The BMW prescription (2020)

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Introduction and motivation

Search for new ϕ in low-energy experiments involving strong interaction effects

- need nonperturbative QCD computations w/ subpercent precision
- must include $O(\alpha)$ and $O(\delta m = m_d - m_u)$ corrections

At this order

$$\mathcal{O}_{\rm SM} = \mathcal{O}_{\rm ISO} + \mathcal{O}_{\rm SIB} + \mathcal{O}_{\rm QED}$$

- $\mathcal{O}_{\rm SM}$ = full SM prediction including $O(\alpha)$ and $O(\delta m)$ effects
  - can be compared directly to experimental measurements at *permil* level
- $\mathcal{O}_{\rm ISO} \sim$ contribution to $\mathcal{O}_{\rm SM}$ w/ renormalized $\alpha = 0$ and $\delta m = 0$ keeping all other renormalized parameters fixed
  - can be compared directly to experimental measurements at *percent* level
- $\mathcal{O}_{\rm SIB} \sim \delta m_\phi \frac{\partial \mathcal{O}_{\rm SM}}{\partial \delta m}\Big|_{\delta m=0,\alpha=0,\dots}$
- $\mathcal{O}_{\rm QED} \sim \alpha_\phi \frac{\partial \mathcal{O}_{\rm SM}}{\partial \alpha}\Big|_{\delta m=0,\alpha=0,\dots}$

Renormalization

- Once $N_f=2+1+\cdots$ QCD$_{\text{ISO}}$ is renormalized by tuning bare $m_{ud}, m_s, \ldots$ and g_s to reproduce *observables* that depend strongly on these parameters

  $\rightarrow$ $\mathcal{O}_{\text{ISO}}$ and $\mathcal{O}_{\text{SIB}}$ both finite

- Calculation of $\mathcal{O}_{\text{QED}}$ introduces $O(\alpha)$ divergences that must be absorbed in bare parameters of QCD

  $\rightarrow$ value of $\mathcal{O}_{\text{ISO,SIB,QED}}$ depends on finite terms absorbed in each contribution (scheme dependence)

  $\rightarrow$ (re-)renormalize parameters of QCD by tuning bare $m_u, m_d, m_s, \ldots$ and g_s to reproduce *precisely measured quantities* that depend strongly on these parameters

  $\rightarrow$ at $O(\alpha)$, α does not renormalize: take e.g. PDG value

  $\rightarrow$ separation into $\mathcal{O}_{\text{ISO,SIB,QED}}$ obtained by tuning bare parameters of QCD$_{\text{ISO}}$ to reproduce as many measured quantities or observables computed in $N_f=1+1+1+\cdots$ QCD+QCD and isolating terms $\propto \alpha, \delta m$

BMW scheme (2020)

Hadronic scheme [BMW '13, '20]:

- M_{qq} = mass of connected $q\bar{q}$ meson as standin for m_q (q=u, d, s, c) [a]
- w_0 [BMW '12] as standin for α_s

Pros:

- very sensitive to individual m_q; χPT expansions [Bijnens et al '07]

$$M^2_{\pi_\chi} \equiv \frac{M^2_{uu} + M^2_{dd}}{2} = B_2(m_u + m_d) + O(m^2_{ud}, \alpha/N_c, (\delta m)^2/N_c)$$

$$\Delta M^2 \equiv M^2_{dd} - M^2_{uu} = B_2 \delta m + \delta m \times O(m_{ud}, \alpha)$$

$$M^2_{ss} = 2B_3 m_s + O(m_s^2, \alpha m_s/N_c, \delta m^2/N_c) \qquad M_{cc} = 2m_c(1 + O(v^2, \alpha/N_c))$$

$\rightarrow$ should be no large αm_s Dashen violations

- w_0 is mostly sensitive to QCD scale
  $\rightarrow$ purely gluonic
  $\rightarrow$ SIB & QED corrections: $(1/N_c) \times O(\alpha, (\delta m)^2)$

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[a] In BMW'20, used m_s / m_c instead of M_{cc}

BMW scheme (2020)

- should be close to GRS scheme [Gasser et al '12] defined via renormalized m_q and α_s
- no need to perform additional renormalization of m_q, $q = u, d, s, c$, and α_s, in QCD and QCD+QED
- M_{qq} and w_0 computable w/ order permil accuracy

Cons:

- NONE!

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Cons:

- M_{qq} and w_0 are not experimentally measurable (though some very close to be)
  - $\rightarrow$ must be computed
  - $\rightarrow$ have done so w/ few permil accuracy [BMW'20]
  - $\rightarrow$ results can be used by others
- If each collaboration uses its own M_{qq} & w_0
  - $\rightarrow$ no common defintion of ISO, SIB & QED contributions
  - $\Rightarrow$ precise comparison not possible
- However could agree on standard values!

