = \frac{\kappa_{xy}}{2}(\partial_x \partial_y \theta)^2 + \frac{\kappa_z}{2}(\partial_z \theta)^2 + 2\sum_I \alpha_I \cos(2\pi f_I)$$

$f_0 = \theta$ vortex monopole

$f_x = a_x \partial_x \theta$

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Integrate fast modes $\theta_>$ with $(\Lambda/b)^2 \leq k_x^2 k_y^2 + k_z^2 \leq \Lambda^2$ s.t.

$$\alpha_0(b) = b^2 \alpha_0 e^{-\frac{1}{2} g_>^{(00)}(0)}$$ where $$g_>^{(00)}(\vec{x}) = (2\pi)^2 \left\langle \theta_>(\vec{x})\, \theta_>(0) \right\rangle_{0>}$$

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$$\left.\frac{d\alpha_0}{d\log b}\right|_{b=1} = \left(2 - \frac{1}{\sqrt{\kappa_{xy}\kappa_z}} \log \frac{2}{\hat{\Lambda}}\right)\alpha_0$$

monopole always irrelevant

RG of Vortex Dipoles

The fractonic dimer plaquette model

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$$\left.\frac{d\alpha_x}{d\log b}\right|_{b=1} = \left(2 - \frac{\pi^2}{2\sqrt{\kappa_{xy}\kappa_z}}\right)\alpha_x$$

dipole irrelevant | dipole relevant

$$\kappa_{xy}^c = \frac{\pi^4}{16\kappa_z}$$

RG of Fractonic Kinetic Coefficient

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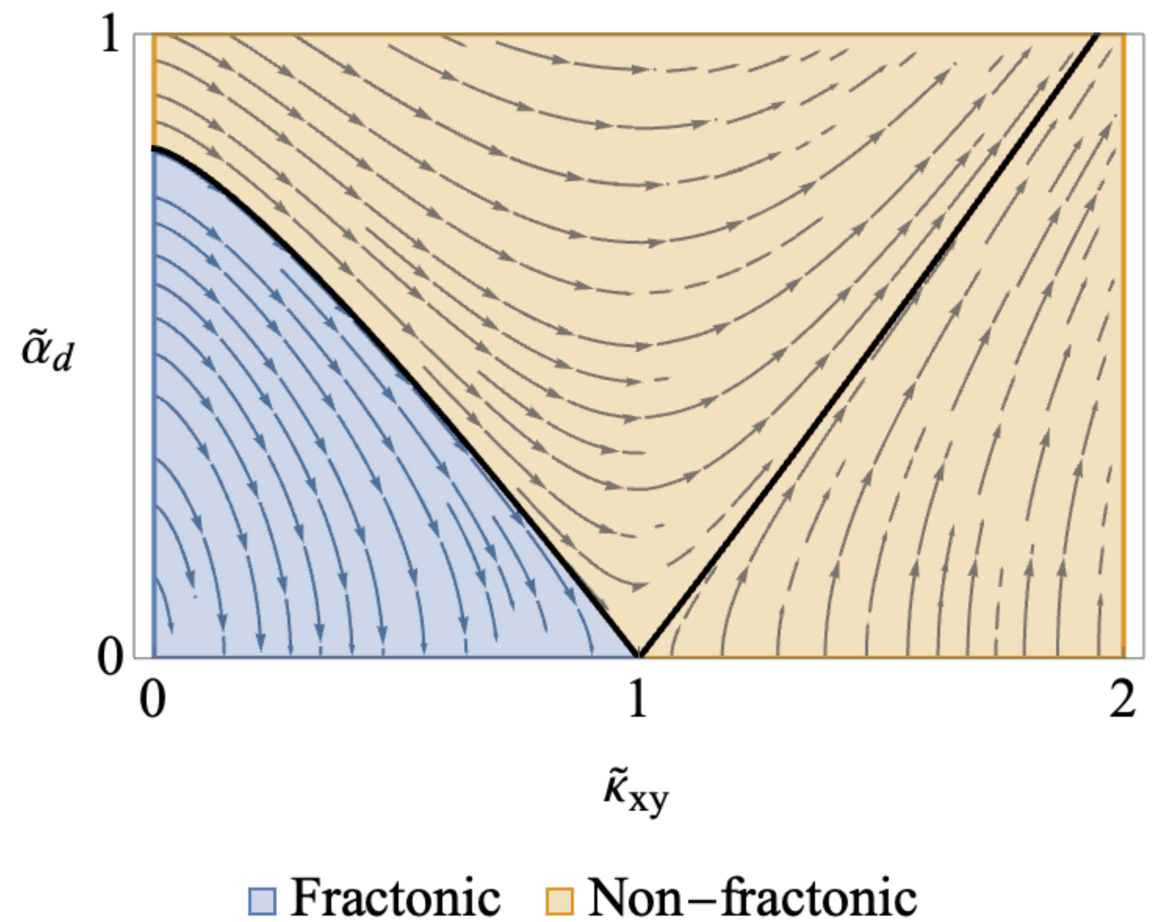
$$\tilde{\kappa}_{xy} = \frac{16\kappa_z \kappa_{xy}}{\pi^4} \qquad \tilde{\alpha}_\mathrm{d} = \frac{64\kappa_z \alpha_\mathrm{d}}{\Lambda^2}$$

$$\left.\frac{d\tilde{\kappa}_{xy}}{d\log b}\right|_{b=1} = \frac{\tilde{\alpha}_\mathrm{d}^2}{\tilde{\kappa}_{xy}}$$

$$\left.\frac{d\tilde{\alpha}_\mathrm{d}}{d\log b}\right|_{b=1} = \left(1 - \frac{1}{\tilde{\kappa}_{xy}^{1/2}}\right)\tilde{\alpha}_\mathrm{d}$$



No Fracton Symmetry Breaking

The vortex monopole term α_0 might have generated a fractonic symmetry-breaking term $(\partial_x\theta)^2 + (\partial_y\theta)^2$, but it doesn't!

