ΔM_{hadron} from gauge (non)-invariant operators

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CENTRE FOR THEORETICAL PHYSICS: Higgs

mostly based on

Nabeebaccus, RZ	JHEP'22	2209.06925	Gauge invariant op.
Rowe, RZ	soon JHEP	2301.04972.	ΔM_{hadron}
Rowe, RZ	in preparation		$B, D \to \ell \nu$

Converging on QCD & QED - 29-30 May 2023 Edinburgh

Overview

I. Universality of soft & collinear IR-logs

- soft-divergences (easy ones)
- coll-divergences (more work)
- Thm on structire dependent collinear divergences

II. A gauge invariant interpolating operator

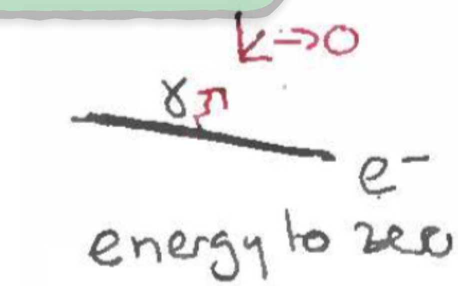
- Modified (non-local) LSZ factor
- When needed. and when not (and relation Dirac dressing)

III. $\Delta M = \Delta M_{QED} + \Delta M_{m_q}$

- Cottingham formula & Feynman Hellmann

Summary

Recap on IR sensitive terms for Rates



- d=4 IR-divergences are **logarithmic**:
  - **"soft"** photon momentum $k \to 0$ (trivial)
  - **"collinear"** photon momentum $k \propto p_{ex}$ (subtle)

    $\alpha \ln m_e/m_b$ can be 10-20% and are **physical effect**

- **Kinoshita-Lee-Nauenberg theorem** (1962)

  Total (decay) rates all **divergences** (IR-logs)

  cancel since physical observables are **finite**

- **Exceptions**:

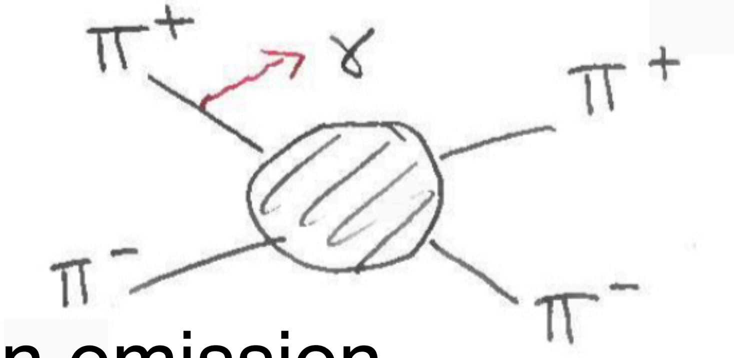
  ia) not photon-inclusive — **soft**+**coll**

  ib) differential not kinematic-inclusive — **coll**

  iii) $\Gamma \supset m_\ell^2 \ln m_\ell$ as finite cancelation not needed — **coll**

  example and exception: leptonic decays with V-A interactions

Soft-Logs: real emission first



- Low's theoremL E_γ-expansion for **"soft"** photon emission

- based on Ward identity (**Low**-Burnett-Kroll-Goldberger-Gell-Mann **thm)**

$$\langle \beta\gamma(k,\lambda)|S|\alpha\rangle = (J_\lambda^{(0)} + J_\lambda^{(1)})\langle\beta|S|\alpha\rangle + \mathcal{O}(E_\gamma)$$

$\mathcal{O}(E_\gamma^{-1})$ ← ; ↓ $\mathcal{O}(E_\gamma^{0})$; ↘

$$J_\lambda^{(0)} = \sum_j \hat{Q}_j \frac{\epsilon^*(k,\lambda)\cdot\hat{p}_j}{k\cdot\hat{p}_j - i0}$$

"charge conservation"

$$J_\lambda^{(1)} = -i\sum_j \hat{Q}_j \frac{\epsilon_\mu^*(k,\lambda)k_\nu J_j^{\mu\nu}}{k\cdot\hat{p}_j - i0}$$

"angular momentum"

non-universal,
structure dependent
resolving hadron

$$\int_{m_\gamma}^{\Delta E} dE_\gamma E_\gamma\left(\frac{1}{E_\gamma^2} + \ldots\right) = \ln\Delta E - \ln m_\gamma$$

