Non-Lorentzian Torus and SL(2,Z) Duality

Anisotropic Compactification of Nonrelativistic M-Theory

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Beyond Lorentzian Geometry

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Funded by Marie Skłodowska-Curie Actions

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  Branched $SL(2,\mathbb{Z})$ Duality 2208.13815

  $SL(2,\mathbb{R})$ Duality in Non-Lorentzian IIB Supergravity to appear

- Stephen Ebert, ZY

  Anisotropic Compactification of Nonrelativistic M-Theory to appear

- Stephen Ebert, Hao-Yu Sun, ZY

  Dual D-Brane Actions in Nonrelativistic String Theory 2112.09316

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Motivation

- DLCQ M-theory & Matrix quantum mechanics [De Wit, Hoppe, Nicolai '88] [Banks, Fischler, Susskind, Shenker '96] [Seiberg '97] [Susskind '97] [Sen '97]
- DLCQ String theory & nonrelativistic string theory [Klebanov, Maldacena '00] [Gomis, Ooguri '00] [Danielsson, Guijosa, Kruczenski '00]

  relativistic physics — nonrelativistic physics

  lightlike circle — spatial circle

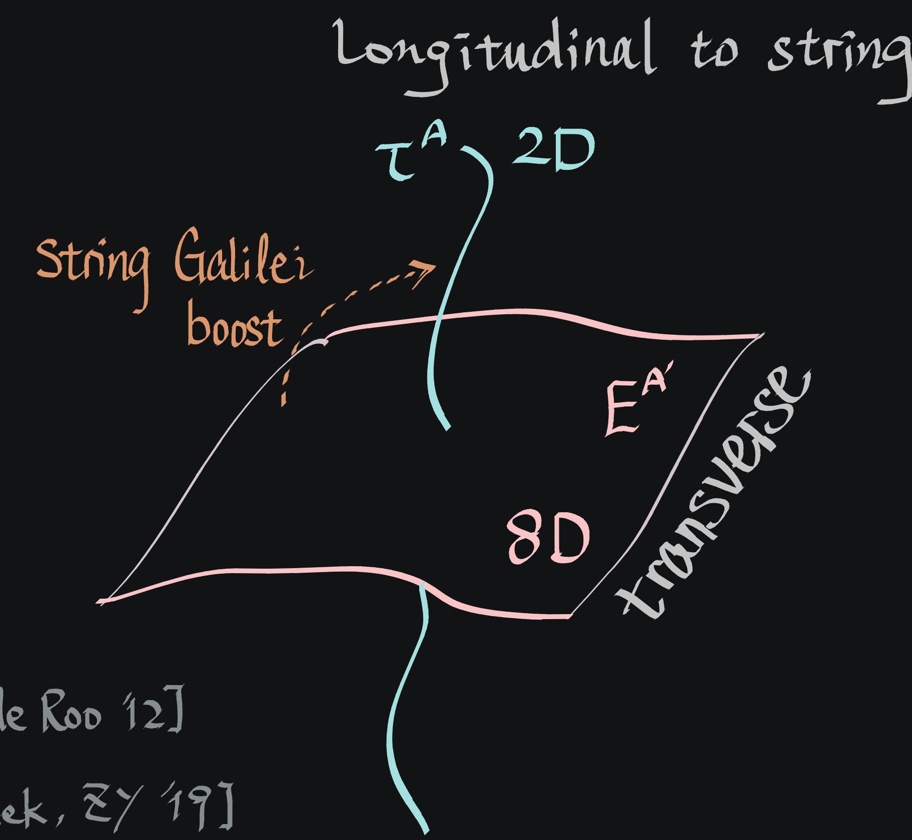
- DLCQ M-theory & nonrelativistic M-theory ?
- U-duality & non-Lorentzian geometry ?
- a first step: $SL(2,\mathbb{Z})$ duality in nonrelativistic string theory

M-Theory on a Torus

- String theory on a circle: $SL(2,\mathbb{Z})$ symmetry
  ↑ isometry group on torus

- $SL(2,\mathbb{Z})$ duality in IIB string theory
  - non-perturbative aspects
  - construction of sugra action
  - solution generating techniques
  - applications to Yang-Mills ...