The vortex dipole term α_x might have generates a fractonic symmetry-breaking term $(\partial_x^2\theta)^2 + (\partial_y^2\theta)^2$, but it doesn't!

Similarly, for higher derivatives.

These are both legitimate concerns that had not been dealt with before our work.

Standard BKT Transition

2D XY Model: $$H = -J \sum_{\langle i,j \rangle} \cos(\theta_i - \theta_j)$$

Dual to sine-Gordon (sG) Model with continuum Lagrangian

$$L = \frac{T}{2J}(\nabla \theta)^2 - 2\alpha \cos(2\pi\theta)$$

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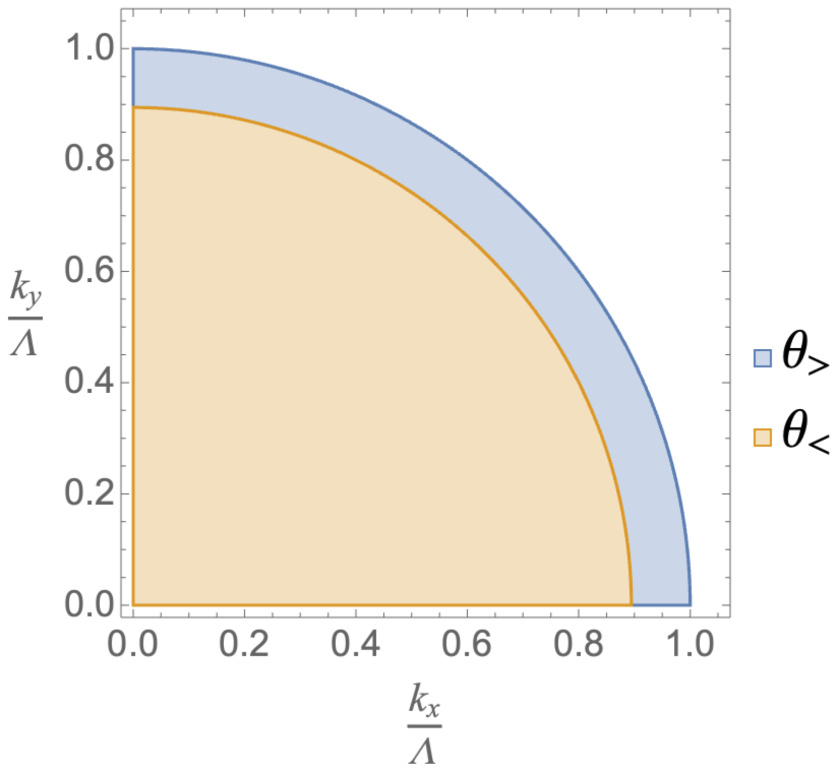

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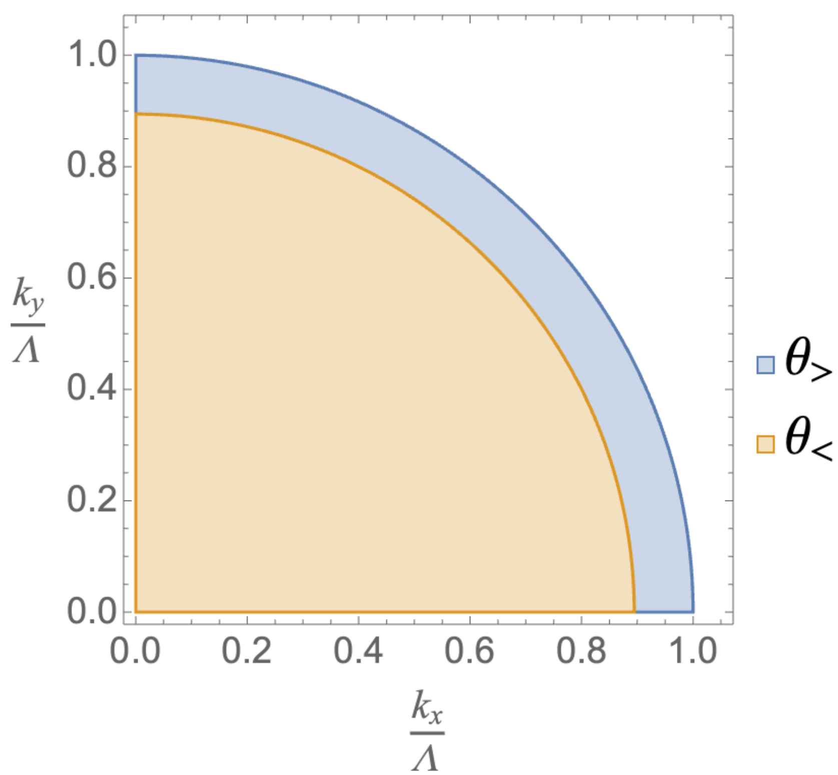

$$\alpha \to \alpha(b) = b^2\alpha e^{-\frac{1}{2}g_>(0)}$$

$$g_>(\vec{x}) = (2\pi)^2\left\langle\theta_>(\vec{x})\,\theta_>(0)\right\rangle_{0>}$$

$$\left.\frac{d\alpha}{d\log b}\right|_{b=1} = \left(2-\frac{\pi J}{T}\right)\alpha$$

