Leading isospin breaking effects in a_μ^{HVP} and in baryon masses - an update of the Mainz effort

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Outline

Mainz setup for Lattice QCD+QED

LO-HVP contribution to the muon anomalous magnetic moment a_μ

Baryon spectrum in QCD+QED

Some remarks on a renormalisation scheme convention

Mainz setup for Lattice QCD+QED

QCD+QED on QCD_{iso} gauge ensembles

- Non-compact QED_L-action[1] for IR regularisation, Coulomb gauge
- QCD+QED quark action from QCD_{iso}:

$$(m_\text{u}^{(0)}, m_\text{d}^{(0)}, m_\text{s}^{(0)}) \to (m_\text{u}, m_\text{d}, m_\text{s}) \qquad m_\text{u}^{(0)} = m_\text{d}^{(0)}$$

$$U^{x\mu} \to W^{x\mu} = U^{x\mu} e^{\text{i}aeQA^{x\mu}} \qquad Q = \text{diag}\left(\frac{2}{3}, -\frac{1}{3}, -\frac{1}{3}\right)$$

- QCD+QED on QCD_{iso} gauge ensembles:
  - QCD+QED parametrised by $\varepsilon = (m_\text{u}, m_\text{d}, m_\text{s}, e^2)$
  - QCD_{iso} parametrised by $\varepsilon^{(0)} = (m_\text{u}^{(0)}, m_\text{d}^{(0)}, m_\text{s}^{(0)}, 0)$
  - Reweighting and leading order perturbative expansion[2] in $\Delta\varepsilon = \varepsilon - \varepsilon^{(0)}$ around $\varepsilon^{(0)}$
  - α_em does not renormalise at leading order
- QCD_{iso} $O(a)$-improved, isospin breaking introduces $O(a)$ lattice artefacts

[1] Hayakawa and Uno 2008.
[2] Divitiis et al. 2013.

Hadronic renormalisation scheme for QCD+QED and $\mathrm{QCD_{iso}}$

- Pseudo-scalar meson masses in χPT + QED[3] give proxies for bare parameters $m_\mathrm{u}+m_\mathrm{d}$, m_s and $m_\mathrm{u}-m_\mathrm{d}$:

$$\hat{m}=\tfrac{1}{2}(m_\mathrm{u}+m_\mathrm{d}) \qquad \varepsilon=\tfrac{\sqrt{3}}{4}\tfrac{m_\mathrm{d}-m_\mathrm{u}}{m_\mathrm{s}-\hat{m}} \quad \pi^0\text{-}\eta \text{ mixing angle}$$

At $O(e^2p^0)$ and $O(\varepsilon)$:

$$m^2_{\pi^0}=2B\hat{m} \qquad m^2_{K^0}=B\Big((m_\mathrm{s}+\hat{m})+\tfrac{2\varepsilon}{\sqrt{3}}(m_\mathrm{s}-\hat{m})\Big)$$

$$m^2_{\pi^+}=2B\hat{m}+2e^2ZF^2 \qquad m^2_{K^+}=B\Big((m_\mathrm{s}+\hat{m})-\tfrac{2\varepsilon}{\sqrt{3}}(m_\mathrm{s}-\hat{m})\Big)+2e^2ZF^2$$

In chiral limit: F pion decay constant, B vacuum condensate parameter, Z dimensionless coupling constant

- At the moment: isospin breaking effects in scale setting disregarded

- Scheme for QCD+QED:

$$\begin{aligned} m^2_{\pi^0} &\propto m_\mathrm{u}+m_\mathrm{d} \\ m^2_{K^+}+m^2_{K^0}-m^2_{\pi^+} &\propto m_\mathrm{s} \\ m^2_{K^+}-m^2_{K^0}-m^2_{\pi^+}+m^2_{\pi^0} &\propto m_\mathrm{u}-m_\mathrm{d} \\ \tfrac{4\pi}{137.035\ldots} &= e^2 \end{aligned}$$

- Scheme for $\mathrm{QCD_{iso}}$:

$$\begin{aligned} m^2_{\pi^0} &\propto m_\mathrm{u}+m_\mathrm{d} \\ m^2_{K^+}+m^2_{K^0}-m^2_{\pi^+} &\propto m_\mathrm{s} \\ 0 &= m_\mathrm{u}-m_\mathrm{d} \\ 0 &= e^2 \end{aligned}$$

[3] Neufeld and Rupertsberger 1996.

