FNAL/MILC QCD+QED Prescription

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Converging on QCD+QED Prescriptions
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Outline

- Introduction
- Prescription for QCD without QED
- Prescription of Isospin Symmetric QCD
- Omega Baryon
- Other Scales
- Gradient Flow
- Adding QED to QCD
- Speculation on Future Calculations

Introduction

- Emphasis on prescriptions to define isospin-breaking corrections to hadronic observables
- FNAL/MILC defined our prescription (scheme) in our 2018 work on meson decay constants
- A. Bazavov *et al.*, PhysRevD.98.074512, *B- and D-meson leptonic decay constants from four-flavor lattice QCD,* [arXiv:1712.09262]
- Also relevant
  - Up-, down-, strange-, charm-, and bottom-quark masses from four-flavor lattice QCD, PhysRevD.98.054517 [arXiv:1802.04248]
  - Lattice computation of the electromagnetic contributions to kaon and pion masses, PhysRevD.99.034503 [arXiv:1807.05556]

QCD Prescription

- How shall we tune the quark masses to best reproduce the world as it would be if the quarks were not charged?
- Not completely well defined as isospin and electromagnetic corrections are both part of real world.
- We use f_π to set the scale and masses of π, K^+, K^0, D_s (B_s) to set quark masses.
  - $N_f = 2 + 1 + 1$ for 5 or 6 lattice spacings

Inputs

- From PDG values of experimental results, we infer values in the QCD only world [PRD98, 074512 (2018)]

TABLE VI. Experimental inputs to our tuning procedure (left side) [66], and the meson masses after adjusting for electromagnetic effects (right side).

Experimental inputs	QCD masses
$f_{\pi^+} = 130.50(1)_{\rm exp}(3)V_{ud}(13)_{\rm EM}$ MeV	
$M_{\pi^0} = 134.9770$ MeV	$(M_\pi)^{\rm QCD} = 134.977$ MeV
$M_{\pi^+} = 139.5706$ MeV	
$M_{K^0} = 497.611(13)$ MeV	$(M_{K^0})^{\rm QCD} = 497.567$ MeV
$M_{K^+} = 493.677(16)$ MeV	$(M_{K^+})^{\rm QCD} = 491.405$ MeV
$M_{K^0} - M_{K^+} = 3.934(20)$ MeV	
$M_{D_s} = 1968.28(10)$ MeV	$(M_{D_s})^{\rm QCD} = 1967.02$ MeV
$M_{B_s} = 5366.89(19)$ MeV	$(M_{B_s})^{\rm QCD} = 5367.11$ MeV

QCD π and K Masses

- Dashen's theorem correction:
- $$\epsilon' \equiv \frac{(M^2_{K^\pm} - M^2_{K^0})^\gamma - (M^2_{\pi^\pm} - M^2_{\pi^0})^{\text{expt}}}{(M^2_{\pi^\pm} - M^2_{\pi^0})^{\text{expt}}}$$
- $\epsilon' = 0.74(1)_{\text{stat}}(^{+\ 8}_{-11})_{\text{syst}}$, and
- $(M^2_{K^0})^\gamma = 44(3)_{\text{stat}}(25)_{\text{syst}}\ \text{MeV}^2$ from PRD99, 034503(2019)
- $$(M^2_{K^+})^{\text{QCD}} = M^2_{K^+} - (1+\epsilon')\left(M^2_{\pi^+} - M^2_{\pi^0}\right) - (M^2_{K^0})^\gamma$$
- $$(M^2_{K^0})^{\text{QCD}} = M^2_{K^0} - (M^2_{K^0})^\gamma$$
- $(M_{\pi^0})^{\text{QCD}} = (M_{\pi^0})^{\text{expt}}$ (scheme dependent)

Isospin Symmetric QED+QCD

- We use the scheme proposed by BMW Collaboration in PRL 111, 252001 (2013)
- Consider neutral mesons containing charged quark and antiquark, but with no annihilation diagram, *i.e.*, quark line connected diagrams only.
  - It is as if the antiquark is another flavor, but with same mass and charge. Denote this meson as xx'
- Isospin symmetry defined by $M_{uu'} = M_{dd'}$.
- To set the overall scale of the quark masses, match to isosymmetric pure QCD

$$M^2_{uu'} = M^2_{dd'} = (M^2_\pi)^{QCD} \equiv M^2_{\pi_0,expt}$$

  - works well as radiative corrections are small for these quantities

Heavy-light Masses

- We use a phenomenological formula
- $M_{H_x}^{\text{expt}} = M_{H_x}^{\text{QCD}} + A e_x e_h + B e_x^2$
- where $A = 4.44\,\text{MeV}, \text{B} = 2.4\,\text{MeV}$ and
- e_x and e_h are the charges of the valence light quark and heavy antiquark, respectively.

