Radiative Leptonic Decays of Pseudoscalar Mesons

"Converging on QCD+QED Prescriptions" workshop at the Higgs Centre for Theoretical Physics

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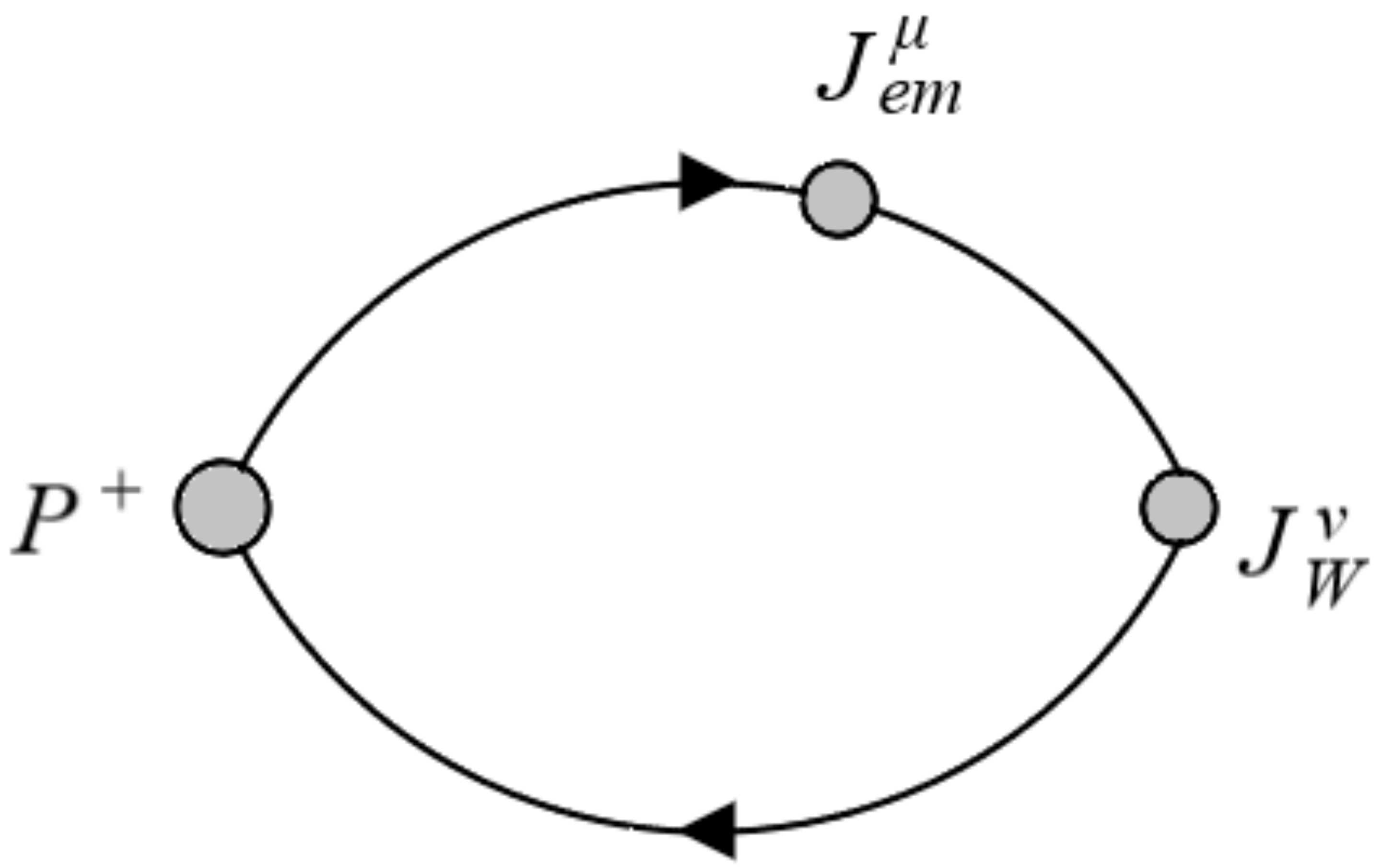
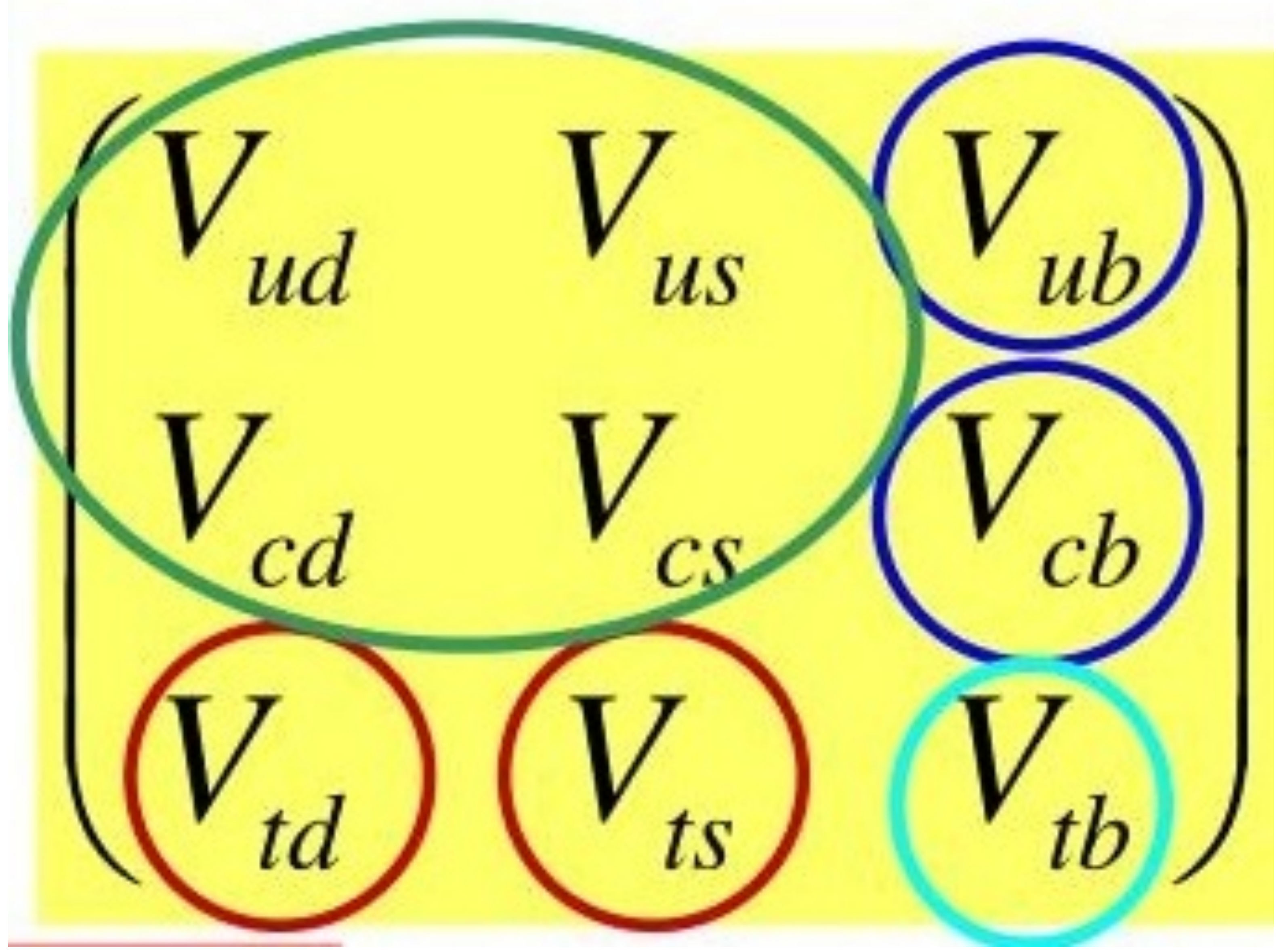


Flavor physics

Cabibbo angle



$B \to \pi \, l \, \nu$

$B \to D^{(*)} \, l \, \nu$

~ 1

parametrized from V_{CKM}

theory restricts the 9 complex values to only four independent parameters

numerous tests of consistency are provided

$B^0_{d,s} - \bar{B}^0_{d,s}$ oscillations

$$\Gamma(B \to \ell\nu) = \frac{G_F^2}{8\pi} \, |V_{ub}|^2 \, m_B^3 \left(\frac{m_\ell}{m_B}\right)^2 \left[1 - \left(\frac{m_\ell}{m_B}\right)^2\right] f_B^2$$

non-perturbative hadronic parameters are the principal source of theoretical uncertainty!

Why QED Corrections?

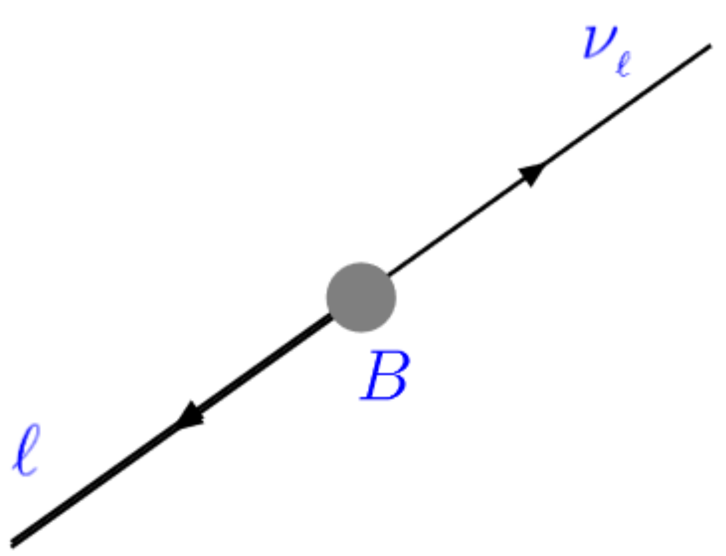
1) Pushing precision under percent level → CKM matrix element extraction from leptonic decays

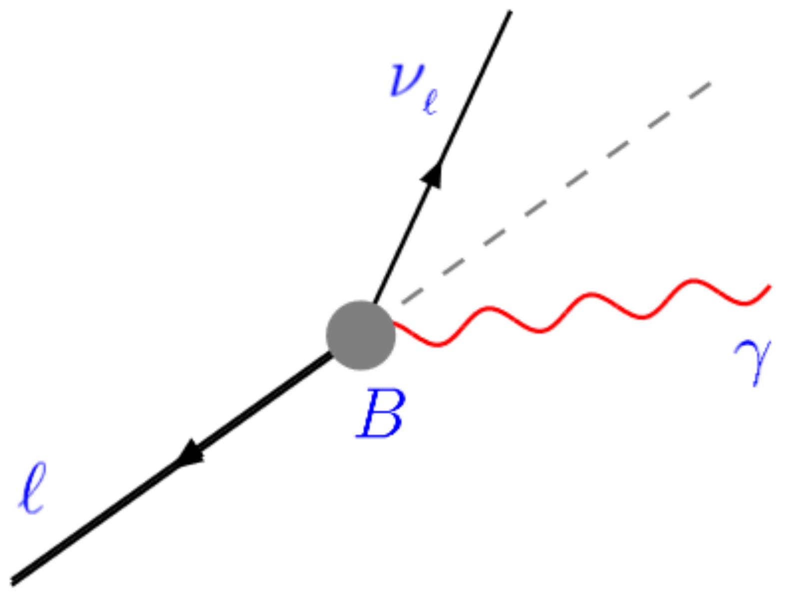
real photon emission has **always** to be included when considering O(α_{em}) corrections **[Block&Nordsiek mechanism]**

[Di Carlo et arXiv:1904.08731 (2019)] and

[Boyle et Al arXiv:2211.12865 (2022)] for K and π

2) Removal of helicity suppression through photon emission

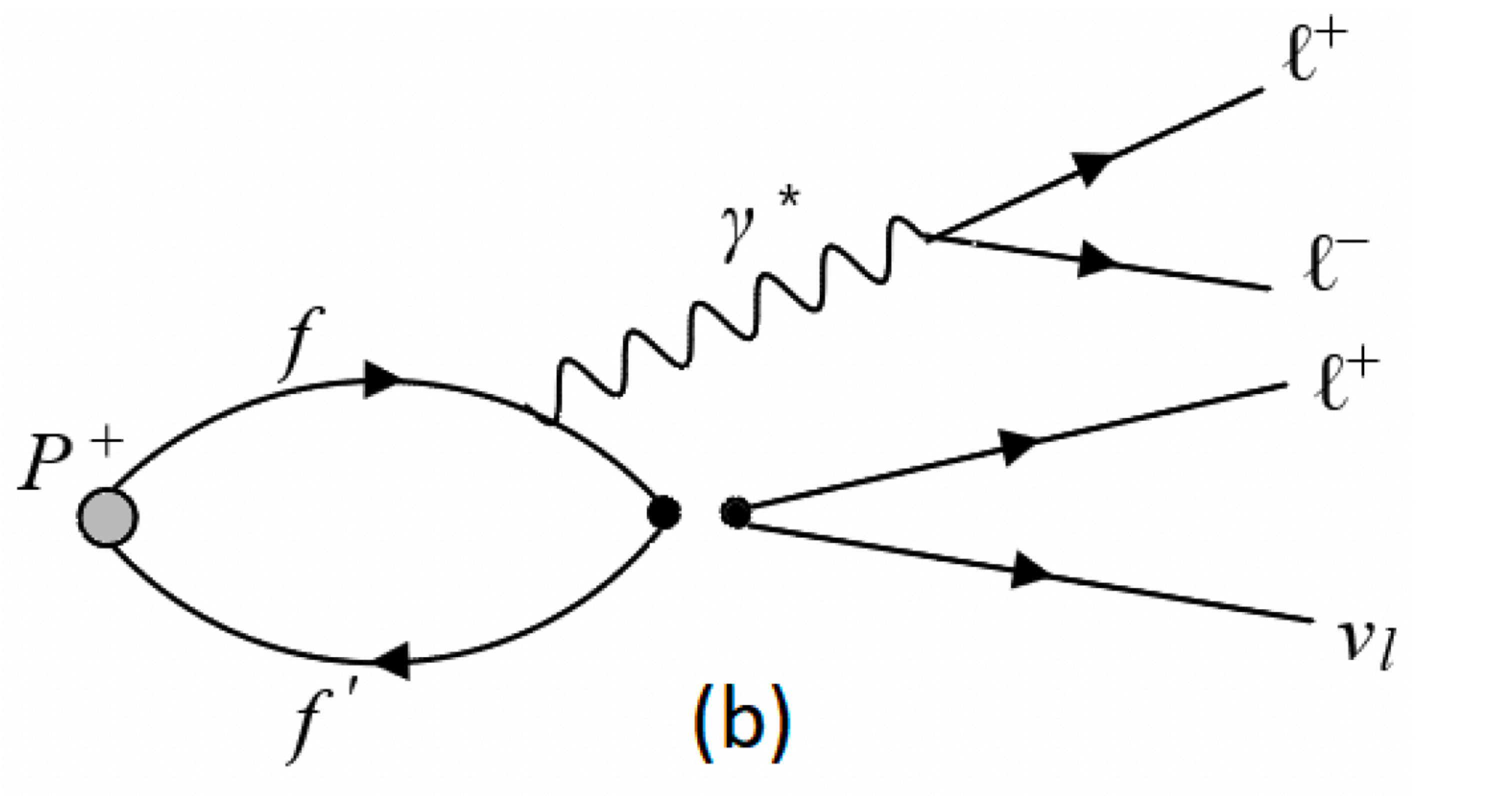




despite α_{em} there is an enhancement of $\left(m_P/m_l\right)^2$

3) Indirect search of New Physics

Operators that parametrize new physics are involved in processes where also QED has to be included.



