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Converging on QC+ED prescriptions

let me start by quoting some of the works in which the issue of the definition of isoQCD has been discussed

J. Gasser, A. Rusetsky and I. Scimemi, Eur. Phys. J. C 32 (2003), 97-114 doi:10.1140/epjc/s2003-01383-1 [arXiv:hep-ph/0305260 [hep-ph]].

G. M. de Divitiis et al. [RM123], Phys. Rev. D 87 (2013) no.11, 114505 doi:10.1103/PhysRevD.87.114505 [arXiv:1303.4896 [hep-lat]]

S.Borsanyi et al.[Budapest-Marseille-Wuppertal],Phys.Rev.Lett.111(2013)no.25,252001 doi:10.1103/PhysRevLett.111.252001 [arXiv:1306.2287 [hep-lat]]

A. Bussone, M. Della Morte, T. Janowski and A. Walker-Loud, PoS LATTICE2018 (2018), 293 doi:10.22323/1.334.0293 [arXiv:1810.11647 [hep-lat]].

M. Di Carlo, D. Giusti, V. Lubicz, G. Martinelli, C. T. Sachrajda, F. Sanfilippo, S. Simula and N. Tantalo, Phys. Rev. D 100 (2019) no.3, 034514 doi:10.1103/PhysRevD.100.034514 [arXiv:1904.08731 [hep-lat]]

S. Borsanyi, Z. Fodor, J. N. Guenther, C. Hoelbling, S. D. Katz, L. Lellouch, T. Lippert, K. Miura, L. Parato and K. K. Szabo, et al. Nature 593 (2021) no.7857, 51-55 doi:10.1038/s41586-021-03418-1 [arXiv:2002.12347 [hep-lat]]

C.T.Sachrajda,ActaPhys.Polon.B52(2021)no.3,175-201 doi:10.5506/APhysPolB.52.175 [arXiv:2104.04312 [hep-lat]].

P. Boyle, M. Di Carlo, F. Erben, V. Gülpers, M. T. Hansen, T. Harris, N. Hermansson- Truedsson, R. Hodgson, A. Jüttner and F. Ó. hÓgáin, et al. doi.org/10.1007/JHEP02(2023)242 JHEP 02 (2023) 242 [arXiv:2211.12865 [hep-lat]]

all FLAG editions

the ones to which I have contributed are

RM123 and RM123+SOTON collaborations

- RM123 collaboration, Leading isospin breaking effects on the lattice, Phys. Rev. D 87 (2013) 114505 [arXiv:1303.4896]
- M. Di Carlo et al., Light-meson leptonic decay rates in lattice QCD+QED, Phys. Rev. D 100 (2019) 034514 [arXiv:1904.08731]

FLAG Scale Setting WG

- Flavour Lattice Averaging Group (FLAG) collaboration, FLAG Review 2021, Eur. Phys. J. C 82 (2022) 869 [arXiv:2111.09849]

- N. Tantalo, Matching lattice QC+ED to Nature, PoS LATTICE2022 (2023) 249 [arXiv:2301.02097] by going trough the steps of an ideal calculation, there is nothing particularly difficult in this game, it is just an exercise in renormalization and perturbation theory by going trough the steps of an ideal calculation, there is nothing particularly difficult in this game, it is just an exercise in renormalization and perturbation theory

of course the exercise has to be done. . .

S-0 let's take $n_f = 2+1$ QCD and let's choose 3 matching observables

$$X_{ud}^{\mathrm{iso}} = \{M_{\pi^0}, M_{\pi^0}(1+\varepsilon_\pi), M_{uu}, \cdots\} \ , \qquad X_{s}^{\mathrm{iso}} = \{M_{K^0}, M_{K^0}(1+\varepsilon_K), M_{ss}, \cdots\} \ ,$$

$$X^{\mathrm{iso}} = \left\{F_\pi, F_K, \frac{1}{w_0}, \frac{1}{\sqrt{t_0}}, \cdots\right\}$$

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$$X^{\rm iso} = \left\{F_\pi, F_K, \frac{1}{w_0}, \frac{1}{\sqrt{t_0}}, \cdots\right\}$$

S-1 at fixed β we then fix $m^{\rm iso}_{ud}$ and $m^{\rm iso}_{s}$ from

$$\left[\frac{aX_u}{aX}\right]\left(\beta, 0, m^{\rm iso}_{ud}(\beta), m^{\rm iso}_{ud}(\beta), m^{\rm iso}_{s}(\beta)\right) = \frac{X^{\rm iso}_u}{X^{\rm iso}} \ , \qquad \left[\frac{aX_s}{aX}\right]\left(\beta, 0, m^{\rm iso}_{ud}(\beta), m^{\rm iso}_{ud}(\beta), m^{\rm iso}_{s}(\beta)\right) = \frac{X^{\rm iso}_s}{X^{\rm iso}} \ ,$$