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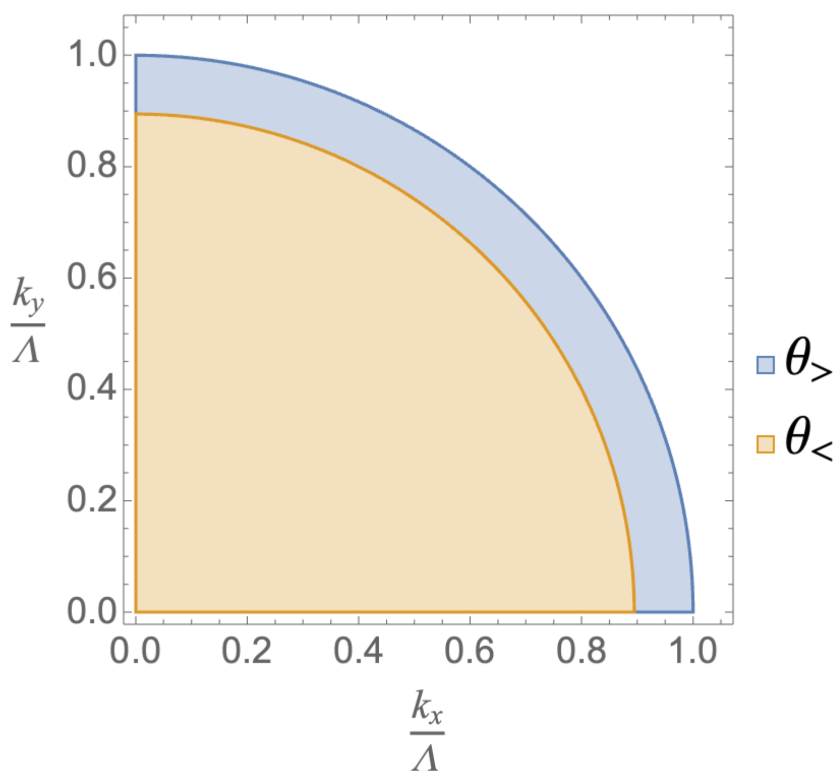

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Fugacity is irrelevant — Fugacity is relevant

Low temperature — $T_{\mathrm{KT}} = \pi J/2$ — High temperature

Standard BKT Transition (cont.)

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sine-Gordon (sG) Model: $L = \frac{T}{2}(\nabla\theta)^2 - 2\alpha\cos(2\pi\theta)$ $(J = 1)$

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$$T(b) = T + \frac{\hat{\alpha}^2}{2T}\log b \qquad \hat{\alpha} = \alpha\,(4\pi/\Lambda)^2$$

$$\left.\frac{dT}{d\log b}\right|_{b=1} = \frac{\hat{\alpha}^2}{2T}$$

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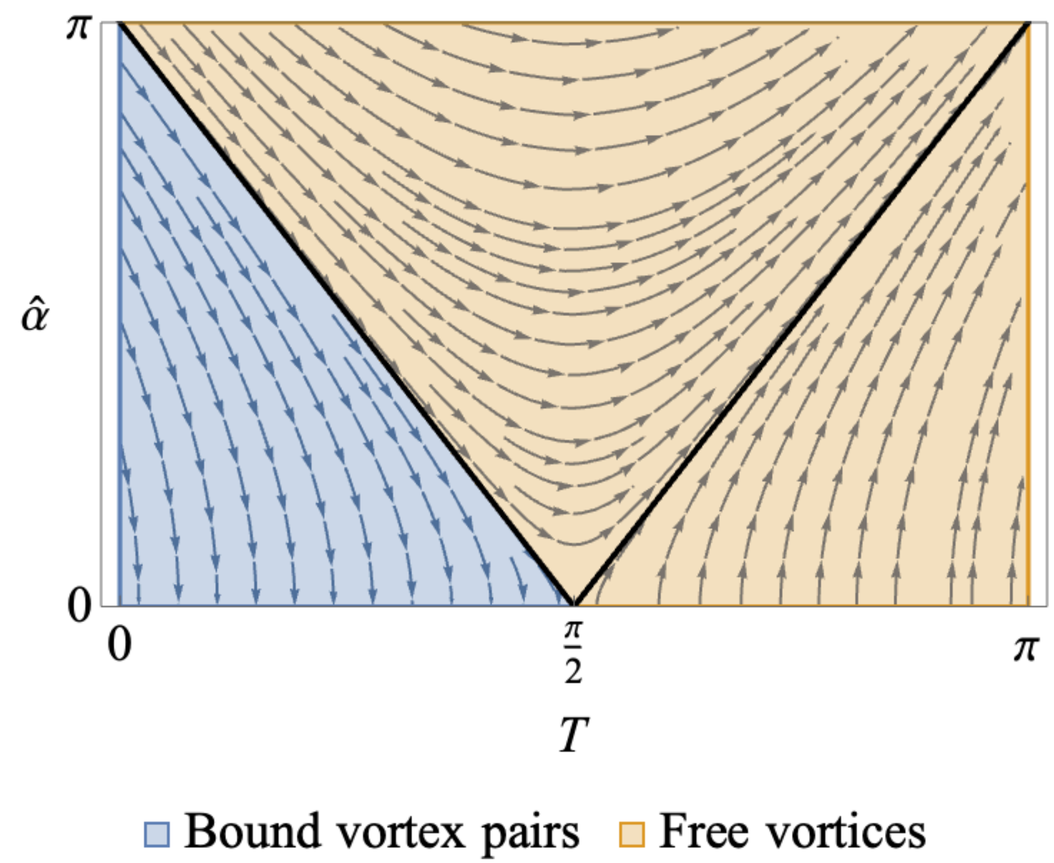

BKT Vortices Story

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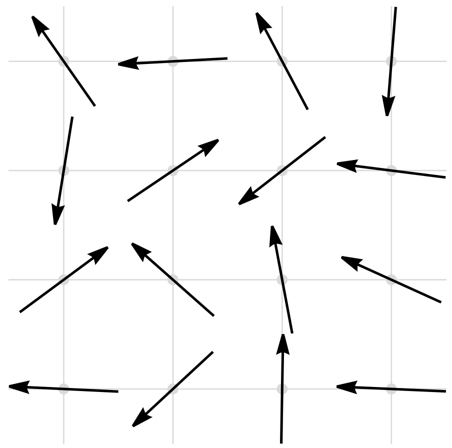

