HVP with C* boundary conditions from lattice QCD(+QED)

Paola Tavella

Institut für Theoretische Physik
ETH Zürich

May 30, 2023

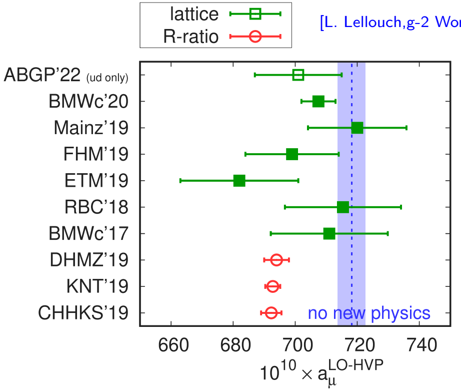
RC* collaboration : *Anian Altherr, Lucius Bushnaq, Isabel Campos-Plasencia, Marco Catillo, Alessandro Cotellucci, Madeleine Dale, Alessandro De Santis, Patrick Fritzsch, Roman Gruber, Tim Harris, Javad Komijani, Jens Luecke, Marina Marinkovic, Sofie Martins, Letizia Parato, Agostino Patella, Joao Pinto Barros, Sara Rosso, Nazario Tantalo & Paola Tavella*

Outline

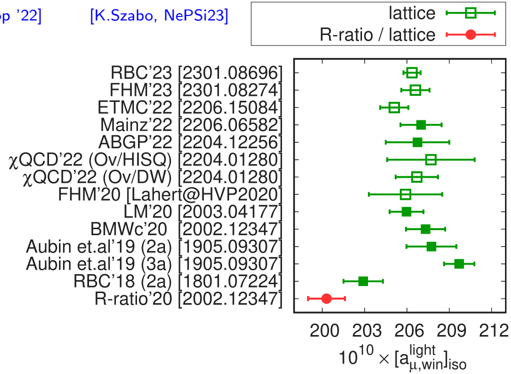
1. Motivation
2. $C^\star$ boundary conditions
   - ⋆ Implementation
   - ⋆ $RC^\star$ ensembles
3. Methods for HVP and status of the work
   - ⋆ Results for QCD
   - ⋆ Isospin breaking effects
   - ⋆ Results for QCD+QED
4. Conclusion & Outlook

Motivation

[L. Lellouch,g-2 Workshop '22]



[K.Szabo, NePSi23]



- new results from the muon g-2 experiment expected soon
- lattice results for the window are in good agreement and in tension with R-ratio

Motivation

Isospin symmetry is violated at the percent level

- strong IBE $\sim \mathcal{O}((m_d - m_u)/\Lambda_{QCD})$
- QED effects $\sim \mathcal{O}(\alpha_{EM})$

IBE effects are important for high-precision calculations of *decay rates of mesons, HVP contribution to g-2, etc.*

Theoretical issue of including QED in a finite periodic box:

- classical picture: Gauss law forbids a net non-zero charge
- path-integral: charged particles' propagation is forbidden due to symmetry under large gauge transformations

QED in lattice simulations

Currently used prescriptions for including QED:

- non-local constraints to remove the zero-modes of the photon [Hayakawa & Uno, 0804.2044]
  e.g QED_L, where $\sum_{\vec{x}} A_\mu(x_0, \vec{x}) = 0$
- infrared regulator m_γ (QED_M) [Endres et al., 1507.08916]
- QED_∞ [Blum et al., 1705.01067; Feng & Jin, 1812.09817]
- QED_C [Kronfeld & Wiese, 1991, 1992; Polley, 1993; Lucini et al., 1509.01636]
- ...

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C* boundary conditions

Local prescription for QED in a finite box → C-periodicity of fields in spatial directions

$$A_\mu(x + L_i \hat{i}) = -A_\mu(x) \qquad U_\mu(x + L_i \hat{i}) = U^*_\mu(x)$$

$$\psi(x + L_i \hat{i}) = C^{-1}\overline{\psi}(x) \qquad \overline{\psi}(x + L_i \hat{i}) = -\psi^T(x) C$$

[+] $A_\mu(x)$ is C-odd $\Longrightarrow \quad p_i = \frac{\pi}{L_i}(2l_i + 1), \quad l_i = 0, 1, ..., L_i - 1$

[+] charged-states propagation is possible

[+] suppressed finite-volume effects [Lucini et al., 1509.01636 ; Martins & Patella, 2212.09565]

[-] violations of flavour and charge conservation (by boundary effects)

[-] more expensive simulations [Bushnaq et al., 2209.13183]