  - Rosner and Wise, PRD 47, 343 (1993), Goity and Jayalath, PL B 650, 22 (2007), Davies *et al*., PRD 82, 114504 (2010).

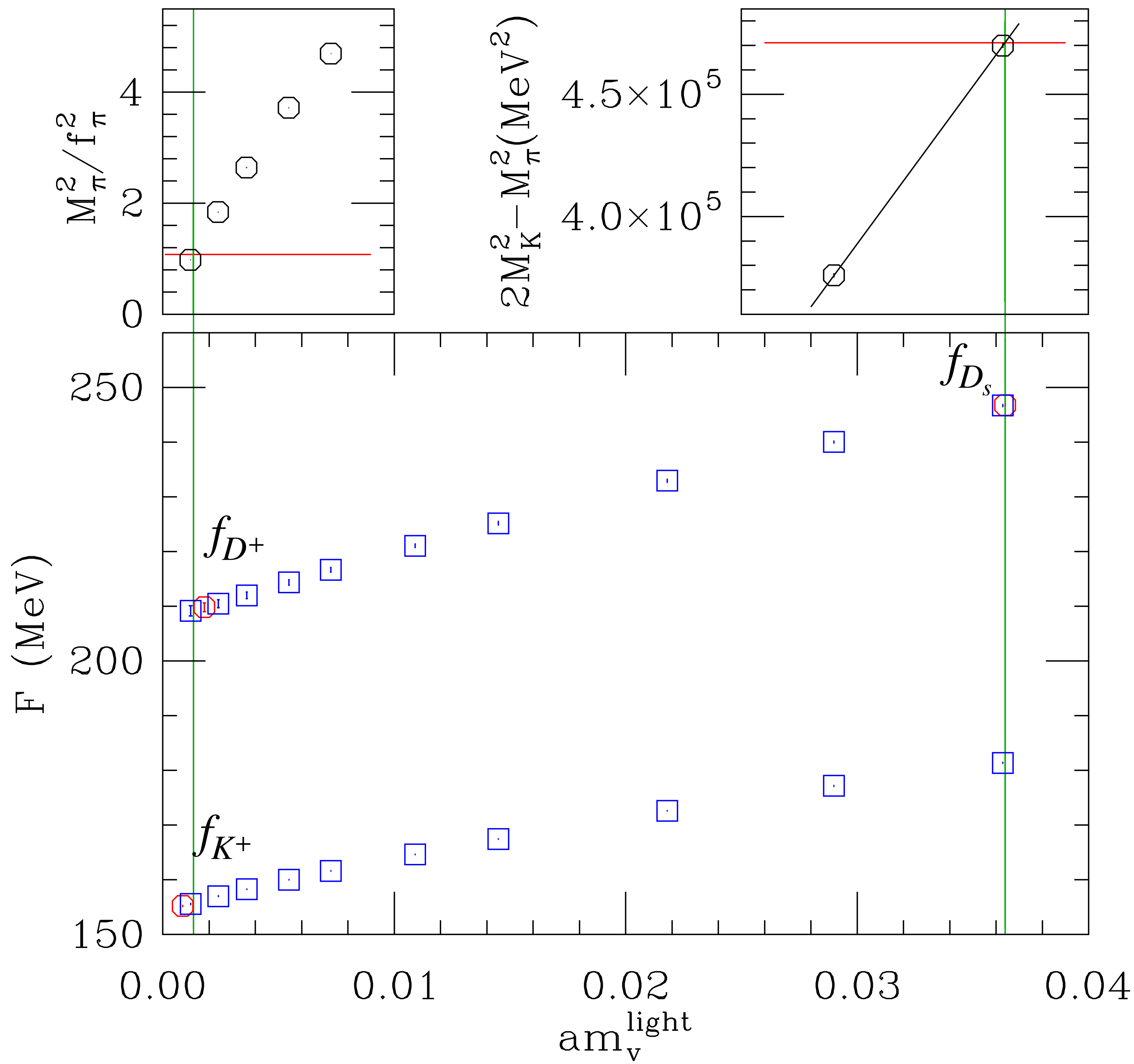
Quark Mass Tuning

✦ For each physical-mass ensemble:

- Tune m_l to achieve desired value of $M_\pi^2/f_{\pi^+}^2$
  - This provides both a and m_l
- Tune m_s to achieve desired value of $2M_K^2 - M_\pi^2$
- Determine $m_d - m_u$ from
  - $$m_d - m_u = \frac{M^2_{K^0_{\text{adj}}} - M^2_{K^+_{\text{adj}}}}{\frac{\partial M_K^2}{\partial m_l}}$$
  - This requires knowledge of the derivative, and adjustment of the masses for finite volume and electromagnetic effects. [PRD90, 074509 (2014)]
- Determine m_c from mass of D_s. (Similarly for m_b.)