$\propto \alpha_{em}^2$

NP is more likely to be detected

To Sum Up

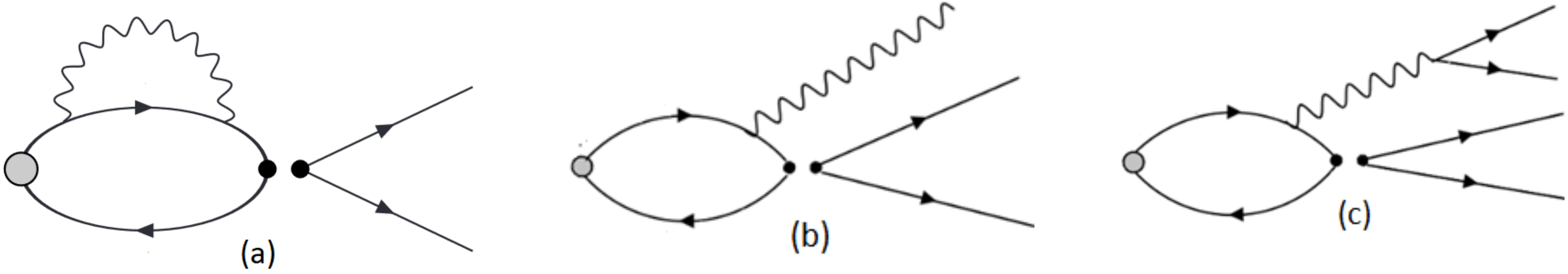
$O(\alpha_{em})$ corrections

NP constraints

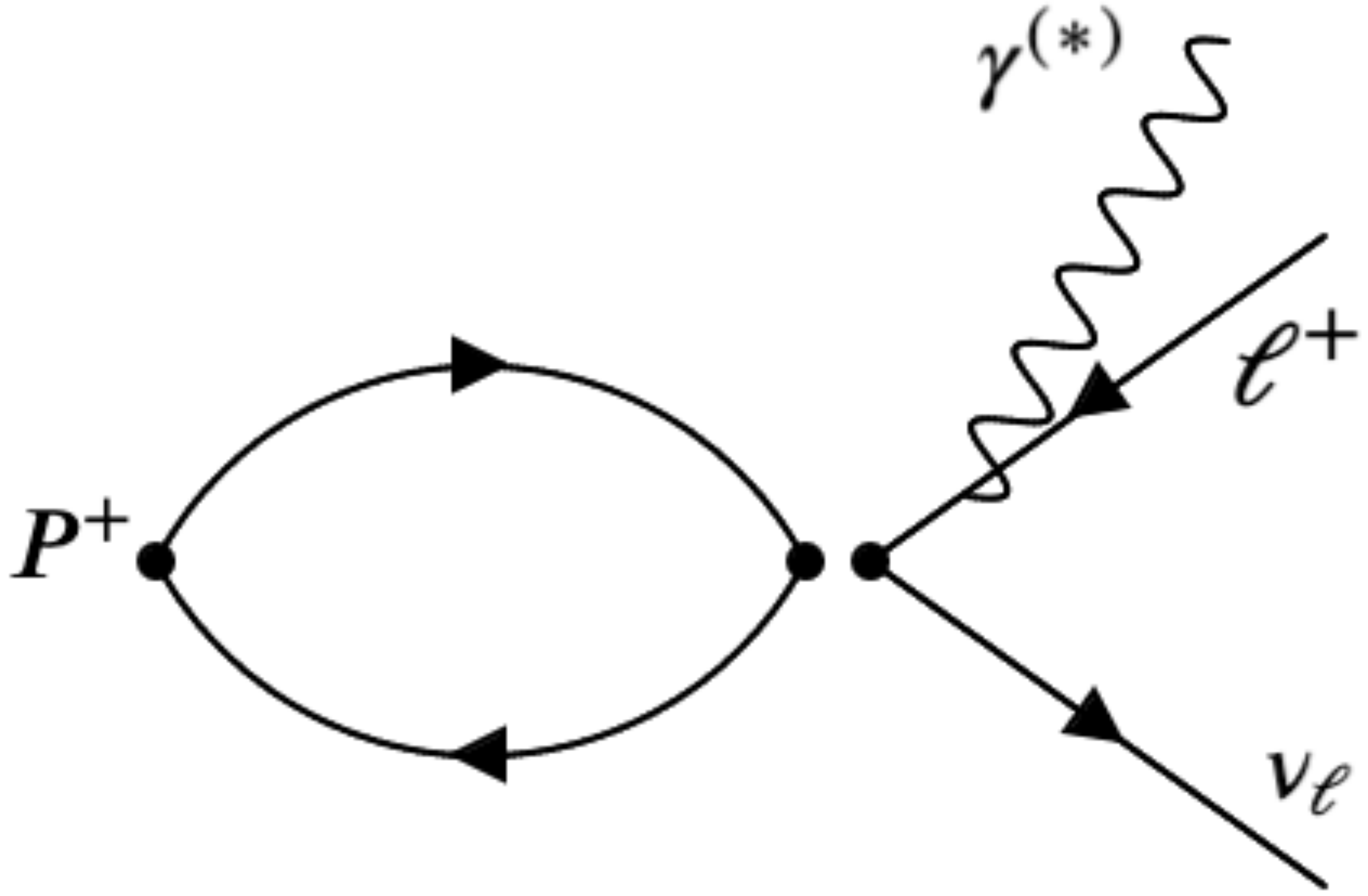
- Virtual corrections (a)
- Real photon emission (b)
- Virtual photon emission (c)

dynamical enhancement despite α_{em} suppression

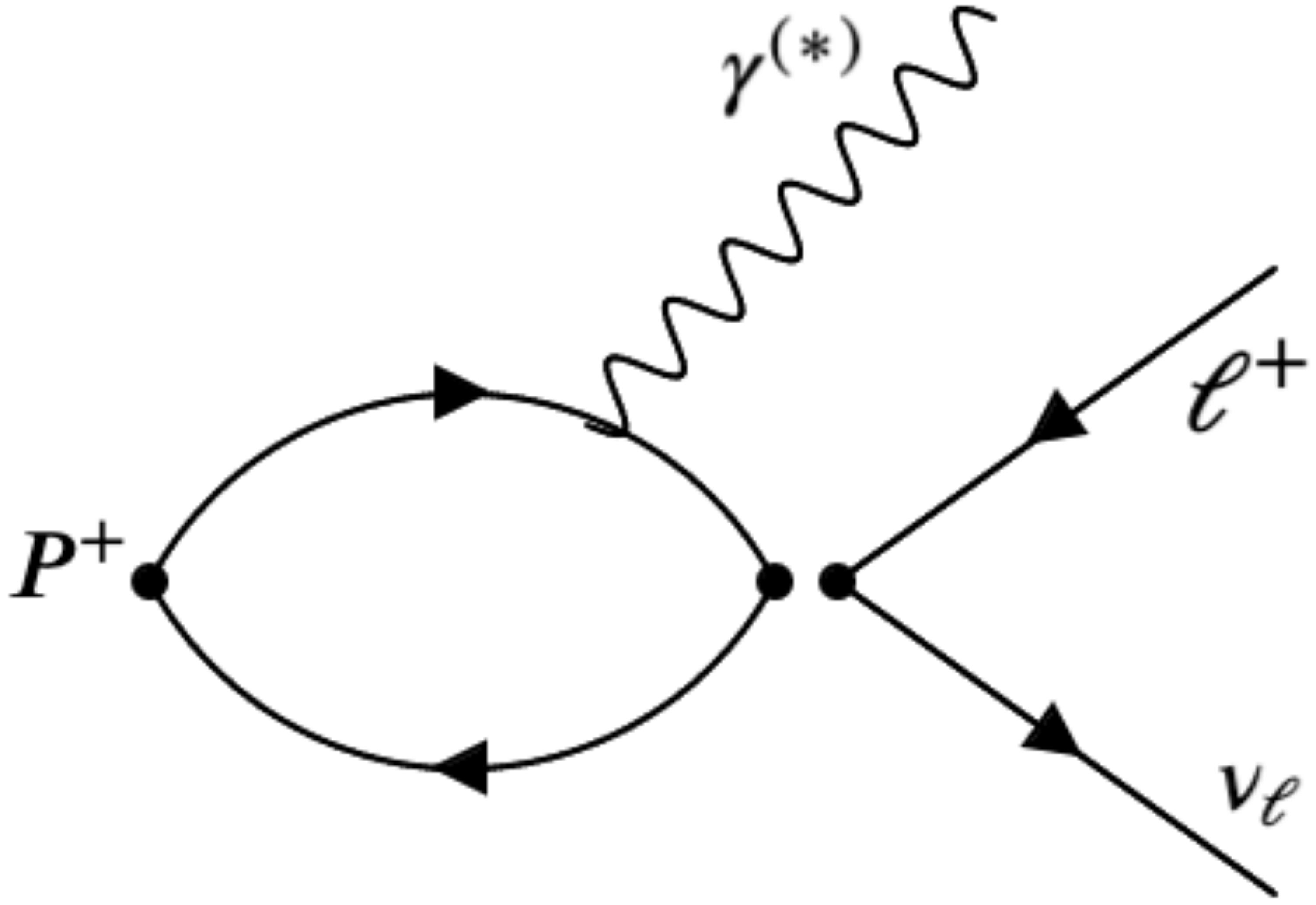
$\propto \alpha_{em}^2$



$P^+ \to \ell^+ \nu_\ell \gamma^{(*)}$ decays



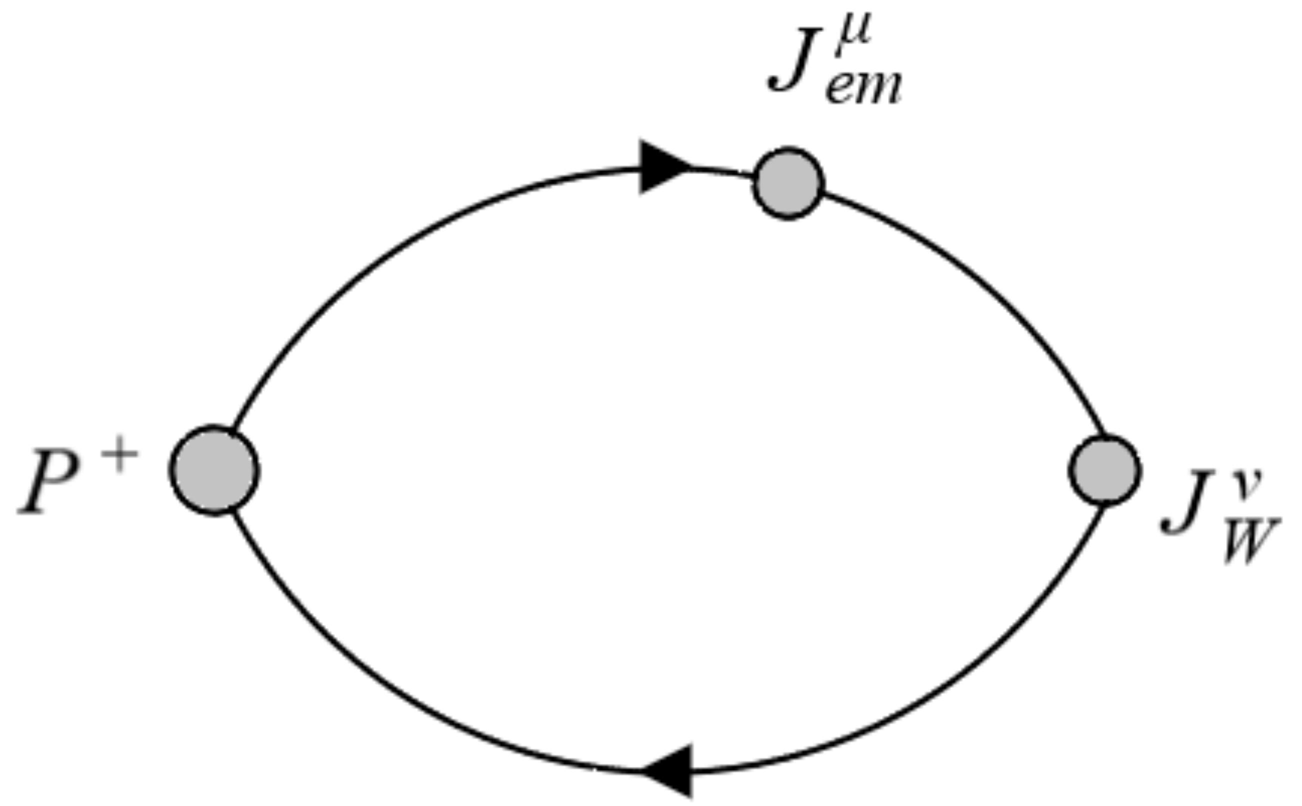
- Can be computed in perturbation theory, by simply knowing f_P



- (Virtual) photon interacts with the internal hadronic structure of P
- Non perturbative strong dynamics encoded in the hadronic tensor

Hadronic Tensor and Form Factors

$$H^{\mu\nu}(k,p) = \int d^4x e^{ik\cdot x} \langle 0|T[J^\mu_{em}(x) J^\nu_W(0)]|P(p)\rangle$$



$$H^{\mu\nu} = H^{\mu\nu}_{pt} + H^{\mu\nu}_{SD}\,,$$

Point-like, IR contribution

$$H^{\mu\nu}_{pt} = f_P\left[g^{\mu\nu} - \frac{(2p-k)^\mu(p-k)^\nu}{(p-k)^2 - m_P^2}\right],$$

SD form factors

$$H^{\mu\nu}_{SD} = \frac{H_1}{m_P}(k^2 g^{\mu\nu} - k^\mu k^\nu) + \frac{H_2}{m_P}\frac{[(k\cdot p - k^2)k^\mu - k^2(p-k)^\mu]}{(p-k)^2 - m_P^2}(p-k)^\nu$$
$$+ \frac{F_A}{m_P}[(k\cdot p - k^2)g^{\mu\nu} - (p-k)^\mu k^\nu] - i\frac{F_V}{m_P}\epsilon^{\mu\nu\alpha\beta}k_\alpha p_\beta\,.$$

Non perturbative functions of k^2 and $(p-k)^2$

For real photon only F_A and F_V contribute!