BMW separation strategy (2020)

1. Calculate $\mathcal{O}_{\text{SM}}$ in $N_f = 4 \times 1$ QCD+QED w/ many m_q bracketing ϕ values & many a, L

2. Obtain $\mathcal{O}_{\text{SM}}$ (Type I fits)

$$\rightarrow \mathcal{O}_{\text{SM}}\left(\frac{M^2_{\pi_\chi}}{M^2_{\Omega^-}}, \frac{M^2_{K_\chi}}{M^2_{\Omega^-}}, \frac{\Delta M^2_K}{M^2_{\Omega^-}}, \frac{M_{\eta_c}}{M_{\Omega^-}}, e_v, e_s, aM_{\Omega^-}, LM_{\Omega^-}\right)$$

w/

$$\begin{aligned} M^2_{\pi_\chi} &= M_{\pi^0}(1+O((\delta m)^2,\alpha^2)) \sim m_u + m_d \\ M^2_{K_\chi} &= \frac{1}{2}(M^2_{K^+} + M^2_{K^0} - M^2_{\pi^+}) \sim m_s \\ \Delta M^2_K &= M^2_{K^0} - M^2_{K^+} \sim m_d - m_u \qquad M_{\eta_c} \sim 2m_c \end{aligned}$$

$$M_{\Omega^-} \sim a \qquad \& \qquad \alpha$$

- interpolate to ϕ masses & α and $aM_{\Omega^-} \rightarrow 0$ & $LM_{\Omega^-} \rightarrow \infty$ $\rightarrow \quad \mathcal{O}^{\phi}_{\text{SM}}$
- obtain thus $\mathcal{O}^{\phi}_{\text{SM}} = M^{\phi,2}_{uu}, M^{\phi,2}_{dd}, M^{\phi,2}_{ss}, w^{\phi}_0$

BMW separation strategy (2020)

3. Now, consider (Type II fit)

$$\mathcal{O}_{\rm SM}\left(w_0^2 M_{uu}^2, w_0^2 M_{dd}^2, w_0^2 \Delta M^2, w_0 M_{cc}, e_v, e_s, a/w_0, L w_0\right)$$

w/ $\Delta M^2 = M_{dd}^2 - M_{dd}^2$

- study as function of its arguments
- interpolate to ϕ masses & α and $a/w_0 \to 0$ & $L/w_0 \to \infty$
- isolate coefficients of ΔM^2, e_v^2, $e_v e_s$, e_s^2 $\to$ finite:

$$\begin{aligned}
\mathcal{O}_{\rm ISO} &= \mathcal{O}_{\rm SM}\left((w_0^2 M_{\pi_\chi}^2)^\phi, (w_0^2 M_{\pi_\chi}^2)^\phi, 0, (w_0 M_{cc})^\phi, 0, 0, 0, \infty\right)\\
\mathcal{O}_{\rm SIB} &= \mathcal{O}_{\rm SM}\left((w_0^2 M_{\pi_\chi}^2)^\phi, (w_0^2 M_{\pi_\chi}^2)^\phi, (w_0^2 \Delta M^2)^\phi, (w_0 M_{cc})^\phi, 0, 0, 0, \infty\right)\\
\mathcal{O}_{\rm QED} &= \mathcal{O}_{\rm SM}\left((w_0^2 M_{\pi_\chi}^2)^\phi, (w_0^2 M_{\pi_\chi}^2)^\phi, 0, (w_0 M_{cc})^\phi, e_\phi, e_\phi, 0, \infty\right)
\end{aligned}$$

Results for K-mass decomposition in BMW'20 scheme

Type I fits yield:

$$w_0^\phi = 0.17236(29)(63)\,\mathrm{fm} \quad (\Delta M^2)^\phi = 0.01317(32)(27)\,\mathrm{GeV}^2 \quad M_{ss}^\phi = 689.89(28)(40)\,\mathrm{MeV}$$

Type II fits for

$$M_{K^{+/0}} = M_{K^{+/0}}^{\mathrm{ISO}} + M_{K^{+/0}}^{\mathrm{SIB}} + M_{K^{+/0}}^{\mathrm{QED}}$$

give (preliminary)

$$M_{K^0}^{\mathrm{ISO}} = 494.55(31)\,\mathrm{MeV} \quad M_{K^0}^{\mathrm{ISO}} = 2.98(14)\,\mathrm{MeV} \quad M_{K^0}^{\mathrm{QED}} = 0.05(7)\,\mathrm{MeV}$$

$$M_{K^+}^{\mathrm{ISO}} = 494.54(31)\,\mathrm{MeV} \quad M_{K^+}^{\mathrm{ISO}} = -3.13(17)\,\mathrm{MeV} \quad M_{K^+}^{\mathrm{QED}} = 2.25(8)\,\mathrm{MeV}$$