C* boundary conditions

- Action in the doublet formulation

$$S_F^{hop} = -\frac{1}{2} \sum_{x \in \Lambda_{phys}, \mu} \overline{\chi}(x) \frac{(1+\gamma_\mu)}{2} \mathbb{V}_\mu^\dagger(x-\mu) \chi(x-\hat{\mu}) + \overline{\chi}(x) \frac{(1-\gamma_\mu)}{2} \mathbb{V}_\mu(x) \chi(x+\hat{\mu})$$

with

$$\chi(x) \equiv \begin{pmatrix} \psi(x) \\ \psi^{\mathcal{C}}(x) \end{pmatrix} \qquad \mathbb{V}_\mu(x) \equiv \begin{pmatrix} U_\mu(x) e^{iqA_\mu(x)} & 0 \\ 0 & U_\mu^*(x) e^{-iqA_\mu(x)} \end{pmatrix}$$

- Fields and their charge-conjugates associated with different space-time points

$$S_F^{hop} = -\frac{1}{2} \sum_{x \in \Lambda_{ext}, \mu} \overline{\psi}(x) \frac{(1+\gamma_\mu)}{2} V_\mu^\dagger(x-\mu) \psi(x-\mu) + \overline{\psi}(x) \frac{(1-\gamma_\mu)}{2} V_\mu(x) \psi(x+\mu)$$

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$C^\star$ boundary conditions

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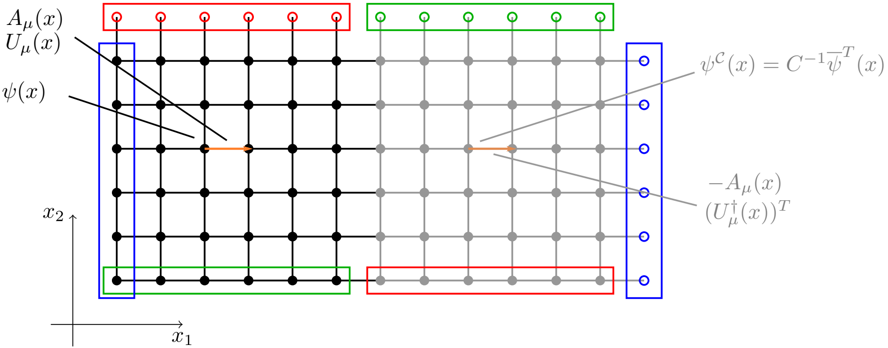
C* boundary conditions



- Lattice is doubled in direction $\hat{1}$: $L_1 = 2L$, $L_k = L$, $\quad k = 2, 3$
- Orbifold construction, e.g $\psi(x + L_k\hat{k}) = \psi(x + \frac{L_1}{2}\hat{1})$
- Effective periodicity of $2L$

Outline

RC* collaboration's program

Seven ensembles generated thus far with the openQ*D code
https://gitlab.com/rcstar/openQxD

First results in Bushnaq et al., 2209.13183

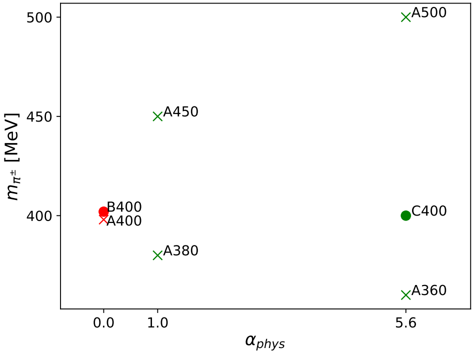
- details about the simulations
- calculations of meson masses, the Ω^- baryon, and the octet baryons
- cost analysis

Our setup allows for two ways of including isospin breaking effects:

- non-isosymmetric configurations at several unphysical values of α_{EM} and $m_u - m_d$
  $+$ extrapolation to the physical point
- isosymmetric configurations with $C^\star$ b.c. $+$ RM123 method

QCD+QED ensembles

- Lüscher-Weisz SU(3) gauge action ($\beta = 3.24$)
- Wilson action for compact U(1) field
- $N_f = 1 + 2 + 1$ of $O(a)$-improved Wilson fermions
- Periodic boundary conditions in time, C* b.c. in space



- Volume: A=64x32, B=80x48, C=96x48
- Lattice spacing $\simeq 0.05$ fm
- Pion mass between 360 and 500 MeV
- $m_\pi L \sim 3$ and $m_\pi L \sim 5$