LO-HVP contribution to the muon anomalous magnetic moment a_μ

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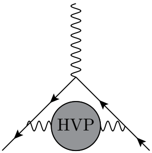
- a_μ^{HVP} in time-momentum representation (TMR)[4]:

$$a_\mu^{\mathrm{HVP}} = \left(\frac{\alpha}{\pi}\right)^2 \int_0^\infty \mathrm{d}t\, \widetilde{K}(t, m_\mu)\, C(t)$$

$$C(x^0)\, \delta^{\mu_2 \mu_1} = \int \mathrm{d}x^3 \langle \mathcal{V}_{\mathrm{R}}^{\gamma x \mu_2} \mathcal{V}_{\mathrm{R}}^{\gamma 0 \mu_1} \rangle$$



- Intermediate window contribution of $a_{\mu,\mathrm{win}}^{\mathrm{HVP}}$ in TMR:

$$a_{\mu,\mathrm{win}}^{\mathrm{HVP}} = \left(\frac{\alpha}{\pi}\right)^2 \int_0^\infty \mathrm{d}t\, \omega_{\mathrm{win}}(t)\, \widetilde{K}(t, m_\mu)\, C(t)$$

$$\omega_{\mathrm{win}}(t) = \Theta(t, t_0, \Delta) - \Theta(t, t_1, \Delta) \qquad \Theta(t, t', \Delta) = \frac{1}{2}\left(1 + \tanh\left(\frac{t - t'}{\Delta}\right)\right)$$

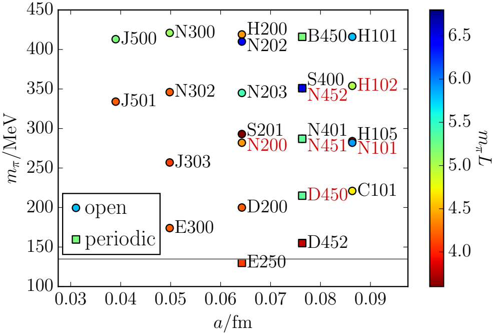
$$t_0 = 0.4\,\mathrm{fm} \qquad t_1 = 1.0\,\mathrm{fm} \qquad \Delta = 0.15\,\mathrm{fm}$$

- Goal: Precise Lattice QCD determination with sub-percent error
  $\Rightarrow$ Isospin breaking effects become relevant!

[4] Bernecker and Meyer 2011; Francis et al. 2013; Della Morte et al. 2017.

QCD+QED on QCD_{iso} gauge ensembles

- QCD_{iso} ensembles (CLS)[5]:
  - Tree-level improved Lüscher-Weisz gauge action
  - $N_\mathrm{f} = 2 + 1$ $O(a)$-improved Wilson fermion action
  - Periodic/open temporal boundary conditions
  - $\mathrm{tr}(M) = \mathit{const.}$



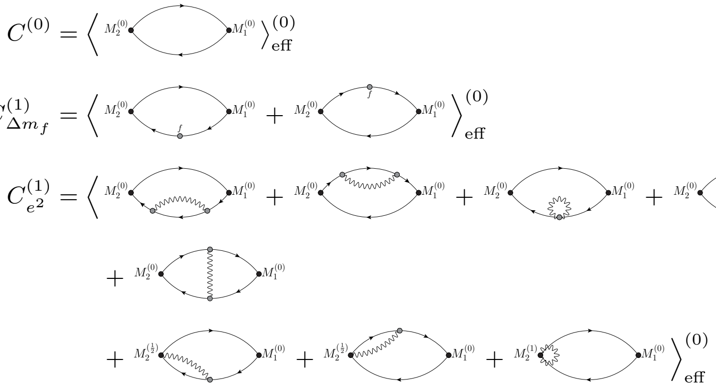
CLS ensembles used to study leading isospin breaking effects in a_μ^{HVP}

