Central emission vertex and pentagon transition

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Iterated integrals and the Regge limit

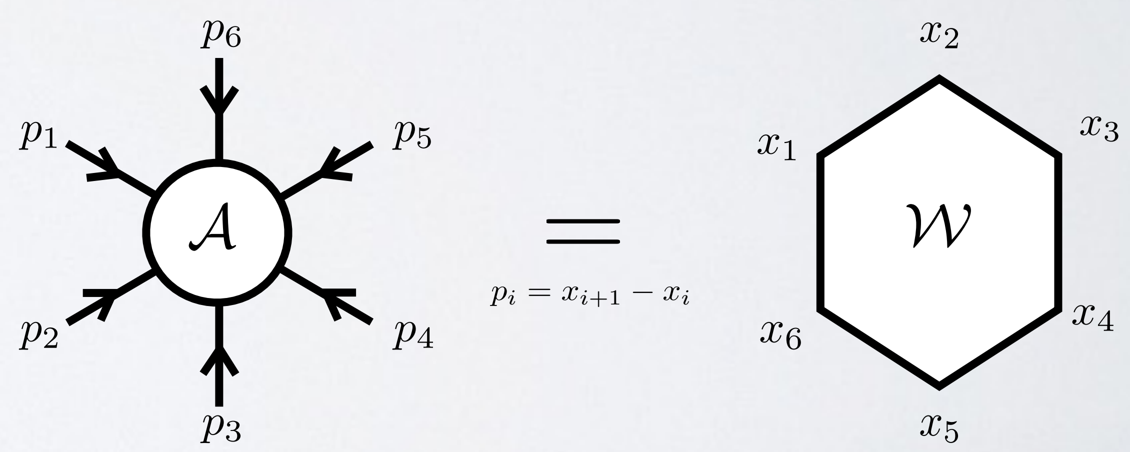
work in progress with Simon Caron-Huot and Amit Sever

Two important corners

Collinear limit (WL side)

Regge limit (SA side)

Rich interplay



[Alday,Maldacena'07]
[Drummond,Korchemsky,Sokatchev'07]
[Brandhuber,Heslop,Travaglini'07]
[Drummond,Henn,Korchemsky,Sokatchev'07]

Two important corners

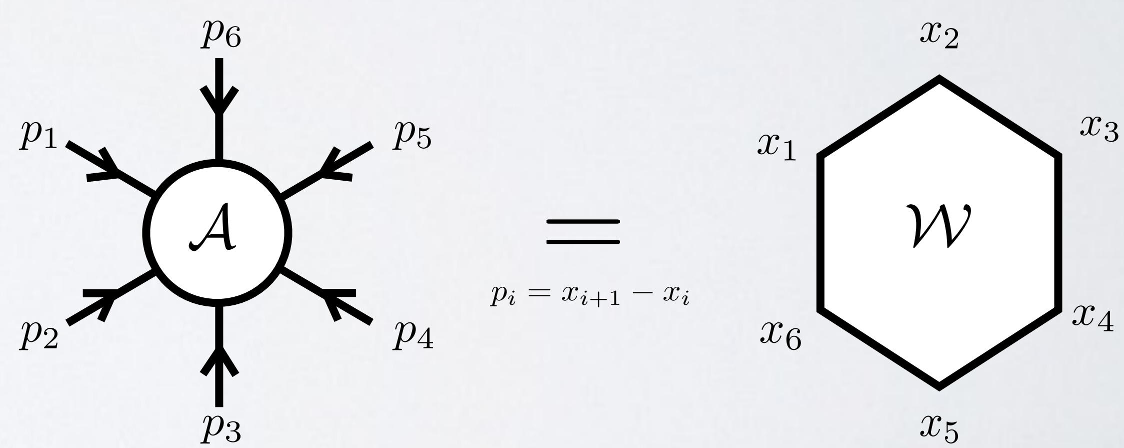
Spin chain (operator/ correlator story)

Integrability

Collinear limit (WL side)

Regge limit (SA side)

Rich interplay



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Plan

Brief review of pentagon OPE

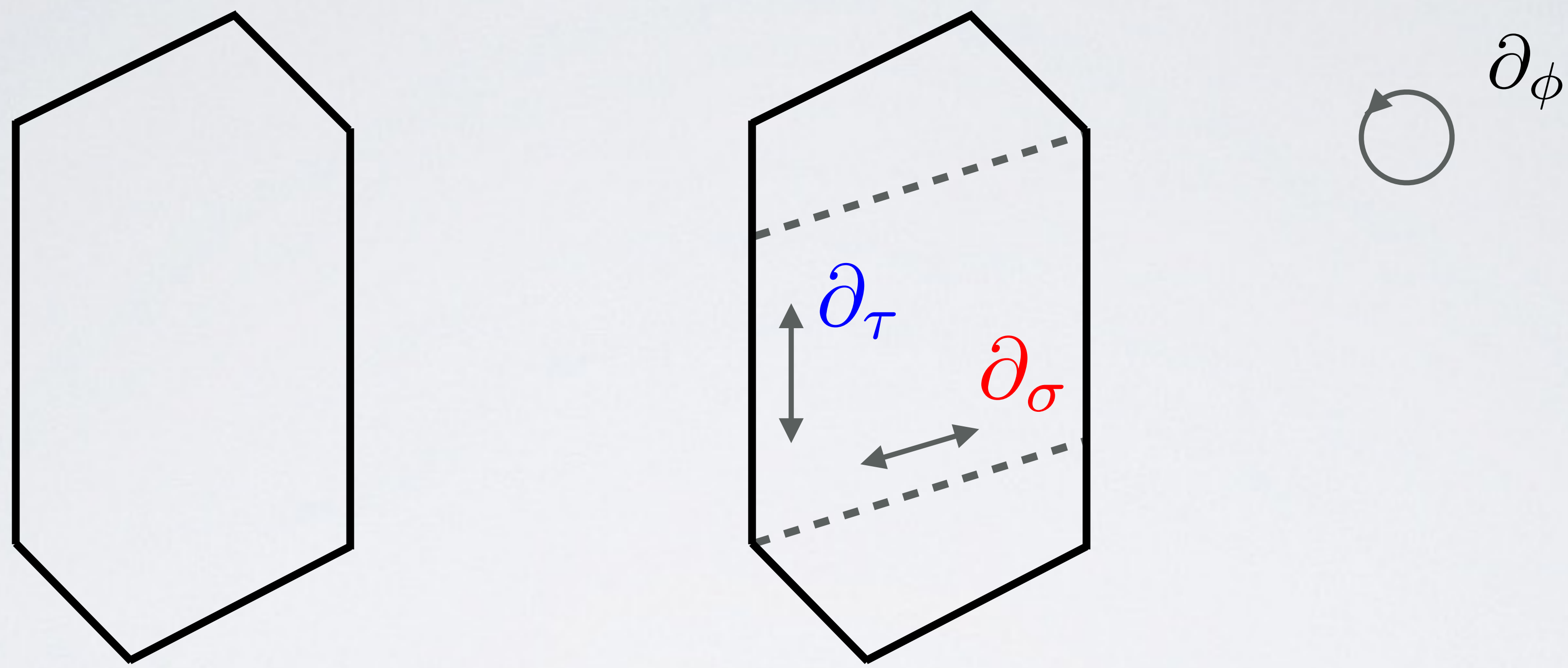
Regge toy model : the BDS/abelian world

The route to the all loop central emission vertex and the multi-Reggeons landscape

Pentagon OPE

OPE

Break polygonal loop into squares along a given channel



Use the 3 abelian symmetries of middle square to parameterize conformally inequivalent loop

$$u_2 = \frac{1}{1+e^{2\tau}} \sim e^{-2\tau} \qquad\qquad u_1 = e^{2\sigma+2\tau} u_2 u_3$$

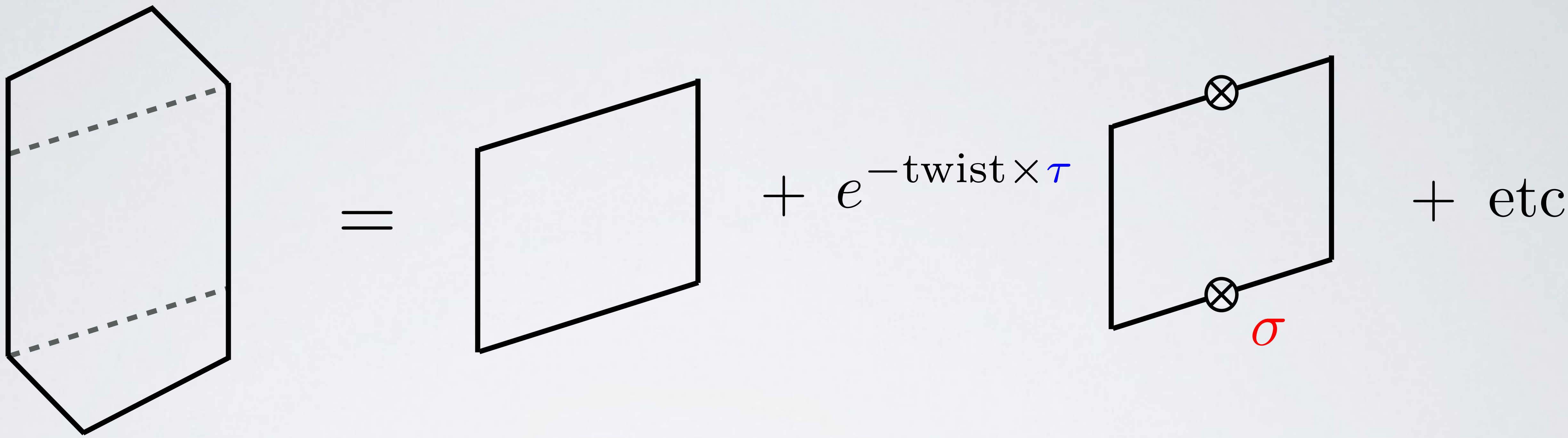
$$u_3 = \frac{1}{1+e^{2\sigma}+2e^{\sigma-\tau}\cos\phi+e^{-2\tau}}$$

τ OPE : large time

OPE

OPE/collinear limit = τ large

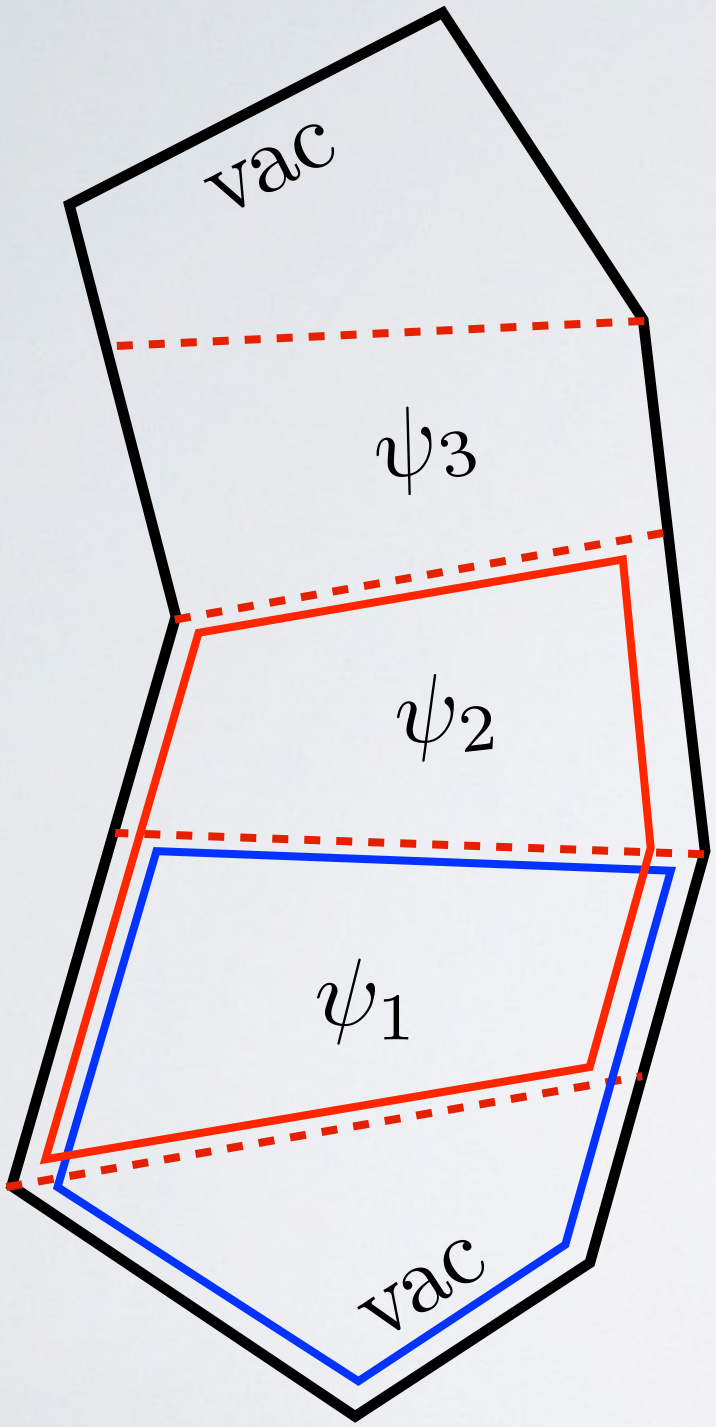
[Alday,Gaiotto,Maldacena,Sever,Vieira'09]