QCD+QED ensembles

- Renormalization scheme [1]: $(8t_0)^{1/2}$, $\alpha_R(t_0)$, ϕ_0, ϕ_1, ϕ_2, ϕ_3

$$\phi_0 = 8t_0(m^2_{K^\pm} - m^2_{\pi^\pm}) \quad \to 0 \quad \text{(fixes } m_s - m_d) \quad [\phi_0^{\text{phys}} = 0.992]$$

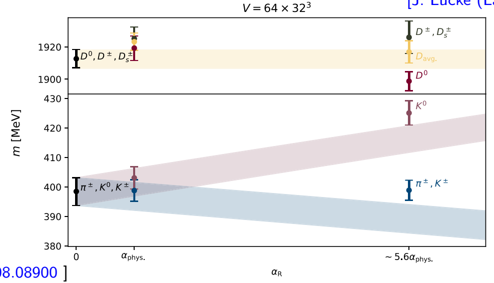
$$\phi_1 = 8t_0(m^2_{K^\pm} + m^2_{\pi^\pm} + m^2_{K^0}) \quad \to 2.11 \quad \text{(fixes } m_s + m_d + m_u) \quad [\phi_1^{\text{phys}} = 2.26]$$

$$\phi_2 = 8t_0(m^2_{K^0} - m^2_{K^\pm})/\alpha_R \quad \to 2.36 \quad \text{(fixes } \delta m_{strong}/\delta_{EM}) \quad [\phi_2^{phys} = 2.36]$$

$$\phi_3 = \sqrt{8t_0}(m^2_{D_s^\pm} + m^2_{D^\pm} + m^2_{D^0}) \to 12.1 \quad \text{(fixes } m_c) \quad [\phi_3^{\text{phys}} = 12.0]$$

ensemble	V	flavor	β	α
A400a00b324	64×32^3	$3+1$	3.24	0
B400a00b324	80×48^3	$3+1$	3.24	0
A450a07b324	64×32^3	$1+2+1$	3.24	0.007299
A380a07b324	64×32^3	$1+2+1$	3.24	0.007299
A500a50b324	64×32^3	$1+2+1$	3.24	0.05
A360a50b324	64×32^3	$1+2+1$	3.24	0.05
C380a50b324	96×48^3	$1+2+1$	3.24	0.05

[J. Lucke (Lattice22)]



[1] we used the CLS $N_f = 2+1$ value of $\sqrt{8t_0} = 0.415$ fm [Bruno et al., 1608.08900]

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First results for the HVP

ensemble	V	flavor	α	a[fm][2]	$m_{\pi^\pm}[MeV]$
A400a00b324	64×32^3	$3+1$	0	0.05393(24)	398.5(4.7)
B400a00b324	80×48^3	$3+1$	0	0.05400(14)	401.9(1.4)
A450a07b324	64×32^3	$1+2+1$	0.007299	0.05469(32)	451.2(4.3)
A380a07b324	64×32^3	$1+2+1$	0.007299	0.05323(28)	383.6(4.4)
A500a50b324	64×32^3	$1+2+1$	0.05	0.05257(14)	495.0(2.8)
A360a50b324	64×32^3	$1+2+1$	0.05	0.05054(27)	358.6(3.7)
C380a50b324	96×48^3	$1+2+1$	0.05	0.050625(79)	386.5(2.4)

- Reported measurements performed on 4 ensembles
- Ref: Altherr et al., 2212.11551, 2301.04385

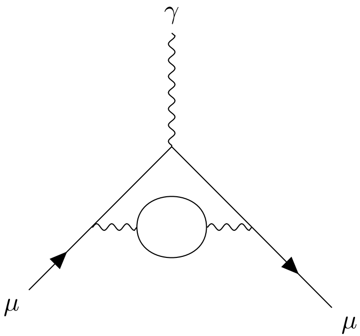
[2] a is determined using the $N_f = 2+1$ value of $\sqrt{8t_0} = 0.415$ fm from Bruno et al., 1608.08900

HVP calculation

- Time-momentum representation

$$G(t) = -\tfrac{1}{3} \sum_{k=1,2,3} \sum_{\vec{x}} \left\langle V_k^{c,l}(x) V_k^l(0) \right\rangle$$

$$a_\mu^{HVP} = \left(\tfrac{\alpha}{\pi}\right)^2 \sum_t G(t) \tilde{K}(t; m_\mu)$$