	$(\frac{L}{a})^3 \times \frac{T}{a}$	a [fm]	m_π [MeV]	m_K [MeV]	$m_\pi L$	L [fm]
H102	$32^3 \times 96$	0.08636(10)	354(5)	438(4)	5.0	2.8
N101	$48^3 \times 128$		282(4)	460(4)	5.9	4.1
N452	$48^3 \times 128$	0.07634(97)	350(5)	440(6)	6.5	3.7
N451	$48^3 \times 128$		287(4)	462(5)	5.3	3.7
D450	$64^3 \times 128$		217(3)	476(6)	5.4	4.9
N200	$48^3 \times 128$	0.06426(76)	282(3)	463(5)	4.4	3.1

[5] Bruno et al. 2015; Bruno et al. 2017.

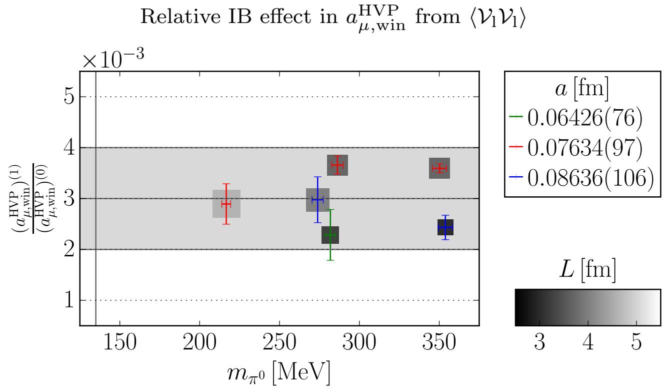
Isospin breaking expansion of mesonic two-point functions

- $\langle \mathcal{O}[U] \rangle_{\mathrm{eff}}^{(0)}$ denotes expectation value regarding $\mathrm{QCD}_{\mathrm{iso}}$ effective gauge action
- Diagrammatic expansion of $C_{\mathcal{M}_2\mathcal{M}_1} = \langle \mathcal{M}_2 \mathcal{M}_1 \rangle$ (quark-connected contributions):



$\Rightarrow$ Isospin breaking effects in sea-quarks not included and quark-disconnected contributions disregarded until now

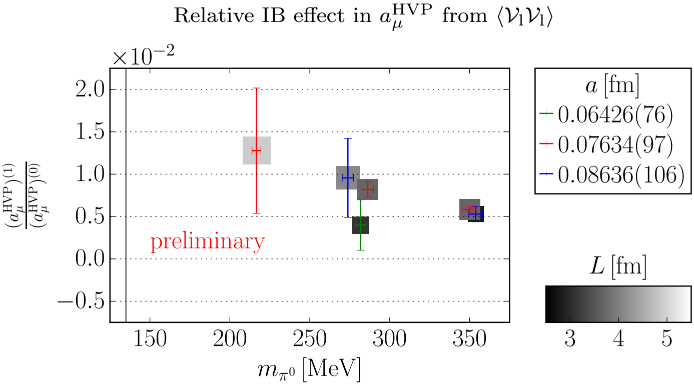
Intermediate window contribution $a_{\mu,\mathrm{win}}^{\mathrm{HVP}}$



- $\frac{(a_{\mu,\mathrm{win}}^{\mathrm{HVP}})^{(1)}}{(a_{\mu,\mathrm{win}}^{\mathrm{HVP}})^{(0)}} = 0.3(1)\%$ for quark-connected contributions
- Uncertainty on relative correction doubled in final results for $a_{\mu,\mathrm{win}}^{\mathrm{HVP}}$ [6]
- New additional data points compatible
- IB in scale setting and QED-FV effects to be included

[6] Cè et al. 2022.

