GATIS

Gauge Theory as an Integrable System

Paths into Multi-Regge Regions

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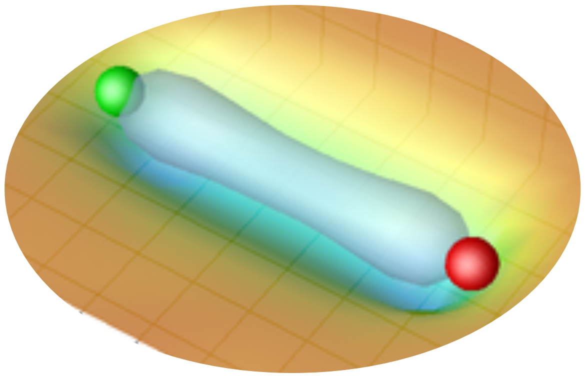
Based on work with Till Bargheer, Vsevolod Chestnov *[in progress]*

and with J. Bartels, J. Kotanski, M. Sprenger.

Epilogue: Gauge/String Correspondence

Recall: Planar N=4 SYM is a classical string theory
genus zero



Solution in terms of its flux tube

= integrable GKP string

[Beisert, Eden, Staudacher] [B. Basso]

Wilson-loop OPE: Efficient formulas in some kinematic regimes, e.g. collinear limit at weak coupling

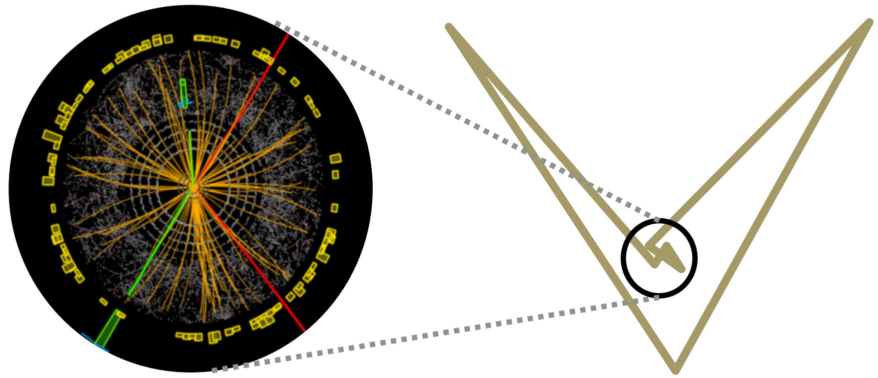
[Basso,Sever,Vieira]

Intro: Multi-Regge Limit…

… typical kinematics

at colliders

n can be large



<u>Strong coupling</u>: Infrared limit of integrable quantum system → MRL of finite remainder can be obtained by solving Bethe Ansatz equations [Bartels,VS,Sprenger]

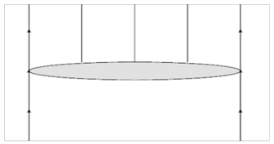
If we knew which solution was relevant

MRL would be solved at strong coupling!

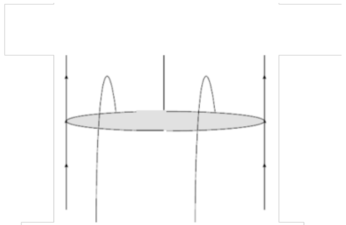
Intro: Analytical Structure

MRL_ϱ is MRL $(u_1, u_2, u_3) \longrightarrow (1, 0, 0)$

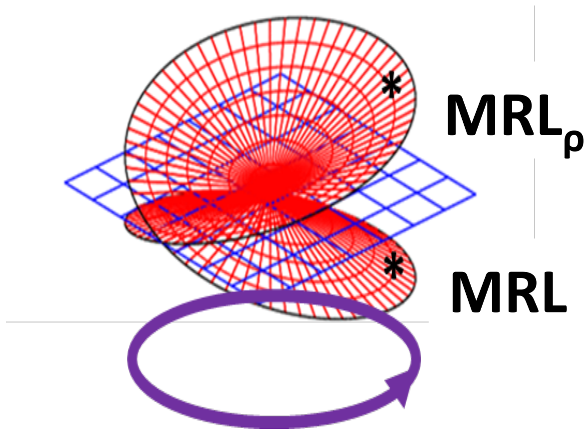
in Mandelstam region $\varrho = (\text{sgn}(E_i))$



$\varrho = (+ + +)$



$\varrho = (- + -)$



$u(\varphi) = e^{i\varphi} u$

BDS on main sheet in MR exact, i.e.

$$R_{n,g}(w, \bar{w}) = [\mathcal{R}_{n,g}(u_1, u_2, u_3)]^{\text{MRL}} = 0$$

$$R_{6,(2)}(w, \bar{w}) = [\mathcal{R}_{6,(2)}(u_1, u_2, u_3)]^{\text{MRL}_{(--)}} = g^4 \ln(1-u) \ln|1+w|^2 \ln\left|1+\frac{1}{w}\right|^2 + \ldots$$

NLL

$u \equiv u_1$

[Goncharov et al.]