$\ln\Delta E$ → residual effect (detector resolution ΔE)

$\ln m_\gamma$ → IR-divergence (regulated) cancels against virtual

Soft-Logs: virtual

- **KLN-thm (or Bloch-Nordsieck 1937)**:
  1) real and virtual soft have to **cancel**.
  2) they do so **point by point** in phase space
  3) textbooks done at diagrammatic level of scalar QED

Beyond scalar QED,/ pt-like
resolving hadrons

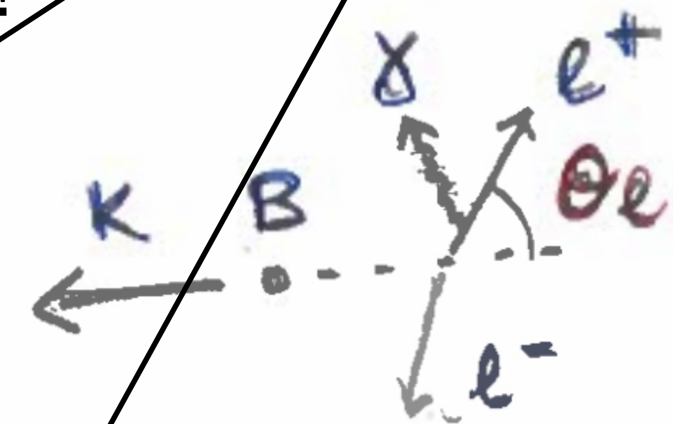
- **Structure-dependent level?**
  Nothing new as soft-divergences do not resolve hadrons

``soft-logs are (relatively) easy"

Collinear-Logs:

hard-coll logs are not easy - **unclear aspects**

- **New elements:** $p_\mu = (E,0,0,E)$ and E can be large!
  - cancellation not point by point
    $\Rightarrow$ not at diff-level in general
  - there are **IR-safe** and **IR-non-safe** kinematics
    Nabeebaccus, Isidori, RZ JHEP 2020 2009.00929



Experiment: e.g. LHCb for R_K cannot always choose

- **Scalar QED:** can be computed and infer as above. All good?

- **Structure dependent level?** hard as not universal
  - since scalar QED logs up 10-20% in $B \to Kee$, $\Delta R_K = \mathcal{O}(10\%)$
    $\Rightarrow$ have to take seriously if large and small scales

however, one can show

Show: coll-logs are universal (with KLN)

eikonal part

1) Real emission in scalar QED: $\mathscr{A} = \hat{Q}_{\ell_1} \frac{a_{\ell_1}}{\ell_1 \cdot k} + \delta\mathscr{A}$

$$\int_\gamma |\mathscr{A}|^2 \propto \int_\gamma \hat{Q}^2_{\ell_1} \left| \frac{a_{\ell_1}}{\ell_1 \cdot k} \right|^2 + \hat{Q}_{\ell_1} \frac{2Re[\delta\mathscr{A} a_{\ell_1}]}{\ell_1 \cdot k} + |\delta\mathscr{A}|^2$$

coll-logs: $O(1) Q^2_{\ell_1} \ln m_{\ell_1}$ **coll-safe*** **coll-safe**

* a) by gauge invariance $\mathscr{A} = \epsilon^\mu \mathscr{A}_\mu \Rightarrow k \cdot \mathscr{A} = 0$
b) in collinear region $\ell_1 \propto k \Rightarrow \cdot \ell_1 \cdot \mathscr{A} = \mathcal{O}(m^2_{\ell_1})$

``the trick"

2) Hence $\delta A \to \delta A + A^{B,K}_{structure}$, no new real collinear logs

3) Since real & virtual cancel (in IR-safe kinematics), by **KLN-thm**
$\Rightarrow$ no new virtual collinear logs either

Gauge invariance acts as **custodian** that sweeps away all the ``**dangerous" hc logs** beyond pt-like app.

Mini-summary:

Gauge invariance controls IR-logs

1) **soft-logs**: no problem

2) **coll-logs**: more subtle - no structure dep. if KLN applies

when does KLN-thm not apply?