Graphical Summary

- Upper left shows tuning for m_l
- Upper right shows tuning for m_s
- Lower plot shows decay constant analysis: red octagon at lower left is f_{K^+}.
- Octagons on higher data set are f_{D^+} and f_{D_s}



Refinements

- ✦ Actually, we have to account for finite volume
- ✦ Targeted values are adjusted to a 5.5 fm box using staggered χpt and continuum χpt.
  - difference is part of systematic error
- ✦ Also need to adjust sea quark masses
  - relies on ensembles with unphysical sea-quark masses to estimate derivatives
  - iterative process to tune valence and sea masses consistently
  - more details in Sec. IV of PRD **90**, 074509 (2014)

Table of Tuned Masses

✦ PRD90, 074509 (2014)

TABLE V. Tuned lattice spacings (using f_{π^+} to set the scale) and quark masses for the physical quark-mass ensembles. The quark-mass entries show the light-, strange- and charm-quark masses in units of the lattice spacing. The column labeled am' gives the run values of the sea-quark masses.

a_{approx} (fm)	a_{tuned} (fm)	am'	am_{tuned}
0.15	0.15089(17)	0.00235/0.0647/0.831	0.002426(8)/0.06730(16)/0.8447(15)
0.12	0.12121(10)	0.00184/0.0507/0.628	0.001907(5)/0.05252(10)/0.6382(8)
0.09	0.08779(8)	0.0012/0.0363/0.432	0.001326(4)/0.03636(9)/0.4313(6)
0.06	0.05676(6)	0.0008/0.0220/0.260	0.000799(3)/0.02186(6)/0.2579(4)

✦ We have retuned ensembles at 0.15 and 0.12 fm.

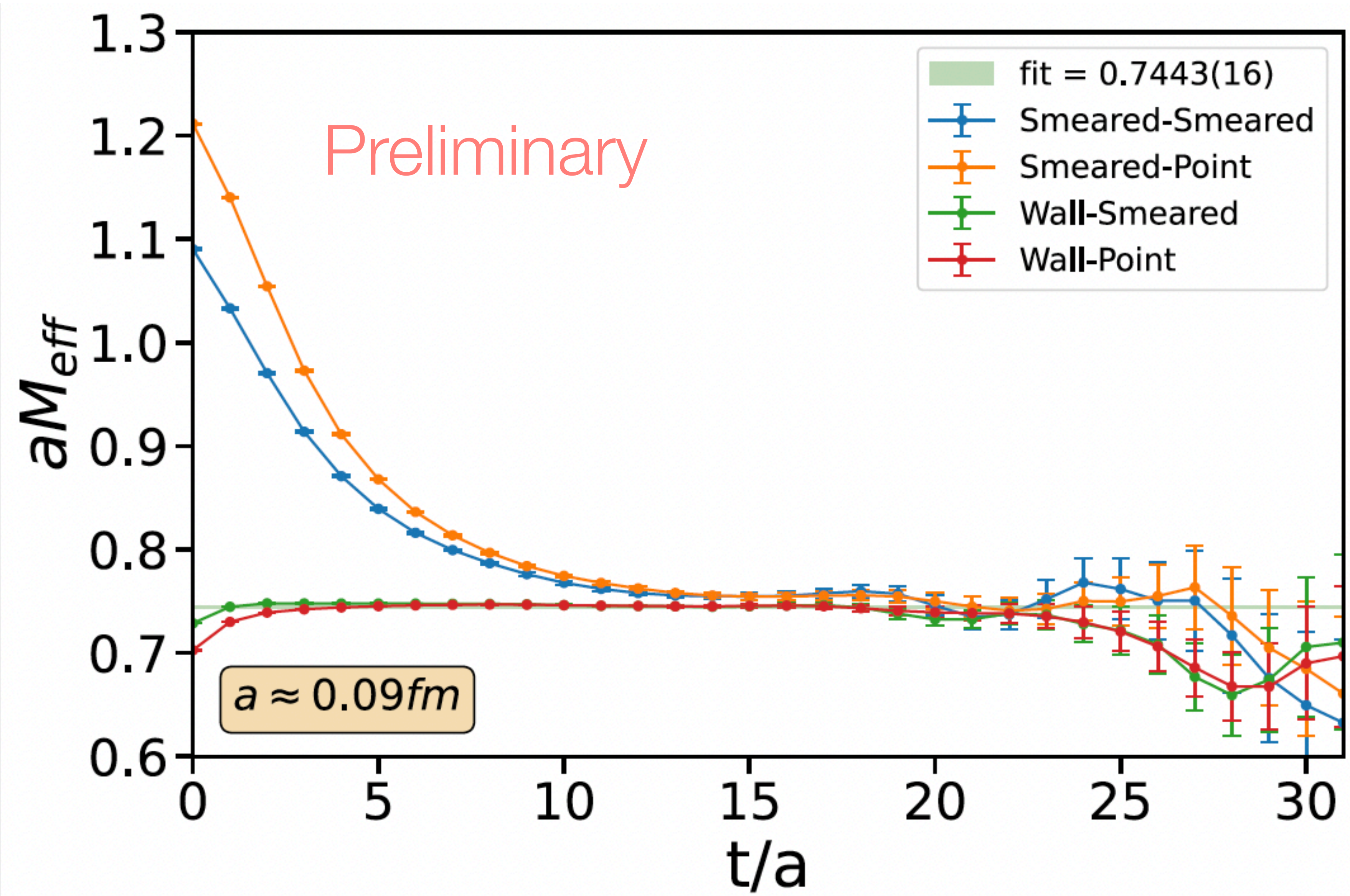
✦ CalLat has generated a retuned 0.09 fm ensemble.