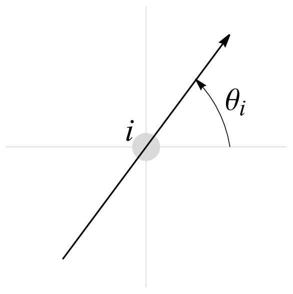

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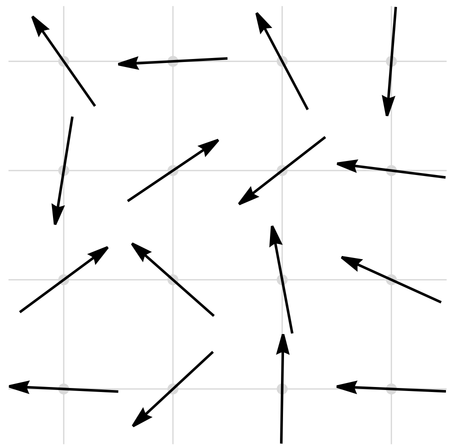

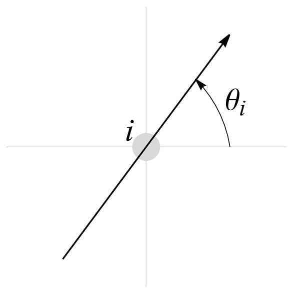

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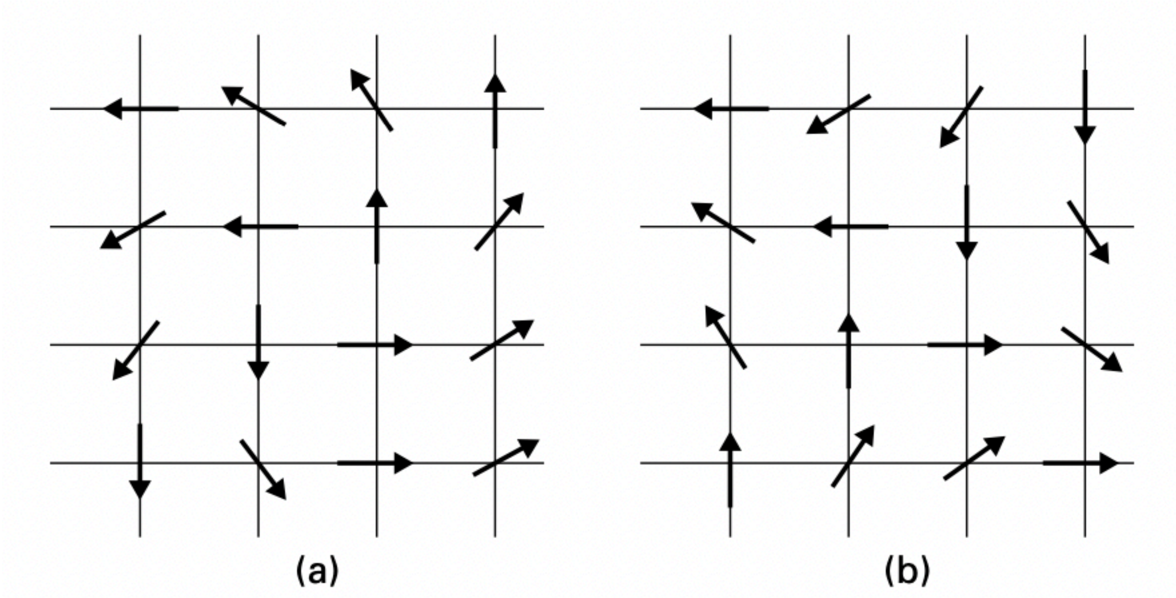
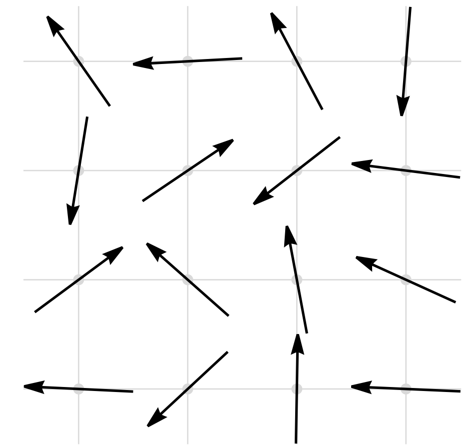
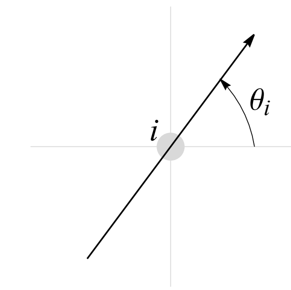


Figure 6.1 Vortex (a) and antivortex (b) on a quadratic lattice.