Sum over a complete basis of local field inserted at position $x = e^{2\sigma}$ along the light-ray

$$\mathcal{W} = \sum_{\text{states } \psi} C_{\text{bot}}(\psi) \times e^{-E(\psi)\tau + ip(\psi)\sigma + im(\psi)\phi} \times C_{\text{top}}(\psi)$$

Pentagon OPE



General OPE aka
multi-collinear factorization [BB,Sever,Vieira'13]

$$\int dp_1 \ldots \prod_i \, e^{-\sum_i E_i\tau_i + ip_i\sigma_i + im_i\phi_i}$$

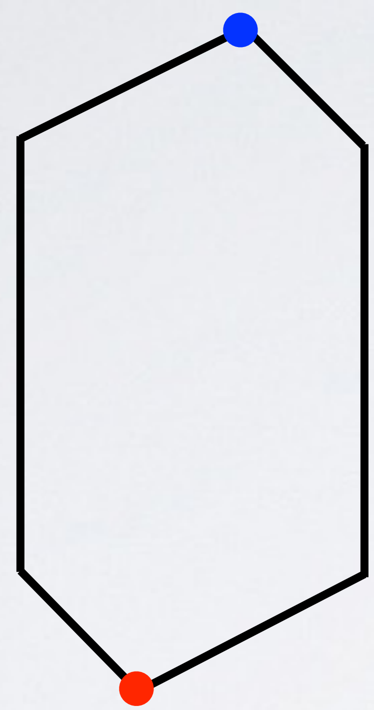
$$\times \; \mu(p_1)P(p_1|p_2)\mu(p_2)P(p_2|p_3)\ldots$$

- Spectrum of states
- Pentagon transitions

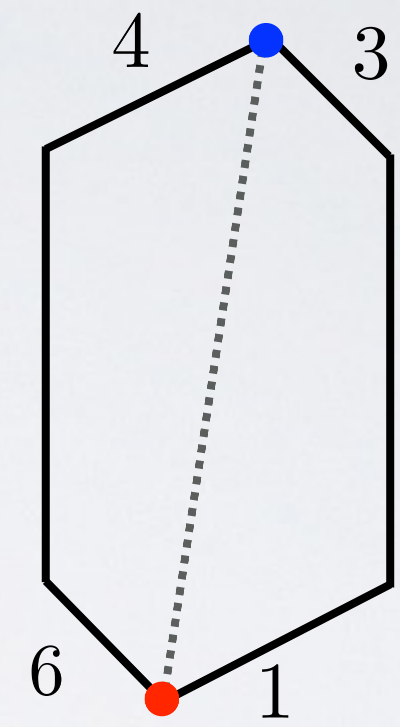
NMHV example I

Scalars inserted at the cusps

$$\mathcal{W}^{(6134)} =$$



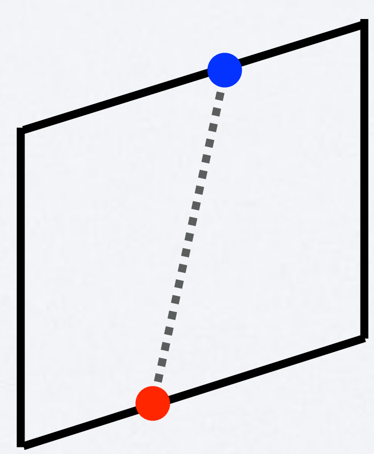
$$=$$



$$= \frac{1}{\langle 6, 1, 3, 4 \rangle}$$

Collinear behaviour

$$\mathcal{W}^{(6134)} = e^{-\mathrm{twist}\times\tau}$$

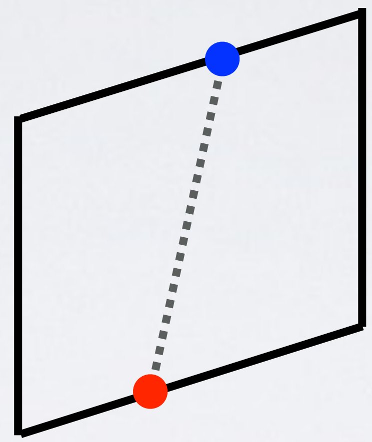


$$= e^{-\tau} \times \frac{1}{e^{\sigma}+e^{-\sigma}}$$

NMHV example I

Collinear behaviour

$$\mathcal{W}^{(6134)} = e^{-\text{twist}\times\tau}$$



$$= e^{-\tau} \times \frac{1}{e^{\sigma}+e^{-\sigma}}$$

Fourier transform in σ

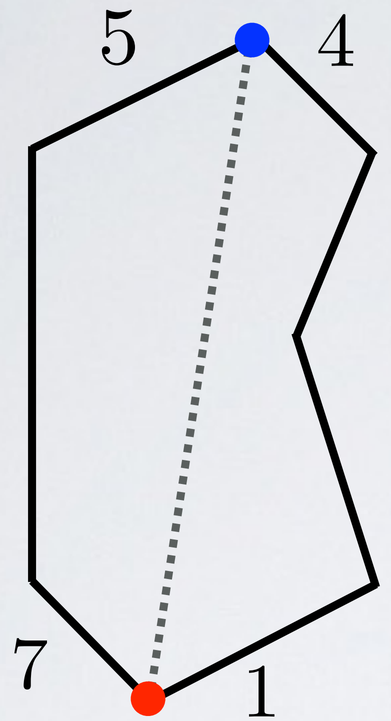
$$\mathcal{W}^{(6134)} = e^{-\tau}\int \frac{dp}{2\pi}\mu(p)e^{ip\sigma} + \ldots$$

Get the (tree-level) measure :

$$\mu(p) = \frac{\pi}{\cosh\left(\pi p/2\right)}$$

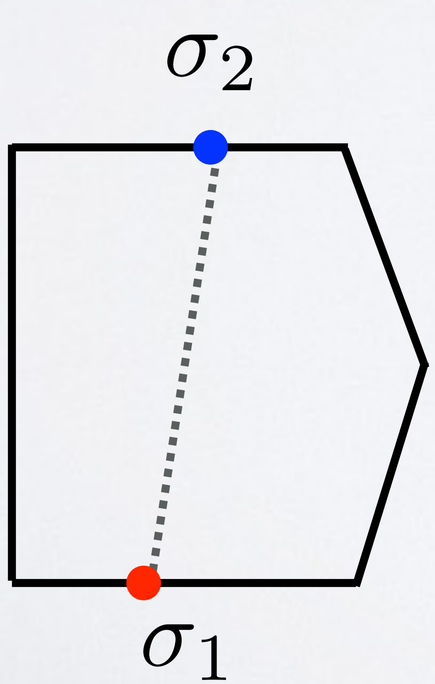
NMHV example II

Heptagon



$$= \frac{1}{\langle 7, 1, 4, 5 \rangle}$$

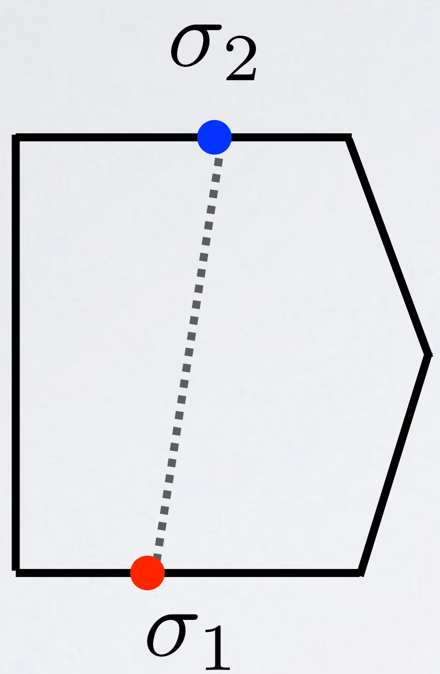
Collinear behaviour $\qquad \tau_{1,2} \to \infty$



$$= \frac{e^{-\tau_1 - \tau_2}}{e^{\sigma_1 - \sigma_2} + e^{\sigma_2 - \sigma_1} + e^{\sigma_1 + \sigma_2}}$$

NMHV example II

Double collinear behaviour $\quad \tau_{1,2} \to \infty$



$$= \frac{e^{-\tau_1-\tau_2}}{e^{\sigma_1-\sigma_2}+e^{\sigma_2-\sigma_1}+e^{\sigma_1+\sigma_2}}$$

Go to momentum space

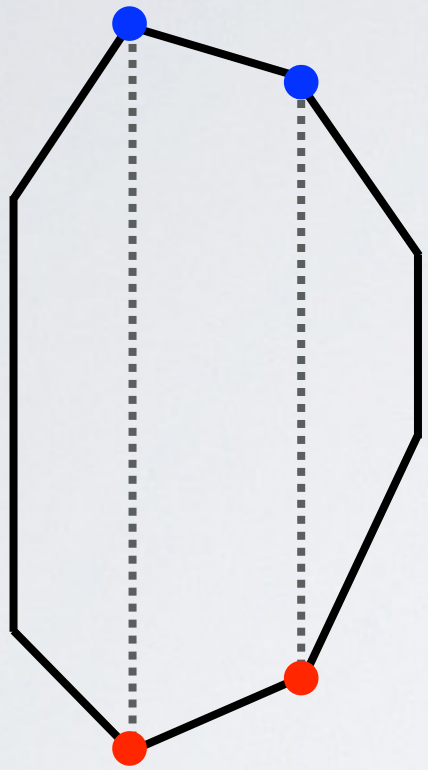
$$e^{-\tau_1-\tau_2}\int\int \frac{dp_1 dp_2}{(2\pi)^2}\mu(p_1)P(p_1|p_2)\mu(p_2)e^{-ip_1\sigma_1+ip_2\sigma_2}$$

Get the (tree-level) pentagon transition :

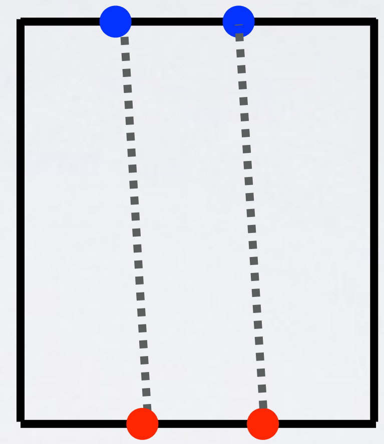
$$\mu_1 P_{1|2}\mu_2 = \Gamma(\tfrac{1}{2}-\tfrac{ip_1}{2})\Gamma(\tfrac{ip_1}{2}-\tfrac{ip_2}{2})\Gamma(\tfrac{1}{2}+\tfrac{ip_2}{2})$$

More NMHV examples

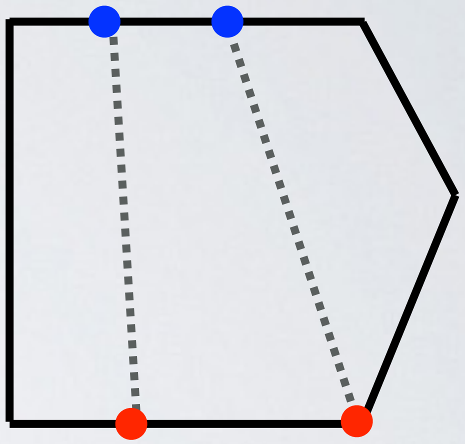
N2MHV octagon



Building blocks



2pt wave functions
overlap



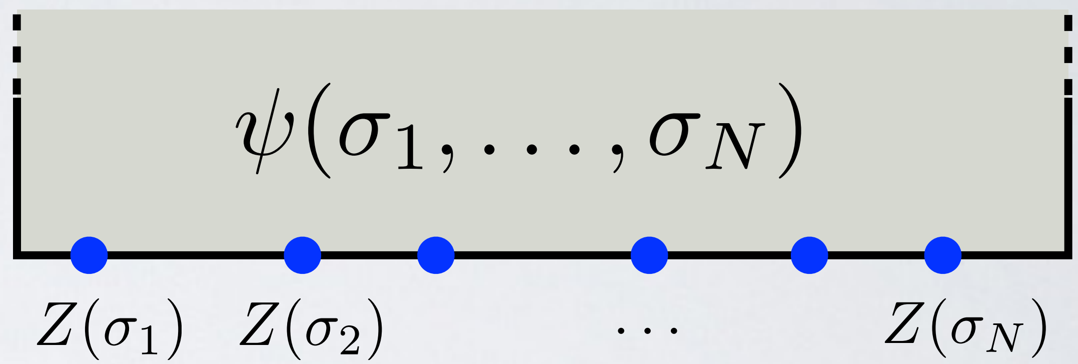
1-to-2 transitions

Pentagon OPE factorization

$$e^{-\sum_i \tau_i} \int \mu(p) P(p|q_1, q_2) \mu(q_1) \mu(q_2) P(q_1, q_2|r) \mu(r) e^{ip\sigma_1 + i(q_1+q_2)\sigma_2 + ir\sigma_3}$$

Wave functions

Problem :
Diagonalize light ray Hamiltonian



Hamiltonian :

$$\mathcal{H} \cdot \psi = 2g^2 \sum_{j=1}^{N} \int_{\sigma_{j-1}-\sigma_j}^{\sigma_{j+1}-\sigma_j} \frac{dt}{\sinh|t|} \left[e^{-|t|}\psi(\ldots) - \psi(\ldots, \sigma_{j-1}, \sigma_j + t, \sigma_{j+1}, \ldots) \right]$$