HVP contribution to the muon anomalous magnetic moment $a_\mu^{\rm HVP}$



- $C(t) = \langle \mathcal{V}^\gamma(t) \mathcal{V}^\gamma(0) \rangle$ exhibits noise problem for large t
  $\Rightarrow$ Apply bounding method[7]:

$$0 < C(t_c) e^{-E_{\rm eff}(t_c)\,(t-t_c)} \le C(t) \le C(t_c) e^{-E_0\,(t-t_c)} \qquad t \ge t_c$$

  E_0 given by m_ρ or $m_{2\pi}$, disregarding $\pi + \gamma$ state in QCD+QED

- IB in scale setting and QED-FV effects to be included

[7] Borsanyi et al. 2017; Blum et al. 2018.

Scale setting for QCD+QED on CLS ensembles

- So far leading isospin breaking effects in scale setting are not included
- CLS scale setting for $\mathrm{QCD_{iso}}$ originally based on $\frac{2}{3}(f_K + \frac{1}{2} f_\pi)$[8]

$$\sqrt{t_0} = 0.1448(15)\,\mathrm{fm}$$

- Computation of decay constants f_K and f_π in QCD+QED is demanding[9]:
  - infrared divergences in intermediate stages of computation
  - cancel taking exchange of virtual photons between quarks and charged decay products as well as emission of real final state photons into account
- RQCD recently computed $\mathrm{QCD_{iso}}$ scale setting on CLS ensembles based on m_Ξ[10]

$$\sqrt{t_0} = 0.1443(7)\,\mathrm{fm}$$

$\Rightarrow$ Scale setting via baryon masses for QCD+QED based on CLS ensembles seems promising

[8] Bruno et al. 2017; Strassberger 2022.
[9] Carrasco et al. 2015; Giusti et al. 2018; Di Carlo et al. 2019; Desiderio 2020.
[10] Bali et al. 2023.

Baryon spectrum in QCD+QED

Baryon spectrum in QCD+QED

- Focus on baryons that are stable in QCD+QED:

$$p \quad n \quad \Sigma^+ \quad \Sigma^- \quad \Xi^0 \quad \Xi^- \quad \Lambda \quad \Omega$$

- Extract baryon masses from $C_{\mathcal{B}\overline{\mathcal{B}}}(t) = \langle \mathcal{B}(t)\overline{\mathcal{B}}(0)\rangle$
- Quasi-local interpolating operators with smeared quark fields for baryons based on Clebsch-Gordan construction in Dirac-Pauli representation[11]
- Smeared quark fields $q_\mu(\vec{x},t) = \sum_{\vec{y}} G(\vec{x},\vec{x}+\vec{y})\,\tilde{q}_\mu(\vec{x}+\vec{y},t)$
  QCD covariant QED non-covariant Wuppertal smearing kernel G on APE-smeared QCD gauge field, radius 0.5 fm for all quark flavours
- Parity and spin operators are diagonal in Dirac-Pauli representation:

$$P q(\vec{x},t) P^{-1} = \gamma_4 q(-\vec{x},t) = \rho q(-\vec{x},t)$$

$$S_z q(\vec{x},t) = \frac{i}{2}[\gamma_2,\gamma_1] q(\vec{x},t) = s q(\vec{x},t)$$

μ	ρ	s
1	+	+
2	+	−
3	−	+
4	−	−

$\Rightarrow$ Each component q_μ has definite parity ρ and spin s
$\Rightarrow$ Direct translation of Dirac index μ to ρ- and s-spin possible

[11] Basak et al. 2005.