Goal of the Work

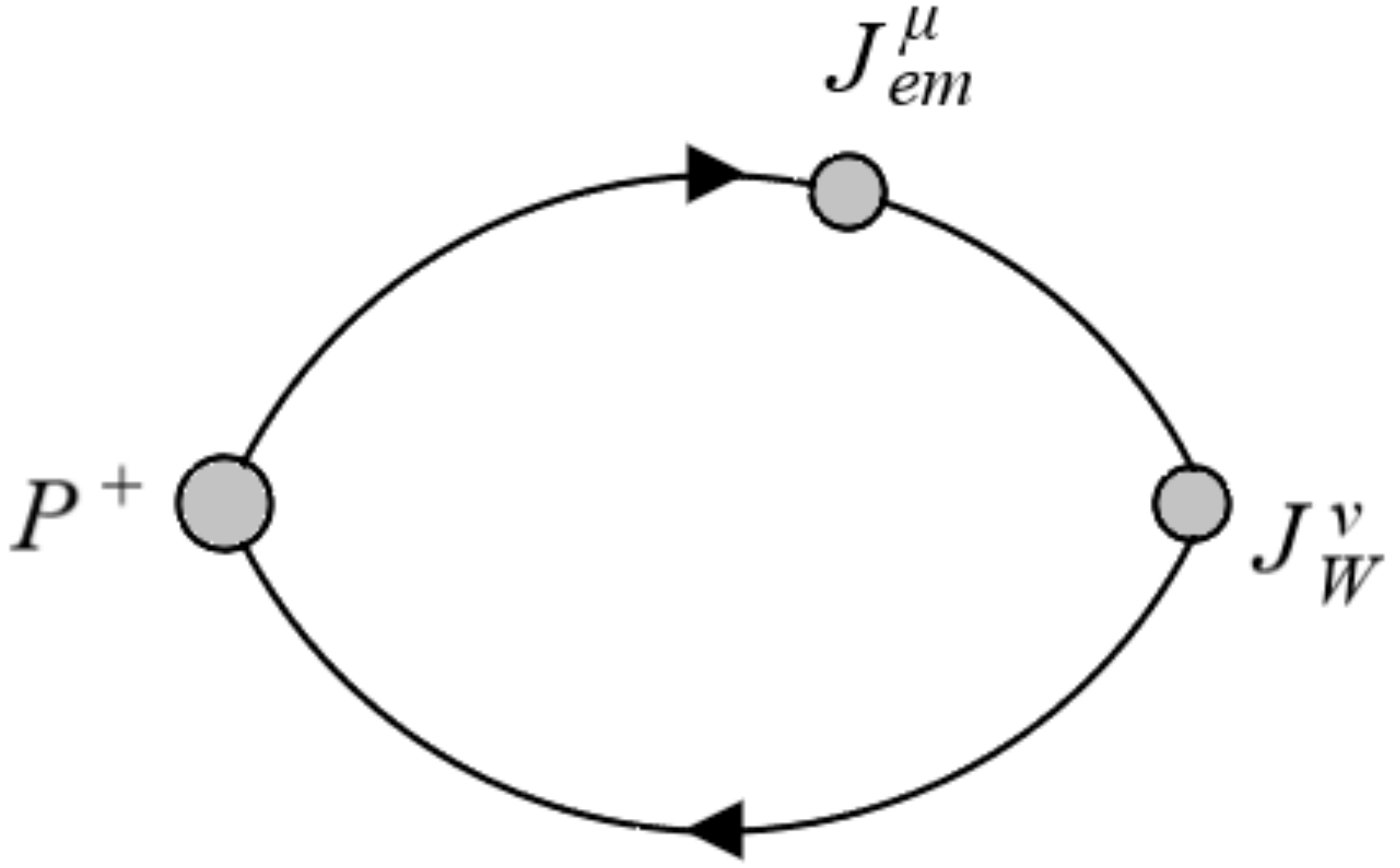
- **Extraction of the SD form factors from suitable lattice Euclidean correlators** →
  - Accounting for QED in lattice simulation
  - Accounting for momentum dependence
  - checking on validity of analytic continuation to Euclidean time
  - Separating point-like contribution
- **Reconstruction of the Branching Ratios for different final states**

Euclidean Correlator

we do not consider the photon on the lattice, only the e.m. current that carries momentum k

finite volume effects are exponentially suppressed (the lighter state is the massive pion)

$$H_E^{\mu\nu}(k,p) = \int d^4y\; d^3x\; e^{t_y E_\gamma - i\mathbf{k}\cdot\mathbf{y} + i\mathbf{p}\cdot\mathbf{x}}\, \mathrm{T}\langle 0| j_W^\alpha(t)\, j_{em}^\mu(y)\, P(0,\mathbf{x})|0\rangle$$



Euclidean extraction of the photon momentum k

source of the pseudoscalar meson with momentum p

Two Delicate Issues

Analytical continuation to Euclidean time

$$-i \sum_{n:\vec{p}_n=\vec{k}} \frac{\langle 0|J^{\mu}_{em}(0)|n\rangle \langle n|J^{\nu}_{W}(0)|P\rangle}{2E_n} \int_0^{+\infty} dt_x e^{-t_x(E_n-E_\gamma)}$$

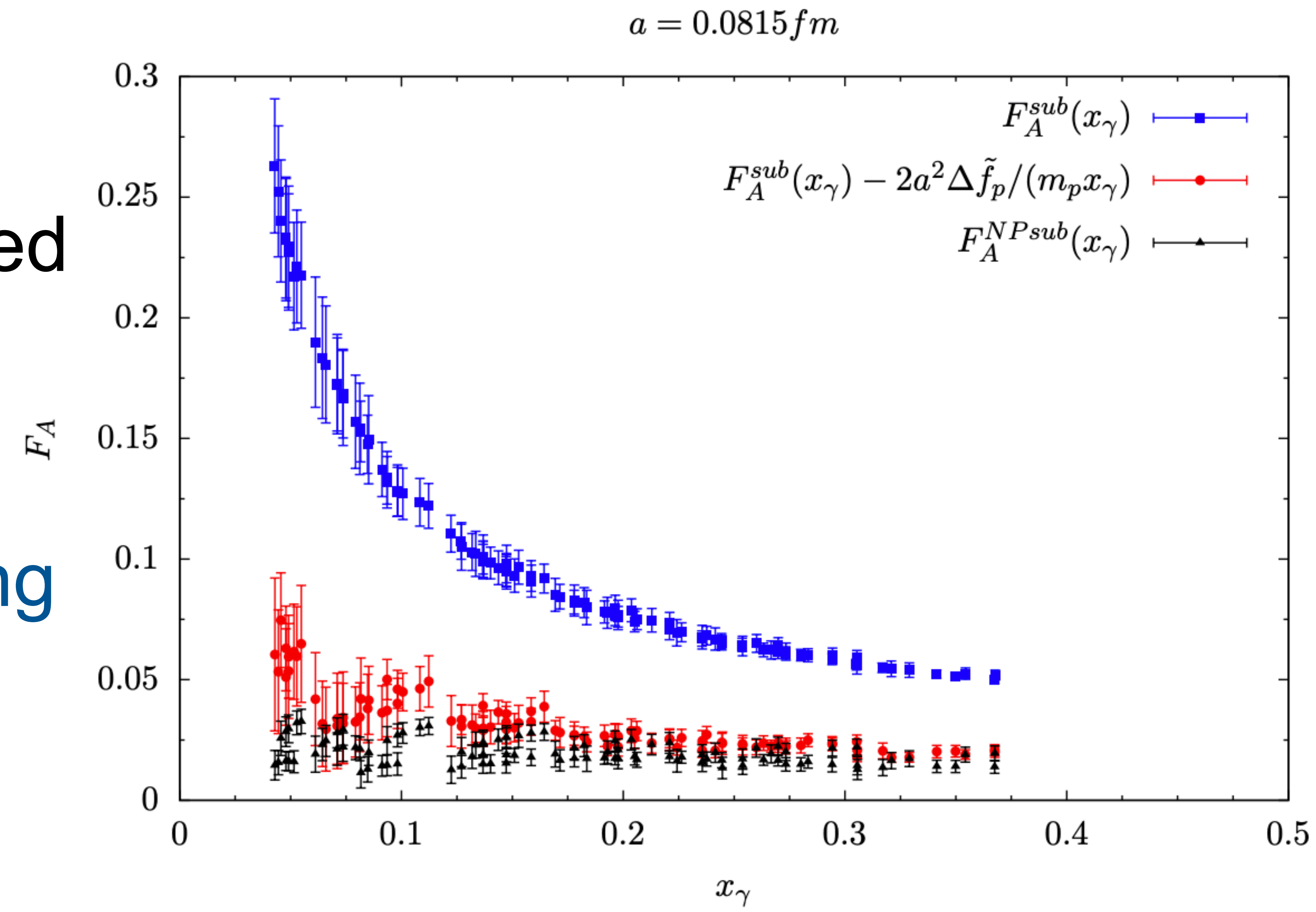
divergent if $E_n < E_\gamma$ ⟷ $k^2 < 4m_\pi^2$

For real photon emission Wick Rotation is always safe

Subtraction of point-like term

Residual discretization errors reflect in SD estimators as enhanced by inverse powers of the photon momentum (IR divergent)

- Three-point function at zero photon momentum is implemented to subtract f_P at all orders in the lattice spacing
- Pathological IR behaviour of correlator due to vanishing mass gap has been properly studied



A look at the past: $P^+ \to \ell^+ \nu_\ell \gamma$ decays

[A. Desiderio et Al ArXiv:2006.05358 (2020)]

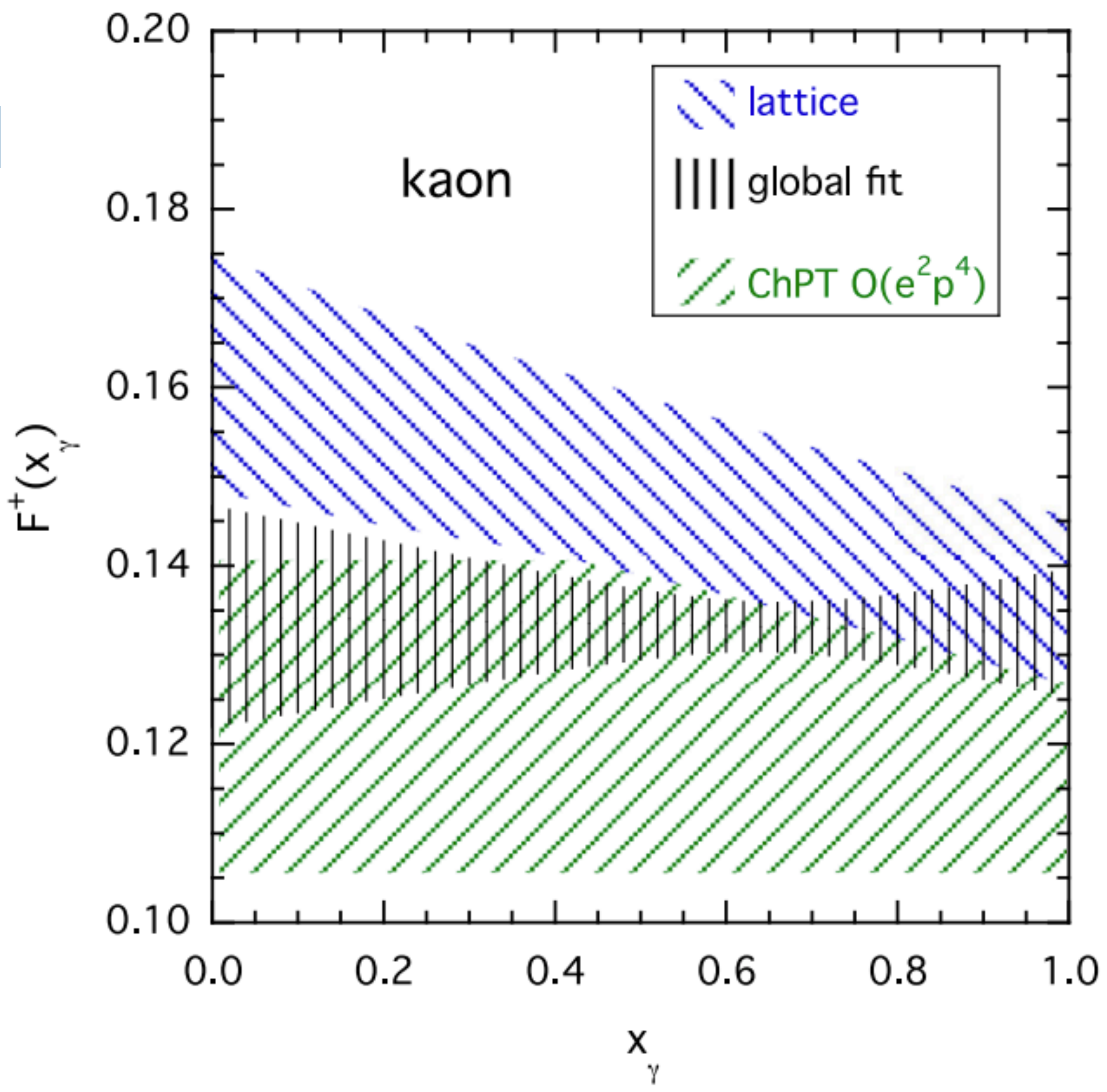
Continuum extrapolated results
for $P^+ = \pi^+, K^+, D, D_s$