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S-1 at fixed β we then fix m_{ud}^{iso} and m_s^{iso} from

$$\left[\frac{aX_u}{aX}\right]\Big(\beta, 0, m_{ud}^{\text{iso}}(\beta), m_{ud}^{\text{iso}}(\beta), m_s^{\text{iso}}(\beta)\Big) = \frac{X_u^{\text{iso}}}{X^{\text{iso}}} \ , \qquad \left[\frac{aX_s}{aX}\right]\Big(\beta, 0, m_{ud}^{\text{iso}}(\beta), m_{ud}^{\text{iso}}(\beta), m_s^{\text{iso}}(\beta)\Big) = \frac{X_s^{\text{iso}}}{X^{\text{iso}}} \ ,$$

S-2 we can now compute any isoQCD observable, e.g.

$$\left[\frac{M_p}{X}\right]^{\text{iso}} = \lim_{\beta\mapsto\infty} \left[\frac{aM_p}{aX}\right]\Big(\beta, 0, m_{ud}^{\text{iso}}(\beta), m_{ud}^{\text{iso}}(\beta), m_s^{\text{iso}}(\beta)\Big) \ , \qquad M_p^{\text{iso}} = \left[\frac{M_p}{X}\right]^{\text{iso}} \times X^{\text{iso}}$$

the same story can be told by using the equivalent language

$$a^{\text{iso}}(\beta) \equiv \frac{[aX]\left(\beta, 0, m_{ud}^{\text{iso}}(\beta), m_{ud}^{\text{iso}}(\beta), m_s^{\text{iso}}(\beta)\right)}{X^{\text{iso}}} \ ,$$

$$\left[\frac{aX_{ud}}{a}\right]\left(\beta, 0, m_{ud}^{\text{iso}}(\beta), m_{ud}^{\text{iso}}(\beta), m_s^{\text{iso}}(\beta)\right) = X_{ud}^{\text{iso}} \ , \qquad \left[\frac{aX_s}{a}\right]\left(\beta, 0, m_{ud}^{\text{iso}}(\beta), m_{ud}^{\text{iso}}(\beta), m_s^{\text{iso}}(\beta)\right) = X_s^{\text{iso}}$$

$$M_p^{\text{iso}} = \lim_{\beta \mapsto \infty} \left[\frac{aM_p}{a}\right]\left(\beta, 0, m_{ud}^{\text{iso}}(\beta), m_{ud}^{\text{iso}}(\beta), m_s^{\text{iso}}(\beta)\right)$$

S-3 let's now take $n_f = 1 + 1 + 1$ QCD+QED and, for simplicity, let's neglect terms of $O(\alpha_{\rm em}^2)$; this allows us to set

$$\alpha_{\rm em} = \frac{1}{137.035999084}$$

S-3 let's now take $n_f = 1+1+1$ QCD+QED and, for simplicity, let's neglect terms of $O(\alpha_{\rm em}^2)$; this allows us to set

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S-4 at the same values of β that we used in our isoQCD calculation we now fix m_u, m_d and m_s by using experimental inputs, e.g.

$$\left[\frac{aM_{\pi^+}}{aM_\Omega}\right](\beta, \alpha_{\rm em}, m_u(\beta), m_d(\beta), m_s(\beta)) = \left[\frac{M_{\pi^+}}{M_\Omega}\right]^{\rm exp} ,$$

$$\left[\frac{aM_{K^0}}{aM_\Omega}\right](\beta, \alpha_{\rm em}, m_u(\beta), m_d(\beta), m_s(\beta)) = \left[\frac{M_{K^0}}{M_\Omega}\right]^{\rm exp} ,$$

$$\left[\frac{aM_{K^0} - aM_{K^+}}{aM_\Omega}\right](\beta, \alpha_{\rm em}, m_u(\beta), m_d(\beta), m_s(\beta)) = \left[\frac{M_{K^0} - M_{K^+}}{M_\Omega}\right]^{\rm exp} ,$$

S-5 let's now expand the previous expressions, e.g.