I. Herbut, "A Modern Approach to Critical Phenomena."

BKT Vortices Story





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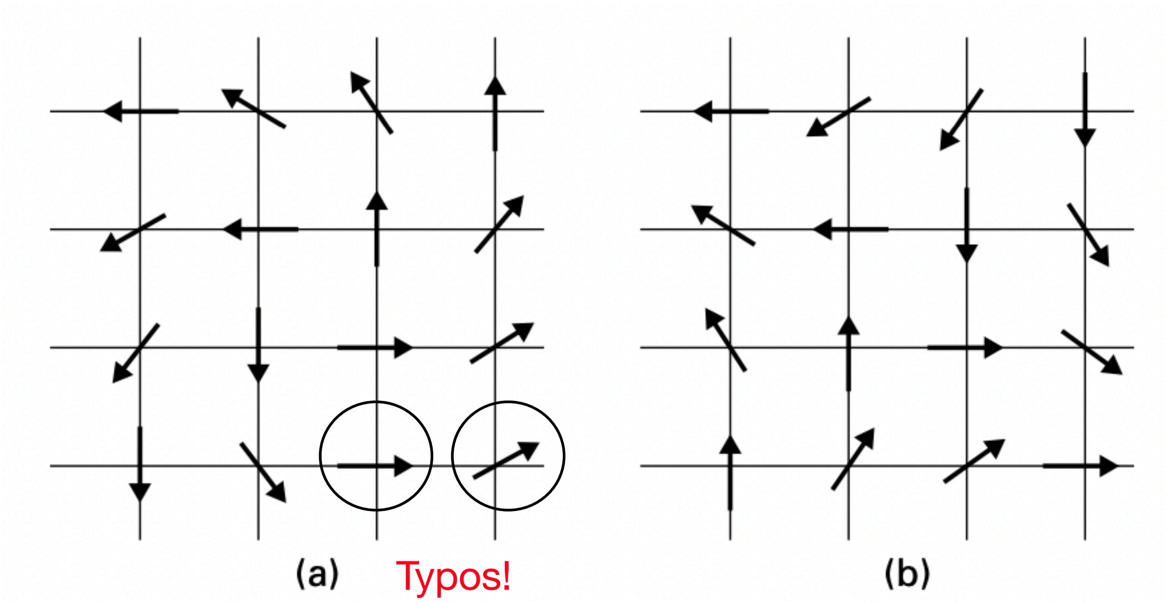

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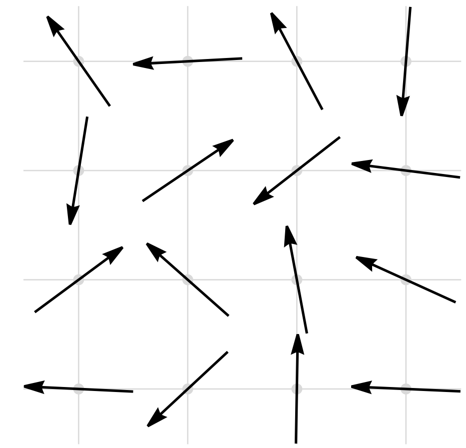

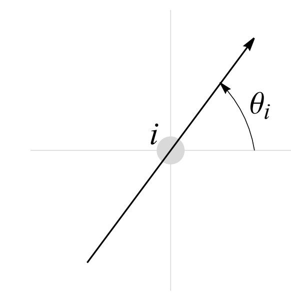

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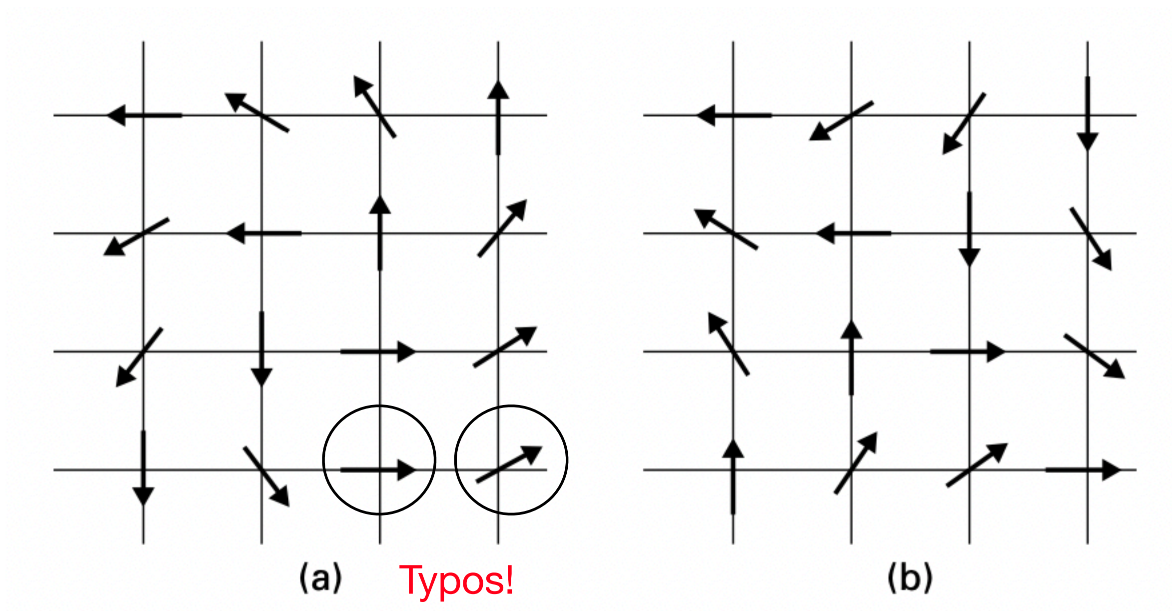

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Vortex solution with vorticity q:

$$\theta(\vec{x}) = q\alpha$$ $\alpha =$ angle w.r.t. fixed axis

$$H \approx \pi J q^2 \ln \frac{R}{r_0}$$

R: system size

r_0: short-dist. cut-off

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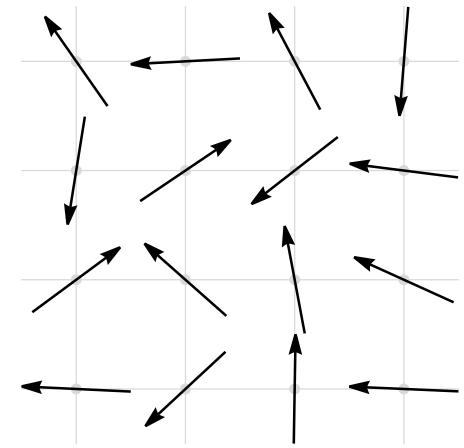

