A Lagrangian for Factorization Violation and Forward Scattering

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with Ira Rothstein (arXiv:1601.04695)
+ ongoing work

Workshop on Iterated integrals and the Regge limit
Higgs Center, Edinburgh, April 2017

Outline

- Introduction to Factorization and
  Soft-Collinear Effective Theory (SCET)

- Fact. Violation: Glauber Operators & Forward Scattering
  Complete Leading Power Glauber Interaction Lagrangian

- Fwd. Scattering: Regge and BFKL (Rapidity RGE)
  Exponentiation & Eikonalization

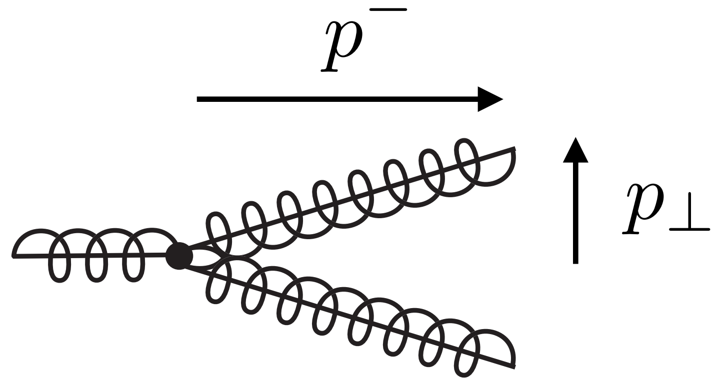
- Fact. Violation: Wilson Line Directions & "Cheshire Glauber"
  Glauber Effects with "Spectator" Partons

- Work in progress: quark Reggeization, small-x DIS

Introduction

Relevant Momentum Regions:

- Collinear Splittings

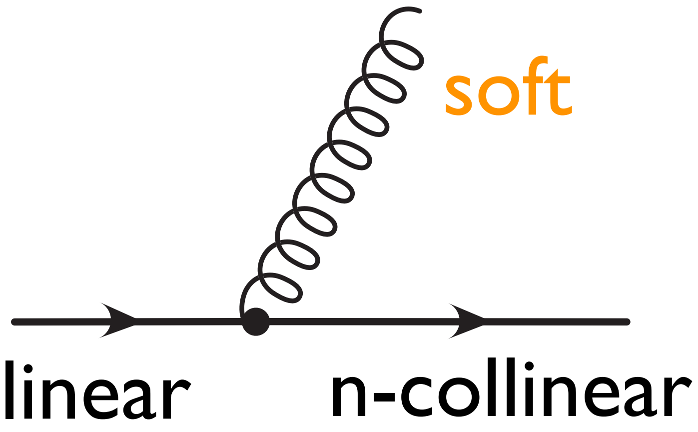


"n-collinear"

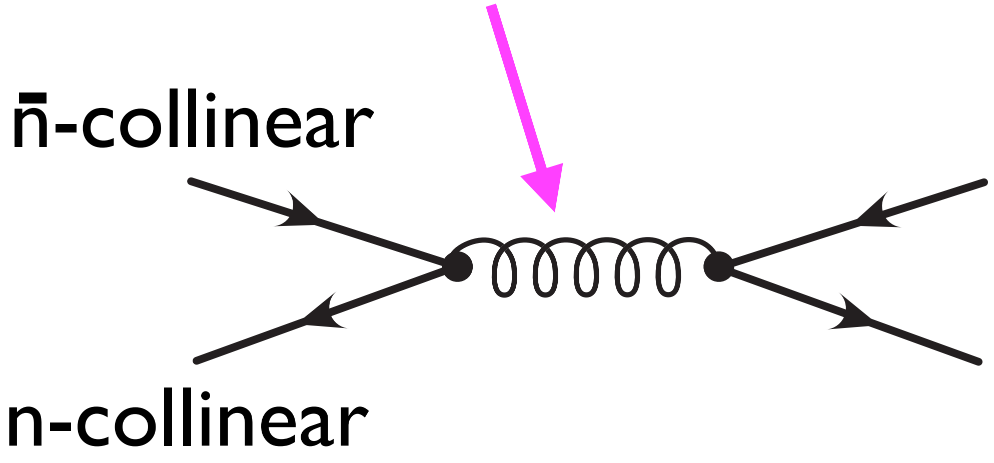
$$p^- \gg p_\perp \gg p^+$$

onshell: $p^+p^- = \vec{p}_\perp^{\,2}$

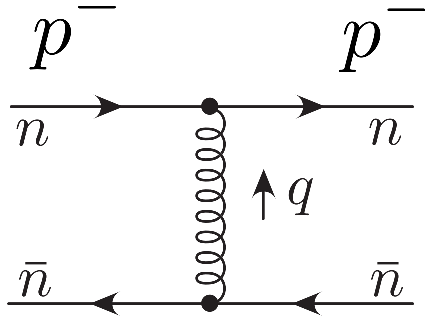
- Soft Emission



- Hard Propagators (short dist.)

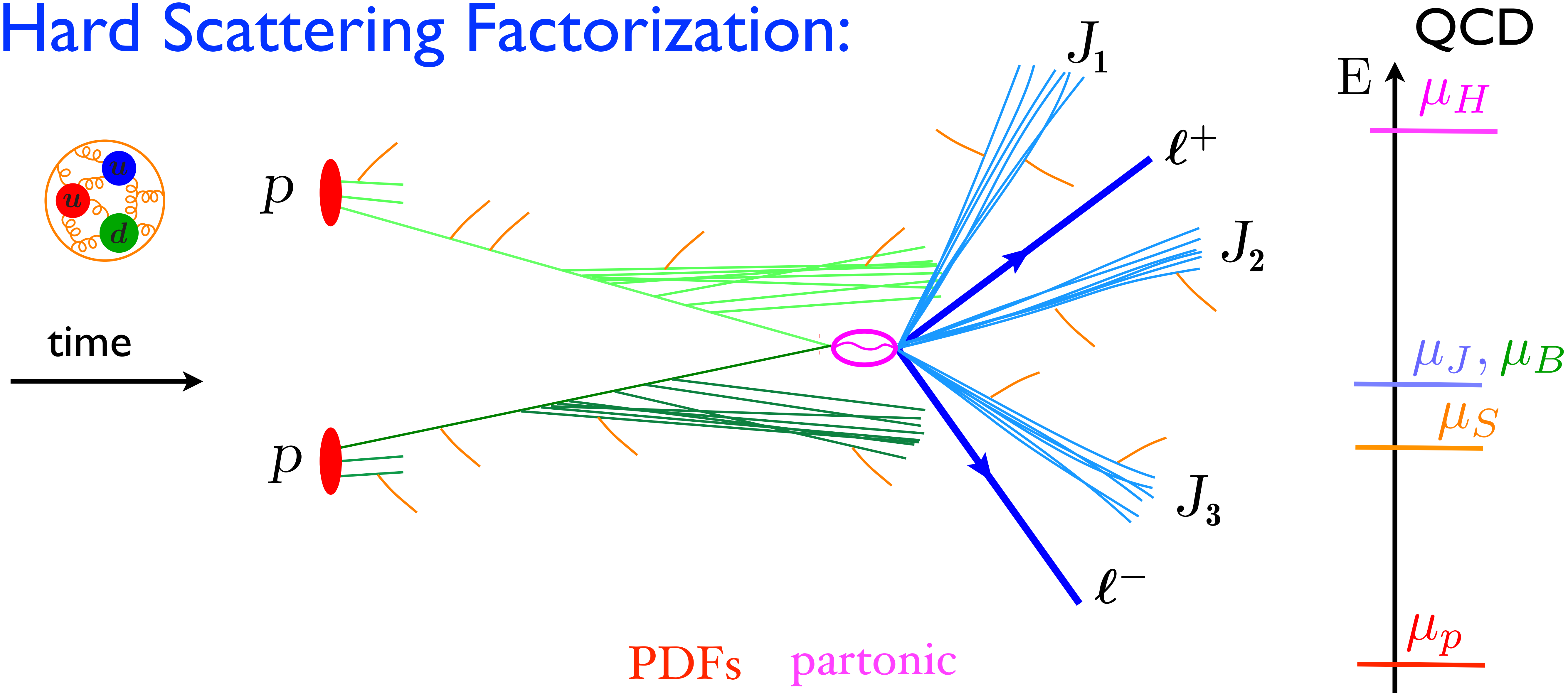


- Glauber Exchange



forward scattering

Hard Scattering Factorization:



PDFs partonic

Nonperturbative: $d\sigma = f_a f_b \otimes \hat{\sigma} \otimes F$ ← hadronization

$\mu_p \simeq \Lambda_{\rm QCD}$

(In some cases by Operators, or is power suppressed)

eg. Perturbative: $\hat{\sigma}_{\rm fact} = \mathcal{I}_a \mathcal{I}_b \otimes H \otimes \prod_i J_i \otimes S$

Used to Sum Logs

Examples of Factorization:

- Inclusive Higgs production $pp \to \text{Higgs} + \text{anything}$

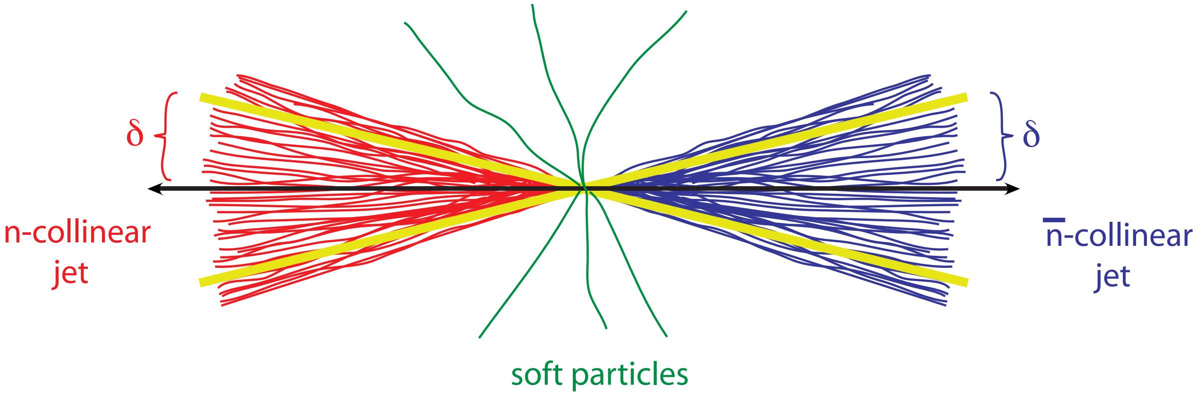
$$d\sigma = \int dY \sum_{i,j} \int \frac{d\xi_a}{\xi_a}\frac{d\xi_b}{\xi_b} f_i(\xi_a,\mu) f_j(\xi_b,\mu)\; H_{ij}^{\text{incl}}\Big(\frac{m_H e^{Y}}{E_{\text{cm}}\xi_a}, \frac{m_H e^{-Y}}{E_{\text{cm}}\xi_b}, m_H, \mu\Big)$$

(CSS = Collins, Soper, Sterman)

(PDFs contribute, No Glaubers, No Softs)

- Dijet production $e^+e^- \to 2\ \text{jets}$

thrust $\tau \ll 1$



$$\frac{d\sigma}{d\tau} = \sigma_0 H(Q,\mu)\, Q \int d\ell\, d\ell'\; J_T(Q^2\tau - Q\ell, \mu) S_T(\ell - \ell', \mu) F(\ell')$$

hard function

jet functions

perturbative soft function

non-perturbative soft function

(No PDFs, No Glaubers, Softs contribute)

- Hard Amplitude Factorization (Quarks)

$$\langle X_1 \cdots X_N; X_s|i\rangle \cong \mathcal{C}(P_i) \frac{\langle X_1|\bar\psi W_1|0\rangle}{\mathrm{tr}\,\langle 0|Y_1^\dagger W_1|0\rangle} \cdots \frac{\langle X_N|W_N^\dagger \psi|0\rangle}{\mathrm{tr}\,\langle 0|W_N^\dagger Y_N|0\rangle} \langle X_s|Y_1^\dagger \cdots Y_N|0\rangle$$

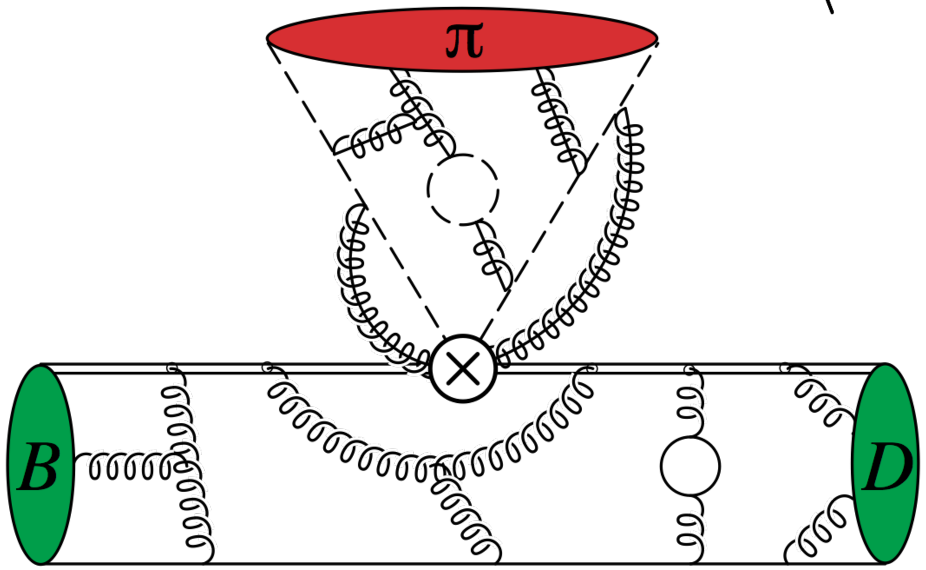
Direct Proof with well separated
Final State Collinear Particles

(Feige, Schwartz)

Simple to prove in SCET if we assume Glaubers are absent

- Exclusive Factorization

$$\langle D\pi|(\bar c b)(\bar u d)|B\rangle = N\,\xi(v\cdot v')\int_0^1 dx\, T(x,\mu)\,\phi_\pi(x,\mu)$$



soft $\langle D^{(*)}|O_s|B\rangle = \xi(v\cdot v')$

n-collinear $\langle \pi|O_n(x)|0\rangle = f_\pi \phi_\pi(x)$

- Higgs with a Jet Veto $p_T^{\rm jet} \le p_T^{\rm cut} \ll m_H$

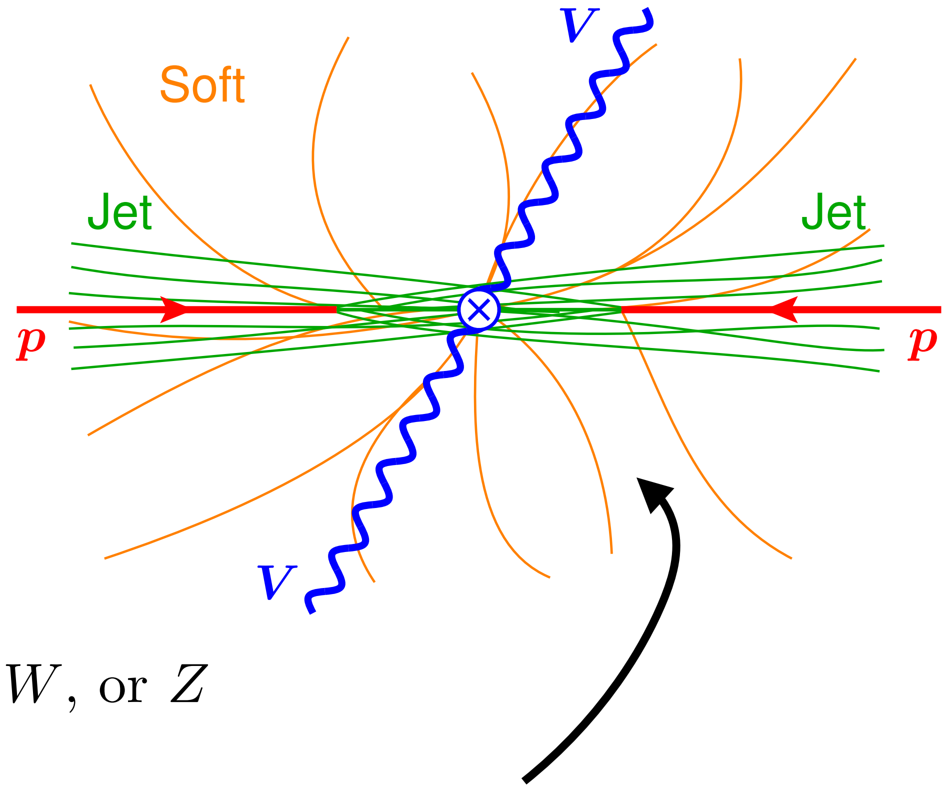
  (anti-kT jets, radius R) $\Lambda_{\rm QCD} \ll p_T^{\rm cut}$

$$\boldsymbol{\sigma_0(p_T^{\rm cut}) = H_{gg}(m_H) \times [B_g(m_H, p_T^{\rm cut}, R)]^2}$$

$$\boldsymbol{\times\, S_{gg}(p_T^{\rm cut}, R)}$$

$$B_g = \mathcal{I}_{gj}(m_H, p_T^{\rm cut}, R) \otimes f_j$$

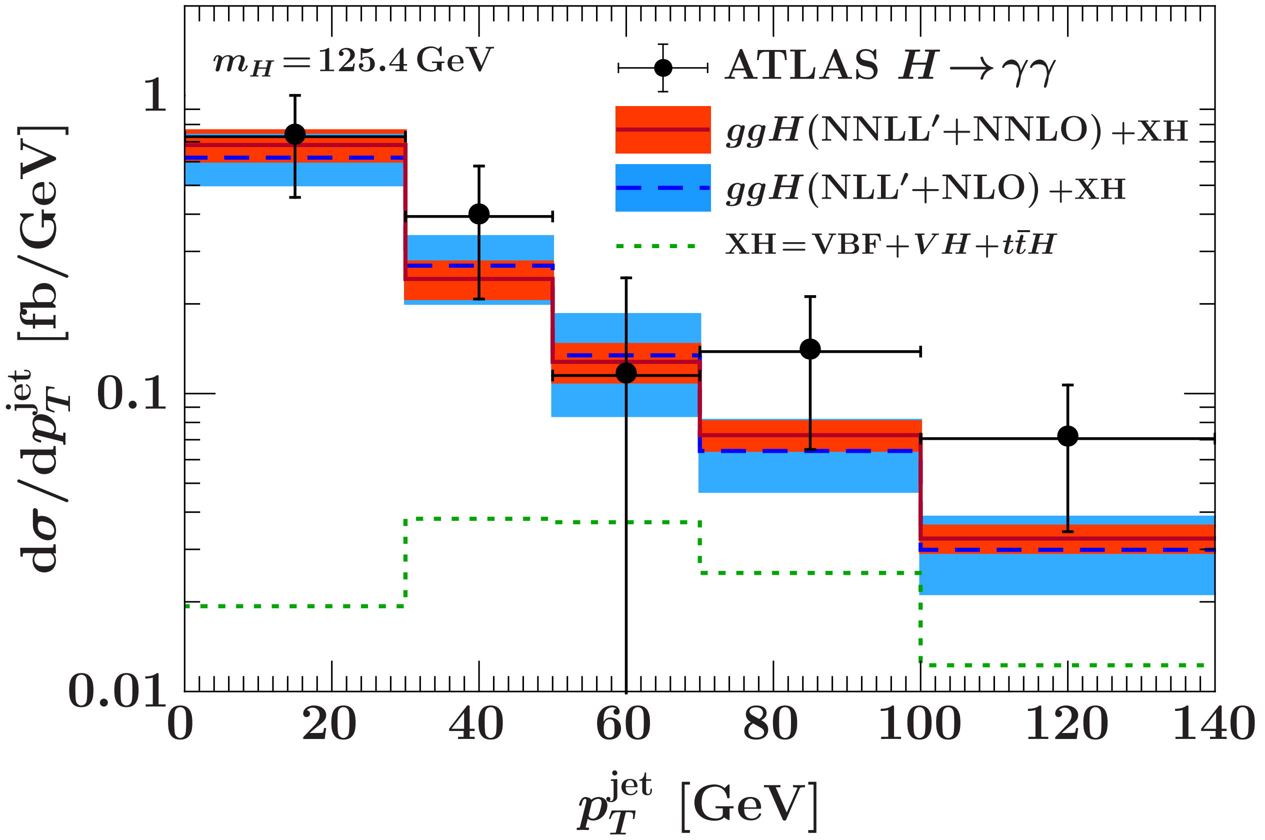
(PDFs and Softs contribute, Glaubers?)



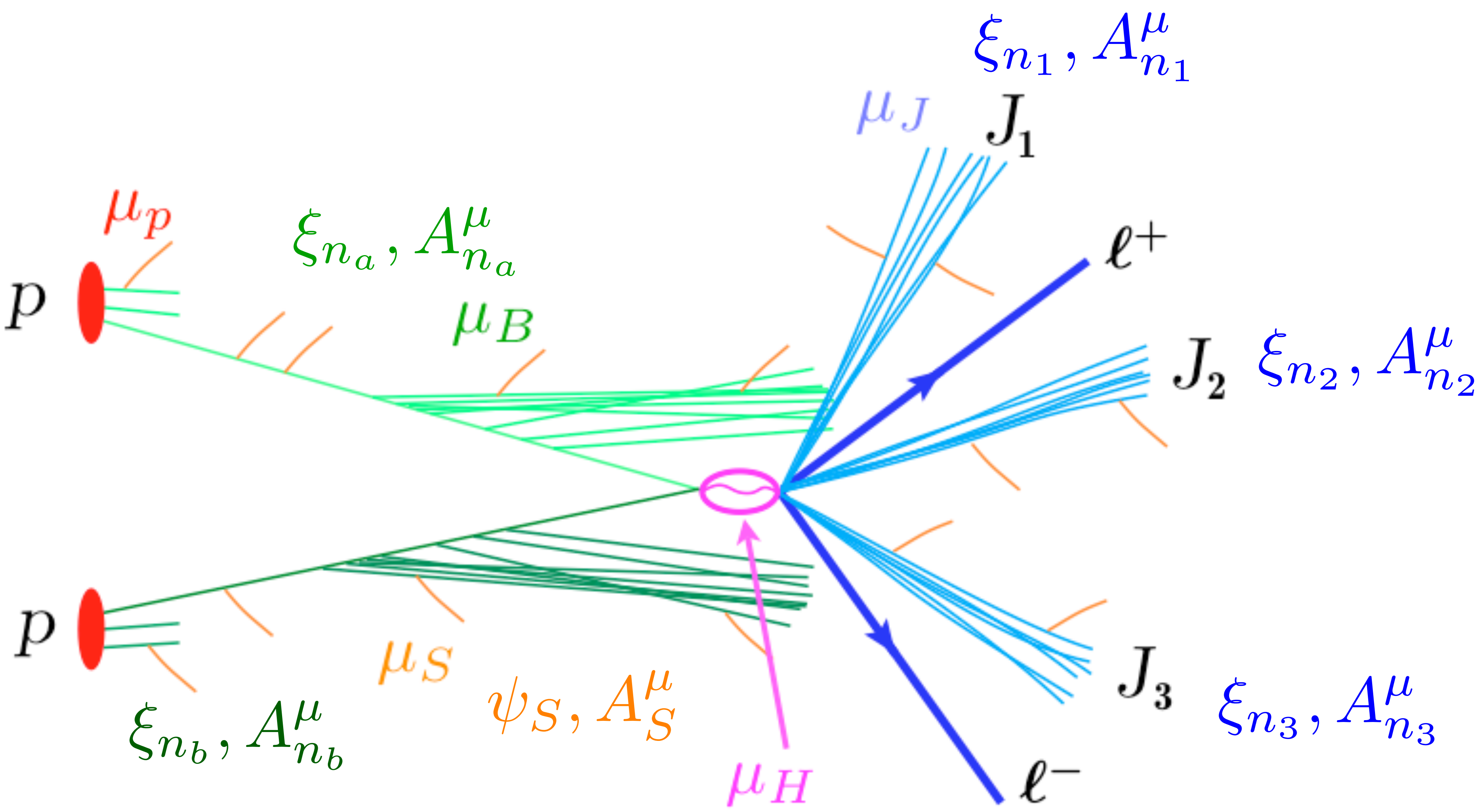
$\mathrm{V}= \gamma,\, W,$ or Z

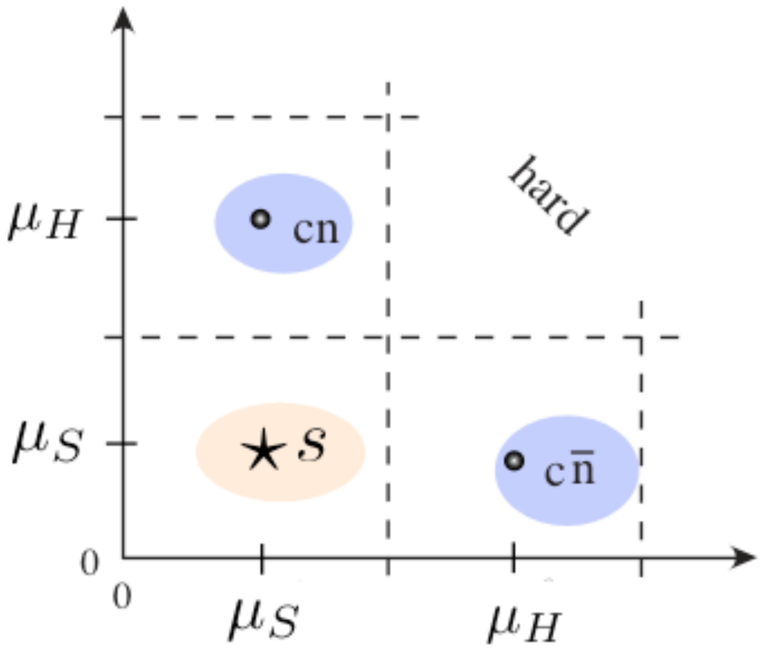
veto anti-kT jets here

data (run-1)



SCET Fields for various Modes





- dominant contributions from isolated regions of momentum space
- use subtractions rather than sharp boundaries to preserve symmetry

Relevant Modes

$\lambda \ll 1$ large Q

Infrared Structure of Amplitudes (Landau eqtns, CSS, ...)