Nonrelativistic String Theory



- closed string sector: string Newton-Cartan geometry

$$\hat{G}_{\mu\nu} = \hat{E}_\mu{}^I \hat{E}_\nu{}^J \eta_{IJ} \qquad I = (\overset{0,1}{A}, \underset{2,\cdots,9}{A'}) \qquad \omega \to \infty$$

$$\hat{E}^A = \omega \tau^A$$

$$\hat{E}^{A'} = E^{A'}$$

$$\hat{B} = -\frac{1}{2}\omega^2 \tau^A \wedge \tau^B \epsilon_{AB} + B$$

$$\hat{C}_q = \frac{1}{2}\omega^2 C_{q-2} \wedge \tau^A \wedge \tau^B \epsilon_{AB} + C_q$$

$$\hat{\Phi} = \Phi + \ln\omega$$

[Andringa, Bergshoeff, Gomis, de Roo '12]
[Bergshoeff, Gomis, Rosseel, Simsek, ZY '19]
[Bidussi, Harmark, Hartong, Obers, Oling '22]
[Ebert, Sun, ZY '21]

- T-dual in longitudinal sector: DLCQ string theory

[Gomis, Ooguri '00]
[Danielsson, Guijosa, Kruczenski '00]
[Bergshoeff, Gomis, ZY '18]

- open string sectors: nonrelativistic/noncommutative open strings / Yang-Mills

[Danielsson, Guijosa, Kruczenski '00] [Gomis, Yu, ZY '20 '20]

SL(2, $\mathbb{Z}$) Duality in IIB Superstring Theory

- SL(2, $\mathbb{Z}$) transformations $\Lambda = \begin{pmatrix} \alpha & \beta \\ \gamma & \delta \end{pmatrix}$ $\alpha, \beta, \gamma, \delta \in \mathbb{Z}$ $\alpha\delta - \beta\gamma = 1$

- relativistic IIB superstring theory $\hat{\tau} = \hat{C}_0 + i e^{-\hat{\Phi}}$

$$\hat{\tau} \to \frac{\alpha \hat{\tau} + \beta}{\gamma \hat{\tau} + \delta} \qquad \begin{pmatrix} \hat{B} \\ \hat{C}_2 \end{pmatrix} = (\Lambda^{-1})^T \begin{pmatrix} \hat{B} \\ \hat{C}_2 \end{pmatrix}$$

- infinite ω limit and nonrelativistic IIB

$$C_0 \to \frac{\alpha C_0 + \beta}{\gamma C_0 + \delta}$$

$$e^{\Phi} \to (\gamma C_0 + \delta)^2 e^{\Phi}$$

SL(2,ℤ) Duality in IIB Superstring Theory

- SL(2, ℤ) transformations $\Lambda = \begin{pmatrix} \alpha & \beta \\ \gamma & \delta \end{pmatrix}$ $\alpha, \beta, \gamma, \delta \in \mathbb{Z}$ $\alpha\delta - \beta\gamma = 1$

- relativistic IIB superstring theory $\hat{\tau} = \hat{C}_0 + i e^{-\hat{\Phi}}$

$$\hat{\tau} \to \frac{\alpha\hat{\tau} + \beta}{\gamma\hat{\tau} + \delta} \qquad \begin{pmatrix} \hat{B} \\ \hat{C}_2 \end{pmatrix} = (\Lambda^{-1})^T \begin{pmatrix} \hat{B} \\ \hat{C}_2 \end{pmatrix}$$

Einstein's frame

- infinite ω limit and nonrelativistic IIB $\ell = \frac{1}{2} \tau^A \wedge \tau^B \epsilon_{AB}$

$$C_0 \to \frac{\alpha C_0 + \beta}{\gamma C_0 + \delta} \qquad \begin{pmatrix} B \\ C_2 \end{pmatrix} \to \left\{ (\Lambda^{-1})^T \left[\begin{pmatrix} B \\ C_2 \end{pmatrix} - \frac{\gamma e^{-\Phi}}{\gamma C_0 + \delta} \begin{pmatrix} 0 \\ \frac{1}{2} e^{-\Phi/2} \ell \end{pmatrix} \right] \right.$$

$$e^{\Phi} \to (\gamma C_0 + \delta)^2 e^{\Phi} \qquad \left. - \frac{\gamma e^{-\Phi}}{(\gamma C_0 + \delta)^2} \begin{pmatrix} 0 \\ \frac{1}{2} e^{-\Phi/2} \ell \end{pmatrix} \right\} \operatorname{sgn}(\gamma C_0 + \delta)$$

A Surprising Simplification

- redefinition of 2-forms

$$\mathcal{B} = e^{-\Phi/2} B \qquad \mathcal{C}_2 = e^{\Phi/2}(C_2 + C_0 B)$$

- $SL(2,\mathbb{Z})$ transformations: polynomial realization

$$\mathcal{B} \to \mathcal{B} - \kappa \mathcal{C}_2 + \tfrac{1}{2}\kappa^2 \ell$$

$$\mathcal{C}_2 \to \mathcal{C}_2 - \kappa \mathcal{B}$$

$$\kappa = \frac{\gamma e^{-\Phi}}{\gamma C_0 + \delta}$$

$$\mathcal{C}_4 \to \mathcal{C}_4 - \kappa \mathcal{B} \wedge \ell + \tfrac{1}{2}\kappa^2 \mathcal{C}_2 \wedge \ell \qquad \mathcal{C}_4 = C_4 + \tfrac{1}{2} B \wedge C_2$$

- same structure for properly defined field strengths!