- When LO amplitude is chirally suppressed:
  e.g. $\mathscr{A}_{P\to\ell\nu} \propto m_\ell$ for V-A interaction ⇒ it's interesting!*
  N.B. $\mathscr{A}_{P\to\ell\nu} \propto \mathcal{O}(1)$ for S-P interaction (Yukawa) no further coll-logs

- Described in notes 2205.06194 RZ & **relation to splitting function**
  applied J/Ψ-resonance in $B \to Kee$ Isidori, Lancierini, Nabeebaccus, RZ 2205.08635

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* seen for $B_s \to \ell\ell$ in Beneke, Bobeth, Szafron'17 (thuogh they do not think in this way…)

The problem: requires interpolating operator

- Standard operator **not gauge invariant** *

$$J_B \equiv \bar{u}\gamma_5 b \to e^{i\lambda Q_B}\bar{u}\gamma_5 b \qquad A \to A + \partial\lambda$$

- Lattice cancel gauge dependence $t_E \to \infty$ (sufficiently large)

$$Z_B(gauge) \times \text{amplitude} \times e^{-E_B t_E} + \ldots$$

- Continuum: rely quark hadron duality with no simple factorisation

(1) **universal IR-logs not reproduced** in $P \to \ell\nu$ (pert. QCD/QED at least)

(2) **observable IR safe** ΔM we can get away with it

Quark-hadron duality & IR-logs do not commute

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* In real emission, that is for the $B^+ \to \gamma$ form factor, we were able to get away with it ….

(1) A solution for IR-sensitive observables

leptonic decay

$P \to \ell \nu$

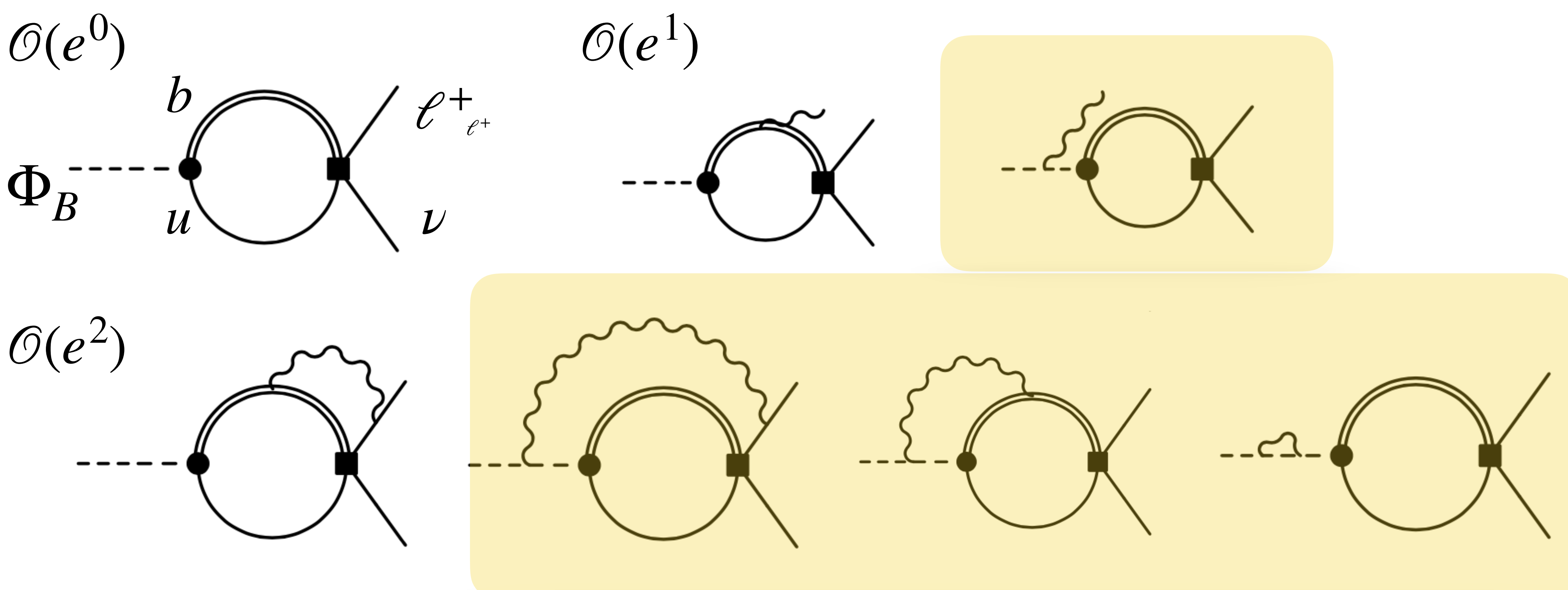
- New gauge interpolating operator (modification of LSZ factor - later)