Method of Regions (Beneke & Smirnov)

mode	fields	p^μ momentum scaling	physical objects
n_a-collinear	$\xi_{n_a},\ A^\mu_{n_a}$	$(n_a\cdot p, \bar{n}_a\cdot p, p_{\perp a}) \sim Q(\lambda^2, 1, \lambda)$	collinear initial state jet a
n_b-collinear	$\xi_{n_b},\ A^\mu_{n_b}$	$(n_b\cdot p, \bar{n}_b\cdot p, p_{\perp b}) \sim Q(\lambda^2, 1, \lambda)$	collinear initial state jet b
n_j-collinear	$\xi_{n_j},\ A^\mu_{n_j}$	$(n_j\cdot p, \bar{n}_j\cdot p, p_{\perp j}) \sim Q(\lambda^2, 1, \lambda)$	collinear final state jet in $\hat{n}_j$
soft	$\psi_S,\ A^\mu_S$	$p^\mu \sim Q(\lambda, \lambda, \lambda)$	soft virtual/real radiation
ultrasoft	$\psi_{us},\ A^\mu_{us}$	$p^\mu \sim Q(\lambda^2, \lambda^2, \lambda^2)$	ultrasoft virtual/real radiation
Glauber	–	$p^\mu \sim Q(\lambda^a, \lambda^b, \lambda),\ a+b>2$	forward scattering potential
hard	–	$p^2 \gtrsim Q^2$	hard scattering

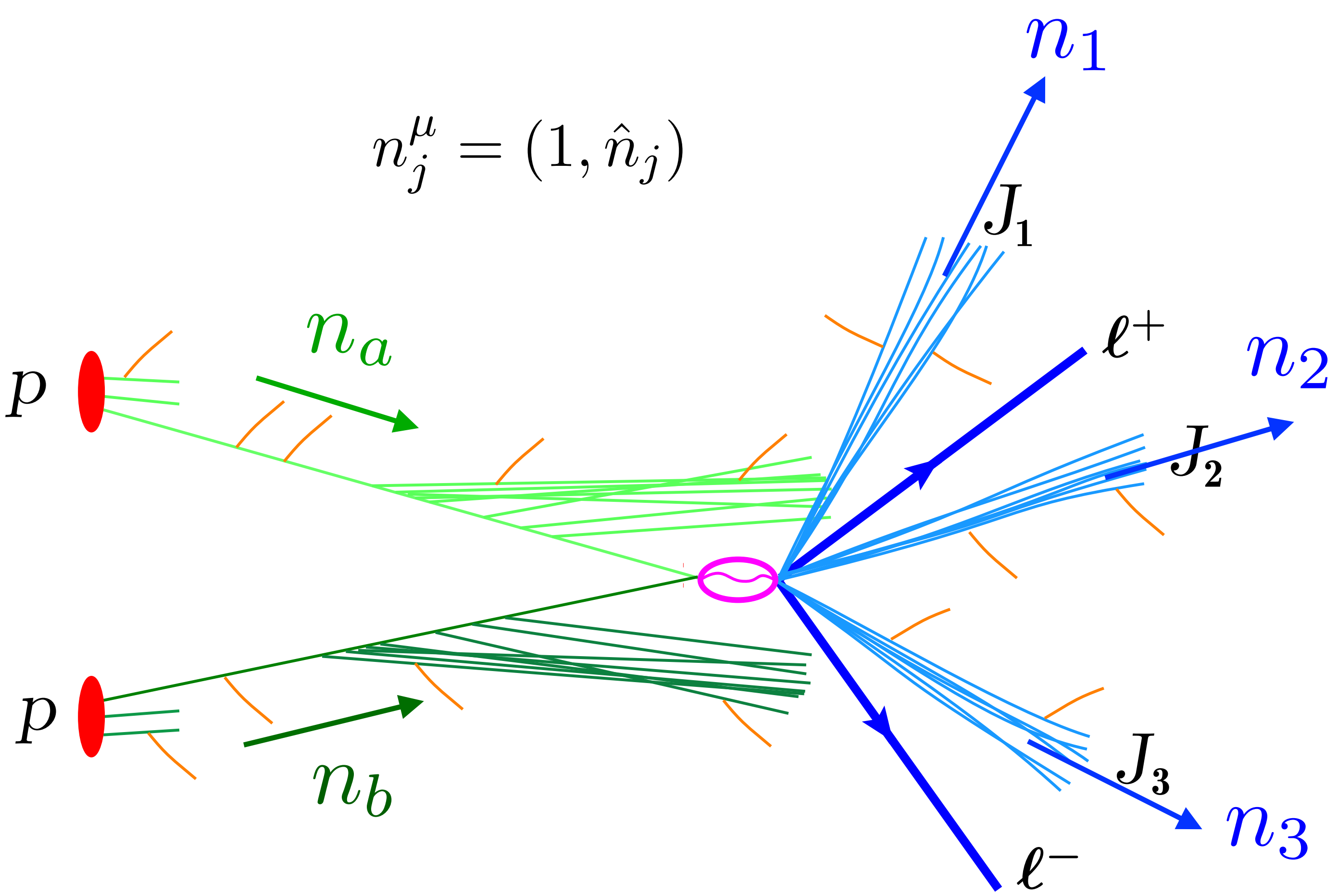
$$p^\mu = \bar{n}_i\cdot p\,\frac{n_i^\mu}{2} + n_i\cdot p\,\frac{\bar{n}_i^\mu}{2} + p_\perp^\mu$$

$$n_i^2 = 0$$

$$\bar{n}_i^2 = 0$$

$$n_i\cdot\bar{n}_i = 2$$

$$n_j^\mu = (1, \hat{n}_j)$$



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Glauber	–	$p^\mu \sim Q(\lambda^a, \lambda^b, \lambda)$, $a + b > 2$	forward scattering potential	offshell
hard	–	$p^2 \gtrsim Q^2$	hard scattering	$\boxed{\text{offshell}}$

Integrate out these modes

$$p^\mu = \bar{n}_i \cdot p \, \frac{n_i^\mu}{2} + n_i \cdot p \, \frac{\bar{n}_i^\mu}{2} + p_\perp^\mu$$

$$n_i^2 = 0$$

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$$n_i \cdot \bar{n}_i = 2$$

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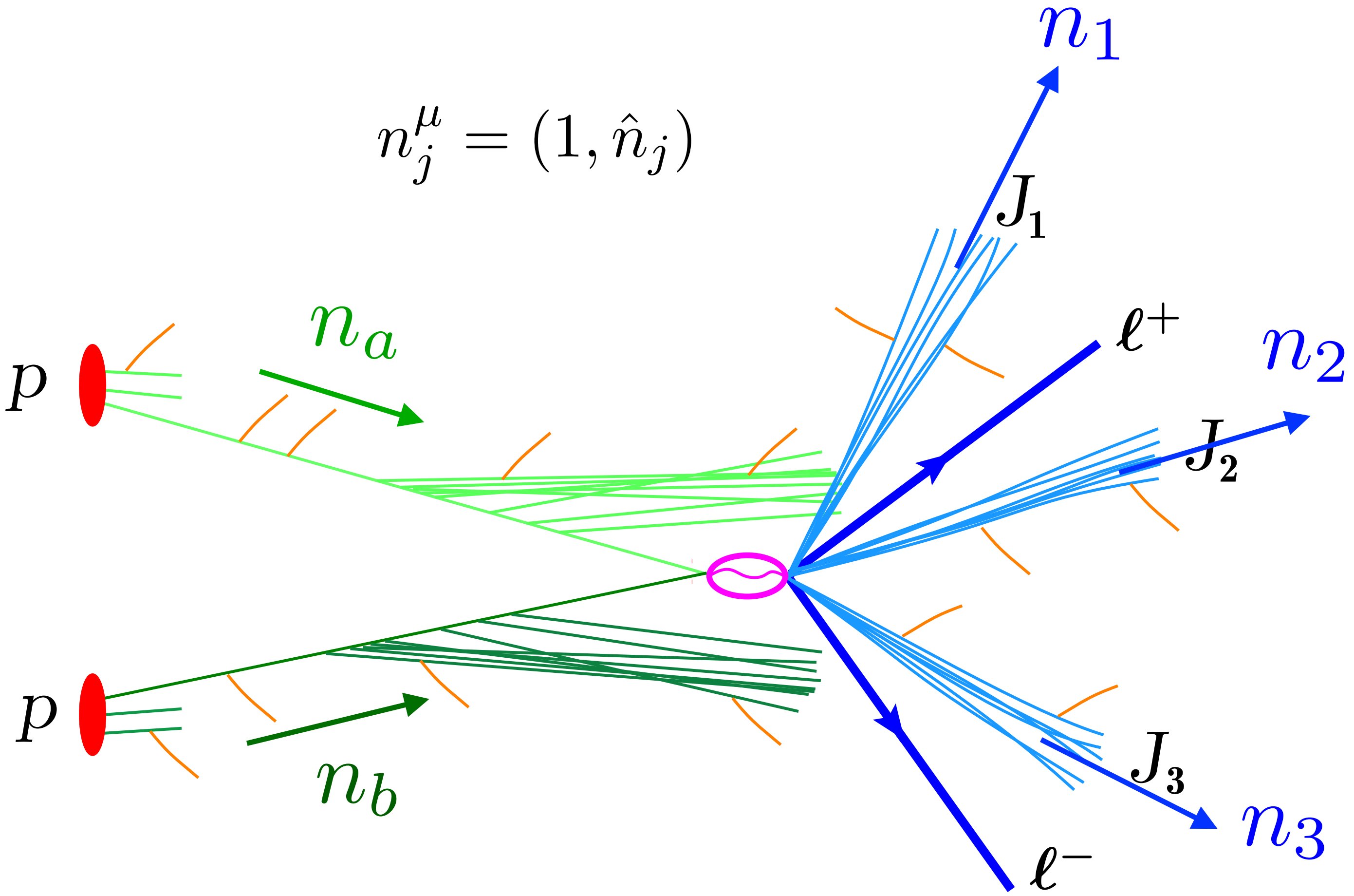
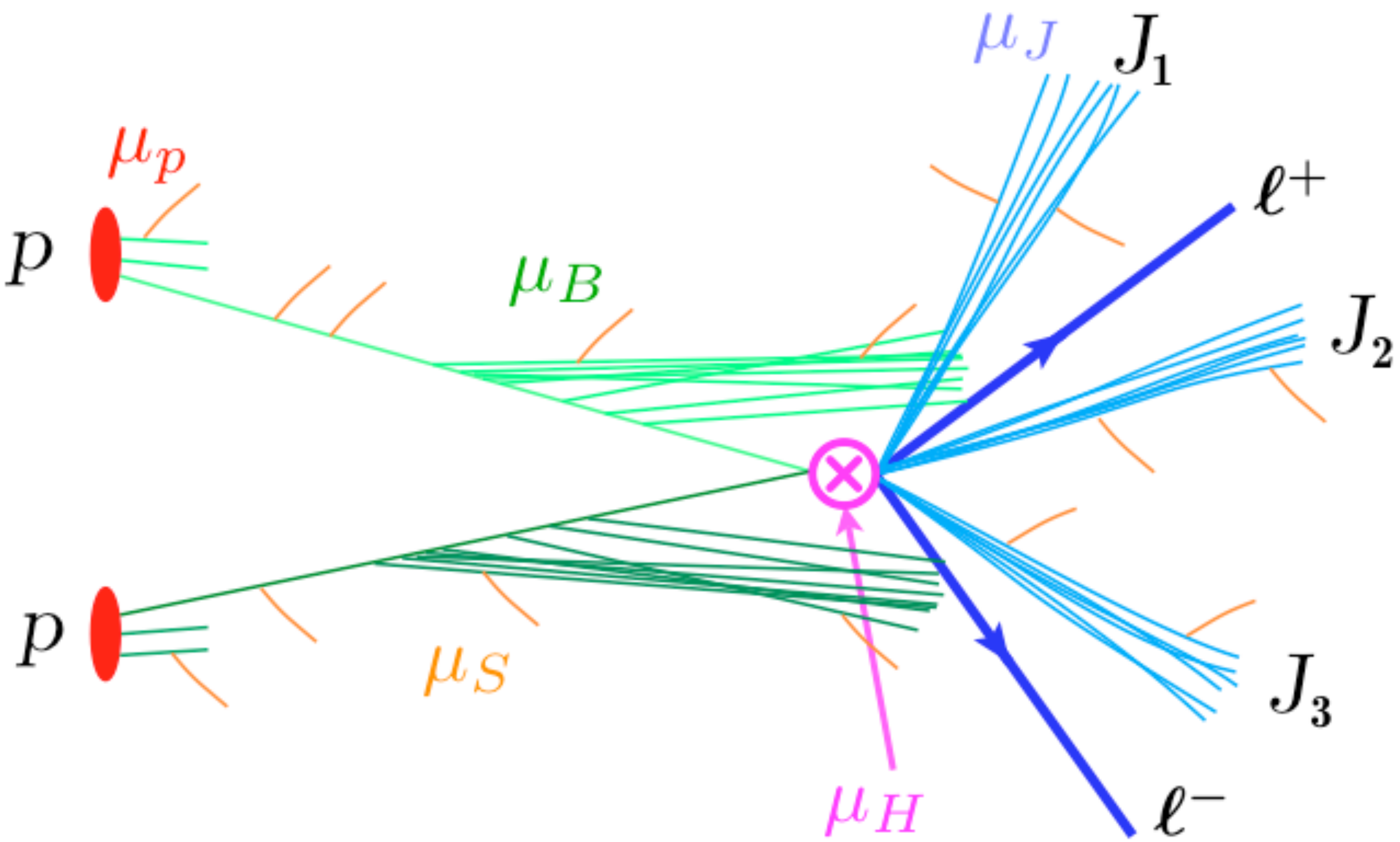
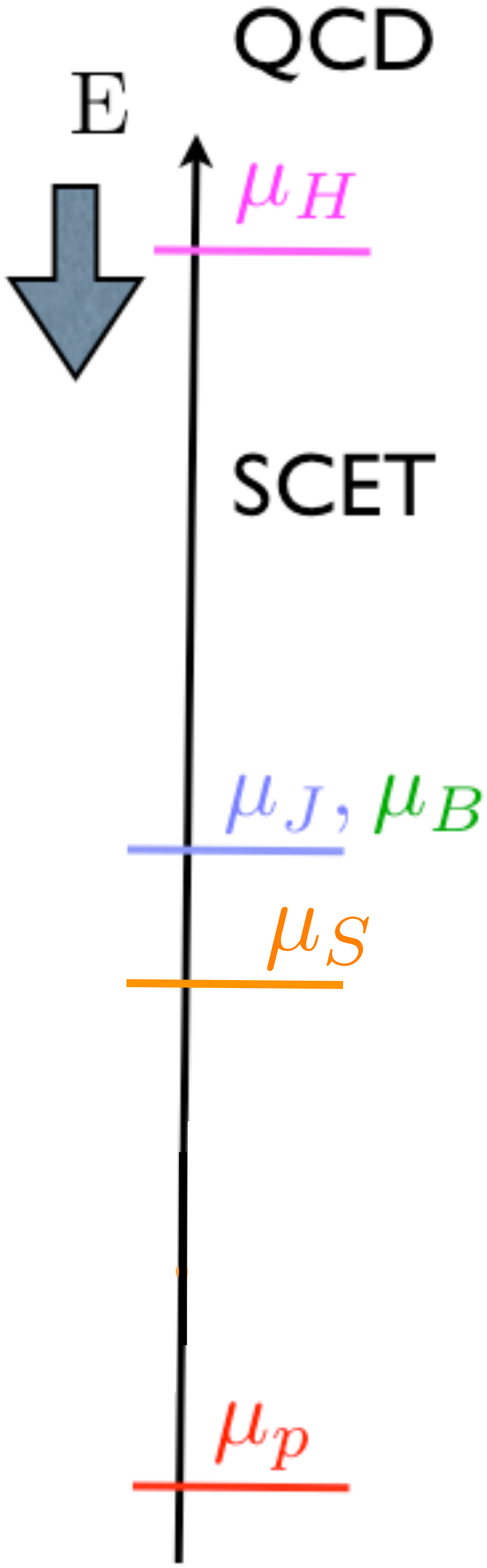


Hard-collinear factorization

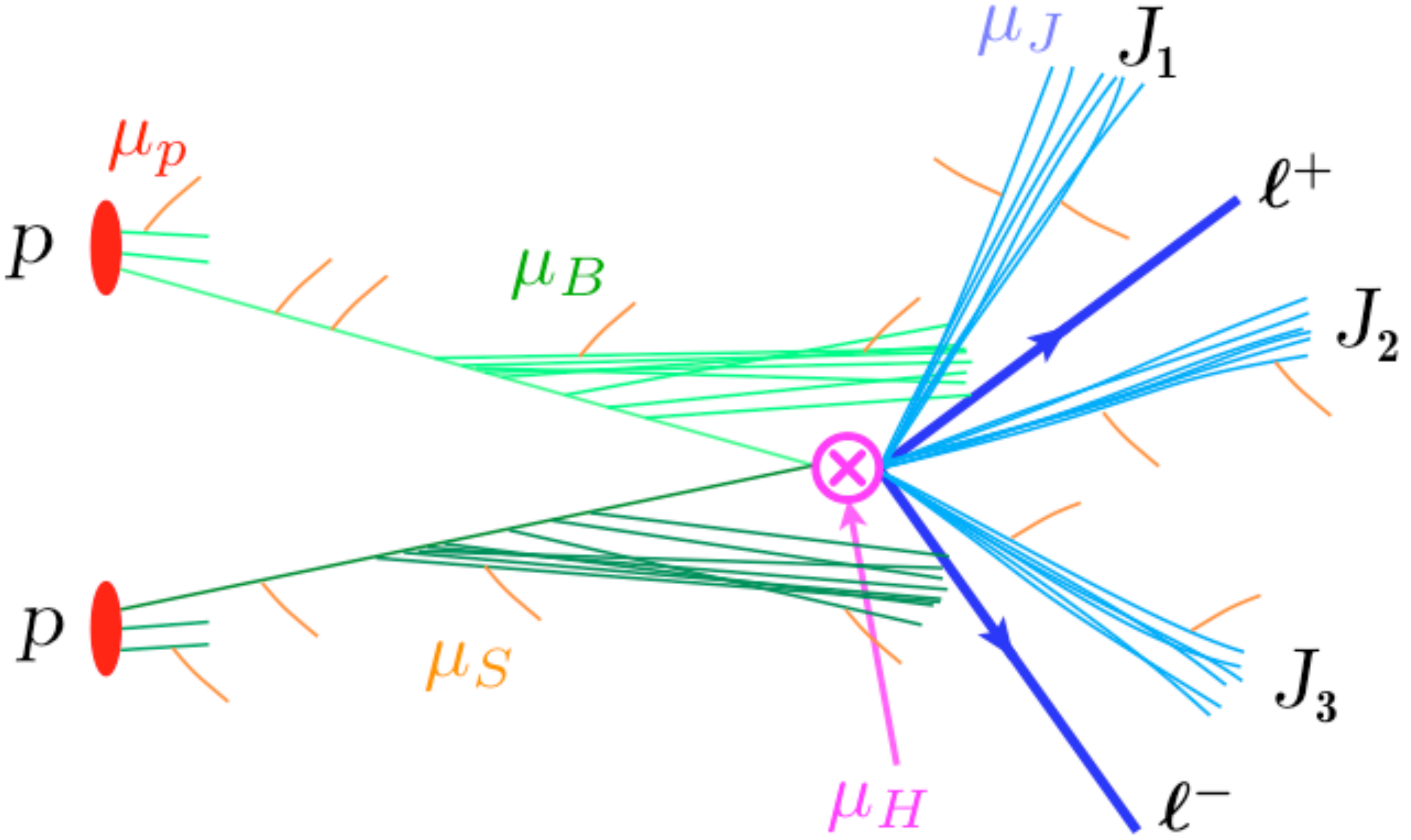


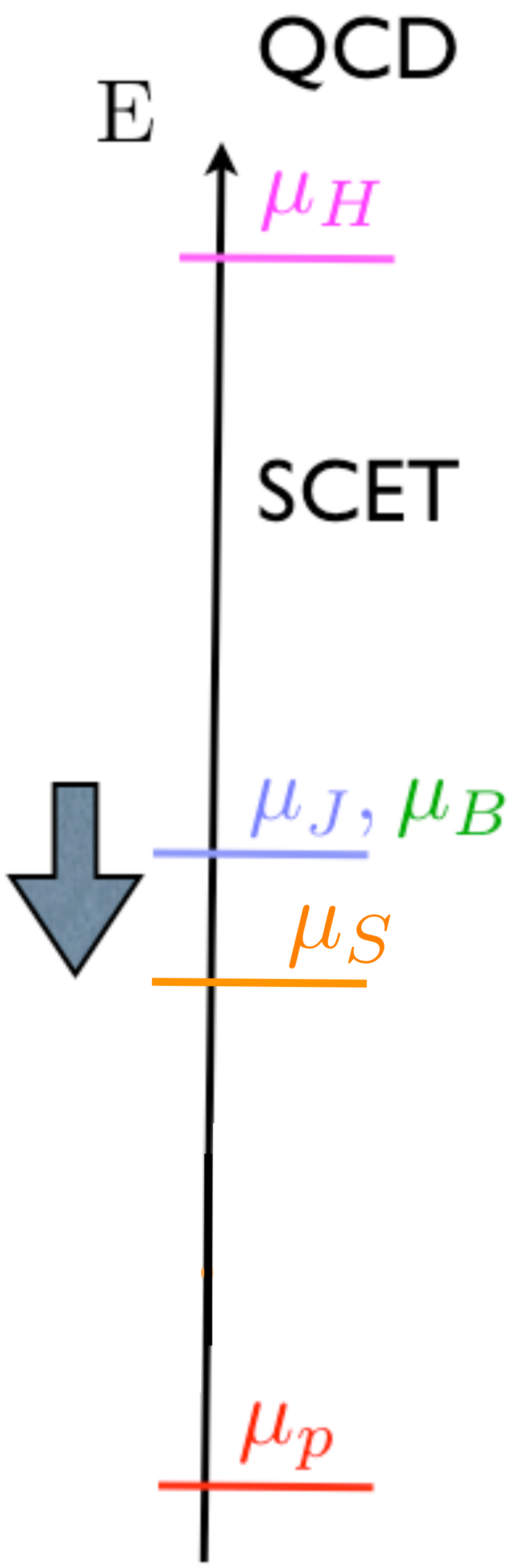


μ_H: Wilson coefficients for SCET Hard Scattering Operators

$$C \otimes \mathcal{O}$$

Hard-collinear factorization





Operators are built of building block fields:

$$\mathcal{O} = (\mathcal{B}_{n_a\perp})(\mathcal{B}_{n_b\perp})(\mathcal{B}_{n_1\perp})(\bar{\chi}_{n_2})(\chi_{n_3})$$

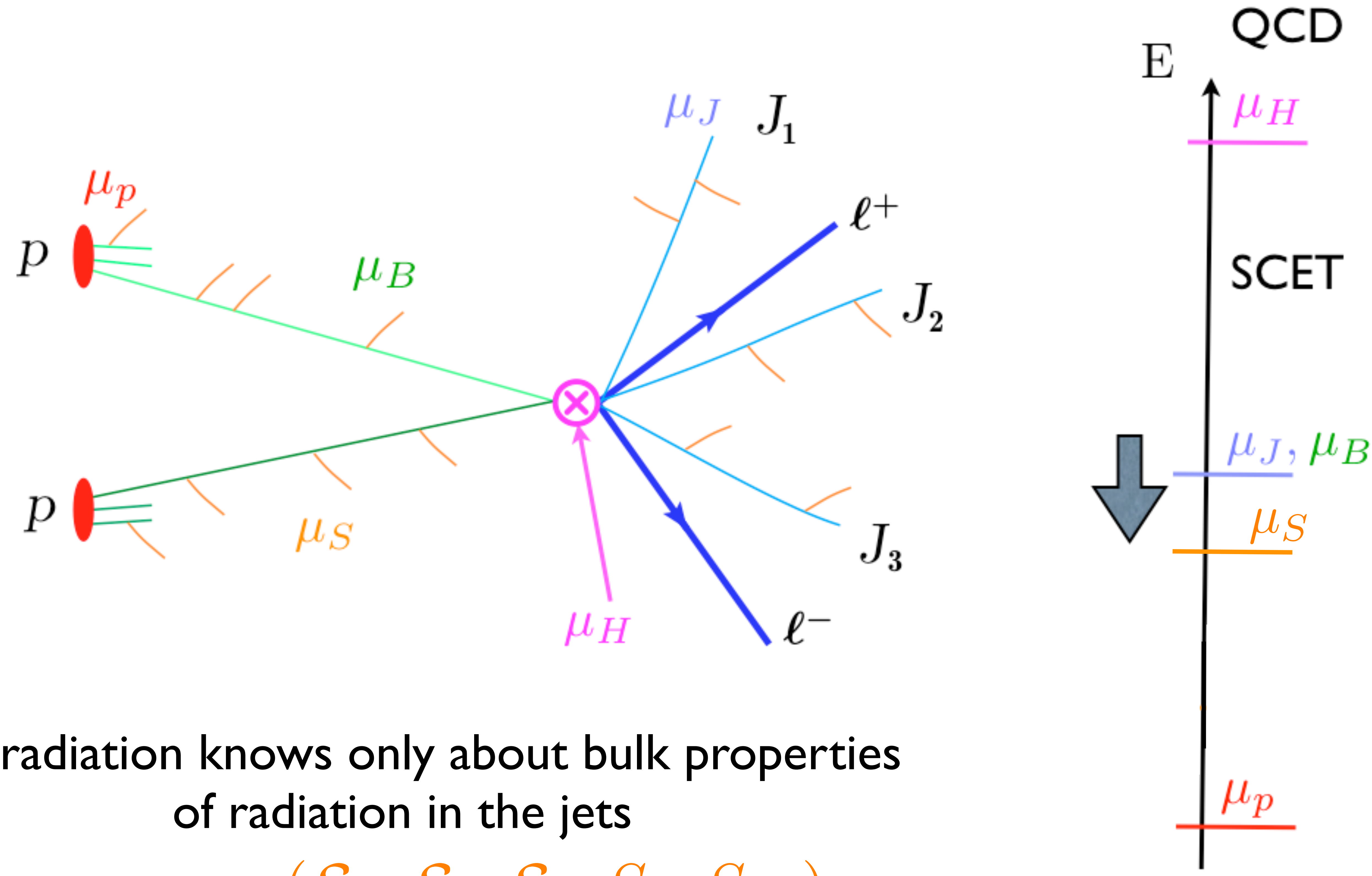
$$\chi_n = (W_n^\dagger \xi_n)$$ “quark jet”

$$\mathcal{B}_{n\perp}^\mu = [W_n^\dagger i D_\perp^\mu W_n]$$ “gluon jet”

Wilson lines

$$W_n = P \exp\left(ig \int_{-\infty}^0 ds\, \bar{n} \cdot A_n(x + \bar{n}s)\right)$$

Soft-collinear factorization

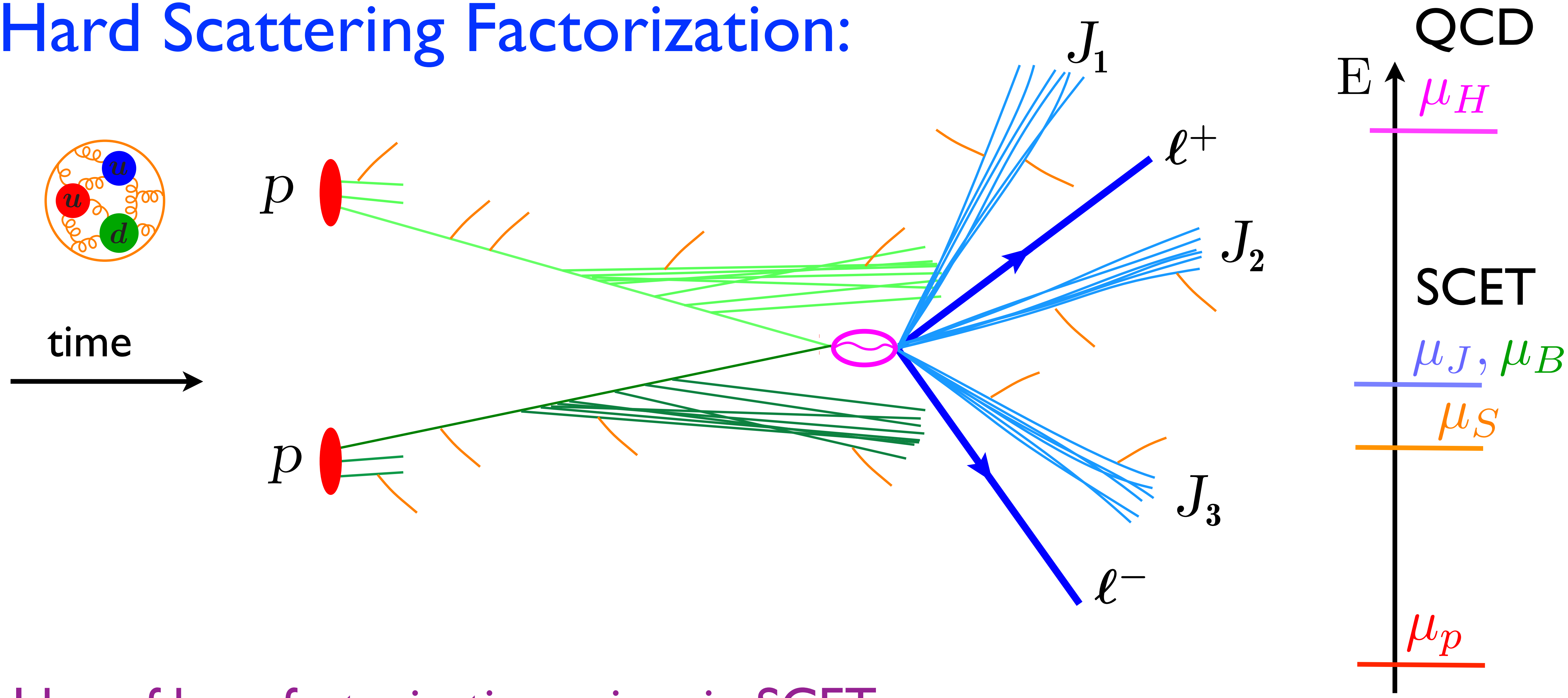


Soft radiation knows only about bulk properties of radiation in the jets

$$(S_{n_a} S_{n_b} S_{n_1} S_{n_2} S_{n_3})$$

Soft Wilson Lines

Hard Scattering Factorization:



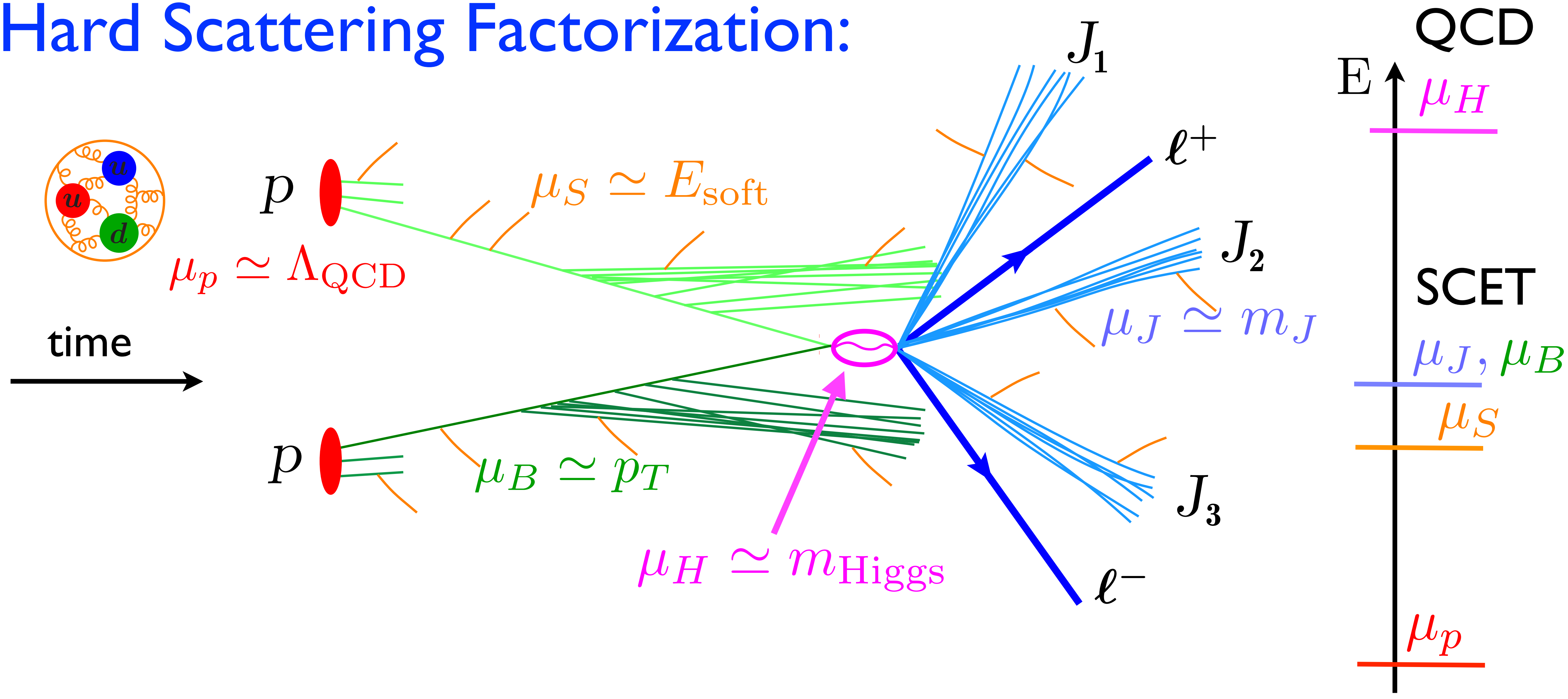
Idea of how factorization arises in SCET:

factorized Lagrangian: $\mathcal{L}^{(0)}_{\mathrm{SCET_{II},S,\{n_i\}}} = \mathcal{L}^{(0)}_S(\psi_S, A_S) + \sum_{n_i} \mathcal{L}^{(0)}_{n_i}(\xi_{n_i}, A_{n_i})$

factorized Hard Ops: $C \otimes (\mathcal{B}_{n_a\perp})(\mathcal{B}_{n_b\perp})(\mathcal{B}_{n_1\perp})(\bar{\chi}_{n_2})(\chi_{n_3})(\mathcal{S}_{n_a}\mathcal{S}_{n_b}\mathcal{S}_{n_1}\mathcal{S}_{n_2}\mathcal{S}_{n_3})$

→ factorized squared matrix elements defining jet, soft, … functions

Hard Scattering Factorization:



Nonperturbative: $d\sigma = f_a f_b \otimes \hat{\sigma} \otimes F$ ← hadronization

$\mu_p \simeq \Lambda_{\rm QCD}$

eg. Perturbative: $\hat{\sigma}_{\rm fact} = \mathcal{I}_a \mathcal{I}_b \otimes H \otimes \prod_i J_i \otimes S$

Used to Sum Logs

	μ_B	μ_H	μ_J	μ_S
Universal Functions:	beam	hard	jet	pert. soft

"Factorization Violation"

My Definition: The expected form for a factorization formula is invalid.

