Carrollian black holes

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Beyond Lorentzian Geometry II, University of Edinburgh, February 2023

based on work in progress w. F. Ecker, J. Hartong, A. Perez, S. Prohazka, and R. Troncoso

Outline

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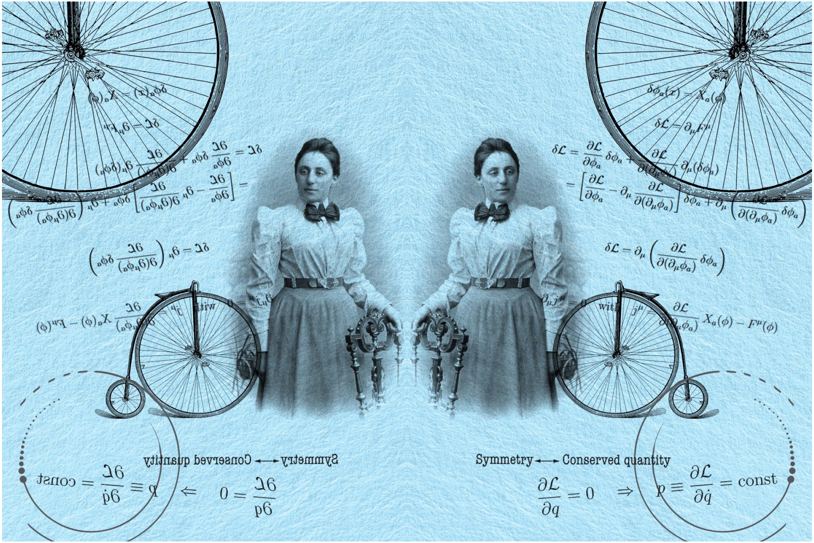
Carrollian symmetries

Symmetries ubiquitious in constraining physics

- Kinematics & Dynamics



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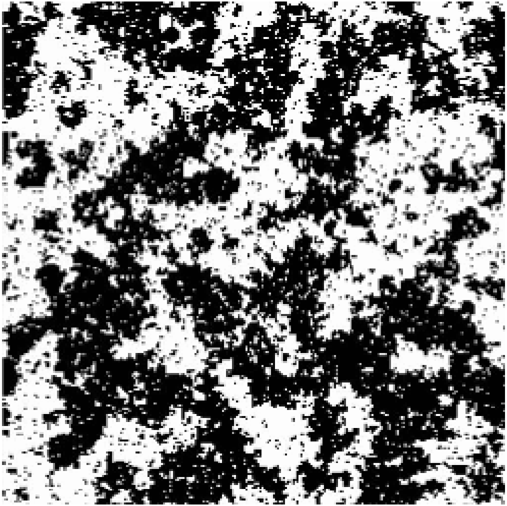

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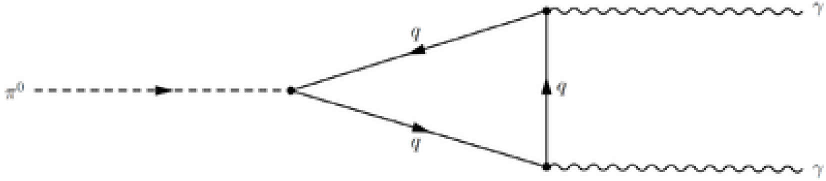

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- Density of states

$$S_{\mathrm{BH}} = S_{\mathrm{Cardy}}$$

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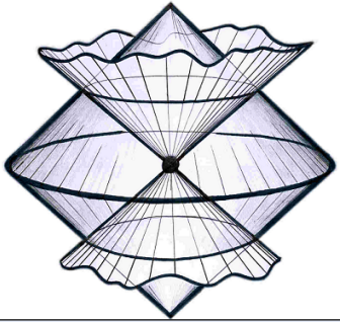