- We are generating a larger volume ensemble with the same masses
- In our original running, m_l was over 10% different from the tuned value for this lattice spacing.

Omega Baryon

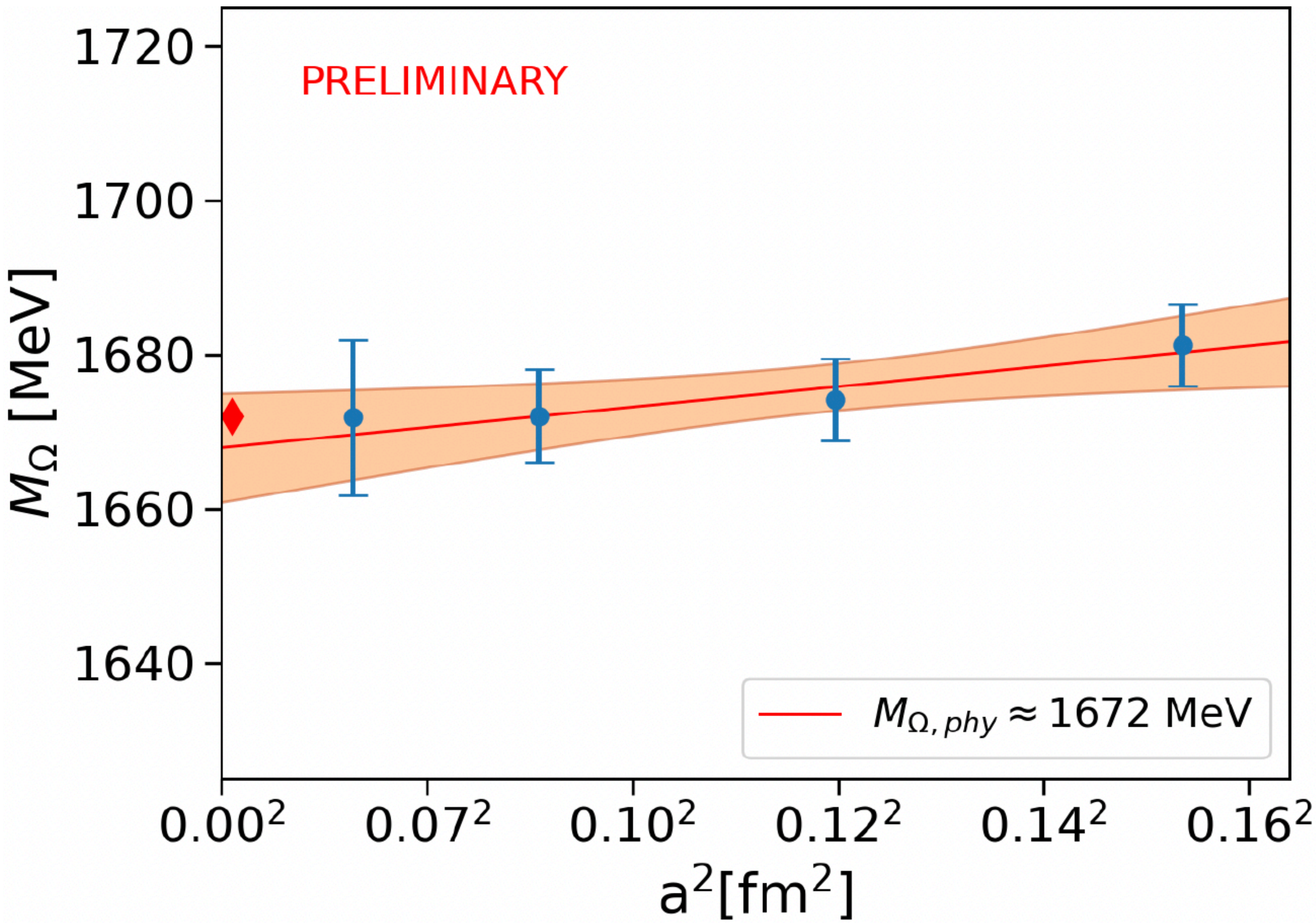
- Omega baryon is a popular quantity for scale determination
  - no light valence quark
  - no reliance on V_{ud} as with f_π
  - electromagnetic effects expected to be small
- We have been measuring Omega baryon mass on physical mass ensembles.
- We have applied for time to add quenched QED to these calculations.
- Yin Lin has been leading this analysis.
  - Plan to compare results using this alternative scale.