Interpolation operators for light baryons[12]

	I_3	S	operator	symmetry
N	$\frac{1}{2}$	0	$\overline{p}_{\mu_1\mu_2\mu_3} = \varepsilon_{abc}\frac{1}{\sqrt{2}}(\overline{u}^a_{\mu_1}\overline{d}^b_{\mu_2} - \overline{d}^a_{\mu_1}\overline{u}^b_{\mu_2})\overline{u}^c_{\mu_3}$	MA
	$-\frac{1}{2}$		$\overline{n}_{\mu_1\mu_2\mu_3} = \varepsilon_{abc}\frac{1}{\sqrt{2}}(\overline{u}^a_{\mu_1}\overline{d}^b_{\mu_2} - \overline{d}^a_{\mu_1}\overline{u}^b_{\mu_2})\overline{d}^c_{\mu_3}$	
Σ	1	-1	$\overline{\Sigma}^+_{\mu_1\mu_2\mu_3} = \varepsilon_{abc}\overline{u}^a_{\mu_1}\overline{u}^b_{\mu_2}\overline{s}^c_{\mu_3}$	MS
	0		$\overline{\Sigma}^0_{\mu_1\mu_2\mu_3} = \varepsilon_{abc}\frac{1}{\sqrt{2}}(\overline{u}^a_{\mu_1}\overline{d}^b_{\mu_2} + \overline{d}^a_{\mu_1}\overline{u}^b_{\mu_2})\overline{s}^c_{\mu_3}$	
	-1		$\overline{\Sigma}^-_{\mu_1\mu_2\mu_3} = \varepsilon_{abc}\overline{d}^a_{\mu_1}\overline{d}^b_{\mu_2}\overline{s}^c_{\mu_3}$	
Ξ	$\frac{1}{2}$	-2	$\overline{\Xi}^0_{\mu_1\mu_2\mu_3} = \varepsilon_{abc}\overline{s}^a_{\mu_1}\overline{s}^b_{\mu_2}\overline{u}^c_{\mu_3}$	MS
	$-\frac{1}{2}$		$\overline{\Xi}^-_{\mu_1\mu_2\mu_3} = \varepsilon_{abc}\overline{s}^a_{\mu_1}\overline{s}^b_{\mu_2}\overline{d}^c_{\mu_3}$	
Λ	0	-1	$\overline{\Lambda}_{\mu_1\mu_2\mu_3} = \varepsilon_{abc}\frac{1}{\sqrt{2}}(\overline{u}^a_{\mu_1}\overline{d}^b_{\mu_2} - \overline{d}^a_{\mu_1}\overline{u}^b_{\mu_2})\overline{s}^c_{\mu_3}$	MA
Ω	0	-3	$\overline{\Omega}_{\mu_1\mu_2\mu_3} = \varepsilon_{abc}\overline{s}^a_{\mu_1}\overline{s}^b_{\mu_2}\overline{s}^c_{\mu_3}$	S

- Clebsch-Gordan construction of flavour content of interpolation operators

[12] Basak et al. 2005.

Interpolation operators for baryons: spin and parity

Interpolation operators for light baryons[13]

P	S_z	Ω	Ξ/Σ	Λ/N
+	$\frac{3}{2}$	$\overline{\Omega}_{111}$		
	$\frac{1}{2}$	$\sqrt{3}\overline{\Omega}_{112}$	$\sqrt{\frac{2}{3}}(\overline{\Xi}/\overline{\Sigma}_{112} - \overline{\Xi}/\overline{\Sigma}_{121})$	$\sqrt{2}\overline{\Lambda}/\overline{N}_{121}$
	$-\frac{1}{2}$	$\sqrt{3}\overline{\Omega}_{122}$	$\sqrt{\frac{2}{3}}(\overline{\Xi}/\overline{\Sigma}_{122} - \overline{\Xi}/\overline{\Sigma}_{221})$	$\sqrt{2}\overline{\Lambda}/\overline{N}_{122}$
	$-\frac{3}{2}$	$\overline{\Omega}_{222}$		
−	$\frac{3}{2}$	$\overline{\Omega}_{333}$		
	$\frac{1}{2}$	$\sqrt{3}\overline{\Omega}_{334}$	$\sqrt{\frac{2}{3}}(\overline{\Xi}/\overline{\Sigma}_{334} - \overline{\Xi}/\overline{\Sigma}_{343})$	$\sqrt{2}\overline{\Lambda}/\overline{N}_{343}$
	$-\frac{1}{2}$	$\sqrt{3}\overline{\Omega}_{344}$	$\sqrt{\frac{2}{3}}(\overline{\Xi}/\overline{\Sigma}_{334} - \overline{\Xi}/\overline{\Sigma}_{443})$	$\sqrt{2}\overline{\Lambda}/\overline{N}_{344}$
	$-\frac{3}{2}$	$\overline{\Omega}_{444}$		