Reasons Factorization can fail:

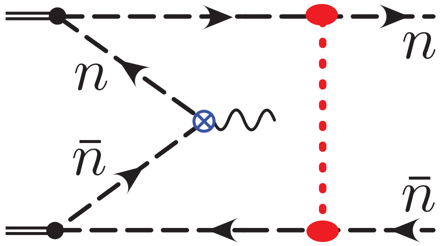
- Measurement doesn't factor: no simple factorization with universal functions. (eg. Jade jet algorithm)

- Divergent convolutions, not controlled by ones regulation procedures. (Requires more careful construction.)

$$\int_0^1 \frac{dx}{x^2}\,\phi_\pi(x,\mu)$$

- Interactions that couple other modes and spoil factorization.

Glauber exchange



spectator-spectator CSS
cancel in proof for Drell-Yan

- Collinear Wilson Line universality fails.

examples studied by Collins, Qiu, Aybat, Rogers, …

$H_3, H_4 \simeq$ back-to-back

$H_1 + H_2 \to H_3 + H_4 + X$

pT dependent

Determine Glauber Lagrangian

$$\mathcal{L}^{(0)}_{\mathrm{SCET_{II}}} = \mathcal{L}^{(0)}_{\mathrm{SCET_{II},S,\{n_i\}}} + \mathcal{L}^{(0)}_{G}\left(\psi_S, A_S, \xi_{n_i}, A_{n_i}\right)$$

- Complete description of factorization violation (any α_s, any power)
- If $\mathcal{L}^{(0)}_{G}$ does not contribute → can derive usual types of Factorization Formulae
- Power suppressed $\mathcal{L}^{(k\geq 1)}(S, n_i)$ do not spoil factorization

Determine Glauber Lagrangian

$$\mathcal{L}^{(0)}_{\mathrm{SCET_{II}}} = \mathcal{L}^{(0)}_{\mathrm{SCET_{II},S,\{n_i\}}} + \mathcal{L}^{(0)}_{G}\left(\psi_S, A_S, \xi_{n_i}, A_{n_i}\right)$$

Key Technical ingredients:

- No double counting (0-bin Subtractions)
- Contributions separately well defined (Rapidity Regulator)

Note: SCET Glauber not simply related to CSS Glauber

(expand first then integrate) vs. (study integrals then expand)

Construction:

$\lambda \ll 1$ large Q

will do calculations with back-to-back collinear particles for simplicity

mode	fields	p^μ momentum scaling	physical objects	type
n-collinear	$\xi_n,\ A_n^\mu$	$(n\cdot p, \bar{n}\cdot p, p_\perp) \sim Q(\lambda^2, 1, \lambda)$	n-collinear "jet"	onshell
$\bar{n}$-collinear	$\xi_{\bar{n}},\ A_{\bar{n}}^\mu$	$(\bar{n}\cdot p, n\cdot p, p_\perp) \sim Q(\lambda^2, 1, \lambda)$	$\bar{n}$-collinear "jet"	onshell
soft	$\psi_S,\ A_S^\mu$	$p^\mu \sim Q(\lambda, \lambda, \lambda)$	soft virtual/real radiation	onshell
ultrasoft	$\psi_{us},\ A_{us}^\mu$	$p^\mu \sim Q(\lambda^2, \lambda^2, \lambda^2)$	ultrasoft virtual/real radiation	onshell
Glauber	–	$p^\mu \sim Q(\lambda^a, \lambda^b, \lambda),\ a+b>2$	forward scattering potential	offshell
		(here $\{a,b\} = \{2,2\}, \{2,1\}, \{1,2\}$)		
hard	–	$p^2 \gtrsim Q^2$	hard scattering	offshell

Integrate out these modes

Glaubers are offshell and must be integrated out (despite having $p^2 \sim \lambda^2$)

Otherwise one has problems with simultaneously having gauge invariant operators and homogeneous power counting

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$\lambda \ll 1$ large Q

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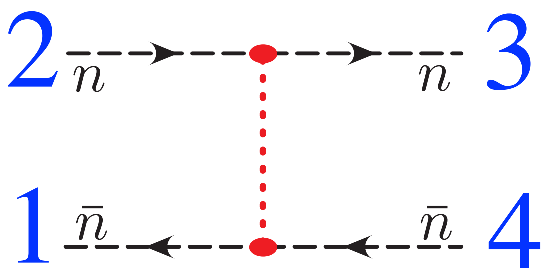
Integrate out

Need 3-types of Glauber momenta:

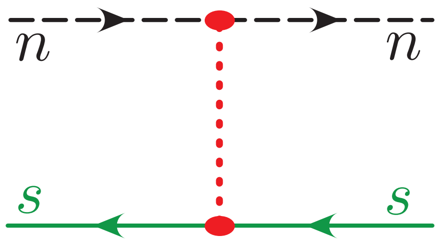
n-$\bar{n}$
fwd. scattering



$$\bar{n}\cdot p_2 = \bar{n}\cdot p_3$$
$$n\cdot p_1 = n\cdot p_4$$

forward
conditions

n-s
fwd. scattering



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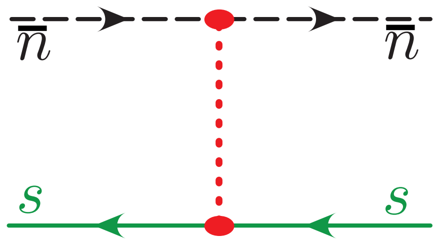

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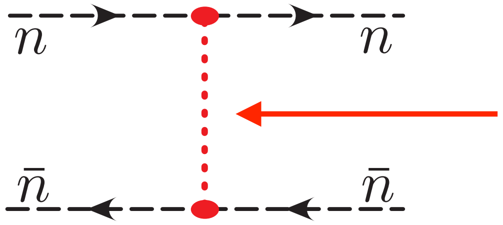
Integrate out

Need 3-types of Glauber momenta:

$(+, -, \perp)$

n-$\bar{n}$ fwd. scattering



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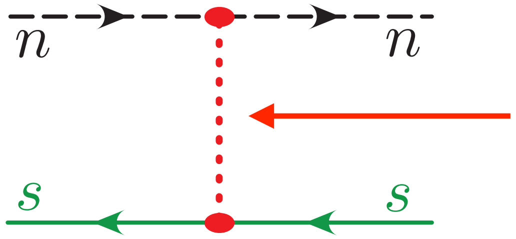
n-s fwd. scattering



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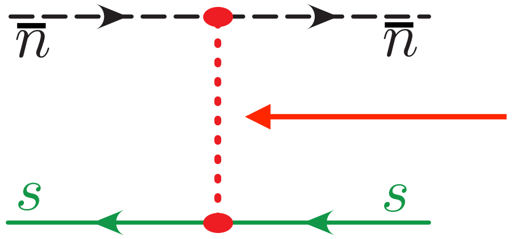
$\bar{n}$-s fwd. scattering



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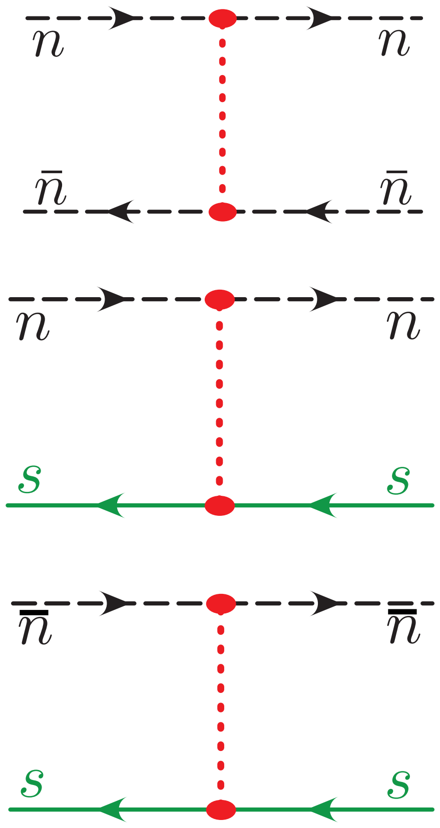
Integrate out

Need 3-types of Glauber momenta:

n-$\bar n$
fwd. scattering

n-s
fwd. scattering

$\bar n$-s
fwd. scattering



- $\frac{1}{k_\perp^2}$ potentials
- instantaneous in x^+, x^- (t and z)

(also scatter forward gluons)

Full Leading Power Glauber Lagrangian:

$$\mathcal{L}_G^{\mathrm{II}(0)} = \sum_{n,\bar{n}} \sum_{i,j=q,g} \mathcal{O}_n^{iB} \frac{1}{\mathcal{P}_\perp^2} \mathcal{O}_s^{BC} \frac{1}{\mathcal{P}_\perp^2} \mathcal{O}_{\bar{n}}^{jC} + \sum_{n} \sum_{i,j=q,g} \mathcal{O}_n^{iB} \frac{1}{\mathcal{P}_\perp^2} \mathcal{O}_s^{j_n B}$$

(3 rapidity sectors)

(2 rapidity sectors)

sum pairwise
on all collinears

sum on all
collinears

- Interactions with more sectors is given by T-products
- No Wilson coefficients ie. no new structures at loop level.

Defining SCET building blocks:

$$\chi_n = W_n^\dagger \xi_n \qquad \psi_s^n = S_n^\dagger \psi_s$$

$$\mathcal{B}_{n\perp}^\mu = \frac{1}{g}\left[W_n^\dagger i D_{n\perp}^\mu W_n\right] \qquad \mathcal{B}_{S\perp}^{n\mu} = \frac{1}{g}\left[S_n^\dagger i D_{S\perp}^\mu S_n\right] \qquad \widetilde{\mathcal{B}}_{S\perp}^{nAB} = -i f^{ABC} \mathcal{B}_{S\perp}^{nC}$$

$$\widetilde{G}_s^{\mu\nu\, AB} = -i f^{ABC} G_s^{\mu\nu\, A}$$

Full Leading Power Glauber Lagrangian:

$$\mathcal{L}_G^{\mathrm{II}(0)} = \sum_{n,\bar n}\sum_{i,j=q,g} \mathcal{O}_n^{iB}\frac{1}{\mathcal{P}_\perp^2}\mathcal{O}_s^{BC}\frac{1}{\mathcal{P}_\perp^2}\mathcal{O}_{\bar n}^{jC} + \sum_{n}\sum_{i,j=q,g} \mathcal{O}_n^{iB}\frac{1}{\mathcal{P}_\perp^2}\mathcal{O}_s^{j_n B}$$

↑ sum pairwise on all collinears

(3 rapidity sectors)

↑ sum on all collinears

(2 rapidity sectors)

- Interactions with more sectors is given by T-products
- No Wilson coefficient ie. no new structures at loop level.

$$\mathcal{O}_n^{qB} = \overline{\chi}_n T^B \frac{\not{\bar n}}{2}\,\chi_n \qquad \mathcal{O}_n^{gB} = \frac{i}{2} f^{BCD}\mathcal{B}^C_{n\perp\mu}\,\frac{\bar n}{2}\cdot(\mathcal{P}+\mathcal{P}^\dagger)\mathcal{B}^{D\mu}_{n\perp}$$

$$\mathcal{O}_{\bar n}^{qB} = \overline{\chi}_{\bar n} T^B \frac{\not{n}}{2}\,\chi_{\bar n} \qquad \mathcal{O}_{\bar n}^{gB} = \frac{i}{2} f^{BCD}\mathcal{B}^C_{\bar n\perp\mu}\,\frac{n}{2}\cdot(\mathcal{P}+\mathcal{P}^\dagger)\mathcal{B}^{D\mu}_{\bar n\perp}$$

$$\mathcal{O}_s^{BC} = 8\pi\alpha_s\Big\{\mathcal{P}_\perp^\mu\mathcal{S}_n^T\mathcal{S}_{\bar n}\mathcal{P}_{\perp\mu} - \mathcal{P}_\mu^\perp g\widetilde{\mathcal{B}}_{S\perp}^{n\mu}\mathcal{S}_n^T\mathcal{S}_{\bar n} - \mathcal{S}_n^T\mathcal{S}_{\bar n}g\widetilde{\mathcal{B}}_{S\perp}^{\bar n\mu}\mathcal{P}_\mu^\perp - g\widetilde{\mathcal{B}}_{S\perp}^{n\mu}\mathcal{S}_n^T\mathcal{S}_{\bar n}g\widetilde{\mathcal{B}}_{S\perp\mu}^{\bar n} - \frac{n_\mu\bar n_\nu}{2}\mathcal{S}_n^T ig\widetilde{G}_s^{\mu\nu}\mathcal{S}_{\bar n}\Big\}^{BC}$$

$$\mathcal{O}_s^{q_n B} = 8\pi\alpha_s\left(\bar\psi_S^n\,T^B\frac{\not{n}}{2}\psi_S^n\right) \qquad \mathcal{O}_s^{g_n B} = 8\pi\alpha_s\left(\frac{i}{2}f^{BCD}\mathcal{B}^{nC}_{S\perp\mu}\,\frac{n}{2}\cdot(\mathcal{P}+\mathcal{P}^\dagger)\mathcal{B}^{nD\mu}_{S\perp}\right)$$

$$\mathcal{O}_s^{q_{\bar n} B} = 8\pi\alpha_s\left(\bar\psi_S^{\bar n}\,T^B\frac{\not{\bar n}}{2}\psi_S^{\bar n}\right) \qquad \mathcal{O}_s^{g_{\bar n} B} = 8\pi\alpha_s\left(\frac{i}{2}f^{BCD}\mathcal{B}^{\bar nC}_{S\perp\mu}\,\frac{\bar n}{2}\cdot(\mathcal{P}+\mathcal{P}^\dagger)\mathcal{B}^{\bar nD\mu}_{S\perp}\right)$$

Construction: $\lambda \ll 1$ large Q

mode	fields	p^μ momentum scaling	physical objects	type
n-collinear	$\xi_n,\ A_n^\mu$	$(n\cdot p, \bar{n}\cdot p, p_\perp) \sim Q(\lambda^2, 1, \lambda)$	n-collinear "jet"	onshell
$\bar{n}$-collinear	$\xi_{\bar{n}},\ A_{\bar{n}}^\mu$	$(\bar{n}\cdot p, \bar{n}\cdot p, p_\perp) \sim Q(\lambda^2, 1, \lambda)$	$\bar{n}$-collinear "jet"	onshell
soft	$\psi_S,\ A_S^\mu$	$p^\mu \sim Q(\lambda, \lambda, \lambda)$	soft virtual/real radiation	onshell
ultrasoft	$\psi_{us},\ A_{us}^\mu$	$p^\mu \sim Q(\lambda^2, \lambda^2, \lambda^2)$	ultrasoft virtual/real radiation	onshell
Glauber	–	$p^\mu \sim Q(\lambda^a, \lambda^b, \lambda),\ a+b>2$ (here $\{a,b\} = \{2,2\}, \{2,1\}, \{1,2\}$)	forward scattering potential	offshell
hard	–	$p^2 \gtrsim Q^2$	hard scattering	offshell

enumerate # of vertices from gauge invariant operators of order $\sim \lambda^k$

V_k^n vertices with only n-collinear fields,

$V_k^{\bar{n}}$ vertices with only $\bar{n}$-collinear fields,

V_k^S vertices with only soft fields,

V_k^{nS} vertices that have both n-collinear and soft fields but do not have $\bar{n}$ fields,

$V_k^{\bar{n}S}$ vertices with both $\bar{n}$-collinear and soft fields but not n fields,

$V_k^{n\bar{n}}$ vertices with both n and $\bar{n}$-collinear fields (with or without soft fields).

Construction:

$\lambda \ll 1$ large Q

mode	fields	p^μ momentum scaling	physical objects	type
n-collinear	$\xi_n,\ A_n^\mu$	$(n\cdot p, \bar n\cdot p, p_\perp) \sim Q(\lambda^2, 1, \lambda)$	n-collinear "jet"	onshell
$\bar n$-collinear	$\xi_{\bar n},\ A_{\bar n}^\mu$	$(\bar n\cdot p, \bar n\cdot p, p_\perp) \sim Q(\lambda^2, 1, \lambda)$	$\bar n$-collinear "jet"	onshell
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hard	–	$p^2 \gtrsim Q^2$	hard scattering	offshell

Power Counting formula for graph (any loop order, any power):

$$\sim \lambda^\delta$$

topological factors

operator insertions

$$\delta = 6 - N^n - N^{\bar n} - N^{nS} - N^{\bar n S} + 2u$$

$$+ \sum_k (k-8)\underbrace{V_k^{us} + (k-4)}_{\text{standard SCET}}\big(V_k^n + V_k^{\bar n} + V_k^S\big) + (k-3)\big(V_k^{nS} + V_k^{\bar n S}\big) + (k-2)V_k^{n\bar n}$$

need $\sim \lambda^3$ $\sim \lambda^2$

Glauber
operators at leading power

Construction:

$\lambda \ll 1$ large Q

mode	fields	p^μ momentum scaling	physical objects	type
n-collinear	$\xi_n,\ A_n^\mu$	$(n\cdot p, \bar{n}\cdot p, p_\perp) \sim Q(\lambda^2, 1, \lambda)$	n-collinear "jet"	onshell
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		(here $\{a,b\} = \{2,2\}, \{2,1\}, \{1,2\}$)		
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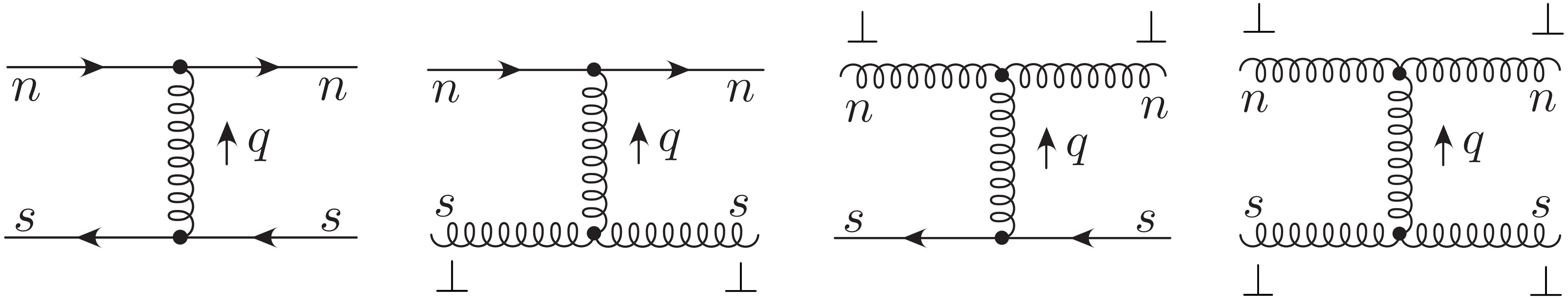
integrated out

n-s fwd. scattering (2 rapidity sectors)

$s \gg t$

calculate



$$\lambda^2 = \frac{t}{s} \ll 1$$

Construction:

$\lambda \ll 1$ large Q

mode	fields	p^μ momentum scaling	physical objects	type
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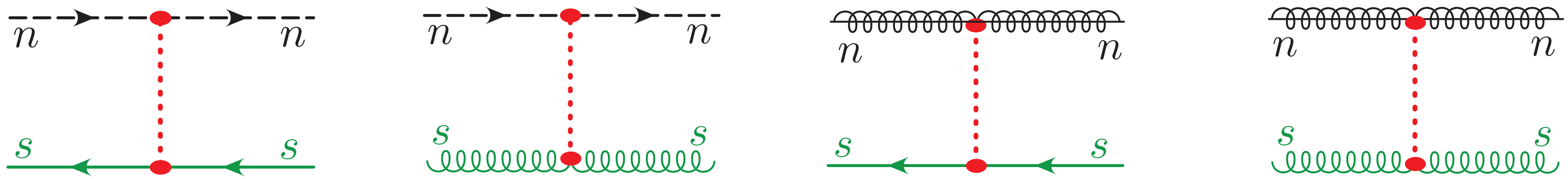
integrated out

n-s fwd. scattering

$s \gg t$

$\lambda^2 = \frac{t}{s} \ll 1$



determine $\mathcal{O}(\lambda^3):$ $$\sum_{i,j=q,g} \underbrace{\mathcal{O}_n^{iB}}_{\lambda^2} \underbrace{\frac{1}{\mathcal{P}_\perp^2}}_{\lambda^{-2}} \underbrace{\mathcal{O}_s^{j_n B}}_{\lambda^3}$$

(2 rapidity sectors)

with bilinear octet operators

$\psi_s^n = S_n^\dagger \psi_s$

$$\mathcal{O}(\lambda^2): \quad \mathcal{O}_n^{qB} = \overline{\chi}_n T^B \frac{\bar{n}\!\!\!/}{2} \chi_n\,,$$

$$\mathcal{O}_n^{gB} = \frac{i}{2} f^{BCD} \mathcal{B}^C_{n\perp\mu} \frac{\bar{n}}{2}\cdot(\mathcal{P}+\mathcal{P}^\dagger)\mathcal{B}^{D\mu}_{n\perp}$$

$$\mathcal{O}(\lambda^3): \quad \mathcal{O}_s^{q_n B} = 8\pi\alpha_s \left(\bar{\psi}^n_S T^B \frac{n\!\!\!/}{2} \psi^n_S\right),$$

$$\mathcal{O}_s^{g_n B} = 8\pi\alpha_s \left(\frac{i}{2} f^{BCD} \mathcal{B}^{nC}_{S\perp\mu} \frac{n}{2}\cdot(\mathcal{P}+\mathcal{P}^\dagger)\mathcal{B}^{nD\mu}_{S\perp}\right)$$

Construction: $\lambda \ll 1$ large Q

mode	fields	p^μ momentum scaling	physical objects	type
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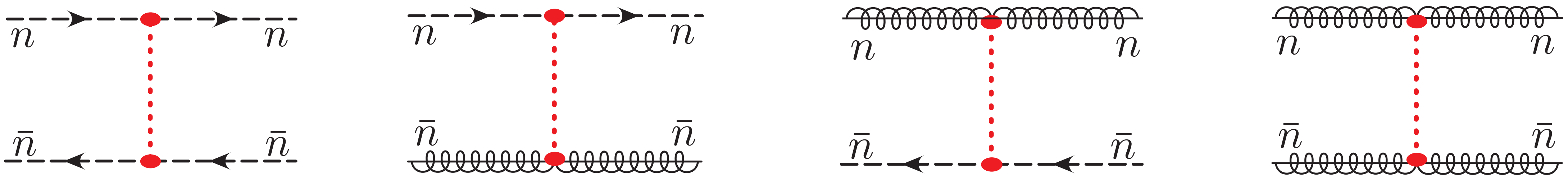
n-$\bar n$ fwd. scattering $s \gg t$



actually $\mathcal{O}(\lambda^2)$:

(3 rapidity sectors)

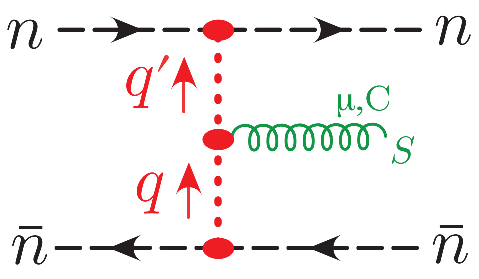
$$\sum_{i,j=q,g} \underset{\lambda^2}{\mathcal{O}_n^{iB}} \underset{\lambda^{-2}}{\frac{1}{\mathcal{P}_\perp^2}} \underset{\lambda^2}{\mathcal{O}_s^{BC}} \underset{\lambda^{-2}}{\frac{1}{\mathcal{P}_\perp^2}} \underset{\lambda^2}{\mathcal{O}_{\bar n}^{jC}}$$

same $\mathcal{O}_n^{iB}$

analogous $\mathcal{O}_{\bar n}^{jC}$

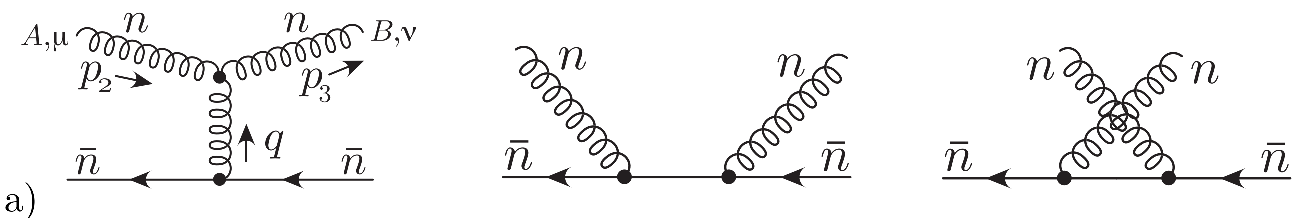
includes soft emission from **between** the rapidity sectors:

$$\mathcal{O}_s^{BC} = 8\pi\alpha_s \mathcal{P}_\perp^2 \delta^{BC} \boxed{+\ \ldots.}$$

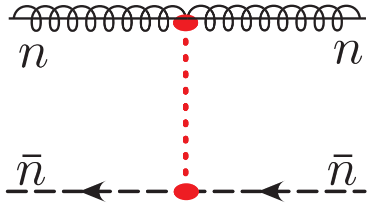


Matching with arbitrary polarizations:

QCD



EFT

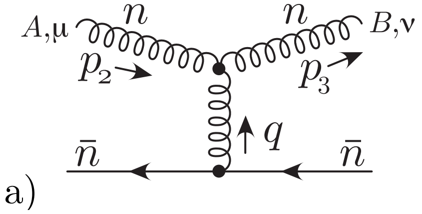


QCD calculation

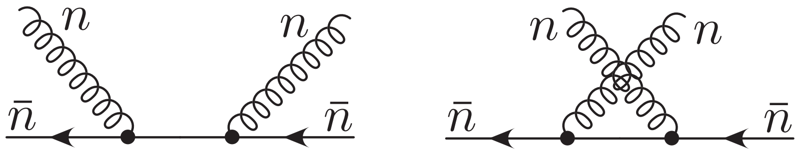
remove

$$0 = p^\mu A_\mu(p) = \frac{1}{2}\bar{n}\cdot p\; n\!\cdot\! A(p) + \frac{1}{2} n\cdot p\; \bar{n}\!\cdot\! A(p) + p_\perp\!\cdot\! A_\perp(p)$$

used equations of motion



$$= \frac{g^2 f^{ABC}}{q^2}\Big[\bar{v}_{\bar{n}}\frac{\not{n}}{2}\bar{T}^C v_{\bar{n}}\Big]\Big\{2\,\bar{n}\cdot p_2\, g_\perp^{\mu\nu} - 2\bar{n}^\mu p_{2\perp}^\nu - 2p_{3\perp}^\mu \bar{n}^\nu - n\!\cdot\!(p_2+p_3)\bar{n}^\mu\bar{n}^\nu\Big\}$$



$$= \frac{g^2 f^{ABC}}{q^2}\Big[\bar{v}_{\bar{n}}\frac{\not{n}}{2}\bar{T}^C v_{\bar{n}}\Big]\Big\{-\frac{q^2}{\bar{n}\cdot p_2}\Big\}\bar{n}^\mu\bar{n}^\nu$$

sum & use equations of motion: $q^2 + \bar{n}\!\cdot\! p_2\, n\!\cdot\!(p_2+p_3) = -2p_{2\perp}\!\cdot\! p_{3\perp}$

$$\frac{2g^2 f^{ABC}}{q^2}\Big[\bar{v}_{\bar{n}}\frac{\not{n}}{2}\bar{T}^C v_{\bar{n}}\Big]\Big\{\bar{n}\cdot p_2\, g_\perp^{\mu\nu} - \bar{n}^\mu p_{2\perp}^\nu - p_{3\perp}^\mu\bar{n}^\nu + \frac{p_{2\perp}\!\cdot\! p_{3\perp}}{\bar{n}\cdot p_2}\bar{n}^\mu\bar{n}^\nu\Big\}$$