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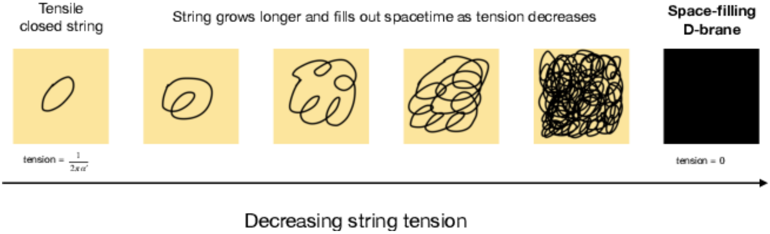

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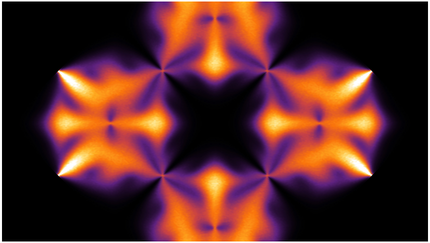

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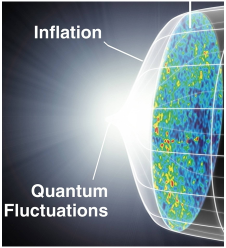

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Following history from SR to GR: natural to gauge Carroll algebra

- Gravity actions (but with Carroll boost invariance)

$$\tilde{\mathcal{L}} = E\left[\frac{1}{4}\Pi^{\mu\nu}\Pi^{\rho\sigma}T_{\mu\rho}T_{\nu\sigma} + \sigma\Pi^{\mu\nu}\overset{(C)}{R}_{\mu\nu} - \sigma^2 T^\mu T^\nu \overset{(C)}{R}_{\mu\nu}\right]$$

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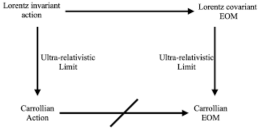

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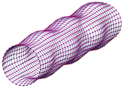

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google "carroll black hole" images; 17th result is song 'Black Hole' by Mackin Carroll

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Can some of the latter be regarded as "Carrollian black holes?"

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- Carrollian holography
- Cond-mat applications
- because it is there

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What?

- generic statements about whole model space vs. specific examples
- focus on simple model
- lowest dimension possible: 1+1
- consider limit from 2d (dilaton) gravity

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No lightcones in Carroll gravity!

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Thanks for your attention! Questions?

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This talk pursues last option: Carrollian extremal surfaces (CES)

Outline

Motivation for Carrollian gravity

Carrollian dilaton gravity in 1+1 dimensions

Carrollian extremal surfaces

Outlook towards Carrollian black holes

JT model

- In first order form: sl$(2,\mathbb{R})$ BF theory

$$I_{\text{JT}} \sim \int \langle \mathcal{X}\, F \rangle \qquad\qquad F = \mathrm{d}A + A \wedge A$$

Expand connection 1-form as

$$A = \omega\, J + e_a\, P^a$$

generators J, P_a obey (A)dS$_2$ algebra

$$[P_a,\, P_b] = \epsilon_{ab}\, \Lambda \qquad\qquad [P_a,\, J] = \epsilon_a{}^b\, P_b$$

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- Expanding in components more familiar Cartan formulation

$$I_{\text{JT}} \sim \int \big(X R + X^a T_a - \epsilon\, \Lambda X \big)$$

with $R = \mathrm{d}\omega$, $T_a = \mathrm{d}e_a + \epsilon_{ab}\, \omega \wedge e^b$, and $\epsilon = \epsilon_{ab} e^a \wedge e^b$
Interpretation of fields:
  - ω: (dualized) Lorentz connection
  - e_a: zweibein (dyad)
  - X: dilaton
  - X^a: Lagrange multipliers for torsion constraints

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- JT/SYK correspondence (Schwarzian, chaos bound, ...)
- JT gravity has black hole solutions

