QCD+QED prescriptions and scale setting

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GENERALITAT VALENCIANA

$\mathrm{QCD}_{N_\mathrm{f}}$ on the lattice

$N_\mathrm{f} + 1$ dimensionless inputs

$$S = -\frac{\beta}{6}\sum_{\mathcal{W}}(U_{\mathcal{W}}) + \sum_{\mathrm{f}}\overline{\psi}D(\hat{m}_\mathrm{f})\psi\,, \qquad (\hat{m} = am_0)\ .$$

- $\beta = \frac{1}{g_0^2}$, $\hat{m}_\mathrm{f} = am_{0,\mathrm{f}}$ are dimensionless

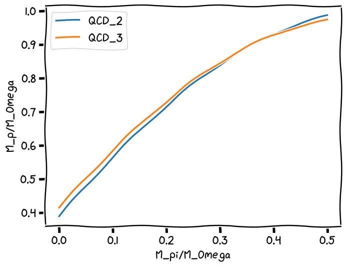
Only dimensionless predictions in the continuum are unambiguous

- N_f **dimensionless** input
- Different theories, different curves
- Since the whole curve is unambiguous, also derivatives are

$$\frac{\partial}{\partial(M_\pi/M_\Omega)}\frac{M_p}{M_\Omega}$$

in contrast to

$$\frac{\partial}{\partial M_\pi}M_p$$



$(\mathrm{QCD}+\mathrm{QED})_{N_\mathrm{f}}$ on the lattice

$N_\mathrm{f}+2$ dimensionless input

- $g_0^2, \hat{m}_\mathrm{f}, \alpha_\mathrm{EM}$
- If working to leading order in α_EM (i.e. ignoring $\mathcal{O}(\alpha_\mathrm{EM}^2)$)

$$\alpha_\mathrm{EM} = \alpha_0 \approx 1/137$$

In principle only dimensionless predictions in the continuum are unambiguous

- N_f **dimensionless** input
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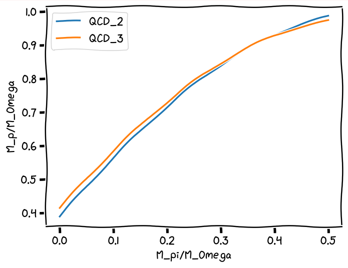
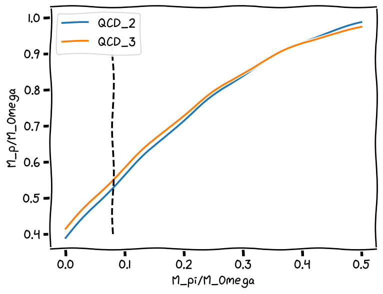


Differences[1] between QCD_{N_f} and $(\mathrm{QCD}+\mathrm{QED})_{N_f}$

QCD_{N_f}



- What to choose for $\frac{M_\pi}{M_\Omega} = ??$
- What input to use (i.e. $M_\Omega, f_\pi, \dots$) to give final results?

$(\mathrm{QCD}+\mathrm{QED})_{N_f}$

- Real world, up to irrelevant precision
- Clear option

$$\frac{M_\pi^+}{M_\Omega} = \frac{[M_\pi^+]^{\mathrm{exp}}}{[M_\Omega]^{\mathrm{exp}}}$$

- Input choice is irrelevant:

$$[M_\Omega]^{\mathrm{exp}} \times \frac{M_p}{M_\Omega}\Big|_{\frac{M_\pi^+}{M_\Omega}=\mathrm{exp}} = [M_\Xi]^{\mathrm{exp}} \times \frac{M_p}{M_\Xi}\Big|_{\frac{M_\pi^+}{M_\Xi}=\mathrm{exp}}$$

[1] We assumme $N_f \geq 3$, and up to some "irrelevant" precision

What are we looking for?

A convention for QCD

Meson masses seem ideal candidates

1. $M_{\pi^0}/\mathcal{S}, M_{K^+}/\mathcal{S}, M_{K^0}/\mathcal{S}$

Ideal candidate for $\mathcal{S}$ (in **subjective** order of importance)

1. Easy to determine on the lattice (easy for QCD!)
2. Direct relation with experiment.
3. Small/under control radiative corrections (**not enough** information at a single a/electroquenched).

Unfortunately no such quantity!