Demystification: a geometric interpretation?

↪ an M-theory uplift!

Nonrelativistic M-Theory

[Gomis, Ooguri '00][Blair, Gallegos, Zinnato '21]
[Gomis, Kamimura, Townsend '04][Ebert, Sun, ZY '21]

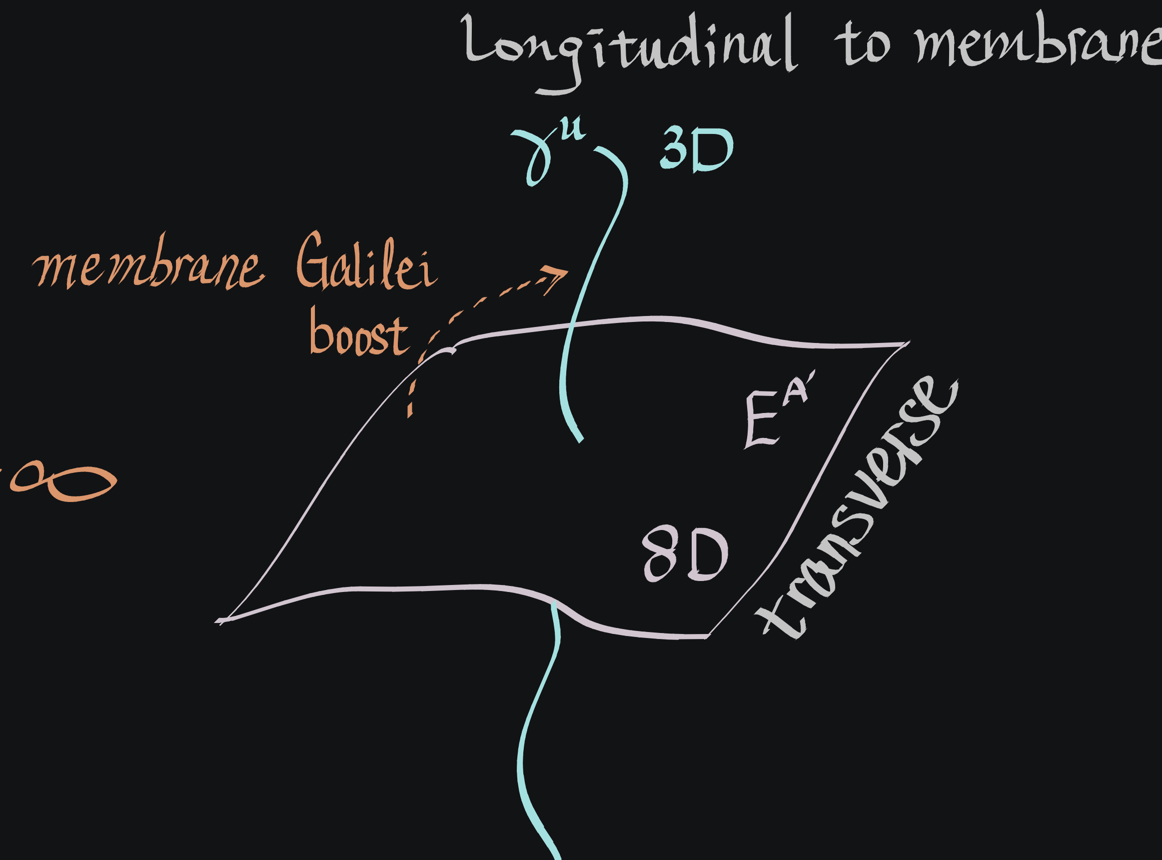
$$\hat{G}_{MN} = \hat{E}_M{}^{\mathbb{I}} \hat{E}_N{}^{\mathbb{J}} \eta_{\mathbb{I}\mathbb{J}} \qquad \mathbb{I} = (\overset{0,1,10}{u}, \underset{2,\cdots,9}{A'})$$

