Regge cut contributions to QCD amplitudes

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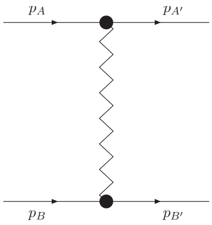
Introduction

One of the remarkable properties of QCD is the gluon Reggeization.
The Reggeization allows to express an infinite number of amplitudes through several effective vertices and gluon trajectory.
It provides a general way for theoretical description of processes at high c.m.s. energy $\sqrt{s}$ and fixed (not growing with energy) momentum transfers.
Validity of the Reggeization is proved now in all orders of perturbation theory in the coupling constant g both in the leading logarithmic approximation (LLA), where in each order of the perturbation theory only terms with the highest powers of $\ln s$ are kept, and in the next-to-leading one (NLLA), where terms with one power less are also kept.
The Reggeization provides a simple derivation of the BFKL equation both in the LLA and NLLA.

Gluon Reggeization

For elastic scattering processes $A + B \to A' + B'$ in the Regge kinematical region: $s \simeq -u \to \infty$, t fixed (i.e. not growing with s) the Reggeization means that scattering amplitudes with the gluon quantum numbers in the t-channel can be presented as

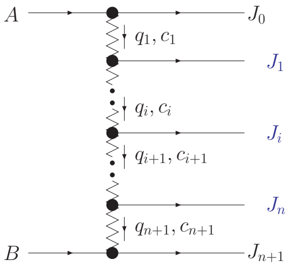


$$\mathcal{A}_{AB}^{A'B'} = \Gamma^{c}_{A'A} \left[\left(\frac{-s}{-t} \right)^{j(t)} - \left(\frac{s}{-t} \right)^{j(t)} \right] \Gamma^{c}_{B'B} \; ;$$

Gluon Reggeization

$\Gamma^{c}_{P'P}$–particle-particle-Reggeon (PPR) vertices or scattering vertices ("c" are colour indices); $j(t) = 1 + \omega(t)$ – Reggeon trajectory.

The Reggeization means definite form not only of elastic amplitudes, but of inelastic amplitudes in the multi-Regge kinematics (MRK) as well. It can be presented by the picture



and written as

$$\Re\mathcal{A}_{AB}^{\tilde{A}\tilde{B}+n} = 2s\,\Gamma_{\tilde{A}A}^{c_1}\left(\prod_{i=1}^{n}\gamma_{c_i c_{i+1}}^{J_i}(q_i,q_{i+1})\left(\frac{s_i}{s_0}\right)^{\omega(t_i)}\frac{1}{t_i}\right)$$

$$\frac{1}{t_{n+1}}\left(\frac{s_{n+1}}{s_0}\right)^{\omega(t_{n+1})}\Gamma_{\tilde{B}B}^{c_{n+1}}$$

Here $\gamma_{c_i c_{i+1}}^{J_i}(q_i,q_{i+1})$ – the Reggeon-Reggeon-particle (RRP) or production vertices.

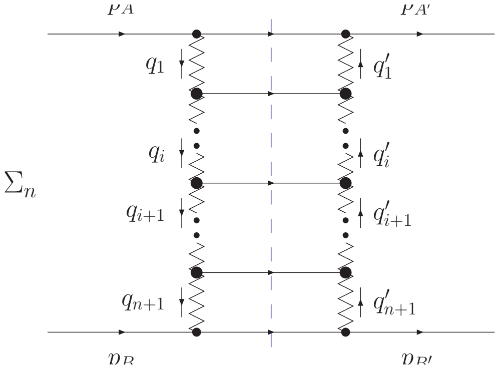
MRK is the kinematics where all particles have limited (not growing with s) transverse momenta and are combined into jets with limited invariant mass of each jet and large (growing with s) invariant masses of any pair of the jets.

The MRK gives dominant contributions to cross sections of QCD processes at high energy $\sqrt{s}$. In the LLA only a gluon can be produced. In the NLA one has to account production of $Q\bar{Q}$ and GG jets.

s-channel unitarity

Amplitudes of processes with all possible quantum numbers in the t–channel are calculated using s-channel unitarity and analyticity .