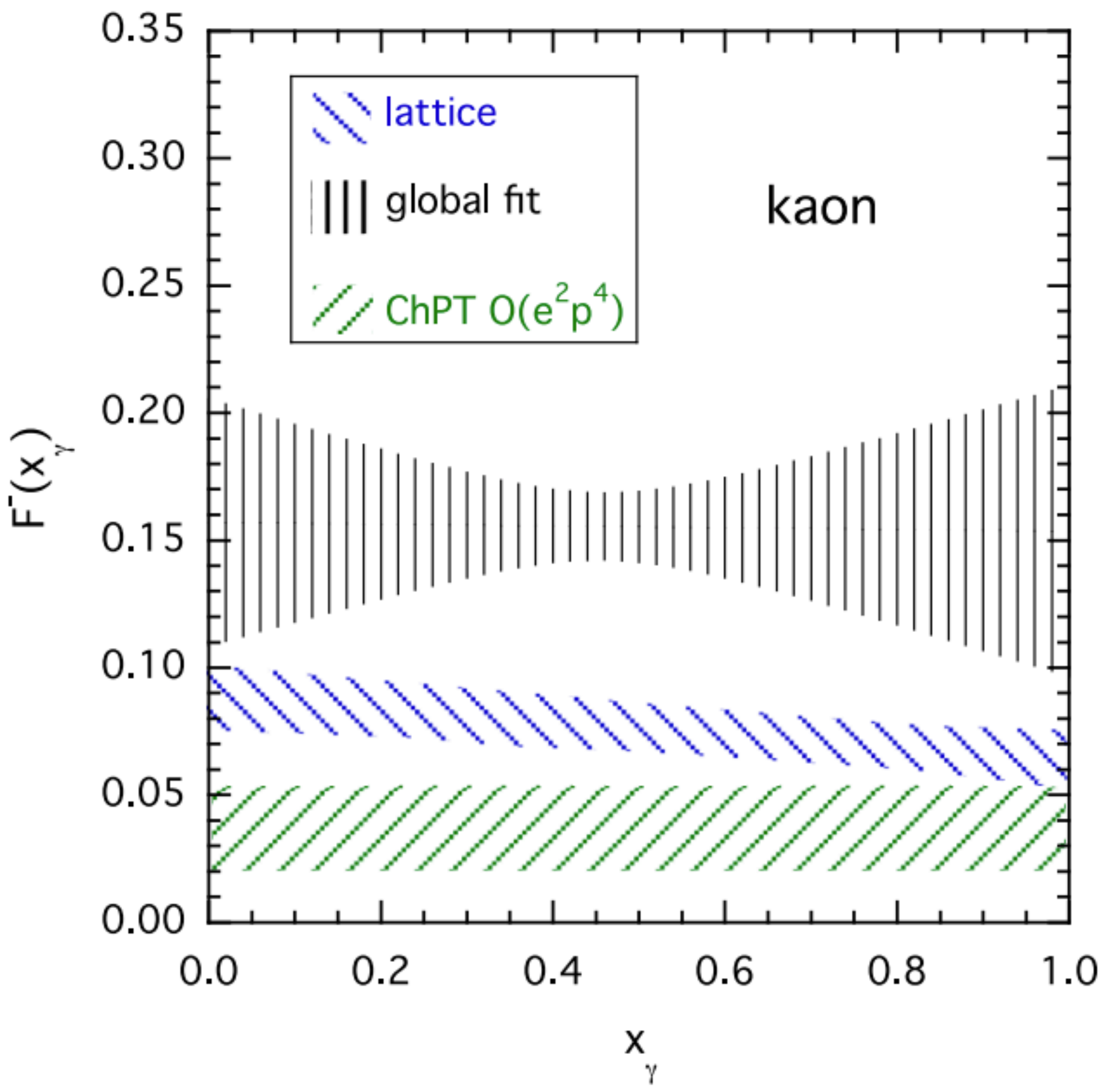
Restricted to less than
a half of phase space

Comparison of lattice results with experimental measurements for $P^+ = \pi^+, K^+$

[R. Frezzotti et Al arXiv:2012.02120 (2021)]

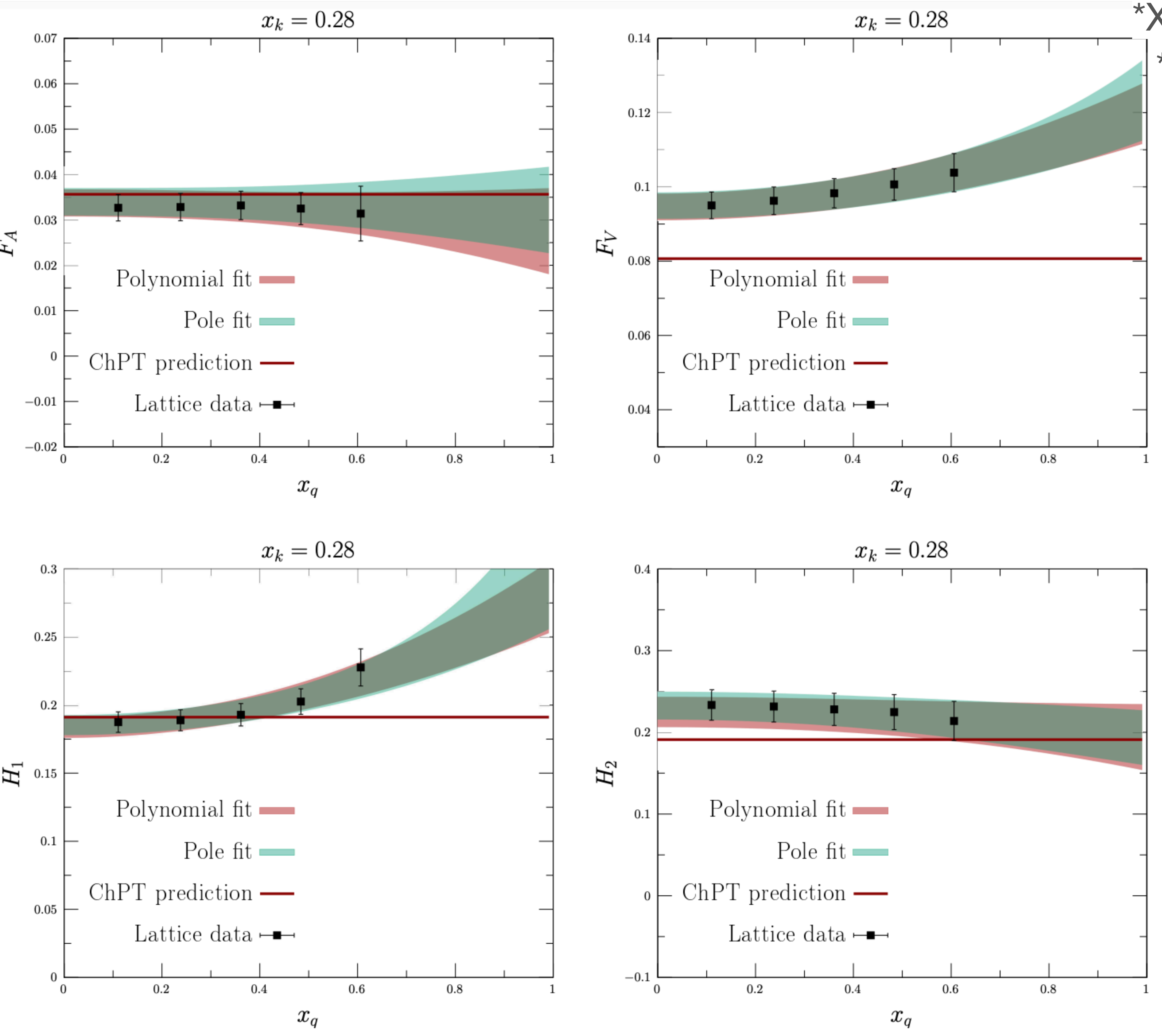
Interesting puzzles
are observed!





A look at the past: $K^+ \to \ell^+ \nu_\ell \ell'^+ \ell'^-$

Channels	our Lattice	Tuo et al.*	ChPT**	experiments
$\mathrm{Br}[K \to \mu\nu_\mu e^+ e^-]$	$8.26(13) \times 10^{-8}$	$10.59(33) \times 10^{-8}$	$9.8 - 8.2 \times 10^{-8}$	$7.93(33) \times 10^{-8}$***
$\mathrm{Br}[K \to e\nu_e \mu^+ \mu^-]$	$0.762(49) \times 10^{-8}$	$0.72(5) \times 10^{-8}$	$1.1 - 0.6 \times 10^{-8}$	$1.72(45) \times 10^{-8}$****
$\mathrm{Br}[K \to e\nu_e e^+ e^-]$	$1.95(11) \times 10^{-8}$	$1.77(16) \times 10^{-8}$	$3.4 - 1.7 \times 10^{-8}$	$2.91(23) \times 10^{-8}$***
$\mathrm{Br}[K \to \mu\nu_\mu \mu^+ \mu^-]$	$1.178(35) \times 10^{-8}$	$1.45(6) \times 10^{-8}$	$1.5 - 1.1 \times 10^{-8}$	——



*X.Y. Tuo et Al arXiv:2103.11331 (2022).

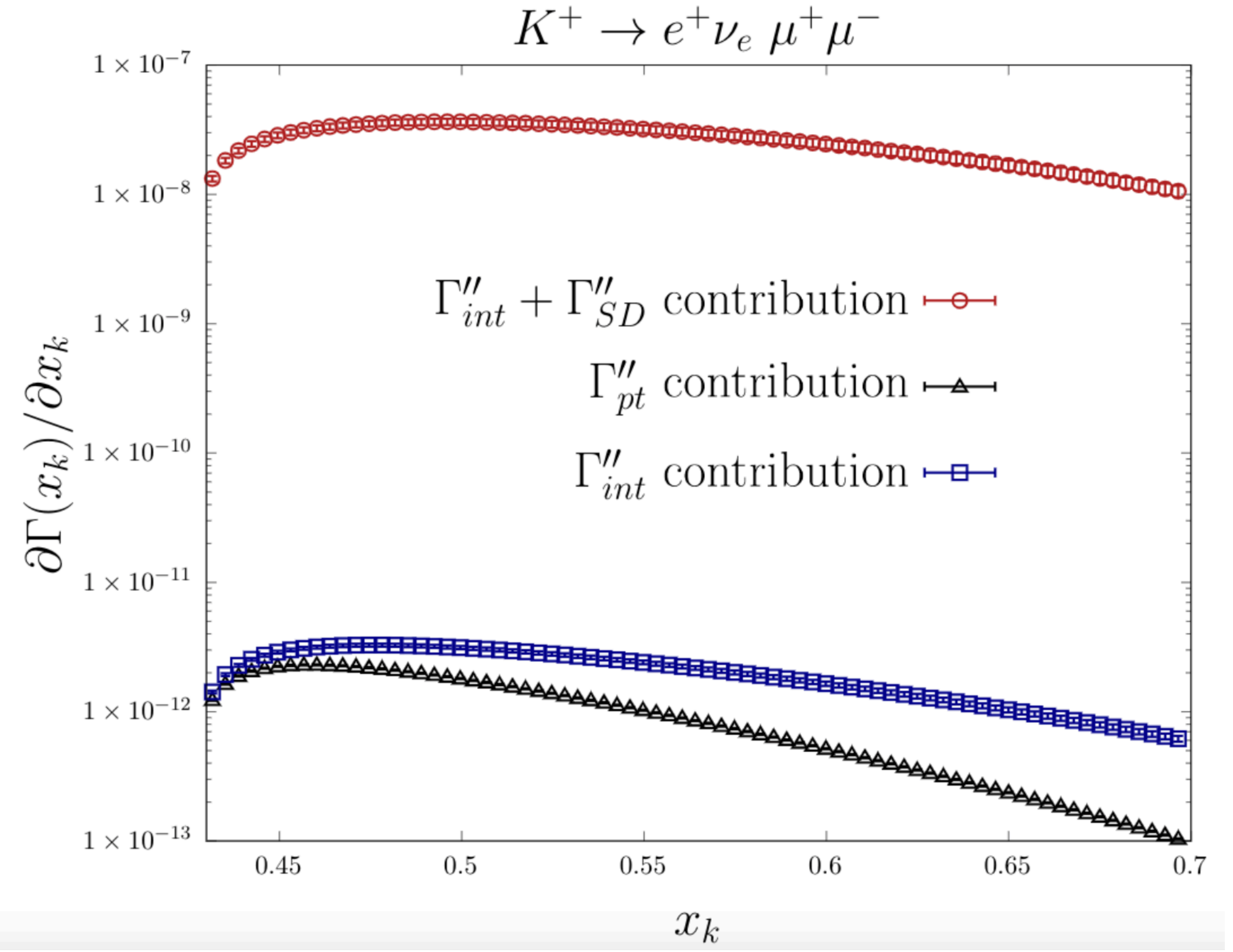
**J. Bijnens et Al arXiv:9411311 (1993)

*** A. A. Poblaguev et Al arXiv: 0204006 (2002)

**** H. Ma et Al arXiv:0505011(2006)

(based on one gauge ensemble only)

[G.Gagliardi et Al arXiv:2202.03833 (2022)]



(Very soon on ArXiv, stay tuned!)

$D_s^+ \to \ell^+ \nu_\ell \gamma$ new results

ETMC ensembles at physical pion mass

ensemble	β	V/a^4	a (fm)	M_π (MeV)	L (fm)
cA211.12.48	1.726	$48^3 \cdot 128$	0.09075 (54)	174.5 (1.1)	4.36
cB211.072.64	1.778	$64^3 \cdot 128$	0.07957 (13)	140.2 (0.2)	5.09
cB211.072.96	1.778	$96^3 \cdot 192$	0.07957 (13)	140.2 (0.2)	7.64
cC211.060.80	1.836	$80^3 \cdot 160$	0.06821 (13)	136.7 (0.2)	5.46
cD211.054.96	1.900	$96^3 \cdot 192$	0.05692 (12)	140.8 (0.2)	5.46

- 2+1+1 dynamical Wilson-Clover twisted mass fermions
- Electro-quenched approximation
- 11 momentum configurations via twisted boundary conditions covering the whole phase space

$D_s^+ \to \ell^+ \nu_\ell \gamma$

$$x_\gamma = \frac{2p \cdot k}{m_{D_s}^2} \to \frac{2E_\gamma}{m_{D_s}}$$

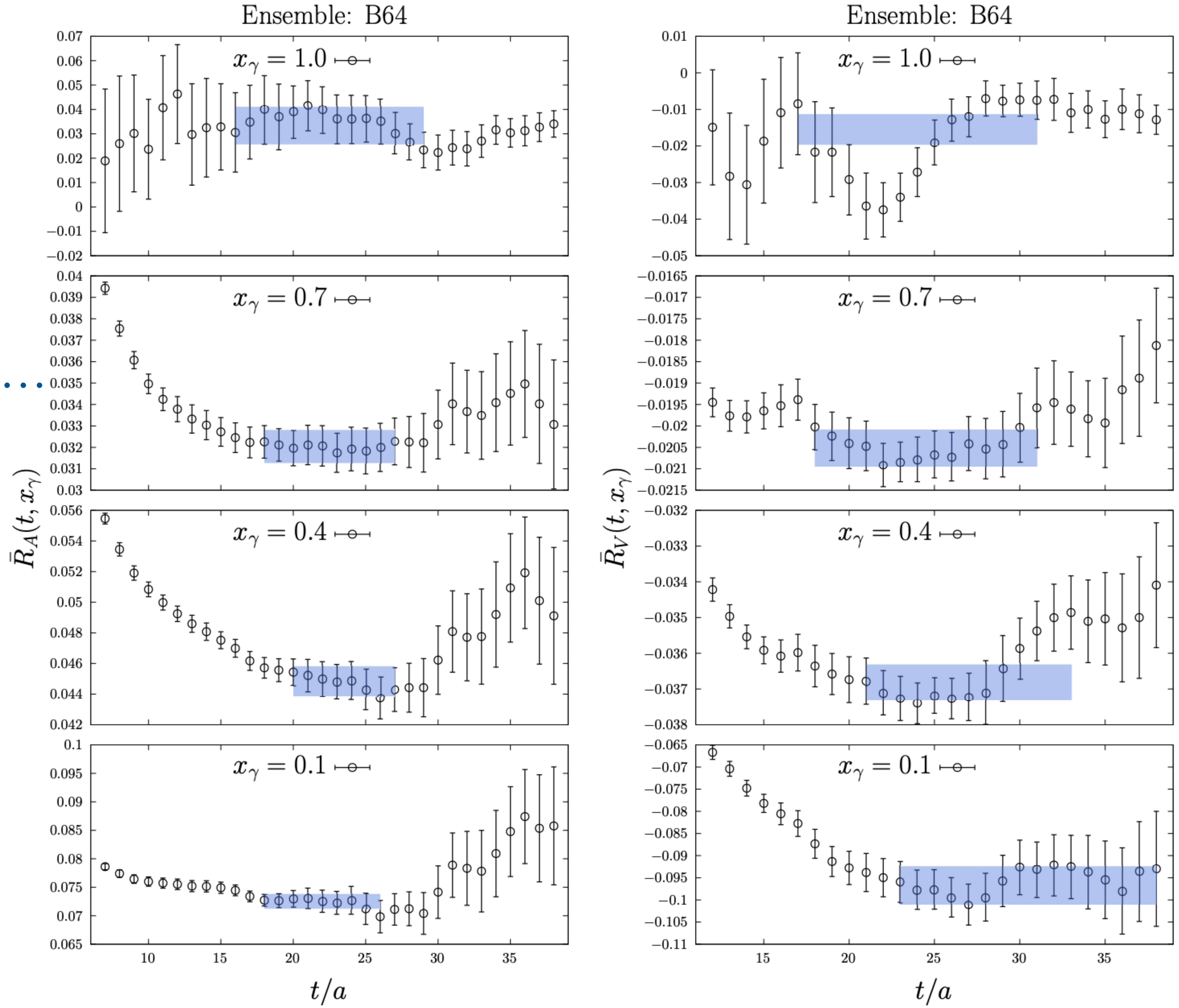
Deterioration of the signal for large $x_\gamma \geq 0.8$