[Bartels et al.]

Intro: Paths & Plan

MRL captures monodromies of finite remainder

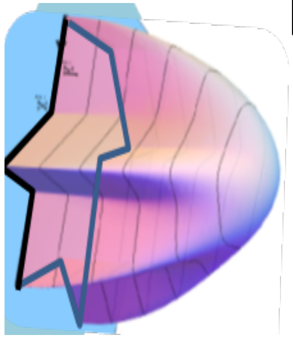
$$R^{\varrho}_{n,g}(w,\bar{w}) \sim \left[\mathcal{C}_{\varrho}\mathcal{R}_{n,g}(u_1,u_2,u_3)\right]^{\mathrm{MRL}}$$

What is $\mathcal{C}_{\varrho}$? **need homotopy not just homology**

I MRL at strong coupling & Bethe Ansatz

II Continuation paths at weak coupling

I.1 Minimal Area & TBA



[Alday,Gaiotto,Maldacena,Sever,Vieira]

amplitude = area of minimal surface in AdS_5

R **= energy of 1D multiparticle quantum system with**

3n-15 parameters (*m*,μ) & integrable interaction $S^{2\to 2}$

$$\log Y_a(\theta) = p_A(\theta) + \sum_B \int d\theta' K_{AB}(\theta - \theta') \log(1 + Y_B(\theta'))$$

Energy: $$\mathcal{E}(m,\mu) = \sum_A \int d\theta |m_A| \cosh\theta \, \log(1 + Y_A(\theta))$$

Solve $u = \left.\frac{Y}{1+Y}\right|_{\theta=\theta_*}$ **… for** $m(u,w), \mu(u,w)$ **to obtain**

$$\mathcal{R}_{n,\infty}(u) \sim \mathcal{E}(m,\mu)$$ **Theorem**

I.2 MRL is 1D Infrared Limit

[Bartels,Kotanski,Sprenger,VS]

MRL corresponds to limit of large masses *m* → ∞

$$\rightarrow \quad [\mathcal{R}_{n,\infty}(u)]^{\mathrm{MRL}} \sim [\mathcal{E}(m,\mu)]^{\mathrm{IRL}} = 0$$

To reach MRL$_\rho$ we must continue $m(u,w), \mu(u,w)$

During this process, excitations are produced in the 1D system by mechanism of [Dorey,Tateo]

$$\rightarrow \quad \mathcal{E}_{\varrho}^{\mathrm{IRL}} \sim \sum_{i} E(\theta_i)$$

θ_i Bethe roots of excitations

I.3 Multi-Regge Bethe Ansatz

In MRL: TBA → BA for rapidities θ of bare excitations

$\theta_\mu^{(A)}, \mu = 1, \ldots, N_A$ $\kappa = -\mathrm{sign}(\mathrm{Im}\theta)$

1 $$e^{p_A(\theta_\mu^{(A)})} = \prod_B \prod_{\nu=1}^{N_B} S_{AB}(\theta_\mu^{(A)} - \theta_\nu^{(B)})^{\kappa_\nu^{(B)}}$$

From solution of BA equations compute the energy

2 $$\mathcal{E}_\varrho^{\mathrm{IRL}} \sim \sum_A \sum_{\nu=1}^{N_A} \tilde{\kappa}_\nu^{(A)} |m_s| \sinh(\theta_\nu^{(A)})$$

3 **Solve** $u_{\alpha\sigma} = \left.\frac{Y_{A_{\alpha\sigma}}}{1 + Y_{A_{\alpha\sigma}}}\right|_{\theta=\theta_{\alpha\sigma}}$ **for** $(m(u), C(u))$ **& <u>compute *R*</u>**

with $\log Y_A(\theta) = p_A(\theta) - \sum_B \sum_{\nu=1}^{N_B} \kappa_\nu^{(B)} \log S_{AB}(\theta - \theta_\nu^{(B)})$