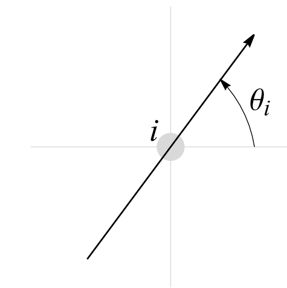

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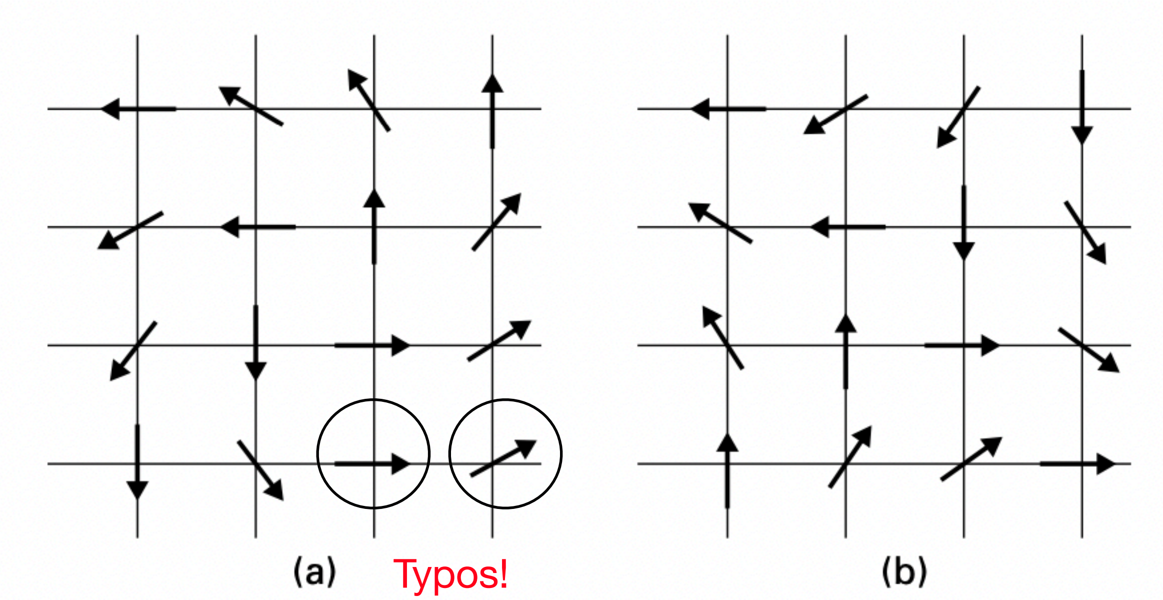

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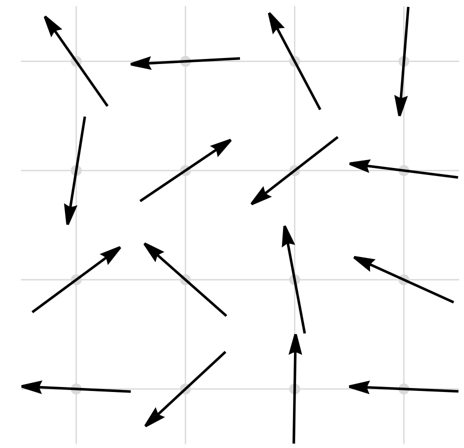
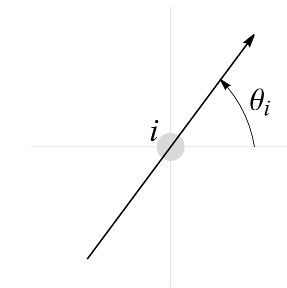
$(R/r_0)^2$ multiplicity for vortex center

BKT Vortices Story





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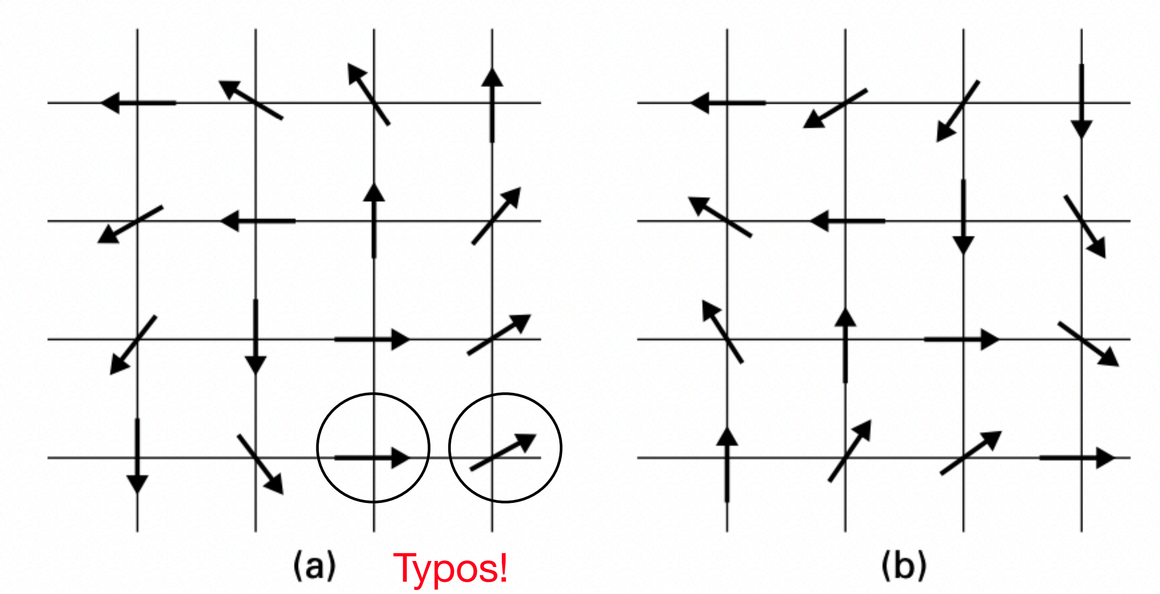

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$\Delta F = H - T\Delta S$ switches sign at $T_{\mathrm{KT}} = \dfrac{\pi}{2} J$ $(k_B = 1)$