- Two discretizations of the vector current

$$V_\mu^l(x) = \sum_f q_f \overline{\psi}_f(x) \gamma_\mu \psi_f(x)$$

$$V_\mu^c(x) = \sum_f \frac{1}{2} q_f \left[\bar{\psi}_f(x+\hat{\mu}) \left(1+\gamma_\mu\right) U_\mu^\dagger(x) \psi_f(x) - \bar{\psi}_f(x) \left(1-\gamma_\mu\right) U_\mu(x) \psi_f(x+\hat{\mu}) \right]$$

- Extrapolation of the signal at large t (single-exponential for now)

HVP calculation

By considering the different Wick contractions:

$$\langle V_k^l(x) V_k^l(0) \rangle = \sum_{f,f'} q_f q_{f'} \,\mathrm{tr}\left[\gamma_k D_f^{-1}(x|x)\right] \cdot \mathrm{tr}\left[\gamma_k D_{f'}^{-1}(0|0)\right] +$$

$$- \sum_f q_f^2 \,\mathrm{tr}\left[\gamma_k D_f^{-1}(x|0) \gamma_k D_f^{-1}(0|x)\right]$$

disconnected ~ 2%

connected

- $D_f^{-1}(x|y)$ quark propagator from y to x
- γ_k Dirac matrices ($k = 1, 2, 3$)

QCD configurations → leading HVP (w/o IBE effects)

QCD+QED configurations → full HVP

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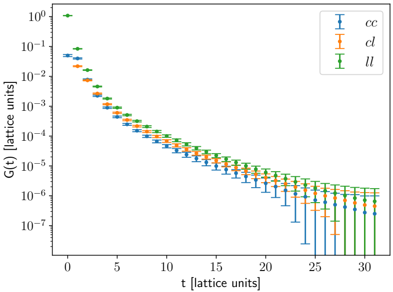
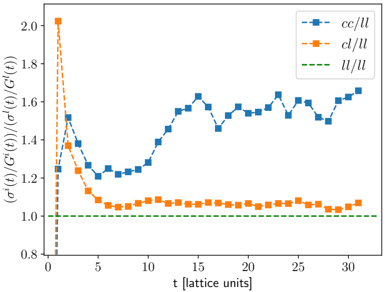
QCD with C⋆ b.c.

ensemble	V	β	α	$\kappa_{u,d,s}$	κ_c	$c_{SW}^{SU(3)}$
A400a00b324	64×32^3	3.24	0	0.1344073	0.12784	2.18859
B400a00b324	80×48^3	3.24	0	0.1344073	0.12784	2.18859

- Study of the signal-to-noise ratio for different discretizations of the correlator: $G^{ll}(t)$, $G^{cl}(t)$, $G^{cc}(t)$





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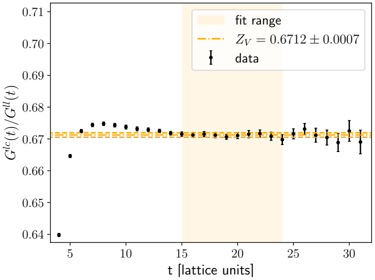
- V^l_μ requires a renormalization constant and $O(a)$-improvement

$$V^R_{\mu,f} = Z_V^{m_f}(V^l_{\mu,f} + ac_V \partial_\nu T_{\mu\nu,f})$$ [Bhattacharya et al., 0511014]

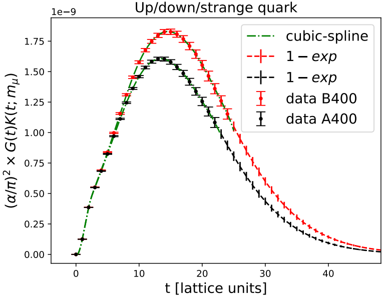
$$R(t) = \frac{\sum_{\vec{x},k} \langle V^c_f(x) V^l_f(0) \rangle}{\sum_{\vec{x},k} \langle V^l_f(x) V^l_f(0) \rangle}$$

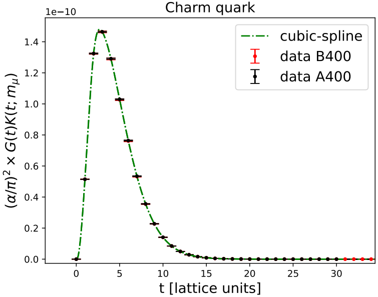
ensemble	$Z_V^{m_{l/s}}$	$Z_V^{m_c}$
A400a00b324	0.6745(12)	0.6066(2)
B400a00b324	0.6752(10)	0.6066(4)



QCD with C* b.c.