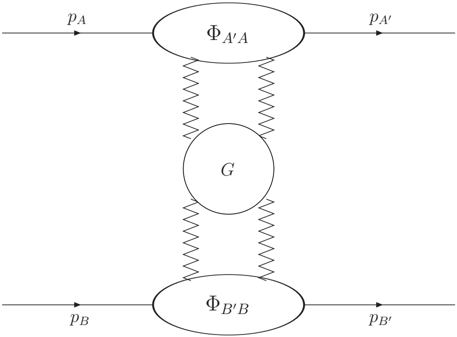
The s-channel discontinuity



s-channel unitarity

The amplitudes are presented in the form :

$$\Phi_{A'A} \otimes G \otimes \Phi_{B'B}.$$



s-channel unitarity

Impact factors $\Phi_{A'A}$ and $\Phi_{B'B}$ describe transitions $A \to A'$ $B \to B'$,
G – Green's function for two interacting Reggeized gluons,

$$\hat{\mathcal{G}} = e^{Y\hat{\mathcal{K}}},$$

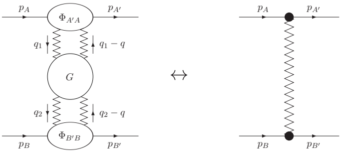
$\hat{\mathcal{K}}$ – BFKL kernel, $Y = \ln(s/s_0)$,

$$\hat{\mathcal{K}} = \hat{\omega}_1 + \hat{\omega}_2 + \hat{\mathcal{K}}_r$$

$$\hat{\mathcal{K}}_r = \hat{\mathcal{K}}_G + \hat{\mathcal{K}}_{Q\bar{Q}} + \hat{\mathcal{K}}_{GG}$$

Energy dependence of scattering amplitudes is determined by the BFKL kernel.
The BFKL kernel and the impact factors are expressed in terms of the Reggeon vertices and trajectory.
The kernel is universal (process independent).

The Reggeized form appeared as a result of calculations in several fixed orders.
The main idea of the proof of this form is based on the *s*-channel unitarity.
The requirement of compatibility of the *s*-channel unitarity with the Reggeized form of amplitudes leads to the bootstrap relations. These relations are quite simple in the elastic case:



$$\frac{1}{-\pi i}\mathrm{disc}_s \mathcal{A}_{AB}^{A'B'} = \frac{\partial}{\partial s}\ \Re\mathcal{A}_{AB}^{A'B'}/s$$

Bootstrap of the gluon Reggeization

Unfortunately, it is not so simple for inelastic amplitudes.
Compatibility of the unitarity with the multi-Regge form leads to bootstrap relations connecting discontinuities of the amplitudes with products of their real parts and gluon trajectories:

$$\sum_{l=j+1}^{n+1} \Delta_{jl} - \sum_{l=0}^{j-1} \Delta_{lj} = \frac{1}{2} \left(\omega(t_{j+1}) - \omega(t_j)\right) \Re\, \mathcal{A}_{AB}^{A'B'+n} \; .$$

Here the s-channel discontinuities must be calculated by inserting the Reggeized form of amplitudes into the unitarity conditions.
Evidently, there is an infinite number of the bootstrap relations, because there is an infinite number of the amplitudes $\mathcal{A}_{AB}^{A'B'+n}$.

Bootstrap of the gluon Reggeization

It occurs that an infinite number of the bootstrap relations is fulfilled if:
1. The impact factors for scattering particles satisfy the conditions

$$|A'A\rangle = g\Gamma_{A'A}|R_\omega(q)\rangle$$

where $q = p_A - p_{A'}$ and
2. $|R_\omega(q)\rangle$ is the universal (process independent) eigenstate of the kernel $\hat{\mathcal{K}}$ with the eigenvalue $\omega(t)$

$$(\hat{\mathcal{K}} - \omega(t))|R_\omega(q)\rangle = 0$$

and the normalization

$$\frac{g^2 t N_c}{2(2\pi)^{D-1}}\langle R_\omega(q)|R_\omega(q)\rangle = \omega(t)\ , \quad t = q^2;$$

3. The Reggeon-gluon impact factors and the gluon production vertices satisfy the condition

$$gt_1\langle R_\omega(q_1)|\hat{\mathcal{G}} + \langle GR_1| = g\gamma^G(q_1, q_2)\langle R_\omega(q_2)|.$$

All these conditions are checked in the NLO and are proved to be satisfied.