$$\hat{E}^u = \omega^{\frac{2}{3}} \gamma^u$$

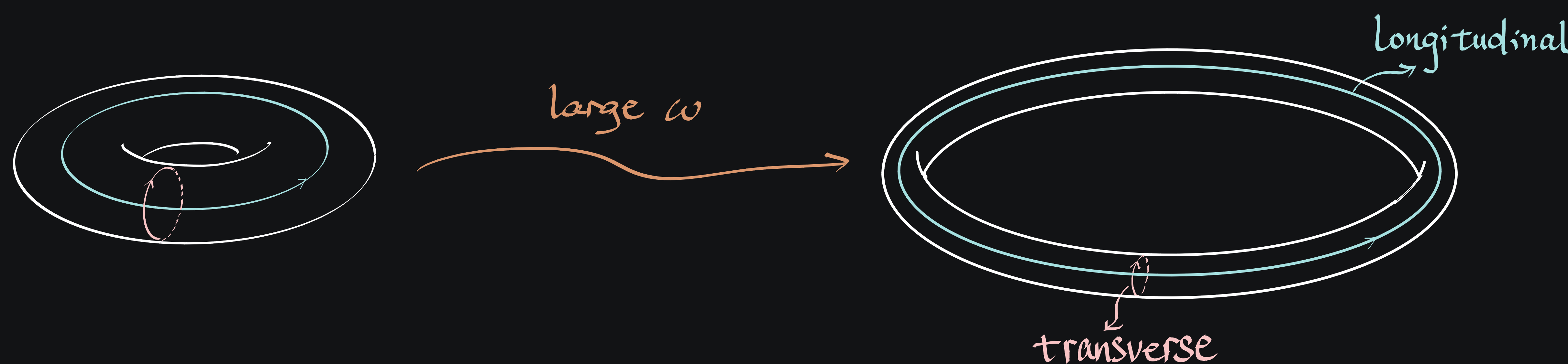
$$\hat{E}^{A'} = \omega^{-\frac{1}{3}} E^{A'}$$

$$\hat{A}_3 = -\frac{1}{2}\omega^2 \gamma^u \wedge \gamma^v \wedge \gamma^w \epsilon_{uvw} + A_3$$

$\omega \to \infty$



- compactification on anisotropic torus



Detour: Relativistic M-Theory Revisited

$SL(2,\mathbb{Z})$ Symmetry from Toroidal Compactification

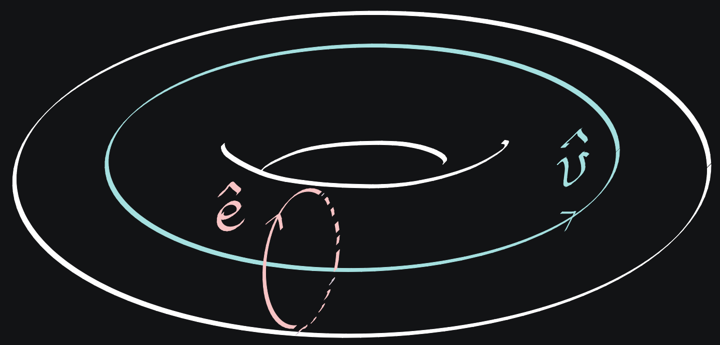
- metric on torus

$$ds^2 = \frac{1}{\hat{\tau}_2}\left|d\hat{x}^1 - \hat{\tau}\, d\hat{x}^2\right|^2 \qquad \hat{\tau} = \hat{\tau}_1 + i\hat{\tau}_2$$

$\hat{\tau}_1 \to \hat{C}_0$, $\quad \hat{\tau}_2 \to e^{-\hat{\Phi}} > 0$

- $SL(2,\mathbb{Z})$ transformations $\quad \alpha, \beta, \gamma, \delta \in \mathbb{Z} \quad \alpha\delta - \beta\gamma = 1$

$$\begin{pmatrix} \hat{x}^1 \\ \hat{x}^2 \end{pmatrix} \to \begin{pmatrix} \alpha & \beta \\ \gamma & \delta \end{pmatrix} \begin{pmatrix} \hat{x}^1 \\ \hat{x}^2 \end{pmatrix} \qquad \hat{\tau} \to \frac{\alpha\hat{\tau} + \beta}{\gamma\hat{\tau} + \delta}$$



- Zweibein formalism

$$\hat{g}_{mn} = \hat{v}_m \hat{v}_n + \hat{e}_m \hat{e}_n \qquad \hat{v}_m = \frac{1}{\sqrt{\hat{\tau}_2}} \begin{pmatrix} 1 \\ -\hat{\tau}_1 \end{pmatrix} \qquad \hat{e}_m = \begin{pmatrix} 0 \\ \sqrt{\hat{\tau}_2} \end{pmatrix}$$

Local Rotation from $SL(2,\mathbb{Z})$

$$\begin{pmatrix} \hat{v}_m d\hat{x}^m \\ \hat{e}_m d\hat{x}^m \end{pmatrix} \to \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} \hat{v}_m d\hat{x}^m \\ \hat{e}_m d\hat{x}^m \end{pmatrix}$$

$$\tan\theta = \frac{\gamma \hat{\tau}_2}{\gamma \hat{\tau}_1 + \delta} \equiv \hat{\kappa}$$