Eigenvalue problem :

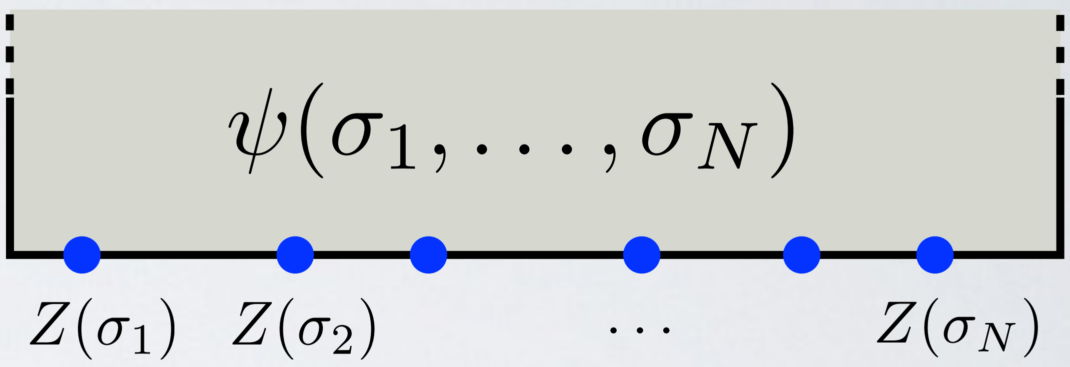
$$\mathcal{H} \cdot \psi_{p_1,\ldots,p_N} = E\psi_{p_1,\ldots,p_N} \qquad E = \sum_i E_i$$

1-body problem (solved by plane wave; thanks to translation symmetry)

$$\psi_p(\sigma) = e^{ip\sigma} \quad \Rightarrow \quad E(p) = 2g^2\left(\psi\left(\tfrac{1}{2} + i\tfrac{p}{2}\right) + \psi\left(\tfrac{1}{2} - i\tfrac{p}{2}\right) - 2\psi(1)\right)$$

Wave functions

Problem :
Diagonalize light ray Hamiltonian



Hamiltonian :

$$\mathcal{H} \cdot \psi = 2g^2 \sum_{j=1}^{N} \int_{\sigma_{j-1}-\sigma_j}^{\sigma_{j+1}-\sigma_j} \frac{dt}{\sinh|t|} \left[e^{-|t|} \psi(\ldots) - \psi(\ldots, \sigma_{j-1}, \sigma_j + t, \sigma_{j+1}, \ldots) \right]$$

Eigenvalue problem :

$$\mathcal{H} \cdot \psi_{p_1,\ldots,p_N} = E \psi_{p_1,\ldots,p_N} \qquad E = \sum_i E_i$$

N-body problem ? Doable because this Hamiltonian is integrable!

Wave functions

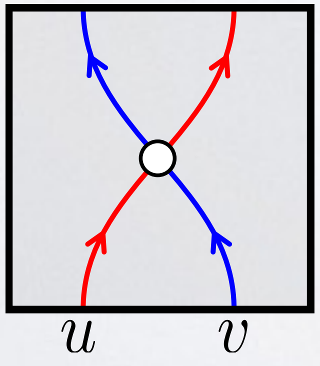
Integrability : waves function must diagonalize an auxiliary complete family of commuting conserved charges
The latter are given as differential operators (much easier)
For 2-magnon we get that the wave function solves an hypergeometric diff eq

$$\psi_{p_1,p_2}(\sigma_1,\sigma_2) = \frac{e^{i(p_1+ip_2)\sigma_2}}{P(q_1|q_2)} e^{\sigma_2-\sigma_1}\ {}_2F_1(\tfrac{1}{2}+i\tfrac{q_1}{2}, \tfrac{1}{2}+i\tfrac{q_2}{2}, 1, 1-e^{2\sigma_2-2\sigma_1})$$

Its asymptotic at large relative distance gives the S-matrix

$$\psi_{p_1,p_2}(\sigma_1,\sigma_2) \sim e^{ip_1\sigma_1+ip_2\sigma_2} + S(p_1,p_2)e^{ip_1\sigma_2+ip_2\sigma_1}$$

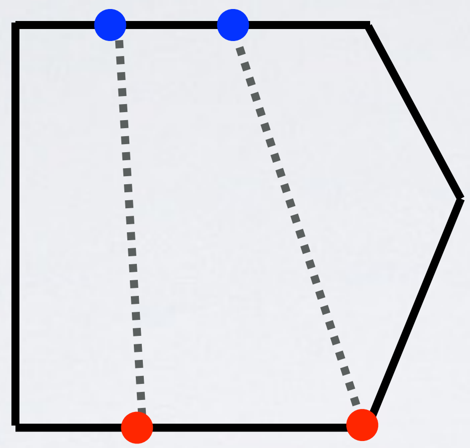
S-matrix :

$S(u,v) =$ 

$$S(p_1,p_2) = \frac{P(p_1|p_2)}{P(p_2|p_1)}$$

Pentagon transitions

Knowing the wave function we can explicitly compute the multi-parton transitions



We find that they factorize into elementary transitions!

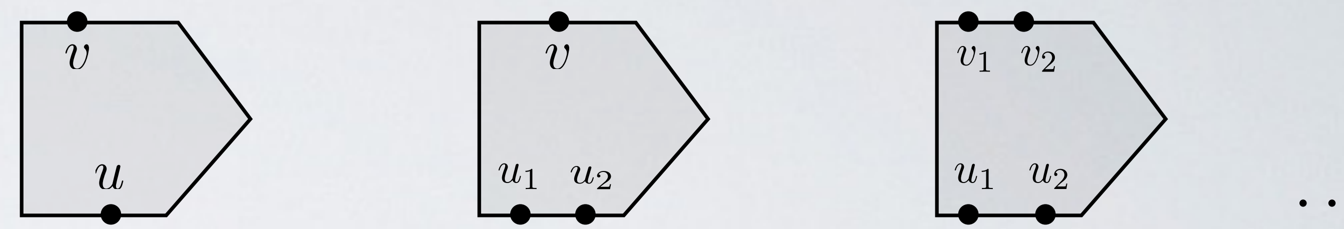
$$P(p|q_1, q_2) = \frac{P(p|q_1)P(p|q_2)}{P(q_1|q_2)}$$

Mathematically this is because

$$\int \mu(p) e^{ip\sigma} \frac{P(p|q_1)P(p|q_2)}{P(q_1|q_2)} \propto \frac{\mu(q_1)\mu(q_2)}{P(q_1|q_2)} \, {}_2F_1\left(\tfrac{1}{2} + i\tfrac{q_1}{2}, \tfrac{1}{2} + i\tfrac{q_2}{2}, 1, -e^{-2\sigma}\right)$$

General

All transitions are known



$P(u|v)$ $\quad$ $P(u_1, u_2|v)$ $\quad$ $P(u_1, u_2|v_1, v_2)$

Main ingredients are the elementary transitions :
multi-particle transitions are conjectured to factorize

[BB,Sever,Vieira'13]
[Belitsky,Derkachov,Manashov'13]
[Belitsky'15]

$$P(\{u_i\}|\{v_i\}) = \frac{\prod_{i,j} P(u_i|v_j)}{\prod_{i>j} P(u_i|u_j) \prod_{i<j} P(v_i|v_j)}$$

at any value of the coupling!

All pentagon transitions are known

$$P_{A|B}(u|v)^2 = \mathcal{F}_{A|B}(u|v) \times \frac{S_{AB}(u,v)}{S_{AB}(u^\gamma, v)}$$

ϕ : scalar

ψ : fermion

F : gluon

[BB,Sever,Vieira'13'14]
[BB,Caetano,Cordova,Sever,Vieira'15]
[Belitsky'14'15]

$$\mathcal{F}_{\phi F}(u|v) = 1\,,$$

$$\mathcal{F}_{\phi\psi}(u|v) = -\frac{1}{(u-v+\frac{i}{2})}\,,$$

$$\mathcal{F}_{\phi\phi}(u|v) = \frac{1}{(u-v)(u-v+i)}\,,$$

$$\mathcal{F}_{FF}(u|v) = \frac{(x^+y^+-g^2)(x^+y^--g^2)(x^-y^+-g^2)(x^-y^--g^2)}{g^2x^+x^-y^+y^-(u-v)(u-v+i)}\,,$$

$$\mathcal{F}_{F\psi}(u|v) = -\frac{(x^+y-g^2)(x^-y-g^2)}{g\sqrt{x^+x^-}y(u-v+\frac{i}{2})}\,,$$

$$\mathcal{F}_{F\bar\psi}(u|v) = -\frac{g\sqrt{x^+x^-}y(u-v+\frac{i}{2})}{(x^+y-g^2)(x^-y-g^2)}\,,$$

$$\mathcal{F}_{F\bar F}(u|v) = \frac{g^2x^+x^-y^+y^-(u-v)(u-v+i)}{(x^+y^+-g^2)(x^+y^--g^2)(x^-y^+-g^2)(x^-y^--g^2)}\,,$$

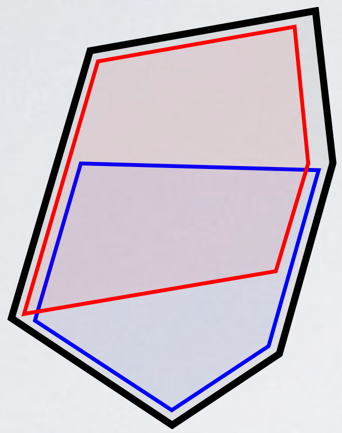
$$\mathcal{F}_{\psi\psi}(u|v) = -\frac{(xy-g^2)}{\sqrt{gxy}(u-v)(u-v+i)}\,,$$

$$\mathcal{F}_{\psi\bar\psi}(u|v) = -\frac{\sqrt{gxy}}{(xy-g^2)}\,,$$

Full 6-gluon amplitude

[BB, Sever, Vieira'15]

OPE series :

$\mathcal{W}_{\rm hex} =$  $= \sum_n \frac{1}{S_n} \int \frac{du_1 \dots du_n}{(2\pi)^n} \Pi(\{u_i\})$

Flux tube integrand :

(everything here is known at any coupling)

$$\Pi(\{u_i\}) = \Pi_{\rm dyn} \times \Pi_{\rm mat}$$

$$\Pi_{\rm dyn} = \prod_i \mu(u_i)\, e^{-E(u_i)\tau + ip(u_i)\sigma + im_i\phi} \prod_{i<j} \frac{1}{|P(u_i|u_j)|^2}$$

Toy model for Regge limit

Toy model

Definition :

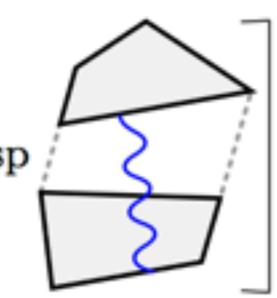
Through away remainder, keep BDS

Equivalently : abelian approximation
Compute WL in U(1) theory with coupling

$$a = \frac{\Gamma_{\text{cusp}}}{4} = g^2 - \frac{\pi^2}{3} g^4 + \frac{11\pi^4}{45} g^6 + \ldots$$

Example : hexagon (OPE) ratio function

$$\mathcal{W}_{\text{hex}}^{\text{BDS}} = \exp\left[\Gamma_{\text{cusp}}\right.$$



$$\left.\right] = \exp\left[\frac{\Gamma_{\text{cusp}}}{4}\{\text{Li}_2(u_2) - \text{Li}_2(1-u_1) - \text{Li}_2(1-u_3) + \log^2(1-u_2) - \log(u_1)\log(u_3) - \log(u_1/u_3)\log(1-u_2) + \frac{\pi^2}{6}\}\right] \qquad (119)$$

BDS/abelian impact factor

Hexagon :

$$W^{\circlearrowright}_{U(1)} = e^{i\pi\delta_6} = \left(\frac{|w|}{|1+w|^2}\right)^{2i\pi a} = \left(\frac{\rho}{1+2\rho\cos\phi+\rho^2}\right)^{2i\pi a}$$

Fourier-Mellin transform :

$$a = \frac{\Gamma_{\text{cusp}}}{4} = g^2 - \frac{\pi^2}{3}g^4 + \frac{11\pi^4}{45}g^6 + \dots$$

$$W^{\circlearrowright}_{U(1)} = \sum_{n\in\mathbb{Z}} e^{in\phi}\int\frac{d\nu}{2\pi}\underbrace{\frac{\Gamma(1-2i\pi a)\Gamma(\frac{n}{2}-i\nu+i\pi a)\Gamma(-\frac{n}{2}+i\nu+i\pi a)}{\Gamma(2i\pi a)\Gamma(1+\frac{n}{2}+i\nu-i\pi a)\Gamma(1-\frac{n}{2}-i\nu-i\pi a)}}_{\mu_n(\nu)_{U(1)}}\rho^{2i\nu}$$

Check : Weak coupling limit : $\lim_{a\to 0}\mu_n(\nu)_{U(1)} = (-1)^n\frac{2i\pi a}{\nu^2+\frac{n^2}{4}}$ Receives contribution from Remainder