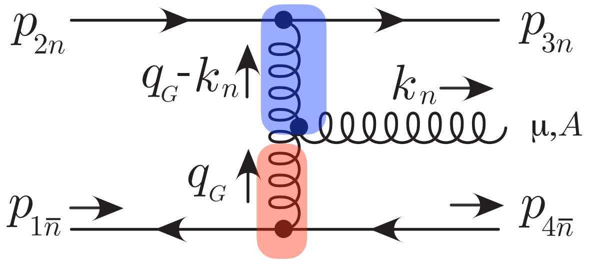
same as Glauber operator

Gluon Operators include Compton graphs in fwd.limit

Wilson Lines in the operators are obtained from Matching:

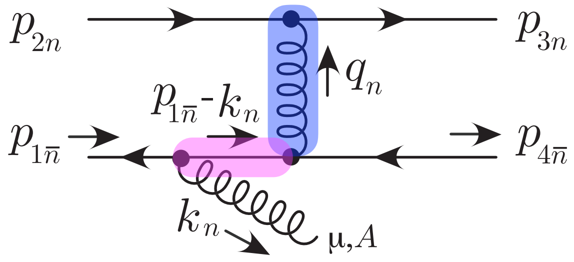
eg. $W_n^\dagger$ in χ_n

QCD



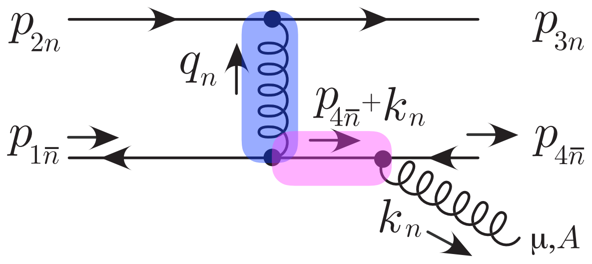
offshell glauber

onshell collinear



offshell hard

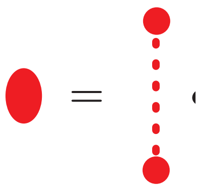
onshell collinear

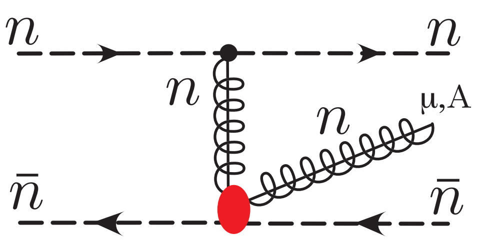


offshell hard

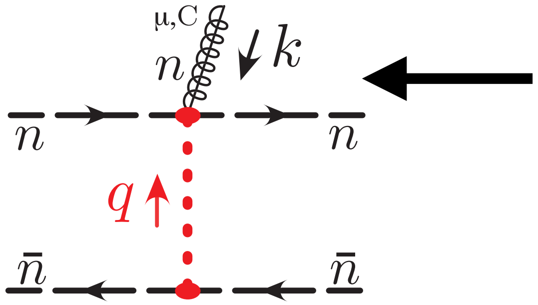
onshell collinear

SCET





has onshell collinear



from W Wilson lines

just offshell Glauber potential

sums agree after using equations of motion

- This calculation differs from hard scattering Wilson lines due to onshell term on EFT side
- Similar for other operators

Soft $\mathcal{O}_s^{BC}$ Operator

$$\mathcal{O}_s^{BC} = 8\pi\alpha_s \sum_i C_i O_i^{BC}$$

basis of $\mathcal{O}(\lambda^2)$ operators allowed by symmetries:

$$O_1 = \mathcal{P}_\perp^\mu \mathcal{S}_n^T \mathcal{S}_{\bar n} \mathcal{P}_{\perp\mu}, \qquad O_2 = \mathcal{P}_\perp^\mu \mathcal{S}_{\bar n}^T \mathcal{S}_n \mathcal{P}_{\perp\mu},$$

$$O_3 = \mathcal{P}_\perp\cdot(g\widetilde{\mathcal{B}}^{n}_{S\perp})(\mathcal{S}_n^T\mathcal{S}_{\bar n})+(\mathcal{S}_n^T\mathcal{S}_{\bar n})(g\widetilde{\mathcal{B}}^{\bar n}_{S\perp})\cdot\mathcal{P}_\perp, \qquad O_4 = \mathcal{P}_\perp\cdot(g\widetilde{\mathcal{B}}^{\bar n}_{S\perp})(\mathcal{S}_{\bar n}^T\mathcal{S}_{n})+(\mathcal{S}_{\bar n}^T\mathcal{S}_{n})(g\widetilde{\mathcal{B}}^{n}_{S\perp})\cdot\mathcal{P}_\perp,$$

$$O_5 = \mathcal{P}^\perp_\mu(\mathcal{S}_n^T\mathcal{S}_{\bar n})(g\widetilde{\mathcal{B}}^{\bar n\mu}_{S\perp})+(g\widetilde{\mathcal{B}}^{n\mu}_{S\perp})(\mathcal{S}_n^T\mathcal{S}_{\bar n})\mathcal{P}^\perp_\mu, \qquad O_6 = \mathcal{P}^\perp_\mu(\mathcal{S}_{\bar n}^T\mathcal{S}_{n})(g\widetilde{\mathcal{B}}^{n\mu}_{S\perp})+(g\widetilde{\mathcal{B}}^{\bar n\mu}_{S\perp})(\mathcal{S}_{\bar n}^T\mathcal{S}_{n})\mathcal{P}^\perp_\mu,$$

$$O_7 = (g\widetilde{\mathcal{B}}^{n\mu}_{S\perp})\mathcal{S}_n^T\mathcal{S}_{\bar n}(g\widetilde{\mathcal{B}}^{\bar n}_{S\perp\mu}), \qquad O_8 = (g\widetilde{\mathcal{B}}^{\bar n\mu}_{S\perp})\mathcal{S}_{\bar n}^T\mathcal{S}_{n}(g\widetilde{\mathcal{B}}^{n}_{S\perp\mu}),$$

$$O_9 = \mathcal{S}_n^T n_\mu \bar n_\nu (ig\widetilde{G}_s^{\mu\nu})\mathcal{S}_{\bar n}, \qquad O_{10} = \mathcal{S}_{\bar n}^T n_\mu \bar n_\nu (ig\widetilde{G}_s^{\mu\nu})\mathcal{S}_{n},$$

octet Wilson line

octet reps

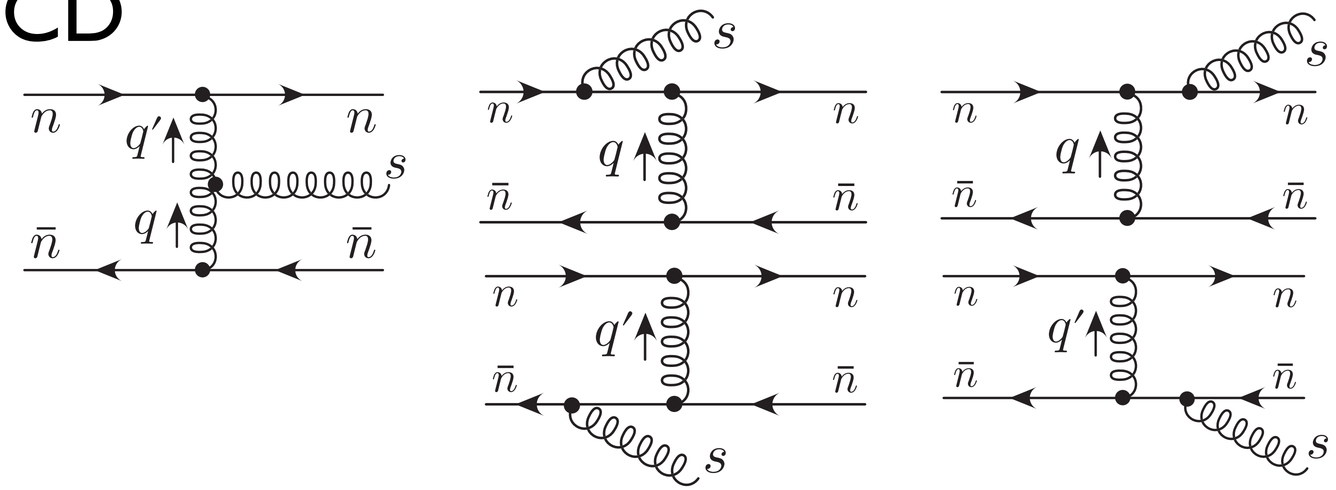
Restricted by: Hermiticity $O_i^\dagger\big|_{n\leftrightarrow\bar n} = O_i$, one $\mathcal{S}_n$, one $\mathcal{S}_{\bar n}$

operator identities: eg. $[\mathcal{P}_\perp^\mu(\mathcal{S}_n^T\mathcal{S}_{\bar n})] = -g\widetilde{\mathcal{B}}^{n\mu}_{S\perp}(\mathcal{S}_n^T\mathcal{S}_{\bar n}) + (\mathcal{S}_n^T\mathcal{S}_{\bar n})g\widetilde{\mathcal{B}}^{\bar n\mu}_{S\perp}$

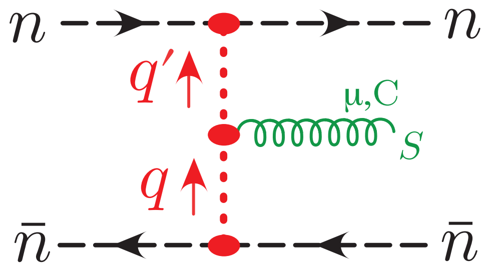
Matching with up to 2 soft gluons fixes all coefficients

One Soft Gluon:

QCD



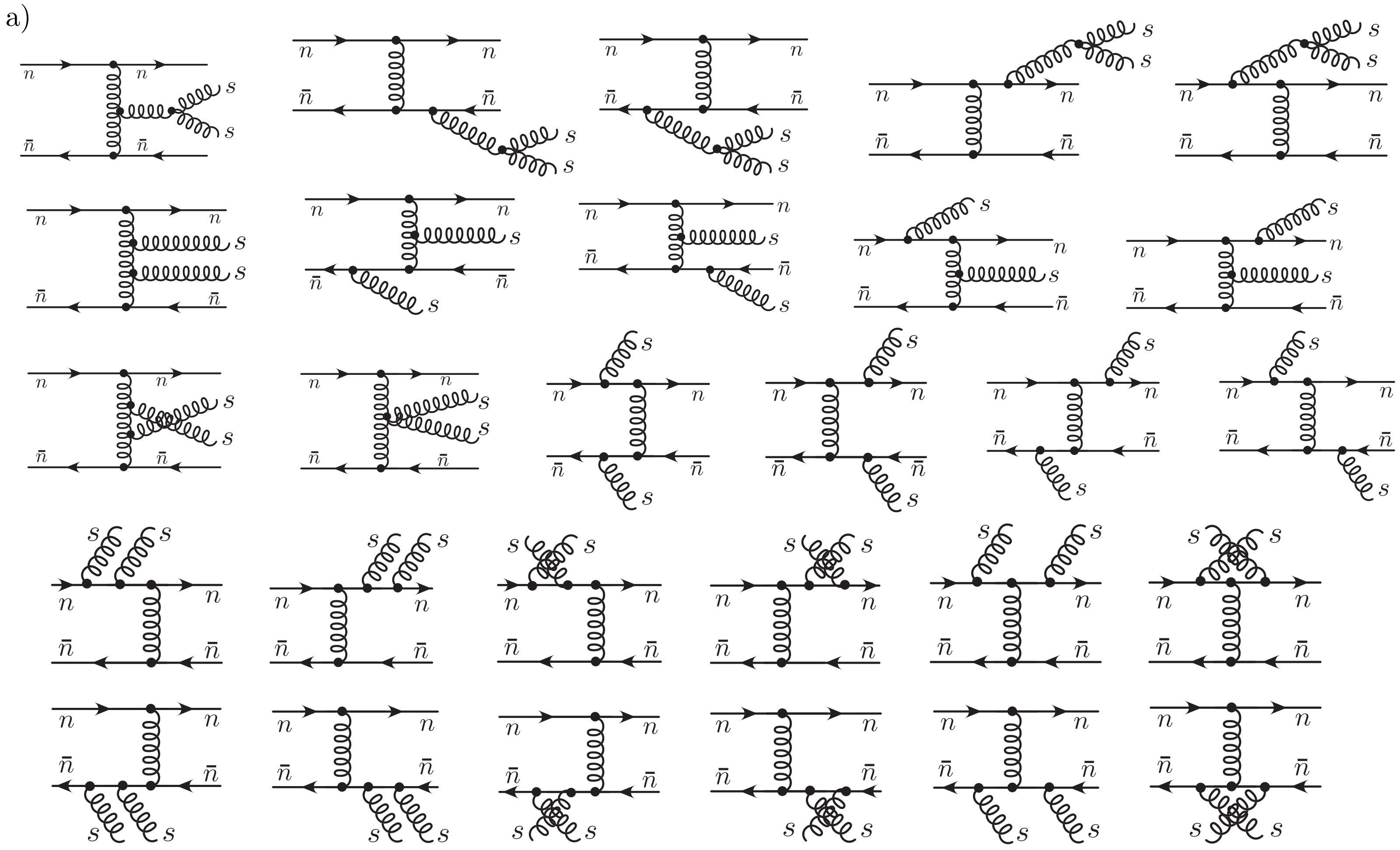
EFT



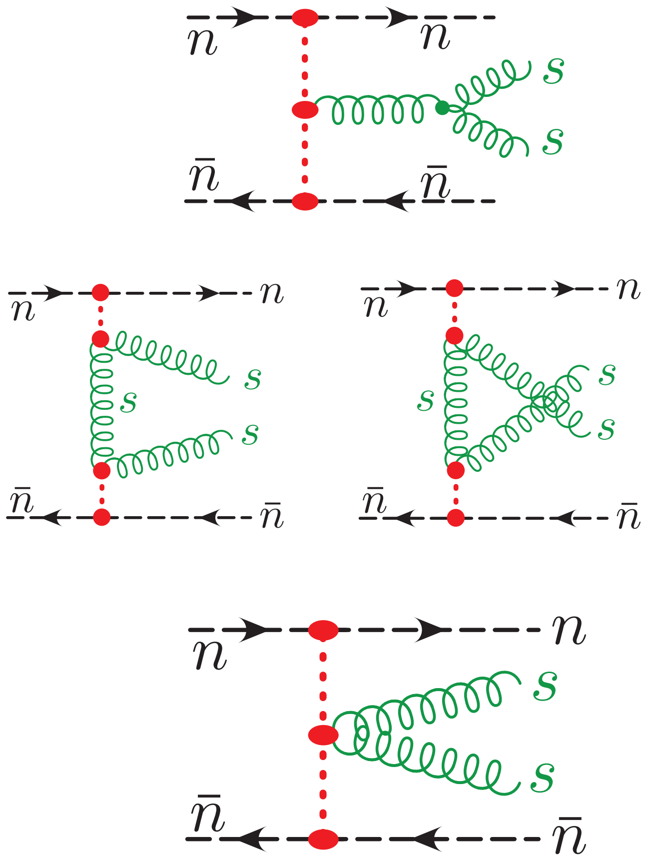
Lipatov vertex

Two Soft Gluons:

QCD



EFT



2 gluon vertex

Find:

$$\boxed{C_2 = C_4 = C_5 = C_6 = C_8 = C_{10} = 0}\,,$$

$$\boxed{C_1 = -C_3 = -C_7 = +1\,, \qquad C_9 = -\frac{1}{2}}$$

$$\mathcal{O}_s^{BC} = 8\pi\alpha_s \Big\{ \mathcal{P}_\perp^\mu \mathcal{S}_n^T \mathcal{S}_{\bar n} \mathcal{P}_{\perp\mu} - \mathcal{P}^\perp_\mu\, g\widetilde{\mathcal{B}}_{S\perp}^{n\mu} \mathcal{S}_n^T \mathcal{S}_{\bar n} - \mathcal{S}_n^T \mathcal{S}_{\bar n}\, g\widetilde{\mathcal{B}}_{S\perp}^{\bar n\mu} \mathcal{P}^\perp_\mu - g\widetilde{\mathcal{B}}_{S\perp}^{n\mu} \mathcal{S}_n^T \mathcal{S}_{\bar n}\, g\widetilde{\mathcal{B}}^{\bar n}_{S\perp\mu}$$
$$- \frac{n_\mu \bar n_\nu}{2} \mathcal{S}_n^T\, ig\widetilde{G}_s^{\mu\nu} \mathcal{S}_{\bar n} \Big\}^{BC} .$$

Form is unique to all loops since there are no hard α_s corrections to this matching (more later)

Full Leading Power Glauber Lagrangian:

$$\mathcal{L}_G^{\mathrm{II}(0)} = \sum_{n,\bar{n}} \sum_{i,j=q,g} \mathcal{O}_n^{iB} \frac{1}{\mathcal{P}_\perp^2} \mathcal{O}_s^{BC} \frac{1}{\mathcal{P}_\perp^2} \mathcal{O}_{\bar{n}}^{jC} + \sum_{n} \sum_{i,j=q,g} \mathcal{O}_n^{iB} \frac{1}{\mathcal{P}_\perp^2} \mathcal{O}_s^{j_n B}$$

(3 rapidity sectors)

(2 rapidity sectors)

sum pairwise
on all collinears

sum on all
collinears

- Interactions with more sectors is given by T-products
- No Wilson coefficient ie. no new structures at loop level. [more later]

$$\mathcal{O}_n^{qB} = \overline{\chi}_n T^B \frac{\bar{\not{n}}}{2} \chi_n \qquad \mathcal{O}_n^{gB} = \frac{i}{2} f^{BCD} \mathcal{B}_{n\perp\mu}^C \frac{\bar{n}}{2} \cdot (\mathcal{P}+\mathcal{P}^\dagger) \mathcal{B}_{n\perp}^{D\mu}$$

$$\mathcal{O}_{\bar{n}}^{qB} = \overline{\chi}_{\bar{n}} T^B \frac{\not{n}}{2} \chi_{\bar{n}} \qquad \mathcal{O}_{\bar{n}}^{gB} = \frac{i}{2} f^{BCD} \mathcal{B}_{\bar{n}\perp\mu}^C \frac{n}{2} \cdot (\mathcal{P}+\mathcal{P}^\dagger) \mathcal{B}_{\bar{n}\perp}^{D\mu}$$

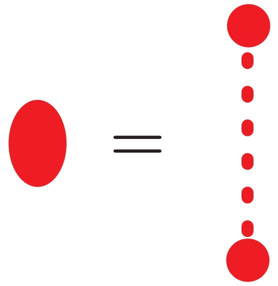
$$\mathcal{O}_s^{BC} = 8\pi\alpha_s \Big\{ \mathcal{P}_\perp^\mu \mathcal{S}_n^T \mathcal{S}_{\bar{n}} \mathcal{P}_{\perp\mu} - \mathcal{P}_\mu^\perp g \widetilde{\mathcal{B}}_{S\perp}^{n\mu} \mathcal{S}_n^T \mathcal{S}_{\bar{n}} - \mathcal{S}_n^T \mathcal{S}_{\bar{n}} g \widetilde{\mathcal{B}}_{S\perp}^{\bar{n}\mu} \mathcal{P}_\mu^\perp - g \widetilde{\mathcal{B}}_{S\perp}^{n\mu} \mathcal{S}_n^T \mathcal{S}_{\bar{n}} g \widetilde{\mathcal{B}}_{S\perp\mu}^{\bar{n}} - \frac{n_\mu \bar{n}_\nu}{2} \mathcal{S}_n^T ig \widetilde{G}_s^{\mu\nu} \mathcal{S}_{\bar{n}} \Big\}^{BC}$$

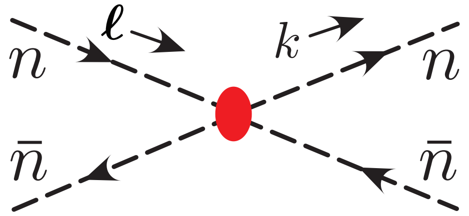
$$\mathcal{O}_s^{q_n B} = 8\pi\alpha_s \left(\bar{\psi}_S^n \, T^B \frac{\not{n}}{2} \psi_S^n \right) \qquad \mathcal{O}_s^{g_n B} = 8\pi\alpha_s \left(\frac{i}{2} f^{BCD} \mathcal{B}_{S\perp\mu}^{nC} \frac{n}{2} \cdot (\mathcal{P}+\mathcal{P}^\dagger) \mathcal{B}_{S\perp}^{nD\mu} \right)$$

$$\mathcal{O}_s^{q_{\bar{n}} B} = 8\pi\alpha_s \left(\bar{\psi}_S^{\bar{n}} \, T^B \frac{\bar{\not{n}}}{2} \psi_S^{\bar{n}} \right) \qquad \mathcal{O}_s^{g_{\bar{n}} B} = 8\pi\alpha_s \left(\frac{i}{2} f^{BCD} \mathcal{B}_{S\perp\mu}^{\bar{n}C} \frac{\bar{n}}{2} \cdot (\mathcal{P}+\mathcal{P}^\dagger) \mathcal{B}_{S\perp}^{\bar{n}D\mu} \right)$$

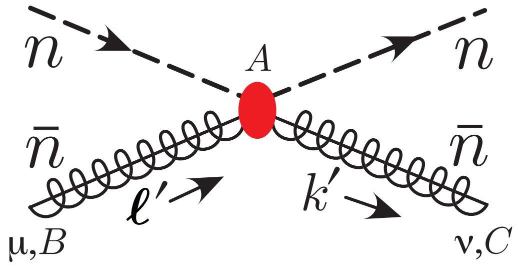
eg. Feynman Rules:

shorthand

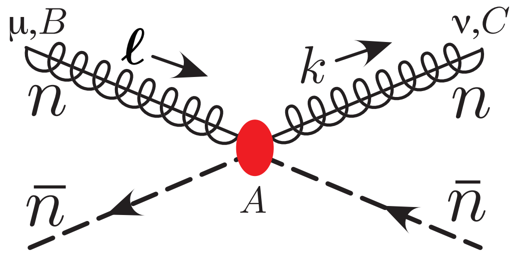




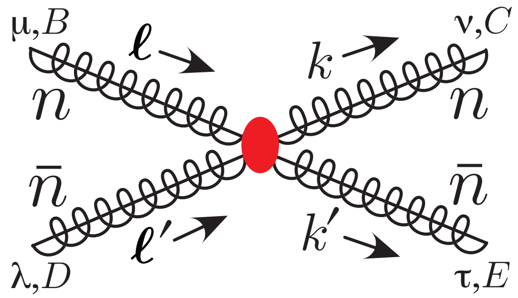
$$= \frac{-8\pi i\alpha_s}{(\vec{\ell}_\perp - \vec{k}_\perp)^2}\Big[\bar{u}_n \frac{\bar{n}\!\!\!/}{2} T^A u_n\Big]\Big[\bar{v}_{\bar{n}} \frac{n\!\!\!/}{2} \bar{T}^A v_{\bar{n}}\Big]$$



$$= \frac{-8\pi\alpha_s f^{ABC}}{(\vec{\ell}'_\perp - \vec{k}'_\perp)^2}\Big[\bar{u}_n \frac{\bar{n}\!\!\!/}{2} T^A u_n\Big]\Big[n\cdot k'\, g_\perp^{\mu\nu} - n^\mu \ell'^\nu_\perp - n^\nu k'^\mu_\perp + \frac{\ell'_\perp\cdot k'_\perp\, n^\mu n^\nu}{n\cdot k'}\Big]$$

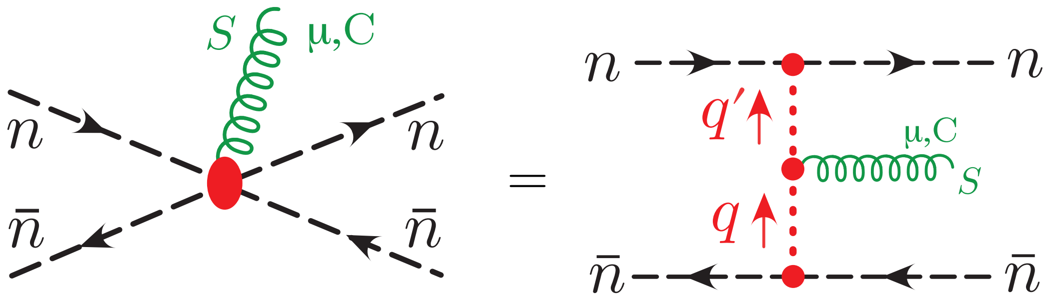


$$= \frac{-8\pi\alpha_s f^{ABC}}{(\vec{\ell}_\perp - \vec{k}_\perp)^2}\Big[\bar{n}\cdot k\, g_\perp^{\mu\nu} - \bar{n}^\mu \ell^\nu_\perp - \bar{n}^\nu k^\mu_\perp + \frac{\ell_\perp\cdot k_\perp\, \bar{n}^\mu \bar{n}^\nu}{\bar{n}\cdot k}\Big]\Big[\bar{v}_{\bar{n}} \frac{n\!\!\!/}{2} \bar{T}^A v_{\bar{n}}\Big]$$



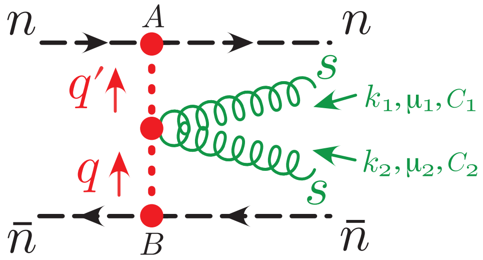
$$= \frac{8\pi i\alpha_s f^{ABC} f^{ADE}}{(\vec{\ell}_\perp - \vec{k}_\perp)^2}\Big[\bar{n}\cdot k\, g_\perp^{\mu\nu} - \bar{n}^\mu \ell^\nu_\perp - \bar{n}^\nu k^\mu_\perp + \frac{\ell_\perp\cdot k_\perp\, \bar{n}^\mu \bar{n}^\nu}{\bar{n}\cdot k}\Big]$$
$$\times\Big[n\cdot k'\, g_\perp^{\lambda\tau} - n^\lambda \ell'^\tau_\perp - n^\tau k'^\lambda_\perp + \frac{\ell'_\perp\cdot k'_\perp\, n^\lambda n^\tau}{n\cdot k'}\Big]$$

More Feynman Rules:



Lipatov vertex

$$= i\Big[\bar{u}_n \frac{\bar{\slashed{n}}}{2} T^A u_n\Big]\Big[\frac{8\pi\alpha_s}{\vec{q}_\perp^{\,2}\,\vec{q}_\perp^{\,\prime 2}}\; igf^{ABC}\Big(q_\perp^\mu + q_\perp^{\prime\mu} - n\cdot q\,\frac{\bar{n}^\mu}{2} - \bar{n}\cdot q'\,\frac{n^\mu}{2} - \frac{n^\mu \vec{q}_\perp^{\,\prime 2}}{n\cdot q} - \frac{\bar{n}^\mu \vec{q}_\perp^{\,2}}{\bar{n}\cdot q'}\Big)\Big]\Big[\bar{v}_{\bar{n}} \frac{\slashed{n}}{2} \bar{T}^B v_{\bar{n}}\Big]$$



$$= i\Big[\bar{u}_n \frac{\bar{\slashed{n}}}{2} T^A u_n\Big]\Big[\bar{v}_{\bar{n}} \frac{\slashed{n}}{2} \bar{T}^B v_{\bar{n}}\Big]\Big(\frac{8\pi\alpha_s}{\vec{q}_\perp^{\,2}\,\vec{q}_\perp^{\,\prime 2}}\Big)$$

$$\times\Big\{g^2 f^{C_1AE} f^{C_2BE}\Big[-g_\perp^{\mu_1\mu_2} - \frac{n^{\mu_1}(2q_\perp^{\mu_2} + k_{2\perp}^{\mu_2})}{n\cdot k_1} + \frac{(2q_\perp^{\prime\mu_1} - k_{1\perp}^{\mu_1})\bar{n}^{\mu_2}}{\bar{n}\cdot k_2} + \frac{\bar{n}^{\mu_1}n^{\mu_2} - n^{\mu_1}\bar{n}^{\mu_2}}{2}$$

$$+\frac{n^{\mu_1}\bar{n}^{\mu_2}}{n\cdot k_1\,\bar{n}\cdot k_2}\Big(\vec{q}_\perp\cdot\vec{q}_\perp^{\,\prime} + \vec{k}_{1\perp}\cdot\vec{k}_{2\perp} + \vec{k}_{1\perp}\cdot\vec{q}_\perp^{\,\prime} - \vec{k}_{2\perp}\cdot\vec{q}_\perp - \tfrac{1}{2}n\cdot k_2\,\bar{n}\cdot k_2 - \tfrac{1}{2}n\cdot k_1\,\bar{n}\cdot k_1\Big)$$

$$+ n^{\mu_1}n^{\mu_2}\Big(\frac{\vec{q}_\perp^{\,\prime 2}}{n\cdot q\,n\cdot k_2} + \frac{\bar{n}\cdot k_2}{2n\cdot k_1}\Big) + \bar{n}^{\mu_1}\bar{n}^{\mu_2}\Big(\frac{-\vec{q}_\perp^{\,2}}{\bar{n}\cdot k_1\,\bar{n}\cdot q'} + \frac{n\cdot k_1}{2\bar{n}\cdot k_2}\Big)\Big]$$

$$+g^2 f^{C_2AE} f^{C_1BE}\Big[-g_\perp^{\mu_1\mu_2} + \frac{\bar{n}^{\mu_1}(2q_\perp^{\prime\mu_2} - k_{2\perp}^{\mu_2})}{\bar{n}\cdot k_1} - \frac{(2q_\perp^{\mu_1} + k_{1\perp}^{\mu_1})n^{\mu_2}}{n\cdot k_2} + \frac{n^{\mu_1}\bar{n}^{\mu_2} - \bar{n}^{\mu_1}n^{\mu_2}}{2}$$

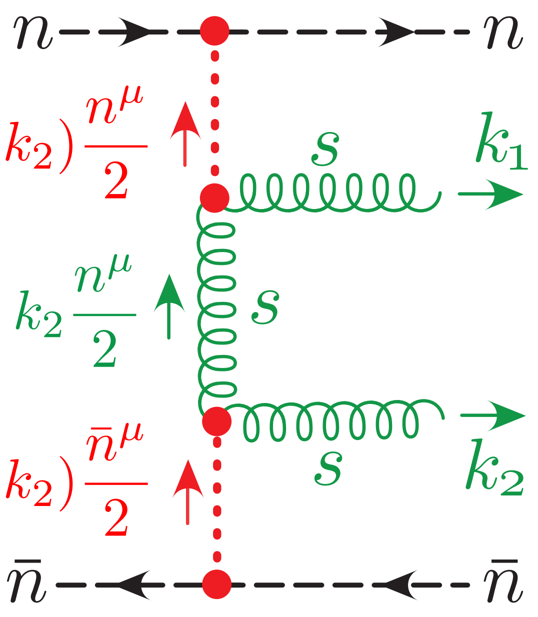
$$+\frac{\bar{n}^{\mu_1}n^{\mu_2}}{n\cdot k_2\,\bar{n}\cdot k_1}\Big(\vec{q}_\perp\cdot\vec{q}_\perp^{\,\prime} + \vec{k}_{1\perp}\cdot\vec{k}_{2\perp} + \vec{k}_{2\perp}\cdot\vec{q}_\perp^{\,\prime} - \vec{k}_{1\perp}\cdot\vec{q}_\perp - \tfrac{1}{2}n\cdot k_2\,\bar{n}\cdot k_2 - \tfrac{1}{2}n\cdot k_1\,\bar{n}\cdot k_1\Big)$$

$$+n^{\mu_1}n^{\mu_2}\Big(\frac{\vec{q}_\perp^{\,\prime 2}}{n\cdot q\,n\cdot k_1} + \frac{\bar{n}\cdot k_1}{2n\cdot k_2}\Big) + \bar{n}^{\mu_1}\bar{n}^{\mu_2}\Big(\frac{-\vec{q}_\perp^{\,2}}{\bar{n}\cdot k_2\,\bar{n}\cdot q'} + \frac{n\cdot k_2}{2\bar{n}\cdot k_1}\Big)\Big]\Big\}$$

plus analogs replacing collinear quarks by gluons

Are there differences between n-$\bar{n}$ and n-s forward scattering?