I.4 Strong Coupling Results

For n=6 gluons: Pair of 1D excitations gives in IRL

$$e^{R_{6,\infty}^{(--)}+i\delta} \sim \left((u-1)\frac{|w|}{|1+w|^2} \right)^{-\omega^\infty}$$

[Bartels, Sprenger, VS]

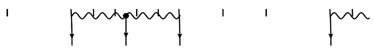
where $\omega^\infty = \frac{\sqrt{\lambda}}{2\pi}\left(\sqrt{2} - \log(1+\sqrt{2})\right)$

Same curve as in 2-loop analysis $u(\varphi) = e^{i\varphi} u$

ω^∞ is indeed the strong coupling limit of the BFKL eigenvalues $\omega(\nu, n)$

[Basso,Caron-Huot,Sever]

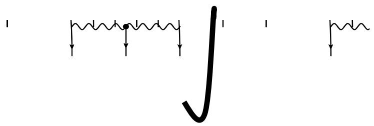
I.5 Beyond the Hexagon



$$e^{R_{7,\infty}^{(---)}+i\delta} \sim \left(\prod_{i=1,2}(u_i-1)\frac{|w_i|}{|1+w_i^2|^2}\right)^{-\omega^\infty}$$

$\varrho = (- \; - \, -)$

$$u_i = u_{1i}(\varphi) = e^{2i\varphi}\left(1-\sqrt{1-e^{-2i\varphi}}\right)u_{1i}$$

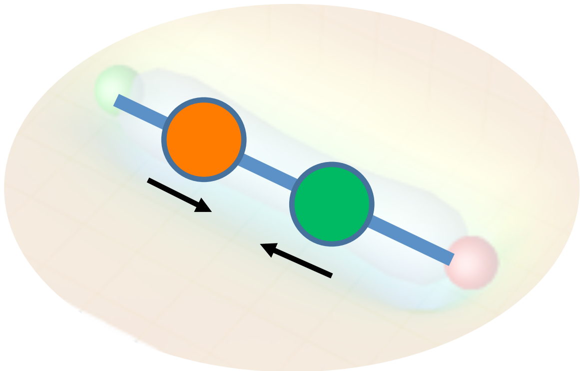


$$e^{R_{7,\infty}^{(-+-)}+i\delta} \overset{?}{\sim} 0$$

$\varrho = (- \; + \; -)$

$$u_{1i}(\varphi) = e^{2i\varphi}u_{1i} \quad u_{2i}(\varphi) = e^{\mp i\varphi}u_{2i} \quad u_{3i}(\varphi) = e^{\pm i\varphi}u_{3i}$$

II.1 The GKP String



Known 1-particle excitations X

$$E_X^g(u) \quad p_X^g(u) \quad m_X$$

Dispersion
***u* = rapidity**

Helicity
U(1) charge

Determined by BES equation **[B. Basso]**

Interact through factorizable scattering ← $S^{2\to 2}$

$$E_\Psi^g = \sum_a E_{X_a}^g(u_a) \quad p_\Psi^g = \sum_a p_{X_a}^g(u_a) \quad m_\Psi = \sum_a m_{X_a}$$