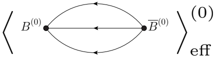
- Selected operators with the best overlap to the ground state
- Consider parity partners for averaging with time-reversed correlation function
- Simple structure of operators leads to computationally cheaper contractions

[13] Basak et al. 2005.

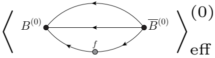
Isospin breaking expansion of baryonic two-point functions

- $\langle \mathcal{O}[U] \rangle_{\mathrm{eff}}^{(0)}$ denotes expectation value regarding $\mathrm{QCD}_{\mathrm{iso}}$ effective gauge action
- Diagrammatic expansion of $C_{\mathcal{B}\overline{\mathcal{B}}} = \langle \mathcal{B}\overline{\mathcal{B}} \rangle$ (quark-connected contributions):

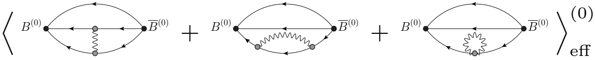
$(C_{\mathcal{B}\overline{\mathcal{B}}})^{(0)} =$



$(C_{\mathcal{B}\overline{\mathcal{B}}})^{(1)}_{\Delta m_f} =$



$(C_{\mathcal{B}\overline{\mathcal{B}}})^{(1)}_{e^2} =$



$\Rightarrow$ Isospin breaking effects in sea-quarks not included until now

- Evaluated with quark point sources and stochastic Z_2 photon sources
- Noise reduction: covariant approximation averaging[14] & truncated solver method[15] (all-mode averaging)

[14] Shintani et al. 2015.
[15] Bali et al. 2010.

Extraction of ground state mass via two state fit

- Baryonic two-point functions suffer from noise problem at large t
  $\Rightarrow$ Apply fit model that is also valid at earlier time slices

- Two state fit model:
$$C(t) = c\, e^{-mt} + c_* e^{-m_* t}$$
  m ground state mass, c_* and m_* absorb excited state effects

- Effective mass:
$$m_{\text{eff}}(t) = -\frac{\mathrm{d}}{\mathrm{d}t} \log(C(t))$$
  Expand in $d\, e^{-\Delta M t}$ with $\Delta M = m_* - m$ and $d = \frac{c_*}{c}$[16]:
$$m_{\text{eff}}(t) = m + \Delta M\, d\, e^{-\Delta M t} + h.o.$$

- First order perturbative expansion $X = X^{(0)} + \sum_l \Delta\varepsilon_l X_l^{(1)}$:
$${m_{\text{eff}}}^{(0)}(t) = m^{(0)} + d^{(0)} \Delta M^{(0)}\, e^{-\Delta M^{(0)} t} + h.o.$$
$${m_{\text{eff}}}_l^{(1)}(t) = m_l^{(1)} + \left(\frac{d_l^{(1)}}{d^{(0)}} + \frac{\Delta M_l^{(1)}}{\Delta M^{(0)}} - \Delta M_l^{(1)} t \right) d^{(0)} \Delta M^{(0)}\, e^{-\Delta M^{(0)} t} + h.o.$$

- In addition: Single state fit

[16] Del Debbio et al. 2007.