Yes. Consider:



$$q'^{\mu} = q'^{\mu}_{\perp} - \bar{n}\cdot(k_1+k_2)\frac{n^\mu}{2}$$

$$q_2^{\mu} = (q_{\perp} - k_{2\perp})^\mu + n\cdot k_1 \frac{\bar{n}^\mu}{2} - \bar{n}\cdot k_2 \frac{n^\mu}{2}$$

$$q^{\mu} = q^{\mu}_{\perp} + n\cdot(k_1+k_2)\frac{\bar{n}^\mu}{2}$$

Scattering is forward in one light-cone momentum at each soft vertex!

Allowed because:

$$n\cdot k_{1,2} \ll n\cdot p_{\bar{n}}$$

$$\bar{n}\cdot k_{1,2} \ll \bar{n}\cdot p_{n}$$

$$\lambda \qquad\qquad \lambda^0$$

there is an allowed routing of soft momentum through collinear lines

One Loop EFT graphs

- QCD topologies will appear more than once (soft, collinear, …)
- Each dominated by one invariant mass scale & one rapidity
- Require invariant mass regulator (dim.reg.)

Requires rapidity regulator for Glauber potential $|2k^z|^{-\eta}\nu^{\eta}$ and for Wilson lines

Use Chiu, Jain, Neill, Rothstein regulator, works like $\overline{\rm MS}$:

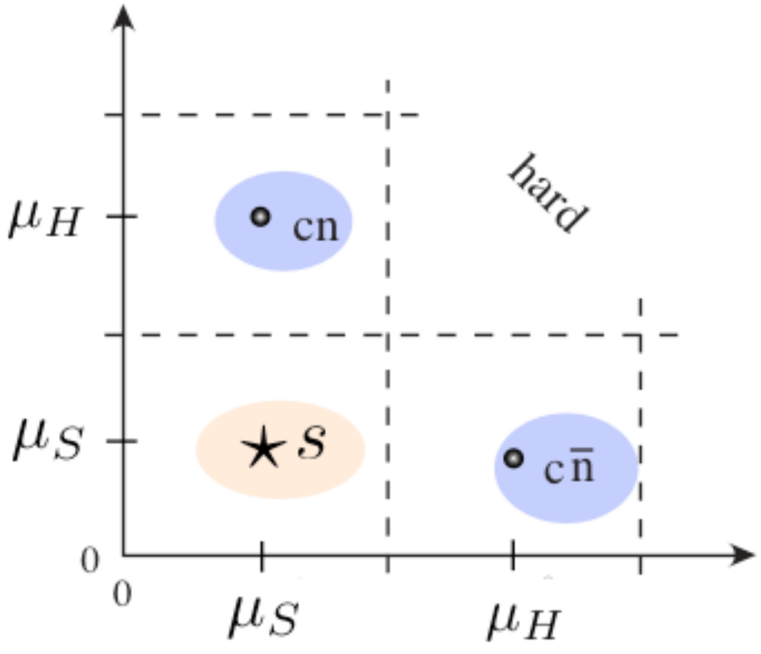
$$S_n = \sum_{\rm perms} \exp\left\{ \frac{-g}{n\cdot\mathcal{P}} \left[\frac{w|2\mathcal{P}^z|^{-\eta/2}}{\nu^{-\eta/2}} n\cdot A_s \right] \right\}$$

$$W_n = \sum_{\rm perms} \exp\Bigg\{ \frac{-g}{\bar{n}\cdot\mathcal{P}} \Bigg] \frac{w^2|\bar{n}\cdot\mathcal{P}|^{-\eta}}{\nu^{-\eta}} \bar{n}\cdot A_n \Bigg] \Bigg\}$$

$\frac{1}{\eta}$ $\ln(\nu)$

$\updownarrow$

$\frac{1}{\epsilon}$ $\ln(\mu)$



- use subtractions rather than sharp boundaries to preserve symmetry

- Zero-bin subtractions, avoid double counting IR regions

1-loop graphs:

$$S = \tilde{S} - S^{(G)}$$

(construction ala Manohar & IS)

naive soft graph

Glauber limit of soft graph

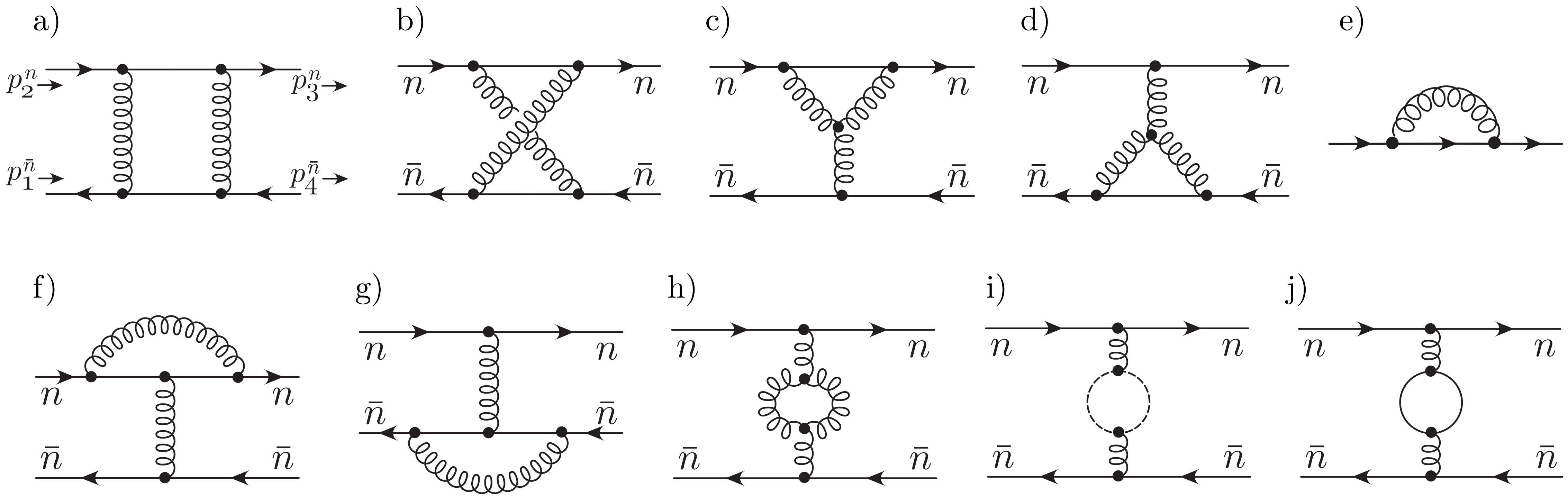
$$C_n = \tilde{C}_n - C_n^{(S)} - C_n^{(G)} + C_n^{(GS)}$$

naive collinear graph

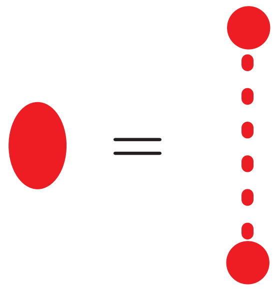
Glauber limit of collinear graph

eq. One Loop $q\bar{q}$ scattering

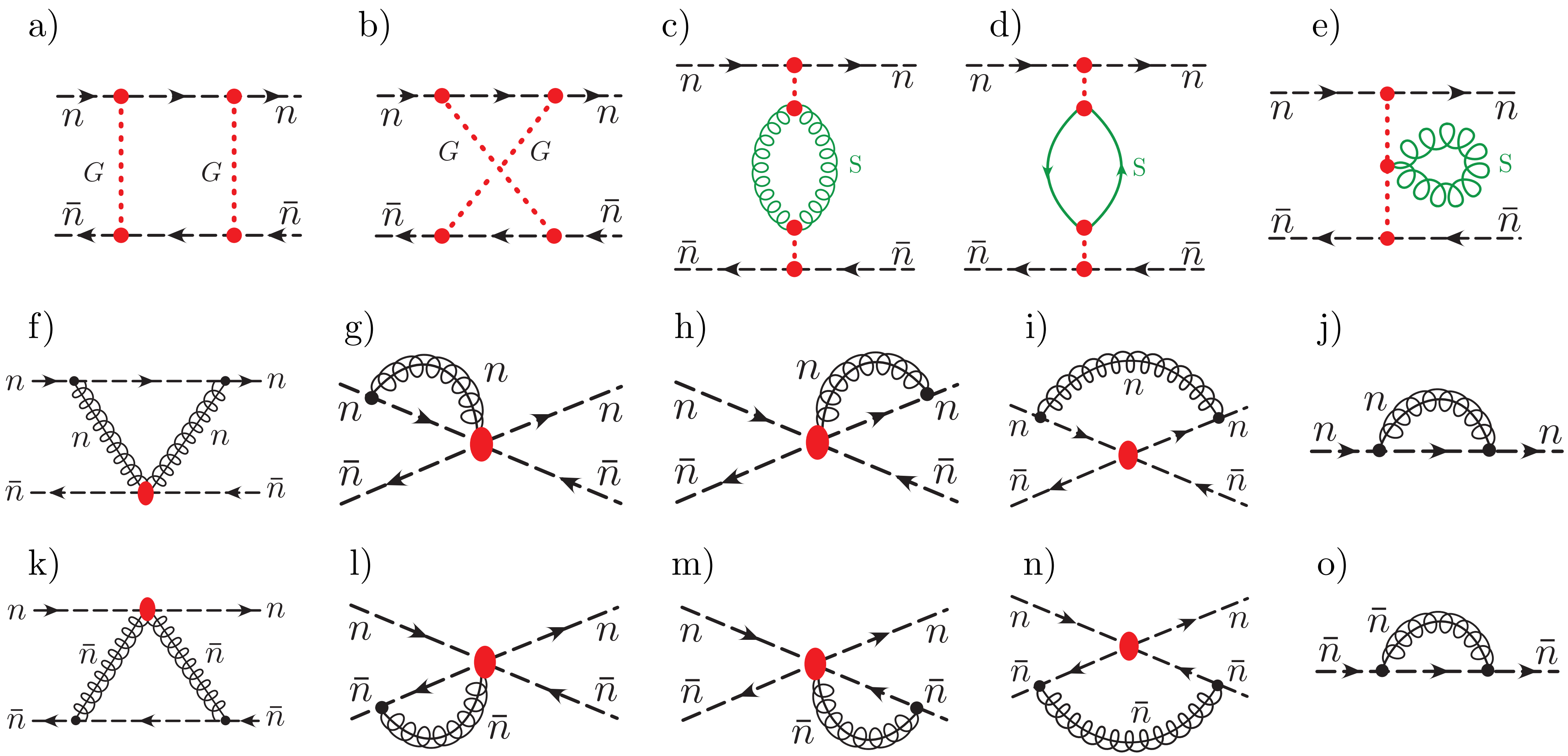
QCD graphs with leading power contributions, $s \gg t$

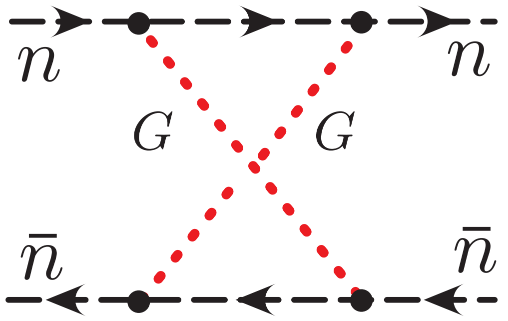


eq. One Loop $q\bar{q}$ scattering



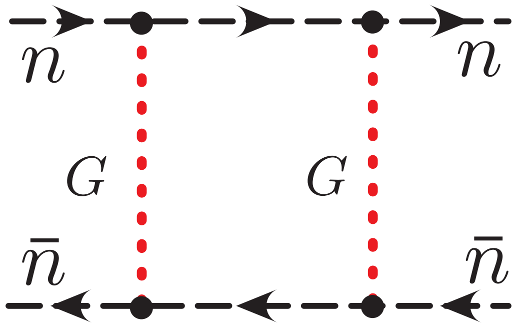
Leading Power EFT graphs (Glauber, Soft, & Collinear Loops)



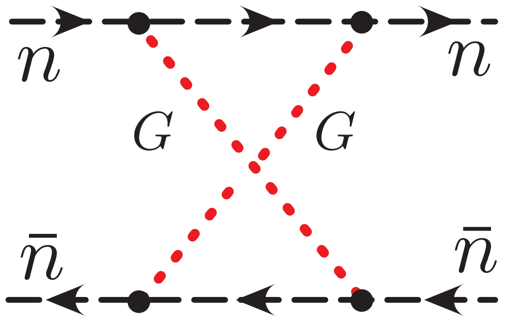


$$I_{\mathrm{Gcbox}} = \int \frac{{\mathchar'26\mkern-12mu d}^{d-2}k_\perp \; {\mathchar'26\mkern-12mu d}k^+ \; {\mathchar'26\mkern-12mu d}k^-}{2(\vec{k}_\perp^{\,2})(\vec{k}_\perp + \vec{q}_\perp)^2 \Big(k^+ + p_3^+ - (\vec{k}_\perp + \vec{q}_\perp/2)^{\,2}/p_2^- + i0\Big)\Big(+k^- + p_1^- - (\vec{k}_\perp + \vec{q}_\perp/2)^{\,2}/p_1^+ + i0\Big)}$$

Must regulate light-cone singularities.



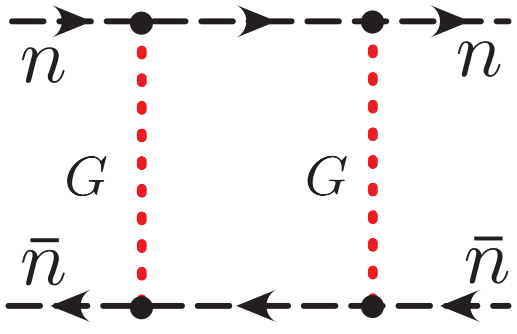
$$I_{\mathrm{Gbox}} = \int \frac{{\mathchar'26\mkern-12mu d}^{d-2}k_\perp \; {\mathchar'26\mkern-12mu d}k^+ \; {\mathchar'26\mkern-12mu d}k^-}{2(\vec{k}_\perp^{\,2})(\vec{k}_\perp + \vec{q}_\perp)^2 \Big(k^+ + p_3^+ - (\vec{k}_\perp + \vec{q}_\perp/2)^{\,2}/p_2^- + i0\Big)\Big(-k^- + p_4^- - (\vec{k}_\perp + \vec{q}_\perp/2)^{\,2}/p_1^+ + i0\Big)}$$



$$I_{\rm Gcbox} = \int \frac{đ^{d-2}k_\perp\ đk^0\ đk^z\ |k^z|^{-2\eta}(\nu/2)^{2\eta}}{(\vec{k}_\perp^{\,2})(\vec{k}_\perp+\vec{q}_\perp)^2\Big(k^+-\Delta_1(k_\perp)+i0\Big)\Big(k^--\Delta_2(k_\perp)+i0\Big)}$$

$$= 0$$

k^0 poles on same side



$$I_{\rm Gbox} = \int \frac{đ^{d-2}k_\perp\ đk^0\ đk^z\ |k^z|^{-2\eta}(\nu/2)^{2\eta}}{(\vec{k}_\perp^{\,2})(\vec{k}_\perp+\vec{q}_\perp)^2\Big(k^+-\Delta_1(k_\perp)+i0\Big)\Big(-k^--\Delta_2(k_\perp)+i0\Big)}$$

$$= -i\int \frac{đ^{d-2}k_\perp đk^z\ |k^z|^{-2\eta}\ (\nu/2)^{2\eta}}{(\vec{k}_\perp^{\,2})(\vec{k}_\perp+\vec{q}_\perp)^2(-2k^z-\Delta+i0)}$$

$$= \frac{-i}{4\pi}\int \frac{đ^{d-2}k_\perp}{(\vec{k}_\perp^{\,2})(\vec{k}_\perp+\vec{q}_\perp)^2}\Big[(\nu/2)^{2\eta}\,(-2i\pi)\csc(2\pi\eta)\sin(\pi\eta)\,(i\Delta)^{-2\eta}\Big]$$

$$= \Big(\frac{-i}{4\pi}\Big)\int \frac{đ^{d-2}k_\perp}{(\vec{k}_\perp^{\,2})(\vec{k}_\perp+\vec{q}_\perp)^2}\ \Big[-i\pi+\mathcal{O}(\eta)\Big]$$

generic,
Glauber phase

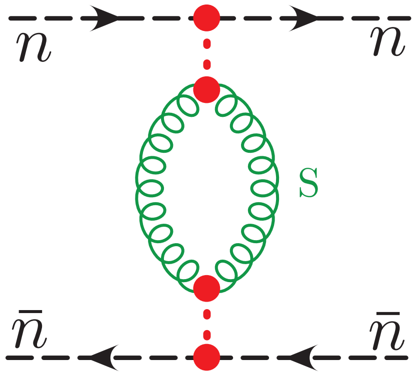
independent of Δ
hence collinears behave
like eikonal propagators
here (but not always!)

Similar for other Glauber boxes (softs, gluons, …).

eg. Rapidity divergences from Wilson lines

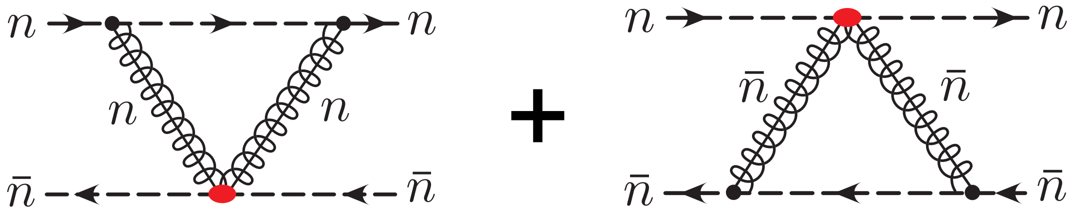
$\mu^2 \sim \nu^2 \sim -t$

rapidity divergent



$$= -\frac{i\alpha_s^2}{t}\,\mathcal{S}_3^{n\bar{n}}\Big\{\frac{8}{\eta}\,g(\epsilon,\mu^2/t)+\frac{4}{\epsilon^2}+\frac{4}{\epsilon}\ln\Big(\frac{\mu^2}{\nu^2}\Big)+\boxed{4\ln\Big(\frac{\mu^2}{\nu^2}\Big)\ln\Big(\frac{\mu^2}{-t}\Big)}-2\ln^2\Big(\frac{\mu^2}{-t}\Big)+\frac{\pi^2}{3}$$
$$+2\Big(-\frac{11}{3\epsilon}-\frac{11}{3}\ln\frac{\mu^2}{-t}-\frac{67}{9}\Big)\Big\}$$

β-function (without ghost graphs)



$$= \frac{i\alpha_s^2}{t}\,\mathcal{S}_3^{n\bar{n}}\Big[\Big\{\frac{4}{\eta}\,g(\epsilon,\mu^2/t)-\frac{4}{\epsilon}\ln\Big(\frac{\nu}{\bar{n}\cdot p_3}\Big)-4\ln\Big(\frac{\nu}{\bar{n}\cdot p_3}\Big)\ln\Big(\frac{\mu^2}{-t}\Big)-\frac{3}{\epsilon}-3\ln\Big(\frac{\mu^2}{-t}\Big)-6+\frac{4\pi^2}{3}\Big\}$$
$$+\Big\{\frac{4}{\eta}\,g(\epsilon,\mu^2/t)-\frac{4}{\epsilon}\ln\Big(\frac{\nu}{n\cdot p_4}\Big)-4\ln\Big(\frac{\nu}{n\cdot p_4}\Big)\ln\Big(\frac{\mu^2}{-t}\Big)-\frac{3}{\epsilon}-3\ln\Big(\frac{\mu^2}{-t}\Big)-6+\frac{4\pi^2}{3}\Big\}\Big]$$
$$= \frac{i\alpha_s^2}{t}\,\mathcal{S}_3^{n\bar{n}}\Big\{\frac{8}{\eta}\,g(\epsilon,\mu^2/t)-\frac{4}{\epsilon}\ln\Big(\frac{\nu^2}{s}\Big)-\boxed{4\ln\Big(\frac{\nu^2}{s}\Big)\ln\Big(\frac{\mu^2}{-t}\Big)}-\frac{6}{\epsilon}-6\ln\Big(\frac{\mu^2}{-t}\Big)-12+\frac{8\pi^2}{3}\Big\}.$$

opposite sign for this rapidity divergence

$\mu^2 \sim -t$

$\nu^2 \sim s$

One Loop Results & Matching

$$\mathcal{S}_1^{n\bar n} = -\Big[\bar u_n T^A T^B \frac{\bar{n}\!\!\!/}{2} u_n\Big]\Big[\bar v_{\bar n} \bar T^A \bar T^B \frac{n\!\!\!/}{2} v_{\bar n}\Big]\,, \qquad \mathcal{S}_2^{n\bar n} = C_F\Big[\bar u_n T^A \frac{\bar{n}\!\!\!/}{2} u_n\Big]\Big[\bar v_{\bar n} \bar T^A \frac{n\!\!\!/}{2} v_{\bar n}\Big]\,,$$

$$\mathcal{S}_3^{n\bar n} = C_A\Big[\bar u_n T^A \frac{\bar{n}\!\!\!/}{2} u_n\Big]\Big[\bar v_{\bar n} \bar T^A \frac{n\!\!\!/}{2} v_{\bar n}\Big]\,, \qquad \mathcal{S}_4^{n\bar n} = T_F n_f\Big[\bar u_n T^A \frac{\bar{n}\!\!\!/}{2} u_n\Big]\Big[\bar v_{\bar n} \bar T^A \frac{n\!\!\!/}{2} v_{\bar n}\Big]\,.$$

m = gluon mass IR regulator

$$\text{Glauber Loops} = \frac{i\alpha_s^2}{t}\,\mathcal{S}_1^{n\bar n}\Big[8i\pi\ln\Big(\frac{-t}{m^2}\Big)\Big]$$

$$\text{Soft Loops} = \frac{i\alpha_s^2}{t}\,\mathcal{S}_3^{n\bar n}\Big\{-\frac{8}{\eta}h(\epsilon,\mu^2/m^2) - \frac{8}{\eta}g(\epsilon,\mu^2/t) - 4\ln\Big(\frac{\mu^2}{\nu^2}\Big)\ln\Big(\frac{m^2}{-t}\Big)$$
$$- 2\ln^2\Big(\frac{\mu^2}{m^2}\Big) + 2\ln^2\Big(\frac{\mu^2}{-t}\Big) - \frac{2\pi^2}{3} + \frac{22}{3}\ln\frac{\mu^2}{-t} + \frac{134}{9}\Big\} \longleftarrow$$
$$+ \frac{i\alpha_s^2}{t}\,\mathcal{S}_4^{n\bar n}\Big[-\frac{8}{3}\ln\Big(\frac{\mu^2}{-t}\Big) - \frac{40}{9}\Big].$$

no $1/\epsilon$ poles

(after coupling renormalization)

$$\text{Collinear Loops} = \frac{i\alpha_s^2}{t}\,\mathcal{S}_3^{n\bar n}\Big\{\frac{8}{\eta}h\Big(\epsilon,\frac{\mu^2}{m^2}\Big) + \frac{8}{\eta}g\Big(\epsilon,\frac{\mu^2}{-t}\Big) + 4\ln\Big(\frac{\nu^2}{s}\Big)\ln\Big(\frac{-t}{m^2}\Big) + 2\ln^2\Big(\frac{m^2}{-t}\Big) + 4 + \frac{4\pi^2}{3}\Big\}$$
$$+ \frac{i\alpha_s^2}{t}\,\mathcal{S}_2^{n\bar n}\Big[-4\ln^2\Big(\frac{m^2}{-t}\Big) - 12\ln\Big(\frac{m^2}{-t}\Big) - 14\Big]$$

$i\pi$ purely from Glauber loop

$$\begin{aligned}\text{Total SCET} &= \frac{i\alpha_s^2}{t}\,\mathcal{S}_1^{n\bar n}\Big[8i\pi\ln\Big(\frac{-t}{m^2}\Big)\Big] + \frac{i\alpha_s^2}{t}\,\mathcal{S}_2^{n\bar n}\Big[-4\ln^2\Big(\frac{m^2}{-t}\Big) - 12\ln\Big(\frac{m^2}{-t}\Big) - 14\Big] \\ &+ \frac{i\alpha_s^2}{t}\,\mathcal{S}_3^{n\bar n}\Big\{-4\ln\Big(\frac{s}{-t}\Big)\ln\Big(\frac{-t}{m^2}\Big) + \frac{22}{3}\ln\frac{\mu^2}{-t} + \frac{170}{9} + \frac{2\pi^2}{3}\Big\} \\ &+ \frac{i\alpha_s^2}{t}\,\mathcal{S}_4^{n\bar n}\Big[-\frac{8}{3}\ln\Big(\frac{\mu^2}{-t}\Big) - \frac{40}{9}\Big]\end{aligned}$$

rapidity divergences cancel

leave behind large log

$$\text{Total SCET} = \text{Total QCD}\ (s \gg t)$$

- IR divergences are all reproduced
- no hard matching (no loops with momenta $\sim s$)

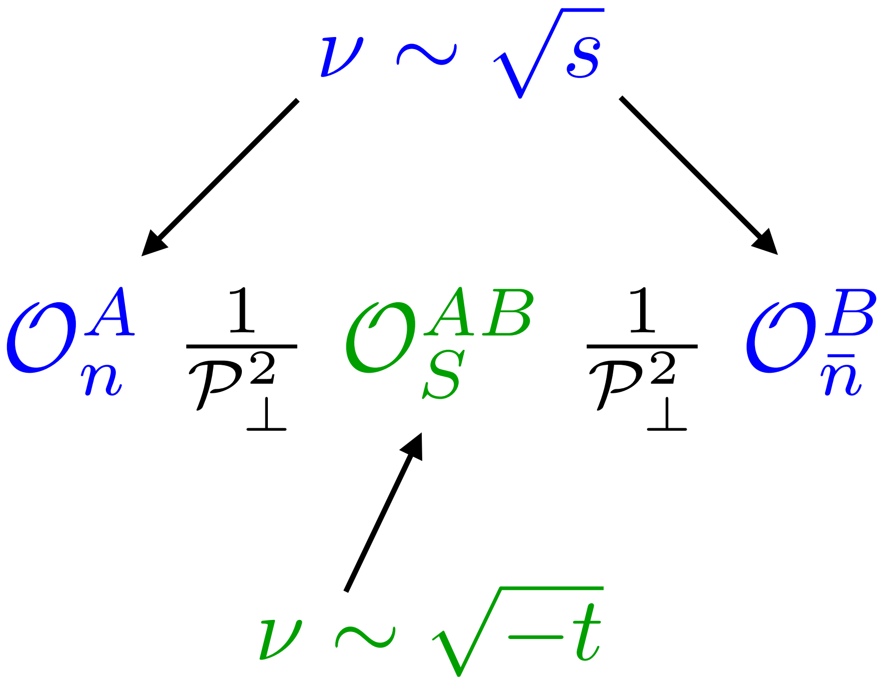
offshell Glauber lines in loop graphs are always sequestered in tree level subcomponents (equal t,z)