Statistical error

$$\sigma_{R_W^{\mu\nu}}(t, \boldsymbol{k}, 0) = \frac{B_W^{\mu\nu}}{|E_\gamma - M_{\bar{q}q}^{\rm PS}|} \exp\{(E_\gamma - M_{\bar{q}q}^{\rm PS})(T/2 - t)\} + \ldots$$

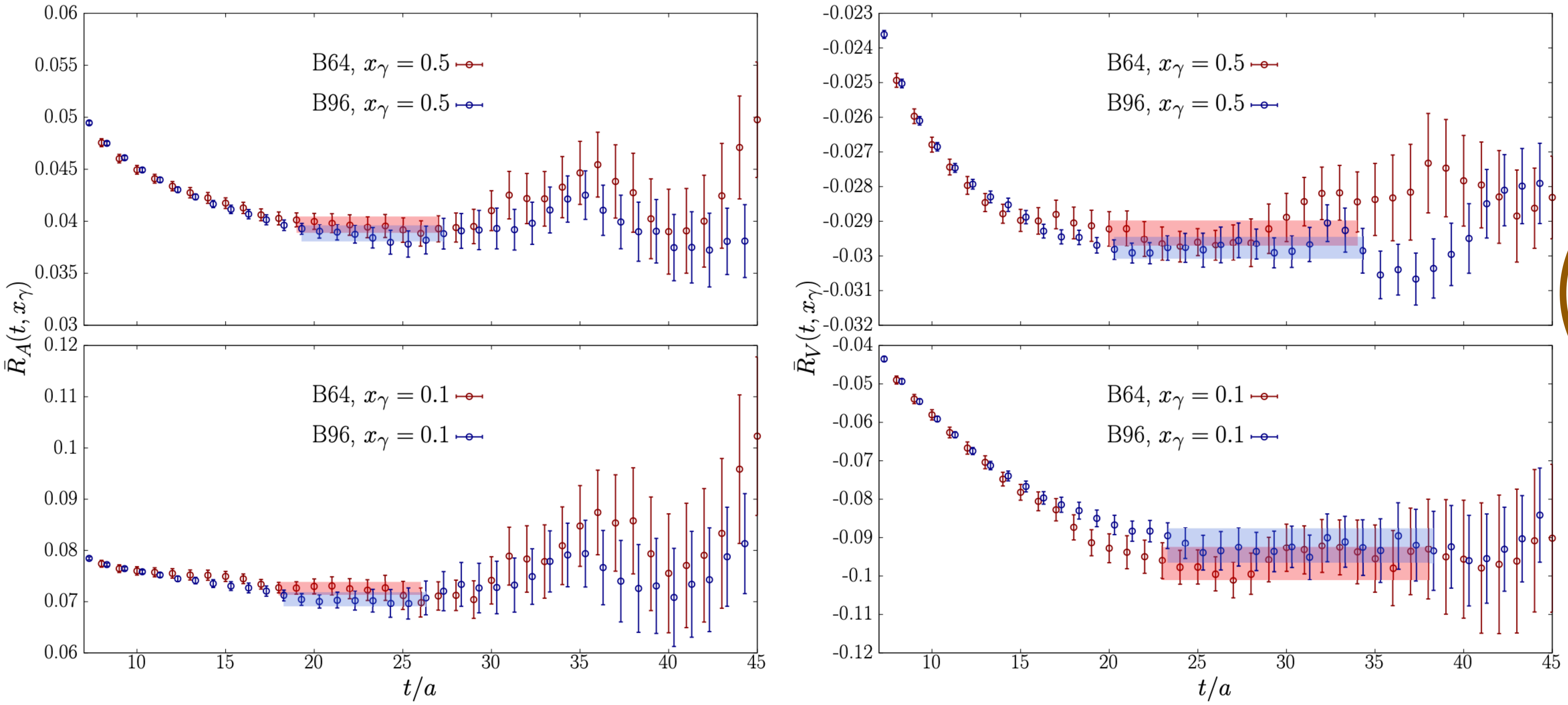
$M_{\bar{q}q}^{\rm PS}$ from lightest pseudoscalar $\bar{q}\gamma^5 q$state $\rightarrow$ $M_{\eta_{ss'}} \simeq 0.69$ GeV, $x_\gamma^{th} \simeq 0.7$

- Enhanced by large T
- Lower x_γ threshold for D and B
- 3d method can help (D. Giusti et Al ArXiv:2302.01298 (2023).)



$D_s^+ \to \ell^+ \nu_\ell \gamma$ **FVEs**

$$\frac{\sigma_W^{\rm FSE}(x_\gamma)}{F_W(x_\gamma)} = \left|\frac{\Delta F_W^L(x_\gamma)}{F_W(x_\gamma, \mathrm{B64})}\right| \mathrm{erf}\left(\frac{\Delta F_W^L(x_\gamma)}{\sqrt{2}\sigma_W^{\rm comb}(x_\gamma)}\right)$$

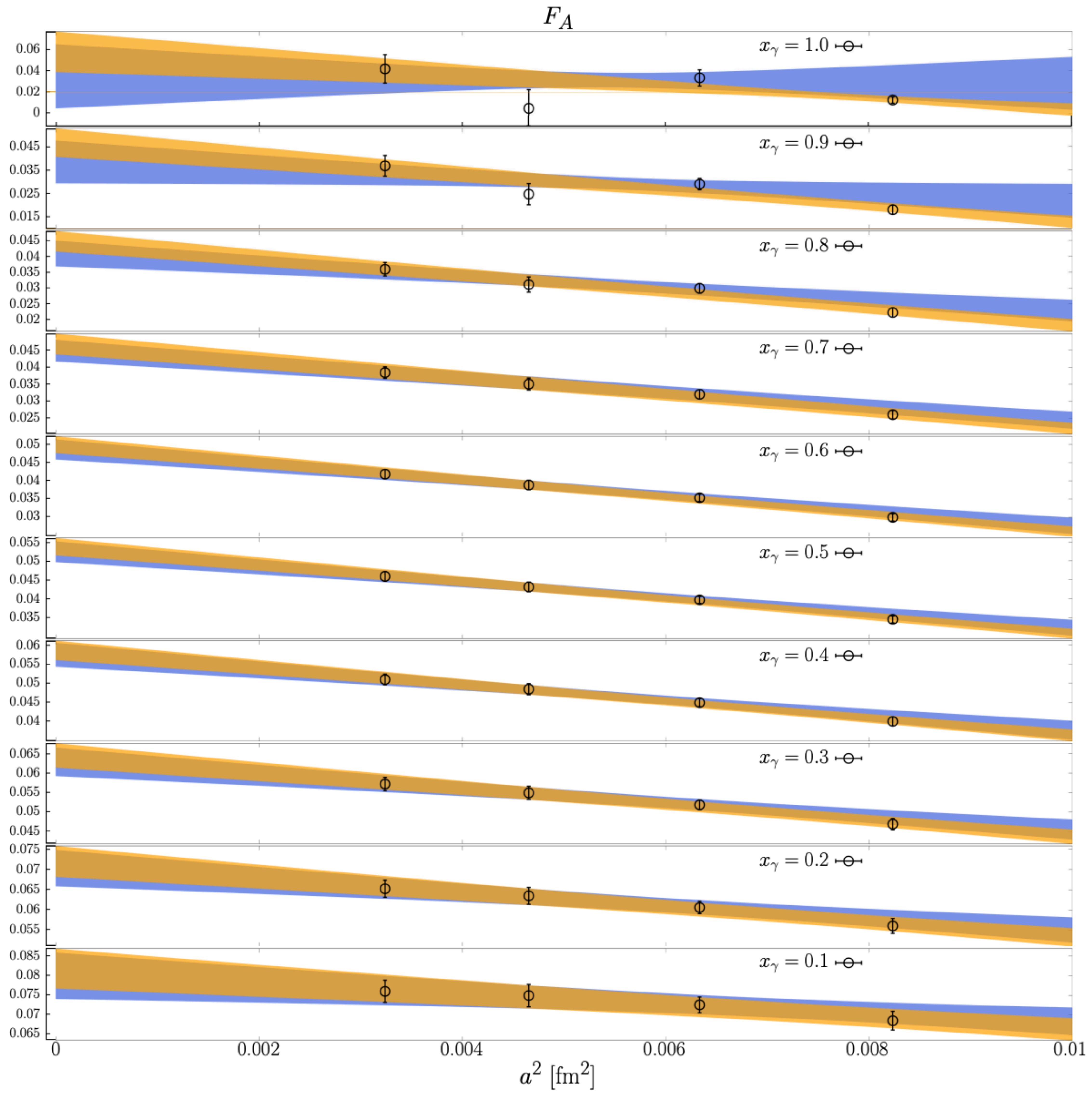


almost no dependence on L

$D_s^+ \to \ell^+ \nu_\ell \gamma$ continuum extrapolation

$$F_W(x_\gamma, a) = F_W(x_\gamma)\left(1 + D_W(x_\gamma)(a\Lambda)^2 + D_{2,W}(x_\gamma)(a\Lambda)^4\right)$$

Tested, but leads to overfit, set equal to zero



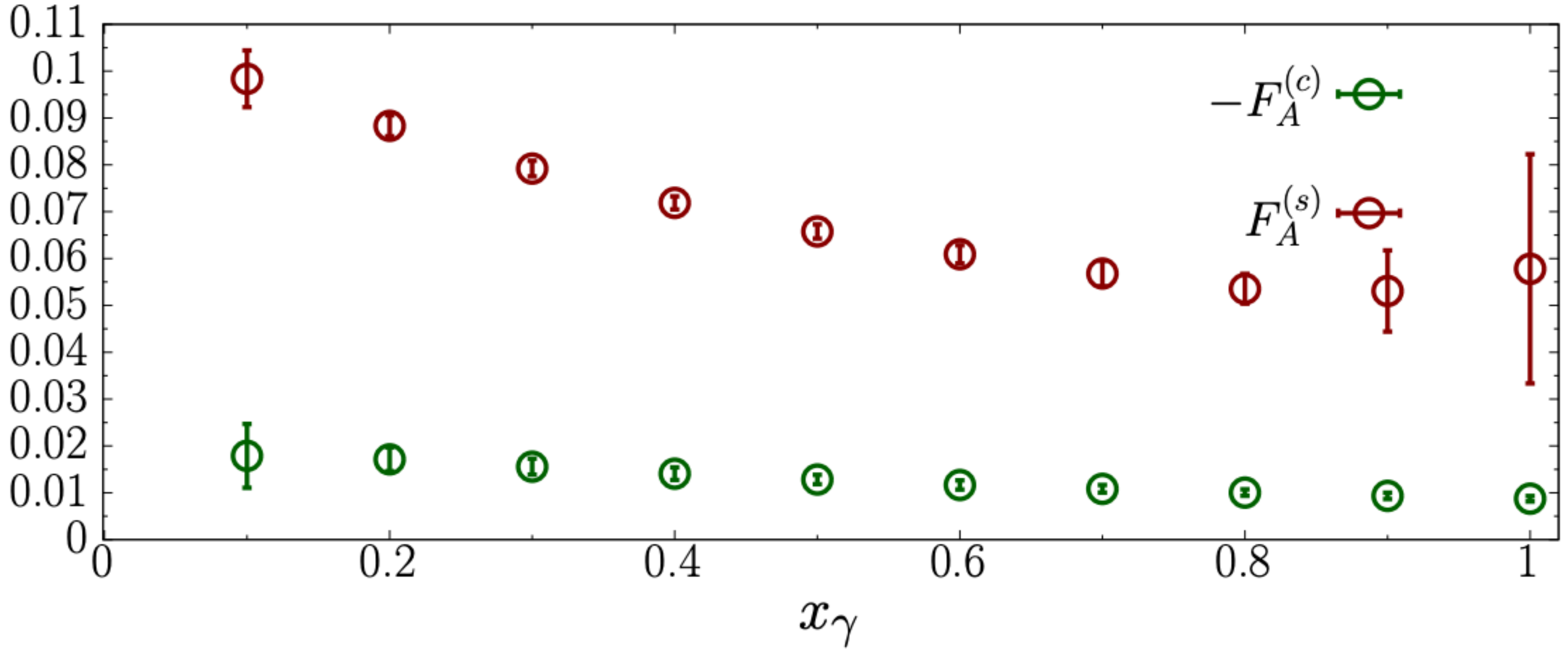
- systematic effects estimated by extrapolating with or without ens A48 (coarsest),