Check : Regge pole (for n=0) : $\mu_{n=0}(\nu\sim\pm\pi a)\sim\frac{\pm i}{\nu\mp\pi a}$ Always true

[It enforces that $W^{\circlearrowright}_{U(1)}\sim\rho^{\pm 2i\pi a}$ in collinear limits $\rho = 0,\infty$ [Simon]]

BDS/abelian emission vertex

Heptagon :

$$W_{U(1)}^{\circlearrowright} = e^{i\pi\delta_7} = \left(\frac{|w_1 w_2|}{|1 + w_1 w_2 + w_2|^2}\right)^{2i\pi a}$$

Fourier-Mellin transform / Regge (OPE) factorization :

$$W_{U(1)}^{\circlearrowright} = \sum_{n_{1,2}\in\mathbb{Z}} \int \frac{d\nu_1 d\nu_2}{2\pi} \mu_{n_1}(\nu_1) P_{n_1|n_2}(\nu_1|\nu_2) \mu_{n_2}(\nu_2) w_1^{i\nu_1+\frac{n_1}{2}} \bar{w}_1^{i\nu_1-\frac{n_1}{2}} w_2^{i\nu_2+\frac{n_2}{2}} \bar{w}_2^{i\nu_2-\frac{n_2}{2}}$$

Abelian/BDS emission vertex :

$$\mu_1 P_{1|2} \mu_2 = \frac{\Gamma(1-2i\pi a)\Gamma(\frac{n_1}{2} - i\nu_1 + i\pi a)\Gamma(\frac{n_2-n_1}{2} + i\nu_1 - i\nu_2)\Gamma(-\frac{n_2}{2} + i\nu_2 + i\pi a)}{\Gamma(2i\pi a)\Gamma(1+\frac{n_1}{2} + i\nu_1 - i\pi a)\Gamma(1+\frac{n_2-n_1}{2} - i\nu_1 + i\nu_2)\Gamma(1-\frac{n_2}{2} - i\nu_2 - i\pi a)}$$

Decoupling pole (soft limit) :

True in full theory as well

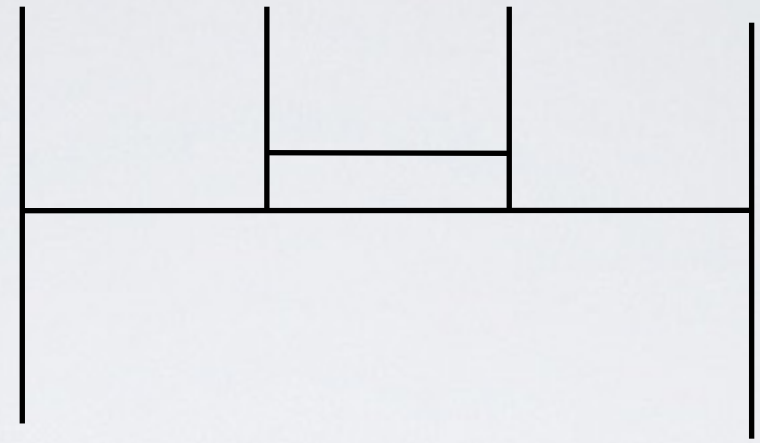
$$P_{n_1|n_2}(\nu_1|\nu_2) \sim \frac{\delta_{n_1,n_2}}{i\mu_{n_1}(\nu_1)(\nu_1-\nu_2)}$$

Interpolation with collinear vacua at $\rho_1 \sim \infty$ and $\rho_2 \sim 0$

$$\mu_0(\nu_1) P_{0|n_2}(\nu_1|\nu_2) \sim \frac{i}{\nu_1 - \pi a} \qquad P_{n_1|0}(\nu_1|\nu_2)\mu_0(\nu_2) \sim \frac{-i}{\nu_2 + \pi a}$$

Regge picture

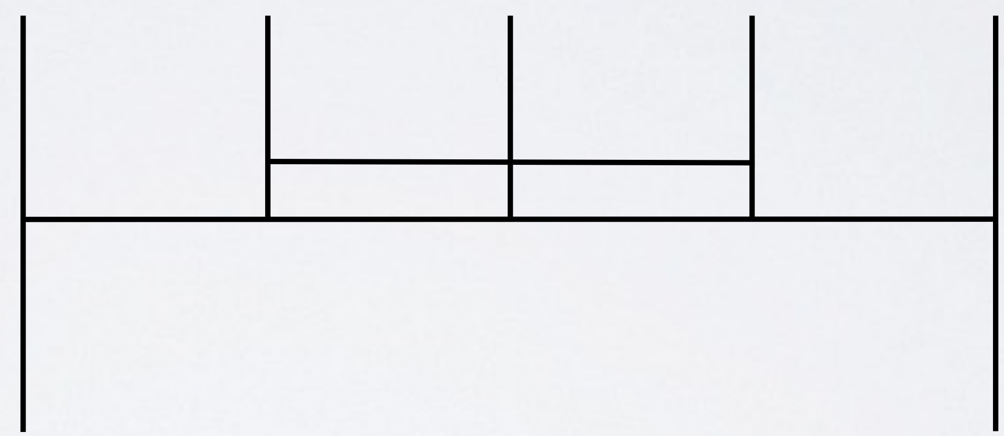
Hexagon gives measure a.k.a impact factor



Diagram

$$\mu(u)$$

Heptagon gives pentagon transition a.k.a central emission vertex



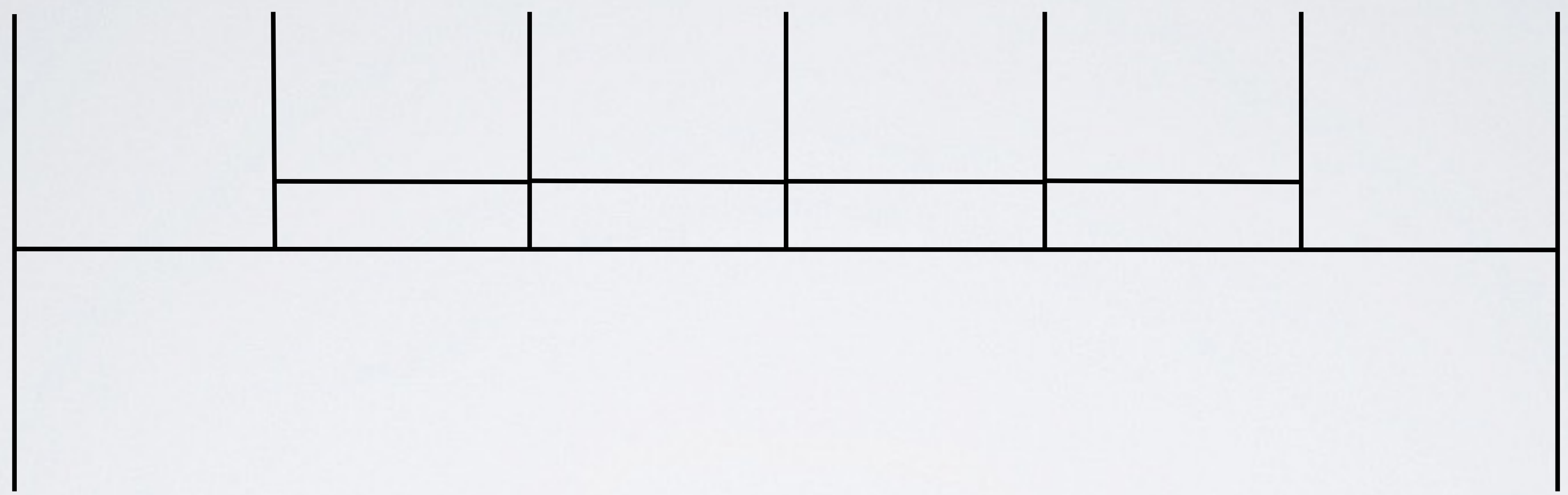
Diagram

$$\mu(u) \times P(u|v) \times \mu(v)$$

Higher polygons

recent discussions : [Bargheer'16], [Del Duca et al.'16]

Linear sequence
(higher n-gon)

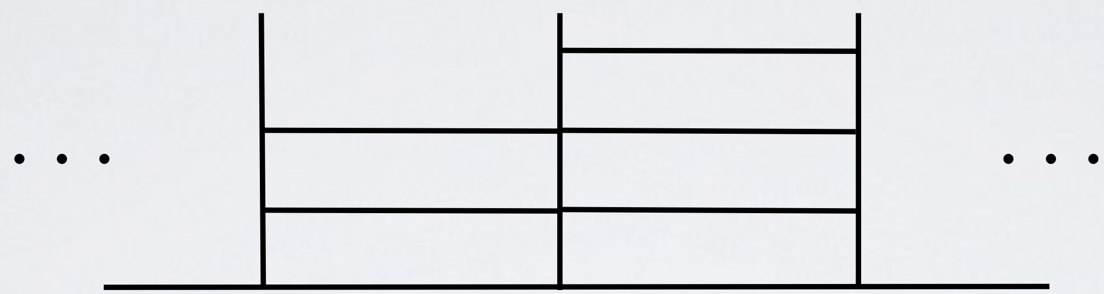


Diagram

$$\mu(u) \times P(u|v) \times \mu(v) \times P(v|w) \times \mu(w) \times P(w|z) \times \mu(z)$$

More Reggeons

Octagon and higher

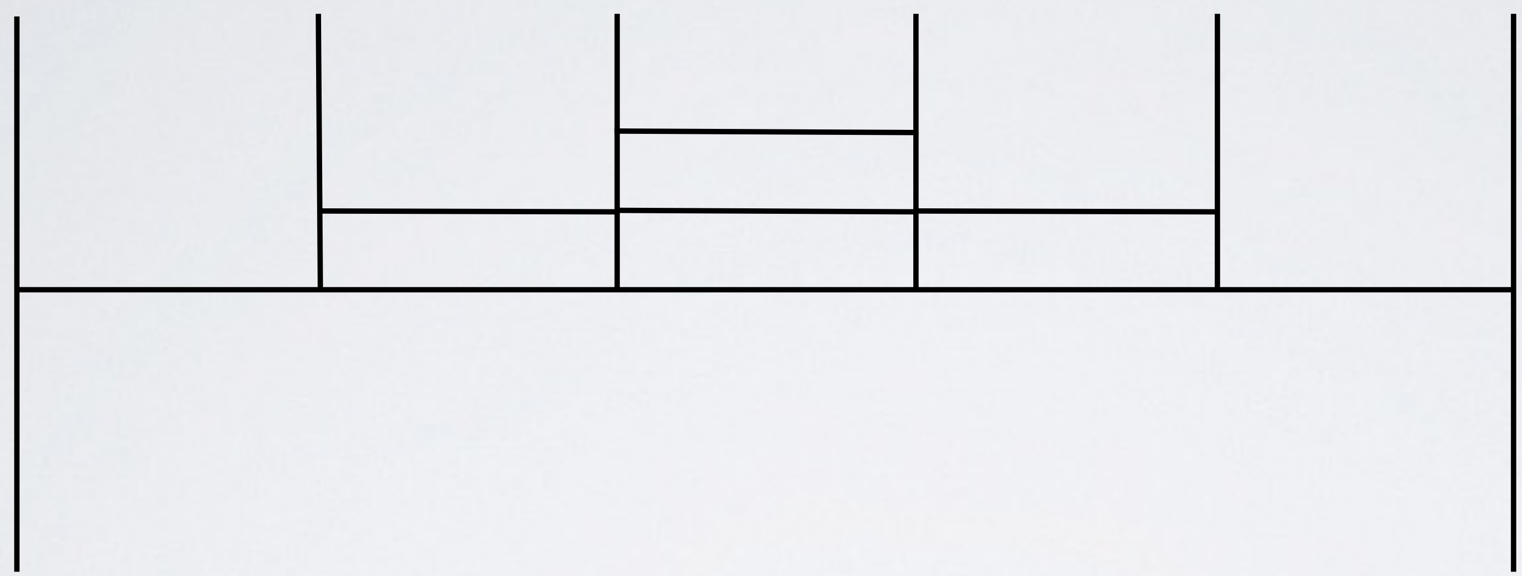


Conjecture (following factorization of OPE transitions) :

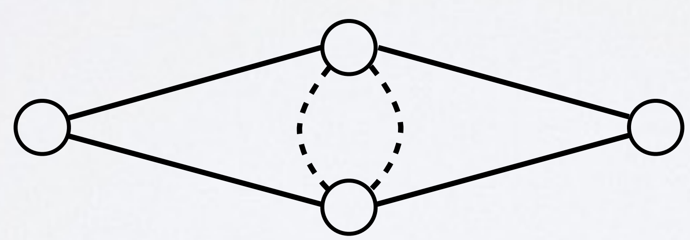
$$\frac{\prod_{i,j} P(u_i|v_j)}{\prod_{i<j} P(u_i|u_j)\prod_{i>j} P(v_i|v_j)}$$