Fit model averaging

- Selection of optimal fit range $[t_{\min}, t_{\max}]$ for $m_{\text{eff}}(t)$ is to some degree ambiguous and subjective
  $\Rightarrow$ Apply fit model averaging

- Commonly applied Akaike information criterion[17] (AIC) favours models that tend to overfit the data
  $\Rightarrow$ introduce penalty term[18]

$$\underbrace{\mathrm{pr}(M|D)}_{\text{model weight}} \propto \exp\left(-\frac{1}{2}\left(\chi^2(a) + 2(k+n)\right)\right)$$

  k number of model parameters, n number of data points not considered in fit

- Optimal parameter $\langle a_i \rangle$ and parameter variance $\sigma^2_{a_i}$:

$$\langle a_i \rangle = \sum_M \langle a_i \rangle_M \, \mathrm{pr}(M|D)$$

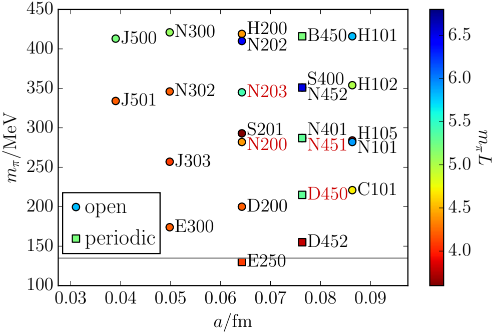
$$\sigma^2_{a_i} = \underbrace{\sum_M \sigma^2_{a_i,M} \, \mathrm{pr}(M|D)}_{\text{stochastic variance}} + \underbrace{\sum_M \langle a_i \rangle^2_M \, \mathrm{pr}(M|D) - \Big(\sum_M \langle a_i \rangle_M \, \mathrm{pr}(M|D) \Big)^2}_{\text{model variance}}$$

[17] Akaike 1974.

[18] Jay and Neil 2021; Neil and Sitison 2022.

QCD+QED on $\mathrm{QCD_{iso}}$ gauge ensembles

- $\mathrm{QCD_{iso}}$ ensembles (CLS)[19]:
  - Tree-level improved Lüscher-Weisz gauge action
  - $N_\mathrm{f} = 2 + 1$ $O(a)$-improved Wilson fermion action
  - Periodic/open temporal boundary conditions
  - $\mathrm{tr}(M) = \mathit{const.}$

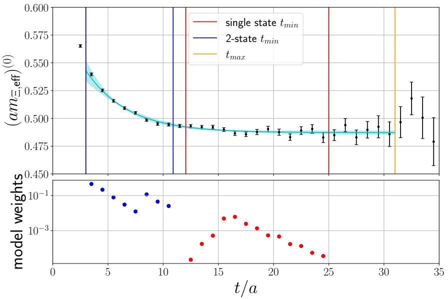


CLS ensembles used to study leading isospin breaking effects in baryon masses

	$(\frac{L}{a})^3 \times \frac{T}{a}$	a [fm]	m_π [MeV]	m_K [MeV]	$m_\pi L$	L [fm]
N451	$48^3 \times 128$	0.07634(97)	287(4)	462(5)	5.3	3.7
D450	$64^3 \times 128$		217(3)	476(6)	5.4	4.9
N203	$48^3 \times 128$	0.06426(76)	346(4)	442(5)	5.4	3.1
N200	$48^3 \times 128$		282(3)	463(5)	4.4	3.1

[19] Bruno et al. 2015; Bruno et al. 2017.

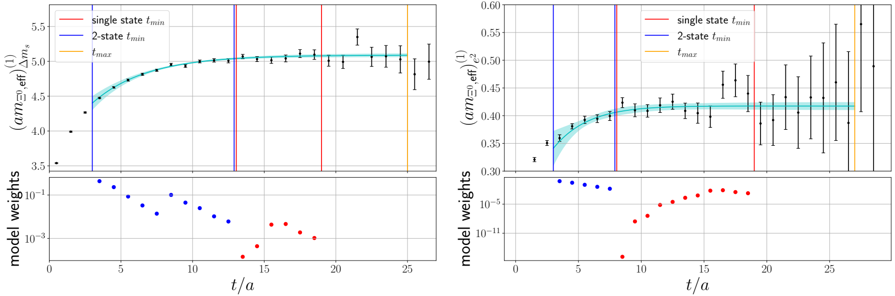
Fit to isosymmetric contribution



Fit to isosymmetric contribution $am_{\text{eff}}{}^{(0)}$ for the Ξ baryon on D450