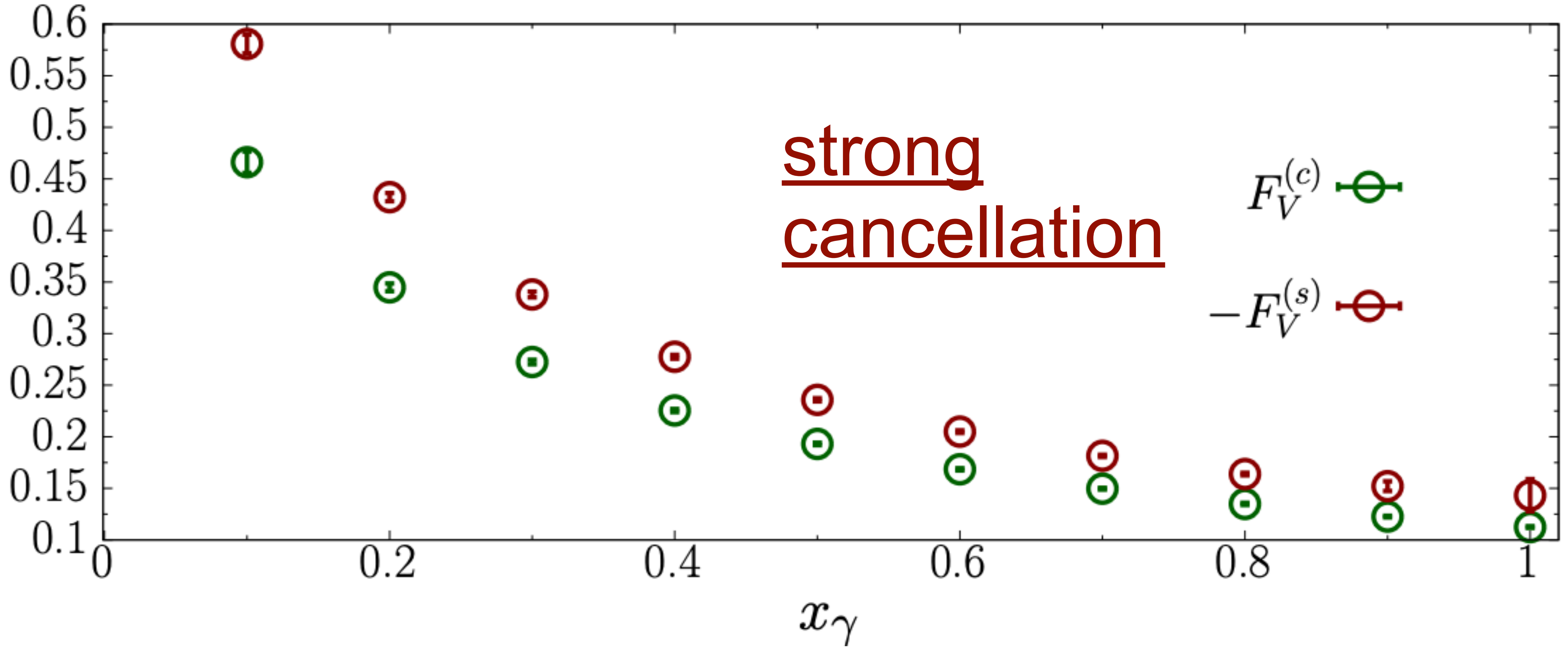
$$\bar{f} = w_A\, f_A + w_B\, f_B, \qquad w_A + w_B = 1\ .$$

$$\sigma^2_{\text{syst}} = \sum_{i=A,B} w_i\,(f_i - \bar{f})^2\ .$$

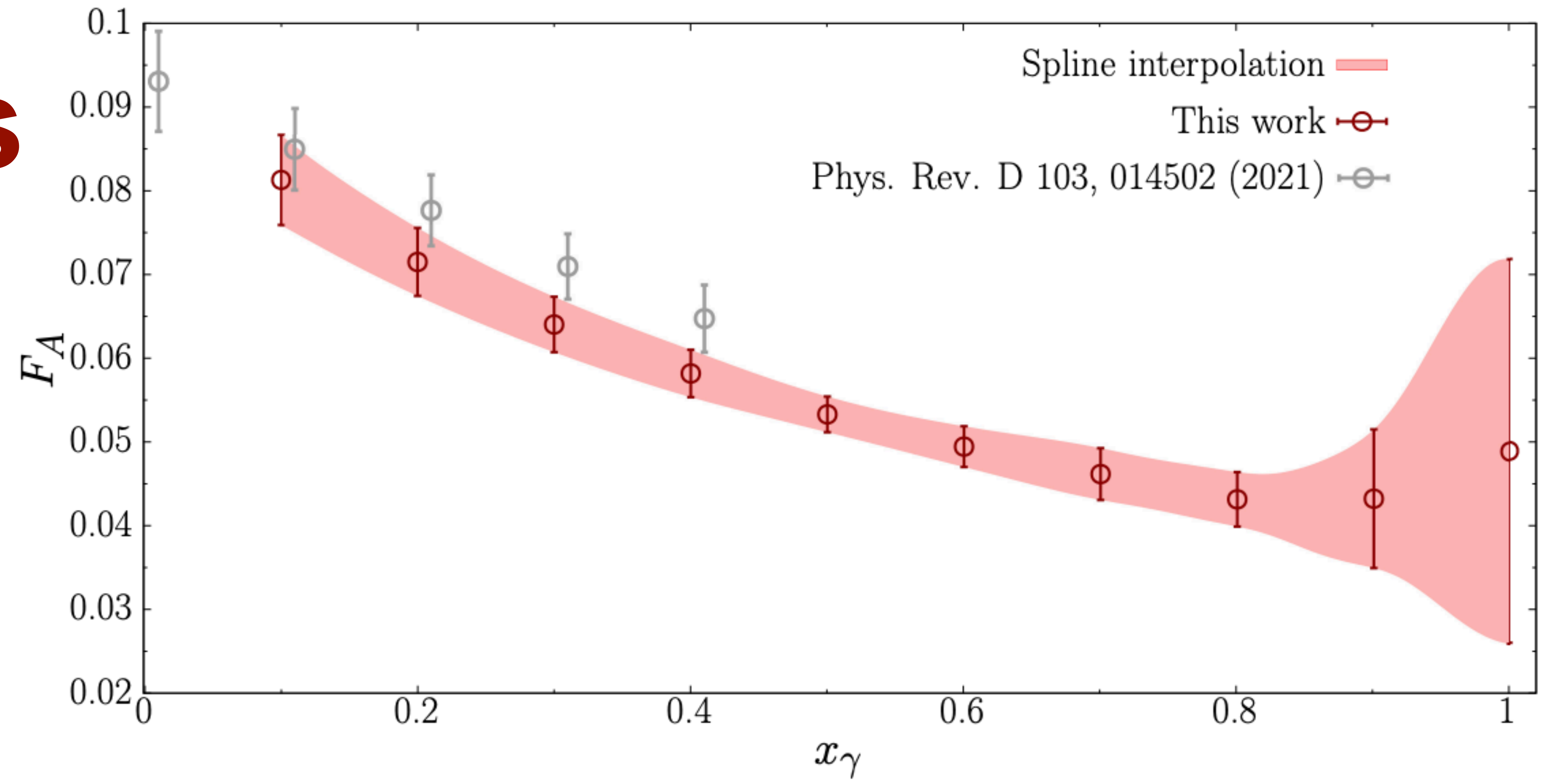
$$w_i \propto e^{-\left(\chi_i^2 + 2N_{\text{pars}}^{(i)} - 2N_{\text{data}}^{(i)}\right)/2}\ ,$$

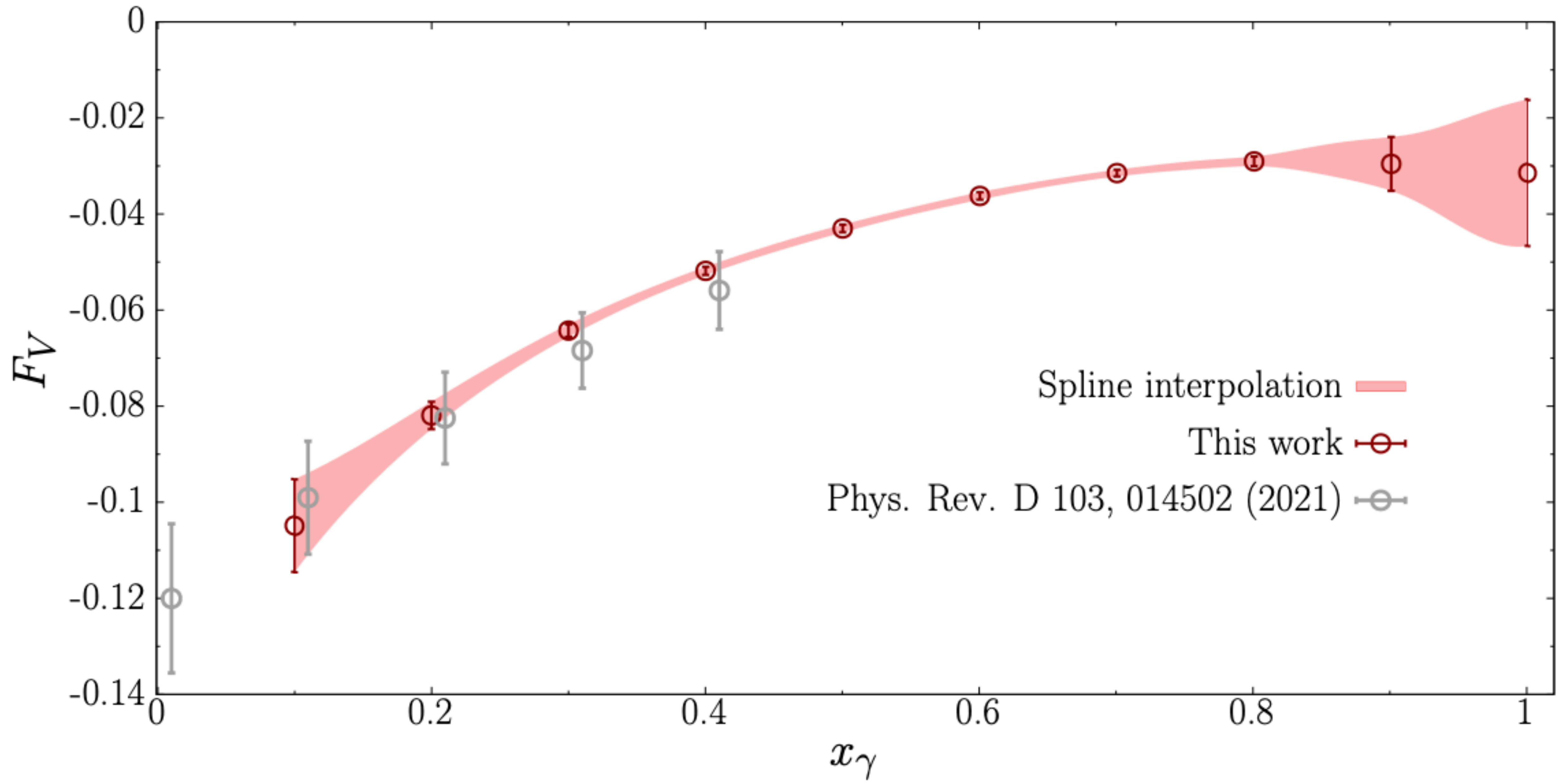
$D_s^+ \to \ell^+ \nu_\ell \gamma$ form factors





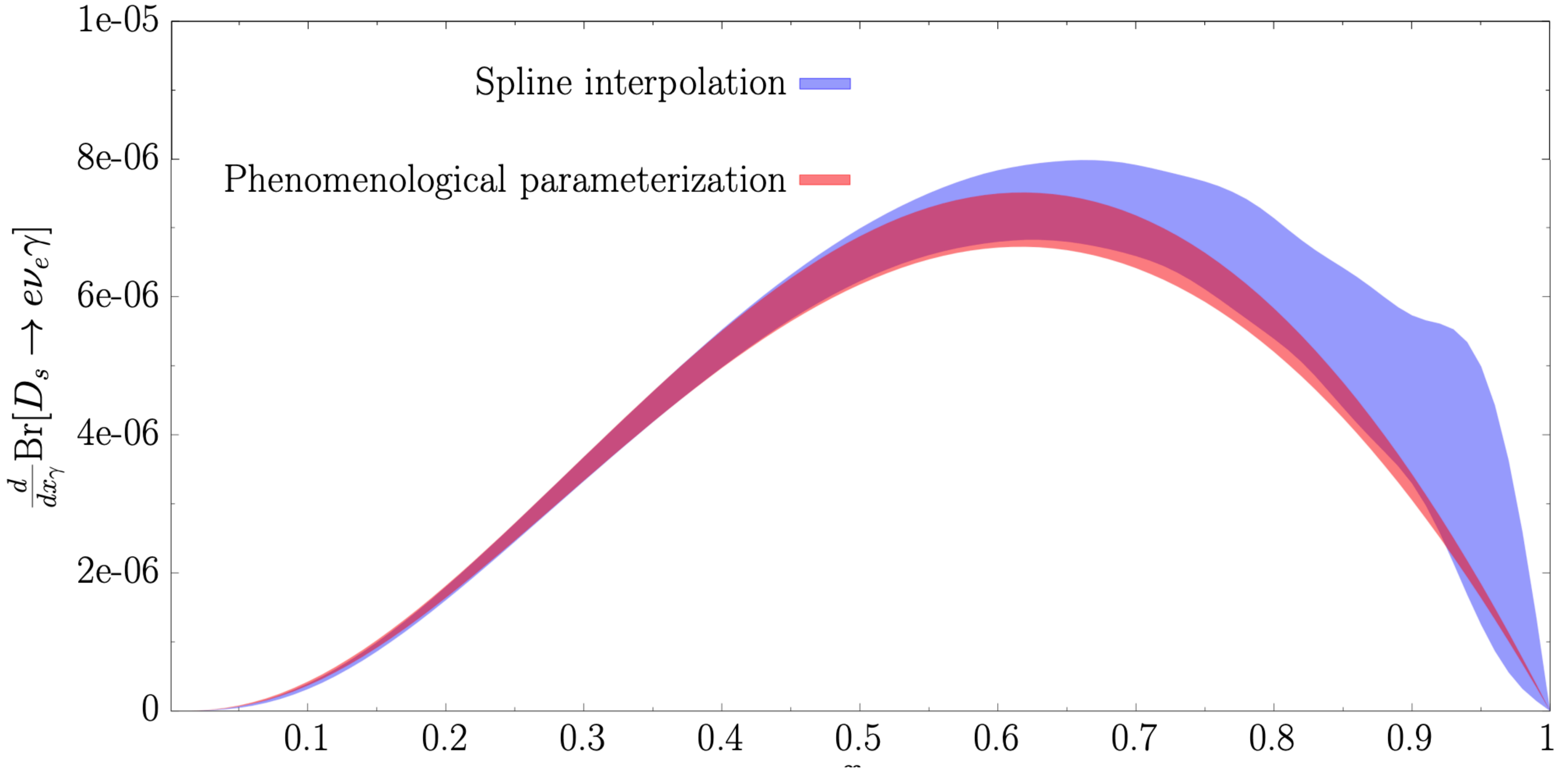
A separate study of the charm and strange contribution was suggested to us by R. Zwicky





$D_s^+ \to e^+ \nu_e \gamma$ decay rate

$$\frac{d\Gamma(D_s \to \ell\nu\gamma)}{dx_\gamma} = \frac{\alpha_{\rm em}}{4\pi}\Gamma^{(0)}\left\{\frac{dR^{\rm pt}}{dx_\gamma} + \frac{dR^{\rm int}}{dx_\gamma} + \frac{dR^{\rm SD}}{dx_\gamma}\right\}$$