More Reggeons

Example of a sequence with up to 3 Reggeons



Diagram

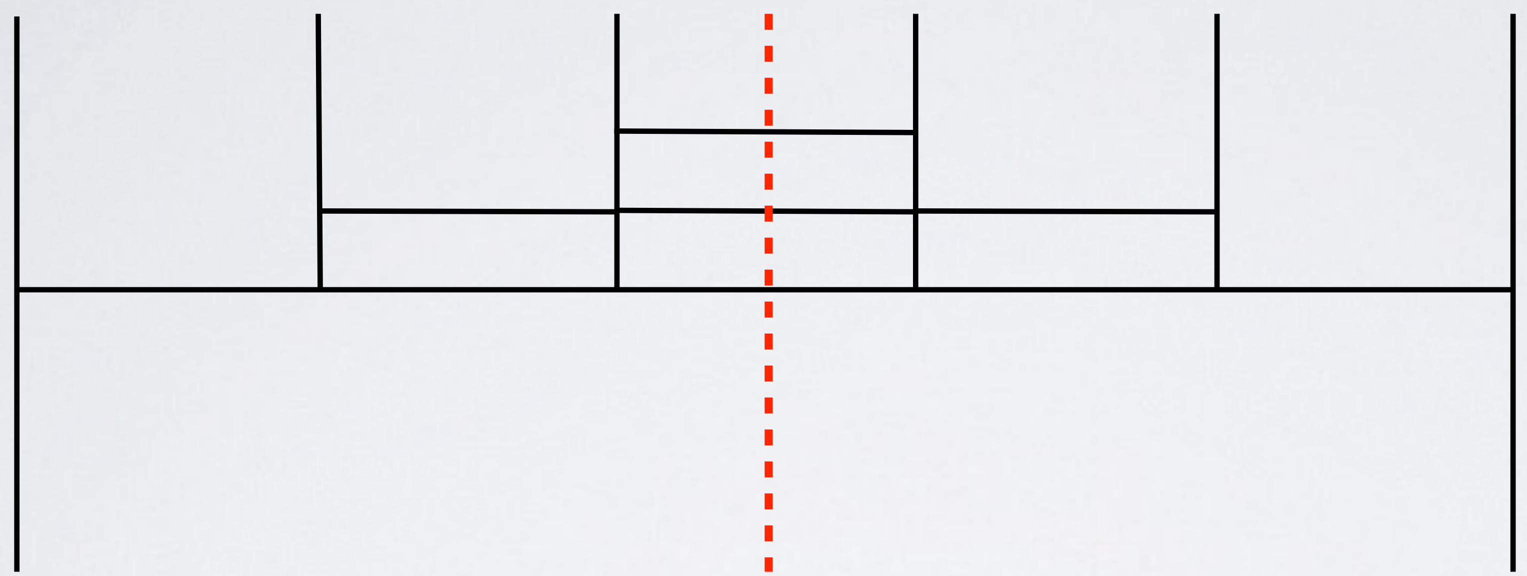


Integrand

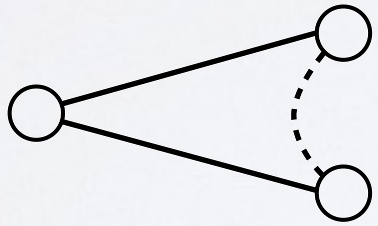
$$P(u|v_1)P(u|v_2) \times \frac{1}{P(v_1|v_2)P(v_2|v_1)} \times P(v_1|w)P(v_2|w) \times P(w|z)$$

Test

Cut in the middle and look at wave function



Diagram



Integral

Right 2-body wave function!

[Bartels,Lipatov,Prygarin'11]

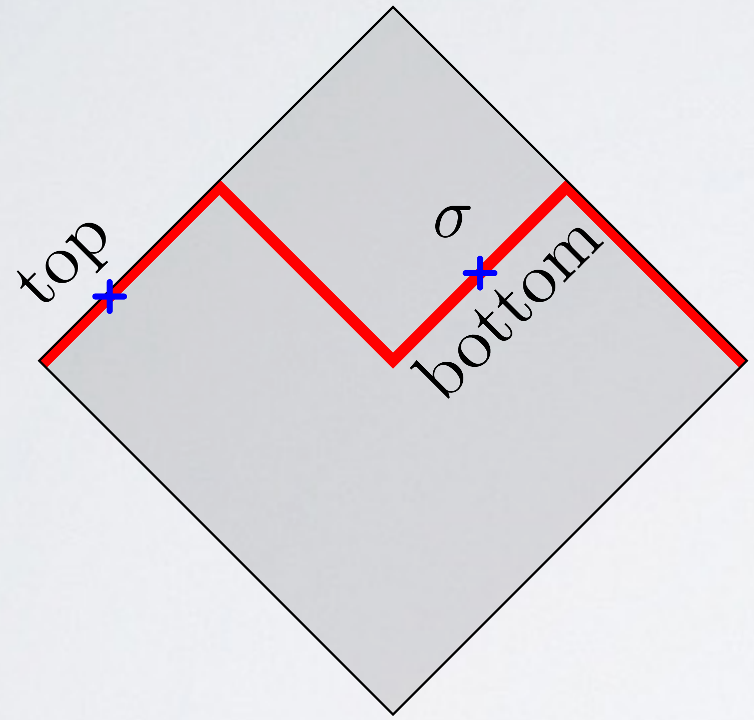
$$\sum_n \int d\nu w^{i\nu+\frac{n}{2}} \bar{w}^{i\nu-\frac{n}{2}} \frac{P_{n|m_1}(\nu|\lambda_1) P_{n|m_2}(\nu|\lambda_2)}{P_{m_1|m_2}(\lambda_1|\lambda_2)} \sim \frac{1}{P_{m_1|m_2}(\lambda_1|\lambda_2)} \, {}_2F_1(\ldots;-w) \times \, {}_2F_1(\ldots;-\bar{w})$$

From the collinear to the Regge limit

Passing to the real kinematics

Hexagon in collinear limit

$$\mathcal{W}_{\text{hex}} = 1 + \sum_m e^{im\phi} \int \frac{du}{2\pi} \mu_m(u) e^{ip_m(u)\sigma - E_m(u)\tau}$$



- Leading twist approximation : bottom/top cusps are replaced by insertions of field strength tensor

- Insertions are spacelike separated

$$\sim \frac{1}{e^{\sigma} + e^{-\sigma}}$$

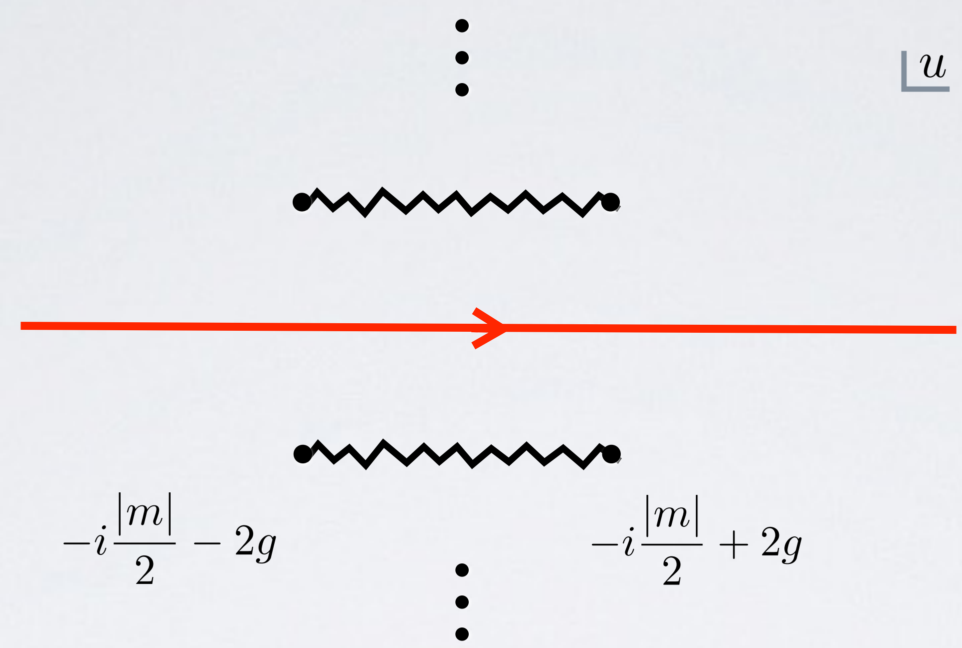
At $\sigma = \pm \frac{i\pi}{2}$ the flat cusps are null separated : a cut starts there

$$F_1(\sigma,\tau) = g^2 e^{-\tau}\left[-(e^{\sigma}+e^{-\sigma})\log(1+e^{2\sigma}) + 2\sigma e^{\sigma}\right] + O(g^4)$$

The (Minkowskian) Regge limit is reached by going to the other side of this cut and sending $\sigma \to \infty$

Discontinuity and contour deformation

OPE contour :



- Contour along real line in rapidity plane

- There is an infinite tower of Zhukowski cuts both in lower and upper half planes

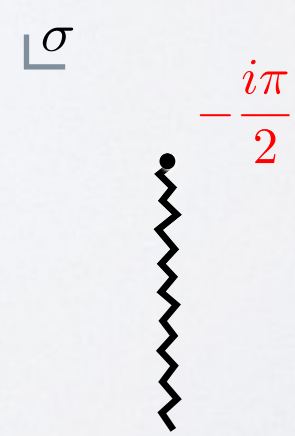
After shifting

$$\sigma \to \sigma - \frac{i\pi}{2} \qquad e^{ip(u)\sigma} \to e^{\pi u} \times e^{ip(u)\sigma}$$

