QED corrections to the HVP contribution of the muon g-2

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May 29, 2023

Motivation

In Phys.Rev.D 100 (2019) 3, 034506 (1902.04223)
FNAL/HPQCD/MILC published

$$10^{10} a_\mu^{HVP,LO,light} = 630.8(8.8)(13)$$

- Second error of first line (13) (corresponds to 2% error.) is from the estimate of strong isospin breaking, electromagnetism, and quark-disconnected diagrams.

The goal is to compute the missing quantities to reduce the 2% error.

- Quenched QED and QCD analysis with an emphasis on schemes
- Start with charm (a_μ^c), then look at strange (a_μ^s) and light (a_μ^l)

Overview of analysis method

Measure connected vector correlator.

$$G_{ff'}(t) = Q_f Q_{f'} \sum_{\vec{x}} Z_V^2 \langle j_f^i(\vec{x}, t) j_{f'}^i(0) \rangle \, .$$

where f is a flavor index and Q_f is the charge.
The contribution to a_μ from $G_{ff'}(t)$ is then given by an integral over time:

$$a_{\mu,ff'}^{\mathrm{HVP}} = \left(\frac{\alpha}{\pi}\right)^2 \int_0^\infty dt \, G_{ff'}(t) K_G(t) \, , \qquad (1)$$

with the kernel $K_G(t)$.

- $G_{ff'}(t)$ gets noisy for large t. Replace $G_{ff'}(t)$ with fit model $G^{fit}(t)$ for $t > t^\star$
- We use Peter's g2tools `https://github.com/gplepage/g2tools` to compute the integral.

Including quenched QED with QCD

Charmonium properties from lattice QCDQCD+QED : Hyperfine splitting, J/ψ leptonic width, charm quark mass, and a_μ^c
HPQCD Collaboration, D. Hatton et al., Phys.Rev.D 102 (2020) 5, 054511, 2005.01845

- Calculation of QED contribution to J/ψ - η_c mass splitting.
- Computation of QED contribution to mass of the charm quark m_c.
- J/ψ leptonic decay constant
- Computation of QED contribution to a_μ^c.

Avantages Good statisical precision for charmonium.

Disavantages Contribution is small to $a_\mu^{\mathrm{HVP,LO}}$

Details of the calculation

- HISQ staggered action 2+1+1 flavours of sea quark mass. Configurations generated by the MILC collaboration.
- 6 lattice spacings: 0.15, 0.12, 0.09, 0.06, 0.045, and 0.03 fm
- Pseudo-scalar and vector correlators. Only charge neutral mesons.
- No disconnected diagrams

Tuning (scheme choice)

- No contribution of QED to scale setting.
- Fix the charm mass from the J/ψ mass either in QCD or QCD+QED.

Comment on η_c mass

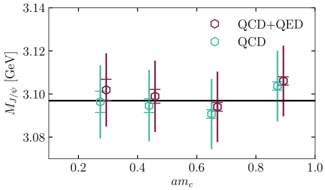
- HPQCD collaboration, $M_{J/\psi} - M_{\eta_c}$ (connected) $= 120.3(1.1)$ MeV
- PDG, $M_{J/\psi} - M_{\eta_c} = 113.0(5)$ MeV

$$\Delta M_{\eta_c}^{\mathrm{annihiln}} = +7.3(1.2) MeV$$

- Better to use J/ψ to fix mass of the charm quark mass in the simulation.
- Computation of disconnected diagrams complicated by η_c and η' mixing (Levkova and DeTar, 1012.1837).

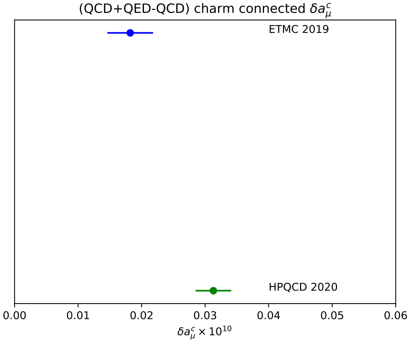
m_c and mass of J/ψ meson

From HPQCD, Hatton et al.,



- Lattice spacing from w_0 from pure QCD simulations.
- Tune m_c charm quark in QCD simulations to fix the J/ψ meson.
- Use m_c from QCD simulations in QCD+QED simulations.

Summary of lattice results for a_μ^c



ETMC 1901.10462, D. Giusti et al., GRS match the quark masses.

HPQCD 2005.01845, tune the charm quark mass in QCD+QED to J/ψ meson.

