Prescriptions for the definition of isospin-breaking effects

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Motivations

- The parameters matching QCD+QED to our world can be **unambiguously determined** by imposing a complete set of experimental hadronic measurement
- The separate determination of isospin-breaking corrections is **prescription dependent**
- **Important phenomenological interest**, for example
  - Comparison of iso-symmetric quantities in theoretical g-2 determinations
  - Radiative corrections to weak decays relatively to QCD decay constants and form factors

Background literature

- Phenomenology

  [Gasser & Leutwyler, Phys. Rep. 87(3), pp. 77-169 (1982)]

  [Gasser, Rusetsky & Scimemi, EPJC 32, pp. 97–114 (2003)]

  [Gasser & Zarnauskas, PLB 693(2), pp. 122-128 (2010)]

- Lattice

  [RM123, Phys. Rev. D 87(11), 114505 (2013)]

  [BMW, Phys. Rev. Lett. 111(25), 252001 (2013)]

  [BMW, Science 347 (6229), pp. 1452-1455 (2015)]

  [QCDSF, JHEP 93 (2016)]

  [BMW, Phys. Rev. Lett. 117(8), 082001 (2016)]

  [Bussone et al., PoS LATTICE2018 293 (2018)]

  [MILC, Phys. Rev. D 99(3), 034503 (2019)]

  [RM123-Soton, Phys. Rev. D 100(3), 034514 (2019)]

  [FLAG, EPJC 80, 113 (2020)]

Generalities

General problem

- For an observable X one ideally wants an expansion (FLAG notation)

$$X^{\phi} = \bar{X} + X_{\gamma} + X_{\mathrm{SU}(2)}$$

strong IB ($X_{\mathrm{SU}(2)}$)
electromagnetic IB (X_{γ})
iso-symmetric ($\bar{X}$)

- A complete set of hadron masses defines X^{ϕ} **unambiguously**

- The separation in 3 contributions requires additional conditions, and are **scheme-dependent**

High-level strategy

- This is quite technical to describe fully, so before anything else…

  The key choices in designing a scheme are

  1) which variables are kept fixed when $\alpha \to 0$
  2) which variable parametrises $\delta m = m_u - m_d$

- Both 1) and 2) define the scheme and are sufficient to define the isospin expansion

First step: finding the physical point

- Tilde quantities: lattice units

- Choose a set of known dimensionless ratios ρ
  e.g. $\rho = (M_{\pi^+}^2/M_{\Omega^-}^2, M_{K^+}^2/M_{\Omega^-}^2, M_{K^0}^2/M_{\Omega^-}^2)$

- Find physical bare quark masses

$$\tilde{m}_0^{\phi} = \tilde{m}_0^{\mathrm{sim}} - \left(\frac{\partial \rho}{\partial \tilde{m}_0}\right)^{-1} \left(\rho^{\mathrm{sim}} - \rho^{\mathrm{exp}} + \alpha \frac{\partial \rho}{\partial \alpha}\right)$$

- Predict any observable at the physical point

$$\tilde{X}^{\phi} = \tilde{X}^{\mathrm{sim}} + \frac{\partial \tilde{X}}{\partial \tilde{m}_0}(\tilde{m}_0^{\phi} - \tilde{m}_0^{\mathrm{sim}}) + \alpha \frac{\partial \tilde{X}}{\partial \alpha}$$

Formal definitions

- Renormalised observable parametrisation

$$X_M(M, \alpha, \Lambda) = \Lambda^{[X]} \tilde{X}_M(M/\Lambda^{[M]}, \alpha)$$

M : renormalised mass variables (hadronic, quarks, …)