Violation of the Regge pole factorization

The first observation of the violation of the Regge factorization was made by
V.Del Duca, N. Glover, 2001
in the consideration of the the high-energy limit of the two-loop amplitudes for parton-parton scattering.
The interference of the tree- and two-loop amplitudes for gg, gq and qq have been explicitly computed
C. Anastasiou, E. W. N. Glover, C. Oleari and M. E. Tejeda-Yeomans, 2001,
E. W. N. Glover, C. Oleari and M. E. Tejeda-Yeomans, 2001.
The discrepancy appears in non-logarithmic two-loop terms. If the Reggeization would be correct in the NNLLA, they should satisfy a definite condition, because
three amplitudes should be expressed in terms of two Reggeon-Particle-Particle vertices.

Violation of the Regge pole factorization

Detailed consideration of the terms responsible for the Regge factorization breaking in the case of two-loop and three-loop quark and gluon amplitudes in QCD was performed by
V. Del Duca, G. Falcioni, L, Magnea and L. Vernazza, 2014
In particular, the non-logarithmic double-pole contribution at two-loops was recovered and all non-factorizing single-logarithmic singular contributions at three loops were found using the techniques of infrared factorization.

For comparison of Regge and infrared factorizations the representation of scattering apmlitudes

$$\mathcal{M}_{\rm rs}^{[8]}\left(\frac{s}{\mu^2},\frac{t}{\mu^2},\alpha_s\right) = 2\pi\alpha_s\, H_{\rm rs}^{(0),[8]}$$

$$\times\left\{C_{\rm r}\left(\frac{t}{\mu^2},\alpha_s\right)\left[A_+\left(\frac{s}{t},\alpha_s\right)+\kappa_{\rm rs}\,A_-\left(\frac{s}{t},\alpha_s\right)\right]C_{\rm s}\left(\frac{t}{\mu^2},\alpha_s\right)\right.$$

$$\left.+\,\mathcal{R}_{\rm rs}^{[8]}\left(\frac{s}{\mu^2},\frac{t}{\mu^2},\alpha_s\right)\right\}, \quad \kappa_{gg}=\kappa_{qg}=0, \quad \kappa_{qq}=(4-N_c^2)/N_c^2\,,$$

was used. $H_{\rm rs}^{(0)[8]}$ represents the tree-level amplitude. $A_+\left(\frac{s}{t},\alpha_s\right)$ and $A_-\left(\frac{s}{t},\alpha_s\right)$ represent contributions of Regge poles with negative and positive signature respectively, $\mathcal{R}_{\rm rs}$ is the non-factorizing remainder.

The results obtained:

$$R_{qq}^{(2),0,[8]} = \left(\frac{\tilde{\alpha}_s}{\pi}\right)^2 \frac{\pi^2}{4\epsilon^2}\,,$$

$$R_{gg}^{(2),0,[8]} = -\left(\frac{\tilde{\alpha}_s}{\pi}\right)^2 \frac{3\pi^2}{2\epsilon^2}\,,$$

$$R_{qg}^{(2),0,[8]} = -\left(\frac{\tilde{\alpha}_s}{\pi}\right)^2 \frac{\pi^2}{4\epsilon^2}\,.$$

$$\tilde{\alpha}_s = \alpha_s \Gamma(1+\epsilon)\frac{\Gamma^2(1-\epsilon)}{\Gamma(1-2\epsilon)}\,,$$