The integral becomes marginally convergent

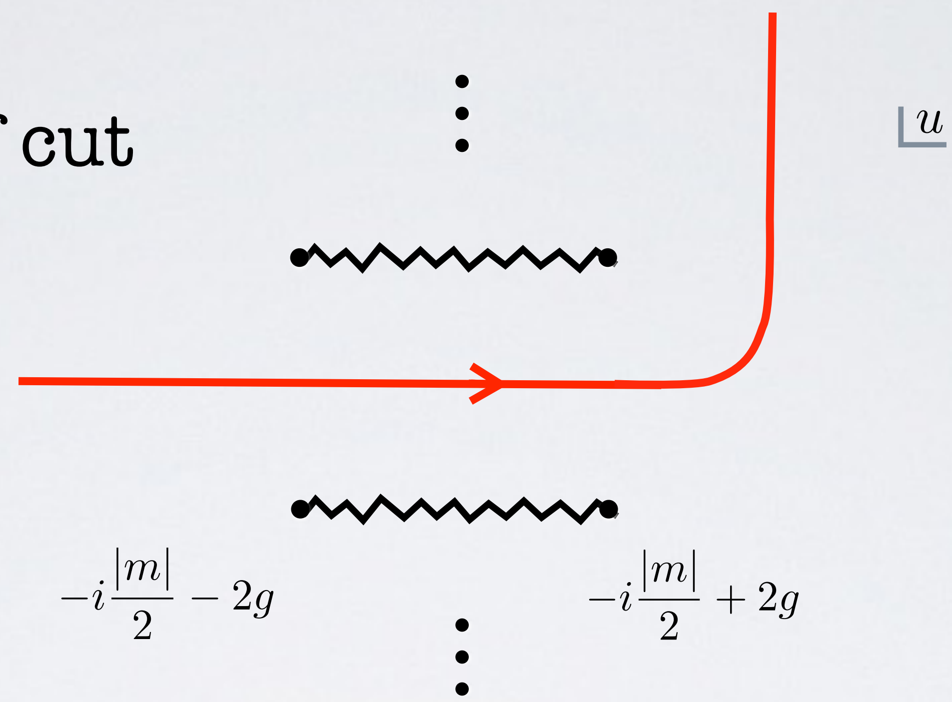
Singularity at

$$\sigma = -\frac{i\pi}{2}$$

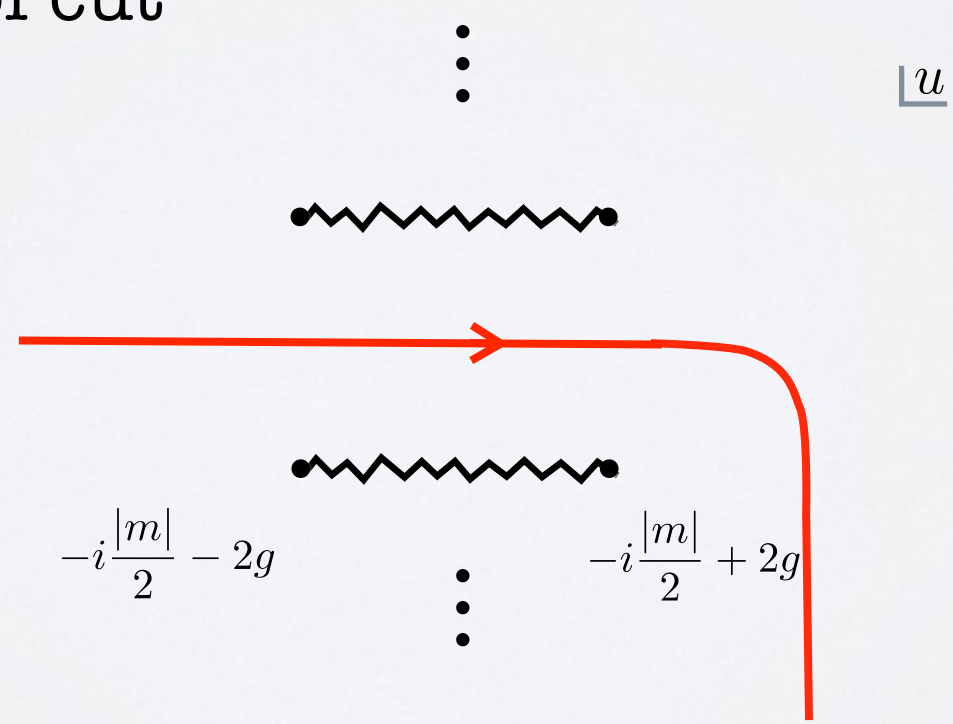


Discontinuity and contour deformation

Move to left side of cut

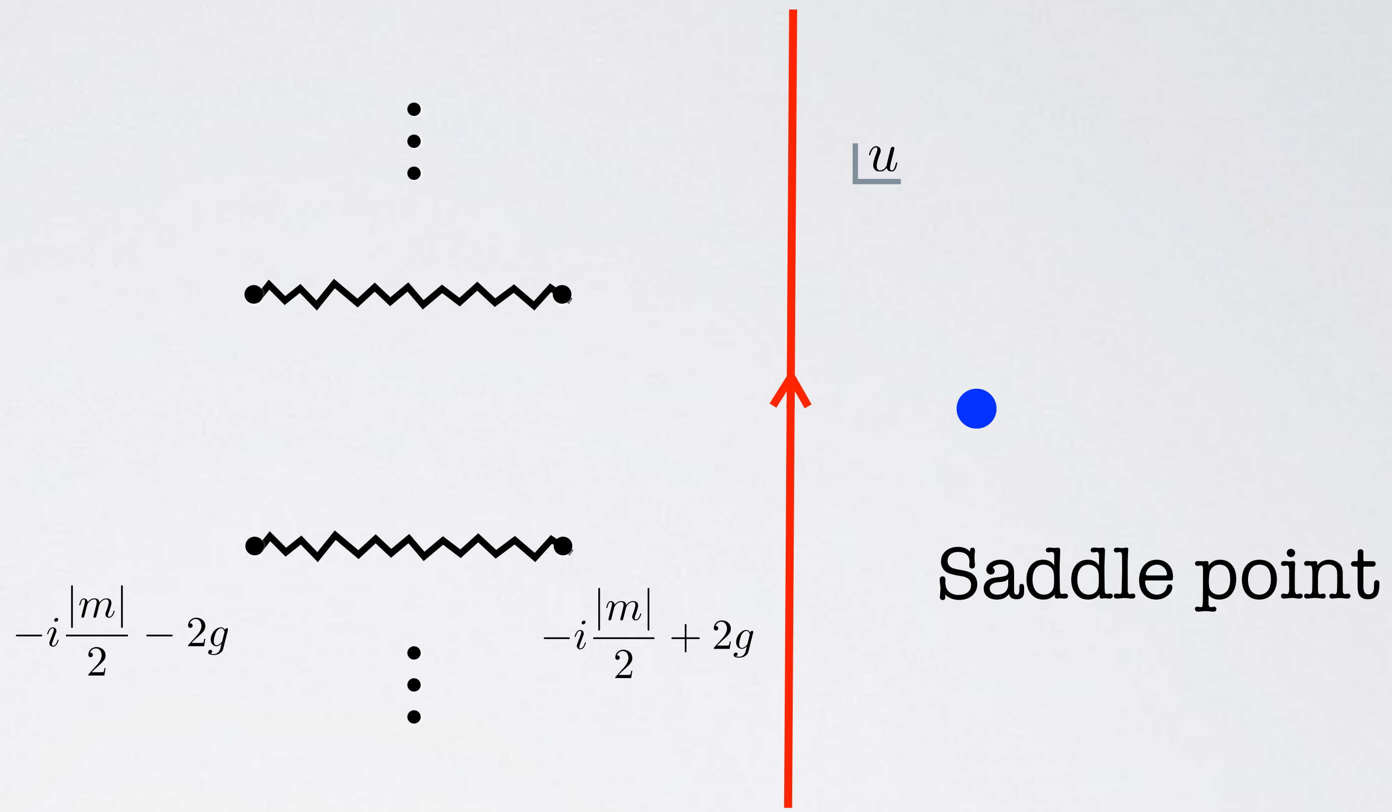


Move to right side of cut



Discontinuity and contour deformation

Difference = discontinuity



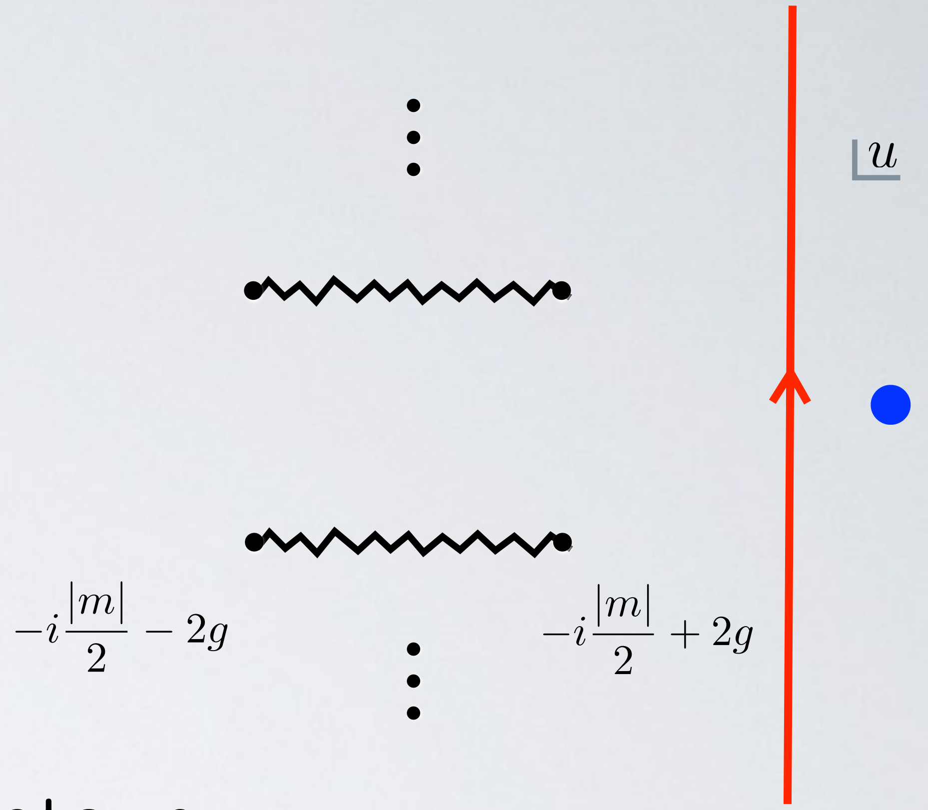
Main message : the discontinuity through the cut is controlled by the same OPE integrand but with a vertical (inverse Laplace like) contour

It is dominated by a saddle point that only exists passed the cut

Discontinuity and contour deformation

Regge limit has to do with enhancement of discontinuity in limit

$$\sigma \rightarrow \pm\infty$$



To study this regime we must rotate the contour (equivalently the saddle point) to lower/upper half plane

Problem : Must avoid infinite sequence of cuts there

Remedy : Redefine the OPE integrand such that this sequence of cuts terminates

Freedom : Vertical contour allows us to add/remove exponentially small terms at large rapidity

[BB,Caron-Huot,Sever'14]

This give rise to a new object : the sister trajectory

Sister map

One loop example : Take energy

$$\psi(1+\frac{|m|}{2}+iu)+\psi(1+\frac{|m|}{2}-iu)-2\psi(1)$$

Use reflection property and drop exponentially small terms

$$\psi(1+\frac{|m|}{2}\pm iu)\to\psi(-\frac{|m|}{2}\mp iu)\pm i\pi+O(e^{-2\pi u})$$

It gives rise to the sister energy in lower/upper half plane

$$\psi(1+\frac{|m|}{2}\pm iu)+\psi(-\frac{|m|}{2}\pm iu)\mp i\pi$$

with infinite sequence of cuts (here poles) in upper/lower plane

Sister map

It is easy to generalize that to all loops since the flux tube data is expressed in terms of psi-function and its derivatives

Before map : all loop dispersion relation for flux tube gluon [BB'10]

$$E_\ell(u) = \ell + \int_0^\infty \frac{dt}{t} K(t) \left(\cos(ut) e^{-\ell t/2} - 1 \right)$$

$$p_\ell(u) = 2u + \int_0^\infty \frac{dt}{t} K(-t) \sin(ut) e^{-\ell t/2}$$

$$K(t) = \frac{2}{1-e^{-t}} \sum_{n \geq 1} (2n) \gamma_{2n} J_{2n}(2gt) - \frac{2}{e^t - 1} \sum_{n \geq 1} (2n-1) \gamma_{2n-1} J_{2n-1}(2gt)$$

(kernel of BES equation)

After map : all loop sister dispersion relation

[BB,Caron-Huot,Sever'14]

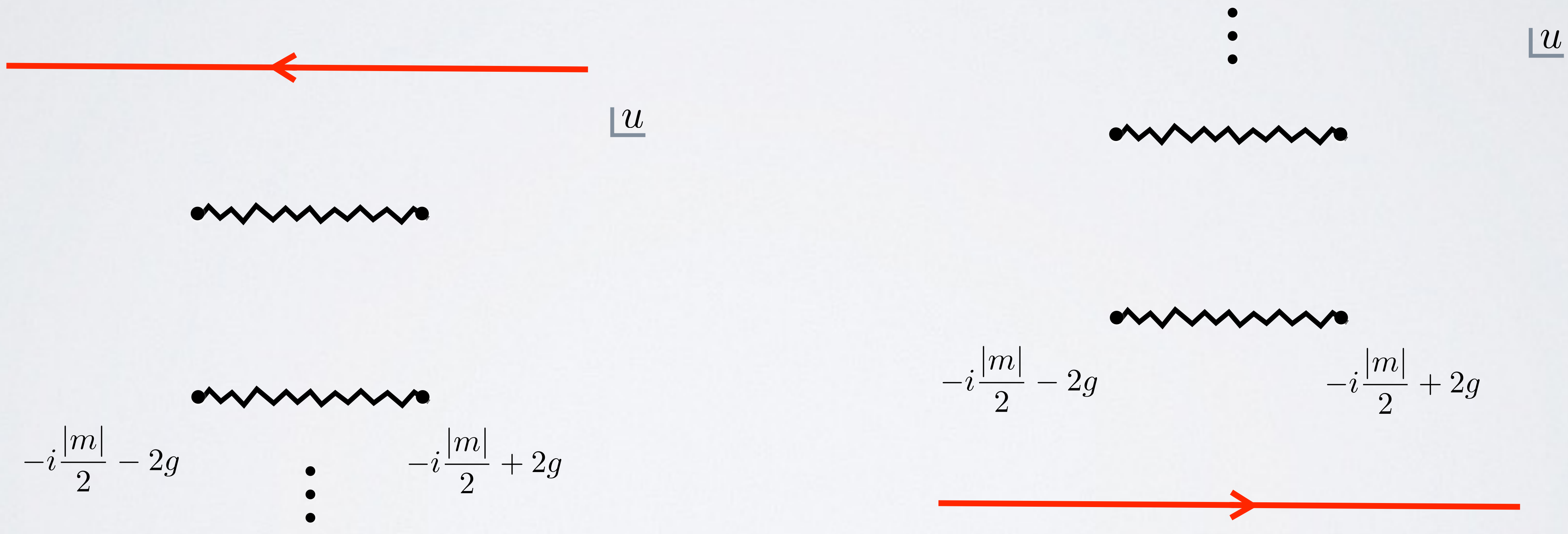
$$\check{E}_\ell(u) = \ell + \frac{i\pi}{2} \Gamma_{\text{cusp}} + \int_0^\infty \frac{dt}{t} \left[K(t) \frac{e^{-iut - \ell t/2} - 2}{2} + K(-t) \frac{e^{-iut + \ell t/2}}{2} \right]$$

$$\check{p}_\ell(u) = 2u + \frac{\pi}{2} \Gamma_{\text{cusp}} - i \int_0^\infty \frac{dt}{t} \left[K(t) \frac{e^{-iut + \ell t/2}}{2} - K(-t) \frac{e^{-iut - \ell t/2}}{2} \right]$$

Sister map

It is easy to generalize that to all loops since the flux tube data is expressed in terms of psi-function and its derivatives