$$\left[\frac{aM_{\pi^+}}{aM_\Omega}\right]^{\text{full}}(\beta,\alpha_{\text{em}}) = \frac{[aM_\pi]^{\text{iso}}(\beta)\left\{1+\delta M_{\pi^+}(\beta,\alpha_{\text{em}})\right\}}{[aM_\Omega]^{\text{iso}}\left\{1+\delta M_\Omega(\beta,\alpha_{\text{em}})\right\}}$$

$$= \left[\frac{aM_{\pi^+}}{aM_\Omega}\right]^{\text{iso}}(\beta)\left\{1+\delta M_{\pi^+}(\beta,\alpha_{\text{em}}) - \delta M_\Omega(\beta,\alpha_{\text{em}})\right\}$$

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$$= \left[\frac{aM_{\pi^+}}{aM_\Omega}\right]^{\text{iso}}(\beta)\left\{1+\delta M_{\pi^+}(\beta,\alpha_{\text{em}}) - \delta M_\Omega(\beta,\alpha_{\text{em}})\right\}$$

where

$$[H]^{\text{iso}}(\beta) \equiv [H]\left(\beta, 0, m_{ud}^{\text{iso}}(\beta), m_{ud}^{\text{iso}}(\beta), m_s^{\text{iso}}(\beta)\right)$$

$$\delta M_H(\beta,\alpha_{\text{em}}) \equiv \frac{1}{[aM_H]^{\text{iso}}(\beta)}\left\{\alpha_{\text{em}}\frac{\partial}{\partial\alpha_{\text{em}}} + \left(m_f - m_f^{\text{iso}}\right)\frac{\partial}{\partial m_f}\right\}[aM_X]\left(\beta, 0, m_{ud}^{\text{iso}}(\beta), m_{ud}^{\text{iso}}(\beta), m_s^{\text{iso}}(\beta)\right)$$

$$\equiv \left\{\alpha_{\text{em}}\frac{\partial}{\partial\alpha_{\text{em}}} + \left(m_f - m_f^{\text{iso}}\right)\frac{\partial}{\partial m_f}\right\}\log\,[aM_X](\beta)$$

$$[H]^{\text{iso}} = \lim_{\beta\mapsto\infty}[H]^{\text{iso}}(\beta)\ , \qquad \delta M_H = \lim_{\beta\mapsto\infty}\delta M_H(\beta,\alpha_{\text{em}})$$

S-6 the mass counter-terms can thus be fixed by using

$$\delta M_{\pi^+}(\beta, \alpha_{\rm em}) - \delta M_{\Omega}(\beta, \alpha_{\rm em}) = \left[\frac{M_{\pi^+}}{M_\Omega}\right]^{\rm exp} \left[\frac{M_\Omega}{M_{\pi^+}}\right]^{\rm iso} - 1 \ ,$$

$$\delta M_{K^0}(\beta, \alpha_{\rm em}) - \delta M_{\Omega}(\beta, \alpha_{\rm em}) = \left[\frac{M_{K^0}}{M_\Omega}\right]^{\rm exp} \left[\frac{M_\Omega}{M_{K}}\right]^{\rm iso} - 1 \ ,$$

$$\delta M_{K^0}(\beta, \alpha_{\rm em}) - \delta M_{K^+}(\beta, \alpha_{\rm em}) = \left[\frac{M_{K^0} - M_{K^+}}{M_\Omega}\right]^{\rm exp} \left[\frac{M_\Omega}{M_{K}}\right]^{\rm iso}$$

S-6 i.e. by solving at fixed β the linear system of equations

$$\left(m_f - m_f^{\mathrm{iso}}(\beta)\right)\frac{\partial}{\partial m_f}\log\left[\frac{aM_{\pi^+}}{aM_\Omega}\right](\beta) = \left\{\left[\frac{M_{\pi^+}}{M_\Omega}\right]^{\mathrm{exp}}\left[\frac{M_\Omega}{M_{\pi^+}}\right]^{\mathrm{iso}} - 1\right\} - \alpha_{\mathrm{em}}\frac{\partial}{\partial\alpha_{\mathrm{em}}}\log\left[\frac{aM_{\pi^+}}{aM_\Omega}\right](\beta)\ ,$$

$$\left(m_f - m_f^{\mathrm{iso}}(\beta)\right)\frac{\partial}{\partial m_f}\log\left[\frac{aM_{K^0}}{aM_\Omega}\right](\beta) = \left\{\left[\frac{M_{K^0}}{M_\Omega}\right]^{\mathrm{exp}}\left[\frac{M_\Omega}{M_K}\right]^{\mathrm{iso}} - 1\right\} - \alpha_{\mathrm{em}}\frac{\partial}{\partial\alpha_{\mathrm{em}}}\log\left[\frac{M_{K^0}}{M_\Omega}\right](\beta)\ ,$$