$$\checkmark \quad M_{K^0}^{\mathrm{ISO}} = M_{K^0}^{\mathrm{ISO}}$$

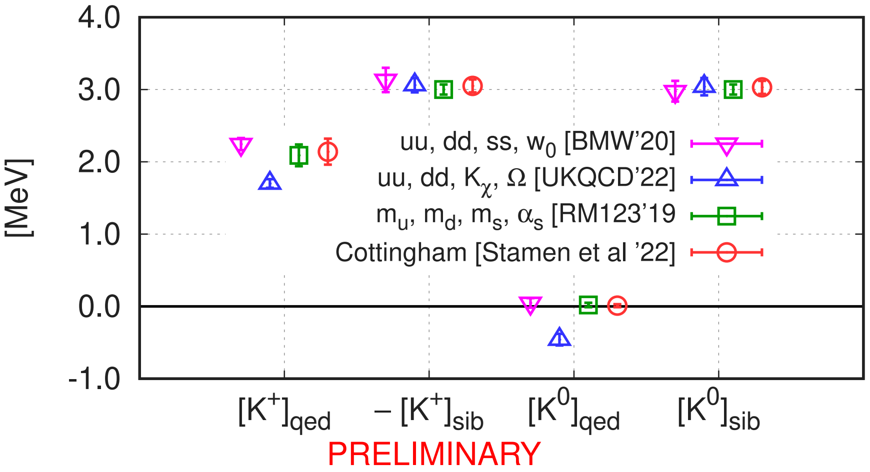
Comparison to other schemes

- Phenomenology
  - Cottingham & narrow width approximation [Cottingham '63, Stamen et al '22]

$$M_\omega,\ \Gamma_\omega,\ M_\phi,\ \Gamma_\phi,\ M_{K^0},\ M_{K^+},\ \alpha$$

- Lattice
  - GRS [Gasser et al 02, RM123 '17,'19]: α & e.g. in $\overline{\mathrm{MS}}$ @ $2\,\mathrm{GeV}$
    - $m_{ud} = \frac{1}{2}(m_u + m_d),\ \delta m = m_d - m_u,\ m_s,\ m_c,\ \alpha_s$
  - BMW '13, UKQCD '22
    - QCD+QED: $M^2_{\pi_\chi},\ \Delta M^2_K,\ M^2_{K_\chi},\ M_{\Omega^-},\ \alpha$
    - Separation: $M^2_{\pi_\chi},\ \Delta M^2,\ M^2_{K_\chi},\ M_{\Omega^-},\ \alpha$
      $\to$ large αm_s Dashen violations
  - [Mainz '22
    QCD+QED: $M^2_{\pi_chi},\ \Delta M^2_K - \Delta M^2_\pi,\ M^2_{K_\chi},\ f_\pi^{\mathrm{ISO}},\ \alpha$
    QCD$_{\mathrm{ISO}}$: $M^2_{\pi_\chi},\ m_u = m_d,\ M^2_{K_\chi},\ f_\pi^{\mathrm{ISO}},\ \alpha = 0$
    $\to$ large αm_s Dashen violations]

Comparison to other schemes



$\rightarrow$ excellent agreement with GRS and Cottingham

$\Rightarrow$ indication of equivalence between BMW'20 and GRS even beyond $K^{+,0}$ mass decomposition

Conclusion

- BMW'20 prescription: decompose $N_f = 4 \times 1$ QCD+QED results via matching of

$$w_0^\phi = 0.17236(29)(63)\,\mathrm{fm} \qquad M_{\pi_\chi}^\phi = \sqrt{\frac{M_{uu}^2 + M_{dd}^2}{2}} = 134.9768(5)\,\mathrm{MeV}$$

$$(\Delta M^2)^\phi = M_{dd}^2 - M_{uu}^2 = 0.01317(32)(27)\,\mathrm{GeV}^2 \qquad M_{ss}^\phi = 689.89(28)(40)\,\mathrm{MeV}$$

$$M_{\eta_c^\phi} = 2.9863(27)\,\mathrm{GeV} \quad \text{[HPQCD '15]} \quad \left(\text{or } \frac{m_c}{m_s} = 11.85\right) \quad \text{[HPQCD '10]}$$

- Suggested prescription: decompose $N_f = 4 \times 1$ QCD+QED results via matching of e.g.

$$w_0^\phi = 0.1730\,\mathrm{fm} \qquad M_{\pi_\chi}^\phi = \sqrt{\frac{M_{uu}^2 + M_{dd}^2}{2}} = 135.0\,\mathrm{MeV}$$

$$(\Delta M^2)^\phi = M_{dd}^2 - M_{uu}^2 = 0.01317\,\mathrm{GeV}^2 \qquad M_{ss}^\phi = 689.9\,\mathrm{MeV}$$

$$M_{\eta_c^\phi} = 2.986\,\mathrm{GeV}$$