Λ : scale

- Physical point M^ϕ unambiguous.
Scheme defined by the choice of two points $\hat{M}, \bar{M}$

$$X^\phi = X_M(M^\phi, \alpha^\phi, \Lambda^\phi) \quad \text{physical point}$$

$$\hat{X} = X_M(\hat{M}, 0, \Lambda^\phi) \quad \text{pure QCD}$$

$$\bar{X} = X_M(\bar{M}, 0, \Lambda^\phi) \quad \text{iso-symmetric QCD}$$

Second step: apply scheme

- Choose a variable set M (masses + scale)
- If M is not known experimentally, predict M^ϕ
  Choose prescription for $\hat{M}, \bar{M}$
- Derivatives in M can be computed using the Jacobian

$$\frac{\partial X_M}{\partial(M,\alpha)} = \frac{\partial X}{\partial(m_0,\alpha)} \left[\frac{\partial(M,\alpha)}{\partial(m_0,\alpha)}\right]^{-1}$$

- Compute IB corrections, for example QED corrections

$$X_\gamma = \frac{\partial X_M}{\partial M}(M^\phi - \hat{M}) + \alpha \frac{\partial X_M}{\partial \alpha}$$

Linear expansion

- Isospin breaking effects are small.
  Up to 1% corrections, **unphysical theories are within a linear correction from the physical point**

$$X_M(M,\alpha) = X^\phi + \frac{\partial X_M}{\partial M}(M - M^\phi) + (\alpha - \alpha^\phi)\frac{\partial X_M}{\partial \alpha}$$

- The space of all possible prescriptions can be explored with the knowledge of the **observable derivatives**

- The variable M can be changed using Jacobians
  Requires knowledge of **variable derivatives**

Numerical examples

Physical point setup

- Same data than leptonic decay IB corrections [Boyle et al, JHEP 02 (2023) 242]
- Physical point DWF ensemble with $a \simeq 0.12\ \mathrm{fm}$
- Bare parameters derivatives through operator insertions
- Electro-quenched $\mathrm{QED_L}$
- Universal FV $\mathrm{QED_L}$ effects subtracted from masses
- Using best AIC fit results, **statistical errors only**

Consistency check: quark masses

- Exercise: find $\overline{\rm MS}$ physical quark masses at $\mu = 2$ GeV

- Using renormalisation constants from RBC-UKQCD and 100% error on undetermined QED corrections

$$m_{ud} = 3.33(2) \text{ MeV} \qquad m_{ud} = 3.38(4) \text{ MeV}$$
$$m_s = 92.7(5) \text{ MeV} \qquad m_s = 92.2(1.0) \text{ MeV}$$
$$m_u/m_d = 0.457(4) \qquad m_u/m_d = 0.485(19)$$

this analysis [FLAG 2021 $N_f = 2 + 1$]

- **This is just a check, not a new result Systematics and continuum limit needed**

Quark mass scheme

- Prescription: take physical renormalised quark masses

$$m^{\phi} = (m^{\phi}_{ud}, m^{\phi}_{s}, m^{\phi}_{u} - m^{\phi}_{d})$$

- Then with $\alpha \to 0$

pure QCD $\hat{m} = (m^{\phi}_{ud}, m^{\phi}_{s}, m^{\phi}_{u} - m^{\phi}_{d})$

iso-symmetric QCD $\bar{m} = (m^{\phi}_{ud}, m^{\phi}_{s}, 0)$

- Generally implicit scheme for EFT calculations

- Introduced in lattice calculations by RM123 as "GRS scheme" for electro-quenched theories
  [RM123, Phys. Rev. D 87(11), 114505 (2013)]

BMW 2013 scheme

[BMW, Phys. Rev. Lett. 111(25), 252001 (2013)] [BMW, Phys. Rev. Lett. 117(8), 082001 (2016)]

- Connected $\bar{q}q$ meson masses as a proxy for quark masses

$$M_{\bar{q}q}^2 = 2B_0 m_q + \mathrm{NLO}$$

[Bijnens & Danielsson, Phys. Rev. D 75(1), 014505 (2007)]

- Variable set

$$M_{ud}^2 = \frac{1}{2}(M_{\bar{u}u}^2 + M_{\bar{d}d}^2) = 2B_0 m_{ud} + \mathrm{NLO}$$

$$\Delta M^2 = (M_{\bar{u}u}^2 - M_{\bar{d}d}^2) = 2B_0(m_u - m_d) + \mathrm{NLO}$$

$$2M_{K_\chi}^2 = M_{K^+}^2 + M_{K^0}^2 - M_{\pi^+}^2 = 2B_0 m_s + \mathrm{NLO}$$