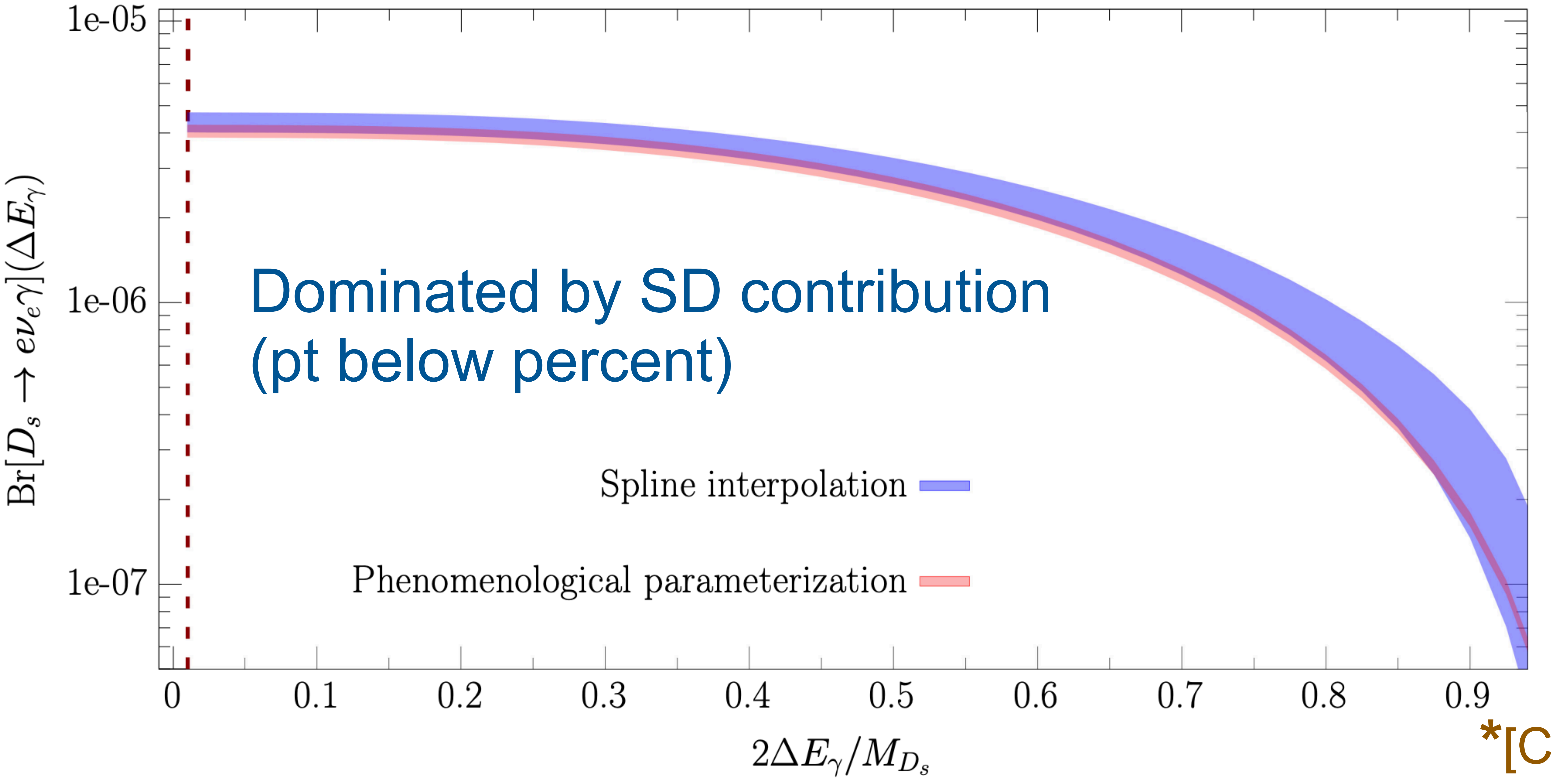
For final electron it dominates the rate

Large contributions from high values of x_γ

$D_s^+ \to e^+ \nu_e \gamma$ branching fraction

$\mathrm{Br}(\Delta E_\gamma = 10\ \mathrm{MeV}) = 4.4(3) \times 10^{-6} \ll 1.3 \times 10^{-4}$

BESIII exp upper bound



Quark Model Predictions
$10^{-5} - 10^{-4}$ *

pQCD+HQEFT predictions
10^{-3} **

*[C.Q. Geng et Al ArXiv:0012066 (2000)] and [C.D. Lu et Al ArXiv:0212363 (2003)]

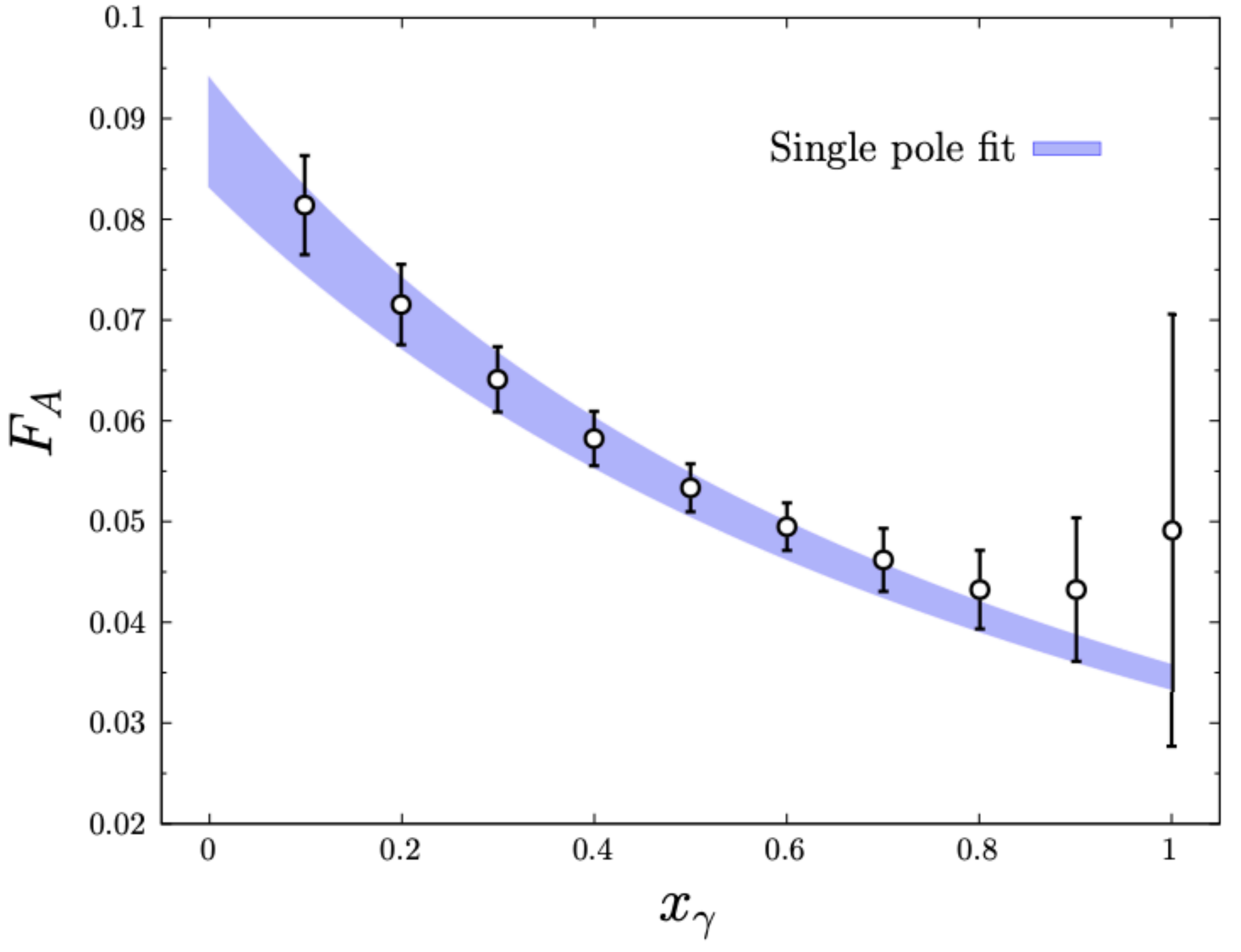
**[G.P. Korchemskyet Al ArXiv:9911427 (2000)]

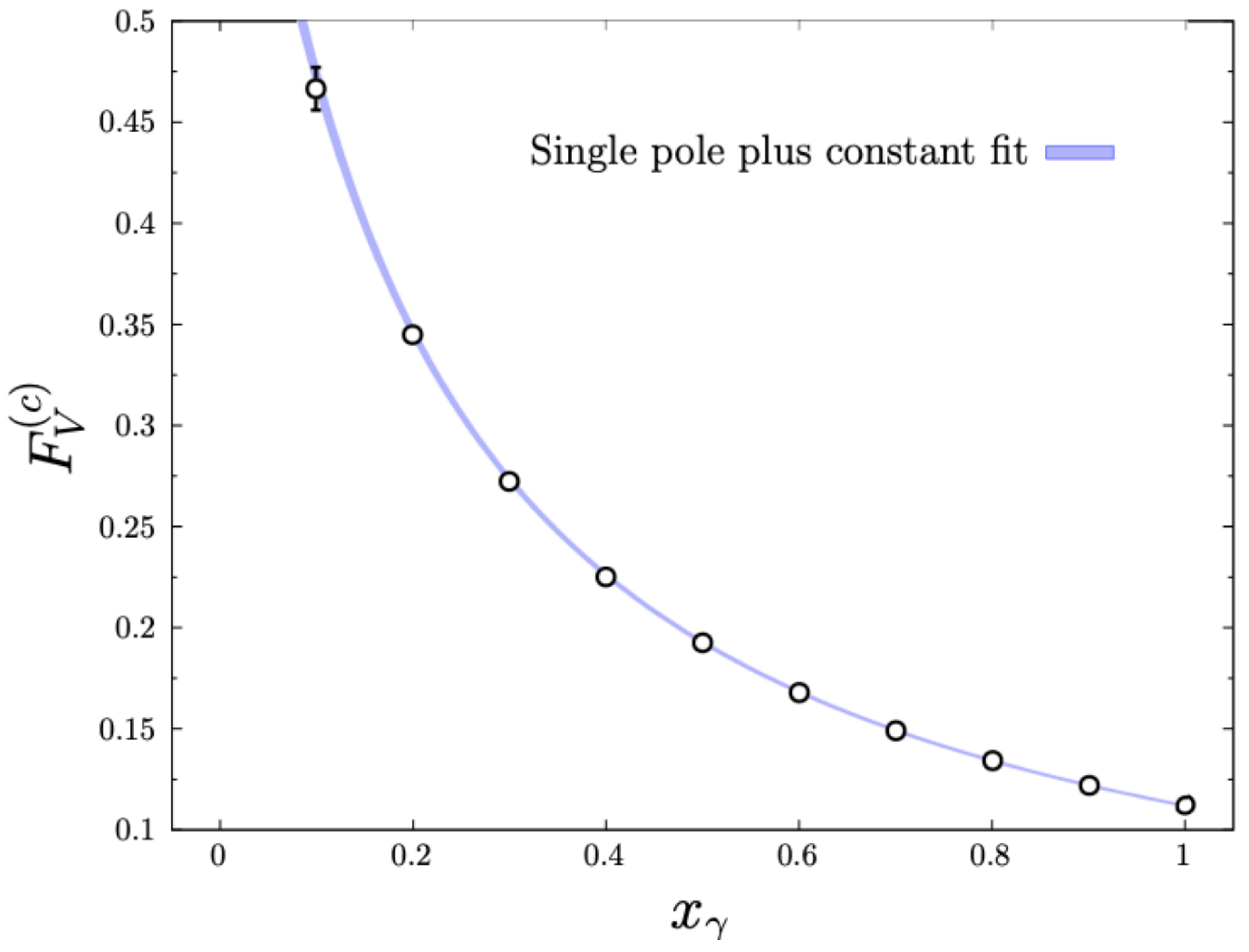
Form Factors Phenomenological Parametrization

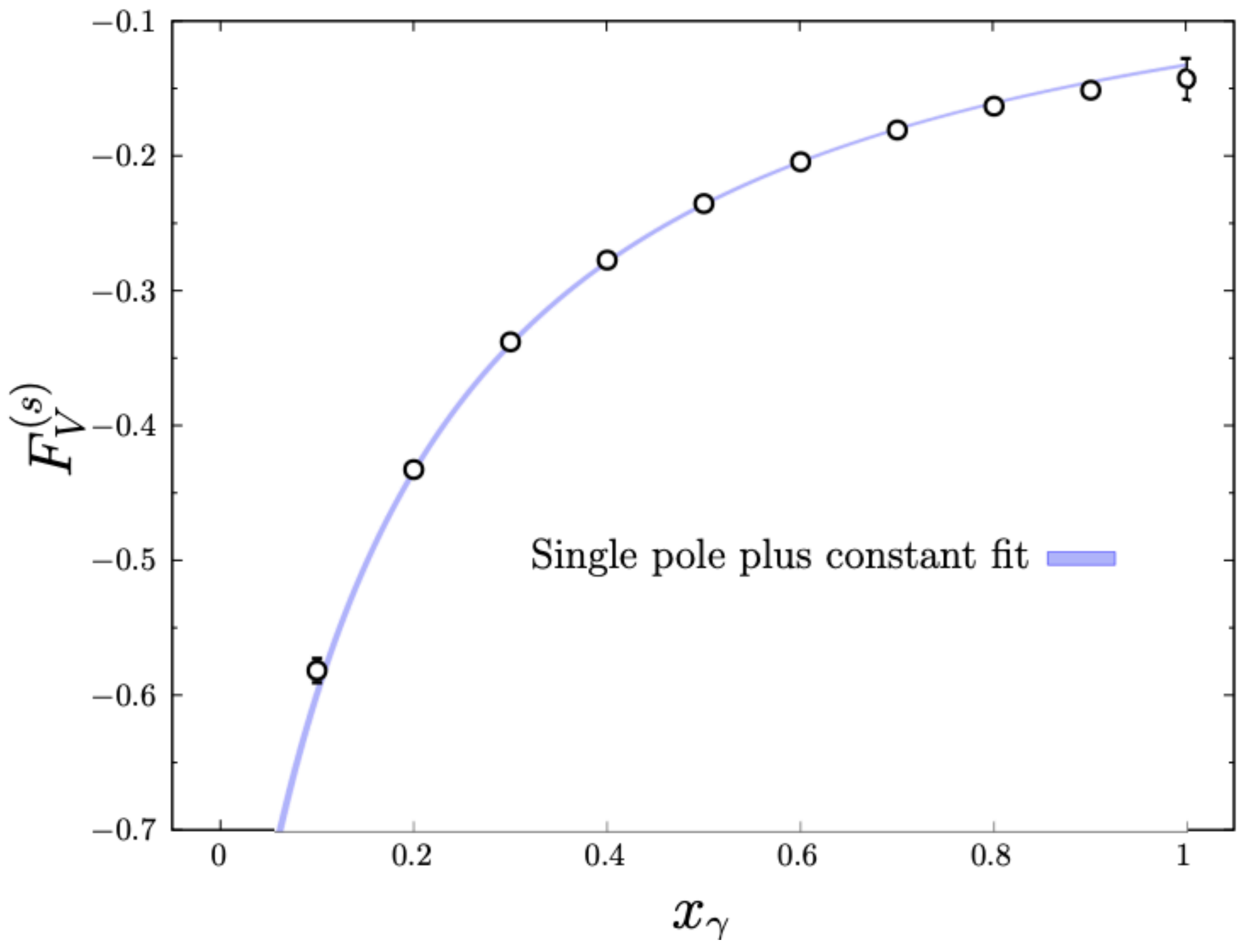
$$F_W(x_\gamma) = \frac{C_W}{\sqrt{R_W^2 + \frac{x_\gamma^2}{4}}\left(\sqrt{R_W^2 + \frac{x_\gamma^2}{4}} + \frac{x_\gamma}{2} - 1\right)}$$