- Connected LO-HVP for lattice volumes 64×32^3 and 80×48^3





ensemble	n. cnfg	type	$am_V^{u/d/s}$	$a_\mu^{u/d/s} \times 10^{-10}$	am_V^c	$a_\mu^c \times 10^{-10}$
A400a00b324	200	*ll*	0.2644(50)	338(8)	0.8463(5)	7.83(8)
		cl	0.2652(55)	334(9)	0.8462(5)	6.18(7)
B400a00b324	108	*ll*	0.2522(33)	402(9)	0.8458(9)	7.81(9)
		cl	0.2530(32)	397(9)	0.8454(8)	6.16(7)

Outline

RM123 method

Isosymmetric QCD configurations are generated with $m_u = m_d$, $\alpha_{em} = 0$

To include the strong and the QED ibe:

- perturbative expansion in $\alpha = e^2/4\pi$ and $\delta m = (m_d - m_u)/\Lambda_{QCD}$
  [De Divitiis et al, 1303.4896]

$$\langle O(U, A, \psi, \bar{\psi}) \rangle_{QCD+QED} = \frac{1}{Z} \int \mathcal{D}U \mathcal{D}A \, \det[U, A] \exp(-S_g) \exp(-S_\gamma) O$$

$$\rightarrow \det[U, A] \simeq \det[U] + e_{sea} det[U, A]' + e_{sea}^2 det[U, A]''$$

$$\rightarrow O_{QCD+QED} \simeq O_{isoQCD} + \delta m O'_{\delta m} + e_{val} O'_e + e_{val}^2 O''_e$$

RM123 method

$$\left\langle V_k^{c,l}(x) V_k^l(0) \right\rangle = \left\langle V_k^{c,l}(x) V_k^l(0) \right\rangle \Big|_{e=0, m_f=\hat{m}} + \frac{1}{2} e^2 \frac{\partial^2}{\partial e^2} \left\langle V_k^{c,l}(x) V_k^l(0) \right\rangle \Big|_{e=0, m_f=\hat{m}} + \\ + (m_f - \hat{m}) \frac{\partial}{\partial m_f} \left\langle V_k^{c,l}(x) V_k^l(0) \right\rangle \Big|_{e=0, m_f=\hat{m}} + \ldots$$

[+] no need to generate new configurations

[+] corrections are measured as $\mathcal{O}(1)$ observables

[-] calculations of many diagrams needed

Photon propagator

- The photon field is Gaussian distributed in momentum space

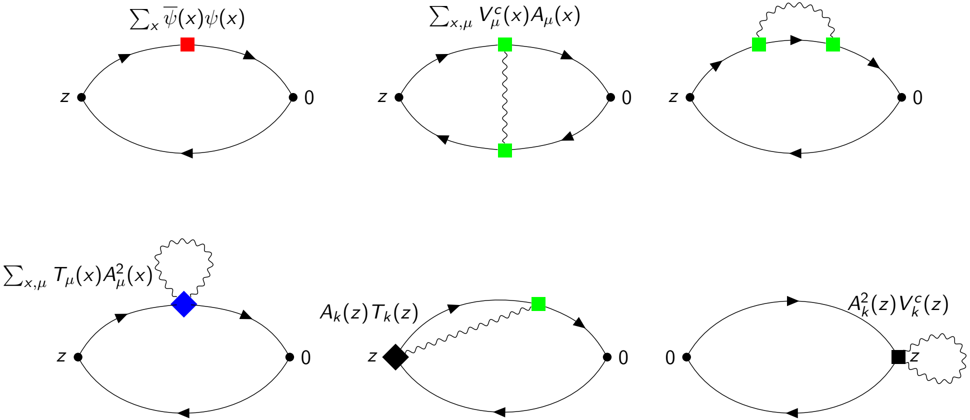
$$S_\gamma^{Feyn.} = \frac{1}{2}\sum_{k,\mu,\nu} \tilde{A}^*_\mu(k)\hat{k}_\nu^2\tilde{A}_\mu(k), \qquad \hat{k}_\mu = 2\sin\left(\frac{k_\mu}{2}\right), \qquad k_\mu = \frac{(2x_\mu + c_\mu)\pi}{L_\mu}$$

- Propagator in the Feynman gauge is stochastically estimated with

$$\hat{A}_\mu(x) = \frac{1}{\sqrt{N}}\sum_k \frac{e^{-ikx}}{\sqrt{\hat{k}^2}}\tilde{B}_\mu(k), \qquad P(B) \propto \exp\big(-B_\mu^2(k)\big)$$