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notation: $A_I = (\omega, e_a) = (\omega, e_+, e_-)$ and $X^I = (X, X^a) = (X, X^+, X^-)$

X^I can be interpreted as (target space) coordinates of a Poisson manifold

$$I_{\text{dil}} = I_{\text{PSM}} \sim \int \left(X^I \, \mathrm{d}A_I + P^{IJ}(X^K)\, A_I \wedge A_J \right) \qquad P^{IJ} = \begin{pmatrix} 0 & -X^+ & X^- \\ X^+ & 0 & \mathcal{V} \\ -X^- & -\mathcal{V} & 0 \end{pmatrix}$$

Ikeda '93; Schaller, Strobl '94

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Have infinite family of (toy) models available!

Selected list of models $\mathcal{V} = V(X) + X^a X_a U(X)$ (see review hep-th/0604049)

Black holes in (A)dS$_2$, asymptotically flat or arbitrary spaces (Wheeler property)

Model	$U(X)$	$V(X)$
1. Schwarzschild (1916)	$-\frac{1}{2X}$	$-\lambda^2$
2. Jackiw–Teitelboim [JT] (1984)	0	ΛX
3. Witten Black Hole (1991)	$-\frac{1}{X}$	$-2b^2 X$
4. CGHS (1992)	0	-2Λ
5. (A)dS$_2$ ground state (1994)	$-\frac{a}{X}$	BX
6. Rindler ground state (1996)	$-\frac{a}{X}$	BX^a
7. Black Hole attractor (2003)	0	BX^{-1}
8. Spherically reduced gravity ($N > 3$)	$-\frac{N-3}{(N-2)X}$	$-\lambda^2 X^{(N-4)/(N-2)}$
9. All above: ab-family (1997)	$-\frac{a}{X}$	BX^{a+b}
10. Liouville gravity	a	$be^{\alpha X}$
11. Reissner–Nordström (1916)	$-\frac{1}{2X}$	$-\lambda^2 + \frac{Q^2}{X}$
12. Schwarzschild-(A)dS	$-\frac{1}{2X}$	$-\lambda^2 - \ell X$
13. Katanaev–Volovich (1986)	α	$\beta X^2 - \Lambda$
14. BTZ/Achucarro–Ortiz (1993)	0	$\frac{Q^2}{X} - \frac{J}{4X^3} - \Lambda X$
15. KK reduced CS (2003)	0	$\frac{1}{2}X(c - X^2)$
16. KK red. conf. flat (2006)	$-\frac{1}{2}\tanh(X/2)$	$A \sinh X$
17. 2D type 0A string Black Hole	$-\frac{1}{X}$	$-2b^2 X + \frac{b^2 q^2}{8\pi}$
18. exact string Black Hole (2005)	lengthy	lengthy

Carrollian JT model DG, Hartong, Prohazka, Salzer '20; Gomis, Hidalgo, Salgado-Rebolledo '20

- Change to sl(2) basis convenient for Carrollian contraction

$$[H,J] = -\delta\, P \qquad [P,J] = -H \qquad [H,P] = -\Lambda\, J$$

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- Write down corresponding BF action

$$I_{\text{CJT}} \sim \int \big(XR + X_{\text{H}} T + X_{\text{P}}\, \mathrm{d}e - \epsilon\, \Lambda X \big)$$

with curvature $R = \mathrm{d}\omega$, torsion $T = \mathrm{d}\tau + \omega \wedge e$, and volume $\epsilon = \tau \wedge e$

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- action of Carrollian boosts on fields

$$\delta_\lambda X = 0 \qquad \delta_\lambda X_{\text{H}} = 0 \qquad \delta_\lambda X_{\text{P}} = X_{\text{H}}\, \lambda$$
$$\delta_\lambda \omega = \mathrm{d}\lambda \qquad \delta_\lambda \tau = -e\, \lambda \qquad \delta_\lambda e = 0$$

invariance of dilaton X, auxiliary scalar X_{H}, and spatial vielbein e

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again PSM interpretation, with Poisson tensor $P^{IJ} = \begin{pmatrix} 0 & 0 & X_{\text{H}} \\ 0 & 0 & \mathcal{V} \\ -X_{\text{H}} & -\mathcal{V} & 0 \end{pmatrix}$

