QED corrections to hadronic observables from Pauli-Villars regulated photon propagators

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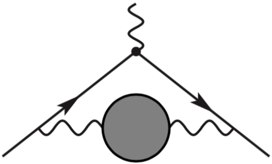
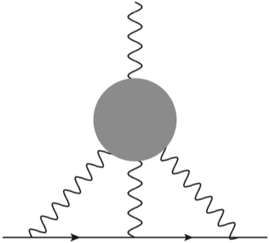
Largely based on article 2209.02149, published in JHEP.

Workshop *Isospin-Breaking Effects on Precision Observables in Lattice QCD*,
MITP Mainz, 22 - 26 July 2024.
(Tom Blum, Marina Marinkovic, Harvey Meyer, Hartmut Wittig)

Motivation: dominant uncertainties in SM prediction for $(g-2)_\mu$



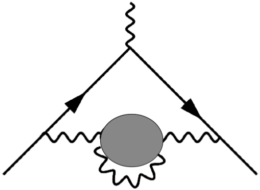
Hadronic vacuum polarisation



Hadronic light-by-light scattering

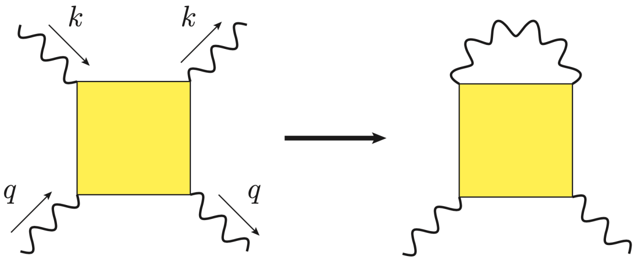
HVP: O(α^2), about $700 \cdot 10^{-10}$
$\Rightarrow$ desirable accuracy: 0.2%

HLbL: O(α^3), about $10 \cdot 10^{-10}$
$\Rightarrow$ desirable accuracy: 10%.



How large are the (overall O(α^3)) QED corrections to the HVP contribution?

E.m. correction to HVP from the forward HLbL amplitude



- the leading correction to HVP is expressible in terms of the forward HLbL amplitude, which by itself is a finite, physical amplitude
- regulating the propagator of the internal photon, e.g. $(1/k^2 - 1/(k^2 + \Lambda^2))$ is sufficient to obtain a finite expression for the e.m. correction to HVP.
- similarly, expressing the e.m. corrections to hadron masses in terms of the forward Compton amplitude, the shifts in the quark masses and the gauge coupling can be determined as a function of Λ.
- after adding the contribution of the quark-mass and gauge-coupling counterterms to the HVP, the $\Lambda \to \infty$ limit can be taken.

E.m. correction to HVP

Thus the strong isospin and e.m. corrections to the HVP
computed on the lattice in isosymmetric QCD
can be estimated unambiguously in the continuum.

Required physics input: (assume here scale-setting via a stable hadron mass)

1. the mass-operator insertions, i.e. $\langle h|\bar{q}q|h\rangle$ for (N_f+1) hadrons, as well as in the HVP
2. the forward Compton amplitudes for these hadrons
3. the forward LbL amplitude.

◇ A lot is known about the forward Compton amplitude in the nucleon
(see e.g. Gasser, Leutwyler, Rusetsky 2008.05806; and Stamen et al. 2202.11106 for the kaon).

◇ Exploratory lattice calculation of the forward hadronic light-by-light amplitude already exists (see 1712.00421).

E.m. correction to HVP: the master formula

If the leading HVP Π_{e^2} is counted as being order e^2,
the leading e.m. correction (of order e^4) is given by

$$\Pi_{4\text{pt}}(Q^2,\Lambda) = \frac{1}{6Q^4(2\pi)^3}\int_0^\infty dK^2\left[\frac{1}{K^2}\right]_\Lambda \int_0^{K^2Q^2} d\nu^2\left(\frac{K^2Q^2}{\nu^2}-1\right)^{1/2}\mathcal{M}(\nu,\,K^2,\,Q^2)$$

where $\nu = k\cdot q$,

$$\left[\frac{1}{K^2}\right]_\Lambda = \frac{1}{K^2} - \frac{1}{K^2+\Lambda^2}$$

and

$$\mathcal{M}(\nu,\,K^2,\,Q^2) = g_{\mu_1\mu_3}g_{\mu_2\mu_4}\mathcal{M}^{\mu_1\mu_2\mu_3\mu_4}(k,q) = 4\mathcal{M}_{TT} - 2\mathcal{M}_{LT} - 2\mathcal{M}_{TL} + \mathcal{M}_{LL}.$$

- First, test the formula in QED ...