- Scheme defined by

$$\hat{M} = (M_{ud}^{2,\phi}, \Delta M^{2,\phi}, 2M_{K_\chi}^{2,\phi}) \quad \text{pure QCD}$$

$$\bar{M} = (M_{ud}^{2,\phi}, 0, 2M_{K_\chi}^{2,\phi}) \quad \text{iso-symmetric QCD}$$

BMW 2013 scheme

- M_{ud}^2 and ΔM^2 are unphysical and need to be determined at the physical point, we found

$$M_{ud}^2 = 18251(15)\ \mathrm{MeV}^2$$
$$\Delta M^2 = -13127(104)\ \mathrm{MeV}^2$$

- Scheme slightly modified in BMW 2022 g-2 calculation

- They obtained

$$\Delta M^2 = -13170(320)(270)\ \mathrm{MeV}^2$$

Mainz scheme

[Mainz, arXiv:2203.08676]

- Identical to BMW 2013 up to the substitution

$$\Delta M^2 \mapsto \Delta_{\mathbf{8}}^2 = M_{K^+}^2 - M_{K^0}^2 - M_{\pi^+}^2 + M_{\pi^0}^2$$

- $2\Delta_{\mathbf{8}}^2$ and ΔM^2 are both equal to $2B_0(m_u - m_d)$ at LO

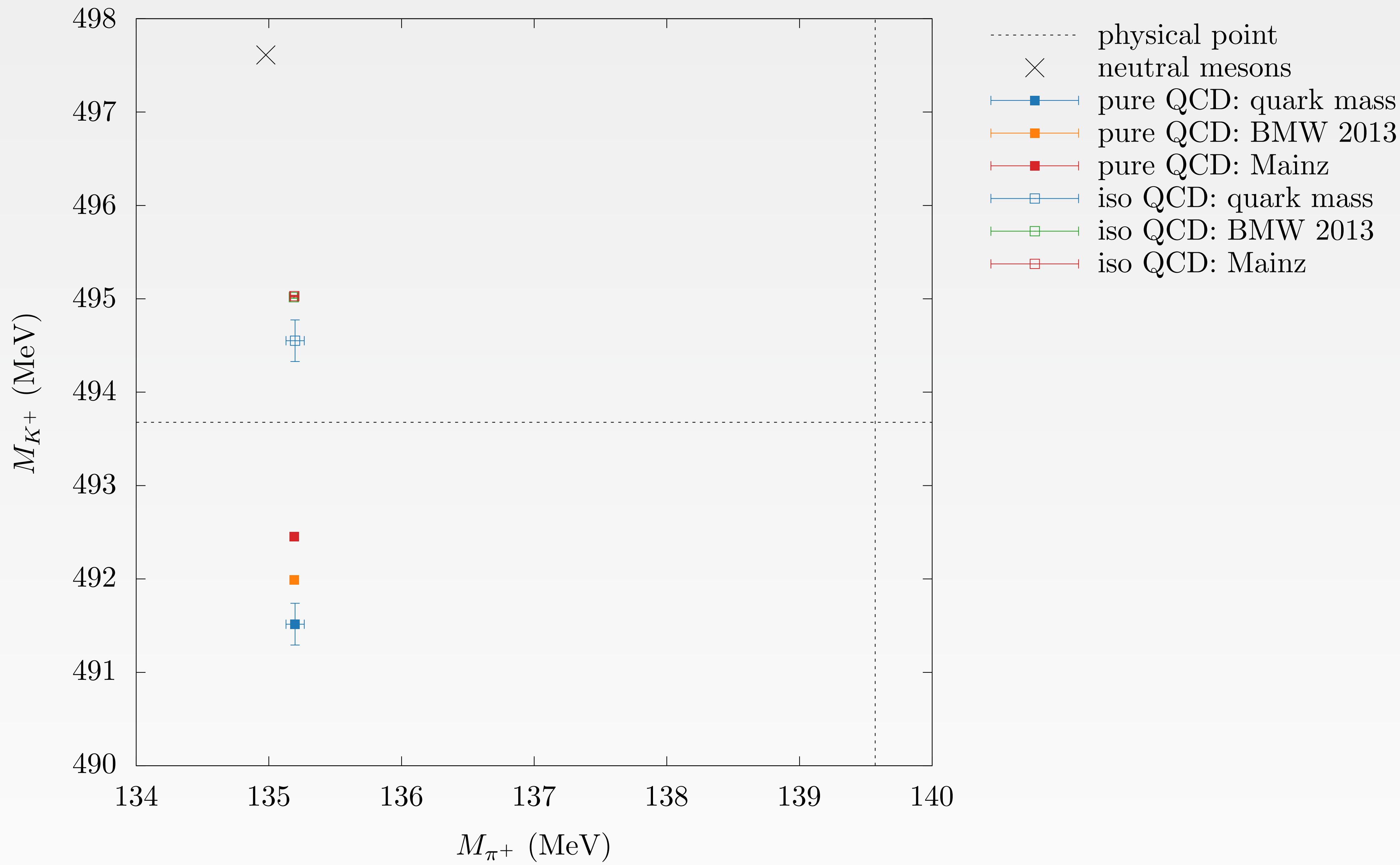
- $\Delta_{\mathbf{8}}^2$ is known experimentally, but potentially receives large corrections at NLO (Dashen theorem violations)