on-shell correlations are gauge invariant

$$J_B = \bar{u}\gamma_5 b \to J'_B = \Phi_B J_B$$

$$Q_{\Phi_B} + Q_B = 0$$

- Some new diagrams (selection)



The main formula and procedure

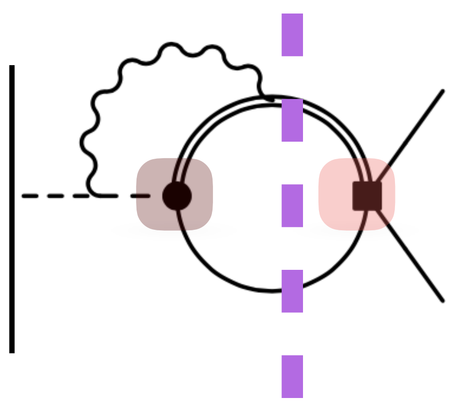
mostly skip

$$\Gamma(B \to \ell\nu)_{\delta_{ex}} = \frac{1}{\langle \Phi_B | B \rangle} \times [\langle \Phi_B | B \rangle \Gamma(B \to \ell\nu)_{\delta_{ex}}] \quad *$$

where both terms are computed separately

- LSZ (dispersion) variable p_B^2, the one between J_B' and H_w

$$[\langle \Phi_B | B \rangle \Gamma(B \to \ell\nu)]_{\delta_{ex}} \propto LSZ(p_B^2, p_{\Phi_B}^2) \left| \quad + \ldots \right|^2$$



$$LSZ(p_B^2, p_{\Phi_B}^2) \propto \lim_{p^2_{B,\Phi_B} \to m_B^2} (p_B^2 - m_B^2)(p_{\Phi_B}^2 - m_B^2)$$

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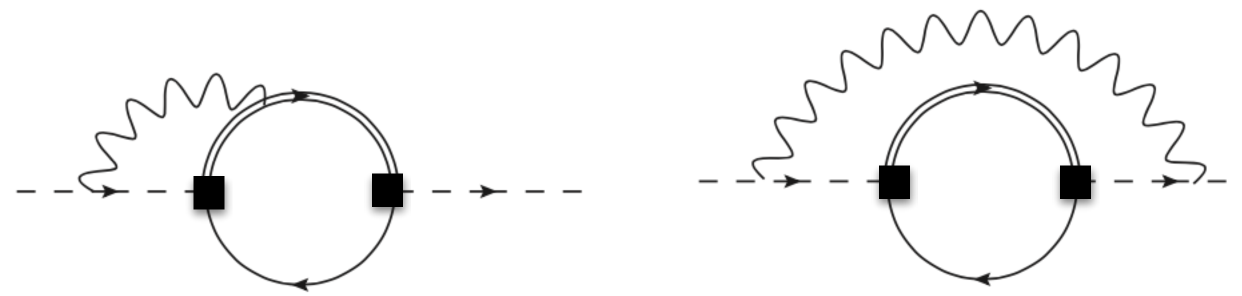
* $\Gamma(B \to \ell\nu)_{\delta_{ex}} = \int_{\delta_{ex}} d\phi_\gamma \left(\Gamma(B \to \ell\nu)\delta(\phi_\gamma) + \frac{d^3}{d\phi_\gamma}\Gamma(B \to \ell\nu\gamma) \right)$ where $m_B \delta_{ex} = 2\Delta E_\gamma$

What about the LSZ denominator $\langle \Phi_B | B \rangle^*$?

mostly skip

- Diagrams (selection below) contain both real and virtual
  Like an **inclusive quantity** and thus **IR finite**

$$|\langle \Phi_B | B \rangle|^2 \propto LSZ(p_B^2, p_{\Phi_B}^2)$$



$+\ldots$

or as off-shell in $p_B^2 \Rightarrow$ **IR finite,** by **Kinoshita-Poggio-Quinn theorem**