- M_Ω: has property 2). Not 1) (signal to noise), not 3)
- f_π: has property 1), not 3), and 2) not as clean as one would like.
- $\sqrt{t_0}, w_0$: has property 1), but not 2), not 3)

THE CASE FOR "f_π"

f_π does not exist, has to be defined

- We **define**

$$f_\pi = \frac{1}{1+\delta_\pi}\sqrt{\frac{\Gamma^{\exp}(\pi\to\mu\bar{\nu}(\gamma))}{\frac{G_F^2}{8\pi}|V_{ud}|^2 M_\pi^{\exp}(m_\mu^{\exp})^2\left[1-\frac{(m_\mu^{\exp})^2}{(M_\pi^{\exp})^2}\right]}}$$

- Definition is **ambiguous**: What M_π, what value for δ_π?

Advantages

- On the lattice easy to determine (i.e. af_π for free when determining aM_π). No correction $\mathcal{O}(m_u - m_d)$
- Relation with experiment requires to fix $M_\pi, \delta_\pi, E_\gamma$ (and $|V_{ud}|$ input).

$$E_\gamma = \frac{M_{\pi^-}}{2}\left[1-\frac{(m_\mu^{\exp})^2}{(M_{\pi^-}^{\exp})^2}\right],$$
$$\delta_\pi = 0.0088(11) \qquad (\chi-\text{PT. Agrees with lattice EQ})$$

- Physical point at

$$M_\pi/f_\pi = 1.0338; \qquad M_K/f_\pi = 3.78122$$

THE CASE FOR t_0, w_0

They are not in the PDG

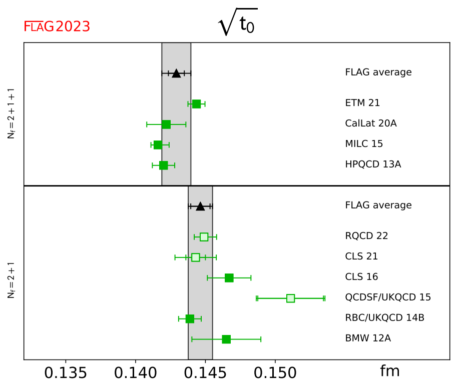
- FLAG averages

$$\sqrt{t_0} = 0.1429(10), 0.14464(87)\ \text{fm}$$
$$w_0 = 0.17236(70), 0.1725(10), 0.17355(92)\ \text{fm}\,.$$

- Sub-percent precision, but differences larger than errors
- Location of physical point

$$\sqrt{t_0}M_\pi = 0.09832; \qquad \sqrt{t_0}M_K = 0.3596$$

- Uncomfortably large systematic, beyond EM/isospin breaking corrections!



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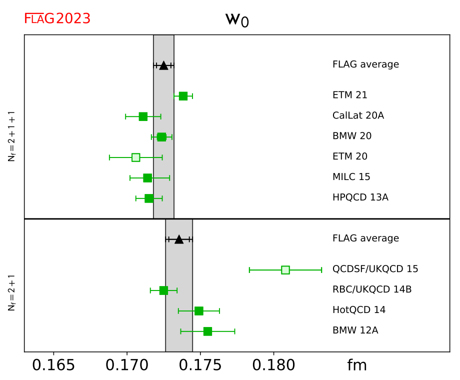

The case for M_Ω

Pros/cons

- Very clean from the QCD+QED perspective
- Small chiral dependence
- Precisión comes from **modeling** correlator at short distances

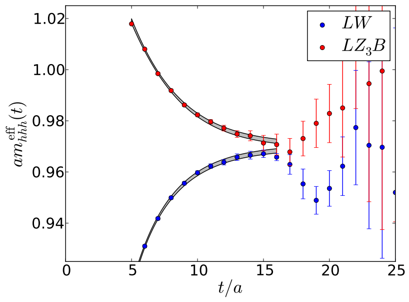

Figure: [RBC/UKQCD]

- This is difficult unless you are into that game
- Lot's of god physics is done **without** facing these difficulties.

Conclusions

We need a convention to compare QCD computations

- The problem
- Do not sacrifice the 99% for the 1%
  - The determination of M_Ω **requires** to model excited state contributions
  - Better to model the 1% than the 99%
- If we want a standard we need a quantity that anyone can determine easily
- Connection with physical world trough derivatives (i.e. Antonin's/Nazario's/Laurent's talks)
- $\Gamma(\pi \to \mu\bar{\nu}(\gamma))$ (i.e. "f_π")
  - no strong IB corrections
  - reasonable radiative corrections (χ-PT + Lattice EQ)
- GF scales: They are ideal, but uncomfortably large discrepancies (**homework!**)

Nazario's and Laurent's point is very relevant

- Past works can be evaluated
- Is QCD-easy

Connect with real world (with IB corrections) when/if precision requires it