→

No loop corrections to $\mathcal{L}_G^{\mathrm{II}(0)}$

Forward Scattering

Gluon Reggeization



$\nu \sim \sqrt{s}$

$$\mathcal{O}_n^A \, \frac{1}{\mathcal{P}_\perp^2} \, \mathcal{O}_S^{AB} \, \frac{1}{\mathcal{P}_\perp^2} \, \mathcal{O}_{\bar{n}}^B$$

$\nu \sim \sqrt{-t}$

Consider separate rapidity renormalization of soft & collinear component operators

Either run collinear operators from $\nu \sim \sqrt{s}$ to $\nu \sim \sqrt{-t}$, or run soft operator.

$$\nu \frac{d}{d\nu} (\mathcal{O}_n^{qA} + \mathcal{O}_n^{gA}) = \gamma_{n\nu} (\mathcal{O}_n^{qA} + \mathcal{O}_n^{gA})$$

$$\gamma_{n\nu} \;=\; \frac{\alpha_s(\mu) C_A}{2\pi} \ln\left(\frac{-t}{m^2}\right)$$

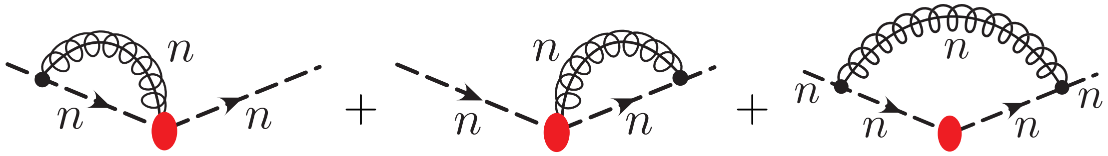
gives: $$\left(\frac{s}{-t}\right)^{-\gamma_{n\nu}}$$

virtual anom.dim. is Regge exponent for gluon

Gluon Reggeization

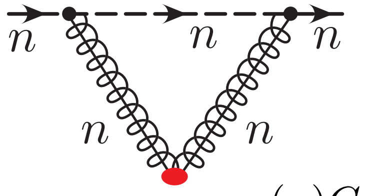
eg. $\mathcal{O}_n^{iA}$ $\qquad \vec{\mathcal{O}}_n^{A\text{bare}} = \hat{V}_{\mathcal{O}_n} \cdot \vec{\mathcal{O}}_n^{A}(\nu, \mu)$

anom.dim. $\quad \hat{\gamma}_{n\nu} = -\hat{V}_n^{-1} \cdot \nu \frac{\partial}{\partial \nu} \hat{V}_n$ $\qquad$ 1-loop: $\gamma_{n\nu}^{ij} = -(\nu d/d\nu)\delta V_n^{ij}$

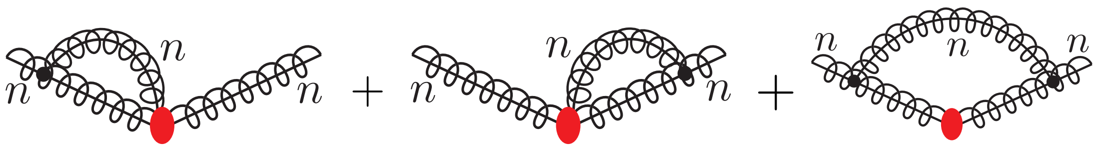


+ w.fn. renorm

$$\gamma_{n\nu}^{qq} = \frac{\alpha_s(\mu) C_A}{2\pi} \ln\left(\frac{\mu^2}{m^2}\right)$$

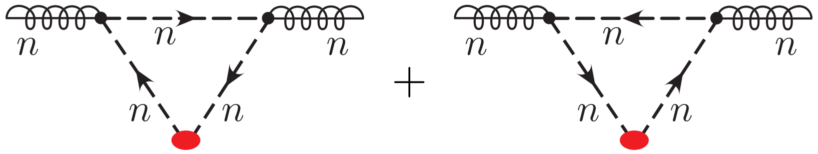


$$\gamma_{n\nu}^{gq} = \frac{\alpha_s(\mu) C_A}{2\pi} \ln\left(\frac{-t}{\mu^2}\right)$$



+ gluon w.fn.

$$\gamma_{n\nu}^{gg} = \frac{\alpha_s(\mu) C_A}{2\pi} \ln\left(\frac{-t}{m^2}\right)$$



$$\gamma_{n\nu}^{gq} = 0$$

$$\boxed{\nu \frac{d}{d\nu}(\mathcal{O}_n^{qA} + \mathcal{O}_n^{gA}) = \gamma_{n\nu}(\mathcal{O}_n^{qA} + \mathcal{O}_n^{gA})}$$

combination remains together under RGE
since no loop-level hard matching coefficient

$$\gamma_{n\nu} \equiv \gamma_{n\nu}^{qq} + \gamma_{n\nu}^{gq} = \gamma_{n\nu}^{gg} + \gamma_{n\nu}^{qg} = \frac{\alpha_s(\mu) C_A}{2\pi} \ln\left(\frac{-t}{m^2}\right)$$

(IR divergent)

Gluon Reggeization

Standard RGE form: $\mathcal{O}(\nu_1) = U_{n\nu}(\nu_1, \nu_0)\mathcal{O}(\nu_0)$

$$(\mathcal{O}_n^{qA} + \mathcal{O}_n^{gA})(\nu_1) = \left(\frac{\nu_0}{\nu_1}\right)^{-\gamma_{n\nu}} (\mathcal{O}_n^{qA} + \mathcal{O}_n^{gA})(\nu_0)$$

same factor from $\bar{n}$

soft: no large logs for $\nu = \sqrt{-t}$

$$(\mathcal{O}_n^{qA} + \mathcal{O}_n^{gA})(\nu = \sqrt{-t})\frac{1}{\mathcal{P}_\perp^2}\mathcal{O}_s^{AB}(\nu = \sqrt{-t})\frac{1}{\mathcal{P}_\perp^2}(\mathcal{O}_{\bar{n}}^{qB} + \mathcal{O}_{\bar{n}}^{gB})(\nu = \sqrt{-t})$$

$$= \boxed{\left(\frac{s}{-t}\right)^{-\gamma_{n\nu}}} (\mathcal{O}_n^{qA} + \mathcal{O}_n^{gA})(\nu = \sqrt{s})\frac{1}{\mathcal{P}_\perp^2}\mathcal{O}_s^{AB}(\nu = \sqrt{-t})\frac{1}{\mathcal{P}_\perp^2}(\mathcal{O}_{\bar{n}}^{qB} + \mathcal{O}_{\bar{n}}^{gB})(\nu = \sqrt{s})$$

Gluon Reggeization from running octet ops.

Forward Scattering & BFKL

Expand time evolution, do soft-collinear factorization term by term:

$$T \exp i \int d^4x \, \mathcal{L}_G^{\mathrm{II}(0)}(x) = \left[1 + i \int d^4y_1 \, \mathcal{L}_G^{\mathrm{II}(0)}(y_1) + \frac{i^2}{2!} \int d^4y_1 \, d^4y_2 \, \mathcal{L}_G^{\mathrm{II}(0)}(y_1) \mathcal{L}_G^{\mathrm{II}(0)}(y_2) + \ldots \right]$$

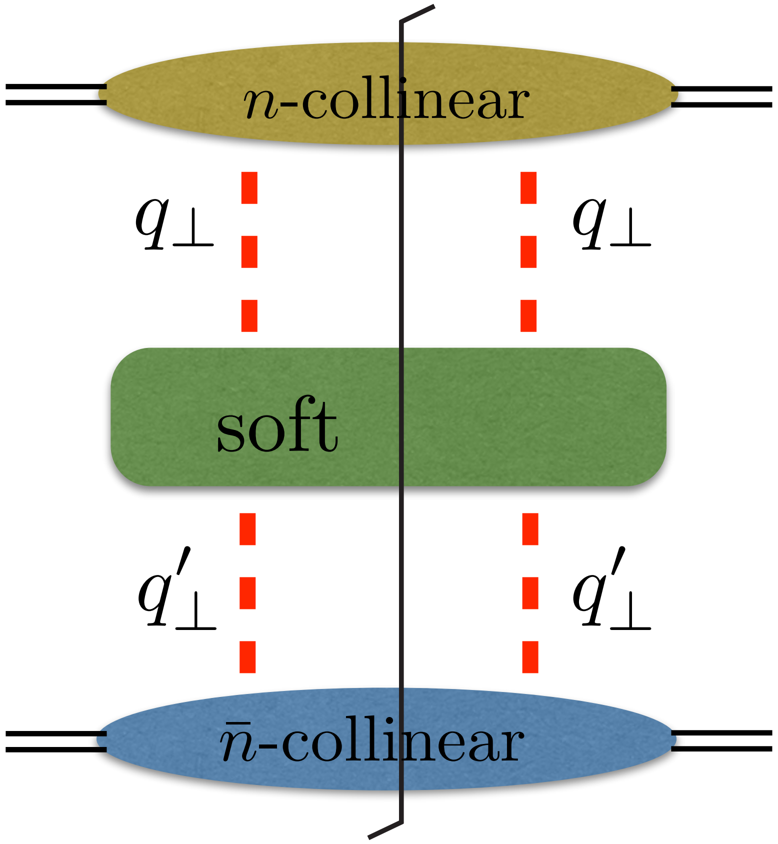
$$\sim 1 + T \sum_{k=1}^{\infty} \sum_{k'=1}^{\infty} \left[\mathcal{O}_n^{jA_i}(q_{i\perp}) \right]^k \left[\mathcal{O}_{\bar{n}}^{j'B_{i'}}(q_{i'\perp}) \right]^{k'} \otimes O_{s(k,k')}^{A_1 \cdot A_k, B_1 \cdots B_{k'}}(q_{\perp 1}, \ldots, q_{\perp k'})$$

$$\equiv 1 + \sum_{k=1}^{\infty} \sum_{k'=1}^{\infty} \; U_{(k,k')}$$

Consider forward scattering with one Glauber exchange,
but all orders in soft and collinear sectors:

$$T_{(1,1)} = \frac{1}{V_4} \sum_X \langle pp' | U_{(1,1)}^\dagger | X \rangle \langle X | U_{(1,1)} | pp' \rangle \quad = \ldots$$

$$= \int d^2q_\perp d^2q'_\perp \; C_n(q_\perp, p^-) S_G(q_\perp, q'_\perp) C_{\bar{n}}(q'_\perp, p'^+)$$



after rapidity renormalization:

$$T_{(1,1)} = \int d^2q_\perp d^2q'_\perp \; C_n(q_\perp, p^-, \nu) S_G(q_\perp, q'_\perp, \nu) C_{\bar{n}}(q'_\perp, p'^+, \nu)$$

collinear and soft functions

n-collinear function:

$$\frac{1}{V_1}\sum_{X_n}\Big\langle p\Big|\sum_{j=q,g}\int dx''^+\mathcal{O}^{jA'}_{n,k'^-}(q''_\perp)\Big(x''^+\frac{\bar n}{2}\Big)\Big|X_n\Big\rangle\Big\langle X_n\Big|\sum_{i=q,g}\int dx^+\mathcal{O}^{iA}_{n,k^-}(q_\perp)\Big(x^+\frac{\bar n}{2}\Big)\Big|p\Big\rangle$$

$$= \delta^{AA'}\,2\,\delta^2(q_\perp - q''_\perp)\,\vec{q}_\perp^{\,2}\,C_n(q_\perp, p^-)$$

Soft function:

$$S_G(q_\perp, q'_\perp) = \frac{(2\pi)^4}{V_2}\frac{\delta^{AA'}\delta^{BB'}}{(\vec{q}_\perp^{\,2}\,\vec{q}_\perp^{\,\prime 2})}\sum_X \langle 0|O^{AB}_{s(1,1)}(q_\perp, q'_\perp)|X\rangle\langle X|O^{\dagger A'B'}_{s(1,1)}(q_\perp, q'_\perp)|0\rangle$$

where the soft operator contains direct and T-product pieces:

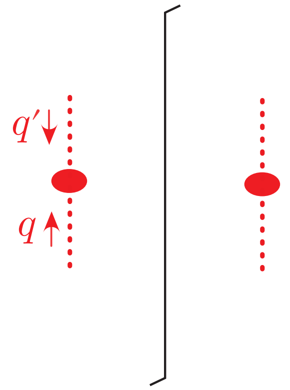
$$O^{AB}_{s(1,1)}(q_\perp, q'_\perp) \equiv \frac{(2\pi)^2}{2}\sum_{k^\pm}\int dx'^+ dx^- O^{AB}_{s(1,1),-k^\pm}(q_\perp, q'_\perp)\Big(x^-\frac{n}{2}, x'^+\frac{\bar n}{2}\Big)$$

$$= \sum_{k^\pm}\mathcal{O}^{AB}_{s,-k^\pm}(q_\perp, -q'_\perp)(\tilde{x}=0)$$

$$+\frac{i}{2}(2\pi)^2\sum_{k^\pm}\int dx'^+ dx^-\, T\sum_{i,j=q,g}\mathcal{O}^{i_n A}_{s,-k^-}(q_\perp)\Big(\frac{n}{2}x^-\Big)\;\mathcal{O}^{j_{\bar n} B}_{s,-k^+}(-q'_\perp)\Big(\frac{\bar n}{2}x'^+\Big)$$

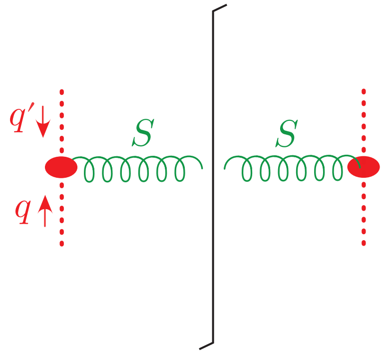
Consider rapidity renormalization for soft function that appears here:

$$S_G(q_\perp, q'_\perp) = \frac{(2\pi)^4}{V_2}\,\frac{\delta^{AA'}\delta^{BB'}}{(\vec{q}_\perp^{\,2}\,\vec{q}_\perp^{\,\prime 2})}\sum_X \langle 0|O_{s(1,1)}^{AB}(q_\perp, q'_\perp)|X\rangle\langle X|O_{s(1,1)}^{\dagger A'B'}(q_\perp, q'_\perp)|0\rangle$$

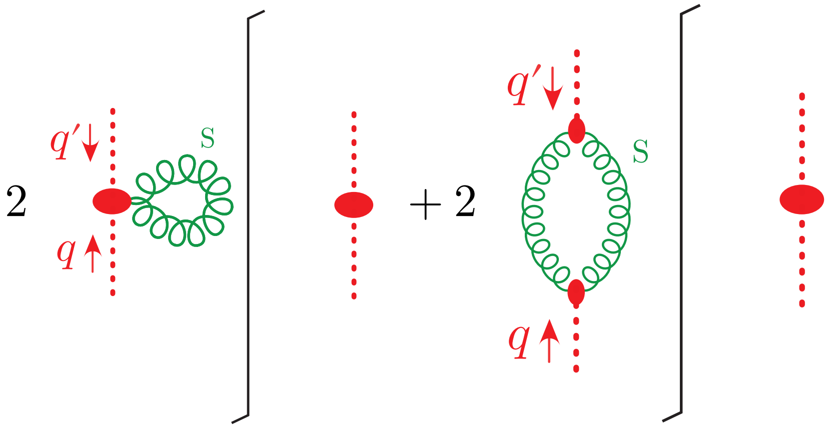


$$= S_G^{(0)}(q_\perp, q'_\perp)$$

$$= (8\pi\alpha_s)^2\delta^{AA}\,(2\pi)^2\delta^2(\vec{q}_\perp + \vec{q}\,'_\perp)$$



$$= \frac{C_A\alpha_s}{\pi^2}\,w^2\Gamma\Big(\frac{\eta}{2}\Big)\int\frac{d^2k_\perp}{(\vec{k}_\perp - \vec{q}_\perp)^2}\,S_G^{(0)}(k_\perp, q'_\perp)$$



$$= -\frac{C_A\alpha_s}{2\pi^2}\,w^2\Gamma\Big(\frac{\eta}{2}\Big)\int d^2k_\perp\frac{\vec{q}_\perp^{\,2}}{\vec{k}_\perp^{\,2}(\vec{k}_\perp - \vec{q}_\perp)^2}\,S_G^{(0)}(q_\perp, q'_\perp)$$

$$S_G(\vec{q}_\perp, \vec{q}\,'_\perp, \nu) = \int d^2k_\perp\, Z_{S_G}(q_\perp, k_\perp)\,S_G^{\rm bare}(k_\perp, q'_\perp)$$

To cancel the $1/\eta$ divergence we require

$$Z(q_\perp, k_\perp) = \delta^2(\vec{q}_\perp - \vec{k}_\perp) - \frac{2C_A\alpha_s(\mu)w^2(\nu)}{\pi^2\,\eta}\Big[\frac{1}{(\vec{k}_\perp - \vec{q}_\perp)^2} - \delta^2(\vec{q}_\perp - \vec{k}_\perp)\int\frac{d^2k_\perp\vec{q}_\perp^{\,2}}{2\vec{k}_\perp^{\,2}(\vec{k}_\perp - \vec{q}_\perp)^2}\Big]$$

$$0 = \nu\frac{d}{d\nu}S_G^{\rm bare}(q_\perp, q'_\perp)$$

➡ $$\nu\frac{d}{d\nu}S_G(q_\perp, q'_\perp, \nu) = \int d^2k_\perp\,\gamma_{S_G}(q_\perp, k_\perp)\,S_G(k_\perp, q'_\perp, \nu)$$

$$= \frac{2C_A\alpha_s(\mu)}{\pi^2}\int d^2k_\perp\Big[\frac{S_G(k_\perp, q'_\perp, \nu)}{(\vec{k}_\perp - \vec{q}_\perp)^2} - \frac{\vec{q}_\perp^{\,2}\,S_G(q_\perp, q'_\perp, \nu)}{2\vec{k}_\perp^{2}(\vec{k}_\perp - \vec{q}_\perp)^2}\Big]$$

evolution given
by
BFKL equation

(also work done by S. Fleming)

RGE consistency of linearized amplitude at LL order implies

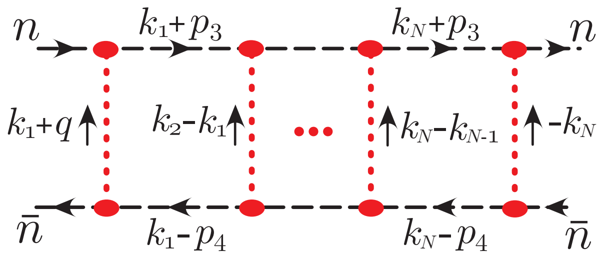
$$\nu \frac{d}{d\nu} C_n(q_\perp, p^-, \nu) = -\frac{C_A \alpha_s}{\pi^2} \int d^2 k_\perp \left[\frac{C_n(k_\perp, p^-, \nu)}{(\vec{k}_\perp - \vec{q}_\perp)^2} - \frac{\vec{q}_\perp^{\,2}\, C_n(q_\perp, p^-, \nu)}{2\vec{k}_\perp^{\,2} (\vec{k}_\perp - \vec{q}_\perp)^2} \right] \qquad -\frac{1}{2}(\mathrm{BFKL})$$

same for $C_{\bar{n}}$

Eikonal Scattering

Rapidity regulator consistent with eikonal phase

Sum up Glauber Boxes



$$= i(-2g^2)^{N+1}\mathcal{S}^{n\bar{n}}_{(N+1)}\, I^{(N)}(q_\perp)\int\frac{đk_1^z\cdots đk_N^z\,\left|2k_1^z(2k_1^z-2k_2^z)\cdots(2k_{N-1}^z-2k_N^z)2k_N^z\right|^{-\eta}\nu^{N\eta}}{2^N(-k_1^z+\Delta_1+i0)\cdots(-k_N^z+\Delta_N+i0)}$$

Fourier transform k_i^z :

$$= 2(-ig^2)^{N+1}\mathcal{S}^{n\bar{n}}_{(N+1)}\, I^{(N)}(q_\perp)\Big(\kappa_\eta\frac{\eta}{2}\Big)^{N+1}\int_{-\infty}^{+\infty}\Big[\prod_{j=1}^{N+1}dx_j\,|x_j|^{-1+\eta}\Big]\;\theta(x_2-x_1)\theta(x_3-x_2)\cdots\theta(x_{N+1}-x_N)\exp\Big[\sum_{m=1}^{N}i\Delta_m(x_{m+1}-x_m)\Big]$$

need $x_j \to 0$

ordered collapse to equal longitudinal postion

$$= -2(ig^2)^{N+1}\mathcal{S}^{n\bar{n}}_{(N+1)}I_\perp^{(N)}(q_\perp)\;\frac{1}{(N+1)!}\Big[1+\mathcal{O}(\eta)\Big]$$

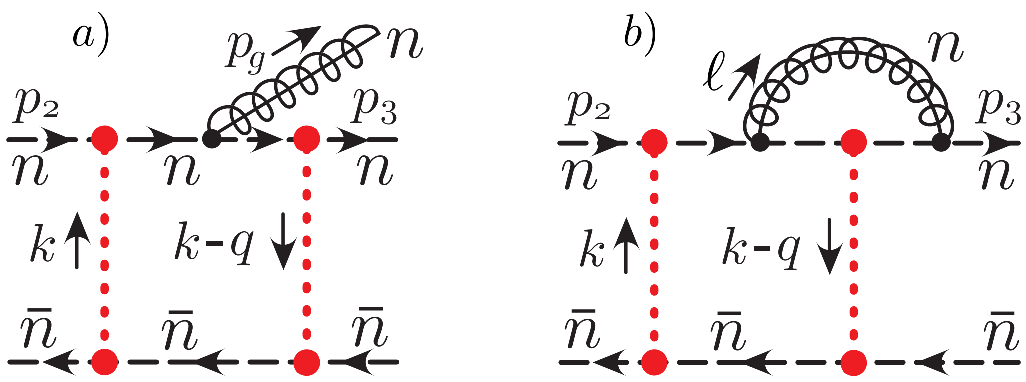
Fourier transform $q_\perp$: $\displaystyle\int đ^{d-2}q_\perp\, e^{i\vec{q}_\perp\cdot\vec{b}_\perp}\sum_{N=0}^{\infty}\text{G.Box}\,{}^{2\to 2}_{N}(q_\perp) = (\tilde{G}(b_\perp)-1)2\mathcal{S}^{n\bar{n}}$

gives classic eikonal scattering result: $\boxed{\tilde{G}(b_\perp) = e^{i\phi(b_\perp)}}$

$$\phi(b_\perp) = -\mathbf{T}_1^A\otimes\mathbf{T}_2^A\, g^2(\mu)\int\frac{đ^{d-2}q_\perp\,(\iota^\epsilon\mu^{2\epsilon})}{\vec{q}_\perp^{\,2}}\,e^{i\vec{q}_\perp\cdot\vec{b}_\perp}$$

Multi-Glauber properties

- When do graphs vanish?
- When do propagators eikonalize?



an interrupted collapse gives a vanishing graph

$$(\text{pre})\int \bar{d}^d k \, \frac{(|k_z|^{-2\eta}\,\nu^{2\eta})\ \text{Num}(k_\perp)}{\vec{k}_\perp^{\,2}(\vec{k}_\perp-\vec{q}_\perp)^2\,[n\cdot k-\Delta_1+i0]\,[n\cdot k-\Delta_2+i0]\,[\bar{n}\cdot k+\bar{\Delta}_1'-i0]} \quad = 0$$

or

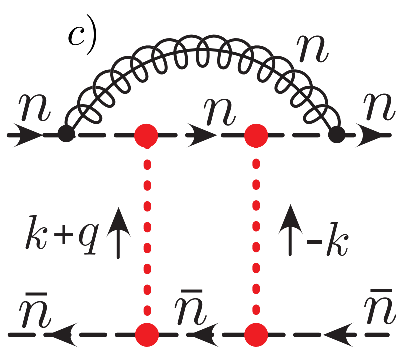
$$= -\frac{i}{4}\Big(\kappa_\eta\frac{\eta}{2}\Big)^2\int \bar{d}^{d-2}k\int_{-\infty}^{+\infty} dx_1 dx_2 d\alpha\,\theta(x_1-\alpha)\theta(\alpha-x_2)|x_1x_2|^{-1+\eta}\big[1+\mathcal{O}(\eta)\big]$$

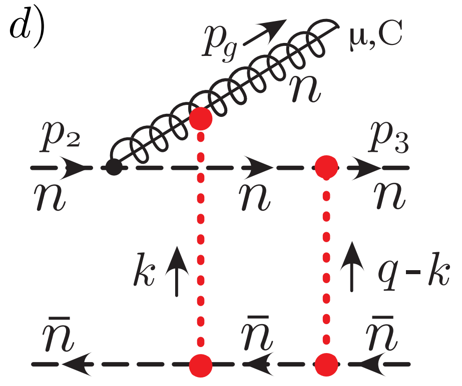
$$= \mathcal{O}(\eta)$$

Graphs with more than one Glauber exchange will vanish unless the exchanges can be moved towards each other unimpeded, so that they all occur at the same longitudinal position x_0 for both sources.

Multi-Glauber properties

- When do graphs vanish?
- When do propagators eikonalize?





nonzero

$$(\text{pre})\int \text{đ}^d k\, \frac{(|k_z|^{-2\eta}\,\nu^{2\eta})\ \text{Num}(k_\perp)}{\vec{k}_\perp^{\,2}(\vec{k}_\perp - \vec{q}_\perp)^2\,[n\cdot k + \Delta_1' - i0]\,[n\cdot k - \Delta_1 + i0]\,[\bar{n}\cdot k + \bar{\Delta}_1' - i0]}$$

$$= \frac{(\text{pre})}{4}\int \text{đ}^{d-2} k_\perp\, \frac{\text{Num}(k_\perp)}{\vec{k}_\perp^{\,2}(\vec{k}_\perp - \vec{q}_\perp)^2(\Delta_1 + \Delta_1' - i0)} \qquad \neq 0$$

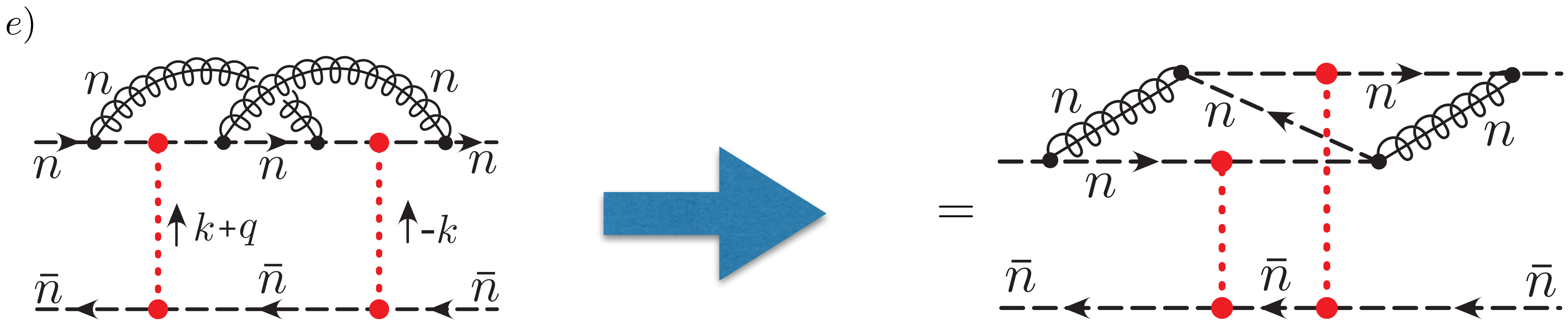
also non-eikonal

$$\Delta_1 = \frac{(\vec{k}_\perp + \vec{p}_{3\perp} - \vec{q}_\perp)^2}{\bar{n}\cdot p_3} - n\cdot p_3$$

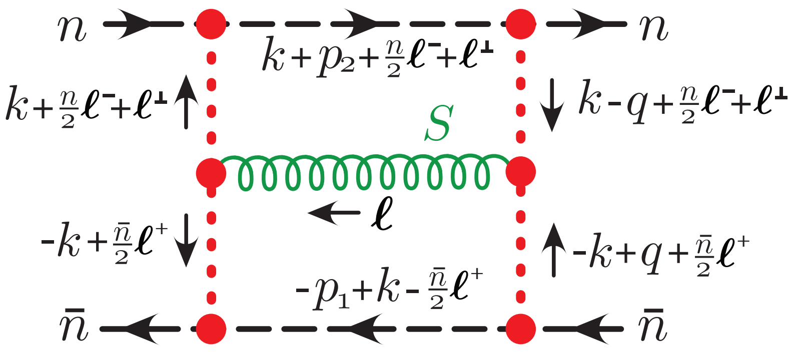
$$\Delta_1' = \frac{(\vec{k}_\perp - \vec{p}_{g\perp})^2}{\bar{n}\cdot p_g} - n\cdot p_g$$

Graphs with more than one Glauber exchange will vanish unless the exchanges can be moved towards each other unimpeded, so that they all occur at the same longitudinal position x_0 for both sources.