- Iwasawa decomposition of local $SL(2,\mathbb{R})$ [Review: Obers, Pioline '98]

$$G = K.A.N$$

$$K \sim \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix} \qquad A \sim \begin{pmatrix} r & 0 \\ 0 & r^{-1} \end{pmatrix} \qquad N \sim \begin{pmatrix} 1 & 0 \\ x & 1 \end{pmatrix}$$

Local Rotation from $SL(2,\mathbb{Z})$

$$\begin{pmatrix} \hat{v}_m d\hat{x}^m \\ \hat{e}_m d\hat{x}^m \end{pmatrix} \to \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} \hat{v}_m d\hat{x}^m \\ \hat{e}_m d\hat{x}^m \end{pmatrix}$$

branching

$$\frac{\mathrm{sgn}(\gamma\hat{\tau}_1+\delta)}{\sqrt{1+\hat{k}^2}} \begin{pmatrix} 1 & -\hat{k} \\ \hat{k} & 1 \end{pmatrix}$$

$$\tan\theta = \frac{\gamma\,\hat{\tau}_2}{\gamma\hat{\tau}_1+\delta} \equiv \hat{k}$$

- relocate the branching

$$\hat{\tau}_2 = e^{-\hat{\Phi}}$$

$$\hat{\Phi} \to \hat{\Phi} + 2\ln|\gamma\hat{\tau}_1+\delta|$$

Local Rotation from $SL(2,\mathbb{Z})$

$$\begin{pmatrix} \hat{v} \\ \hat{e} \end{pmatrix} \to \frac{1}{\sqrt{1+\hat{\kappa}^2}} \begin{pmatrix} 1 & -\hat{\kappa} \\ \hat{\kappa} & 1 \end{pmatrix} \begin{pmatrix} \hat{v} \\ \hat{e} \end{pmatrix}$$

$$\hat{\kappa} = \frac{\gamma\,\hat{\tau}_2}{\gamma\hat{\tau}_1 + \delta} \longrightarrow \text{boost velocity if Wick rotated}$$

- relocate the branching

$$\hat{\tau}_2 = e^{-\hat{\Phi}}$$

$$\hat{\Phi} \to \hat{\Phi} + 2\ln(\gamma\hat{\tau}_1 + \delta)$$

$$= \hat{\Phi} + 2\ln|\gamma\hat{\tau}_1 + \delta| + \pi i\left[1 + \mathrm{sgn}(\gamma\hat{\tau}_1 + \delta)\right]$$