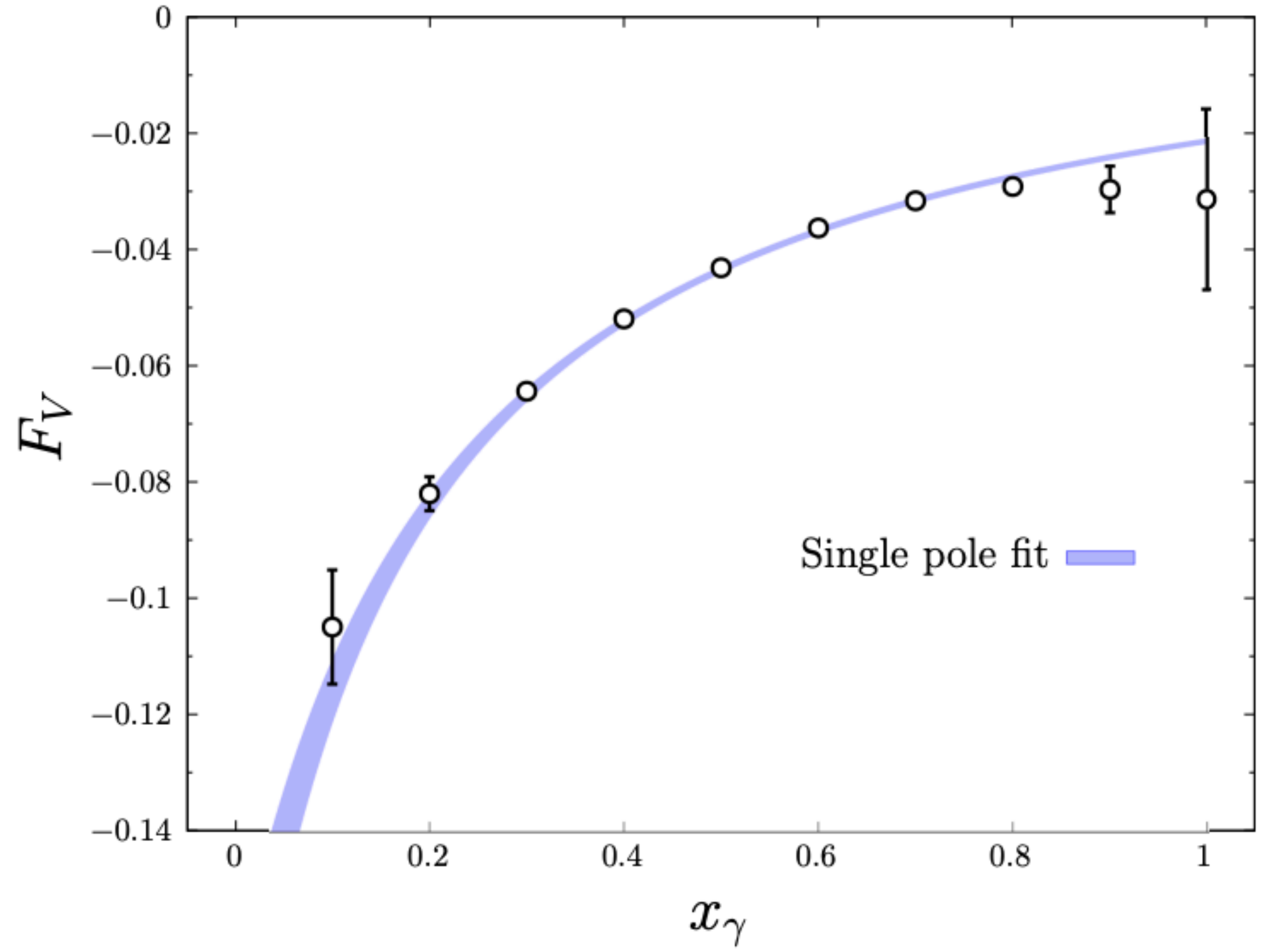
Vector Meson Dominance Inspired!

- $R_V \simeq R_{D_s^*} \quad R_A \neq R_{D_{s1}}$ Vector channel characterized by near, isolated resonance









$$F_W(x_\gamma) = \frac{C_W}{\sqrt{R_W^2 + \frac{x_\gamma^2}{4}}\left(\sqrt{R_W^2 + \frac{x_\gamma^2}{4}} + \frac{x_\gamma}{2} - 1\right)} + B_W$$

However constant background correction is needed to parametrize F_V individual contribution (it cancels in their sum)

Relating $F_V \to g_{D_s D_s^* \gamma}$

$$= -\frac{M_{D_s^*} f_{D_s^*} g_{D_s^* D_s \gamma}}{2 M_{Ds}}$$

$$F_V(x_\gamma) = \frac{C_V}{\sqrt{R_{D_s^*}^2 + \frac{x_\gamma^2}{4}} \left(\sqrt{R_{D_s^*}^2 + \frac{x_\gamma^2}{4}} + \frac{x_\gamma}{2} - 1 \right)}$$

Properly reproduced by lattice data

$\frac{R_V - R_{D_s^*}}{R_{D_s^*}} < 3\,\%$ 1.5σ compatibility

- Striking agreement with direct HPQCD calculation
- no compatibility with LCSR calculation, traced down to $g^{(s)}$

	LCSR *	HPQCD **	This work
$g_{D_s^* D_s \gamma}$ [GeV^{-1}]	0.60(19)	0.10(2)	0.118(13)
$g^{(s)}_{D_s^* D_s \gamma}$ [GeV^{-1}]	1.0	0.50(3)	0.532(15)
$g^{(c)}_{D_s^* D_s \gamma}$ [GeV^{-1}]	−0.4	−0.40(2)	−0.415(16)
$\frac{g^{(s)}}{g^{(c)}}$	−2.5	−1.25(10)	−1.282(61)

*[B. Pullin et Al ArXiv:2106.13617 (2021)]

**[G. C. Donald ArXiv:1312.5264 (2014)]

Conclusions

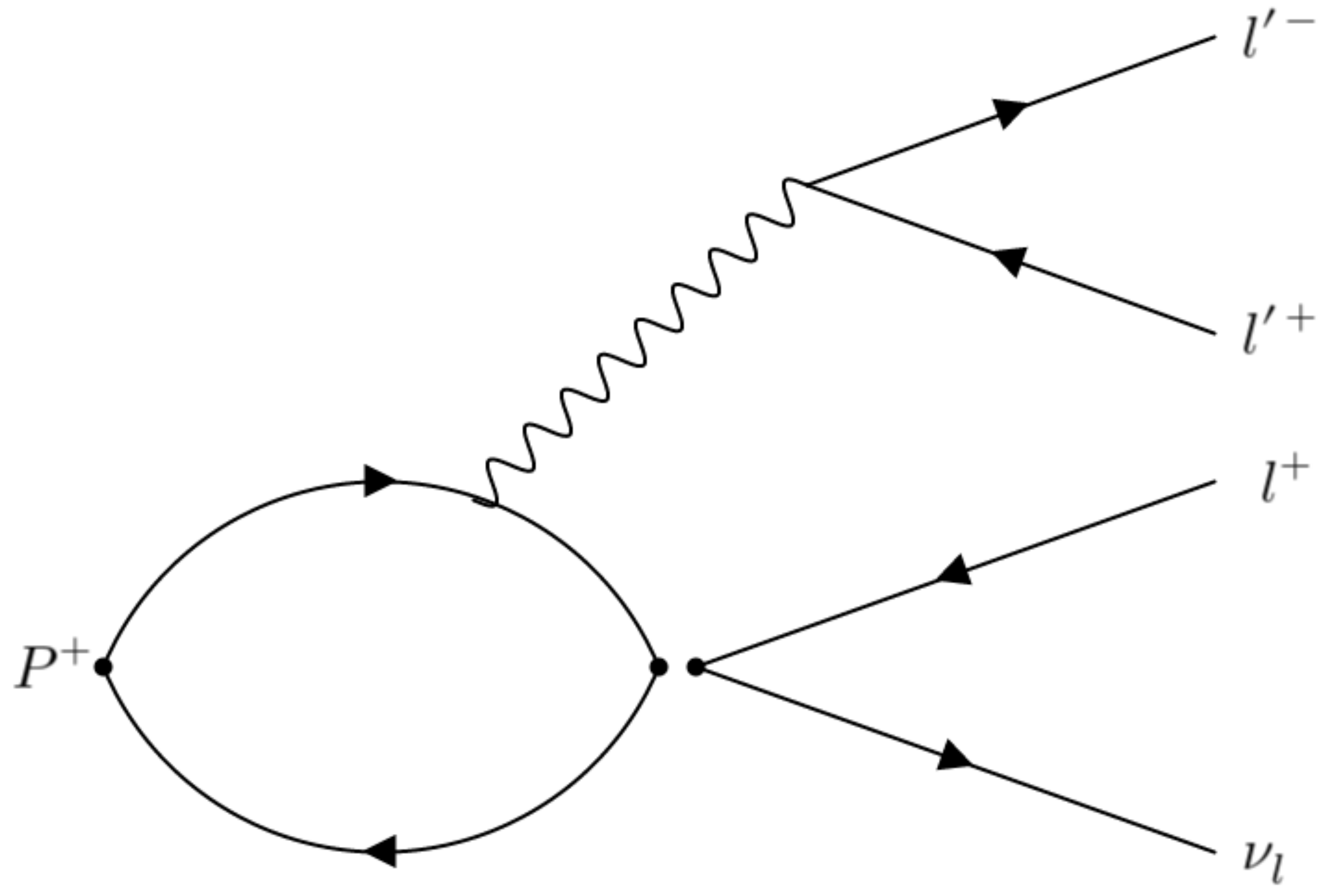
- High-precision, continuum-extrapolated lattice results for $D_s \to \ell \nu_\ell \gamma$ radiative form factors F_A and F_V over the whole kinematical range are provided

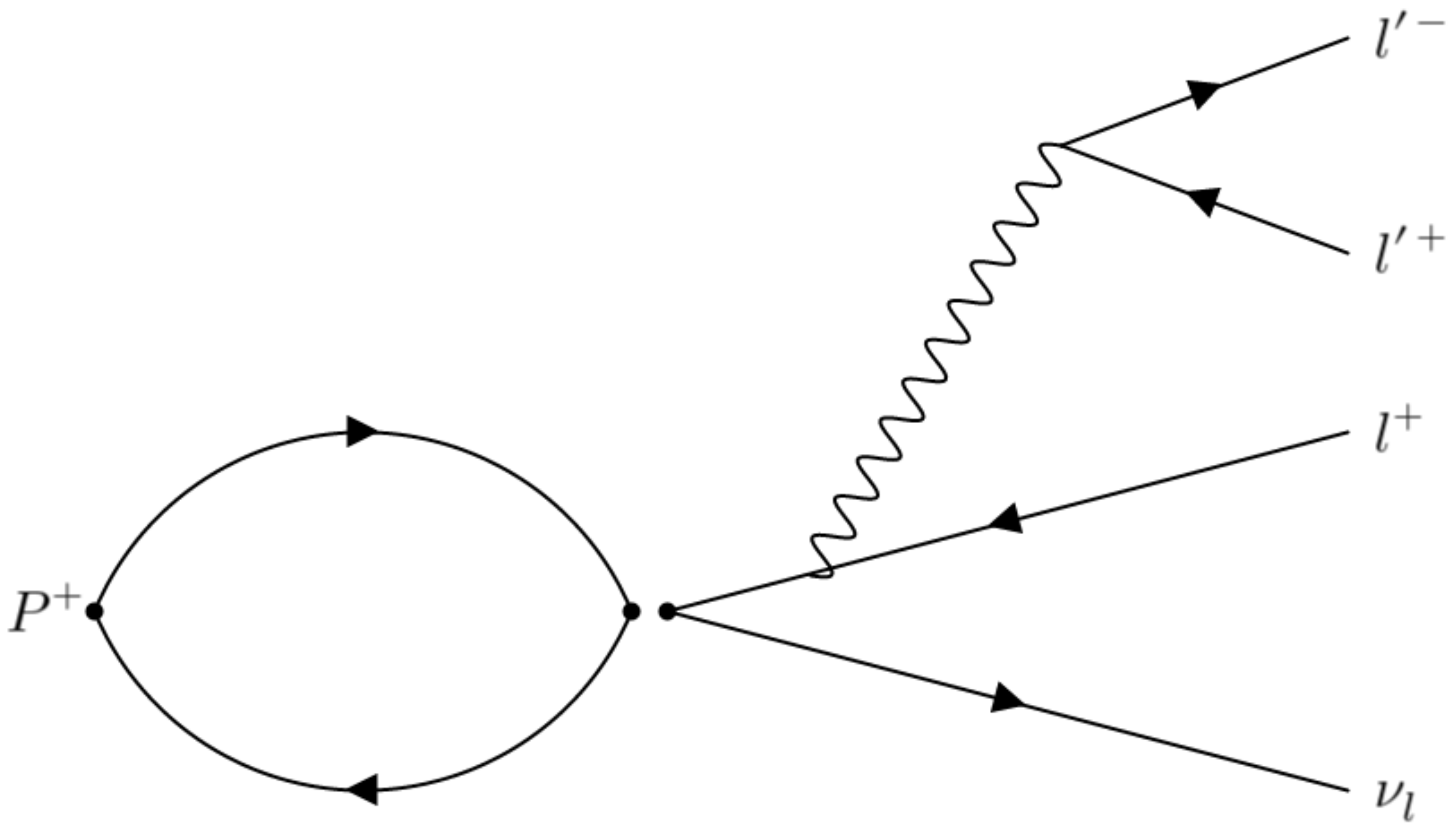
- For $\ell = e$ process is dominated by SD contribution → sensitive tests on NP are allowed!

- Branching fraction well below experimental upper bound from BESIII → improved experimental precisions is needed!

- Results are different with respect to previous calculations (LCSR, pQCD+HQEFT, quark models)

- Vector Meson Dominance parametrization has been checked → Valid only for total F_V !





Thanks for your attention!