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notice that:

- the isoQCD matching quantities appear only through $m_f^{\rm iso}(\beta) \equiv m_f^{\rm iso}(\beta, X_{ud}^{\rm iso}, X_s^{\rm iso}, X^{\rm iso})$
- if the isoQCD matching observables are unphysical, their IB corrections δX_{ud}, δX_s and δX are not needed
- conversely, both the isoQCD value and the associated IB corrections of the physical quantity used to set the scale in QCD+QED, here M_Ω, will be needed

S-7 the proton mass can now be computed, by separating the isoQCD and IB corrections in such a way that these are both UV-finite, according to

$$M_p = M_\Omega^{\mathrm{exp}} \times \lim_{\beta \mapsto \infty} \left[\frac{aM_p}{aM_\Omega} \right] (\beta, \alpha_{\mathrm{em}}, m_u(\beta), m_d(\beta), m_s(\beta))$$

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$$= X^{\rm iso} \left[\frac{M_p}{X}\right]^{\rm iso} \times \frac{M_\Omega^{\rm exp}}{M_\Omega^{\rm iso}} \{1 + \delta M_p - \delta M_\Omega\}$$

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$$= X^{\rm iso} \left[\frac{M_p}{X} \right]^{\rm iso} \times \{1 + \delta M_p + \delta a\} , \qquad \delta a = \frac{M_\Omega^{\exp} - M_\Omega^{\rm iso} \{1 + \delta M_\Omega\}}{M_\Omega^{\rm iso}}$$

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- when, subsequently, lattice QCD+QED is matched to nature, one needs

$$\left[\frac{M_{\pi^+}}{X}\right]^{\text{iso}}, \quad \left[\frac{M_{K^0}}{X}\right]^{\text{iso}}, \quad \left[\frac{M_{K^+}}{X}\right]^{\text{iso}}, \quad \left[\frac{M_{\Omega}}{X}\right]^{\text{iso}}, \quad \cdots$$

  but not the IB corrections δX_{ud}, δX_s, δX to the isoQCD matching observables if these are unphysical quantities that will not be calculated in QCD+QED

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  but not the IB corrections δX_{ud}, δX_s, δX to the isoQCD matching observables if these are unphysical quantities that will not be calculated in QCD+QED

- the formula to compute a generic observable at $O(\alpha_{\rm em})$ is

$$O = \left(X^{\rm iso}\right)^{d_O} \left[\frac{O}{X^{d_O}}\right]^{\rm iso} \times \{1 + \delta O + d_O \delta a\}$$

  where

$$\delta a = \left\{\frac{M_\Omega^{\rm exp}}{X^{\rm iso}} - \left[\frac{M_\Omega}{X}\right]^{\rm iso}(1+\delta M_\Omega)\right\}\left[\frac{X}{M_\Omega}\right]^{\rm iso} \qquad \xrightarrow{X^{\rm iso}=M_\Omega^{\rm exp}} \qquad \delta a = -\delta M_\Omega$$

in my opinion,

- an agreement can be found because any scheme is legitimate and because even if we decide to use unphysical observables (theory scales) their IB corrections are not needed
- moreover, baryons can be avoided to define isoQCD
- on the other hand, if a lattice collaboration is also interested in computing IB corrections, the calculation of the isoQCD value and of the associated IB corrections to a baryon mass can hardly be avoided
- therefore, why not choosing $X^{\mathrm{iso}} = \{M_\Omega^{\mathrm{exp}}, M_p^{\mathrm{exp}}, M_\Xi^{\mathrm{exp}}\}$ from the very beginning?
- if we really want to find an agreement, we have to be ready to give-up some of the choices that we made in the past

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as far as ETMC/RM123 is concerned, $n_f = 2+1+1$ isoQCD has been defined by using

$$X_{ud} = M_\pi^{\rm iso} = 135.0\ MeV\ , \quad X_s = M_K^{\rm iso} = 494.2\ MeV\ , \quad X_c = M_{D_s}^{\rm iso} = 1969.0\ MeV\ ,$$

$$X = F_\pi = 130.4\ MeV$$

but also the theory scales w_0, $\sqrt{t_0}$, $M_{ss}^{(\star)}$, $M_{cc}^{(\star)}$ and the baryon masses m_p, m_Ω and m_{Λ_c} have been considered