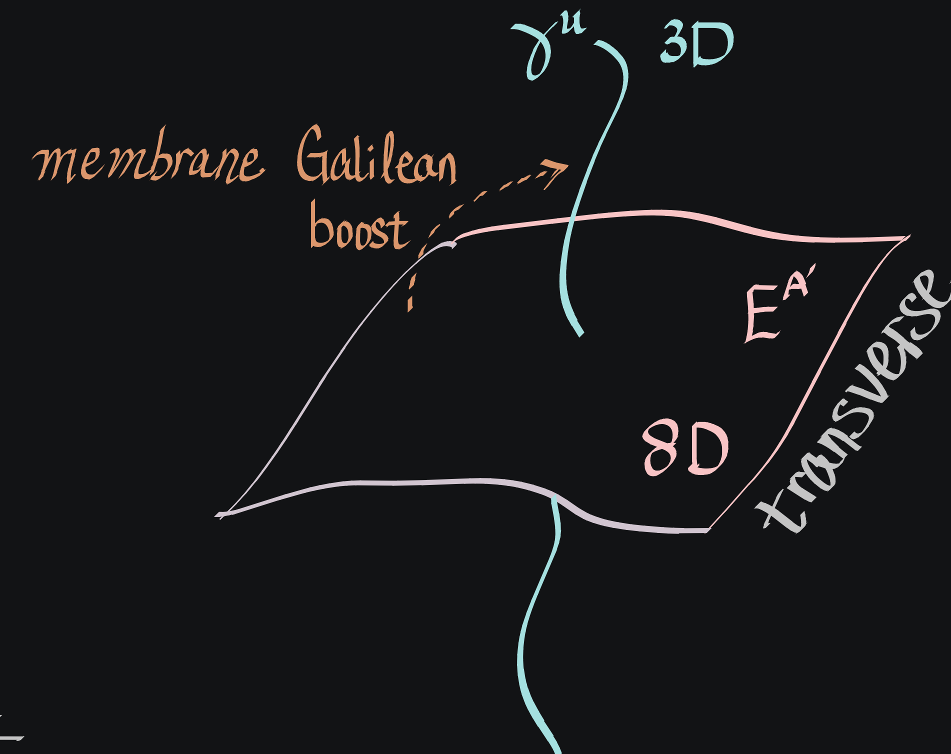
Blowing Up the Torus

- nonrelativistic membrane limit

$$\hat{E}^{u} = \omega^{\frac{2}{3}} \gamma^{u}$$

$$\hat{E}^{A'} = \omega^{-\frac{1}{3}} E^{A'}$$

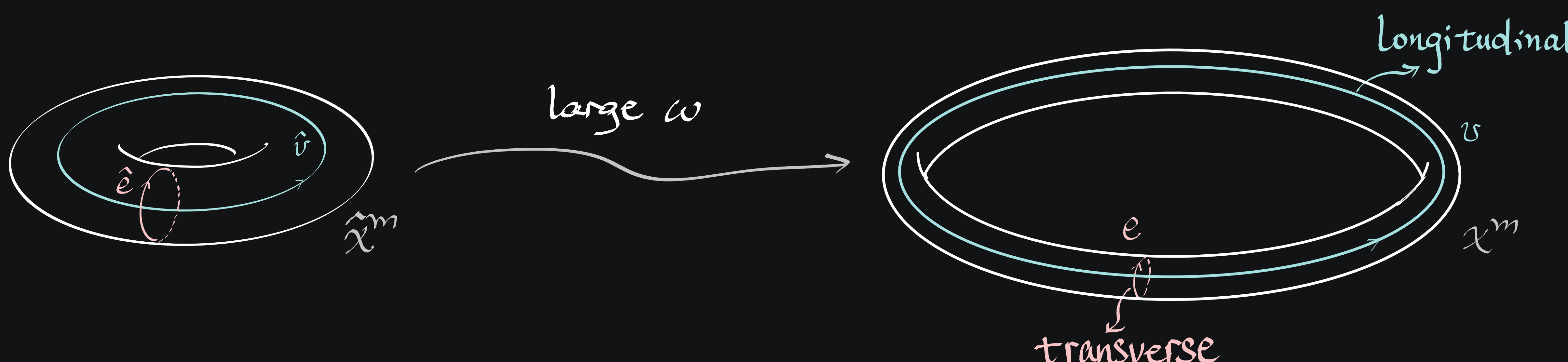
$$\omega \to \infty$$



- anisotropic torus

$$\hat{v} = \omega^{\frac{2}{3}} v \qquad \hat{\tau}_1 = \tau_1 \qquad \hat{\tau}_2 = \frac{\tau_2}{\omega}$$

$$\hat{e} = \omega^{-\frac{1}{3}} e \qquad \tilde{x}^m = \omega^{\frac{1}{6}} x^m$$



Galilei Rotation from $SL(2,\mathbb{Z})$

$$\begin{pmatrix} v \\ e \end{pmatrix} \to \frac{1}{\sqrt{1+\frac{\kappa^2}{\omega^2}}} \begin{pmatrix} 1 & -\frac{\kappa}{\omega^2} \\ \kappa & 1 \end{pmatrix} \begin{pmatrix} v \\ e \end{pmatrix} \qquad \omega \to \infty$$

$$\kappa = \frac{\gamma t_2}{\gamma t_1 + \delta}$$ boost velocity if $\omega = ic$

- Iwasawa decomposition of local $SL(2,\mathbb{R})$

$$G = K.A.N$$

$$K \sim \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix} \qquad A \sim \begin{pmatrix} r & 0 \\ 0 & r^{-1} \end{pmatrix} \qquad N \sim \begin{pmatrix} 1 & 0 \\ x & 1 \end{pmatrix}$$

Galilei Rotation from $SL(2,\mathbb{Z})$

$$\begin{pmatrix} v \\ e \end{pmatrix} \to \frac{1}{\sqrt{1-\frac{\kappa^2}{c^2}}} \begin{pmatrix} 1 & \frac{\kappa}{c^2} \\ \kappa & 1 \end{pmatrix} \begin{pmatrix} v \\ e \end{pmatrix} \qquad c \to \infty$$

$$\kappa = \frac{\gamma t_2}{\gamma t_1 + \delta}$$ Lorentz "boost velocity"

- Iwasawa decomposition of local $SL(2,\mathbb{R})$

$$G = K.A.N$$

$$K \sim \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix} \qquad A \sim \begin{pmatrix} r & 0 \\ 0 & r^{-1} \end{pmatrix} \qquad N \sim \begin{pmatrix} 1 & 0 \\ x & 1 \end{pmatrix}$$