Bound vortex/anti-vortex pairs | Gas of free vortices

Low temperature — T_{KT} — High temperature

Outline

1. Intro to fractons and some history.

2. Fractonic hydro.

3. Fractons on curved manifolds.

4. Fractonic RG.

5. Outlook.

Summary and Outlook

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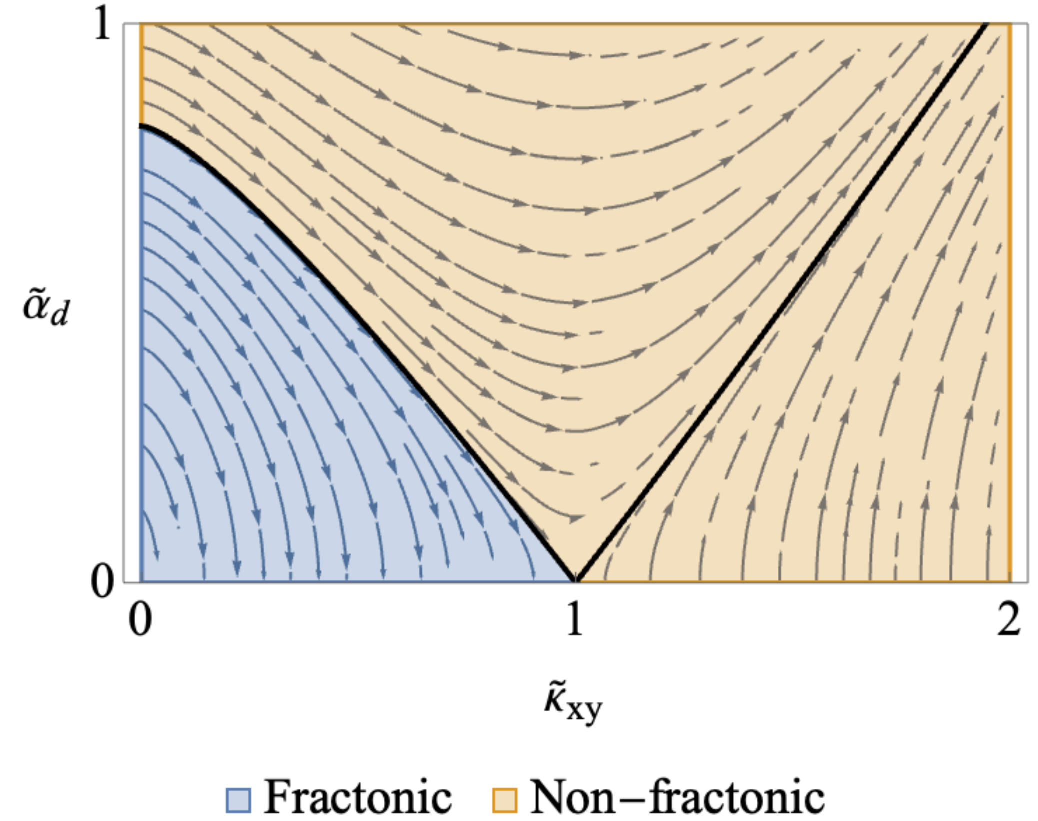

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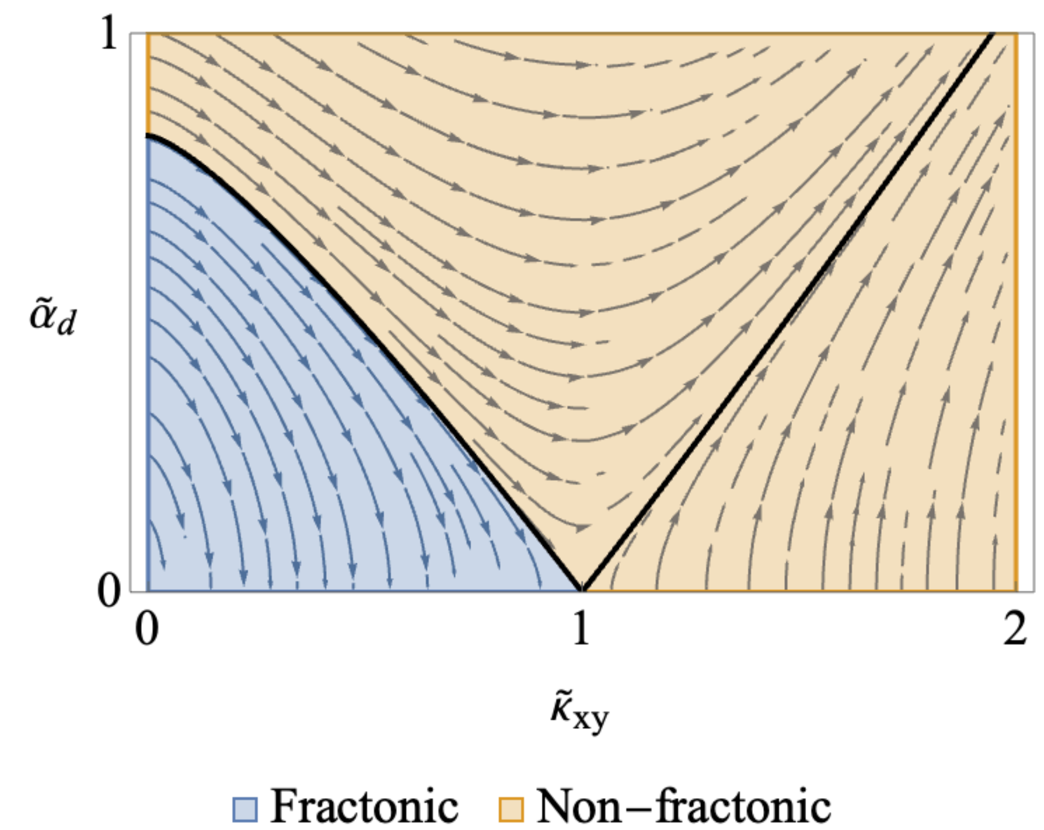