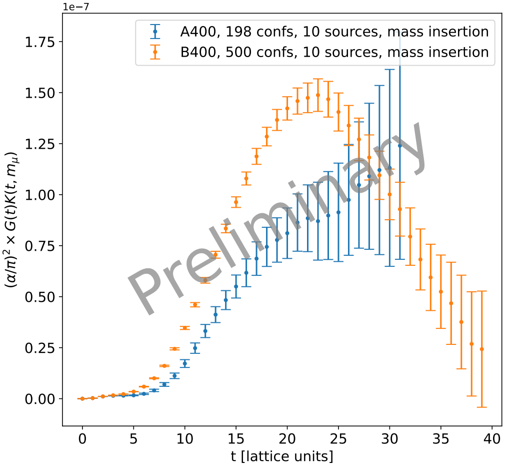
$$\Lambda_{\mu\nu}(x-y) = \frac{\delta_{\mu\nu}}{N}\sum_k \frac{e^{ik(x-y)}}{\hat{k}^2} \simeq \frac{1}{n_{src}}\sum_{i=1}^{n_{src}} \hat{A}^i_\mu(x)\hat{A}^i_\nu(y)$$

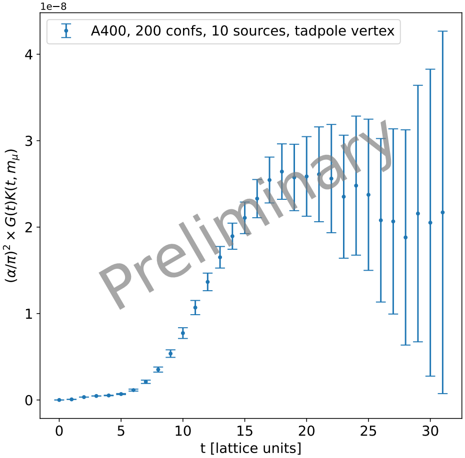
- Leading IB effects in the electro-quenched approximation $\Longrightarrow$ $e_{sea} = 0$



RM123 method

Work in progress...





Outline

QCD+QED with C* b.c.

Simulation details:

ensemble	V	β	α	κ_u	$\kappa_{d,s}$	κ_c	$c_{SW}^{SU(3)}$	$c_{SW}^{U(1)}$
A380a07b324	64×32^3	3.24	0.007299	0.13459164	0.13444333	0.12806355	2.18859	1
A360a50b324	64×32^3	3.24	0.05	0.135560	0.134617	0.129583	2.18859	1

- Wilson action with compact U(1) field

$$S_{g,U(1)} = \frac{1}{8\pi q_{el}^2 \alpha} \sum_x \sum_{\mu\neq\nu} [1 - P_{\mu\nu}^{U(1)}(x)]$$

  with q_{el} elementary charge, $P_{\mu\nu}^{U(1)}$ plaquette
- $c_{SW}^{SU(3)}$ correct up to $O(\alpha)$ terms, tree level improvement for U(1)
- $N_f = 1 + 2 + 1 \rightarrow$ non-physical degenerate d and s quarks

QCD+QED with C* b.c.

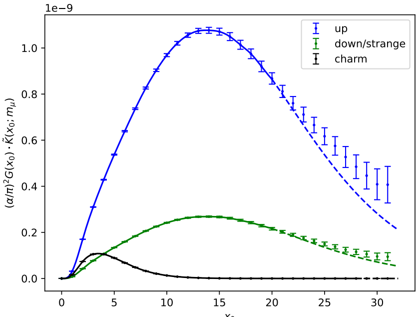


Figure: Integrand of the connected HVP contribution for the `A380a07b324` ensemble

- Results for the mixed correlator $G^{cl}(x_0)$
- Ren. constants are not considered yet

$$V_\mu^{\mathrm{R}}(x) = V_\mu^{\mathrm{c}}(x) + \mathcal{O}\left(\partial_\nu F^{\nu\mu}\right)$$ [Collins et al., 0512187]

- signal-to-noise ratio similar to the QCD case

ensemble	n. cnfg	α_R	flavor	am_V	$a_\mu^{\mathrm{HVP}} \times 10^{10}$
A360a50b324	181	0.040633(80)	up	0.267(8)	309(11)
			down/strange	0.262(7)	77(2)
			charm		10.62(11)
A380a07b324	200	0.007081(19)	up	0.266(4)	331(7)
			down/strange	0.265(6)	83(2)
			charm		9.78(10)

Outline

1. Motivation
2. C* boundary conditions
   - $\star$ Implementation
   - $\star$ RC* ensembles
3. Methods for HVP and status of the work
   - $\star$ Results for QCD
   - $\star$ Isospin breaking effects
   - $\star$ Results for QCD+QED
4. Conclusion & Outlook