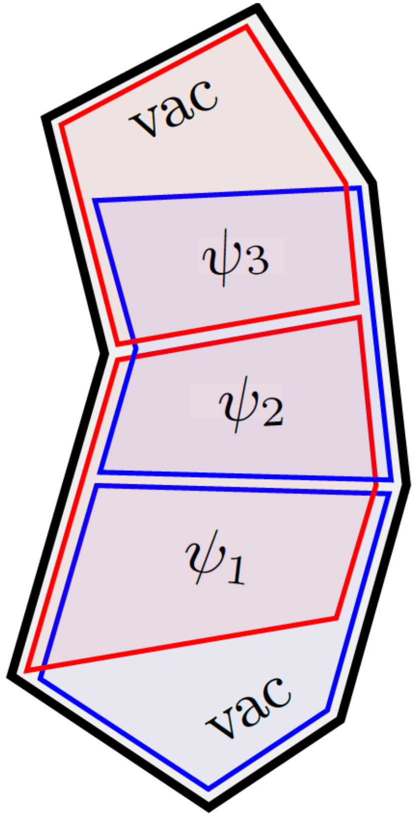
II.2 Wilson loop OPE

Introduce new variables, e.g. for n=6

$$u_1 = \frac{S^2}{(1+T^2)(1+S^2+T^2+2ST\cos\varphi)}$$

$$u_2 = \frac{T^2}{1+T^2} \qquad u_3 = \frac{1}{1+S^2+T^2+2ST\cos\varphi}$$

$$T = e^{-\tau}\ , \quad S = e^{\sigma}\ , \quad F = e^{i\varphi}$$



Remainder can be computed from GKP string through

$$e^{\mathcal{R}_g} \sim \sum_{\Psi_i} \left[\prod_{i=1}^{n-5} e^{E_i^g \tau_i + i p_i^g \sigma_i + i m_i \varphi_i} \right] P_g(0|\Psi_1) P_g(\Psi_1|\Psi_2) \ldots P_g(\Psi_{n-5}|0)$$

from $S^{2\to2}$

[Basso,Sever,Vieira]

Sum over single particle content, integral over rapidities

II.3 Collinear limit

In collinear limit only single particle excitations

w. X = $F^{\pm}$ contribute → $\mathcal{R}^{\text{CL}}_{n,(\ell)}$ simple to evaluate

$$\mathcal{R}_{6,(2)} \sim \mathcal{R}^{\text{BDS}}_{6,(2)} + 2T\cos(\varphi)(f_2^{(0)}(S) + \log(T) f_2^{(1)}(S)) + \ldots$$

Possesses branch points where S^2, $1+S^2$ vanish

Fundamental group of complex S^2 plane\{0,-1}

generated by $\mathcal{C}_1$ $\mathcal{C}_2$

$$u_1 = \frac{S^2}{1+S^2} + O(T^2) \quad u_2 = O(T^2) \quad u_3 = \frac{1}{1+S^2} + O(T^2)$$

II.4 Finding the Path

List all multiple discontinuities $\Delta_{a_1} \cdots \Delta_{a_M} \mathcal{R}^{\mathrm{CL}}_{(\ell)}$

MRL

$$\left[R^{\varrho}_{(\ell)}\right]^{\mathrm{CL}} \overset{?}{=} \Big(\sum_a c_a \Delta_a + \sum_{a,b} c_{a,b} \Delta_a \Delta_b + \dots \Big) \mathcal{R}^{\mathrm{CL}}_{(\ell)} =$$

$$\overset{?}{=} (1 + \Delta_{a_1})(1 + \Delta_{a_2}) \cdots (1 + \Delta_{a_M}) \mathcal{R}^{\mathrm{CL}}_{(\ell)} = \left[\mathcal{C}_\varrho \mathcal{R}^{\mathrm{CL}}_{(\ell)}\right]^{\mathrm{MRL}}$$

→ $\mathcal{C}_\varrho = \mathcal{C}_{a_1} \circ \mathcal{C}_{a_2} \circ \cdots \circ \mathcal{C}_{a_M}$

Hexagon: Equality holds for $\mathcal{C}_{(--)} = \overline{\mathcal{C}}_1$ unique in semigroup

II.5 Beyond the Hexagon

$\mathcal{R}^{\text{CL}}_{7,(2)}$ possesses branch points where following functions vanish $S_1^2\,,\quad 1+S_1^2\,,\quad S_2^2\,,\quad 1+S_2^2\,,\quad S_1^2+S_2^2+S_1^2S_2^2$

Fundamental group generated by five elements

$\mathcal{C}_1$ u_{25} $\mathcal{C}_2$ u_{36} $\mathcal{C}_3$ u_{15} $\mathcal{C}_4$ u $\mathcal{C}_5$

Analysis of $R_{7(2)}$ gives following continuation paths

$$\mathcal{C}_{(+--)} = \overline{\mathcal{C}}_1 \quad \mathcal{C}_{(--+)} = \overline{\mathcal{C}}_2 \quad \mathcal{C}_{(---)} \overset{\text{LLA}}{=} \overline{\mathcal{C}}_5$$

LLA → the curve $\mathcal{C}_{(-\mp-)}$ cannot be built out of $\mathcal{C}_1$ $\mathcal{C}_2$ $\overline{\mathcal{C}}_5$ alone, but needs either $\mathcal{C}_3$ or $\mathcal{C}_4$

Conclusions / Open problems

Push analysis of paths at weak coupling to higher orders in loops and legs and higher orders in T.

Identify the relevant solution of the Regge Bethe Ansatz at strong coupling.

ω_3 BFKL beyond leading log from octagon

Repeat the analysis of [Basso, Caron-Huot, Sever] beyond hexagon.