Ω Baryon 0.09 fm



- Effective mass and fit for physical-mass ensemble with 0.09 fm
- Using two sources and two sinks, so four sets of correlators.
- 0.2% error
- This is preliminary

Ω Continuum Extrapolation



- Simple extrapolation including a^2 and a^4 terms
- Pion decay-constant used to set scale
- Red point to left is physical Ω mass
- Error on our value for continuum is 7 MeV or 0.4%
- Plan to increase statistics

Other Scales

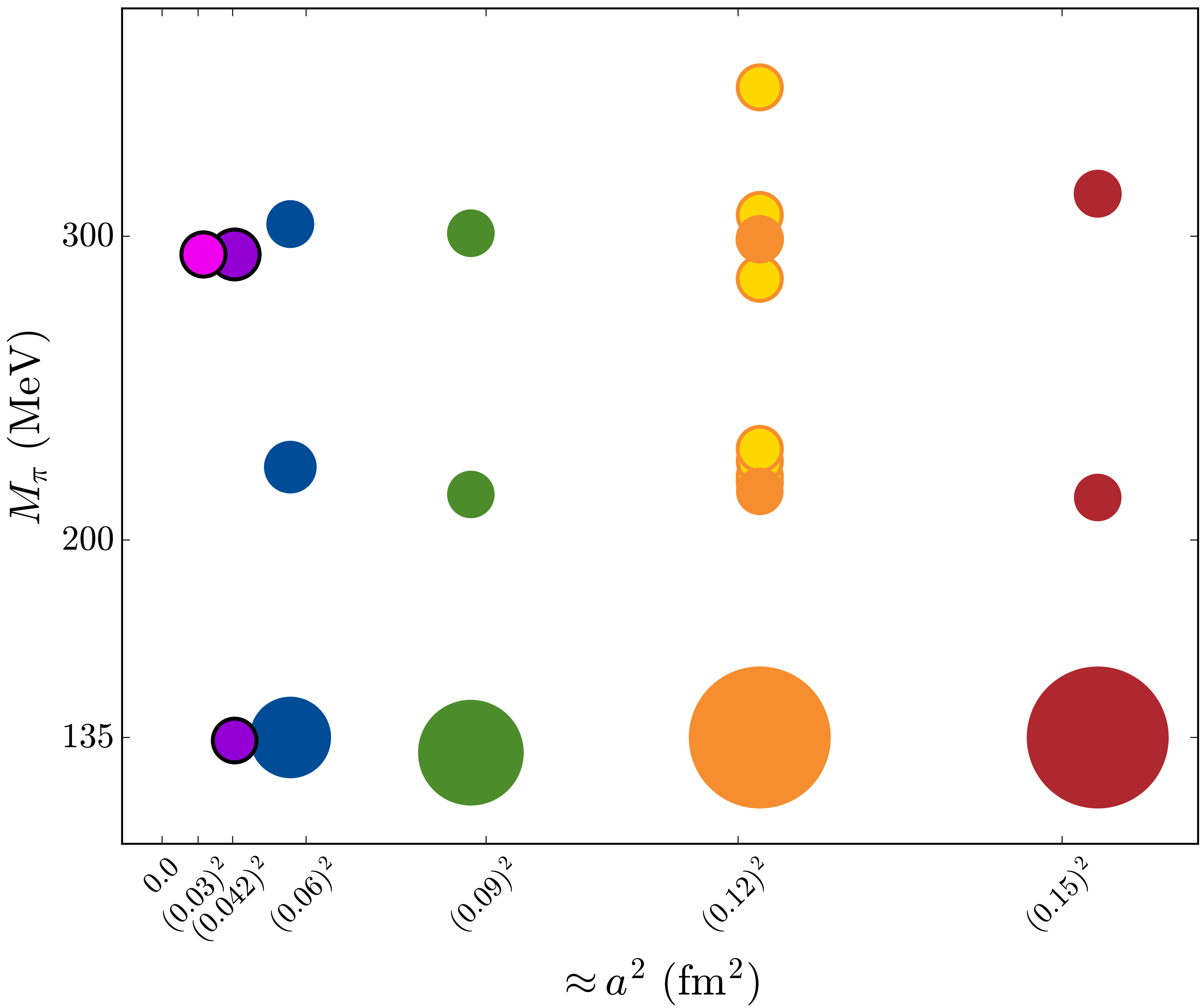
- Heavy-quark potential: r_0 or r_1
  - we measure many Wilson loops as part of generation, but no one actively analyzing
- Alternative to f_π: F_{p4s} is a non-physical quantity with valence quark mass of $0.4m_s$
  - can be calculated with high precision
  - natural part of decay constant analysis
  - not a physical observable, but related to f_π
  - f_π itself has disadvantage of relying on V_{ud} from nuclear β-decay which has been recalculated recently and changed