Conclusion & Outlook

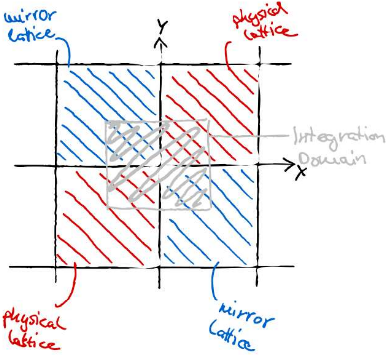
- We computed the connected HVP on two QCD and two QCD+QED ensembles with $C^{\star}$ b.c.
- Signal-to-noise ratio good for QCD and at the physical α_{EM}
- The results are not extrapolated: $a \sim 0.05$, $360 < m_{\pi} < 400$ MeV
- Ongoing work:
  - ⋆ RM123 method for QCD ensembles
  - ⋆ disconnected diagrams for QCD and QCD+QED
  - ⋆ variance-reduction techinques: low-mode averaging [De Grand & Schaefer, 0401011]
- Long-term:
  - ⋆ generation of new ensembles
  - ⋆ extrapolation to the physical point

Backup slides

HVP with C* bcs

- C* boundaries in $\hat{i} \implies p_i = \frac{\pi}{L}(2\mathbb{Z}+1)$ for A_μ
- the vector current is a C-odd operator
- zero-momentum projection is not possible
- the spatial integration domain in TMR should be set to

$$\left(-\frac{L}{2}, \frac{L}{2}\right)^3$$



[Sofie Martins(Lattice22)]

HVP with C* bcs

Vector current in the doublet formulation

$$V_\mu^{loc}(x) := -\chi^T(x) K C \frac{\gamma_\mu}{2} \tau_3 \chi(x), \qquad \chi(x) \equiv \begin{pmatrix} \psi(x) \\ \psi^{\mathcal{C}}(x) \end{pmatrix}, \quad K = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \tau_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

HVP with C* bcs

Vector current in the doublet formulation

$$V_\mu^{loc}(x) := -\chi^T(x) K C \frac{\gamma_\mu}{2} \tau_3 \chi(x), \qquad \chi(x) \equiv \begin{pmatrix} \psi(x) \\ \psi^{\mathcal{C}}(x) \end{pmatrix}, \quad K = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \tau_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

The (conn.) two-point functions has an extra Wick contraction

$$\overline{\chi(x)\chi}(y) = -D^{-1}(x|y) K C^{-1}, \quad \left\langle \chi^T(x) K C \frac{\gamma_\mu}{2} \tau_3 \chi(x) \chi^T(y) K C \frac{\gamma_\nu}{2} \tau_3 \chi(y) \right\rangle$$

HVP with C* bcs

Vector current in the doublet formulation

$$V_\mu^{loc}(x) := -\chi^T(x) K C \frac{\gamma_\mu}{2} \tau_3 \chi(x), \qquad \chi(x) \equiv \begin{pmatrix} \psi(x) \\ \psi^{\mathcal{C}}(x) \end{pmatrix}, \quad K = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \tau_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

The (conn.) two-point functions has an extra Wick contraction

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HVP with C* bcs

Vector current in the doublet formulation

$$V_\mu^{loc}(x) := -\chi^T(x)KC\frac{\gamma_\mu}{2}\tau_3\chi(x), \qquad \chi(x) \equiv \begin{pmatrix}\psi(x)\\ \psi^{\mathcal{C}}(x)\end{pmatrix}, \quad K = \begin{pmatrix}0 & 1\\ 1 & 0\end{pmatrix}, \quad \tau_3 = \begin{pmatrix}1 & 0\\ 0 & -1\end{pmatrix}$$

The (conn.) two-point functions has an extra Wick contraction

$$\overbrace{\chi(x)\chi}(y) = -D^{-1}(x|y)KC^{-1}, \quad \left\langle \chi^T(x)KC\frac{\gamma_\mu}{2}\tau_3\chi(x)\chi^T(y)KC\frac{\gamma_\nu}{2}\tau_3\chi(y)\right\rangle$$