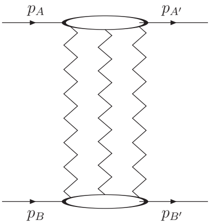
$$R_{qq}^{(3),1,[8]} = \left(\frac{\tilde{\alpha}_s}{\pi}\right)^3 \frac{\pi^2}{\epsilon^3} \frac{2N_c^2 - 5}{12N_c} + \mathcal{O}\left(\epsilon^0\right),$$

$$R_{gg}^{(3),1,[8]} = -\left(\frac{\tilde{\alpha}_s}{\pi}\right)^3 \frac{\pi^2}{\epsilon^3} \frac{2}{3} N_c + \mathcal{O}\left(\epsilon^0\right),$$

$$R_{qg}^{(3),1,[8]} = -\left(\frac{\tilde{\alpha}_s}{\pi}\right)^3 \frac{\pi^2}{\epsilon^3} \frac{N_c}{24} + \mathcal{O}\left(\epsilon^0\right),$$

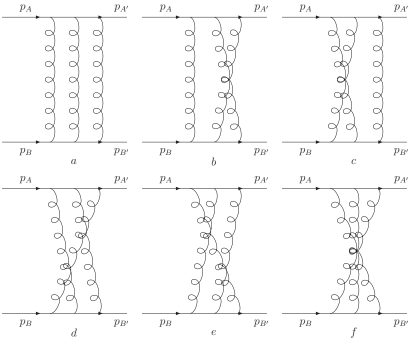
Regge cut contributions

It is well known that Regge poles generate Regge cuts.
Due to the signature conservation the cut responsible for the violation has to be 3-Reggeon one



Regge cut contributions

Since our Reggeon is the Reggeized gluon, the cut starts with the diagrams with three t-channels gluons.



Regge cut contributions

Particles A, A' and B, B' can be quarks or gluons.

The colour structures of the diagrams can be expanded in the basis of the states of irreducible representations of the colour group in the t-channel. We are interested in the adjoint representation of the colour group in the t-channel. In the case of the three-gluon states in the t-channel there are two adjoint representations; we are interested in the antisymmetric one, i.e. in the colour structure

$$C_{AB}^{A'B'} = \langle A'|T^a|A\rangle\langle B'|T^a|B\rangle \ .$$

The diagrams contain this colour structure with the coefficients C_{ij}^{α}, where $\alpha = a, b, c, d, e, f$ and $ij = gg, gq$ and qq for gluon-gluon, gluon-quark and quark-quark scattering correspondingly.

After some colour algebra, we obtain,

$$C^a_{gg} = \frac{3}{2} + \frac{N_c^2}{8}\,, \quad C^a_{gq} = \frac{1}{4} + \frac{N_c^2}{8}\,, \quad C^a_{qq} = \frac{1}{4}\left(1 + \frac{3}{N_c^2}\right)\,,$$

$$C^b_{gg} = C^c_{gg} = C^d_{gg} = C^e_{gg} = C_{gg} = \frac{3}{2}\,,$$

$$C^b_{gq} = C^c_{gq} = C^d_{gq} = C^e_{gq} = C_{gq} = \frac{1}{4}\,,$$

$$C^b_{qq} = C^c_{qq} = C^d_{qq} = C^e_{qq} = C_{qq} = \frac{1}{4}\left(-1 + \frac{3}{N_c^2}\right)\,,$$

$$C^f_{gg} = \frac{3}{2} + \frac{N_c^2}{8}\,, \quad C^f_{gq} = \frac{1}{4} + \frac{N_c^2}{8}\,, \quad C^f_{qq} = \frac{1}{4}\left(N_c^2 - 3 + \frac{3}{N_c^2}\right)\,.$$

Regge cut contributions

The contribution $A^{Fig.1}$ of the diagrams with the three-gluon exchanges to the scattering amplitudes with account of the colour structure can be written as

$$A_{ij}^{Fig.1} = \langle A'|T^a|A\rangle\langle B'|T^a|B\rangle \Bigg[C_{ij}A_{ij}^{(eik)} + \frac{N_c^2}{8}\left(A_{ij}^a + A_{ij}^f\right)$$

$$+\delta_{i,q}\delta_{j,q}\frac{4-N_c^2}{8}\left(A_{ij}^a - A_{ij}^f\right)\Bigg] ,$$

where A_{ij}^α is the contribution of the diagram α with omitted colour factors (as in QED) and $A_{ij}^{eik} = \sum_\alpha A_{ij}^\alpha$.
The last term here is the contribution of the positive signature in the quark-quark scattering, and the second term can can be assigned to the Reggeized gluon contribution. On the contrary, the first term can not be assigned to the Reggeized gluon contribution, because