$$(\Delta M^2)_{\mathrm{LO}} = -13459(756)\ \mathrm{MeV}^2$$ [FLAG 2021] [RBC-UKQCD, PRD 93(7), 074505 (2016)]

$$\Delta M^2 = -13127(104)\ \mathrm{MeV}^2$$ *this analysis*

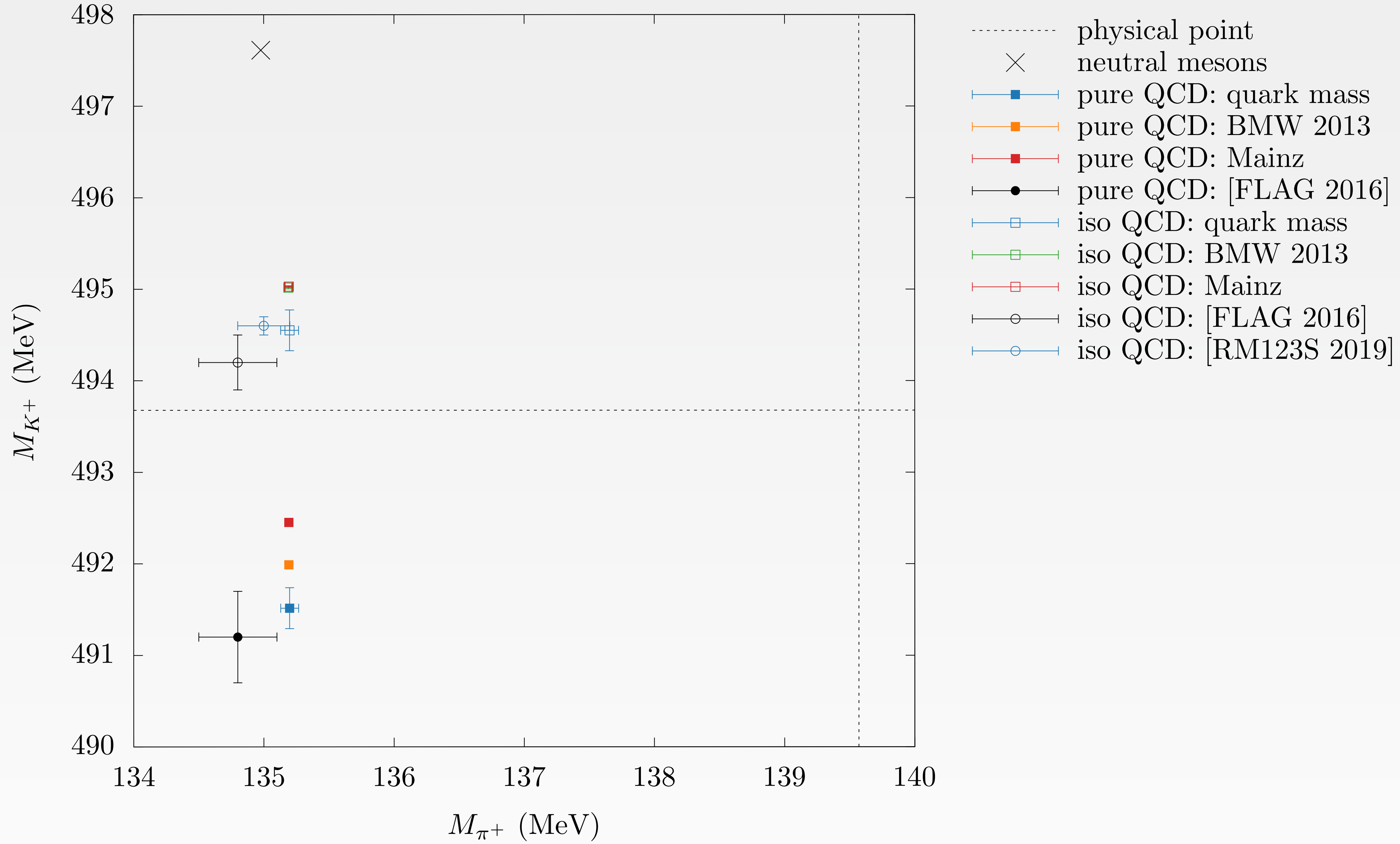
$$2\Delta_{\mathbf{8}}^2 = -10322(41)\ \mathrm{MeV}^2$$ [PDG 2022]