Multi-Glauber rules



nonzero



$$\ell^\mu \sim \lambda$$

$$k_\perp \sim \lambda$$

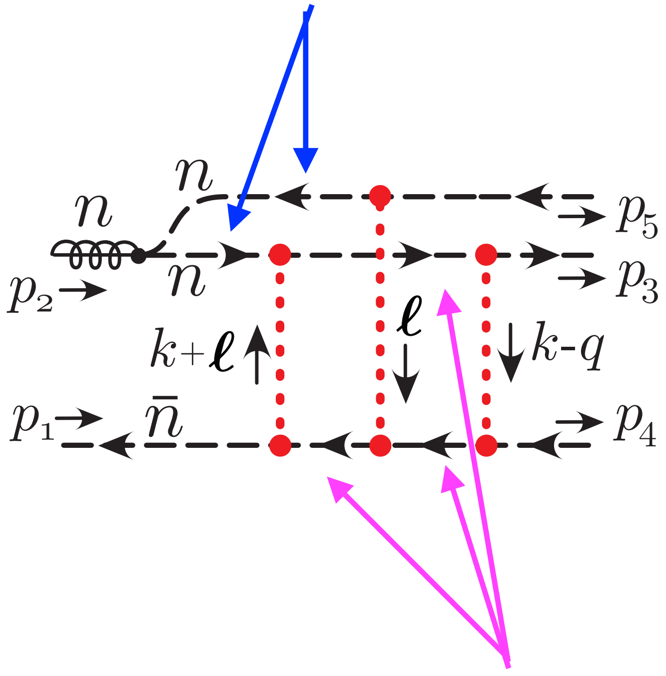
$$k^\pm \sim \lambda^2$$

nonzero

Graphs with more than one Glauber exchange will vanish unless the exchanges can be moved towards each other unimpeded, so that they all occur at the same longitudinal position x_0 for both sources.

Multi-Glauber rules

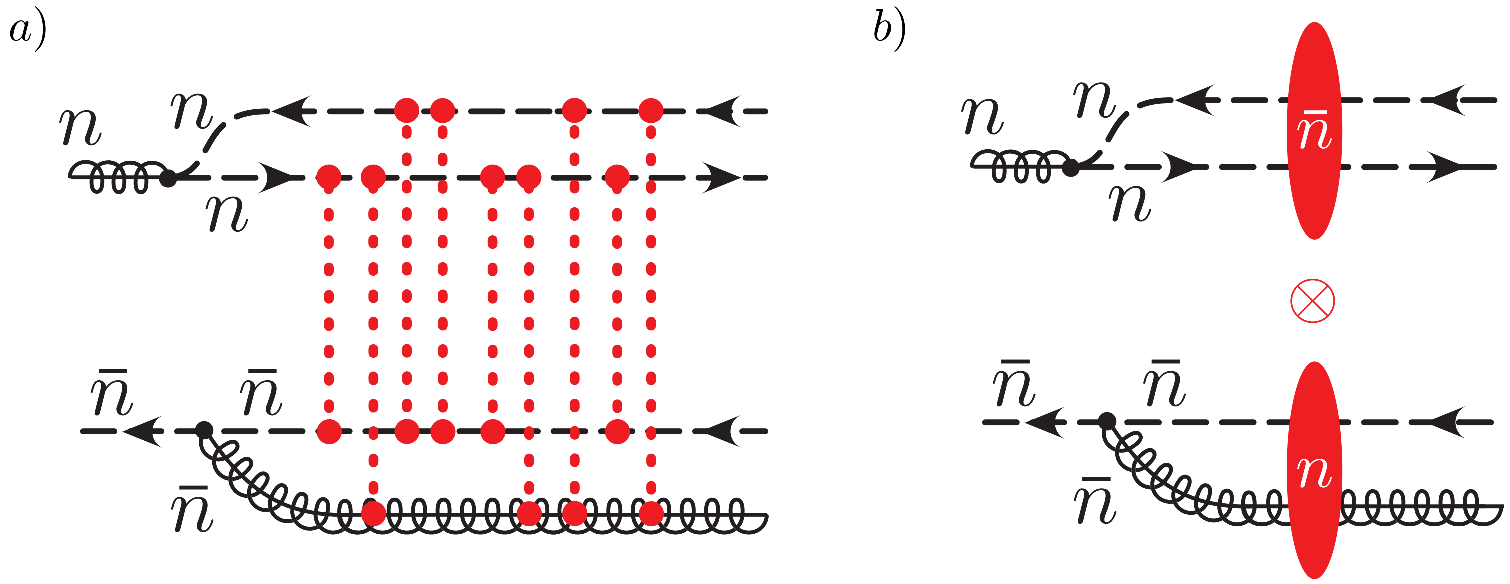
Glauber propagators are (effectively) eikonal
if and only if they are log-divergent



not eikonal

eikonal

Together these rules lead to the picture
of multiple eikonal Wilson lines crossing a shockwave:



connection to:

Balitsky's Wilson line EFT

Factorization

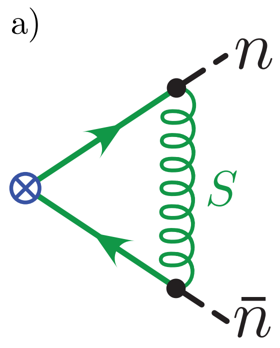
1) Wilson Line Directions
2) Spectator Interactions

Hard Scattering $\otimes$

The Cheshire Glauber

e.g. $J_\Gamma = (\bar{\xi}_n W_n) S_n^\dagger \Gamma S_{\bar{n}} (W_{\bar{n}}^\dagger \xi_{\bar{n}})$ **Active-Active and Soft Overlap**

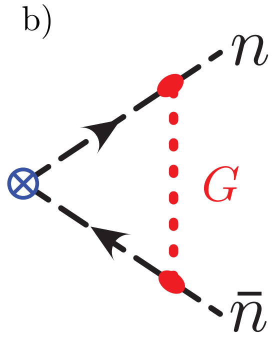
naive soft:



$$\tilde{S} = -2ig^2 C_F \mathcal{S}_\Gamma \int \bar{d}^d k \, \frac{(\iota^\epsilon \mu^{2\epsilon} \, |k_z|^{-\eta} \, \nu^\eta)}{[k^2 - m^2][n \cdot k + i0][\bar{n} \cdot k - i0]}$$
$$= \mathcal{S}_\Gamma \frac{C_F \alpha_s}{2\pi} \Big\{ \Big[\frac{-2h(\epsilon, \mu^2/m^2)}{\eta} + \ln \frac{\mu^2}{\nu^2} \Big(\frac{1}{\epsilon} + \ln \frac{\mu^2}{m^2} \Big) + \frac{1}{\epsilon^2} - \frac{1}{2} \ln^2 \frac{\mu^2}{m^2} - \frac{\pi^2}{12} \Big]$$
$$+ \Big[(i\pi) \Big(\frac{1}{\epsilon} + \ln \frac{\mu^2}{m^2} \Big) \Big] \Big\}$$

with physical directions for soft Wilson lines in hard scattering

true soft: includes 0-bin subtraction $S = \tilde{S} - S^{(G)}$ has no $i\pi$ term



Glauber:

$$G = S^{(G)} = \bar{u}_n \Gamma v_{\bar{n}} \frac{C_F \alpha_s}{2\pi} \Big[(i\pi) \Big(\frac{1}{\epsilon} + \ln \frac{\mu^2}{m^2} \Big) \Big]$$

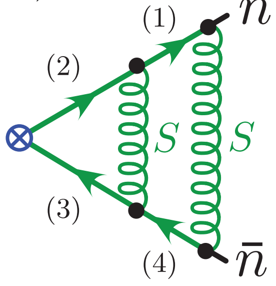
Glaubers give $(i\pi)$ terms

BUT $(\tilde{S} - S^{(G)}) + G = \tilde{S}$

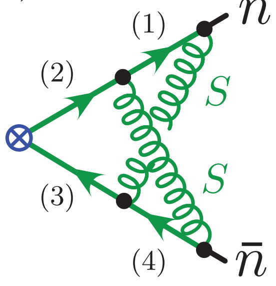
- so we don’t see Glauber in Hard Matching
- can absorb this Glauber into Soft Wilson lines if they have proper directions

This continues at higher orders:

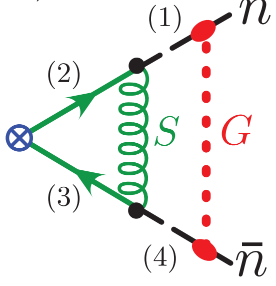
$S_1)$



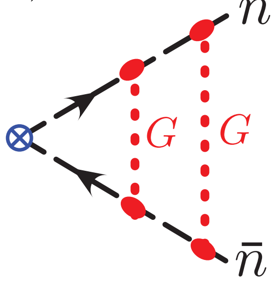
$S_2)$



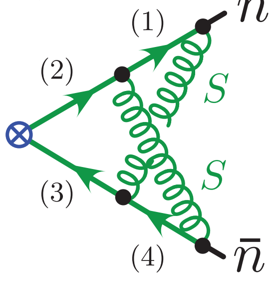
$SG)$



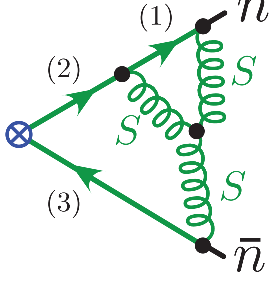
$G)$



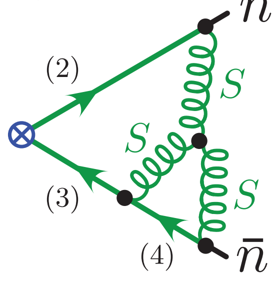
$S_3)$



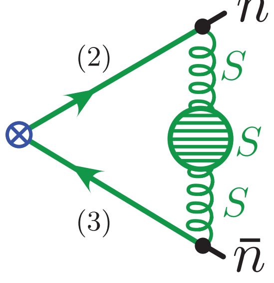
$S_4)$

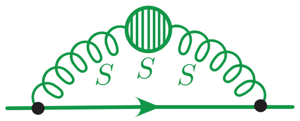


$S_5)$

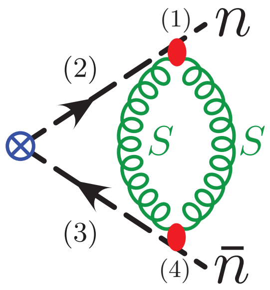


$S_6)$

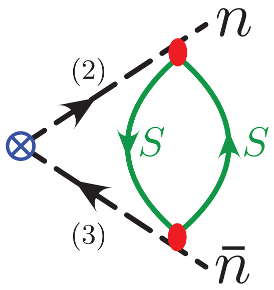




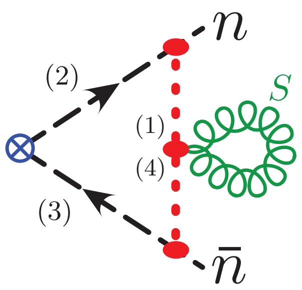
$GS_1)$



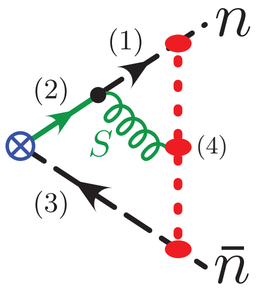
$GS_2)$



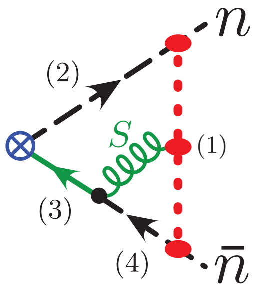
$GS_3)$



$SL_1)$

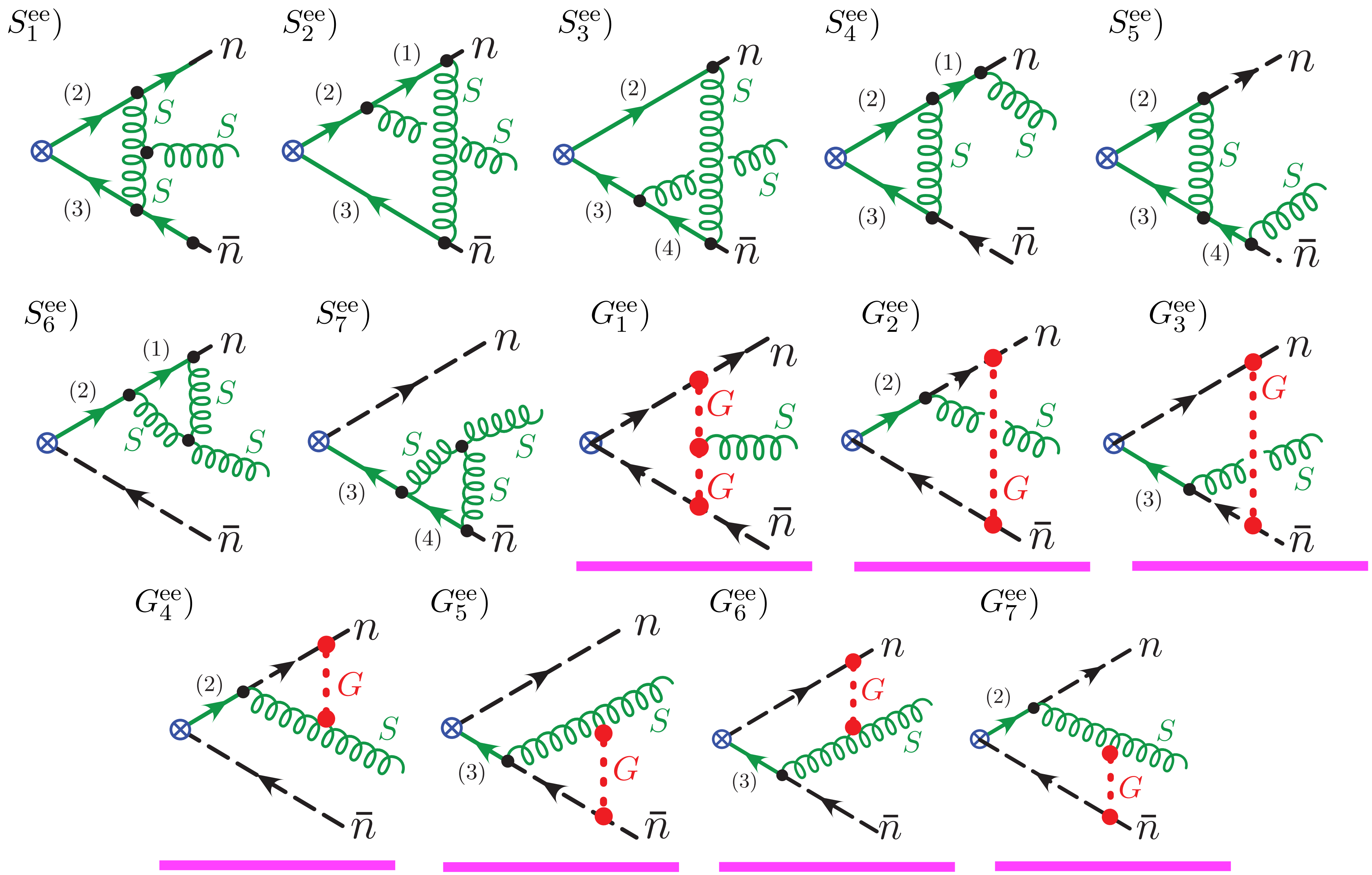


$SL_2)$



Also true in the presence of additional emissions:

e^+e^-

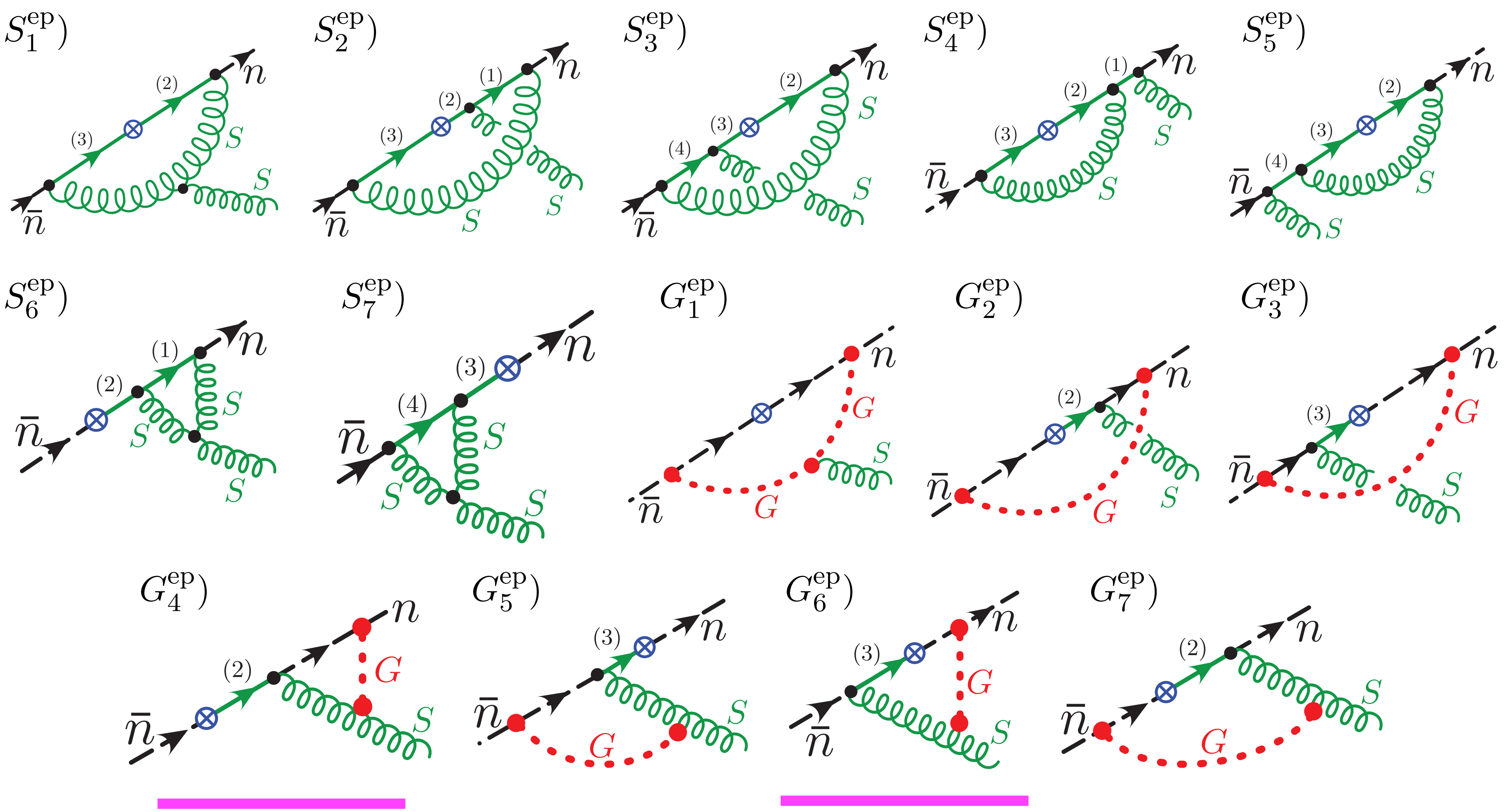


physical:

Glauber again gives all $(i\pi)$ terms here.

Also true in the presence of additional emissions:

e^-p

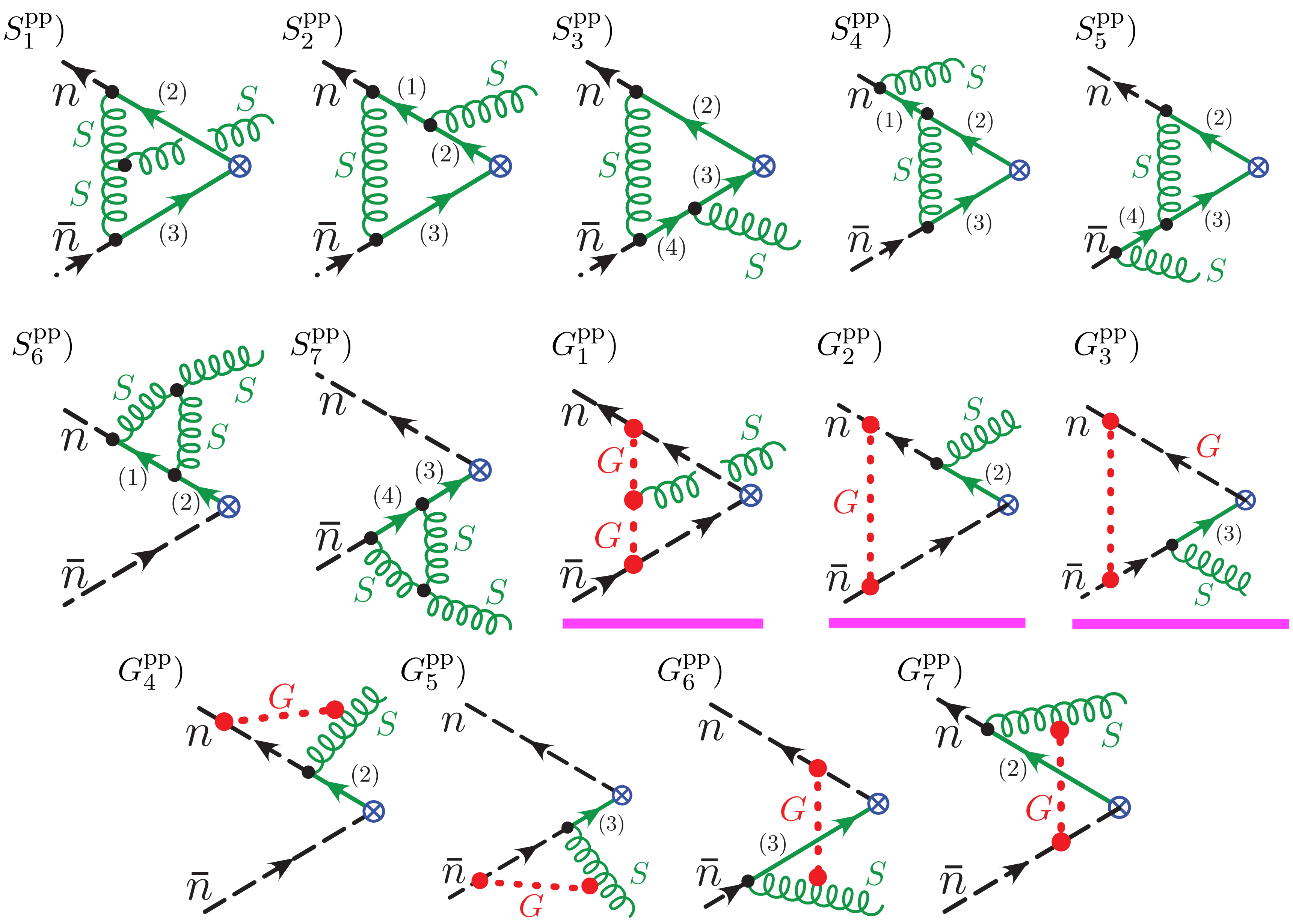


physical:

Glauber again gives all $(i\pi)$ terms here.

Also true in the presence of additional emissions:

pp



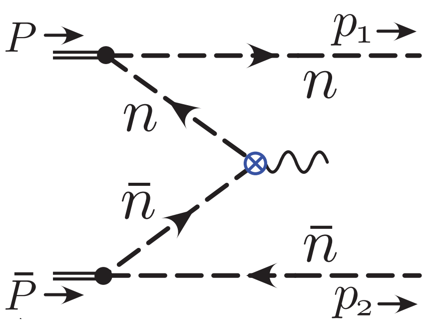
physical:

Glauber again gives all $(i\pi)$ terms here.

agrees with soft-current calculation
Catani, Grazzini 2000 (~SCET$_{\rm II}$)

Spectator Scattering

Add interpolating fields for initial state hadrons.

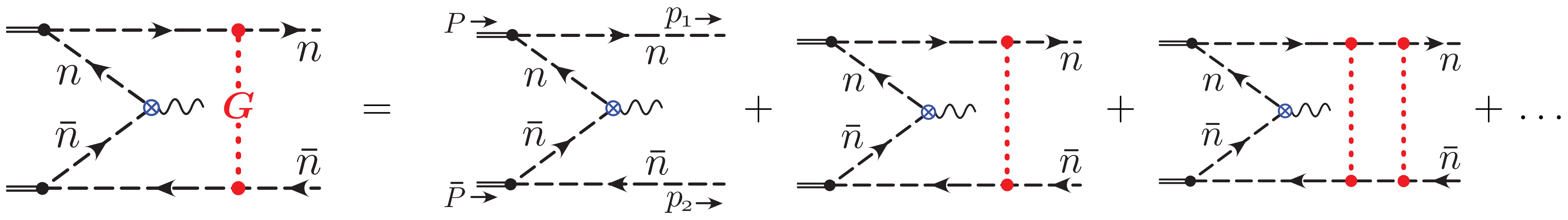


$$= \mathcal{S}^{\gamma}\left[\frac{1}{\vec{p}_{1\perp}^{\,2}}\,\frac{1}{\vec{p}_{2\perp}^{\,2}}\right]\left[\frac{\bar{n}\cdot p_1\,\bar{n}\cdot(P-p_1)}{\bar{n}\cdot P}\,\frac{n\cdot p_2\,n\cdot(\bar{P}-p_2)}{n\cdot\bar{P}}\right]$$

$$\equiv S^{\gamma}\,E(p_{1\perp},p_{2\perp}),$$

$$\mathcal{S}^{\gamma}=\bar{u}_n\gamma_{\perp}^{\mu}v_{\bar{n}}^{*}$$

"an end E"



$$G(k_{\perp}) = \text{Fourier Transform of } e^{i\phi}$$

$$= -\mathcal{S}^{\gamma}\int \text{đ}^{d-2}k_{\perp}\;G(k_{\perp})\;E(p_{1\perp}+k_{\perp},p_{2\perp}-k_{\perp})$$

$$= -\mathcal{S}^{\gamma}\int \text{đ}^{d-2}k_{\perp}\;G(k_{\perp})\;E\Big(k_{\perp}-\Delta p_{\perp}-\frac{q_{\perp}}{2},\Delta p_{\perp}-k_{\perp}-\frac{q_{\perp}}{2}\Big)$$

$$q_{\perp} = -p_{1\perp}-p_{2\perp}$$

$$\Delta p_{\perp} = (p_{2\perp}-p_{1\perp})/2$$

$$\equiv -\mathcal{S}^{\gamma}\int \text{đ}^{d-2}k_{\perp}\;G(k_{\perp})\;E'(\Delta p_{\perp}-k_{\perp},q_{\perp})$$

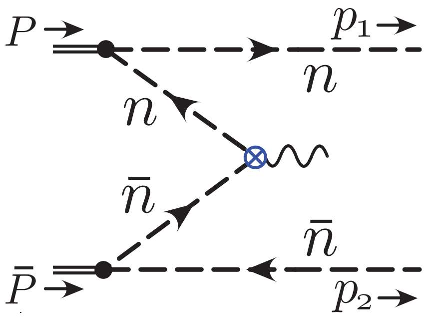
$$= -\mathcal{S}^{\gamma}\int \text{đ}^{d-2}k_{\perp}\int d^{d-2}b_{\perp}\;e^{-i\vec{k}_{\perp}\cdot\vec{b}_{\perp}}\;\tilde{G}(b_{\perp})\int d^{d-2}b'_{\perp}\;e^{-i(\Delta\vec{p}_{\perp}-\vec{k}_{\perp})\cdot\vec{b}'_{\perp}}\,\tilde{E}'(b'_{\perp},q_{\perp})$$

$$= -\mathcal{S}^{\gamma}\int d^{d-2}b_{\perp}\;e^{-i\Delta\vec{p}_{\perp}\cdot\vec{b}_{\perp}}\;\tilde{E}'(b_{\perp},q_{\perp})\;e^{i\phi(b_{\perp})}\;\equiv\;\mathcal{A}_{SS}(\Delta p_{\perp},q_{\perp})$$

Spectator Scattering

Add interpolating fields for initial state hadrons.