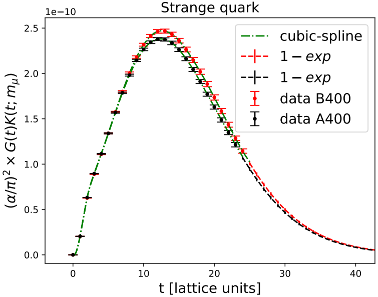
In terms of Dirac spinors

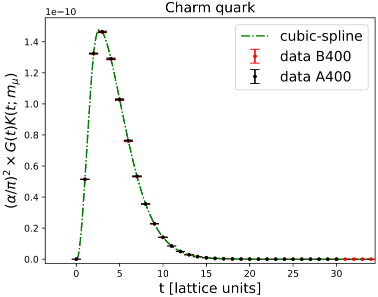
$$-\operatorname{tr}_{CD}[D^{-1}(x_1|y)\gamma_\mu\gamma_5 D^{-1\dagger}(x_1|y)\gamma_5\gamma_\nu - (D^{-1})(x_2|y)\gamma_\mu\gamma_5(D^{-1})^\dagger(x_2|y)\gamma_5\gamma_\nu],$$

$$\vec{x_1} \in (-L, L)^3, \qquad \vec{x_2} \in ((-2L, L) \cup (L, 2L))^3$$

QCD with C* b.c.: strange and charm contributions

- Connected LO-HVP for lattice volumes 64×32^3 and 80×48^3



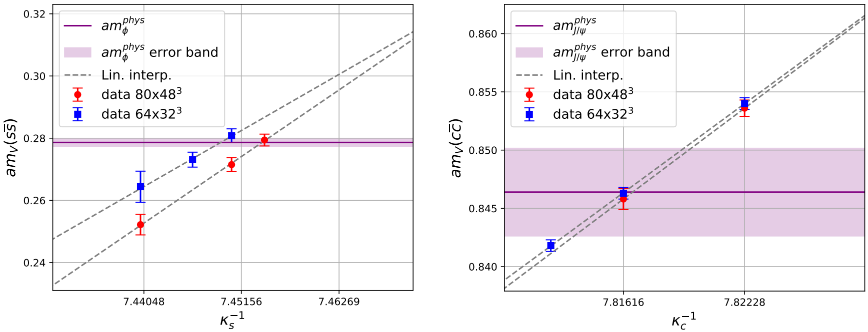


ensemble	n. cnfg	type	am_V^s	$a_\mu^s \times 10^{-10}$	am_V^c	$a_\mu^c \times 10^{-10}$
A400a00b324	200	*ll*	0.2808(22)	46.7(7)	0.8463(5)	7.83(8)
		cl	0.2796(29)	46.2(7)	0.8462(5)	6.18(7)
B400a00b324	108	*ll*	0.2794(19)	48.5(7)	0.8458(9)	7.81(9)
		cl	0.2791(20)	48.0(7)	0.8454(8)	6.16(7)

QCD with C* b.c.: strange and charm contributions

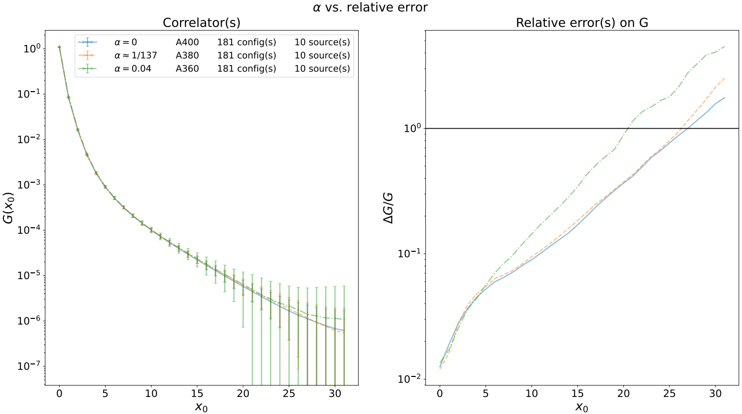
- tuning of $\kappa_{s,c}^{\mathrm{val}}$: change the valence hopping parameters such that $m_V^{s\bar{s}}$, $m_V^{c\bar{c}}$ match (disconnected and QED effects are neglected)

$$m_\phi^{phys} = 1019.461(20)\mathrm{MeV} \qquad m_{J/\psi}^{phys} = 3096.900(6)\mathrm{MeV}$$



QCD+QED with C* b.c.

- comparison of the relative error for three values of α



ensemble	lattice	flavor	α	a [fm]	$m_{\pi^\pm}[MeV]$
A400a00b324	64×32^3	$3+1$	0	0.05393(24)	398.5(4.7)
A380a07b324	64×32^3	$1+2+1$	0.007299	0.05323(28)	383.6(4.4)
A360a50b324	64×32^3	$1+2+1$	0.05	0.05054(27)	358.6(3.7)