The forward LbL amplitude at one-loop in QED

Setting the lepton mass to unity:

$$\mathcal{M}(\nu,K^2,Q^2)=16\alpha^2\Bigg(6-\Bigg\{\frac{2\log\left[\frac{1}{2}Q\left(\sqrt{Q^2+4}+Q\right)+1\right]}{\sqrt{Q^2+4}}$$

$$\times\Big(-4\nu^2Q^2\left[\left(K^2-2\right)\left(K^2+1\right)Q^4+\left(K^2+2\right)\left(7K^2-2\right)Q^2+6K^4+52K^2+16\right]$$

$$+K^2Q^4\left(K^2+Q^2+4\right)^2\left[K^2\left(Q^2+4\right)-2Q^2+4\right]+96\nu^4\Big)\Big/\Big(K^4Q^5\left(K^2+Q^2+4\right)^2$$

$$+16\nu^4Q-4K^2\nu^2Q^3\left[K^2\left(Q^2+2\right)+2\left(Q^2+4\right)\right]\Big)+\{K\leftrightarrow Q\}\Bigg\}$$

$$+\Bigg\{\frac{2\sqrt{1+\frac{4}{K^2+2\nu+Q^2}}\log\left[\frac{1}{2}\left(\sqrt{\left(K^2+2\nu+Q^2\right)\left(K^2+2\nu+Q^2+4\right)}+K^2+2\nu+Q^2+2\right)\right]}{K^2Q^2\left(K^2+Q^2+2\nu+4\right)-4\nu^2}$$

$$\times\Big(K^2Q^2(K^2+Q^2+2\nu)-2(K^2+Q^2)(\nu-1)-(K^4+Q^4)-2\nu(\nu+2)\Big)$$

$$+\frac{(K^2+Q^2)^2+2\nu(K^2+Q^2)+2\nu(\nu-2)-4}{\nu}\,C_0\left(-K^2,-Q^2,-K^2-2\nu-Q^2;1,1,1\right)$$

$$+\{\nu\to-\nu\}\Bigg\}\Bigg),$$

where $C_0(p_1^2,p_2^2,(p_1+p_2)^2;m_1^2,m_2^2,m_3^2)$ is the scalar one-loop integral [hep-ph/9807565]. Inserting this expression into the master formula gives the same result for $\overline{\Pi}_{4\mathrm{pt}}(Q^2)$ as the standard dispersive expression $\overline{\Pi}(Q^2)=-\frac{Q^2}{\pi}\int_{4m_\ell^2}^{\infty}\frac{dt}{t(t+Q^2)}\,\mathrm{Im}\Pi(t)$ using the 1955 Källen-Sabry next-to-leading-order spectral function $\frac{1}{\pi}\mathrm{Im}\Pi(t)$.

Prediction of the operator product expansion (1)

What is the behaviour of $\Pi_{4\text{pt}}(Q^2,\Lambda)$ for large Λ in QCD?

$$\tilde{\Pi}_{\sigma\lambda}(z,k^2) \equiv \int \frac{d\Omega_k}{2\pi^2} \Big\langle \int d^4x \int d^4y\, e^{ik(x-y)}\, V^{\rm em}_\mu(x) V^{\rm em}_\mu(y) V^{(1)}_\sigma(z) V^{(2)}_\lambda(0) \Big\rangle$$
$$\stackrel{k^2\to\infty}{=} \frac{3}{k^2} \sum_f \mathcal{Q}_f^2 \Big[2\Big(1+\frac{\alpha_s}{3\pi}\Big) m_f \Big\langle \int d^4x\, \bar\psi_f \psi_f V^{(1)}_\sigma(z) V^{(2)}_\lambda(0) \Big\rangle$$
$$-\frac{1}{24\pi^2 b_0}\Big(1+g^2\big(\tfrac{7}{24\pi^2}-\tfrac{b_1}{b_0}\big)\Big) \Big\langle \int d^4x\, \theta(x) V^{(1)}_\sigma(z) V^{(2)}_\lambda(0) \Big\rangle \Big],$$