Other Scales II

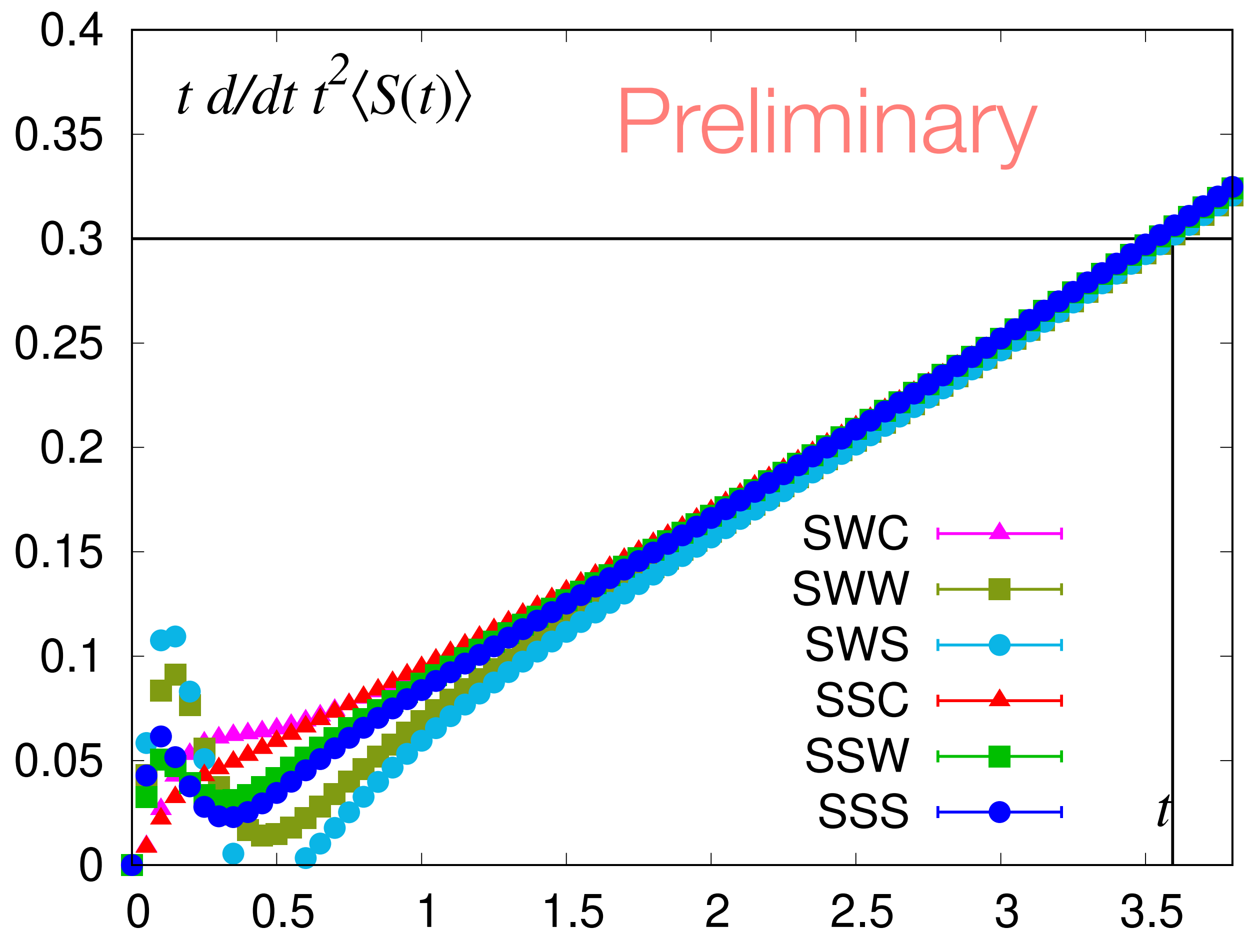
- Gradient flow: w_0, again not a physical observable
  - can be calculated with high precision on each ensemble
  - CalLat has analyzed 10 of our ensembles with $a \geq 0.09$ fm and 12 of their own [PRD 103, 054511 (2021)]
  - we're completing remeasurement/analysis of all (>30) of our ensembles
- We have completely redone our analysis
  - use both Wilson (W) and Symanzik (S) actions for flow
  - observables include clover (C), Wilson (W), and tree-level improved Symanzik (S) definitions of action density
  - every configuration is included