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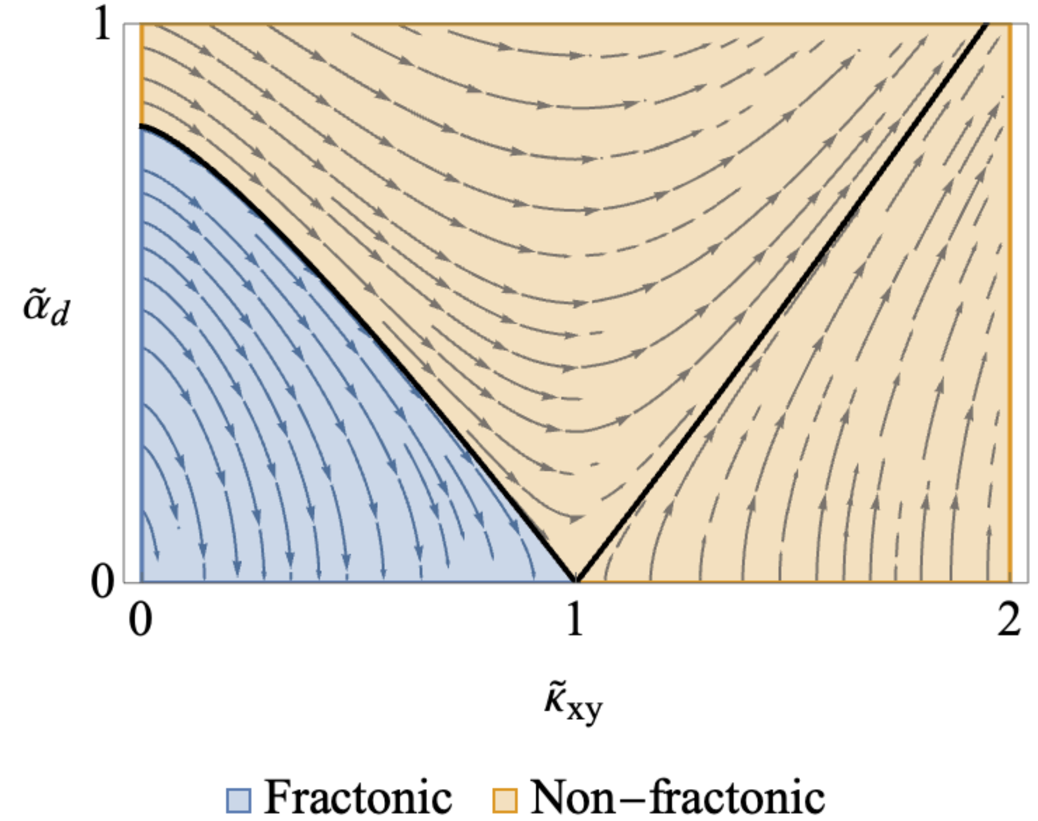

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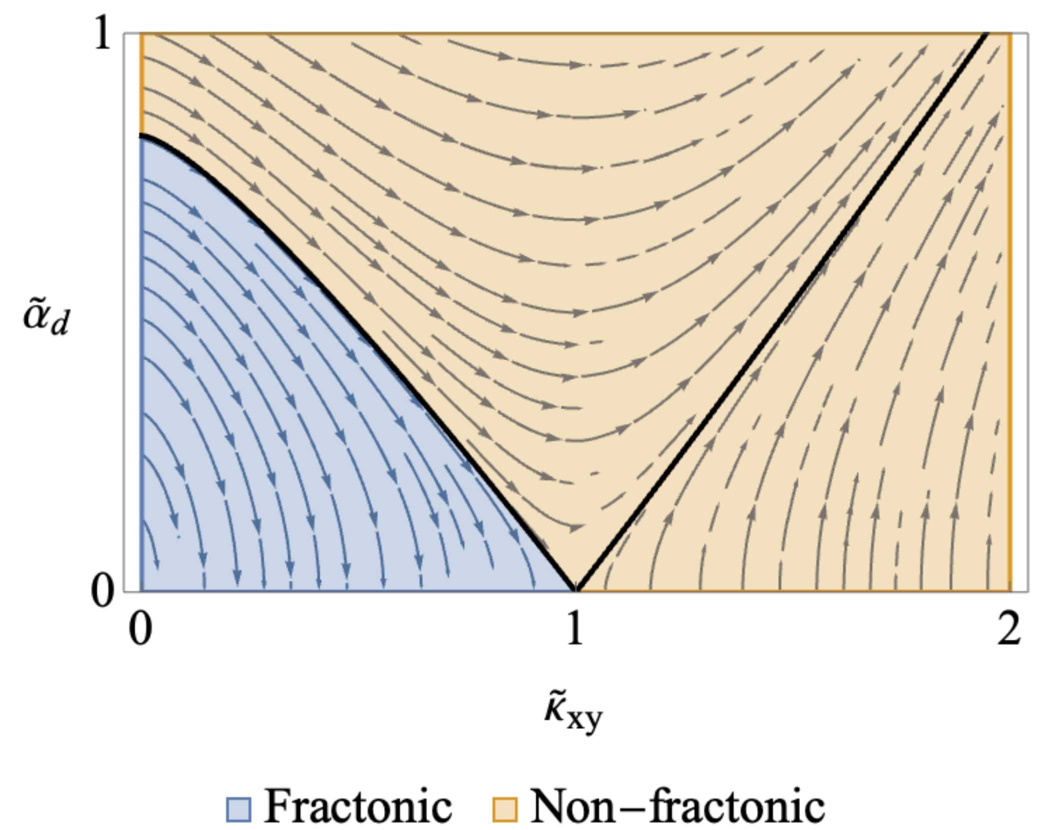

Thank you!