- $(\mathcal{Q}_f, m_f) \equiv$ quark charges and masses
- $b_i \equiv$ the coefficients of the perturbative beta function.
- Written in terms of the (renormalization group invariant) trace anomaly, $\theta(x)$, related to the gluon Lagrangian by $\theta(x) = \frac{2\beta(g)}{g}\mathcal{L}_{\rm g}$.
- The $1/k^2$ behaviour of $\tilde{\Pi}_{\sigma\lambda}(z,k^2)$, leads to $\int_{\mu_{\rm IR}}^{\infty} d|k|\, |k|^3 \cdot \frac{1}{k^2} \cdot \Big(\frac{1}{k^2} - \frac{1}{k^2+\Lambda^2}\Big) \sim \log\Lambda$.

Prediction of the operator product expansion (2)

From previous slide, compute the subtracted vacuum polarisation, $\overline{\Pi}(Q^2) \equiv \Pi(Q^2) - \Pi(0)$:

$$\overline{\Pi}_{4\mathrm{pt}}(Q^2,\Lambda) \overset{\Lambda\to\infty}{=} \frac{3e^2}{8\pi^2}\sum_f \mathcal{Q}_f^2 \Big[\Big(\log\Big(\frac{\Lambda}{\mu_{\mathrm{IR}}}\Big) + \frac{1}{24\pi^2 b_0}\log\Big(\frac{\alpha_s(\mu_{\mathrm{IR}})}{\alpha_s(\Lambda)}\Big)\Big) m_f \frac{\partial}{\partial m_f}$$

$$+\frac{1}{48\pi^2 b_0}\Big(\log\Big(\frac{\Lambda}{\mu_{\mathrm{IR}}}\Big) + \frac{1}{2b_0}\big(\tfrac{7}{24\pi^2} - \tfrac{b_1}{b_0}\big)\log\Big(\tfrac{\alpha_s(\mu_{\mathrm{IR}})}{\alpha_s(\Lambda)}\Big)\Big)\Big(2q^2\tfrac{\partial}{\partial q^2} + \textstyle\sum_{f'} m_{f'}\tfrac{\partial}{\partial m_{f'}}\Big)\Big]\overline{\Pi}_{e^2}(Q^2).$$

◇ To remove the UV divergence, one must adjust $m_f \to m_f - \delta m_f$ and $g \to g - \delta g$; the divergent part of the counterterms is predicted,

◇ $\delta m_f = \frac{3e^2\, m_f}{8\pi^2}\Big(\log\Big(\frac{\Lambda}{\mu_{\mathrm{IR}}}\Big) + \frac{1}{24\pi^2 b_0}\log\Big(\frac{\alpha_s(\mu_{\mathrm{IR}})}{\alpha_s(\Lambda)}\Big)\Big)$,

◇ $\delta g = \frac{e^2}{8\pi^2}\frac{\beta(g)}{16\pi^2 b_0}\Big(\log\Big(\frac{\Lambda}{\mu_{\mathrm{IR}}}\Big) + \frac{1}{2b_0}\big(\frac{7}{24\pi^2} - \frac{b_1}{b_0}\big)\log\Big(\frac{\alpha_s(\mu_{\mathrm{IR}})}{\alpha_s(\Lambda)}\Big)\Big)$.

◇ The hadronic scale μ_{IR} parametrizes the finite part of the correction, which is determined by the chosen ‘scheme’ $\equiv$ the choice of (N_f+1) quantities matched between pure isosymmetric QCD and full (QCD+QED).

2209.02149 (JHEP), using results from Chetyrkin, Gorishny, Spiridonov PLB 160 (1985) 149.