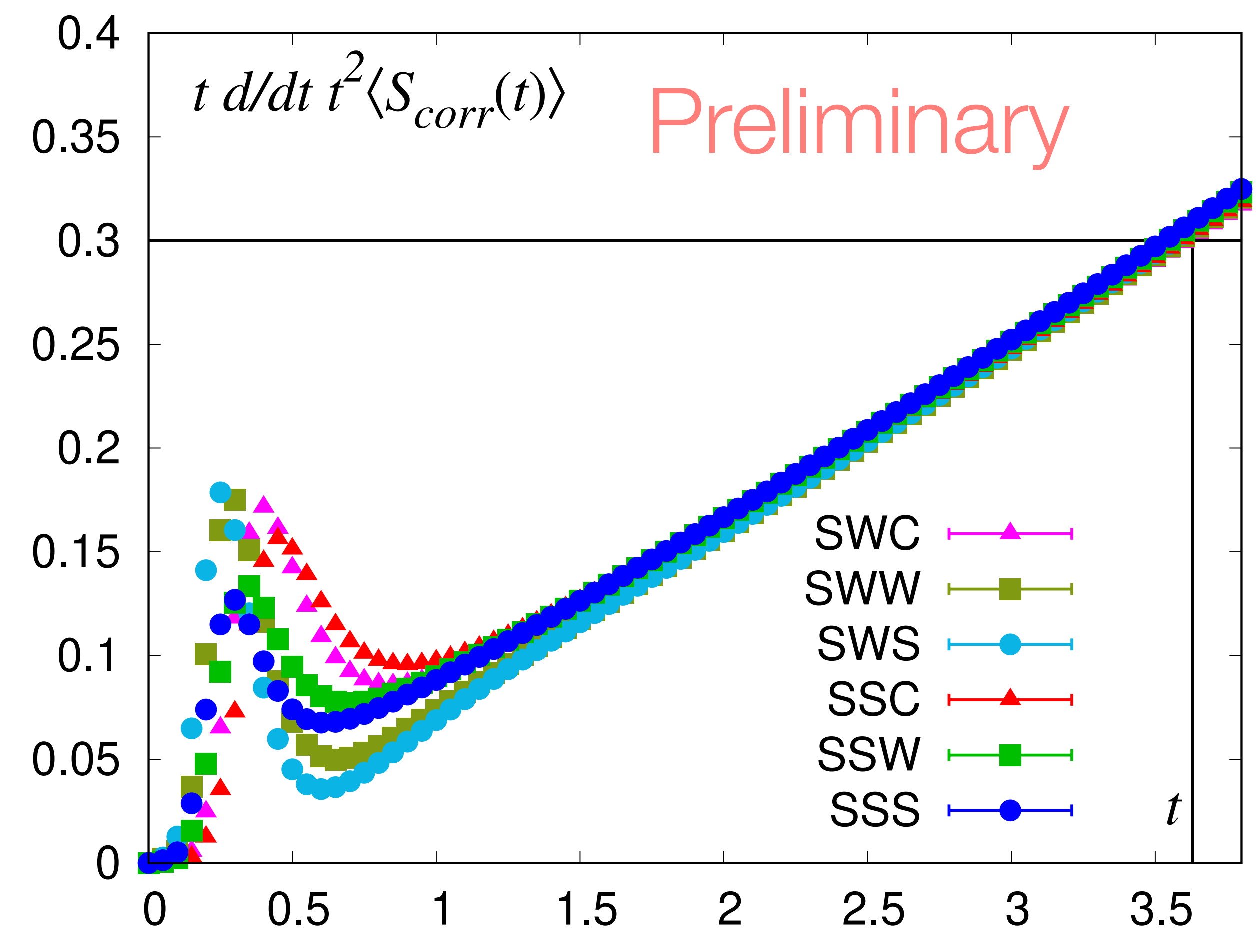
Configurations

- Most ensembles have 1000 configurations
- For physical-mass 0.15, 0.12 fm have about 10,000 configs.
- For 0.09, >5,000
- For 0.06, >3,000
- Also configurations with reduced strange quark mass not shown