Galilei Rotation from $SL(2,\mathbb{Z})$

$$\begin{pmatrix} v \\ e \end{pmatrix} \to \begin{pmatrix} 1 & 0 \\ k & 1 \end{pmatrix} \begin{pmatrix} v \\ e \end{pmatrix}$$

$$k = \frac{\gamma t_2}{\gamma t_1 + \delta}$$ Galilei "boost velocity"

- Iwasawa decomposition of local $SL(2,\mathbb{R})$

$$G = K.A.N$$

$$K \sim \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix} \qquad A \sim \begin{pmatrix} r & 0 \\ 0 & r^{-1} \end{pmatrix} \qquad N \sim \begin{pmatrix} 1 & 0 \\ x & 1 \end{pmatrix}$$

SL(2, $\mathbb{Z}$) Transformations of Three-Form

- 3-form potential in M-Theory

$$\hat{A}_{\mu\nu m} = \begin{pmatrix} \hat{B}_{\mu\nu} \\ \hat{C}_{\mu 2} \end{pmatrix} \quad \text{Kaluza-klein reduction}$$

$$\begin{pmatrix} \hat{A}_m \hat{v}^m \\ \hat{A}_m \hat{e}^m \end{pmatrix} = \begin{pmatrix} \hat{\mathcal{B}} \\ \hat{\mathcal{C}}_2 \end{pmatrix} \longrightarrow \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} \hat{\mathcal{B}} \\ \hat{\mathcal{C}}_2 \end{pmatrix}$$

$\hat{\mathcal{B}} \to e^{-\hat{\Phi}/2}\hat{B}$, $\quad \hat{\mathcal{C}}_2 \to e^{\hat{\Phi}/2}(\hat{C}_2 + \hat{C}_0 \hat{B})$

- $\omega \to \infty$ limit and polynomial realization of SL(2, $\mathbb{Z}$) $\qquad \ell = \frac{1}{2}\tau^A \wedge \tau^B \epsilon_{AB}$

$$\hat{\mathcal{B}} = -\omega^2 \ell + \mathcal{B}$$

$$\hat{\mathcal{C}}_2 = \mathcal{C}_2$$

$$\begin{pmatrix} \ell \\ \mathcal{C}_2 \\ \mathcal{B} \end{pmatrix} \to \begin{pmatrix} 1 & 0 & 0 \\ -K & 1 & 0 \\ \frac{1}{2}K^2 & -K & 1 \end{pmatrix} \begin{pmatrix} \ell \\ \mathcal{C}_2 \\ \mathcal{B} \end{pmatrix}$$

Global Symmetries in Non-Lorentzian Theories

- relativistic theory $\begin{cases} \text{operators: } \hat{\mathcal{O}} \in \hat{\Phi} \\ \text{global symmetry group: } g \in G \end{cases}$

- reparametrizations $\quad \hat{\mathcal{O}} = \omega^2 \mathcal{O}_0 + \mathcal{O}$

  group action $\quad \hat{\mathcal{O}}' = g\,\hat{\mathcal{O}} \implies \begin{cases} \mathcal{O}_0' = g\,\mathcal{O}_0 + O(\omega^{-2}) \\ \mathcal{O}' = g\,\mathcal{O} + K(g, \Phi) + O(\omega^{-2}) \end{cases}$

- non-Lorentzian Limit $\quad \omega \to \infty$

$$g \circ \mathcal{O} = g\,\mathcal{O} + K(g, \Phi)$$

Global Symmetries in Non-Lorentzian Theories

Singlet case: $g\circ\mathcal{O} = \mathcal{O} + K(g,\Psi)$

- Identity: $\mathbb{1}\circ\mathcal{O} = \mathcal{O}$ $\qquad K(\mathbb{1},\Psi) = 0$
- Compatibility $g'\circ(g\circ\mathcal{O}) = (g'g)\circ\mathcal{O}$ $\qquad K(g'g,\Psi) = K(g', g\circ\Psi) + K(g,\Psi)$
- Applications to $SL(2,\mathbb{Z})$ in nonrelativistic IIB

"boost velocity" $\kappa = \dfrac{\gamma\, e^{-\Phi}}{\gamma C_0 + \delta}$ is a singlet K!