Lattice implementation aspects

The split-up of the internal photon propagator

$$\frac{1}{k^2} = \frac{1}{k^2+\Lambda^2} + \left(\frac{1}{k^2} - \frac{1}{k^2+\Lambda^2}\right)$$

can be useful to separate the issue of the UV divergence from the IR effects.

The first term can be implemented by placing the photon on the lattice ('standard method', but with a photon mass $\Lambda \sim 400\,\mathrm{MeV}$).

The second term can be implemented with coordinate-space methods, similar to $a_\mu^{\rm HLbL}$,

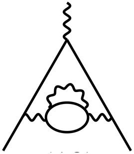
$$\overline{\Pi}_{4\rm pt}(Q^2,\Lambda) = -\frac{e^4}{2}\delta_{\mu\nu}\int_{x,y,z} H_{\lambda\sigma}(z)(G_0-G_\Lambda)(y-x)\,\Big\langle V_\sigma^{\rm em}(z)V_\nu^{\rm em}(y)V_\mu^{\rm em}(x)V_\lambda^{\rm em}(0)\Big\rangle,$$

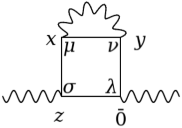
with $H_{\lambda\sigma}(z)$ known analytically [1706.01139] and $G_\Lambda(x) \equiv \int \frac{d^4p}{(2\pi)^4}\frac{e^{ip\cdot x}}{p^2+\Lambda^2}$.

- Natural formulation on large lattices
- Avoids power-law finite-size effects.

2209.02149.

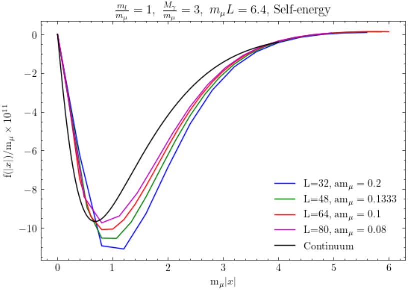
Reproducing QED two-loop VP on the lattice (1): self-energy diag.

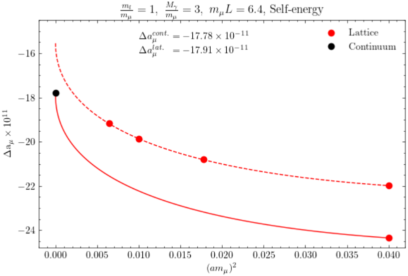




$$a_\mu^{\mathrm{NLOVP}}(m_l = m_\mu,\ \Lambda \equiv M_\gamma) = \int_0^\infty d|x|\ f(|x|),$$

$$f(|x|) = -\frac{e^4}{2}(2\pi^2|x|^3)\int_{y,z} H_{\lambda\sigma}(z)(G_0 - 2G_{\Lambda/\sqrt{2}} + G_\Lambda)(y-x)\ \left\langle V_\sigma^{\mathrm{em}}(z)V_\mu^{\mathrm{em}}(y)V_\mu^{\mathrm{em}}(x)V_\lambda^{\mathrm{em}}(0)\right\rangle$$



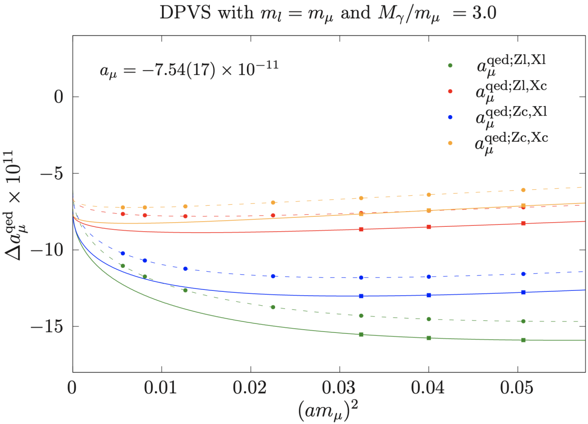


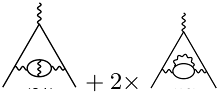
Wilson fermions; four local vector currents;

'double Pauli-Villars' regularisation: $(G_0 - 2G_{\Lambda/\sqrt{2}} + G_\Lambda)$ is finite at $x = y$.