It allows us to write an all loop integral representation for the discontinuity

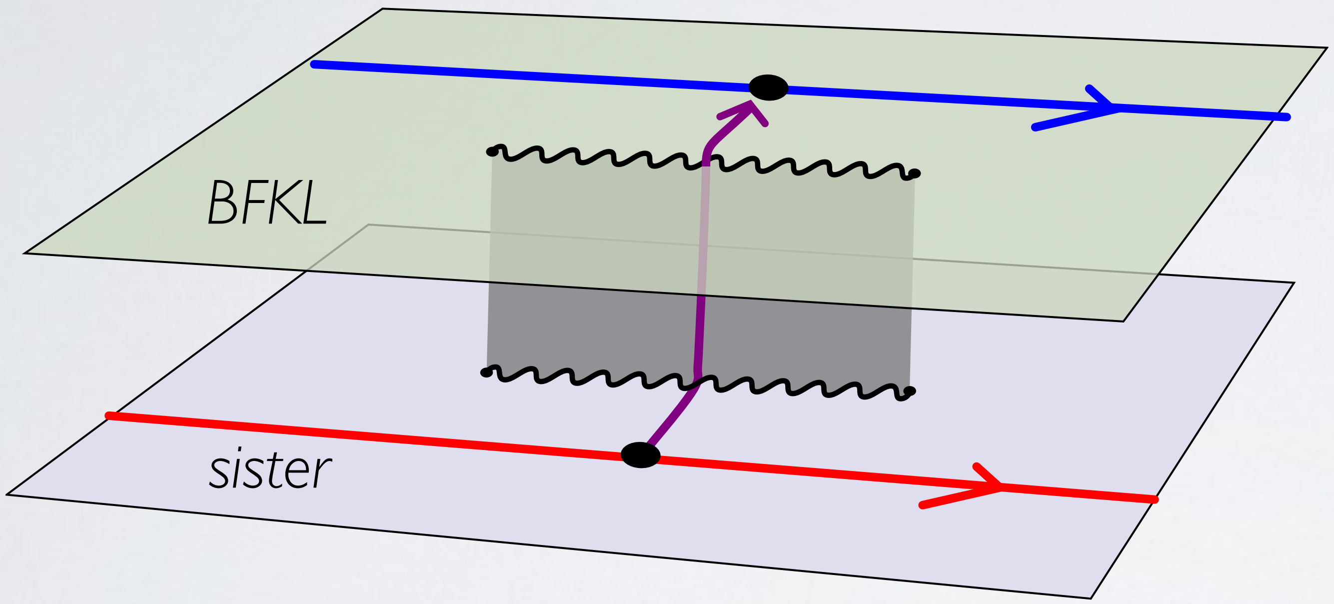


The integrand is just the sister version of the OPE one

Comment : so far all steps can be done order by order in PT

From sister to BFKL

Follow saddle from sister to BFKL regime [BB,Caron-Huot,Sever'14]



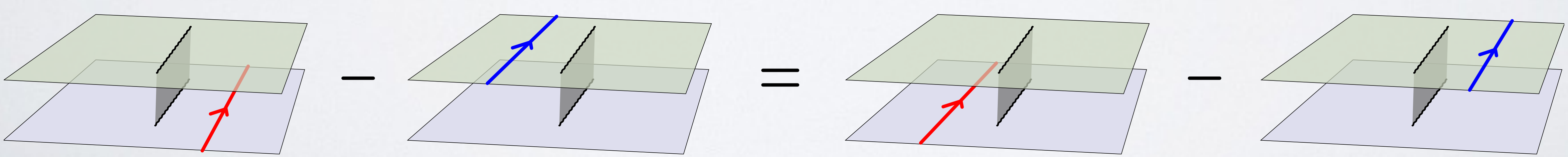
$$\frac{\sigma}{\tau} = \frac{1}{i}\frac{d\check{E}}{dp}(p_*)$$

$$\frac{\tau-\sigma}{\tau+\sigma} = \frac{1}{i}\frac{d\omega}{d\nu}(\nu_*)$$

$$\omega(\nu_*) = \frac{i}{2}\left[p_* + i\check{E}(p_*)\right]$$

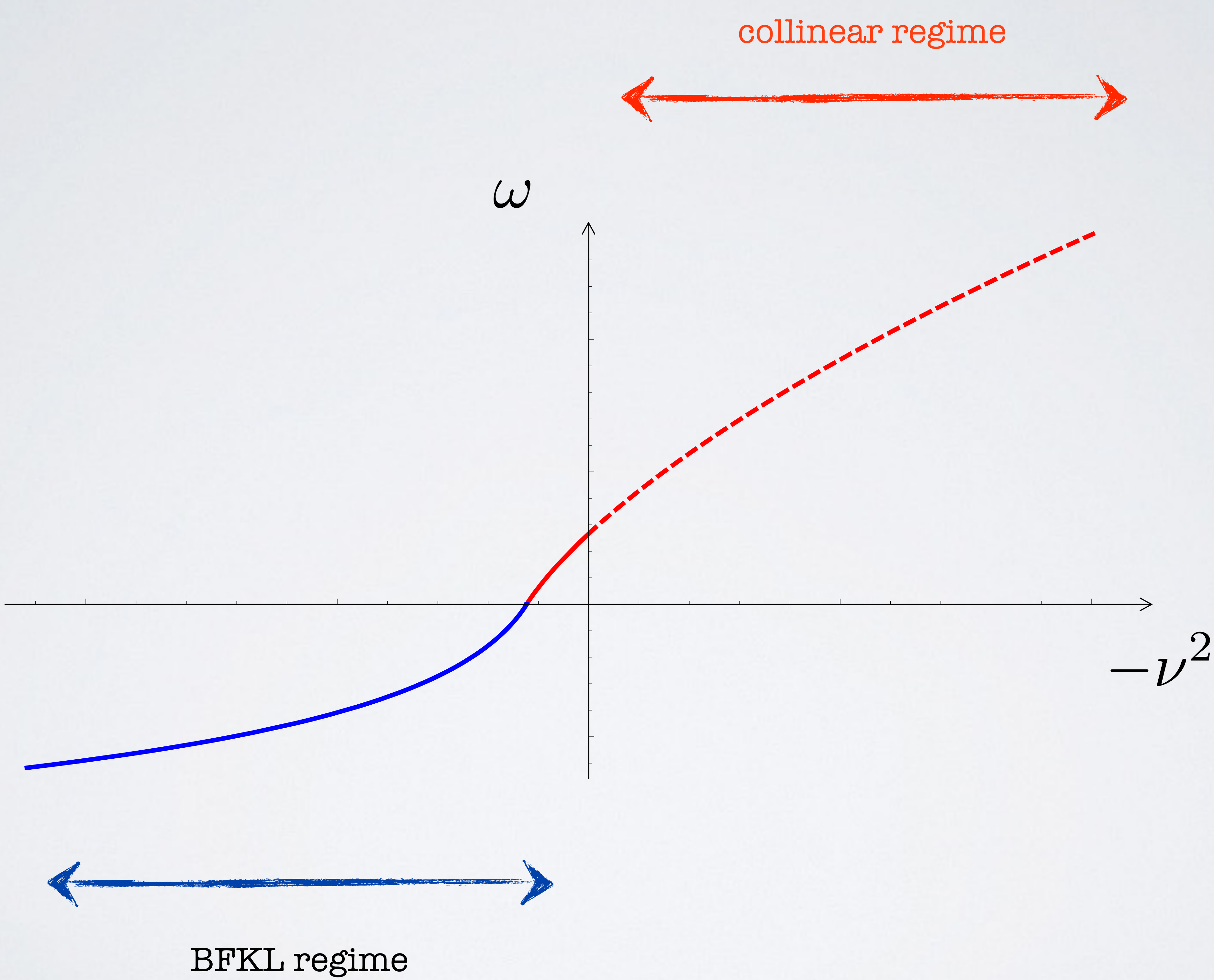
$$\nu_* = \frac{1}{2}\left[p_* - i\check{E}(p_*)\right] - \frac{\pi}{2}\Gamma_{\text{cusp}}$$

Equivalently : wrap contour around the cut and move it to 2nd sheet



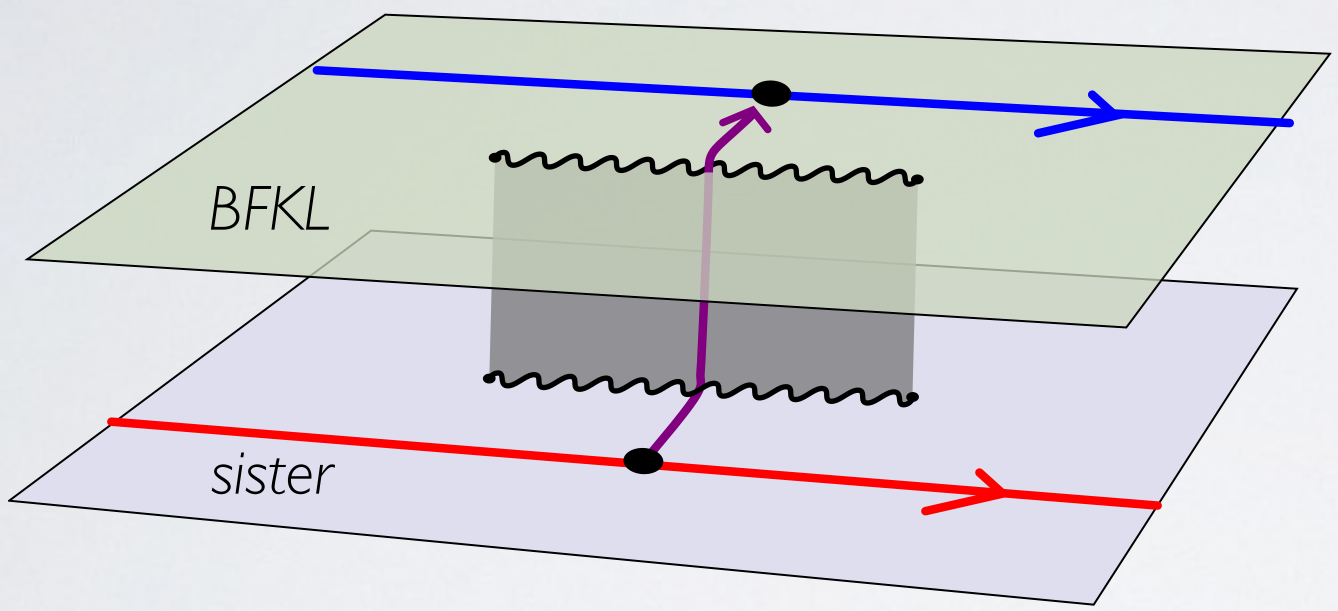
OPE − BFKL = energy suppressed − twist suppressed

Chew-Frautschi plot



From sister to BFKL

Follow saddle from sister to BFKL regime



At finite coupling it is easy to navigate between the two pictures

Give BFKL eigenvalue from the sister flux tube energy to all loops

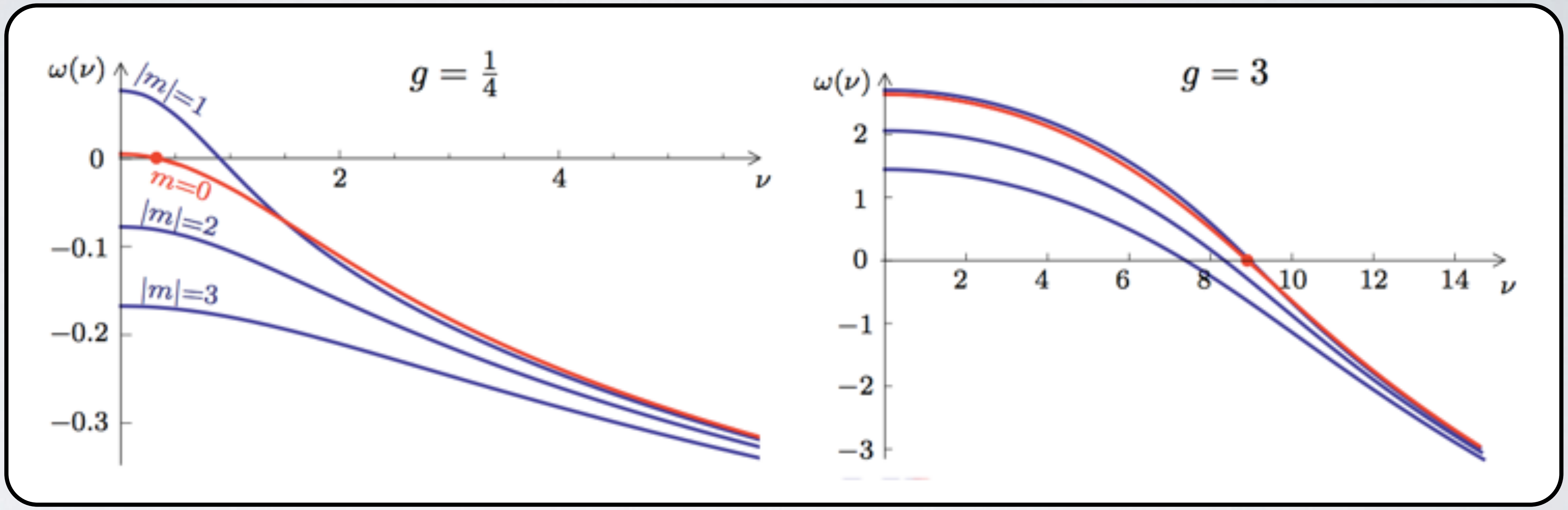
K = BES kernel

$$\omega(u,m) = \int_0^\infty \frac{dt}{t}\left(K(t) - \frac{K(-t)+K(t)}{2}\cos(ut)e^{-|m|t/2}\right)$$