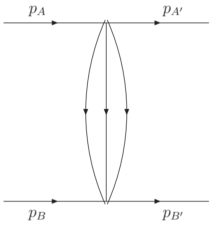
$$2C_{gq} \neq C_{qq} + C_{gg} .$$

Regge cut contributions

The amplitudes A_{ij}^{eik} can be easily found:

$$A_{ij}^{eik} = g^2 \frac{s}{t} \left(\frac{-4\pi^2}{3} \right) g^4 \, \vec{q}^{\,2} A_{\perp}^{(3)} ,$$

where $A_{\perp}^{(3)}$ is presented by the diagram



in the transverse momentum space.

Regge cut contributions

It is given by the integral

$$A_{\perp}^{(3)} = \int \frac{d^{2+2\epsilon} l_1 \, d^{2+2\epsilon} l_2}{(2\pi)^{2(3+2\epsilon)} \vec{l}_1^{\,2} \vec{l}_2^{\,2} (\vec{q} - \vec{l}_1 - \vec{l}_2)^2} =$$

$$= 3C_\Gamma^2 \frac{4}{\epsilon^2} \frac{(\vec{q}^{\,2})^{2\epsilon}}{\vec{q}^{\,2}} \frac{\Gamma^2(1+2\epsilon)\Gamma(1-2\epsilon)}{\Gamma(1+\epsilon)\Gamma^2(1-\epsilon)\Gamma(1+3\epsilon)} ,$$

$$C_\Gamma = \frac{\Gamma(1-\epsilon)\Gamma^2(1+\epsilon)}{(4\pi)^{2+\epsilon}\Gamma(1+2\epsilon)} .$$

The infrared behaviour

$$\frac{\Gamma^2(1+2\epsilon)\Gamma(1-2\epsilon)}{\Gamma(1+\epsilon)\Gamma^2(1-\epsilon)\Gamma(1+3\epsilon)} = 1 + O(\epsilon^3) .$$

Regge cut contributions

However, one can not affirm that it is given entire by the three-Reggeon cut.

Indeed, it can be given partly also by the Reggeized gluon. The problem of separation of these two contribution can be solved by consideration of logarithmic radiative corrections to them.

In te case of the Reggeized gluon contribution the corrections come solely from the Regge factor, so the first order correction is $\omega(t)\ln s$, where

$$\omega(t) = -g^2 N_c \vec{q}^{\,2} \int \frac{d^{2+2\epsilon} l}{2(2\pi)^{(3+2\epsilon)} \vec{l}^{\,2} (\vec{q} - \vec{l})^2} = -g^2 N_c C_\Gamma \frac{2}{\epsilon} (\vec{q}^{\,2})^\epsilon .$$

In the case of the three-Reggeon cut, one has to take into account the Reggeization of each of the three gluons and the interaction between them. The Reggeization gives $\ln s$ with the coefficient $3C_R$, where

$$C_R = -g^2 N_c C_\Gamma \frac{2}{\epsilon} \int \frac{d^{2+2\epsilon} l_1 \, d^{2+2\epsilon} l_2}{(2\pi)^{2(3+2\epsilon)} \vec{l}_1^{\,2} \vec{l}_2^{\,2} (\vec{q} - \vec{l}_1 - \vec{l}_2)^{1-\epsilon}} \left(A_\perp^{(3)}\right)^{-1}$$

$$= -g^2 N_c C_\Gamma \frac{4}{3\epsilon} (\vec{q}^{\,2})^\epsilon \frac{\Gamma(1-3\epsilon)\Gamma(1+2\epsilon)\Gamma(1+3\epsilon)}{\Gamma(1-\epsilon)\Gamma(1-2\epsilon)\Gamma(1+\epsilon)\Gamma(1+4\epsilon)} \, .$$