Pion/kaon plane landcape



Open symbols: iso QCD / Full symbols: pure QCD

Pion/kaon plane landcape

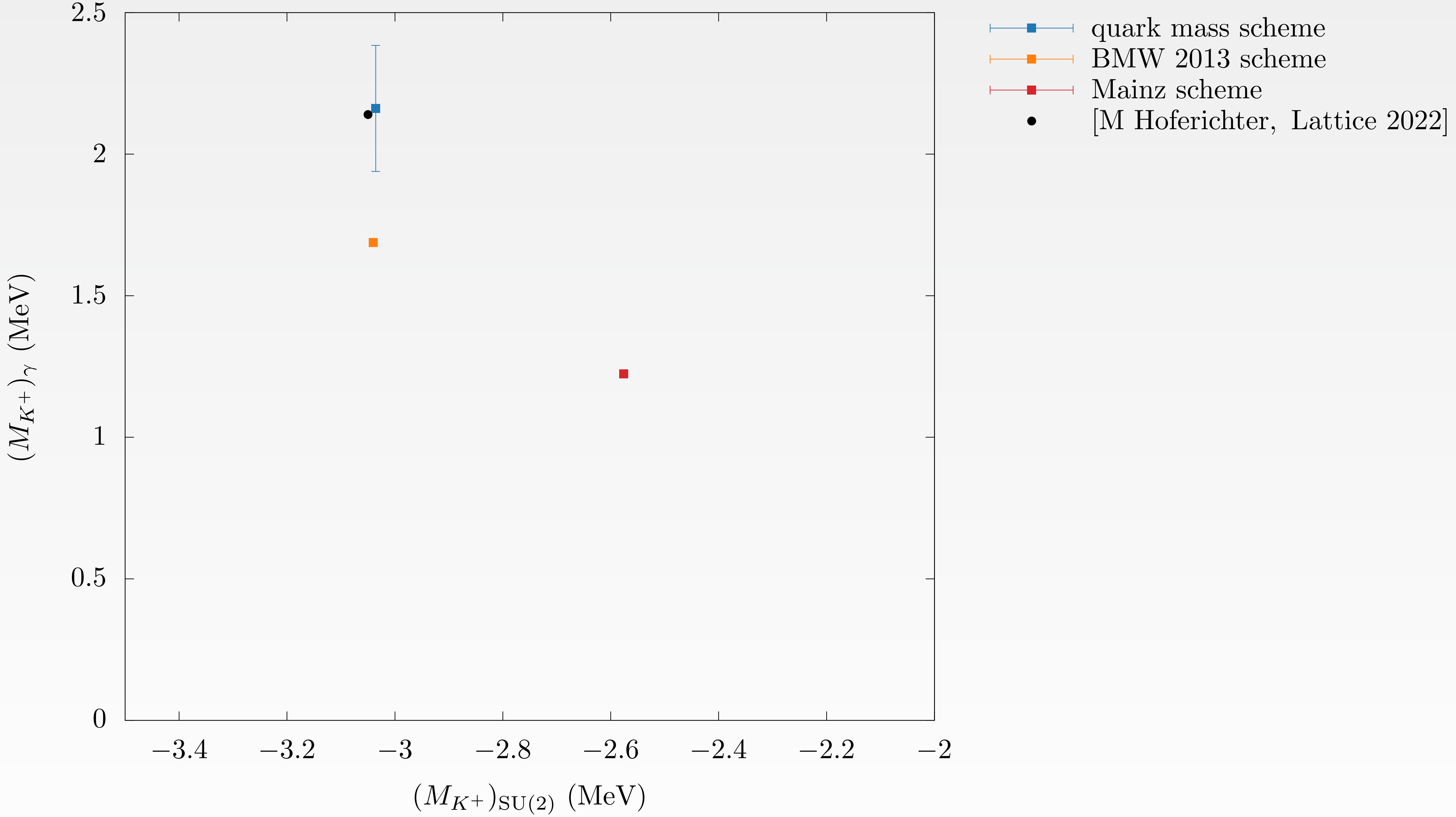


Open symbols: iso QCD / Full symbols: pure QCD

[RM123S 2019]: equivalent to quark mass scheme (electro-quenched GRS)

[FLAG 2016]: equivalent to quark mass scheme (pheno estimate)

Charged kaon decomposition



Outlook

Prescription proposal

- Proximity to the quark mass scheme is desired to make contact with phenomenology
- **Prescription proposal** (both with $\alpha = 0$)

Pure QCD	*Iso-symmetric QCD*
$\hat{M}_{\pi^+} = 135.0\ \mathrm{MeV}$	$\bar{M}_{\pi} = 135.0\ \mathrm{MeV}$
$\hat{M}_{K^+} = 491.6\ \mathrm{MeV}$	$\bar{M}_{K} = 494.6\ \mathrm{MeV}$
$\hat{M}_{K^0} = 497.6\ \mathrm{MeV}$	$\bar{M}_{D_s} = 1967\ \mathrm{MeV}$
$\hat{M}_{D_s} = 1967\ \mathrm{MeV}$	

- Scale setting with $\hat{M}_{\Omega^-} = \bar{M}_{\Omega^-} = 1672.45\ \mathrm{MeV}$ possibly using a theory scale as proxy

Comments

- This agrees with the FLAG 2016 prescription, but it is **self-consistently determined by a lattice calculation**
- Coming back to the FLAG 2016 numbers is non-trivial, it **establishes agreement between lattice and phenomenology** on IB corrections to the meson spectrum

Discussion points

- Convergence on community prescriptions
  Potential recommendation(s) for FLAG and g-2 TI

- Scale setting quantity choice,
  challenges regarding computing QED corrections

- Lattice QED matters, finite-volume, electro-quenching, etc…

Pure QCD

$$\hat{M}_{\pi^+} = 135.0 \text{ MeV}$$

$$\hat{M}_{K^+} = 491.6 \text{ MeV}$$

$$\hat{M}_{K^0} = 497.6 \text{ MeV}$$

$$\hat{M}_{D_s} = 1967 \text{ MeV}$$

Iso-symmetric QCD

$$\bar{M}_{\pi} = 135.0 \text{ MeV}$$

$$\bar{M}_{K} = 494.6 \text{ MeV}$$

$$\bar{M}_{D_s} = 1967 \text{ MeV}$$

$$\hat{M}_{\Omega^-} = \bar{M}_{\Omega^-} = 1672.45 \text{ MeV}$$

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