$\Rightarrow$ decay rate has no memory of its interpolating operator
& reproduces correct logs, **collinear** and $\ln \delta_{\mathbf{ex}}$ **-terms**

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* More precisely $\langle \Phi_B | B \rangle$ is $\langle \Phi_B | J'_B | B \rangle$ and reduces to $\propto f_B$ with no Φ_B

Conceptual remarks

- Things that **were not clear** at beginning Φ_B-scalar:

1) mass m_{Φ_B}: turns out it is m_B, which makes sense

2) does Φ_B-scalar make α_{QED} run?
No: can understand Dirac dressing* where it clear (backup)

* can and is used for C*-boundary approach e.g Lucini, Patella, Ramos, Tantallo'15 + ….

(1) ΔM, an IR-safe observable

$$\Delta m_H = m_{H^+} - m_{H^0} \ , \qquad H = B, D, K, \pi, p \ ,$$

- At our level of precision (20%) the following split is good enough:

$$\Delta m_B = \Delta m_B|_{\text{QED}} + \Delta m_B|_{m_q}$$

Cottingham Formula (1961) **Feynman-Hellman thm**

- Used QCD sum rules double dispersion relation. Why not earlier?
  1) intrerpolating operators not understood
  2) cuts are subtle (and we gained experience from leptonic case)

Cottingham Formula & QCD sum rules

$$\Delta m_B|_{\rm QED} \equiv \delta m_{B^+}|_{\rm QED} - \delta m_{B^0}|_{\rm QED}$$

$$\delta m_B|_{\rm QED} = \frac{-i\alpha}{2m_B(2\pi)^3}\int d^4q\, T^{(B)}_{\mu\nu}(q)\Delta^{\mu\nu}(q) + \mathcal{O}(\alpha^2)$$

- **hadronic object:** Cottingham tensor needs evaluation

$$T^{(B)}_{\mu\nu}(q) = i\int d^4x e^{-iq\cdot x}\langle B|Tj_\mu(x)j_\nu(0)|B\rangle$$

- **Remarks:**
  - Cottingham's contribution, euclideanisation relating it to strcut. fcts
  - Formula in doubt until '79 Collins showed how renormalisation works
  - spelled in more detail deDivitiis 1303.4896 "Roman paper"
  - Recently pion by Feng, Jin ,Riberdy 2108.05311 @1% level

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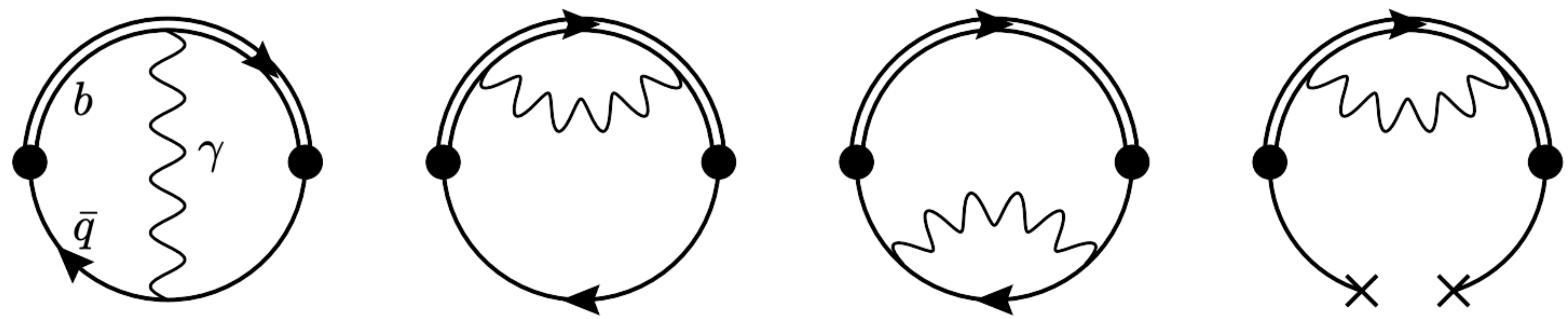
* Q: Why not Cottingham for m_q-effects as well? A:no good for sum rule approximation