Regge cut contributions

Interaction between two Reggeons with transverse momenta $\vec{l}_1$ and $\vec{l}_2$ and colour indices c_1 and $c_1$is given by the real part of the BFKL kernel

$$\left[\mathcal{K}_r(\vec{q}_1, \vec{q}_2; \vec{k})\right]_{c_1 c_2}^{c_1' c_2'} = T^a_{c_1 c_1'} T^a_{c_2 c_2'} \frac{g^2}{(2\pi)^{D-1}} \left[\frac{\vec{q}_1^{\,2} \vec{q}_2^{\,\prime\,2} + \vec{q}_2^{\,2} \vec{q}_1^{\,\prime\,2}}{\vec{k}^{\,2}} - \vec{q}^{\,2}\right],$$

where $\vec{k}$ is the momentum transferred from one Reggeon to another in the interaction, $\vec{q}_1'$ and $\vec{q}_2'$ (c_1' and c_2') are the Reggeon momenta (colour indices) after the interaction, $\vec{q}_1' = \vec{q}_1 - \vec{k}, \;\; \vec{q}_2' = \vec{q}_2 + \vec{k}$, and $\vec{q} = \vec{q}_1 + \vec{q}_2 = \vec{q}_1' + \vec{q}_2'$.

It occurs that for the colour structure which we are interested in account of interactions between all pairs of the Reggeons leads in the sum to the colour coefficients which differ from the coefficients C_{ij} only by the common factor N_c.

Therefore, the first order correction in the case of the three-Reggeon cut is presented as $(-4C_R - C_3)\ln s$, where the term with C_R (C_3) comes from the first two terms (the last term) in the square brackets.

$$C_3 = g^2 N_c C_\Gamma \frac{4}{\epsilon} \int \frac{d^{2+2\epsilon} l_1 \, d^{2+2\epsilon} l_2 (l_1 + l_2)^{2\epsilon}}{(2\pi)^{2(3+2\epsilon)} \vec{l}_1^{\,2} \vec{l}_2^{\,2} (\vec{q} - \vec{l}_1 - \vec{l}_2)^{1-\epsilon}} \left(A_\perp^{(3)}\right)^{-1}$$

$$= g^2 N_c C_\Gamma \frac{32}{9\epsilon} (\vec{q}^{\,2})^{\epsilon} \frac{\Gamma(1-3\epsilon)\Gamma(1-\epsilon)\Gamma^2(1+3\epsilon)}{\Gamma^2(1-2\epsilon)\Gamma(1+2\epsilon)\Gamma(1+4\epsilon)} \, .$$

Therefore, the first order correction in the case of Reggeized gluon is $\omega(t)\ln s$, where

$$\omega(t) = -g^2 N_c C_\Gamma \frac{2}{\epsilon}(\vec{q}^{\,2})^\epsilon \ ,$$

and in the case of the the-Reggeon cut is $(-C_R - C_3)\ln s$, with C_R and C_3 given above. If to present the coefficients C_{ij} as the sum

$$C_{ij} = C_{ij}^R + C_{ij}^C, \ \ 2C_{gq}^R = C_{qq}^R + C_{gg}^R \ ,$$

and to require agreement with
V. Del Duca, G. Falcioni, L. Magnea and L. Vernazza, 2015
one has

$$C_{gg}^R = 3 \ , \ C_{gg}^C = -\frac{3}{2} \ , \ C_{gq}^R = \frac{7}{4} \ ,$$

$$C_{gq}^C = -\frac{3}{2} \ , \ C_{qq}^R = \frac{1}{2} \ , \ \ C_{qq}^C == \frac{3(1-N_c^2)}{4N_c^2} \ .$$

Summary

- One of remarkable properties of QCD is the gluon Reggeization
- It provides a general way for theoretical description of large s and fixed t processes.
- In the LLA and in the NLLA the Reggeization provides a simple factorized form of QCD amplitudes
- This form is violated in the NNLLA by the 3-Regge cut
- It seems, that the cut is the only source of the violation