A Polynomial Realization of $SL(2,\mathbb{Z})$

- n-dim. realization of $SL(2,\mathbb{Z})$

$$K_n(s,\Psi) = s_n \kappa^n + s_{n-1}\kappa^{n-1} + \cdots + s_1 \kappa \qquad \kappa = \frac{\gamma e^{-\Phi}}{\gamma c_0 + \delta}$$

$$\Longrightarrow g \circ \begin{pmatrix} s_1 \\ s_2 \\ \vdots \\ s_n \end{pmatrix} = \begin{pmatrix} 1 & & & \mathbb{0} \\ \kappa & 1 & & \\ \vdots & & \ddots & \\ \frac{\kappa^{n-1}}{(n-1)!} & & \cdots & 1 \end{pmatrix} \begin{pmatrix} s_1 \\ s_2 \\ \vdots \\ s_n \end{pmatrix}$$

Classical Invariant Theory of Binary Form

- 2D polynomial realization of $SL(2,\mathbb{Z})$

$$\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} 1 & k \\ 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix}$$

- n-dim. polynomial realization via a binary form

$$\mathcal{P}(x,y) = \sum_{i=1}^{n+1} \frac{n!}{(n-i)!} S_i \, x^{n-i+1} y^i$$

$$\mathcal{P}'(x,y) = \mathcal{P}(x',y') \Longrightarrow g \circ \begin{pmatrix} S_1 \\ S_2 \\ \vdots \\ S_n \end{pmatrix} = \begin{pmatrix} 1 & & & \mathbb{O} \\ k & 1 & & \\ \vdots & & \ddots & \\ \frac{k^{n-1}}{(n-1)!} & & \cdots & 1 \end{pmatrix} \begin{pmatrix} S_1 \\ S_2 \\ \vdots \\ S_n \end{pmatrix}$$

Non-Lorentzian IIB Supergravity

- sugra data in binary forms

$$P_4^{(7)} = S_1^{(3)} x^4 + 4 S_2^{(3)} x^3 y + 12 S_3^{(3)} x^2 y^2 + 24 S_4^{(3)} x y^3 + 24 S_5^{(3)} y^4$$

$$P_2^{(7)} = S_1^{(5)} x^2 + 2 S_2^{(5)} x y + S_3^{(5)} y^2$$

$$S^{(1)} = \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} \frac{1}{2} d\Phi \\ \mathcal{F}_1 \end{pmatrix} \qquad S^{(3)} = \begin{pmatrix} 3\mathcal{F}_1 \wedge \ell_2 \\ d\ell_2 - \frac{3}{2} d\Phi \wedge \ell_2 \\ \mathcal{F}_3 \\ -\mathcal{H}_3 \\ \mathcal{A}_3 \end{pmatrix} \qquad S^{(5)} = \begin{pmatrix} \mathcal{F}_3 \wedge \ell_2 \\ -\mathcal{H}_3 \wedge \ell_2 \\ \mathcal{F}_5 \end{pmatrix}$$

$$\mathcal{H}_3 = e^{-\Phi/2} dB_2 \qquad \mathcal{F}_1 = e^{\Phi} dC_0 \qquad \ell_2 = \frac{1}{2} \tau^A \wedge \tau^B \epsilon_{AB}$$

$$\mathcal{F}_3 = e^{\Phi/2} (dC_2 + C_0 dB_2) \qquad \mathcal{F}_5 = dC_4 + C_2 \wedge dB_2$$

Quadratic Invariants and Non-Lorentzian IIB Action

- classification of quadratic invariants

$$I^{(\ell)}_{2m} = \sum_{j=1}^{2m-1} (-1)^j \langle S^{(\ell)}_j, S^{(\ell)}_{2m-j} \rangle \qquad m = 1, 2, 3, \cdots \qquad \ell = 1, 3, 5$$

- together with other symmetries (dilatation, string Galilei boost)

a unique non-Lorentzian IIB action!

$$S_{\text{IIB}} = \frac{1}{2\kappa^2}\int d^{10}x\, E\left[R_{\text{SNC}} - \frac{1}{3}(d\ell)_{ABA'}(d\ell)^{AB}{}_{A'} + \frac{1}{4}\left(-\frac{2}{9}I^{(1)}_2 + \frac{2}{3}I^{(3)}_4 + 2I^{(3)}_6 - I^{(3)}_8 + I^{(5)}_4 + I^{(5)}_6\right)\right]$$

$$+ \frac{1}{4\kappa^2}\int \left(C_4 \wedge \mathcal{H}_3 \wedge \mathcal{F}_3 - \mathcal{A}_3 \wedge \mathcal{F}_5 \wedge \ell_2\right)$$

- self-duality constraint from integrating out $\mathcal{A}_3$: $\mathcal{F}_3 = \star(\mathcal{F}_5 \wedge \ell_2)$

- nonrelativistic D-brane solutions from dualities
- black/worm holes in nonrelativistic string theory?
- top-down nonrelativistic holography?
- bootstrapping higher-order supergravity?
- nonrelativistic M5-brane and D3-brane
- DLCQ M-Theory from U-duality?

Thank You!