Calculating the QED correction to the hadronic vacuum polarisation on the lattice, Gaurav Ray et al., 2212.12031, Lattice 2022 presentation.

Including quenched QED with QCD

- Non-compact $A_\mu(k)$ generated in Feynman gauge for each QCD gluon field configuration.
- Electroquenched.
- Use the QED_L formulation to deal with zero modes.

$$U_\mu^{QCD+QED} = \exp(ieQA_\mu) U_\mu^{QCD}$$

- We are currently computing **connected** QED+QCD correlators.
- The value of Z_V has been computed including quenched QED using the RI-SMOM scheme. (Hatton et al., HPQCD, PhysRevD.100.114513)

Follow HPQCD (2005.01845) study QED contributions to charmonium mesons.

Details of the quenched QED+QCD calculation

- Random sources wall sources.
- The analysis is blinded
- Truncated solver method with 16 sloppy and 1 fine inversion on each lattice.
- Use charge averaging over $+Q, -Q$

We measure neutral vector and pseudo-scalar correlators.

Ensemble	$L^3 \times T$	a[fm]	no. meas	masses
very coarse	$32^3 \times 48$	0.15	1844	m_u m_d $3/5/7m_l$ m_s
coarse	$48^3 \times 64$	0.12	967	$3/5/7m_l$ m_s
fine	$64^3 \times 96$	0.09	596	$3/5/7m_l$ m_s

Ensembles have physical pion masses, but because of noise increasing we use $3/5/7m_l$ valence quark masses (following BMW) and extrapolate to m_l.

Definitions of δa_μ^s

QED correction to the $a_\mu^{\mathrm{HVP,LO}}$, δa_μ^f,

$$\delta a_\mu^{(f)} \equiv a_\mu^f(m_f, Q_f) - a_\mu^f(m_f, 0),$$

where f labels the quark flavour and the difference is evaluated at **equal renormalised quark mass.**

The QED correction to the connected strange $a_\mu^{\mathrm{HVP,LO}}$ is then

$$\delta a_\mu^{(s)} = a_\mu^s(m_s, -1/3e) - a_\mu^s(m_s, 0),$$

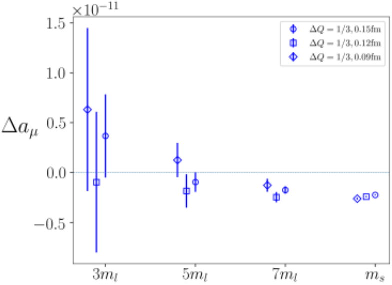
We originally extract QED corrections to a_μ at fixed bare quark mass (Δa_μ) and then convert to δa_μ using

$$\delta a_\mu = \Delta a_\mu - \delta m_q \frac{\partial a_\mu}{\partial m_q}$$

- We are not currently including QED in the lattice spacing determination.

Noise in Δa_μ^q

- The error increases in Δa_μ^q as the light quark mass is reduced
- It is not clear that the noise follows a Lepage like argument for the signal to noise.
- The noise on the up quark contribution is much larger than for the down quark contribution.



Dashen-like scheme

- We have initially used a Dashen-like scheme developed by MILC (1807.05556) and BMW
- We did explore using the GRS scheme (QCD+QED and QCD results compared at fixed renormalized mass) used by the ETM collaboration.
- We followed the prescription used by MILC/FNAL lattice (1807.05556)

$$M^2_{uu'} = M^2_{dd'} = M^2_{nn'} \equiv M^2_{\pi^0}$$
$$(M^2_{uu'})^\gamma = 0 = (M^2_{dd'})^\gamma$$

Dashen-like scheme

We define the mass m_l as the common u, d mass such that the charged pion in our pure QCD simulations has mass $(M_\pi)^{\mathrm{QCD}}$.

$$m_u = m_l(1 - \delta_u)$$

$$m_d = m_l(1 - \delta_d)$$

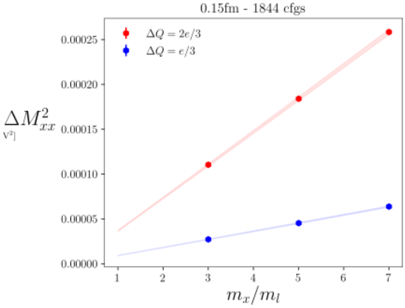
The parameters δ_u and δ_d selected so that EM contributions to $M^2_{uu'}$ and $M^2_{dd'}$ vanish

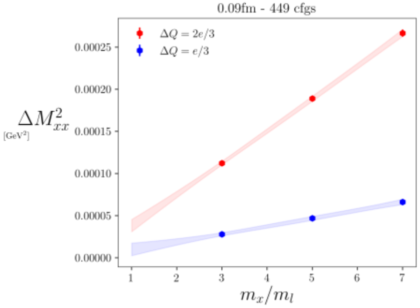
$$(M^2_{uu'})^\gamma = 0 = (M^2_{uu'})^\gamma$$

Use the same δ_d for the strange quark.