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$$\lambda_I^{\text{Cboost}} = (\lambda, 0, 0) \qquad\qquad \lambda_I^{\text{diffeo}} = A_{I\mu}\, \xi^\mu$$

notation: $A_I = (\omega, \tau, e)$ and $X^I = (X, X_{\text{H}}, X_{\text{P}})$

Have infinite family of Carrollian (toy) models available!

Selected list of Carrollian models $\mathcal{V} = V(X)$

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EOM (with their suggested names*):

$$\begin{aligned} \text{Carrollian curvature:} \quad & \mathrm{d}\omega + \partial_X V(X,\, X_{\mathrm{H}})\, \tau \wedge e = 0 \\ \text{Carrollian torsion:} \quad & \mathrm{d}\tau + \omega \wedge e + \partial_{X_{\mathrm{H}}} V(X,\, X_{\mathrm{H}})\, \tau \wedge e = 0 \\ \text{No intrinsic torsion:} \quad & \mathrm{d}e = 0 \\ \text{Carrollian expansion:} \quad & \mathrm{d}X + X_{\mathrm{H}}\, e = 0 \\ \text{Carrollian Casimir:} \quad & \mathrm{d}X_{\mathrm{H}} + V(X,\, X_{\mathrm{H}})\, e = 0 \\ \text{Boost non-invariant:} \quad & \mathrm{d}X_{\mathrm{P}} - V(X,\, X_{\mathrm{H}})\, \tau - X_{\mathrm{H}}\, \omega = 0 \end{aligned}$$

* please speak up now if you object to some of these names

All classical solutions

Follow Lorentzian algorithm DG, Kummer, Vassilevich '02

1. Constant dilaton vacua

$$X_{\rm H} = 0 \qquad\qquad X = \text{const.} \quad \text{s.t.}\ \mathcal{V}(X, 0) = 0$$

constant curvature, $R = \partial_X \mathcal{V}$; slightly boring sector

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   - curvature $R = \partial_X V$ not necessarily constant

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   same solution in metric-like variables:

$$\mathrm{d}s^2 = g_{\mu\nu}\,\mathrm{d}x^\mu\,\mathrm{d}x^\nu = \mathrm{d}r^2 = \frac{\mathrm{d}X^2}{X_{\mathrm{H}}^2} = \frac{\mathrm{d}X^2}{2(U(X)-M)} \qquad v^\mu\partial_\mu = \frac{1}{X_{\mathrm{H}}}\,\partial_t$$

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- Global structure?
- Singularities?
- Special surfaces? (horizon, trapping, **extremal**, ...)

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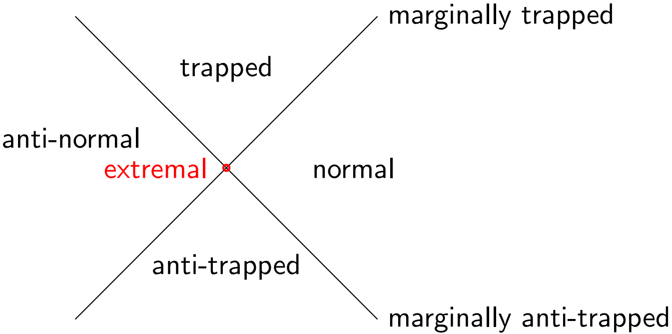

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Extremal surfaces are boost invariant loci!