The computation

1) Cutkowsky rules cuts
2) spurious momenta to distinguish two cuts



main part | cancels | $\mathcal{O}(m_q^2)$ | suppressed

$$\delta_{qq'} m_B = \frac{1}{Z_B^2} \int_{m_+^2}^{\bar{\delta}^{(a)}(m_+^2)} ds\, e^{\frac{(m_B^2 - s)}{M^2}} \int_{m_+^2}^{\bar{\delta}^{(a)}(s)} d\tilde{s}\, e^{\frac{(m_B^2 - \tilde{s})}{M^2}} \rho_{\Gamma_{qq'}}(s, \tilde{s}) \;,$$

compact expressions

$$\rho_{\Gamma_{bq}} = \frac{N_c \alpha Q_q Q_b m_+^2}{32\pi^3 m_B} \cdot \frac{\sqrt{\lambda\tilde{\lambda}}}{s\tilde{s}} \left(A + \frac{B}{\mathtt{b}} \ln\left(\frac{\mathtt{a}+\mathtt{b}}{\mathtt{a}-\mathtt{b}}\right)\right)$$

$$\mathtt{a} = m_q^2 - \frac{1}{4\sqrt{s\tilde{s}}}(s\tilde{s} + (m_+ m_-)^2) + \left\{q \leftrightarrow b\right\}, \quad \mathtt{b} = \frac{1}{2}\sqrt{\frac{\lambda\tilde{\lambda}}{s\tilde{s}}}, \quad A = m_-^2 \;,$$

$$B = \left\{ Y\tilde{Y} s\tilde{s} + \frac{1}{2} m_q^2 \sqrt{s\tilde{s}}(Y + \tilde{Y}) - \frac{1}{4} m_-^2 \left(s + \tilde{s} + 4 m_b m_q + 2 m_q^2 \right) - \frac{1}{4} m_+^2 \sqrt{s\tilde{s}} \right\} + \left\{ q \leftrightarrow b \right\}$$

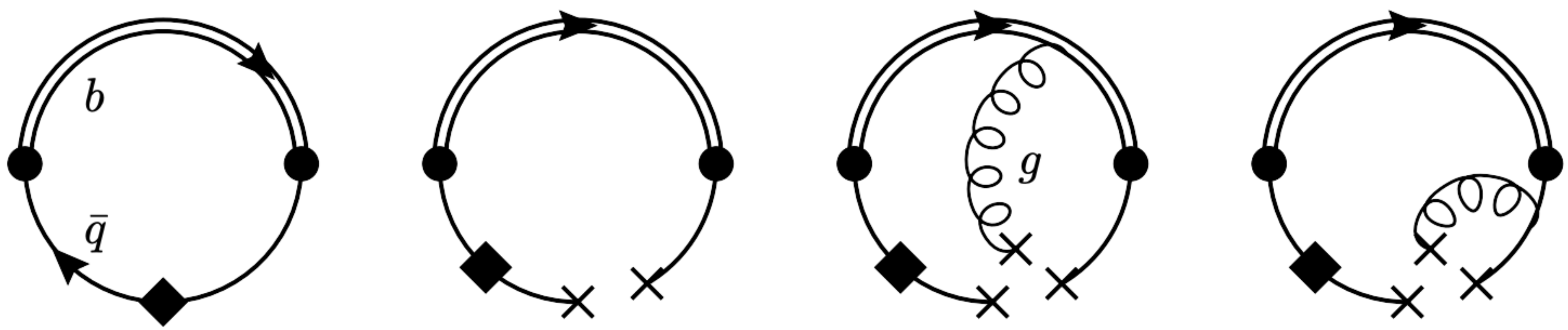
$$m_\pm = m_b \pm m_q \;, \quad \lambda = \lambda(s, m_b^2, m_q^2) \;, \quad Y = \frac{s - m_+ m_-}{2s}$$

Mass-effect

- **Feynman-Hellmann thm:**

hadronic object

$$\Delta m_B \Big|_{m_q} = \frac{(m_u - m_d)}{2m_B} \langle B|\bar{q}q|B\rangle + \mathcal{O}((m_u - m_d)^2)$$