Lattice calculation by Dominik Erb

Reproducing QED two-loop VP on the lattice (2)



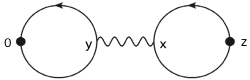


Continuum calculation:
$a_\mu^{\mathrm{NLOVP}}(m_l = m_\mu,\ M_\gamma = 3m_\mu) = -7.5 \cdot 10^{-11}$.

Lattice calculation by Antoine Gérardin, continuum calculation by Volodymyr Biloshytskyi.

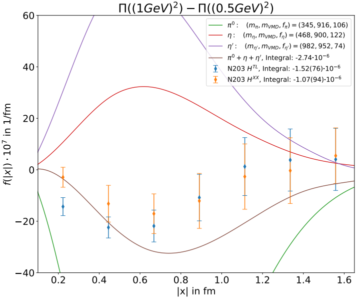
A first application in lattice QCD: a UV-finite diagram

The (1 photon irreducible) disconnected diagram



$$\Pi(Q^2) - \Pi(\bar{Q}^2) = \int_0^\infty d|x|\ f(|x|),$$

$$f(|x|) = -\frac{e^4}{2}(2\pi^2|x|^3)\int_{y,z} H_{\lambda\sigma}(z)G_0(y-x)\ \mathrm{Tr}\{\gamma_\lambda S(0,y)\gamma_\mu S(y,0)\}\ \mathrm{Tr}\{\gamma_\mu S(x,z)\gamma_\sigma S(z,x)\}$$



- $48^3 \times 128$ ensemble (N203): $m_\pi = 346\,$MeV, $m_K = 442\,$MeV, $a = 0.064\,$fm.
- estimate of pseudoscalar exchange explains the size of the integrand.
- weight factor $-25/9$ for π^0 exchange, unity for (η, η')

Calculation by Julian Parrino.

Conclusion

The leading e.m. and strong isospin-breaking effects by which the vacuum polarisation in isosymmetric QCD differs from full (QCD+QED)...

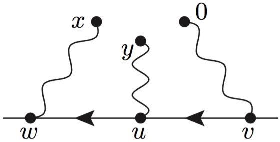
1. ...can be computed unambiguously in the continuum, with sufficient knowledge of the 'sigma terms' and forward Compton amplitudes for $(N_f + 1)$ hadrons, and of the forward HLbL amplitude.
   The divergent part of the counterterms $(\delta m_f, \delta g)$ is predicted by the OPE.

2. On the lattice, regulating the internal photon at a scale $\Lambda \ll a^{-1}$ allows for making helpful comparisons with continuum predictions and for applying covariant coordinate-space methods.

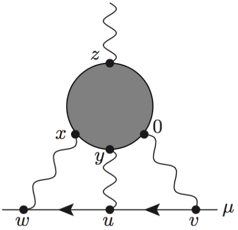
3. For UV-finite diagrams, the coordinate-space methods can even be applied without regulating the photon propagator.

Hadronic light-by-light contribution: coordinate-space approach

QED kernel $\bar{\mathcal{L}}_{[\rho,\sigma];\mu\nu\lambda}(x,y)$



$\Rightarrow$



$$a_\mu^{\mathrm{HLbL}} = \frac{me^6}{3} \underbrace{\int d^4y}_{=2\pi^2|y|^3d|y|} \Big[\int d^4x \, \underbrace{\bar{\mathcal{L}}_{[\rho,\sigma];\mu\nu\lambda}(x,y)}_{\mathrm{QED}} \; \underbrace{i\widehat{\Pi}_{\rho;\mu\nu\lambda\sigma}(x,y)}_{=\mathrm{QCD\ blob}} \Big].$$

$$i\widehat{\Pi}_{\rho;\mu\nu\lambda\sigma}(x,y) = -\int d^4z \, z_\rho \left\langle \, j_\mu(x) \, j_\nu(y) \, j_\sigma(z) \, j_\lambda(0) \right\rangle.$$

- $\bar{\mathcal{L}}_{[\rho,\sigma];\mu\nu\lambda}(x,y)$ computed in the continuum & infinite-volume
- no power-law finite-volume effects & only a 1d integral to sample the integrand in $|y|$.

[Asmussen, Gérardin, Green, HM, Nyffeler 1510.08384, 1609.08454]