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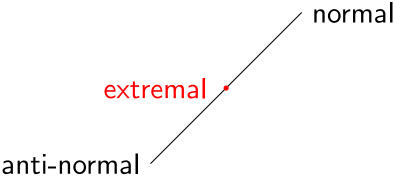

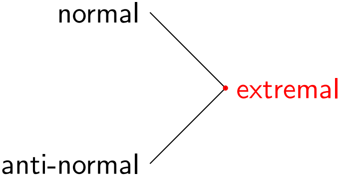

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- definition above conceivably generalizes to higher dimensions

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Carrollian black holes are defined to have all of these properties:

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Address these issues in remainder of talk

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- implement solution algorithm for CJT:

$$
\begin{aligned}
X &= \frac{1}{2}\,e^{r/\ell} + M\ell^2\,e^{-r/\ell} & \qquad \omega &= -\frac{X}{\ell^2}\,\mathrm{d}t \\
X_{\mathrm{H}} &= \frac{1}{2}\,e^{r/\ell} - M\ell^2\,e^{-r/\ell} & \qquad \tau &= X_{\mathrm{H}}\,\mathrm{d}t \\
X_{\mathrm{P}} &= 0 & \qquad e &= \mathrm{d}r
\end{aligned}
$$

$r \to \pm\infty$: asymptotic regions; $r \to \frac{\ell}{2}\,\ln(2M\ell^2)$: CES (for $M > 0$)

note similarities to AdS_2 black holes! suggestive to impose Brown–Henneaux-like boundary conditions

$$
\begin{aligned}
X &= \frac{1}{2}\,e^{r/\ell} + \mathcal{O}(e^{-r/\ell}) & \qquad \omega &= \Big(-\frac{1}{2\ell^2}\,e^{r/\ell} + \mathcal{O}(e^{-r/\ell})\Big)\,\mathrm{d}t \\
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CJT black holes require positive mass M

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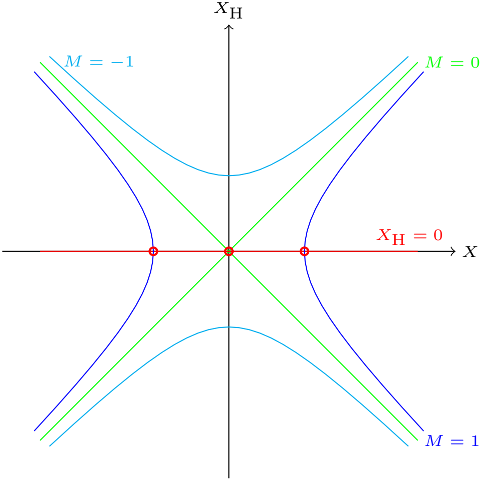

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1. defined using covariant phase space methods

$$\delta Q_\lambda = \tfrac{k}{2\pi}\lambda_I\,\delta X^I\Big|_{\partial M} \quad \Rightarrow \quad E = Q_{\lambda(\partial_t)} = \tfrac{k}{2\pi}\,\delta M \;✓$$

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     $2\pi T := P^{IJ} * (A_I \wedge A_J)|_{\text{extremal}}$ ✓
   - define Carrollian surface gravity
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   - exploit holonomy condition ?
3. know the result is Wald entropy (so that $\mathrm{d}E = T\,\mathrm{d}S$)

$$S = kX|_{\text{extremal}}$$

but not yet how to derive this from first principles ?

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Apply definitions to CJT black holes set $\ell = 1$

$$X = \frac{1}{2}\,e^{r} + M\,e^{-r} \qquad v = \frac{1}{\frac{1}{2}\,e^{r} - M\,e^{-r}}\,\partial_t \qquad \mathrm{d}s^2 = \mathrm{d}r^2$$

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Apply definitions to CJT black holes set $\ell = 1$

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first law holds

$$\mathrm{d}E = T\,\mathrm{d}S$$

evidence suggests: CJT black holes are thermal states with (large) entropy

CES or better definition of Carrollian black holes?

Assessment of Carrollian extremal surface definition

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No final verdict on Carrollian black holes
but incentive to continue research