$\mathcal{O}(m_q)$ leading sizeable

$$\langle \bar{B}|\bar{q}q|\bar{B}\rangle = -\frac{4m_+^2 m_b^2 \langle\bar{q}q\rangle}{Z_B^2} e^{\frac{2(m_B^2 - m_b^2)}{M^2}}$$

compact expressions

$$\langle \bar{B}|\bar{q}q|\bar{B}\rangle = -\frac{m_+^2 \langle\bar{q}\sigma s_g g G q\rangle}{Z_B^2} e^{\frac{2(m_B^2 - m_b^2)}{M^2}} \left((1 - \frac{3m_b^2}{M^2}) + (\frac{5}{8} + \frac{2m_b^2}{M^2} - \frac{4m_b^4}{M^4})\right)$$

Finally .. results

opposite HQ limit

H	$\Delta m_H\vert_{\text{QED}}$	$\Delta m_H\vert_{m_q}$	Δm_H	$\Delta m_H\vert_{\text{PDG}[29]}$
B	$+1.58(24)$ MeV	$-1.88(60)$ MeV [a]	$-0.30(65)$ MeV	$-0.32(5)$ MeV
D	$+2.25(70)$ MeV	$+2.7(1.4)$ MeV [a]	$+4.9(1.6)$ MeV	$+4.822(15)$ MeV
K	$+1.85(54)$ MeV	*$-6.7(1.1)$ MeV* [b]	$-4.9(1.2)$ MeV	$-3.934(20)$ MeV
π	*$+4.8(1.2)$ MeV* [c]	*$+0.16(5)$ MeV* [b]	$+5.0(1.2)$ MeV	$+4.5936(5)$ MeV

isospin suppressed Donoghue Perez'96

- Italic ones are not ours: π use double soft-pion thm on Cottingham*
  π, K GMOR better than Feynman-Hellmann
- Uncertainty 20% work ok, central values accidentally good, proof of principle and of course not competitive with BMW et al uncertainty: quark mass & duality parameterisation

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* Goldstones challenge quark hadron duality (also direct instantons relevant)

Summary

- Gauge invariance governs IR-logs
- **Thm**: **no structure-dependent coll. logs**
  unless chiral suppression as in $P \to \ell\nu$ with V-A interaction
- Presented new **gauge invariant interpolating operator** $J'_B = \Phi_B J_B$
  which **reproduces** all **IR-sensitive terms** (in $B \to \ell\nu$ in preparation)
- ΔM is **IR-safe** and **gauge variant operator works**
  Results within 20%, it was fun to do!

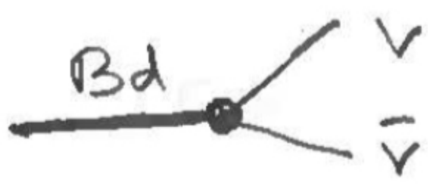
Thanks for your attention

BACKUP

The issue with charged meson & interpolating operators

scalar-QED (pt-like)	GI $\ln m_\gamma$	interpolating-operator (structure)	GI $\ln m_\gamma$

- 1[st] step: consider $\bar{d}\gamma_5 b \bar{\nu}\nu$ - interaction



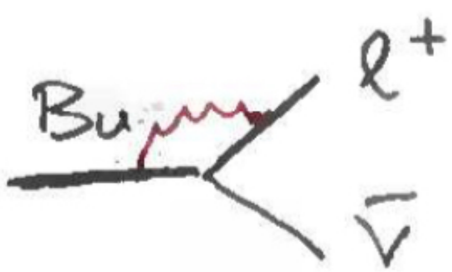
(✓) (✓)

no QED effects

✓ (✓)

$$\mathscr{A} \propto f_B^{-1} \lim_{p_B^2 \to m_B^2} (p_B^2 - m_B^2)\,\Pi(p_B^2)$$

- 2[nd] step: leptonic decay (charged meson)



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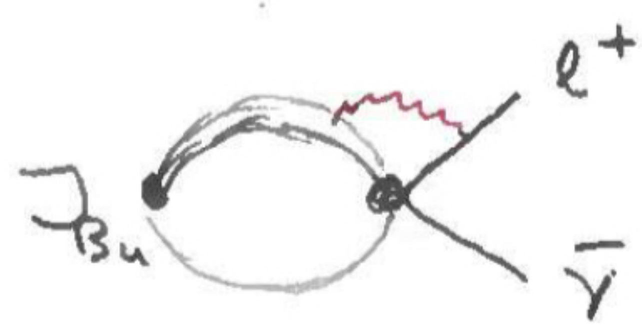