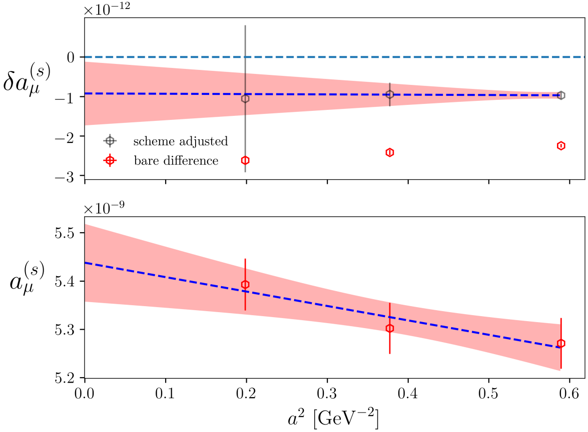
$$m_S = m_s(1 - \delta_d)$$

$$\Delta M_{xy}^2 \equiv M_{xy}^2 - M_{xy}^2\Big|_{q_x=q_y=q_u=q_d=q_s=0} \qquad \text{[fixed bare mass]},$$





From lattice 2022, 2212.12031



- Choice of scheme has increased the errors.

Continum Chiral extrapolation of δa_μ^l

- Extrapolate from heavier masses to the physical mass (m_q)
- Individually extrapolate up and down correlators and the combine.

Combined chiral and continnum extrapolation

$$\delta a_\mu^{(d)}(a^2, m_q/m_l) = c_0^{(d)} \left(1 + c_1^{(d)}(a\Lambda)^2 + c_2^{(d)} m_q/m_l\right)$$
$$\delta a_\mu^{(u)}(a^2, m_q/m_l) = c_0^{(u)} \left(1 + c_1^{(u)}(a\Lambda)^2 + c_2^{(u)} m_q/m_l\right),$$

$\Lambda = 0.5$ GeV for the typical QCD scale.

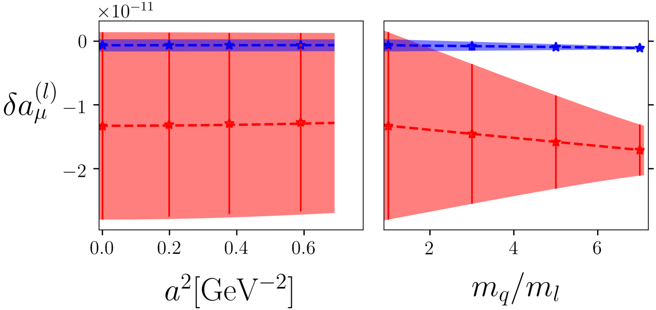
- This is the chiral fit model that BMW used.
- I have not found any chiral perturbation/effective field theory fit forms for the quark mass extrapolations.

Continuum limit of δa_μ^l

Preliminary results

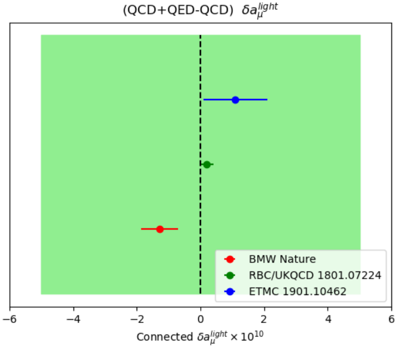
blue down quark contribution

red up quark contribution



Summary of QED contribution to δa_μ^l

- The green band is the estimate for QED contribution by FNAL/HPQCD/MILC
- The results below are in different schemes.



Conclusions

- Status of results on including quenched QED with $a_\mu^{\mathrm{HVP,LO}}$
- Two critical issues: noise and the choice of scheme to compare QCD+QED with QCD.

TODO list

- Increasing the quenched QED statistics on the physical 0.09 fm ensemble.
- Develop better schemes for comparing $a_\mu^{\mathrm{HVP,LO}}$ in QCD and QCD+QED. **Using suggestions from this workshop.**
- Including QED in the disconnected diagrams. (In progress)
- Estimate of QED on setting the lattice spacing (Steve's talk)