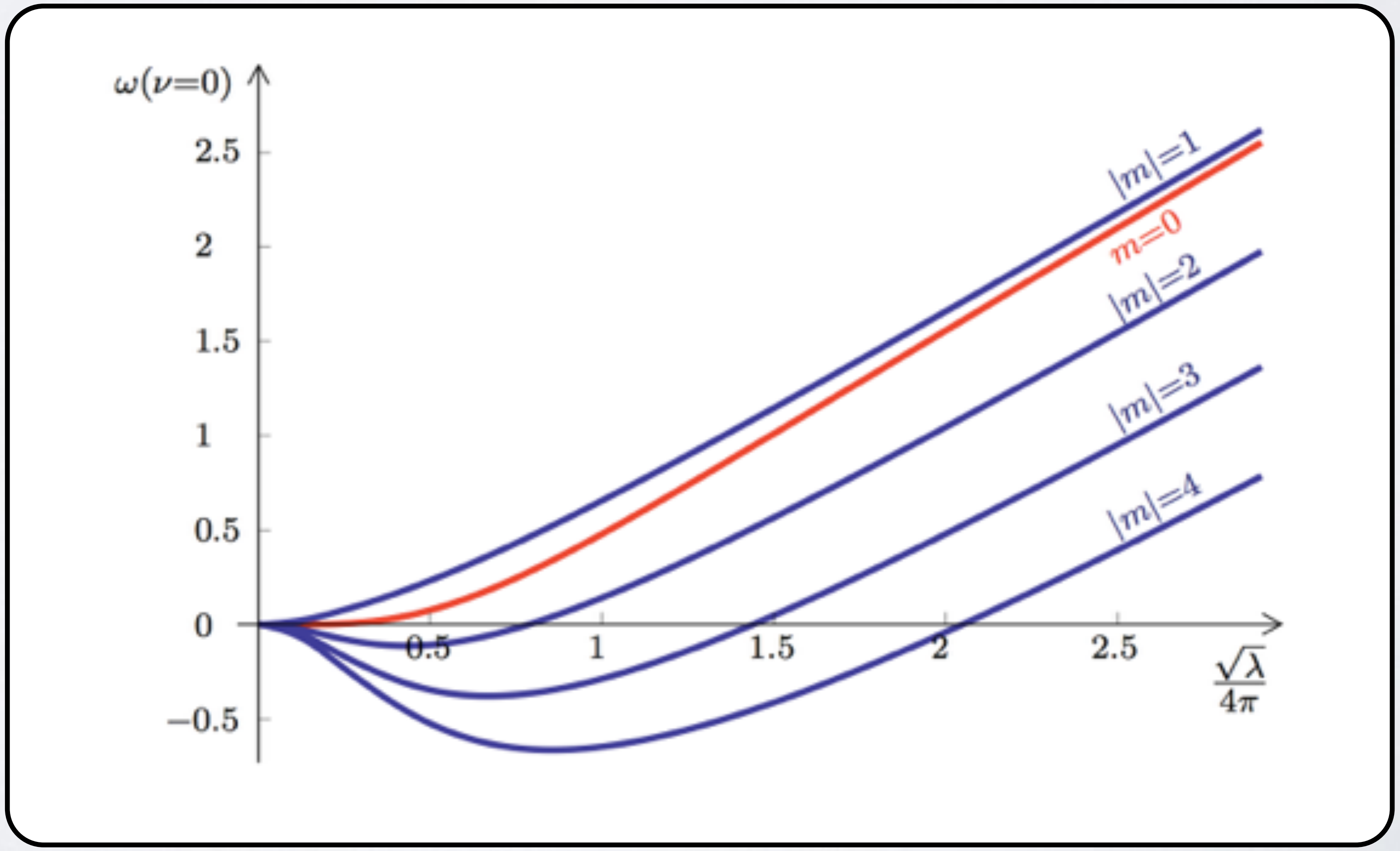
$$\nu(u,m) = 2u + \int_0^\infty \frac{dt}{t}\,\frac{K(-t)-K(t)}{2}\sin(ut)e^{-|m|t/2}$$

Adjoint eigenvalues at finite coupling

Eigenvalues :

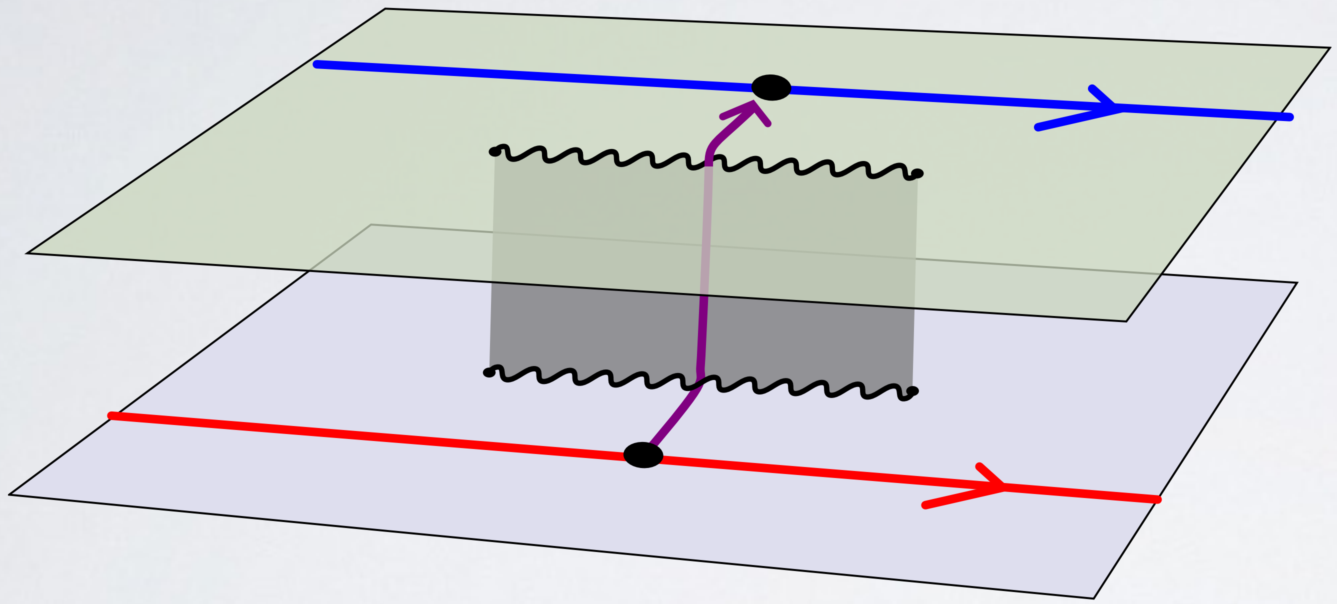


Intercepts :



From sister to BFKL

Follow saddle from sister to BFKL regime



At finite coupling
it is easy to
navigate between
the two pictures

Same applies to measure a.k.a
impact factor

$$\mu_m^{\mathrm{OPE}}(u) \quad \to \quad \mu_m^{\updownarrow}(u) \quad \to \quad \mu_m^{\mathrm{BFKL}}(u)$$

Heptagon

OPE pentagon transition

$$\int \frac{du dv}{(2\pi)^2} \mu(u)\mu(v) e^{-ip(u)\sigma_1 + ip(v)\sigma_2} P(u - i0|v + i0)$$

Gluon transition

$$P(u|v) = -\frac{(\frac{1}{2} - iu)\Gamma(iu - iv)(\frac{1}{2} + iv)}{g^2 \Gamma(\frac{1}{2} + iu)\Gamma(\frac{1}{2} - iv)}$$

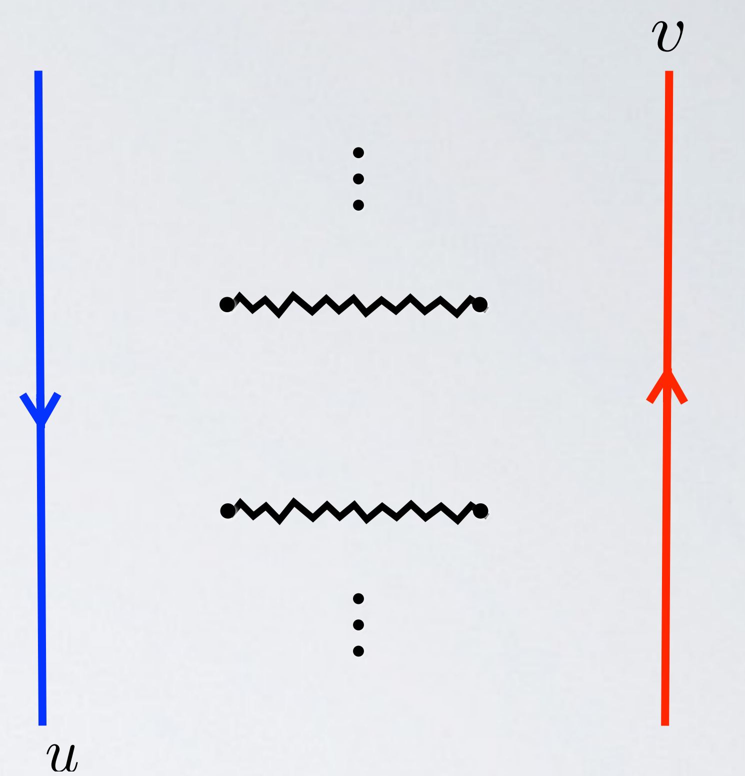
Position space

$$P(\sigma_1|\sigma_2) = \frac{e^{\sigma_1 + \sigma_2}}{2} \log \frac{\left(e^{2\sigma_1} + 1\right)\left(e^{2\sigma_2} + 1\right)}{e^{2\sigma_1} + e^{2\sigma_2} + e^{2\sigma_1 + 2\sigma_2}}$$
$$+ e^{\sigma_2 - \sigma_1} \log \frac{e^{2\sigma_2}\left(e^{2\sigma_1} + 1\right)}{e^{2\sigma_1} + e^{2\sigma_2} + e^{2\sigma_1 + 2\sigma_2}} + (\sigma_1 \leftrightarrow \sigma_2)$$

Heptagon

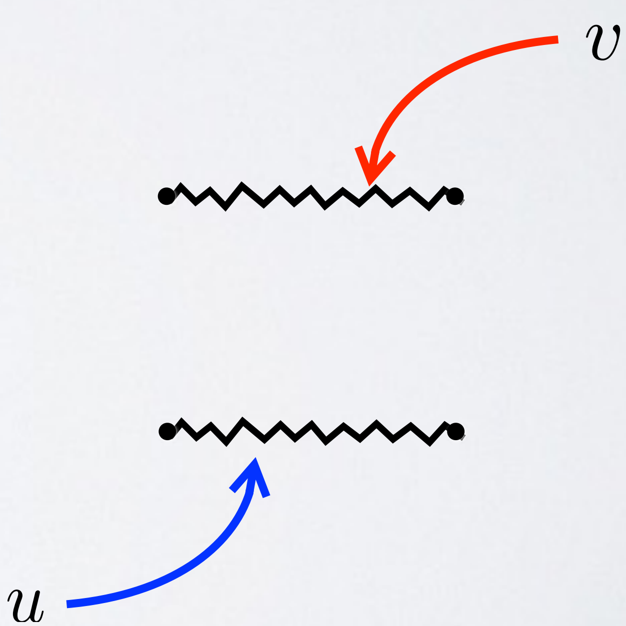
Double shift

$$\sigma_i \to \sigma_i - \frac{i\pi}{2}$$



We should “sister map both of them” and take Regge limit

$$\sigma_i \to -\infty \qquad \tau_i \to \infty \qquad \tau_i + \sigma_i \ \text{fixed}$$



We get the Regge pentagon a.k.a central emission vertex at an coupling

known at weak coupling from [Bartels,Kormilitzin,Lipatov,Prygarin'12]

Regge pentagon

Structure is essentially the same as for pentagon transitions

$$\text{tree} \times \exp\left[\text{bilinears in } \psi \text{ functions and derivatives}\right]$$

Few important properties :

- Decoupling pole

$$P(u|v) \sim \frac{1}{i\mu(u)(u-v)}$$

- Reggeon zeroes for mode zero

$$\lim_{\nu(u)\to\frac{\pi}{2}\Gamma_{\text{cusp}}} P(u|v) = 0$$

$$\lim_{\nu(v)\to-\frac{\pi}{2}\Gamma_{\text{cusp}}} P(u|v) = 0$$

Conclusions

Conclusion

Regge and OPE regimes are the two sides of a same story

BFKL and collinear eigenvalues the two “branches” of a same function

This only becomes manifest and fully tractable at finite coupling

Crossing the kinematics then becomes equivalent to crossing a cut in internal momentum / rapidity plane

Conclusion

Clear route from OPE to Regge; OPE / Regge dictionnary

Following it we can derive the eigenvalue, impact factor and emission vertex directly from the OPE / pentagon data at any coupling

Pushing the analogy further hints at higher cuts with totally factorized structure, like for multiparticle pentagon transitions

Many questions remain : Completeness of states? Can new states appear for n>=8? Can we go all the way from collinear to Regge for higher n-gon? Are there Regge islands we cannot reach?

Possible tests

String coupling saddle point for higher n-gon : looks doable; we could test the factorizability and the presence or not of new stuff (bound states of n>2 Reggeons)

heptagon strong coupling study : [Bartels,Schomerus,Sprenger'14]

3-reggeons at weak coupling : 2 loop 8 points; comparison with Simon's symbol? or with a function?

recent progress : [Bargheer,Papathanasiou,Schomerus'15], [Bargheer'16],[Broedel,Sprenger,Torres Orjuela'16], [Del Duca et al.'16]

N-reggeons analysis using integrable spin chain?

[Bartels,Lipatov,Prygarin'11]

Expect structure of multi-particle wave functions and transitions to be the same as collinear limit

[BB,Sever,Vieira'13]

[Belitsky,Derkachov,Manashov'14]

THANK YOU!