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* GI = gauge invariance, $\ln m_\gamma$ soft logs (known Low theorem)

Relation to Dirac-dressing

- Dirac'55 proposed to take current $\mathcal{J}_\mu$ satisfying $\partial \cdot \mathcal{J} = \delta^{(4)}(x)$ then

$$\Psi_{\mathcal{J}} \equiv U_{\mathcal{J}}(x)\Psi(x) \,, \qquad U_{\mathcal{J}}(x) = \exp[iQ_\psi \int d^4y\, A(y)\cdot \mathcal{J}(x-y)]$$

is gauge invariant, as $U_{\mathcal{J}}(x) \to \exp(-i\lambda(x)Q_\Psi)\, U_{\mathcal{J}}(x)$ under $A \to A + \partial\lambda$

- Specific realisations* of $\mathcal{J}$

$$\mathcal{J} = \partial\,\varphi(\vec{x}) \,,\ \Box_4\varphi(\vec{x}) = \delta^{(4)}(x) \,, \qquad \vec{\mathcal{J}} = \delta(x_0)\vec{\partial}\varphi(\vec{x}) \,,\ \Box_3\varphi(\vec{x}) = \delta^{(3)}(x) \,,$$

$$U_{\mathcal{J}} = e^{-iQ_\psi \int d^4y\, \partial\cdot A(y)\varphi(x-y)} \qquad U_{\mathcal{J}} = e^{-iQ_\psi \int d^3y\, \vec{\partial}\cdot\vec{A}(y)\varphi(x-y)}$$

$$\to 1 \,,\ \partial A = 0 \,,\ \textbf{Lorenz gauge} \qquad \to 1 \,,\ \vec{\partial}\vec{A} = 0 \,,\ \textbf{Coulomb gauge}$$

Nice **duality** between $\mathcal{J}_a$ and $gauge_a$ (just a trick…)

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* can and is used for C*-boundary approach e.g Lucini, Patella, Ramos, Tantallo'15 + ….

…. summary

- May use dressed gauge invariant operator and “dual” gauge (as gauge invariant) to simplify computation.

$$J_B \to \hat{J}_B(\mathcal{J}_a) = \bar{u}_{\mathcal{J}_a} \gamma_5 b_{\mathcal{J}_a}$$

(before $J'_B = \Phi_B J_B$)

- Q1: are all $\hat{J}_B(\mathcal{J}_a)$ equally valid interpolating operators?

  Seems to me the answer is no, as **IR-logs** have to be **reproduced***
  - Lorenz gauge, we do not see soft logs
  - Coulomb gauge: might be there (did not look too closely …)

- Q2: is there a relation to J'_B interpolating current with Φ_B-scalar? Yes ..

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* For IR-insensitive observables such as mass shifts probably all ok

$J'_B = \Phi_B J_B$ as Dirac dressing

$(p_{\phi_B} = p\ , m_B = m, p^2 = m^2)$

- The current $\mathcal{J}^{(\Phi_B)}$ realises Φ_B-field as $\varphi(x) = -\,i\Delta_F(x, m)$ is propagator

$$\mathcal{J}^{(\Phi_B)} = (\partial - i2p)e^{ixp}\,\varphi(x)\ ,\ (\Box_4 + m^2)\varphi(x) = \delta^{(4)}(x)\ ,$$

and the $U_{\mathcal{J}}$ turns into interaction with correct scalar QED Feynman rule

$$U_{\mathcal{J}^{(\Phi_B)}}(x) = \exp(-Q_B \int d^4y e^{i(x-y)p}(2p - i\partial)\cdot A(y)\Delta_F(x-y, m))$$

(N.B. for higher order need iterated integrals; GI works out ok)

- Q2b Is there a dual gauge that trivialises $U_{\mathcal{J}^{(\Phi_B)}} = 1$?

Yes, it is a special axial gauge

$$\Delta_{\mu\nu}\Big|_{\Phi_B-\text{gauge}} = \frac{1}{k^2}\left(-g_{\mu\nu} - n^2\frac{k_\mu k_\nu}{(n\cdot k)^2} + \frac{k_{\{\mu}n_{\nu\}}}{n\cdot k}\right)\ ,\quad n = 2p - k$$