Gradient Flow on 0.09 fm

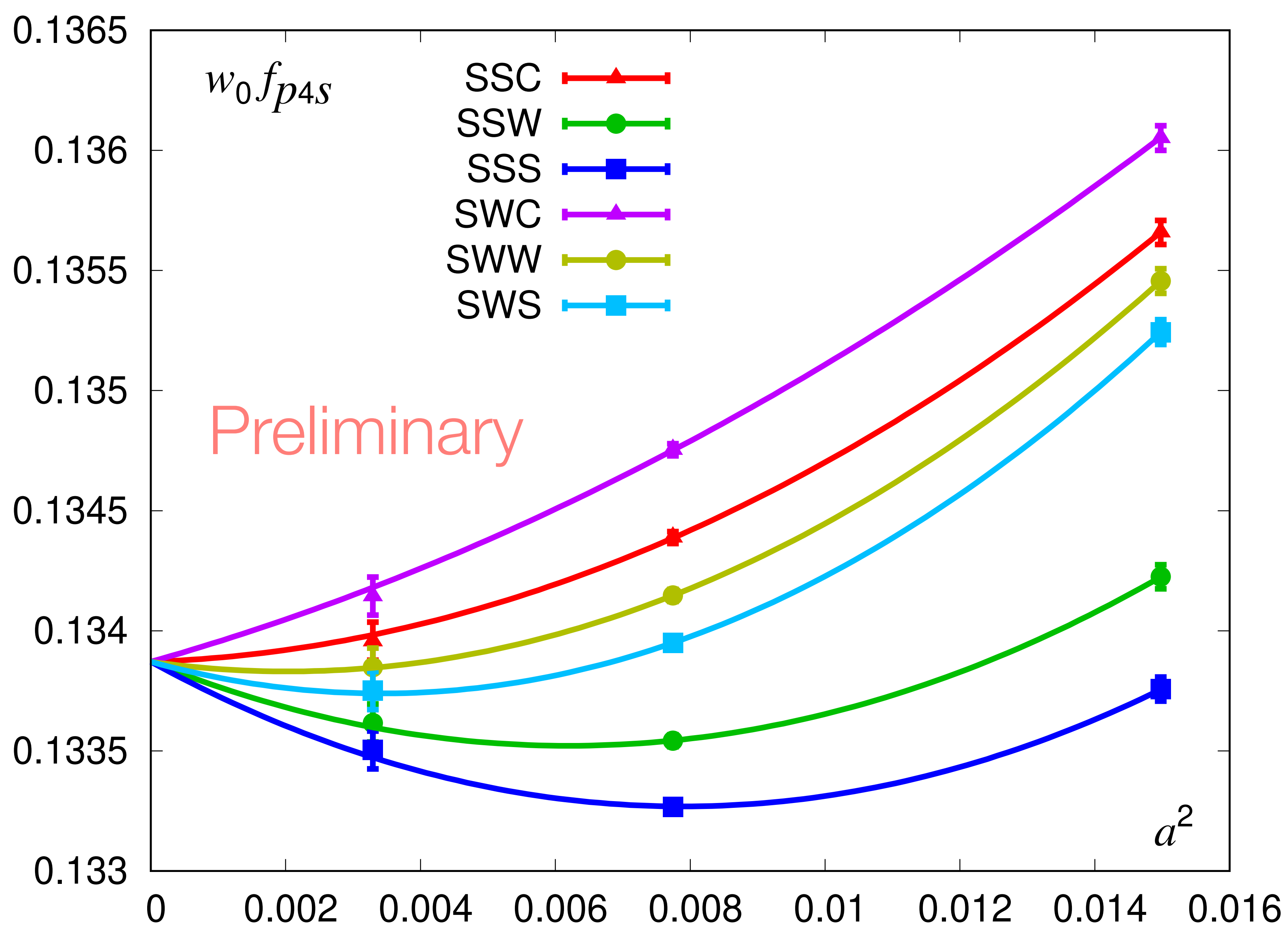




- $t\frac{d}{dt}(t^2\langle S(t)\rangle)\,|_{w_0} = 0.3\ ; \qquad t^2 S_{corr}(t) = \dfrac{t^2 S(t)}{1 + \sum_{m=1}^{4} C_m(a^{2m}/t^m)}$
- See Lüscher, JHEP **08**:071,2010 and Fodor et al., JHEP **09**:018,2014

Continuum Limit

- Six combinations of flow action and observable
- Lattice spacings 0.12, 0.09, 0.06 fm
- Simple fit including a^2 and a^4 terms with common intercept, *i.e.*, 13 parameters for 18 points.



Adding QED to QCD

- ✦ MILC has looked at meson masses using quenched QED PRD99, 034503 (2019)
- ✦ Craig McNeile will report next on current work from FNAL/HPQCD/MILC with quenched QED
- ✦ We have code for both compact and non-compact U(1)
  - We have not decided to generate $SU(3) \times U(1)$ ensembles
  - Will be using a perturbative approach for our HVP calculations, similar to approach of BMW Collaboration

Speculation on Future Calculations

- Have begun equilibration of $192^3 \times 384$ physical mass ensemble with $a \approx 0.03$ fm.
  - original motivation was B meson physics
  - not completely clear that this is needed or practical given computing requirement for the physics program
- Exploring finite volume effects at 0.09 fm
- Precision of calculations in the field has reached the stage where we need to include QED
  - see previous slide