- Fit isosymmetric contribution to obtain $(am, d, a\Delta M)^{(0)}$ for each model
- Perform model averaging
- Add Gaussian noise to $(am, d, a\Delta M)^{(0)}$ to account for model variance
- 2-state fits starting at early time slices dominate

Fit to first-order contribution



Fit to first-order contributions $am_{\mathrm{eff}}{}^{(1)}_{a\Delta m_s}$ and $m_{\mathrm{eff}}{}^{(1)}_{e^2}$ for the Ξ baryon on D450

- Insert model averaged values for $(am, d, a\Delta M)^{(0)}$ into first order fit model and determine $(am, d, a\Delta M)^{(1)}_l$
- Same procedure as in isosymmetric case
- 2-state fits starting at early time slices dominate

Relative precision of baryon masses in QCD+QED

Ensemble	a [fm]	m_π [MeV]	N	Λ	Σ	Ξ	Ω
N451	0.076	287	0.25%	0.72%	0.70%	0.32%	0.24%
D450		217	0.49%	0.30%	0.33%	0.29%	0.32%
N203	0.064	346	0.26%	0.20%	0.21%	0.17%	0.28%
N200		282	1.00%	0.32%	0.30%	0.18%	0.27%

Statistical precision of isosymmetric contribution $m^{(0)}$ to baryon masses

- Ξ most promising candidate on the finer and lighter ensembles D450 and N200

Relative precision of baryon masses in QCD+QED

Ensemble	Ξ^0			Ξ^-			Ω^-	
	e^2	Δm_u	Δm_s	e^2	Δm_d	Δm_s	e^2	Δm_s
N451	1.3%	1.7%	0.5%	0.9%	1.7%	0.5%	1.5%	1.6%
D450	1.8%	2.5%	0.5%	1.0%	2.5%	0.5%	1.4%	1.4%
N203	1.0%	1.0%	0.7%	0.8%	1.0%	0.7%	1.1%	1.0%
N200	1.4%	1.9%	1.0%	1.0%	1.9%	1.0%	1.3%	1.4%

Statistical precision of first order contribution $m_l^{(1)}$ to baryon masses

- Ξ^- and Ξ^0 possess noisier light quark derivatives, in Ω absent
- Ξ^- slightly more precise than Ξ^0
- Caveat: neglected isospin breaking effects from sea-quarks contributions

Some remarks on a renormalisation scheme convention

Some remarks on a renormalisation scheme convention

- 4-flavour scheme should be an extension of a 3-flavour scheme
  $\Rightarrow$ Simplifies comparisons of calculations with and without quenched charm quark
- D mesons should not be used to set the light quark mass splitting
  $\Rightarrow$ Avoid induction of lattice artefacts from charm quark to light quark masses
- Is m_{π^0} a good candidate for a renormalisation scheme?
  - QED correction to quark-disconnected contributions
  - $\pi^0 \to 2\gamma$ decay

Conclusions and Outlook

Isospin breaking effects a_μ^{HVP}:

- Study scheme dependence of isospin breaking effects
- Determination of $Z_{\mathcal{V}_{\mathrm{l,R}}\mathcal{V}_{\mathrm{l}}}$ by means of vector Ward identity including LO isospin breaking effects
- Investigate finite volume effects and include LO QED finite volume corrections
- Include quark-disconnected and sea-quark contributions

Baryon spectrum in QCD+QED:

- Currently most promising candidate for scale setting: Ξ
- Need to refine analysis
- Investigate finite volume effects and include LO QED finite volume corrections
- Include sea-quark contributions

References I

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