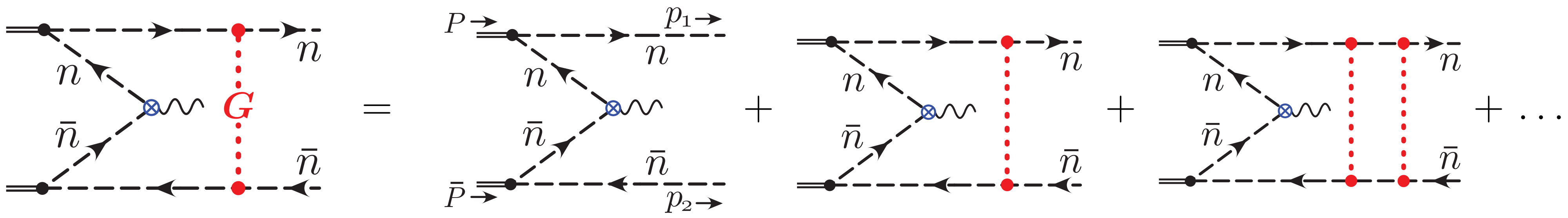
$$\mathcal{S}^\gamma = \bar{u}_n \gamma_\perp^\mu v_{\bar{n}}^*$$



$$= \mathcal{S}^\gamma \left[\frac{1}{\vec{p}_{1\perp}^{\,2}} \frac{1}{\vec{p}_{2\perp}^{\,2}}\right] \left[\frac{\bar{n}\cdot p_1\, \bar{n}\cdot(P-p_1)}{\bar{n}\cdot P} \frac{n\cdot p_2\, n\cdot(\bar{P}-p_2)}{n\cdot \bar{P}}\right]$$

$$\equiv S^\gamma\, E(p_{1\perp}, p_{2\perp}),$$

"an end E"



$$q_\perp = -p_{1\perp} - p_{2\perp}$$
$$\Delta p_\perp = (p_{2\perp} - p_{1\perp})/2$$

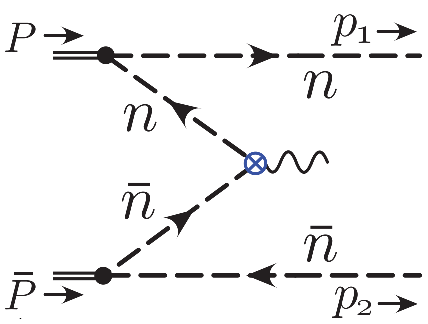
$$= -\mathcal{S}^\gamma \int d^{d-2}b_\perp\, e^{-i\Delta\vec{p}_\perp\cdot\vec{b}_\perp}\; \tilde{E}'(b_\perp, q_\perp)\; e^{i\phi(b_\perp)} \;\equiv\; \mathcal{A}_{SS}(\Delta p_\perp, q_\perp)$$

phase cancels IF we integrate over $\Delta p_\perp$

$$\int \text{đ}^{d-2}\Delta p_\perp \left|\mathcal{A}_{SS}(\Delta p_\perp, q_\perp)\right|^2$$
$$= |\mathcal{S}^\gamma|^2 \int \text{đ}^{d-2}\Delta p_\perp \int d^{d-2}b_\perp\, d^{d-2}b'_\perp\, e^{i\Delta\vec{p}_\perp\cdot(\vec{b}'_\perp - \vec{b}_\perp)}\; \tilde{E}'(b_\perp, q_\perp)\tilde{E}'^\dagger(b'_\perp, q_\perp)\; e^{i\phi(b_\perp) - i\phi(b'_\perp)}$$
$$= |\mathcal{S}^\gamma|^2 \int d^{d-2}b_\perp \left|\tilde{E}'(b_\perp, q_\perp)\right|^2$$
$$= |\mathcal{S}^\gamma|^2 \int \text{đ}^{d-2}\Delta p_\perp \left|E'(\Delta p_\perp, q_\perp)\right|^2$$

Spectator Scattering

Add interpolating fields for initial state hadrons.

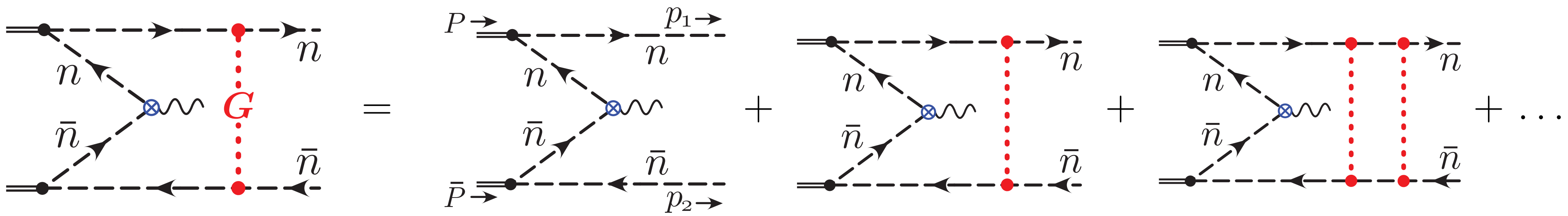


$$= \mathcal{S}^\gamma \left[\frac{1}{\vec p_{1\perp}^{\,2}}\,\frac{1}{\vec p_{2\perp}^{\,2}}\right]\left[\frac{\bar n\cdot p_1\,\bar n\cdot(P-p_1)}{\bar n\cdot P}\,\frac{n\cdot p_2\,n\cdot(\bar P-p_2)}{n\cdot \bar P}\right]$$

$$\equiv S^\gamma\, E(p_{1\perp},p_{2\perp}),$$

$$\mathcal{S}^\gamma = \bar u_n \gamma_\perp^\mu v_{\bar n}^*$$

"an end E"



$$q_\perp = -p_{1\perp} - p_{2\perp}$$

$$\Delta p_\perp = (p_{2\perp} - p_{1\perp})/2$$

$$= -\mathcal{S}^\gamma \int d^{d-2}b_\perp\, e^{-i\Delta\vec p_\perp\cdot\vec b_\perp}\ \tilde E'(b_\perp,q_\perp)\ e^{i\phi(b_\perp)} \ \equiv\ \mathcal{A}_{SS}(\Delta p_\perp,q_\perp)$$

phase cancels IF we integrate over $\Delta p_\perp$

Measurements (like beam thrust & transverse thrust) that disrupt this integration can cause a non-cancellation.

(cf. Gaunt; Zeng)

Underlying Event

- Radiation not described by primary hard scattering.
- Modeled by Multiple Particle Interactions (MPI) in Monte Carlos

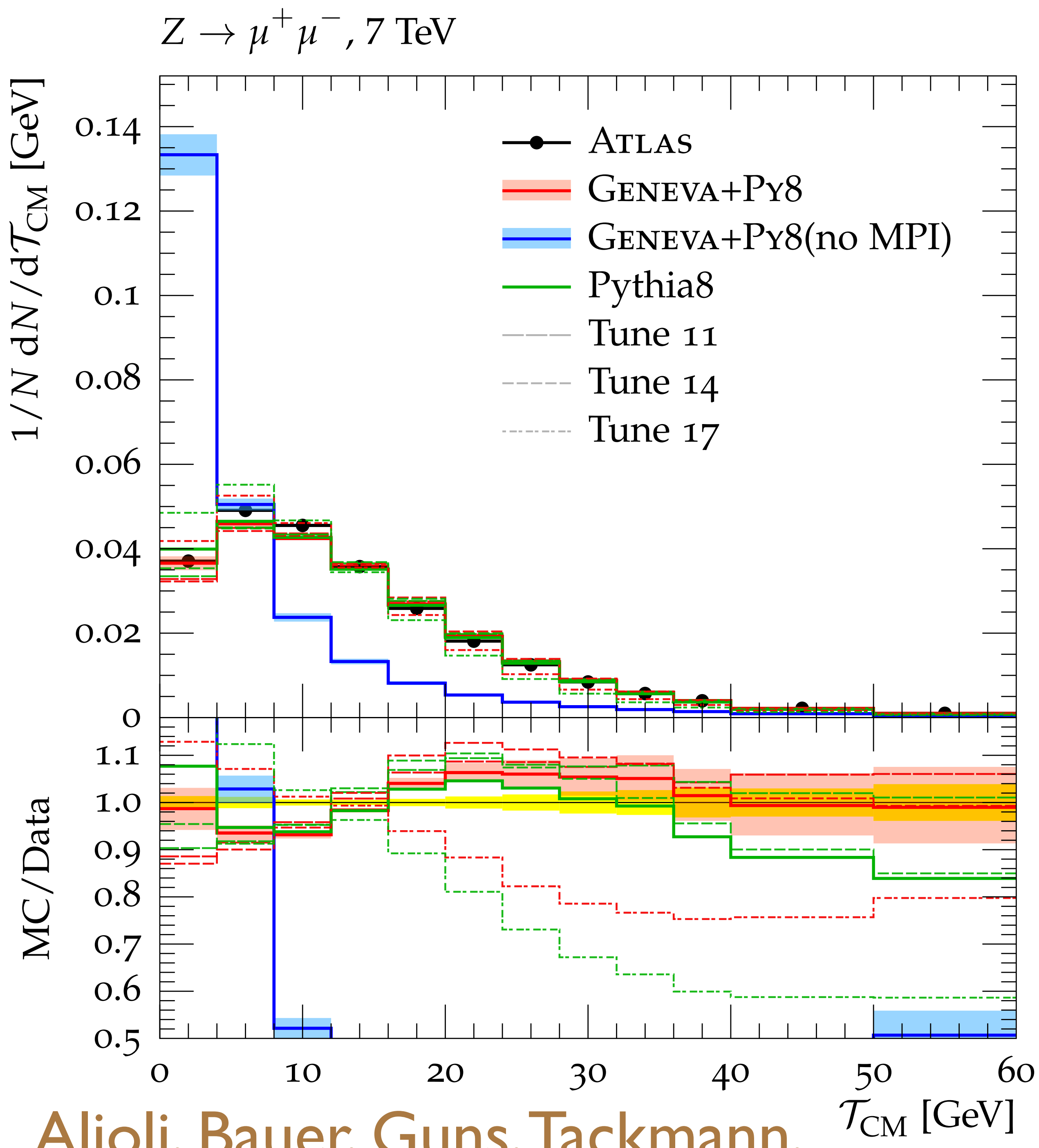
Some observables are sensitive:

beam thrust,

$$\mathcal{T}_{\rm CM} = \sum_i p_{T,i}\; e^{-|\eta_i|}$$

transverse thrust, …

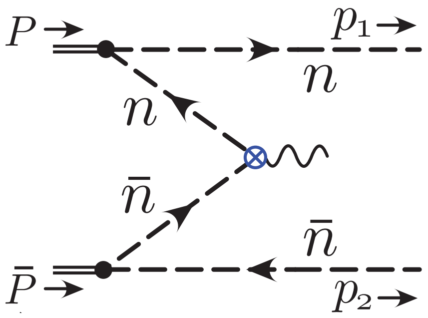
Connection between fact. violation, and small-x dynamics may allow us to directly calculate these effects.



Alioli, Bauer, Guns, Tackmann, 1605.07192

Spectator Scattering

Add interpolating fields for initial state hadrons.

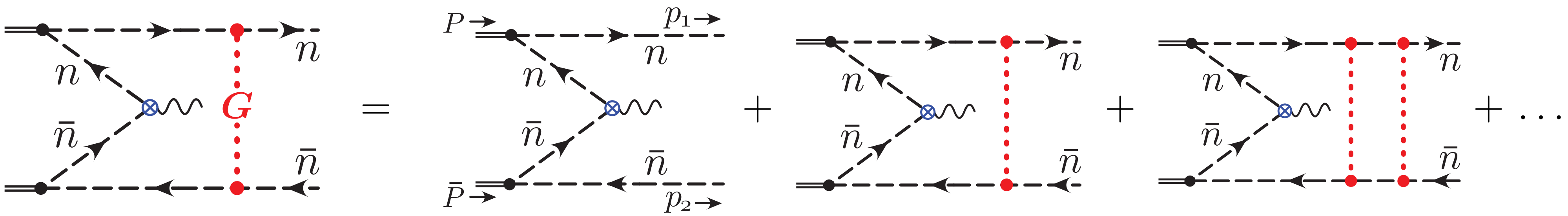


$$= \mathcal{S}^\gamma \left[\frac{1}{\vec{p}_{1\perp}^{\,2}}\,\frac{1}{\vec{p}_{2\perp}^{\,2}}\right]\left[\frac{\bar n\cdot p_1\,\bar n\cdot(P-p_1)}{\bar n\cdot P}\,\frac{n\cdot p_2\,n\cdot(\bar P-p_2)}{n\cdot\bar P}\right]$$

$$\equiv \mathcal{S}^\gamma\, E(p_{1\perp},p_{2\perp}),$$

$$\mathcal{S}^\gamma = \bar u_n \gamma_\perp^\mu v_{\bar n}^*$$

“an end E”



$$q_\perp = -p_{1\perp} - p_{2\perp}$$

$$\Delta p_\perp = (p_{2\perp} - p_{1\perp})/2$$

$$= -\mathcal{S}^\gamma \int d^{d-2}b_\perp\, e^{-i\Delta\vec{p}_\perp\cdot\vec{b}_\perp}\ \tilde{E}'(b_\perp, q_\perp)\ e^{i\phi(b_\perp)} \ \equiv\ \mathcal{A}_{SS}(\Delta p_\perp, q_\perp)$$

phase cancels IF we integrate over $\Delta p_\perp$

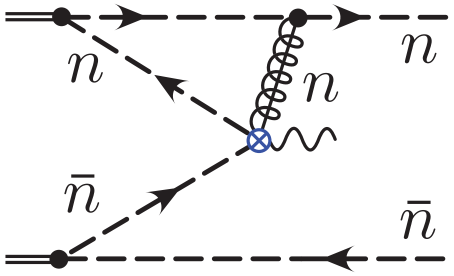
Single scale SCET:

$\Delta p_\perp \sim \Lambda_{\rm QCD} \ll \mathcal{T}$ cancel as in inclusive DY, up to power corrections $\dfrac{\Lambda_{\rm QCD}}{\mathcal{T}} \ll 1$ (cf. Aybat & Sterman)

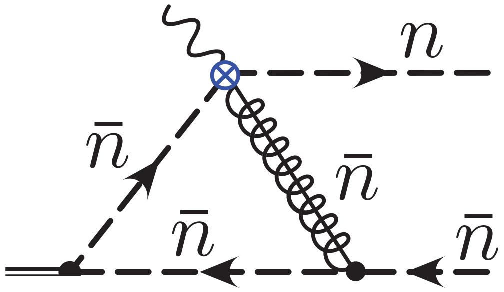
$\Delta p_\perp \sim \mathcal{T}, \sqrt{Q\mathcal{T}}$ starts at $\mathcal{O}(\alpha_s^4)$, calculable factorization violation $(\mathcal{I}\mathcal{I}) \otimes f \otimes f$

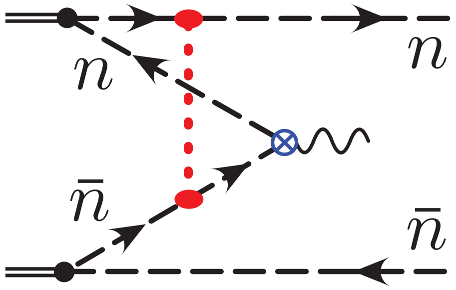
(cf. Gaunt; Zeng)

Active-Spectator and the Collinear Overlap

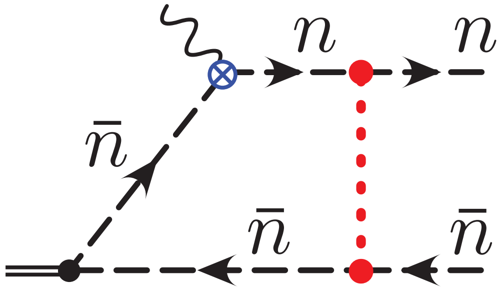


$$C_n = \tilde{C}_n - C_n^{(G)}$$





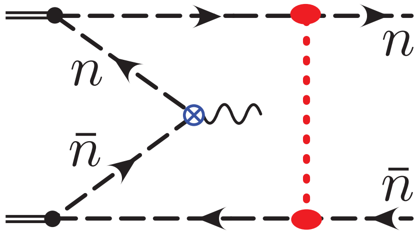
$$G = C_n^{(G)}$$



- can absorb this Glauber into the Collinear Wilson line with proper (physical) directions (note: connection to eikonalization)
- cancel? $= \frac{1}{2} S^{\gamma} \int {\mathchar'26\mkern-12mu d}^{d'}k_\perp \, G^0(k_\perp) E(p_{1\perp}+k_\perp, p_{2\perp})$ now need to integrate over $p_{i\perp}$

Active-Active cancel or absorb Glauber into Soft Wilson lines

Spectator-Spectator



no analogous soft or collinear diagrams at leading power

Glauber Related Examples of Factorization Violation

- Violation of Cross Section factorization, for example PDFs entangled

  $|pp\rangle$, not $|p\rangle|p\rangle$ (Collins, Soper, Sterman; Bodwin; Bodwin, Brodsky, Lepage)

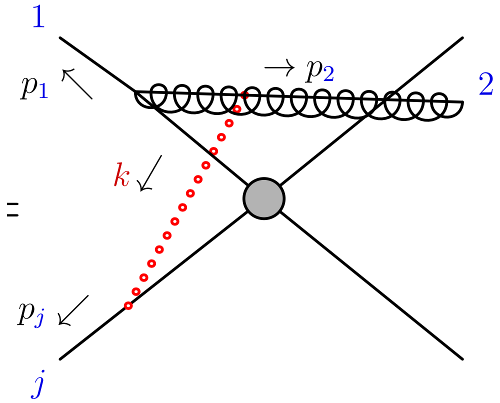
- Violation of Collinear Amplitude Factorization (Catani,de Florian,Rodrigo) (Forshaw, Seymour, Siodmok)

$$
\begin{aligned}
|\mathcal{M}^{(1)}(p_1,p_2,\ldots,p_n)\rangle \ &\simeq\ \boldsymbol{Sp}^{(1)}(p_1,p_2;\widetilde{P};p_3,\ldots,p_n)\ |\mathcal{M}^{(0)}(\widetilde{P},\ldots,p_n)\rangle \\
&+\ \boldsymbol{Sp}^{(0)}(p_1,p_2;\widetilde{P})\ |\mathcal{M}^{(1)}(\widetilde{P},\ldots,p_n)\rangle\ .
\end{aligned}
$$

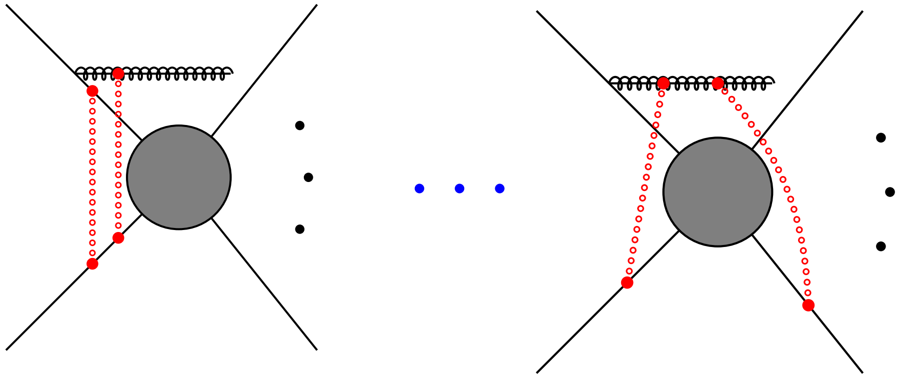
for space-like collinear limits (collinear incoming/outgoing particles)

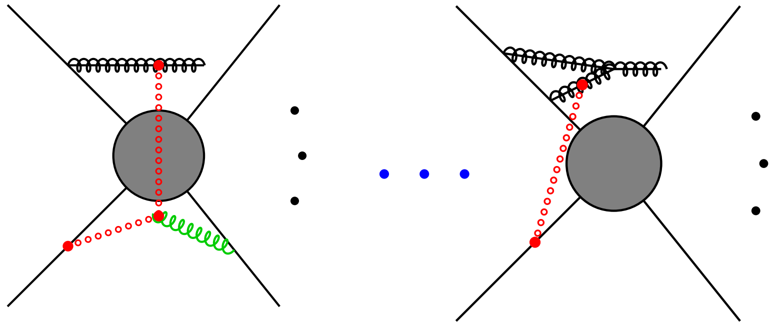
Reproduced with SCET Glauber Ops

Schwartz, Yan, Zhu (1703.08572)



$$
= \pm\frac{\alpha_s}{2\pi}(4\pi e^{-\gamma_E})^{\epsilon}(i\pi)\left(\frac{1}{\epsilon}+\ln\frac{\mu^2}{\vec{p}^{\,2}_{2,\perp}}+\mathcal{O}(\epsilon)\right)(\mathbf{T}_2\cdot\mathbf{T}_j)\,\mathbf{Sp}^0\cdot\overline{\mathcal{M}}^0
$$





$$
-\frac{\alpha_s^2}{4\epsilon^2}\sum_{j=4}^{m} i f_{abc}\,\mathbf{T}_2^a\,\mathbf{T}_3^b\,\mathbf{T}_j^c\ \ \mathbf{Sp}^0\,\overline{\mathcal{M}}^0
$$

$(i\pi)^2$ term

- Regge Amplitude Factorization at NLL

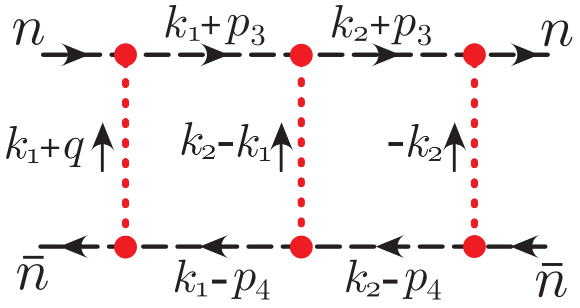
$$s \gg |t|$$

$$\mathcal{M}^{[8]}_{\rm rs}\left(\frac{s}{\mu^2}, \frac{t}{\mu^2}, \alpha_s\right) = 2\pi\alpha_s \, H^{(0),[8]}_{\rm rs} \; C_{\rm r}\left(\frac{t}{\mu^2}, \alpha_s\right) \left[A_+\left(\frac{s}{t}, \alpha_s\right) + \kappa_{\rm rs}\, A_-\left(\frac{s}{t}, \alpha_s\right)\right] C_{\rm s}\left(\frac{t}{\mu^2}, \alpha_s\right)$$

$$A_\pm\left(\frac{s}{t}, \alpha_s\right) = \left(\frac{-s}{-t}\right)^{\alpha(t)} \pm \left(\frac{s}{-t}\right)^{\alpha(t)}$$

corrected at NNLL by $\dfrac{(i\pi)^2\,\alpha_s^2}{\epsilon^2}$

(Del Duca, Glover; Del Duca, Falcioni, Magnea, Vernazza)



Board

Appendix / Backup

Proof of Power Counting formula:

enumerate vertices from gauge invariant operators of order $\sim \lambda^k$

V_k^n vertices with only n-collinear fields,

$V_k^{\bar n}$ vertices with only $\bar n$-collinear fields,

V_k^S vertices with only soft fields,

V_k^{nS} vertices that have both n-collinear and soft fields but do not have $\bar n$ fields,

$V_k^{\bar n S}$ vertices with both $\bar n$-collinear and soft fields but not n fields,

$V_k^{n\bar n}$ vertices with both n and $\bar n$-collinear fields (with or without soft fields).

(includes scaling for external fields)

Proof of Power Counting formula:

count # Loops of various types, and # Internal Propagators

L^n : n–collinear loops with $k^\mu \sim Q(\lambda^2, 1, \lambda)$ loop momenta,

$L^{\bar{n}}$: $\bar{n}$–collinear loops with $k^\mu \sim Q(1, \lambda^2, \lambda)$ loop momenta,

L^S : soft loops with $k^\mu \sim Q(\lambda, \lambda, \lambda)$ loop momenta,

L^{nS} : soft-collinear Glauber loops with $k^\mu \sim Q(\lambda^2, \lambda, \lambda)$ loop momenta ,

$L^{\bar{n}S}$: soft-collinear Glauber loops with $k^\mu \sim Q(\lambda, \lambda^2, \lambda)$ loop momenta ,

$L^{n\bar{n}}$: n–$\bar{n}$ Glauber loops with $k^\mu \sim Q(\lambda^2, \lambda^2, \lambda)$ loop momenta ,

I^n : internal n–collinear propagators,

$I^{\bar{n}}$: internal $\bar{n}$–collinear propagators,

I^S : internal soft propagators.

$$\delta = \sum_k k\Big(V_k^n + V_k^{\bar{n}} + V_k^S + V_k^{nS} + V_k^{\bar{n}S} + V_k^{n\bar{n}}\Big)$$

$$+4L^n + 4L^{\bar{n}} + 4L^S + 5L^{nS} + 5L^{\bar{n}S} + 6L^{n\bar{n}} - 4I^n - 4I^{\bar{n}} - 4I^S$$

Proof of Power Counting formula:

Simplify with topological identities:

overall Euler:

$$1 = \sum_k \left(V_k^n + V_k^{\bar{n}} + V_k^S + V_k^{nS} + V_k^{\bar{n}S} + V_k^{n\bar{n}}\right)$$
$$+ L^n + L^{\bar{n}} + L^S + L^{nS} + L^{\bar{n}S} + L^{n\bar{n}} - I^n - I^{\bar{n}} - I^S$$

count disconnected components when we erase modes:

$\bar{n}$ $$N^{nS} = \sum_k \left(V_k^n + V_k^S + V_k^{nS} + V_k^{\bar{n}S} + V_k^{n\bar{n}}\right) + L^n + L^S + L^{nS} - I^n - I^S\,,$$

n $$N^{\bar{n}S} = \sum_k \left(V_k^{\bar{n}} + V_k^S + V_k^{nS} + V_k^{\bar{n}S} + V_k^{n\bar{n}}\right) + L^{\bar{n}} + L^S + L^{\bar{n}S} - I^{\bar{n}} - I^S\,,$$

s $$N^{n\bar{n}} = \sum_k \left(V_k^n + V_k^{\bar{n}} + V_k^{nS} + V_k^{\bar{n}S} + V_k^{n\bar{n}}\right) + L^n + L^{\bar{n}} + L^{n\bar{n}} - I^n - I^{\bar{n}}\,.$$

$n, \bar{n}$ $$N^S = \sum_k \left(V_k^S + V_k^{nS} + V_k^{\bar{n}S} + V_k^{n\bar{n}}\right) + L^S - I^S\,,$$

$s, \bar{n}$ $$N^n = \sum_k \left(V_k^n + V_k^{nS} + V_k^{n\bar{n}}\right) + L^n - I^n\,,$$

s, n $$N^{\bar{n}} = \sum_k \left(V_k^{\bar{n}} + V_k^{\bar{n}S} + V_k^{n\bar{n}}\right) + L^{\bar{n}} - I^{\bar{n}}\,,$$

final result:

$$\delta = 6 - N^n - N^{\bar{n}} - N^{nS} - N^{\bar{n}S} + \sum_k (k-4)\left(V_k^n + V_k^{\bar{n}} + V_k^S\right) + (k-3)\left(V_k^{nS} + V_k^{\bar{n}S}\right) + (k-2)V_k^{n\bar{n}}$$

Proof of Power Counting formula:

Simplify with topological identities:

overall Euler:

$$1 = \sum_k \left(V_k^n + V_k^{\bar n} + V_k^S + V_k^{nS} + V_k^{\bar n S} + V_k^{n\bar n}\right) + L^n + L^{\bar n} + L^S + L^{nS} + L^{\bar n S} + L^{n\bar n} - I^n - I^{\bar n} - I^S$$

count disconnected components when we erase modes:

$\bar n$ $$N^{nS} = \sum_k \left(V_k^n + V_k^S + V_k^{nS} + V_k^{\bar n S} + V_k^{n\bar n}\right) + L^n + L^S + L^{nS} - I^n - I^S\,,$$

n $$N^{\bar n S} = \sum_k \left(V_k^{\bar n} + V_k^S + V_k^{nS} + V_k^{\bar n S} + V_k^{n\bar n}\right) + L^{\bar n} + L^S + L^{\bar n S} - I^{\bar n} - I^S\,,$$

s $$N^{n\bar n} = \sum_k \left(V_k^n + V_k^{\bar n} + V_k^{nS} + V_k^{\bar n S} + V_k^{n\bar n}\right) + L^n + L^{\bar n} + L^{n\bar n} - I^n - I^{\bar n}\,.$$

$n, \bar n$ $$N^S = \sum_k \left(V_k^S + V_k^{nS} + V_k^{\bar n S} + V_k^{n\bar n}\right) + L^S - I^S\,,$$

$s, \bar n$ $$N^n = \sum_k \left(V_k^n + V_k^{nS} + V_k^{n\bar n}\right) + L^n - I^n\,,$$

s, n $$N^{\bar n} = \sum_k \left(V_k^{\bar n} + V_k^{\bar n S} + V_k^{n\bar n}\right) + L^{\bar n} - I^{\bar n}\,,$$

final result for SCET$_{\mathrm{I}}$ and SCET$_{\mathrm{II}}$:

$$\delta = 6 - N^n - N^{\bar n} - N^{nS} - N^{\bar n S} + 2u$$
$$+ \sum_k (k-8) V_k^{us} + (k-4)\left(V_k^n + V_k^{\bar n} + V_k^S\right) + (k-3)\left(V_k^{nS} + V_k^{\bar n S}\right) + (k-2) V_k^{n\bar n}$$

Construction:

$\lambda \ll 1$ large Q

mode	fields	p^μ momentum scaling	physical objects	type
n-collinear	$\xi_n,\ A_n^\mu$	$(n\cdot p, \bar{n}\cdot p, p_\perp) \sim Q(\lambda^2, 1, \lambda)$	n-collinear "jet"	onshell
$\bar{n}$-collinear	$\xi_{\bar{n}},\ A_{\bar{n}}^\mu$	$(\bar{n}\cdot p, \bar{n}\cdot p, p_\perp) \sim Q(\lambda^2, 1, \lambda)$	$\bar{n}$-collinear "jet"	onshell
soft	$\psi_S,\ A_S^\mu$	$p^\mu \sim Q(\lambda, \lambda, \lambda)$	soft virtual/real radiation	onshell
ultrasoft	$\psi_{us},\ A_{us}^\mu$	$p^\mu \sim Q(\lambda^2, \lambda^2, \lambda^2)$	ultrasoft virtual/real radiation	onshell
Glauber	–	$p^\mu \sim Q(\lambda^a, \lambda^b, \lambda),\ a+b>2$	forward scattering potential	offshell
		(here $\{a,b\} = \{2,2\}, \{2,1\}, \{1,2\}$)		
hard	–	$p^2 \gtrsim Q^2$	hard scattering	offshell

Power Counting formula for graph (any loop order, any power):

$\sim \lambda^\delta$ (gauge invariant)

$$\delta = 6 - N^n - N^{\bar{n}} - N^{nS} - N^{\bar{n}S} + 2u\,,$$

$$+\sum_k \underbrace{(k-8)V_k^{us} + (k-4)\big(V_k^n + V_k^{\bar{n}} + V_k^S\big)}_{\text{standard SCET}} + (k-3)\big(V_k^{nS} + V_k^{\bar{n}S}\big) + (k-2)V_k^{n\bar{n}}$$

need $\sim \lambda^3$ $\sim \lambda^2$

Glauber

operators at leading power